

Creedy, John

**Working Paper**

## Labour Supply Incentives in Alternative Tax and Transfer Schemes: A Diagrammatic Introduction

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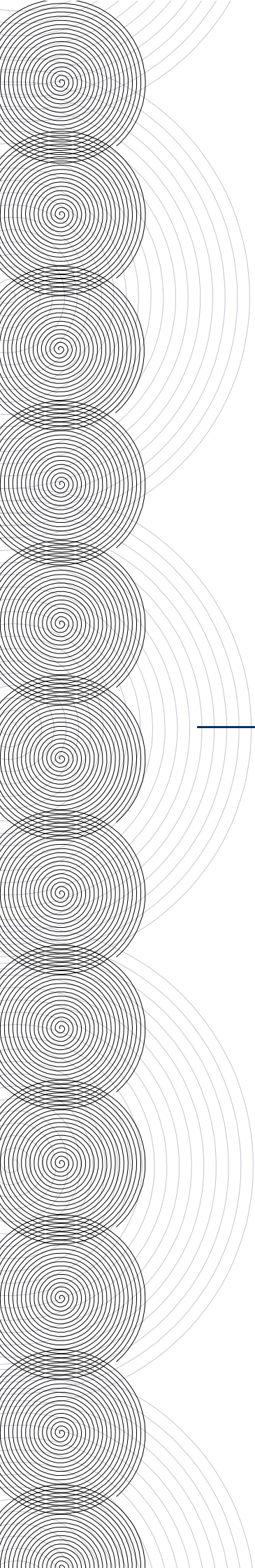
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# Labour Supply Incentives in Alternative Tax and Transfer Schemes: A Diagrammatic Introduction

John Creedy

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Labour Supply Incentives in Alternative Tax and Transfer Schemes:  
A Diagrammatic Introduction

**M O N T H / Y E A R**

June 2003

**A U T H O R / S**

John Creedy  
New Zealand Treasury  
PO Box 3724  
Wellington 6008  
NEW

Email john.creedy@treasury.govt.nz

Telephone 64-4-479-5009

Fax [64-473-1151]

**A C K N O W L E D G E M E N T S**

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**N Z T R E A S U R Y**

New Zealand Treasury  
PO Box 3724  
Wellington 6008  
NEW ZEALAND

Email information@treasury.govt.nz

Telephone 64-4-472 2733

Website [www.treasury.govt.nz](http://www.treasury.govt.nz)

**D I S C L A I M E R**

The views expressed in this Working Paper are those of the author(s) and do not necessarily reflect the views of the New Zealand Treasury. The paper is presented not as policy, but with a view to inform and stimulate wider debate.

# Abstract

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The aim of this paper is to illustrate some of the complexities involved in modelling the incentive effects of taxes and transfers, using only basic diagrammatic methods. It describes a range of diagrams which are helpful in thinking about the design of tax and transfer systems and their potential effects on labour supply behaviour. Emphasis is given to the role of highly nonlinear budget constraints and the resulting wide range of labour supply responses.

**JEL CLASSIFICATION** [H31]

**KEYWORDS** Labour supply; budget constraints, taxes and transfers

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# Labour Supply Incentives in Alternative Tax and Transfer Schemes: A Diagrammatic Introduction

## 1 Introduction

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The aim of paper is to discuss some of the complexities involved in modelling the incentive effects of taxes and transfers, using only basic diagrammatic methods of analysis. Tax and transfer systems in most countries are highly complex, consisting of a wide range of benefit payments, each with its own set of withdrawal rates and thresholds. Often the different components are not well integrated, so that unplanned effects can arise, especially where piecemeal adjustments to benefit levels and income thresholds are made over a period of time. Policy reforms are typically concerned with balancing the aims of providing income support with the desire to avoid excessive disincentive effects of taxes. In designing reforms, it is useful to have in mind a number of relationships which can conveniently be discussed in diagrammatic terms.

This paper therefore explores a range of diagrams which are helpful in thinking about the design of tax and transfer systems and their potential effects on labour supply behaviour. Emphasis is given to the role of nonlinear budget constraints and the resulting wide range of labour supply responses. The analysis of labour supply in the presence of nonlinear budget constraints rapidly becomes complicated; the aim here is to steer a path between oversimplified text book treatments and highly technical research papers.<sup>1</sup>

The use of diagrams relating gross and net incomes, for specified groups, is described in Section 2. Section 3 introduces budget constraints and utility maximisation in a very simple labour supply framework. The general implications of piecewise linear budget constraints are discussed in Section 4. The effect of a non-recoverable cost of applying for benefits and its joint effect on take-up and labour supply are briefly examined in Section 5. Brief conclusions are in Section 6.

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<sup>1</sup>For more technical treatments and further references see, for example, Creedy (1996, 2001).

## 2 Net and gross incomes

Tax and transfer systems are typically specified in terms of a set of gross income thresholds and associated marginal tax, or benefit withdrawal (sometimes called 'taper'), rates applying above the thresholds. Such rates and thresholds generally refer to a particular type of individual or household, depending on its composition. For this reason a popular way of illustrating the nature of the system is to use a diagram with gross income, from all non-benefit sources, on the horizontal axis and net income on the vertical axis, as in Figure 1.

**Figure 1 – Net and gross income**

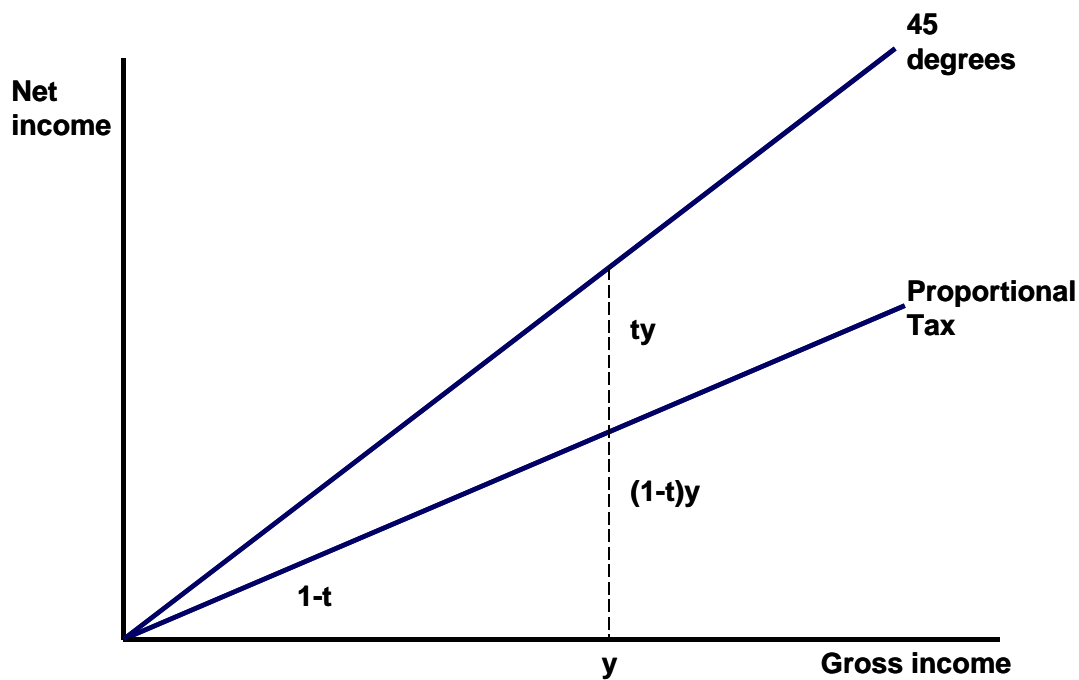
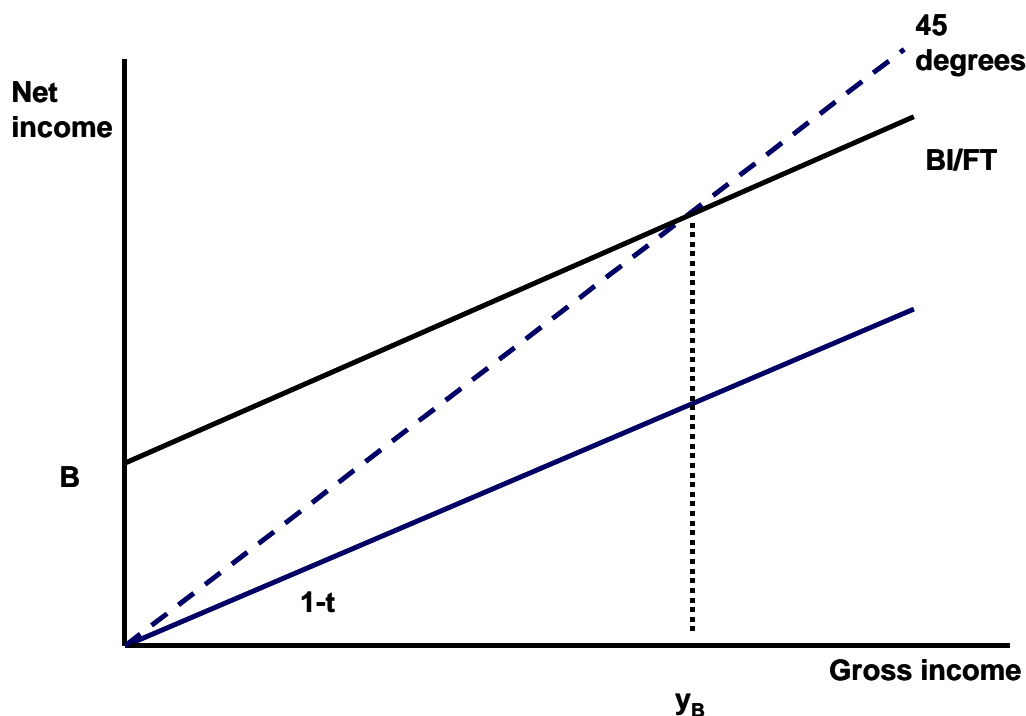


Figure 2 – A basic income - flat tax



This type of diagram has the advantage that it displays the tax thresholds and rates clearly and applies to all those in the specified demographic group. Along the  $45^\circ$  line, net and gross incomes are equal. Figure 1 also shows a simple proportional income tax applied to all income at the rate  $t$ . The vertical distance between the tax schedule and the  $45^\circ$  line, at any gross income level, shows the amount of tax paid,  $ty$ , and the vertical height of the schedule shows net income,  $(1-t)y$ . The slope of the line therefore measures the proportion of each \$ retained after tax, that is  $(1-t)$ .

## 2.1 A basic income – flat tax

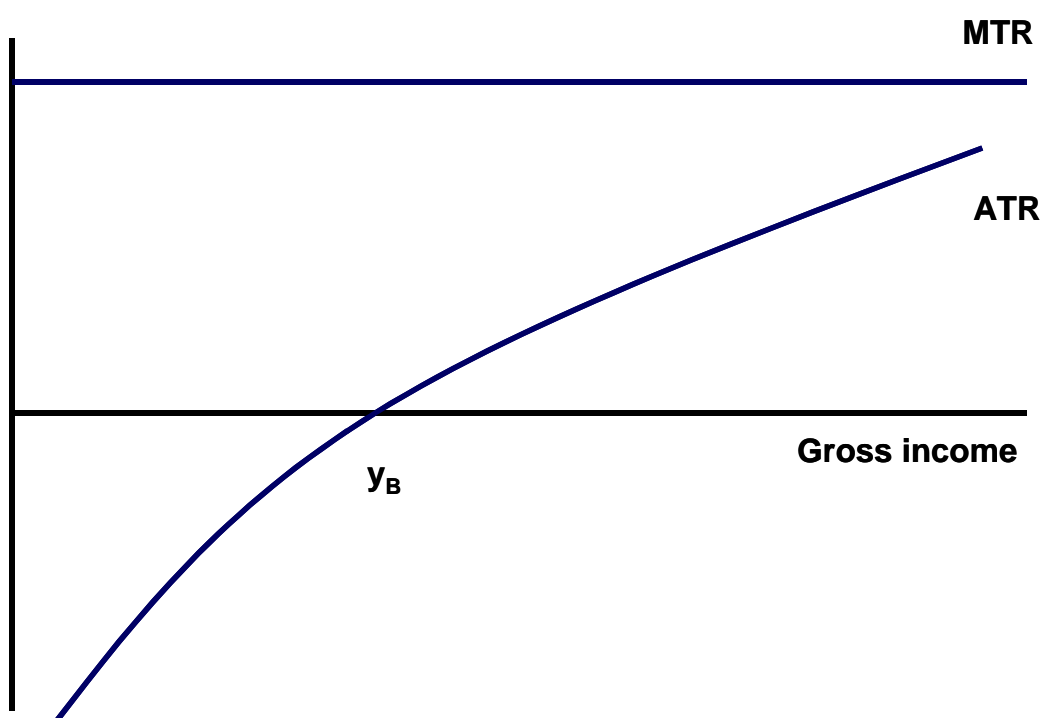
Figure 2 combines the proportional income tax with an unconditional transfer payment equal to  $B$ . Hence the relationship between gross and net income is shifted up by an amount,  $B$ , at all points. This is the simplest possible tax and transfer system imaginable; it is referred to as a basic income - flat tax, or BI/FT, scheme. This system, despite its simplicity, nevertheless gives rise to many complexities. It has been referred to by several other names, including negative income tax (NIT), social dividend, and linear tax scheme.

The figure shows immediately the 'break-even' income level,  $y_B$ , where net and gross income are equal; all those below  $y_B$  receive a net gain. The BI/FT need not necessarily be administered as an unconditional benefit combined with a proportional tax. It may involve an integrated system in which those below  $y_B$  pay no tax and receive the vertical distance between the tax schedule and the  $45^\circ$  line, while those above the break-even point only pay tax, determined as a fixed proportion of income measured in excess of  $y_B$ , so that tax is  $t(y - y_B)$ .



This comparison shows that the measurement of ‘total expenditure’ and ‘total taxation’ in a tax/transfer system is rather arbitrary; gross expenditure may be very high (in this case it is  $nB$ , where  $n$  is the number of people), while total expenditure less total taxation may be negligible. Nevertheless, critics of the BI/FT scheme argue that it requires a high value of  $t$  relative to existing income tax rates (though not benefit withdrawal rates). There are two policy instruments,  $B$  and  $t$ , but they cannot be chosen independently because of the existence of a government budget constraint; for example, given the choice of  $B$ ,  $t$  is determined by the budget constraint. In a ‘pure transfer’ system, where all tax revenue is redistributed in transfer payments, the tax rate is simply  $t = B/\bar{y}$ , or the ratio of the social dividend to arithmetic mean income. Hence the break-even point is the arithmetic mean,  $y_B = \bar{y}$ .

**Figure 3 – Marginal and average tax rates**

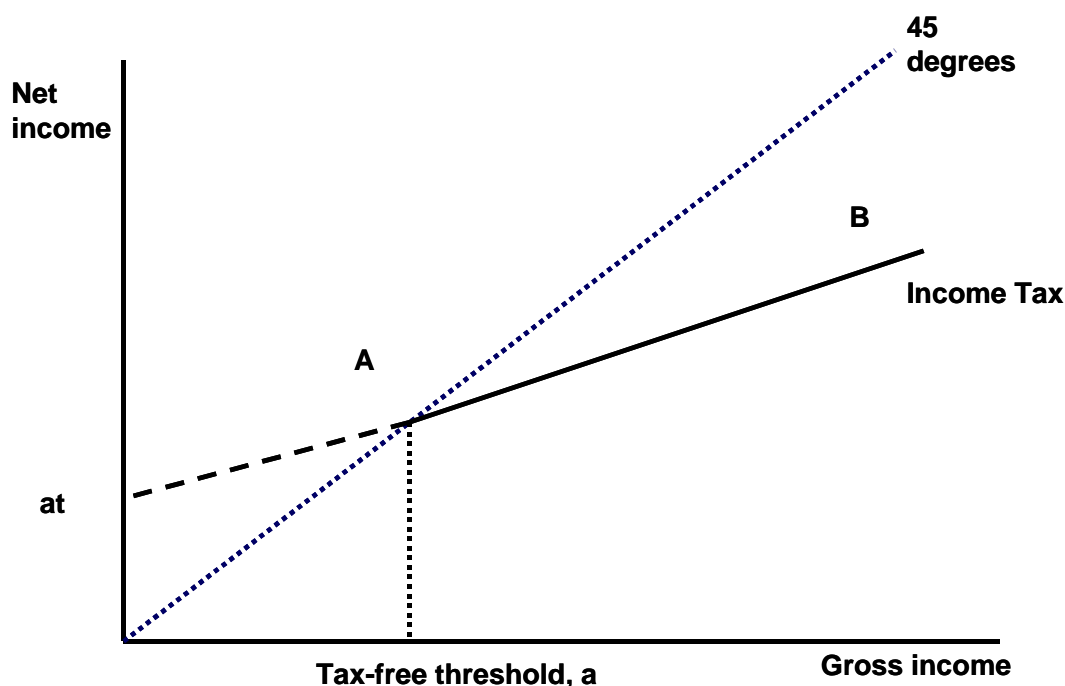


A generous BI/FT scheme therefore appears to require a relatively high marginal tax rate,  $t$ , but it should be remembered that the average tax rate (ATR) is less than the marginal tax rate (MTR) for all individuals, and is negative for all those below  $y_B$ . The marginal and average rate schedules are shown in Figure 3.

This demonstrates that considerable redistribution can be achieved with a tax system having a constant, rather than increasing, marginal rate structure. A progressive tax structure requires only an increasing average tax rate at all income levels. Indeed, in the limit such a system can achieve complete equality if  $t=1$  and  $B = \bar{y}$ . Indeed, with a positively skewed gross income distribution, this would be brought about by a system of majority voting over taxation, since all those (the majority) with  $y < \bar{y}$  unambiguously favour  $t=1$  while all those with  $y \geq \bar{y}$  favour  $t=0$ . Of course, this type of argument has implicitly made the clearly false assumption that gross incomes are unaffected by the tax structure. When allowance is made for labour supply incentives, there are strong limits to

redistribution (even the amount desired by a policy-maker with a high degree of inequality aversion) and majority voting is considerably complicated.<sup>2</sup> This raises the ‘trade-off’ between ‘equity and efficiency’ that cannot be avoided when labour supply effects exist.

**Figure 4 – A tax-free threshold**



Many income tax structures have a tax-free area, and Figure 4 shows an income tax structure of this kind, where taxation above the threshold,  $a$ , is equal to  $t(y - a)$ . The tax schedule thus follows the 45 degree line from the origin until A, and is then AB. However, if AB is extended to the left, it hits the vertical axis at a net income of  $at$ . This demonstrates that, for taxpayers (for whom  $y > a$ ), the tax-free threshold system is equivalent to a BI/FT scheme. An increase in the threshold takes a few individuals out of the tax ‘net’ but provides an increase in the effective basic income for all taxpayers. This result is quite general. Any multi-rate income tax structure, even in the absence of a tax-free area, looks, for those above the first tax rate, like a single rate structure with a tax-free threshold (which can be expressed as a function of the earlier rates and thresholds).

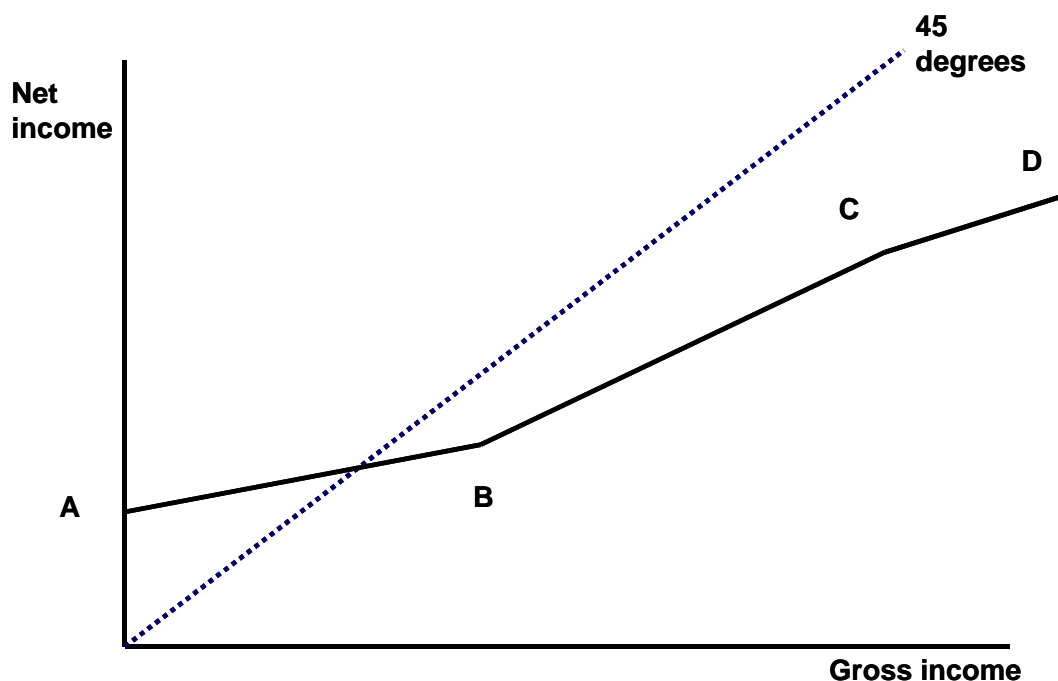
## 2.2 Means-tested benefits

Typically, tax and transfer systems have a degree of means-testing whereby benefits are withdrawn as individuals obtain higher gross incomes. A typical system is shown in Figure 5, where the tax schedule ABCD has a range AB where there is a benefit taper, or withdrawal rate, and a further range CD where the marginal income tax rate is increased. However, the highest marginal effective tax rate is faced by the lowest income recipients, as is evident from the lower gradient of the section AB of the tax schedule. The loss of a degree of freedom in policy choices as a result of the government’s budget constraint has already been mentioned. In the means-tested system of Figure 5, a further degree of

<sup>2</sup>This is partly because individuals’ preferences over the tax rate are no longer ‘single-peaked’. That is, they do not unambiguously prefer low to high rates (or vice versa), preferring low rates when working but high rates when not working (or below a tax-free threshold).

freedom is lost because of the need to avoid a discontinuity, particularly at the point B. Such a discontinuity would produce a sudden drop in net income, or large increase, for a small increase in gross income. The means-testing thresholds and rates must be set alongside those of the income tax system, to avoid discontinuities. However, in practice, because various benefits are planned independently and thresholds are often adjusted independently of others, they often exist.

**Figure 5 – A typical multi-rate system**



A means-tested system is said to achieve high ‘target efficiency’, compared with the BI/FT, where the main - indeed, only - aim of a transfers system is thought to be the avoidance of poverty. However, it raises the problem of adverse labour supply incentives.

The use of a diagram showing the relationship between net and gross income implied by a tax and transfer system is convenient from the point of view of design, such as avoiding discontinuities, and can relate to many individuals, as well as making the thresholds and rates transparent. However, it has serious limitations as a device for examining labour supply incentive effects of taxes. With this in mind, it is necessary to turn to individual budget constraints.

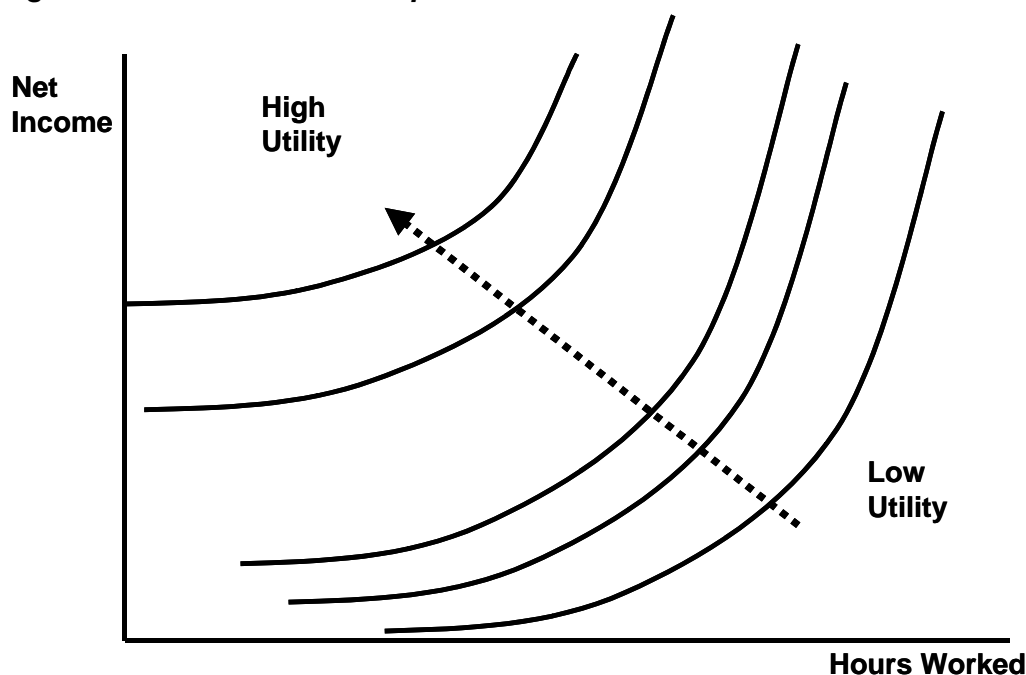
### 3 Taxes and labour supply

Labour supply considerations require the use of budget constraints relating net income to the number of hours worked, rather than the type of diagram used in the previous section. Furthermore, the net/gross income diagram does not distinguish income from earnings and other sources.

### 3.1 The framework of analysis

The discussion in this section assumes that the individual has a fixed gross wage (though there may be an overtime premium) in a single job. Each individual is assumed to maximise utility, which is regarded as a function of net income and leisure. An indifference map is shown in Figure 6: indifference curves that are further to the north west represent higher utility. In this simple static model, although hours of work are considered, consumption does not take time, there is no household production and leisure is simply enjoyed as time not spent working. The individual trades, at the margin, the disutility arising from working an extra hour with the utility obtained from the consumption of the resulting net income.

**Figure 6 – An indifference map**



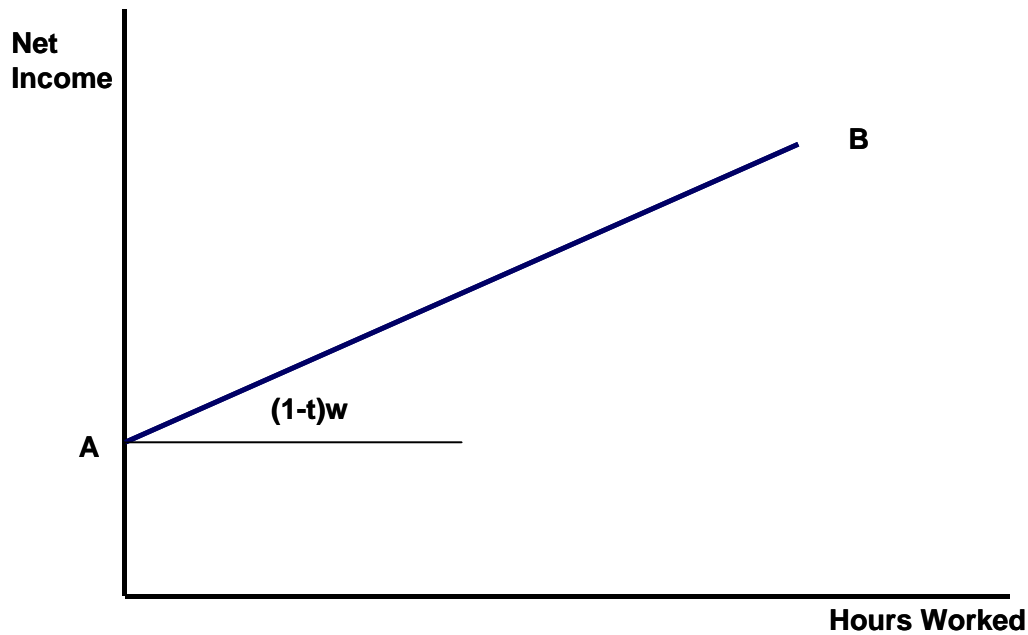
Utility is maximised subject to a budget constraint. The concept of the budget constraint necessarily applies to single individuals, rather than groups of individuals (though it may also apply to individuals with working or non-working partners). This is because each budget constraint depends fundamentally on the individual's wage rate, as well as some other characteristics, and is thus unique. A basic requirement of any model designed to examine taxes in the presence of labour supply responses is therefore that it is capable of producing the budget constraints for all individuals in the data set on which the model is constructed.

### 3.2 The basic income – flat tax

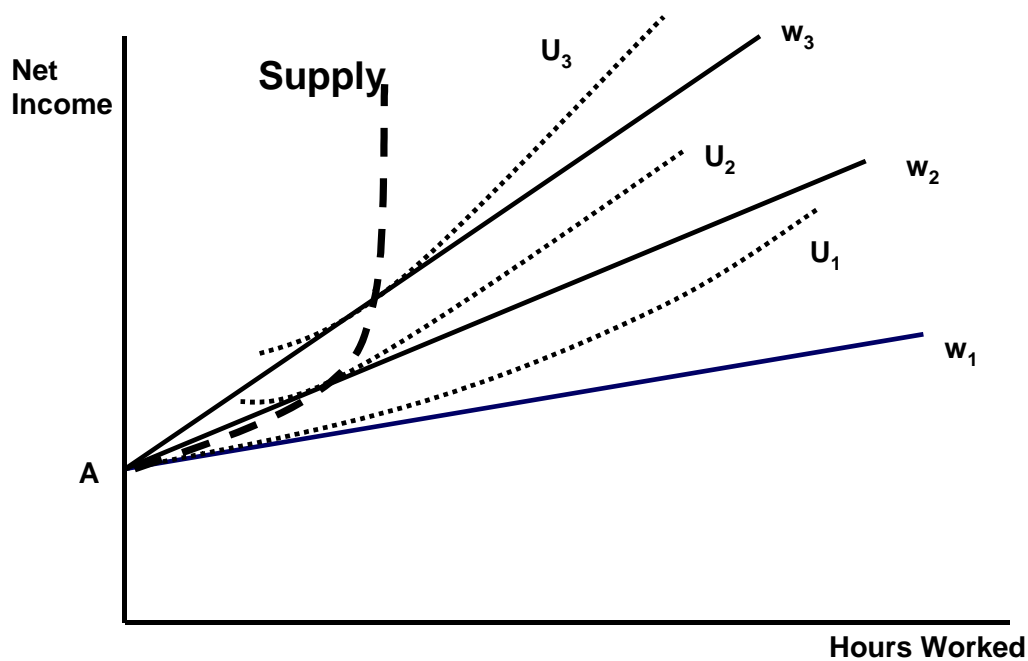
Figure 7 shows the budget constraint for an individual under the BI/FT structure, and facing a gross wage rate per hour of  $w$ . The starting point of the constraint, where hours worked equal zero,  $h = 0$ , corresponds to net income at point A. This includes any benefits, along with all sources of non-wage income (and if the labour supply decision of couples is assumed to involve some kind of joint maximisation process, this may include the net income of the partner). The slope of the budget constraint measures the extra net

income obtained from an extra hour of work, and is thus equal to  $w(1-t)$ , the net wage rate.

**Figure 7 – Individual budget constraint: BI/FT**



**Figure 8 – Wages and hours worked**

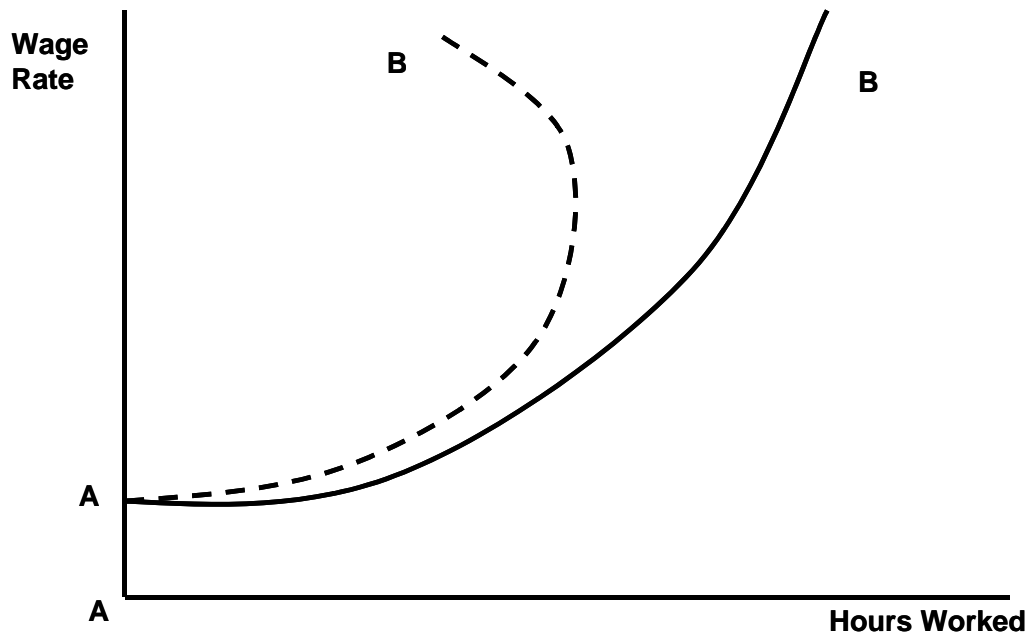


The maximisation of utility, subject to the budget constraint, may therefore lead either to a corner solution at A, where the individual does not work, or a tangency solution somewhere along AB. Within this framework, the work participation decision is associated with movement away from the corner solution. This corner is clearly more likely to be chosen, the lower is the gross wage and the higher is the tax rate, since both tend to make the constraint flatter.

Figure 8 shows the effect on labour supply of varying the wage rate. For the relatively low wage rate of  $w_1$ , the optimal position is at the non-participation corner A on indifference curve  $U_1$ . As the wage increases to  $w_2$ , and then to  $w_3$ , the individual moves to tangency

positions on indifference curves  $U_2$  and  $U_3$  respectively. The gross wage is of course not transparent in Figure 8, because the slope of the budget line is the net wage. It is therefore useful to transfer the information to a separate diagram, the labour supply curve.

**Figure 9 – Labour supply with the BI/FT**



The supply curve associated with this linear budget constraint is shown in Figure 9 as AAB. As usual in economics, the ‘price’ - here the gross wage - is shown on the vertical axis while hours worked are shown on the horizontal axis (although strictly the relevant price here is that of leisure, which is reflected in the net rather than the gross wage rate). The range AA is associated with the kink point A in Figure 7 and reflects wage rates for which the individual remains at the ‘non-participation’ kink. For higher wages, labour supply expands along the range AB as this pivots about A. Depending on the individual’s preferences, the supply curve may bend ‘backwards’ as shown by the dashed line, which may be said to represent the standard elementary ‘text book’ labour supply curve.

## 4 Piecewise-linear budget constraints

This section extends the previous analysis to deal with budget constraints made up of several linear segments, each associated with a given marginal tax rate and earnings threshold. Such multi-segment constraints are referred to as piecewise-linear budget constraints.

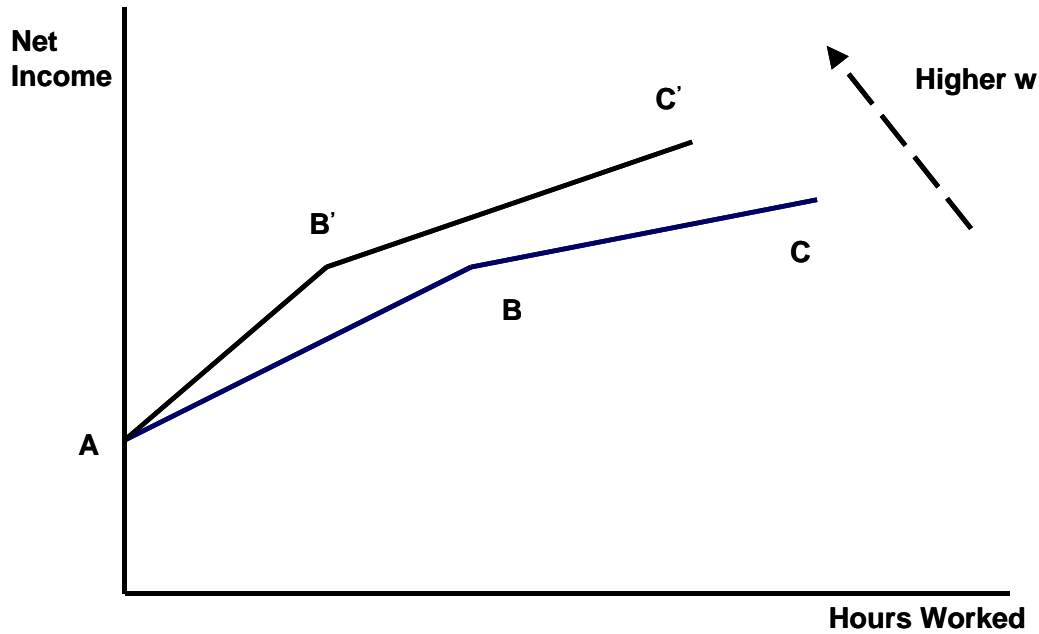
### 4.1 A convex budget set

The effect of an increase in the gross wage on a more complex budget constraint is shown in Figure 10. The constraint ABC has a kink at B, reflecting the presence of an earnings threshold where the marginal effective tax rate increases. The budget set is said

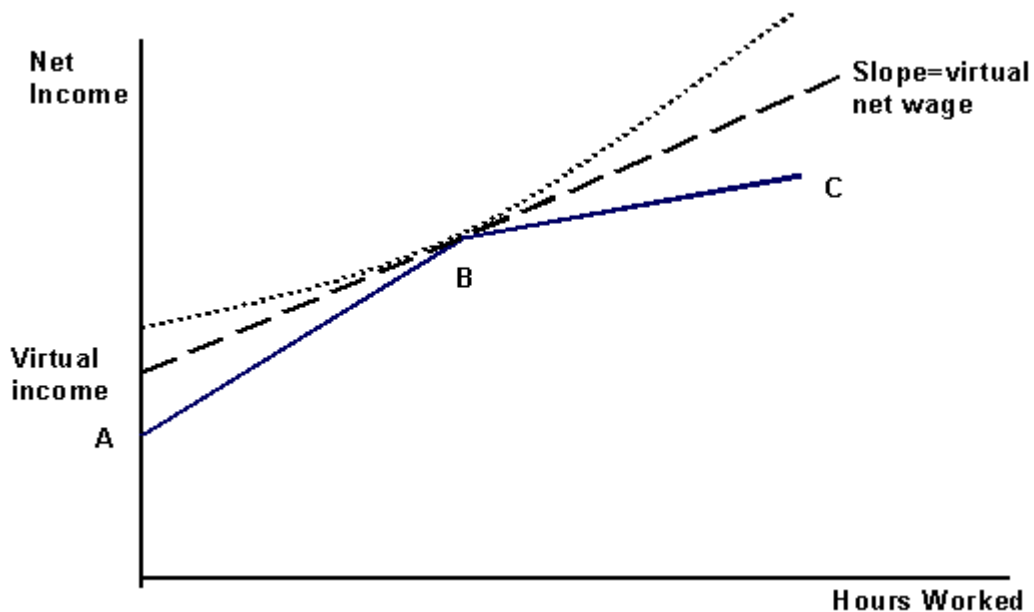
to be convex: a straight line joining any two points is associated with a feasible position, that is a net income less than or equal to the net income along the budget line.

The earnings threshold is not evident from the budget constraint, unlike the net/gross income diagram, since the hours level at which it occurs depends on the wage rate. For a higher wage rate, the budget constraint pivots to  $AB'C'$ , and the kink point  $B'$  is to the left of  $B$ . This is because a lower hours level is required, at the higher wage, to reach the earnings threshold where the marginal rate increases; gross and thus net income remain constant at the kink.

**Figure 10 – A convex budget set**



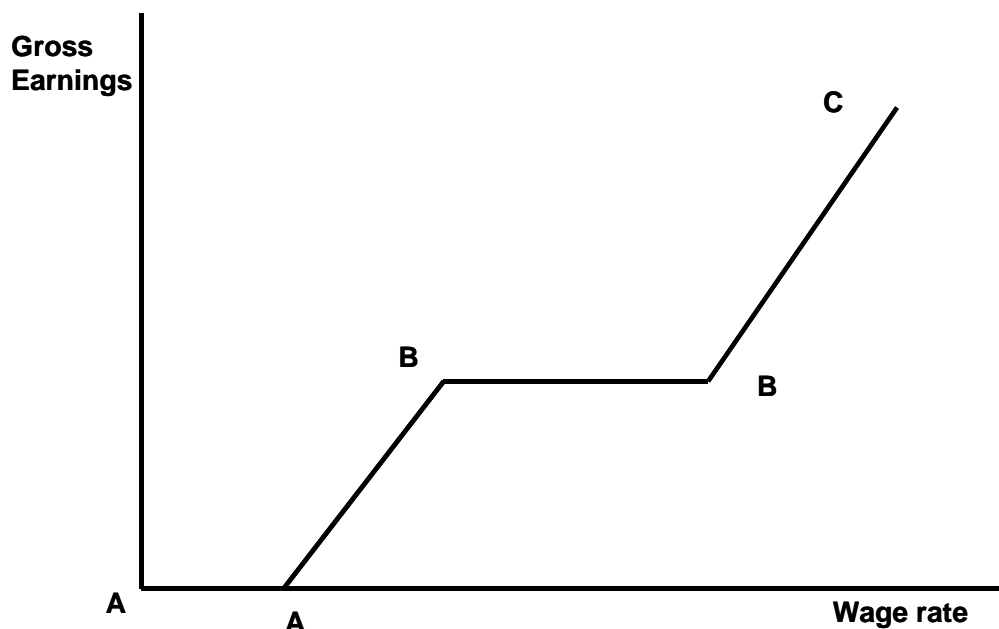
**Figure 11 – A corner solution**



A corner solution is shown in Figure 11, where the highest indifference curve just touches the budget constraint at point B. However, the optimal position for the individual can nevertheless be represented as if it were a tangency solution. The artificial budget line

can be drawn which is tangential to the indifference curve at the kink, B. This generates the important concepts of the virtual wage and virtual income. These concepts are taken from the theory of rationing, where corner solutions are important, and are particularly useful when considering the welfare changes arising from tax reforms.

**Figure 12 – Gross earnings and the wage rate**

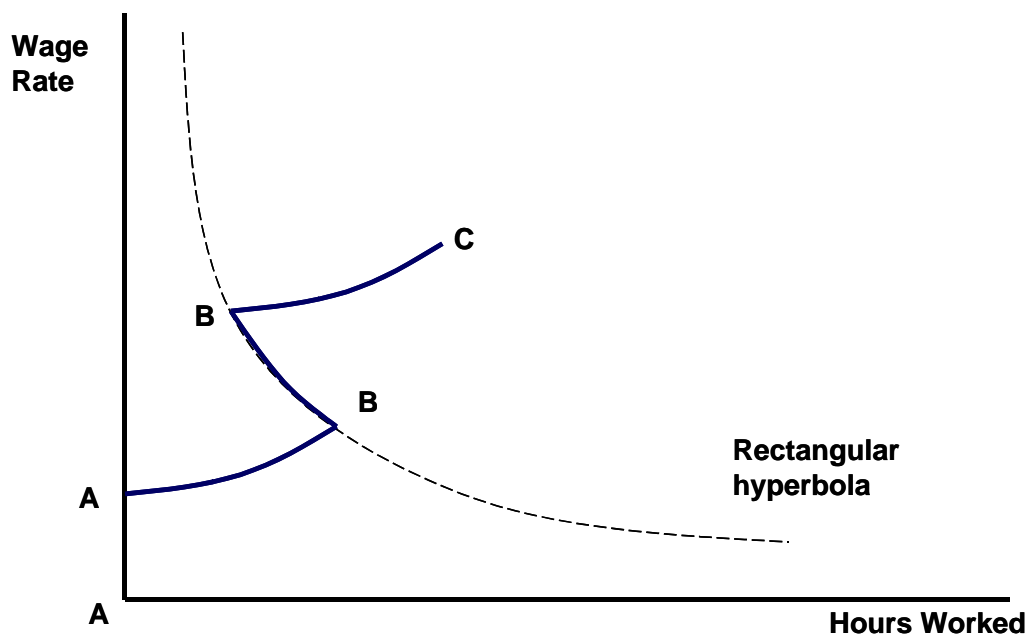


Before considering the labour supply function implied by the budget constraint in Figure 10, it is useful first to examine the relationship between gross earnings (the product of hours worked and the gross wage, that is,  $wh$ ) and the wage rate. This is shown in Figure 12. At very low wages, utility maximisation gives rise to the corner solution at A. When the wage rate exceeds some level (as the section AB of the constraint pivots about A), the individual moves to a tangency position. Increases in the wage induce higher labour supply until the gross earnings threshold is reached at which the marginal effective tax rate increases. A characteristic of this kind of kink in the budget constraint - where there is no tangency solution but a position where the highest indifference curve touches the relevant corner - is that the individual 'sticks' at the corner for a range of wage rates. In this case the gross earnings remain constant as the wage rate rises over a range, although of course the associated hours level falls. Eventually, for a sufficiently high wage rate, the individual moves to a tangency along the range BC of the constraint. The lengths of the flat sections AA and BB, and the nature of the rising sections AB and BC, depend on the individual's preferences. A higher preference for consumption over leisure (flatter indifference curves) gives rise to shorter sections AA and BB in Figure 12.

Individuals may face the same tax rates and thresholds, but the variation in non-wage incomes, wage rates and preferences means that each budget constraint and gross earnings/wage schedule is unique. However, the existence of the kink may suggest that some 'bunching' of individuals around the threshold in the distribution of gross earnings. This kind of phenomenon is nevertheless only observed in particular cases - a tax threshold need not produce a 'spike' in the earnings distribution, and modes may in practice correspond to tangency solutions. Hence, the distribution of earnings need not necessarily provide any information about the extent of the labour supply effects of taxation.



**Figure 13 – Labour supply curve**



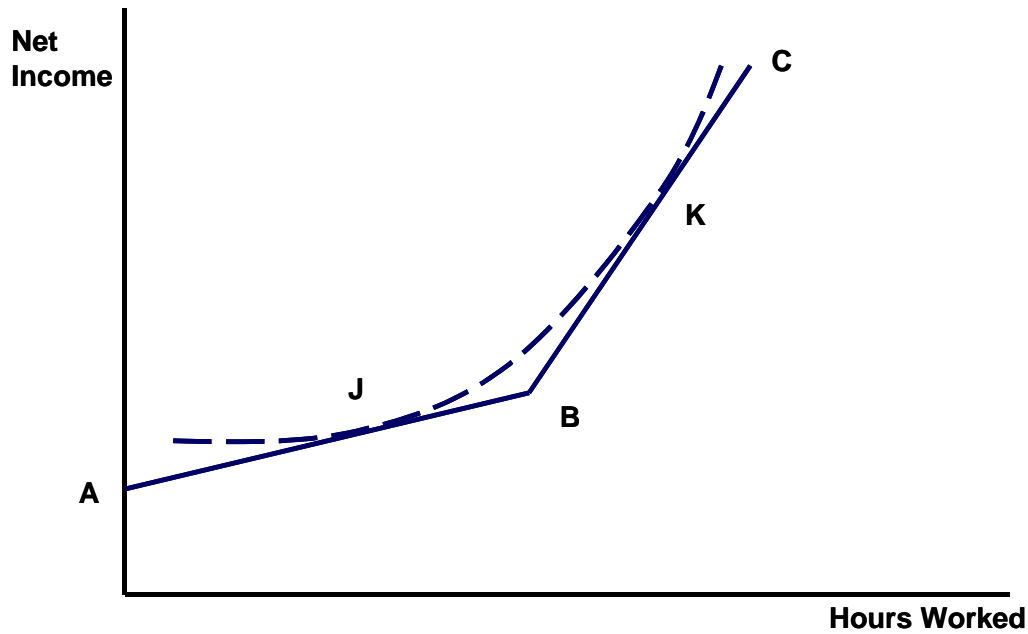
The range BB in Figure 12 is associated with a fixed level of earnings, so that  $wh$  is constant. This implies that, in a graph of hours worked plotted against the wage rate, the hours of work would follow a rectangular hyperbola over the relevant range. The labour supply function is shown in Figure 13. This property, that the labour supply curve turns sharply backwards, following a rectangular hyperbola, as the wage rate increases over a range, is entirely general and applies to any kink in the budget constraint associated with an increase in the marginal effective tax rate at a threshold level of earnings.

Two important general lessons emerge from this discussion. First, the number of hours of work supplied and the net wage are jointly determined. Second, it makes little sense to attempt to describe the labour supply function (hours supplied as a function of the gross wage) in terms of a single elasticity. Even if the ranges AB and BC have a constant elasticity, large variations occur at the kink points, and of course the elasticity changes sign twice.

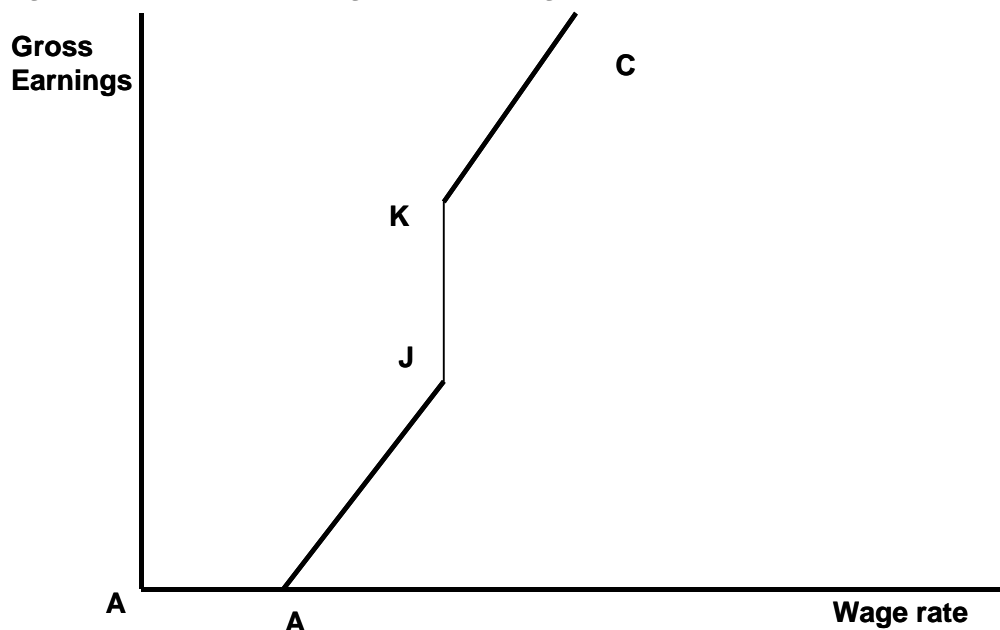
## 4.2 A non-convex budget set

An example of a budget constraint with a means-tested benefit is given in Figure 14, as ABC. The benefit is gradually withdrawn at a relatively high rate until it is exhausted at B, when the individual only pays income tax (where integration of the benefit and tax systems avoids a discontinuity). Here the budget set is said to be non-convex. This raises the possibility of an indifference curve being simultaneously tangential to the two sections of the constraint, for a particular wage rate; this is shown in Figure 14 by the two tangencies at J and K. A small increase in the wage rate would therefore produce a discrete jump in hours worked from J to K.

**Figure 14 -- Means-testing: A non-convex budget set**

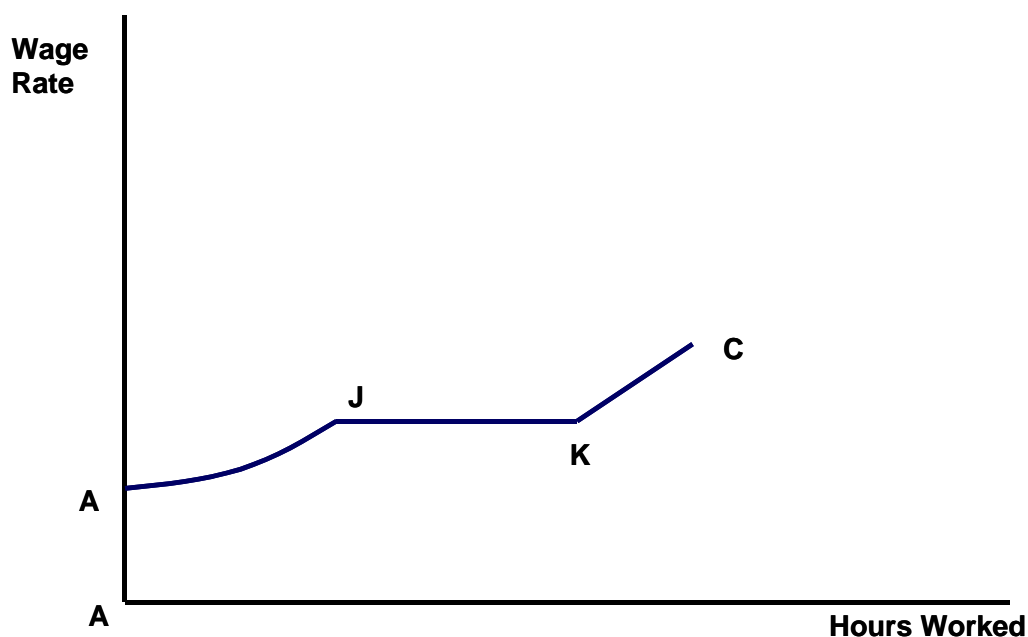


**Figure 15 -- Gross earnings and the wage rate**



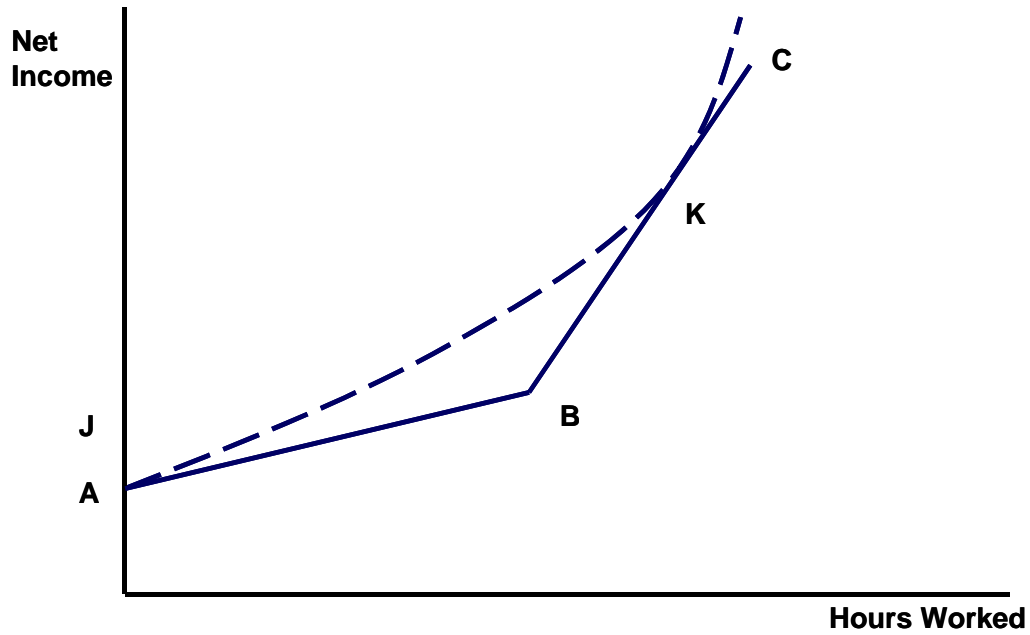
A discrete jump of this kind is reflected in the relationship between gross earnings and the wage rate, shown in Figure 15. When the wage rate is increased, earnings become positive after a particular wage has been passed and the individual increases labour supply along the range AB. But at a particular wage, the ‘switching’ wage, the individual jumps to point K along the range BC in Figure 15. Further increases in the wage produce a gradual increase in labour supply along KC. The associated labour supply curve is shown in Figure 16, where the vertical jump in Figure 15 translates into a horizontal range of the supply curve. There is a range where the individual is working and receiving benefits along part of AJ; but this is just a proportion of the hours range over which the individual is actually eligible for benefits.

**Figure 16 – Labour supply curve**



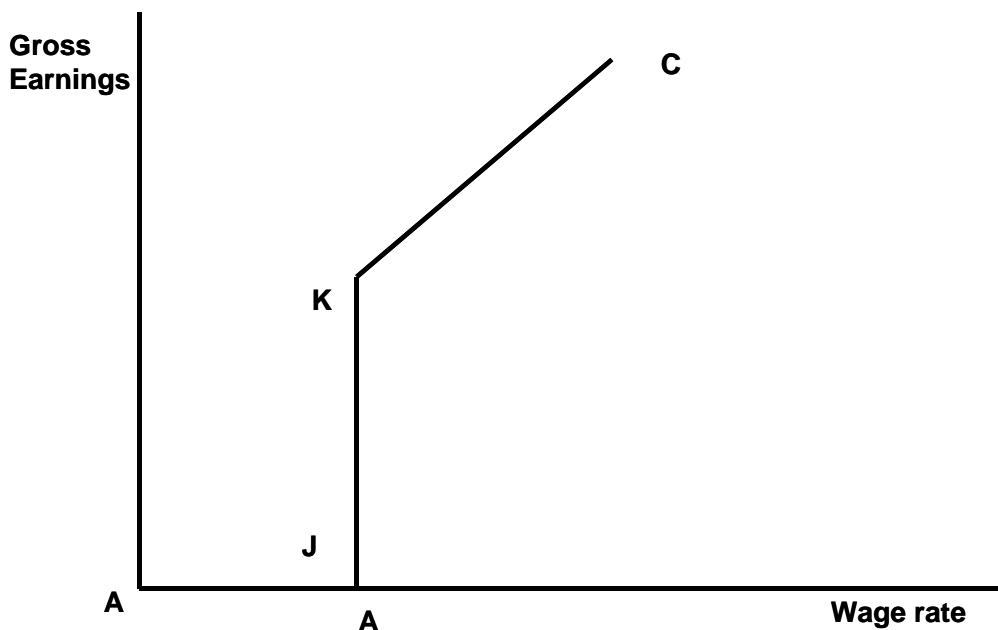
These properties of the relationship between the shape of the budget constraint and the labour supply curve are completely general: a kink in the budget constraint associated with a rise in the marginal tax rate is associated with a 'backward bending' range along a rectangular hyperbola (where gross earnings are constant), and a kink associated with a fall in the effective marginal tax rate produces a horizontal range in the supply curve. In the first case the kink produces a degree of rigidity (no movement from the 'travelling' kink as the wage changes), while in the second case the kink produces a discrete jump in labour supply. The phenomenon of a jump may suggest that the earnings distribution may have a 'gap' or antimode - but (as with the other type of kink) the distribution of earnings is generated by substantial heterogeneity in wages and preferences, and such a gap may not necessarily be observed.

**Figure 17 – Corner and tangency solutions**

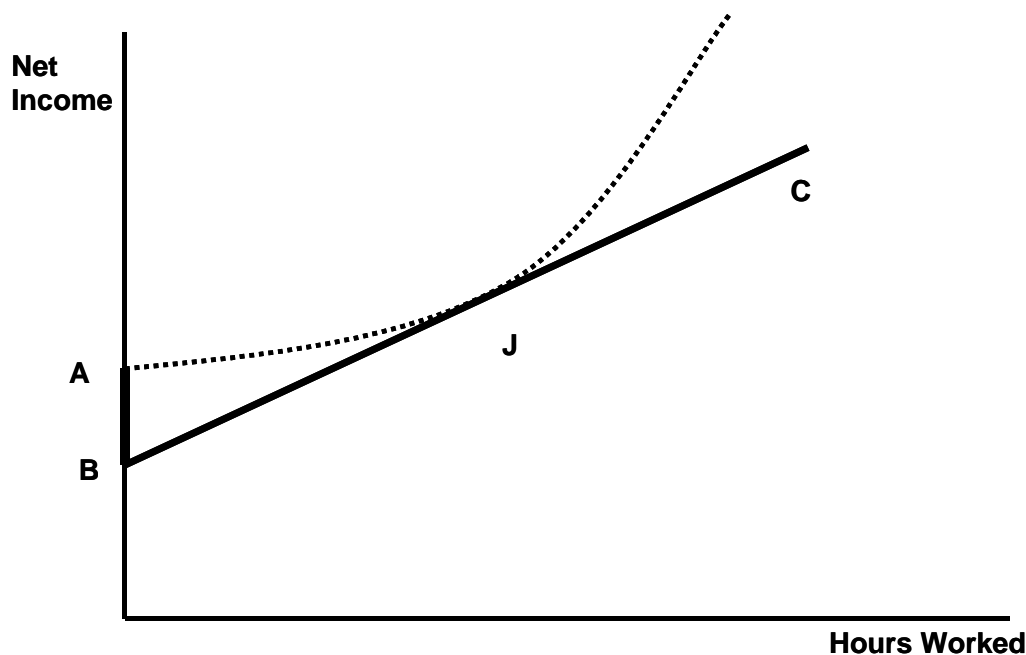


The existence of means-testing may actually rule out a complete section of the budget line from the point of view of labour supply. Such a possibility is shown in Figure 17, where instead of two tangency positions, the highest indifference curve is shown to give rise simultaneously to a corner solution at A and a tangency at K. Hence the whole range of AB is ruled out, as well as the range BK of BC. The relationship between gross earnings and the wage is shown in Figure 18 where the ‘switching’ wage produces a jump from zero labour supply and thus zero earnings to the point K. This individual would not be observed both working and receiving the means-tested benefit, that is, would not be among the ‘working poor’.

**Figure 18 – Gross earnings and the wage rate**



**Figure 19 – Fixed costs of working**



In addition to means testing, a non-convex range of the budget set may arise from the existence of fixed costs of working. This is shown in Figure 19, where fixed costs are shown as the length AB. Their existence substantially raises the minimum wage above which the individual works, and implies that low hours levels are not chosen at all.

### 4.3 A multi-rate tax structure

Figure 20 contains a budget constraint having four marginal tax rates. Means-testing of benefits involves an increase in the marginal rate after point B, and a subsequent reduction in the marginal rate once benefits have been exhausted at point C. The range DE reflects an increase in the marginal income tax rate beyond D. This degree of nonlinearity of the budget constraint generates the possibility of multiple local optima on different ranges of the constraint. Figure 21 shows a situation where indifference curve  $U_1$  is tangential to the budget constraint along CD, while curve  $U_2$  gives a corner solution at point B where higher taper or benefit withdrawal rates start to apply. This does not introduce any new principles, but raises complications when modelling labour supply, as care must be taken to select the global optimum.

Figure 20 – A multi-rate system

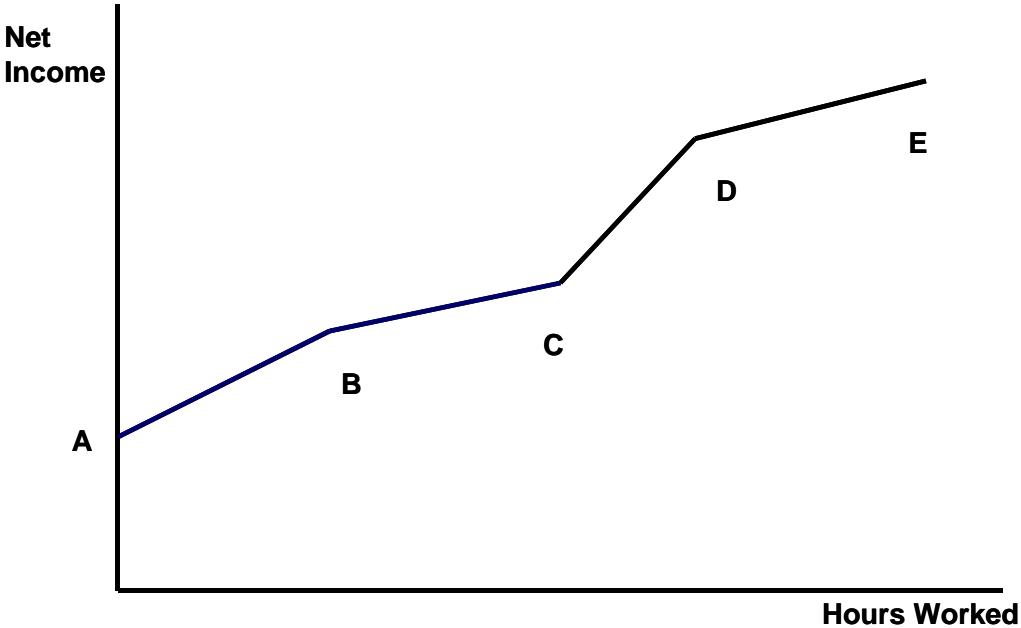
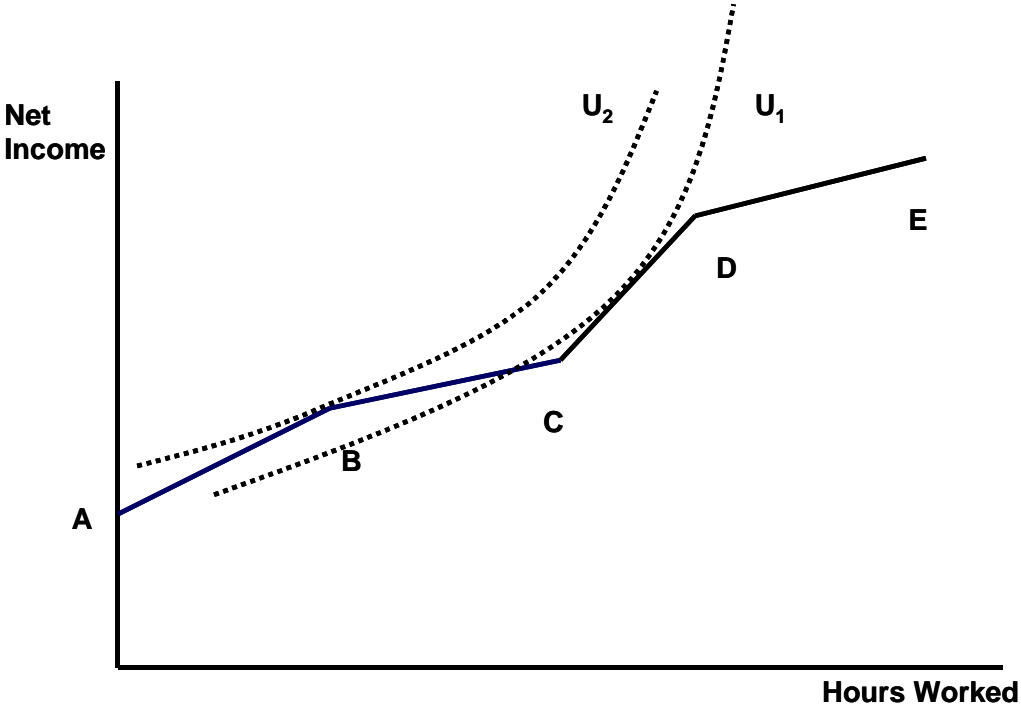
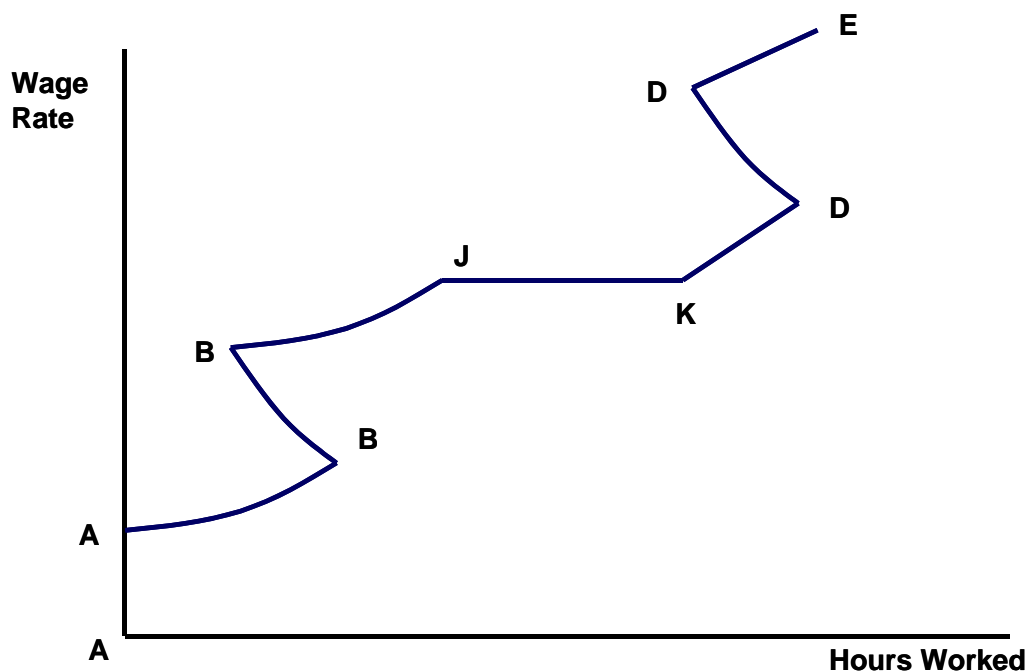


Figure 21 – Multiple local optima



**Figure 22 – Labour supply curve with multi-rate tax system**

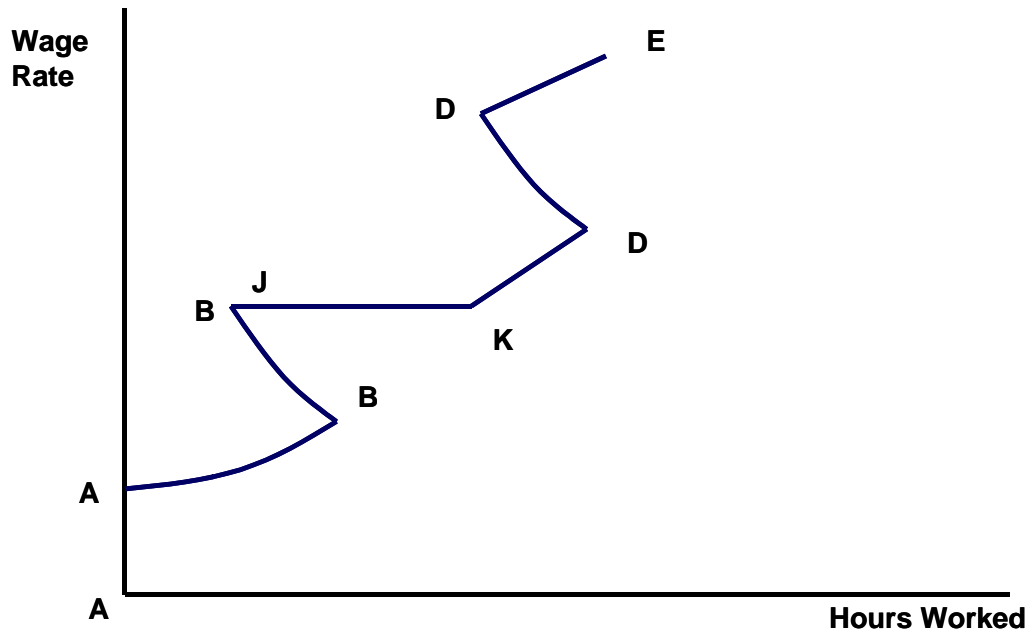


The labour supply curve resulting from this four-rate tax structure is shown in Figure 22. This reflects the general properties obtained above. Hence the two corners B and D of the constraint (and the earnings thresholds associated with them) in Figure 20 generate the two ranges BB and DD of Figure 22 which involve reductions in labour supply along rectangular hyperbolae. The flat section of the supply curve JK is associated with the jump from a tangency along BC to a tangency along DC of the budget constraint. The rising sections of the supply curve are associated with tangency solutions.

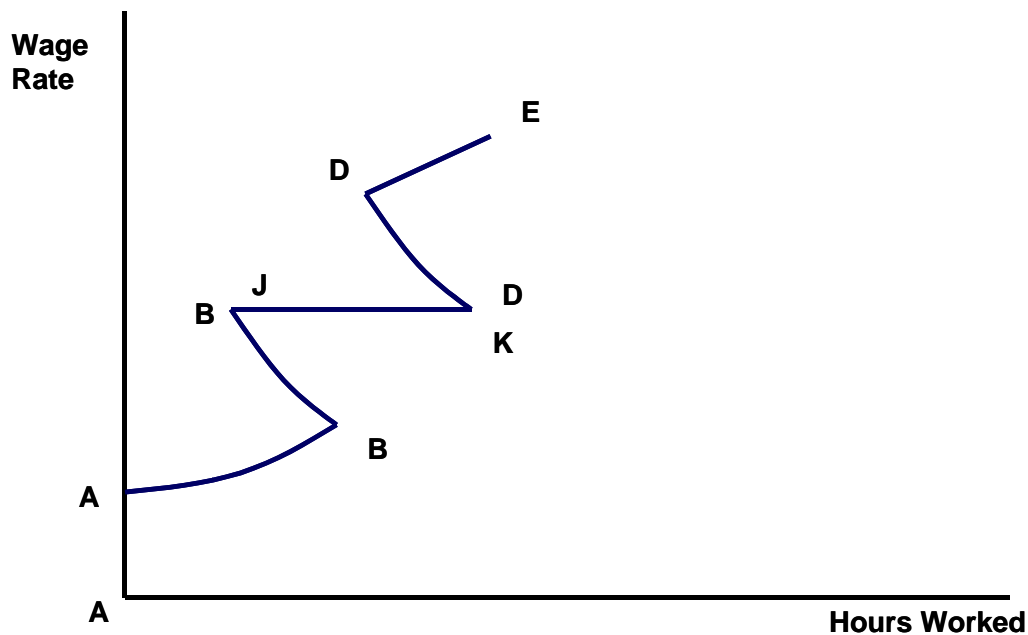
The supply curve need not necessarily take the precise form shown in Figure 22. Consider again the budget constraint shown in Figure 20. The supply curve may involve, for a small increase in the wage rate, a jump directly from a corner solution at point B on the budget constraint to a tangency along the length CD of the constraint.<sup>3</sup> This is more likely to occur, the higher is the benefit withdrawal rate (the flatter the segment AB in Figure 20). This alternative is shown in Figure 23.

<sup>3</sup>This corresponds to the type of jump illustrated in Figure 17.

**Figure 23 – Alternative labour supply curve**



**Figure 24 – Another alternative labour supply curve**



Furthermore, it is quite possible that a jump could occur from the kink in the budget constraint of Figure 20 at B directly to the kink at D, thereby leaving out the whole of the range of the constraint from B to D. The resulting labour supply curve is illustrated in Figure 24.

## 5 Benefit take-up

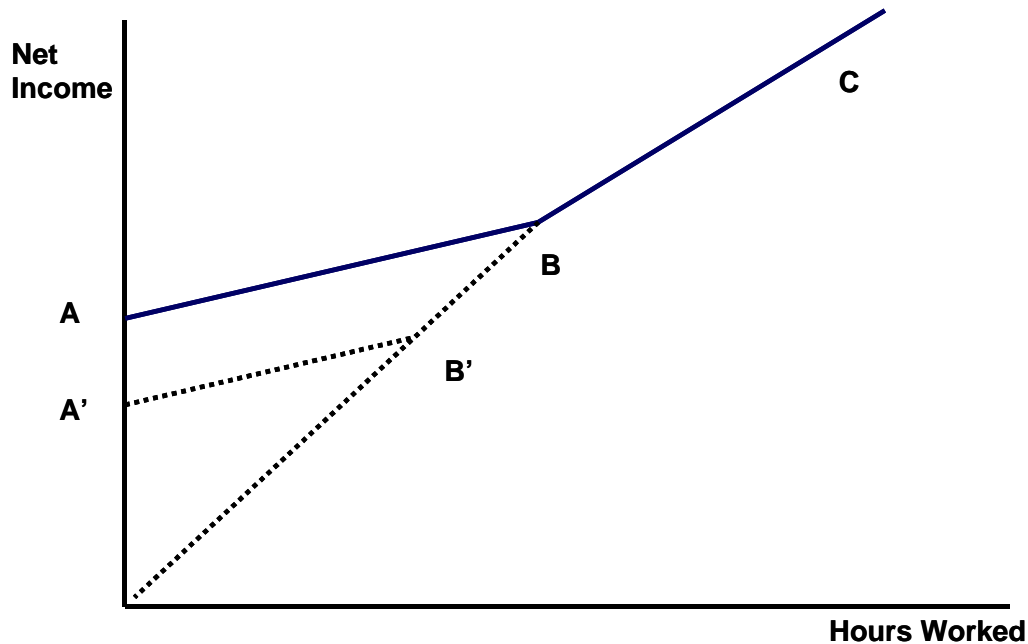
All the above discussion has made the implicit assumption that there is 100 per cent take-up of benefits by eligible individuals. However, suppose that there is a non-recoverable fixed cost of claiming benefits, and consider the simple case of a means-tested benefit



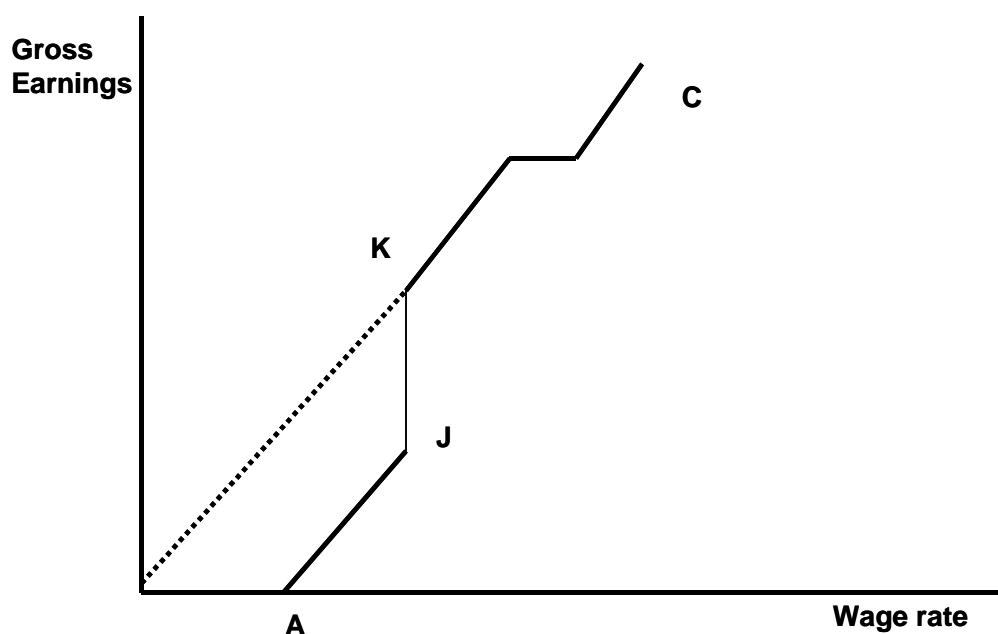
with a single taper rate, combined with an income tax, also imposed at a fixed rate above the tax-free threshold corresponding to the earnings level where the benefit is exhausted. The basic budget constraint is shown as ABC in Figure 25. The line from the origin to point B indicates net wage income, excluding any benefit, although the individual is entitled to claim the benefit: hence the slope of B B is equal to the gross wage rate.

The net income obtained by the individual after claiming the benefit is reduced by the fixed cost of claiming, so that the section AB drops down by a vertical distance equal to the cost of claiming. The new effective budget constraint is the line A' B' BC. For hours of work beyond the point B the individual is better off by not claiming the benefit because the reduced amount of benefit, as a result of means testing, is less than the fixed cost of claiming. Hence along the new range B' B, the individual neither claims the benefit nor earns enough to pay income taxation. The effect is to introduce a further kink in the effective budget constraint, in this case arising from the increase in the effective marginal tax rate (from zero along B' B to the income tax rate as the individual starts to pay income tax at B).

**Figure 25 – A fixed cost of claiming a benefit**



**Figure 26 – Earnings with incomplete take-up**



One possible resulting relationship between gross earnings and the wage rate is shown in Figure 26. In this case the individual jumps from working and receiving benefits along  $A B$  to working and not claiming benefits, or paying income tax, along  $B B$ . This is shown as the jump from  $J$  to  $K$  in Figure 26. Beyond this point the individual works, until reaching the kink where there is an associated horizontal section. There are other possibilities. For example, an indifference curve may simultaneously touch points  $A$  and  $B$  in Figure 26. This means that there is no range where the individual works and receives the means-tested benefit; take up is complete at the non-participation corner at  $A$ . Or an indifference curve may be tangential to the constraint along  $A B$  and touch the corner at  $B$ . In this case there is also no range where take-up is incomplete, though there is a range where the individual works and receives the benefit.

## 6 Conclusion

This paper has illustrated some of the complexities involved in modelling the incentive effects of taxes and transfers, using only basic diagrammatic methods of analysis. A wide range of labour supply responses are shown to result from highly nonlinear budget constraints. A tax or benefit change which improves the labour supply incentives for one group of workers, without making them worse off, may at the same time introduce disincentives facing another group of workers. Tax policy design inevitably involves difficult trade-offs between the desire to avoid labour supply disincentives and the desire to provide income support.

## References

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