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Why do cloud providers prefer renting to selling?

A supply side perspective

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Abstract

Expansion of the cloud computing market, a major reform in information and communication technology (ICT), has attracted wide attention. From the perspective of companies that need cloud services, if access to cloud spreads is available on rent instead of sales, initial investment cost will decline and the number of companies currently adopting the cloud system in the form of renting servers will increase. From the supply side perspective, what are the advantages of renting cloud services? To analyze this question, we consider a duopoly cloud market under licensing and examine the optimal strategy for providers. We find that in a two-part licensing contract, which includes high royalty and fixed fee charged upfront, when the cost-saving effect is high, both firms prefer renting to increase their revenue, but when the effect is low, each firm makes a different choice.

JEL classification: D43, L13, L68

Keywords: Durable goods; Licensing contract; Selling; Renting; Cloud market

Highlights:

Licensing and investment change providers' choices.

The effect of cost saving for customers in the cloud market and investment introduce renting.

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Introduction

Expansion of the cloud computing market has attracted attention as a major reform in information and communication technology (ICT). According to Gartner Inc., the worldwide infrastructure as a services (IaaS) market grew 29.5% in 2017 to total \$23.5 billion, up from \$ 18.2 billion in 2016.¹ Synergy Research Group says that the public cloud infrastructure as a service (IaaS) and platform as a service (PaaS) account for the bulk of market expansion and shows that spending on cloud infrastructure jumped 46% from the final quarter 2016. Moreover, aggressive growth witnessed in Amazon (AWS) and Microsoft is central to the expansion of IaaS and PaaS services.² With the introduction of cloud technology, all the relevant sectors are promoting efficiency by enabling the use of high-performance computer capability through external companies (vendors) for work that was earlier processed by computers of their own company. If the cloud technology can be accessed on a rental basis instead of sales, fixed costs necessary for initially introducing the cloud system can be converted into variable cost (fees for pay-per-use basis), and the initial investment cost of SMEs (small and medium-sized enterprises) entering the market would decline moreover, this will encourage the entry of new entries into the market. Furthermore, by adopting the rental basis, companies introducing cloud services can outsource variable tasks, which would reduce educational costs and personnel expenditure. for facility management. For this reason, companies are currently adopting the cloud storage system by renting servers.

¹ See <https://www.gartner.com/en/newsroom/press-releases/2018-08-01-gartner-says-worldwide-iaas-public-cloud-services-market-grew-30-percent-in-2017>.

² See Synergy research Group HP (URL: <https://www.srgresearch.com/articles/cloud-growth-rate-increases-amazon-microsoft-google-all-gain-market-share>). Amazon had the largest market share, at 34%. Followed by (Microsoft 11%, IBM 8% and Google 5% amounting to a total 24%) in Q2, 2017. The market share gain of MS in the last four quarters is 3%, the largest increase among the four major players.

As described above, the rapid spread of the cloud service has been explained mainly from the viewpoint of demand side, company's cost reduction. But are there any advantages for providing cloud service in rental basis from the viewpoint of supply side? In order to expect the sustainable growth of cloud market, it is also necessary for supply side companies (i.e. Amazon, Microsoft) to feel that it is beneficial to provide cloud service in a rental basis. Based on such questions, this paper refers a theoretical model on durable goods company, analyze in what case vendors prefer rental basis to sales setting various conditions and consider the background that diffusion has progressed. The cloud market is divided into three service layers: IaaS, PaaS and SaaS. SaaS is software as a service. In this paper, we consider cloud as an infrastructure service. Thus, we focus on PaaS and IaaS, and assume that investment in cloud infrastructure with selling for providers are greater than renting.

Our results show that licensor's choice depends on the facility and cost saving effects using licensing and investment. On the one hand, the licensee's choice depends on the degree of royalty. When royalty is high, the licensee prefers renting. However, when royalty is low, the licensee chooses selling. Thus, a two-part tariff contract combines high royalty and fixed fee; therefore, when facility and cost saving effects are high, both firms choose renting. However, if the facility and cost saving effects are low, the licensor chooses selling and the licensee chooses renting.

The remainder of this paper is organized as follows. In Section 2, we review previous research related to our theoretical model. After describing our model and assumptions we analyze a non-licensing game as a benchmark for establishing an optimal licensing strategy in Section 3, and look for optimal means for both firms with ad valorem royalty in Section 4. We

discuss two-part licensing in Section 5, and conclude the paper in Section 6.

2. Literature review

Focusing on the demand side incentives in the cloud market, Keung and Kwork (2012) and Kloch and Madsen (2011) showed that renting is an important tool to minimized SMEs' investment.³ While investing the supply side incentives of renting, Ojala (2012) shows that software renting is a strategic tool to compete in the cloud infrastructure market. Ojala (2013) shows licensing strategy for software vendors. Bulow (1982) formalized that for monopolists renting is superior to selling, but Chien and Chu (2008) showed selling is superior under the network effect. According to Bucovetsky and Chilton (1986), selling for a monopolist, who expects future competitors, is a strategic way. Desai and Purohit (1998), Bhaskaran and Gilbert (2009) examined a firm's choice in the channel structure. Poddar (2004) pointed out that the prisoner's dilemma is the best choice in situation when firms prefer selling. From an environmental perspective, Agrawal et al. (2012) examined a firm's choice. There is a vast amount of literature analyzing a firm's strategy in selling. Bond and Samuelson (1984) and Bulow (1986) investigated the tendency of planned obsolescence. Utaka (2011), Waldman (1993) and Choi (1994) examined the timing of upgrade and incompatible upgrade. Regarding the enclosed method, Choudhary et al. (1998) derived that the combination of selling and renting may be the optimal choice, while Gilbert and Jonnalagedda (2011) examined the lock in effect. Hendel and Lizzeri (1999), Johson (2011) made the complementary role of secondary markets clear in selling. Liu and Zang (2013) examined dynamic pricing competition in selling.

³ Currently, banks and big companies use a private cloud computing.

Postmus et al. (2009) examined the profitability of two strategies in licensing, which are fixed fee and pay-per-use. Ferrante (2006) examined how software licensing affects the organization and consumers. San Martín and Saracho (2010) and Heywood et al. (2014) examined per unit vs. ad valorem royalties in a duopoly market either under complete or incomplete information. Unfortunately, the literature on cloud providers' choice of, selling or renting is limited.

The development of Cloud infrastructure is drawn through innovations to provide high cost performance to customers with externality and investment. Therefore, we consider that licensing, externality and investment are important factor for providers, who prefer selling or renting in the cloud market. To examine why providers in cloud market prefer renting to selling, we setup a duopoly market by referring to San Martín and Saracho (2010) and Xue and Su (2011). Indeed, the cloud market is an oligopoly market, but we believe that the duopoly model clarifies the major factor of competition in the market. We believe that the reexamination of selling or renting cloud infrastructure provides a new perspective the durable goods market, and contributes toward the development in the market.

3. Model

In this section, we describe the basic set up of our model and present our assumptions about the product, firms, and consumers. First, we assume that Firm 1, which is an existing firm, does not have any patent, but provides Good 1, which is cloud infrastructure with renting using free OS, and monopolizes the cloud market. We assume that Firm 1 already has large production facilities, and high productivity. If Firm 2 enters the cloud market, Firm 2 provides Good 2 as cloud infrastructure with own OS, but Firm 2 has to invest for production facilities. From

Keung and Kwork (2012), we assume that if Firm 2 chooses renting, the initial investment is half compared with selling. We use a two-period model following Bulow (1982). Both firms have the same marginal cost, c , and adopt Cournot competition. As both the firms face production capacity constraints for their goods, the assumption of Cournot competition seems reasonable. Further, the goods do not depreciate over time to avoid the influence of durability and upgrade,⁴ thus Good i ($=1,2$) provide in both the periods are identical. There is no time discounting for the firms. If Firm 2 offers to Firm 1 the licensing with ad valorem royalty, Firm 1 can have compatible with Firm 2's OS. The value of the good for both firms in the licensing have a same the spillover. We assume a linear inverse demand function, and consider a four-stage game with the following time structure:

- Stage 1: Firm 2 decides selling or renting and an ad valorem royalty s to license the patent. At the same time, chooses the investment for cloud infrastructure level e ,
- Stage 2: In the first period, Firm 1 decides to sell or rent, and to accept or reject Firm 2's offer. If Firm 1 rejects the offer, the situation becomes similar to producing without a license. If Firm 1 accepts the offer, both firms provide the goods of same value.
- Stage 3: In the first period, both firms produce their outputs simultaneously.

⁴ Bulow (1986) studied the influence of durability in the selling market.

- Stage 4: In the second period, given the use in the first period, both firms decide on their respective outputs simultaneously.

To determine the sub-game perfect equilibrium, we use the backward induction method.

3.1 Demand side

We consider that the consumers' willingness to pay increases through the cost saving effect in cloud infrastructure. How much do firms increase consumers' willingness to pay? We assume consumers are rational because they anticipate the future value of the goods purchased today.

Consumers have unit demand. We consider δ as consumers' basic willingness to pay for Good i , and this is taken as the valuation of Good i for a given period. Further, we assume a continuum of consumers who are heterogeneous. Therefore, δ varies across consumers and is assumed to be uniformly distributed between $-\infty$ and a (where $a > 0$) with density one.⁵

Following Foros (2004), we allow for negative values of δ to avoid corner solution when all the consumers enter the market. However, consumers are homogenous in their valuation of cloud infrastructure quality. Hence, consumers' valuation of cloud infrastructure quality is $\delta + b_i e$. The parameter b_i is demand side spillover based on Firm i 's facility e (> 0). We assume that $b_i \in (0,1)$. In non-licensing case, type δ consumers' willingness to pay for Good 1 in a given period is $\delta + b_1 e$ and that of Good 2 is $\delta + b_2 e$. If $b_1 = b_2$, consumers services will be identical.

We assume that $b_2 > b_1$. Thus, Good 2 provides higher consumers' willingness to pay than Good 1. The demand structure is similar to that in Katz and Shapiro (1985) where the quality of cloud

⁵ Katz and Shapiro (1985) show that the consumers' valuation at $s + v(q_i^e)$ depends on the number of consumers who are expected to be connected to the firm.

infrastructure depends on the number of expected consumers connected to firm i . If both firms sell the goods, consumers consider the timing of purchase in the first period, which is better value of Good i for consumers, or second period. When a consumer buys Good i in the first period, he or she retains it until the end of the second period. That is, type δ consumers who buy Good 1 at price p_{11} gain a surplus in both the periods, which is denoted by $2(\delta + b_1 e) - p_{11} > 2(\delta + b_2 e) - p_{12}$ and $2(\delta + b_1 e) - p_{11} > \delta + b_1 e - \hat{p}_{21} > 0$. p_{1i} denotes the price charged for Good i in the first period, and \hat{p}_{2i} denotes the expected price for Good i in the second period. If a type δ consumer buys Good 2 at price p_{12} , it denotes $\delta + b_2 e - p_{12} + \hat{p}_{22} > \delta + b_1 e - p_{11} + \hat{p}_{21} > 0$. If $\delta + b_1 e - p_{11} + \hat{p}_{21} < 0$ or $\delta + b_2 e - p_{12} + \hat{p}_{22} < 0$, then the consumer will not buy Good i . If both firms are active in the first period, in equilibrium, the prices adjusted by $b_i e$ must be:⁶

$$p_{11} - b_1 e - \hat{p}_{21} = p_{12} - b_2 e - \hat{p}_{22} = P_1. \quad (1)$$

Equation (1) is formed as long as both firms are active and there is demand side spillover from the firms to customers. Thus, the two firms generate total output of $z_1 = q_{11} + q_{12}$ ($q_{1i} > 0$) in the first period, where q_{1i} is the demand for Good i in the first period. P_1 denotes the value for the marginal consumer's basic willingness to pay in the first period. For a given P_1 , type δ consumers with willingness to pay for two periods as $\delta \geq P_1$, enter the market.⁷ As we assume a uniform distribution, the prices must ensure that $z_1 = a - P_1$ as long as both firms are active in the first period. Let $a_1 = a + b_1 e$, $a_2 = a + b_2 e$. From equations (1), the inverse demand functions in the first period are given by:

⁶ If firms are active in the second period, in equilibrium, the price is $\hat{p}_{21} - b_1 e = \hat{p}_{22} - b_2 e$.

⁷ The demand for goods in the first period is represented by $z_1 = \int_{\beta}^a d\phi = a - P_1$.

$$p_{1i} = a_i - q_{11} - q_{12} + \hat{p}_{2i} \quad i = 1, 2 \quad (2)$$

Given the sales in the first period, if both firms are active in the second period, the prices adjusted by $b_i e$ must be the same, that is, $p_{21} - b_1 e = p_{22} - b_2 e = P_2$, where, p_{2i} denotes the price charged for Good i in the second period, and P_2 denotes the value of basic willingness to pay for the marginal consumer. For the given P_2 , consumers with $a - z_1 \geq P_2$ enter the market. As we assume uniform distribution, there are active consumers in the market in the second period. They generate a total output of $z_2 = q_{21} + q_{22}$ ($q_{2i} > 0$), where q_{2i} denotes the demand for Good i in the second period.⁸ When the two firms supply the total quantity $Q = z_1 + z_2$, prices in each period must ensure that $z_1 = a - P_1$ and $z_2 = a - z_1 - P_2$. In such a case, the inverse demand function in the second period is given by:

$$p_{2i} = a_i - q_{11} - q_{12} - q_{2i} - q_{2j} \quad (3)$$

If both firms choose renting, goods are returned to firms at the end of the period. In such a case, the inverse demand function Firm i ($= 1, 2$) faced during the period t ($= 1, 2$), is given by:

$$p_{ti} = a + b_i e - q_{ti} - q_{tj} \quad (4)$$

3.2 Supply side

Given the consumers' demand, both firms decide on selling or renting, and play an output game to maximize their profits during both the periods. Firm i faces maximization problem in the first period as follows:

$$\max_{q_{11}} \Pi_1 = q_{11} \cdot p_{11} + \hat{q}_{21} \cdot \hat{p}_{21} \quad \text{and} \quad (5)$$

⁸ This equation is as follows: $z_2 = \int_{\rho}^{a-z_1} d\phi = a - z_1 - P_2$.

$$\max_{q_{12}} \Pi_2 = q_{12} \cdot p_{12} + \hat{q}_{22} \cdot \hat{p}_{22} - fe^2. \quad (6)$$

Here, \hat{q}_{2i} denotes demand at the expected price \hat{p}_{2i} under $q_{11} + q_{12}$. fe^2 denotes Firm 2's investment, and f denotes a coefficient of investment. We assume $1 < f < 1.5$ from $\Pi_2 > 0$. If Firm 2 chooses renting, the investment denotes $fe^2/2$.

Thus, if both firms choose selling, for a given q_{2j} and $q_{11} + q_{12}$, Firm i 's maximization problem in the second period is

$$\max_{q_{2i}} \pi_{2i} = q_{2i} \cdot p_{2i} = q_{2i} (a_i - q_{11} - q_{12} - q_{2i} - q_{2j}). \quad (7)$$

If both firms choose renting, Firm i 's maximization problem in the second period is

$$\max_{q_{2i}} \pi_{2i} = q_{2i} \cdot p_{2i} = q_{2i} (a_i - q_{2i} - q_{2j}). \quad (8)$$

3.3 Non-Licensing

Both firms provide Good i through selling or renting for the two periods with marginal costs $c = 0$ in Cournot competition.⁹ The equilibrium in non-licensing will be the benchmark for both firms to respond to the offer. We examine firms' preference for either selling or renting, from the following.

3.3.1: Both Firms selling

Fourth stage, Maximization problem in the second period

Solving Firm i 's the maximization problems from Equation (7), we have

⁹ Bulow (1982) pointed out that high marginal costs are a signal of lower future output, and thus, higher future prices. However, the current study does not investigate this impact. Hence, we set $c=0$.

$$q_{2i}^{ns*} = (a + e(2b_i - b_j) - q_{1i} - q_{1j})/3. \quad (9)$$

ns denotes that both firms choose selling in non-licensing.

We assume $2b_i - b_j \geq 0$, where $i, j = 1, 2, i \neq j$. Substituting q_{2i}^{ns*} into Equation (3), we get

$$p_{2i}^{ns*} = (a + e(2b_i - b_j) - q_{11} - q_{12})/3 \quad (10)$$

Third stage, Maximization problem in the first period

When both firms are active during the second period, expectation price is $\hat{p}_{2i} = p_{2i}^{10}$.

Substituting p_{2i}^{ns*} into Equation (2), we get

$$p_{1i}^{ns} = a_i - q_{1i} - q_{1j} + \hat{p}_{2i} = (4a + e(5b_i - b_j) - 4q_{11} - 4q_{12})/3. \quad (11)$$

For a given q_{1j} , from Equation (5), (6) and (11), Firm i solves the following problem:

$$\max_{q_{11}} \Pi_1^{ns} = q_{11}(4a + e(5b_1 - b_2) - 4q_{11} - 4q_{12})/3 + (a + e(2b_1 - b_2) - q_{11} - q_{12})^2/9 \text{ and} \quad (12)$$

$$\max_{q_{12}} \Pi_2^{ns} = q_{12}(4a + e(5b_2 - b_1) - 4q_{11} - 4q_{12})/3 + (a + e(2b_2 - b_1) - q_{11} - q_{12})^2/9 - fe^2. \quad (13)$$

$$\text{s.t } \Pi_2^{ns} \geq 0$$

3.3.2. Both Firms renting

Solving Firm i 's the maximization problems in Equation (8), we have

$$q_{2i}^{nr*} = (a + e(2b_i - b_j))/3. \quad (14)$$

nr denotes that both firms choose renting in non-licensing.

From Equation (5) and (6), for a given q_{1j} , Firm i solves the following problem in the first period.

¹⁰ Xue and Su (2011) noted that to complete the model, it was necessary to assume that consumers correctly anticipate the prices charged in the second period: $\hat{p}_2 = p_2$ by Tirole (1988, pp. 81).

$$\max_{q_{12}} \Pi_1^{nr} = q_{11} (a + b_1 e - q_{11} - q_{12}) + ((a + e(2b_1 - b_2))/3)^2 \text{ and} \quad (15)$$

$$\max_{q_{12}} \Pi_2^{nr} = q_{12} (a + b_2 e - q_{11} - q_{12}) + ((a + e(2b_2 - b_1))/3)^2 - fe^2/2. \quad (16)$$

3.3.3. Firm i selling, Firm j renting

Since the second period is the last period, the condition for both firms is the same. Thus, in equilibrium we have:

$$q_{2i}^{nas*} = (a + e(2b_i - b_j) - q_{1i})/3 \text{ and} \quad (17)$$

$$q_{2j}^{nar*} = (a + e(2b_j - b_i) - q_{1i})/3. \quad (18)$$

nas denotes that Firm i chooses selling and nar denotes Firm j chooses renting in non-licensing.

From $\hat{p}_{2i}^{nas} = p_{2i}^{nas*}$, Firm i 's price in the first period is¹¹

$$p_{1i}^{nas} = a_i - q_{1i} - q_{1j} + \hat{p}_{2i} = (4a + e(5b_i - b_j) - 4q_{1i} - 3q_{1j})/3. \quad (19)$$

From Equation (4), Firm j 's rental price is $p_{1j}^{nar} = a + b_j e - q_{1i} - q_{1j}$

The maximization problem for Firm i in the first period is as follows:

$$\max_{q_{1i}} \Pi_i^{nas} = q_{1i} (4a + e(5b_i - b_j) - 4q_{1i} - 3q_{1j})/3 + (a + e(2b_i - b_j) - q_{1i})^2/9 \text{ and} \quad (20)$$

$$\max_{q_{1j}} \Pi_j^{nar} = q_{1j} (a + b_j e - q_{1i} - q_{1j}) + (a + e(2b_j - b_i) - q_{1i})^2/9 - fe^2/2. \quad (21)$$

The above four types of equilibrium values are presented in Table 1.

TABLE 1
Equilibrium Values Duopoly market in the four case

Non licensing				
Firm i (Firm j)	q_{1i}	p_{1i}	q_{2i}	p_{2i}
Selling(Selling)	$(10a + e(21b_i - 11b_j))/32$	$(2a + e(5b_i - 3b_j))/4$	$(2a + e(9b_i - 7b_j))/16$	$(2a + e(9b_i - 7b_j))/16$
Selling (Renting)	$11(a + e(2b_i - b_j))/35$	$4(a + e(2b_i - b_j))/7$	$8(a + e(2b_i - b_j))/35$	$8(a + e(2b_i - b_j))/35$

¹¹ $\hat{p}_{2i}^{nas*} = (a + e(2b_i - b_j) - q_{1i})/3$, $p_{2j}^{nar*} = (a + e(2b_j - b_i) - q_{1i})/3$.

Renting (Selling)	$(12a+e(23b_i-11b_j))/35$	$(12a+e(23b_i-11b_j))/35$	$(8a+e(27b_i-19b_j))/35$	$(8a+e(27b_i-19b_j))/35$
Renting (Renting)	$(a+e(2b_i-b_j))/3$	$(a+e(2b_i-b_j))/3$	$(a+e(2b_i-b_j))/3$	$(a+e(2b_i-b_j))/3$
	$\Pi_1^{ns} = (44a^2 - 44ae(3b_1$ $- 5b_2) + e^2(115b_1^2 - 362$ $b_1b_2 + 291b_2^2)/256$	$\Pi_1^{nas} = 284(a + e(2b_1$ $- b_2))^2/1225$	$\Pi_1^{nar} = 2(104a^2$ $+ 4ae(123b_1 - 71b_2)$ $+ e^2(629b_2^2 - 766b_1b_2$ $+ 241b_1^2)/1225$	Π_1^{nr} $= 2(a + e(2b_1 - b_2))^2/9$
	$\Pi_2^{ns} = (44a^2 - 44ae(5b_1$ $- 3b_2) + e^2(291b_1^2 - 362$ $b_1b_2 + 115b_2^2)/256 - e^2f$	$\Pi_2^{nar} = 2(104a^2$ $+ 4ae(123b_2 - 71b_1)$ $+ e^2(629b_1^2 - 766b_1b_2 +$ $241b_2^2)/1225 - e^2f/2$	$\Pi_2^{nas} = 284(a + e(2b_2$ $- b_1)^2/1225 - e^2f$	Π_2^{nr} $= 2(a + e(2b_2 - b_1))^2/9$ $- e^2f/2$

We set $\hat{e}(b_1, b_2, f) \equiv_{\text{def}} \{ e : \Pi_2^{ns*}(b_1, b_2, f) \geq 0 \}$.¹² Thus, the degree of cost saving for Firm

2 to enter the market with the investment is $0 < e \leq \hat{e}(b_1, b_2, f)$.

Second stage: Firm 1's decision

If Firm 2 chooses selling, $\Pi_1^{ns} > \Pi_1^{nar}$, Firm 1 also chooses selling. If Firm 2 chooses renting, $\Pi_1^{nas} > \Pi_1^{nr}$, Firm 1 chooses selling. Firm 1 always chooses selling.

First stage: Firm 2's decision

We set $e^*(b_1, b_2) \equiv_{\text{def}} \{ e : \Pi_2^{ns*}(b_1, b_2, e) = \Pi_2^{nar*}(b_1, b_2, e) \}$.¹³ When Firm 1 chooses selling and $0 < e < e^*(b_1, b_2)$, $\Pi_2^{ns} > \Pi_2^{nar}$, Firm 2 chooses selling. If $e^*(b_1, b_2) \leq e \leq \hat{e}(b_1, b_2)$, $\Pi_2^{ns} \leq \Pi_2^{nar}$, Firm 2 chooses renting. Therefore, we have the following Lemma.

¹² See Appendix (1)

¹³ See Appendix (2)

Lemma 1. If $0 < e < e^*(b_1, b_2)$, both firms choose selling. If $e^*(b_1, b_2) \leq e \leq \hat{e}(b_1, b_2)$, Firm 1 chooses selling, and Firm 2 chooses renting.

Lemma 1 tells that Firm 1 always chooses selling to gain a bigger share in the market. When e is small, competition becomes intense. In such a case, Firm 2 also chooses selling to acquire a greater share in the market. On the one hand, when e is high, as $b_2 > b_1$, Firm 2's cost-saving effect is higher than Firm 1. Thus, Firm 2 chooses renting to increase prices and control investment.

4. Licensing

In this section, Firm 2 offers a licensing with an ad valorem royalty $s \in (0, 1)$ based on the licensee's revenue. We set parameter $(b_1, b_2) = (b, b)$, and assume $b > b_2$.

4.1 Both firms choose selling

The maximization problems for both firms in the second period are

$$\max_{q_{21}} \pi_{21}^{ls} = (1-s)q_{21}(a + be - q_{11} - q_{12} - q_{21} - q_{22}) \text{ and} \quad (22)$$

$$\max_{q_{22}} \pi_{22}^{ls} = (q_{22} + s \cdot q_{21})(a + be - q_{11} - q_{12} - q_{21} - q_{22}). \quad (23)$$

ls denotes that both firms chooses selling under the licensing.

Solving the maximization problems, $q_{2i}^{ls*} = (a + be - q_{11} - q_{12})/(3 - s)$. Thus, $p_2^{ls*} = (a + be - q_{11} - q_{12})/(3 - s)$. From $p_2^{ls*} = \hat{p}_2$, the inverse demand function that firms faces in the first period is:

$$p_1^{ls} = a + be - q_{11} - q_{12} + \hat{p}_2 = (4 - s)(a + be - q_{11} - q_{12})/(3 - s) \quad (24)$$

Thus, the maximization problems for firms in the first period are

$$\max_{q_{11}} \Pi_1^{ls} = (1-s)[q_{11}(4-s)(a + be - q_{12} - q_{11})/(3-s)] + (a + be - q_{11} - q_{12})^2/(3-s)^2 \text{ and} \quad (25)$$

$$\max_{q_{12}} \Pi_2^{ls} = (q_{12} + s \cdot q_{11})(4 - s)(a + be - q_{12} - q_{11})/(3 - s) + (a + be - q_{11} - q_{12})^2/(3 - s)^2 - fe^2. \quad (26)$$

4.2 Both firms choose renting

The inverse demand function that firms face in second period is Equation (4). Thus, the

maximization problems for firms in the second period are

$$\max_{q_{21}} \pi_{21}^{lr} = (1 - s)q_{21}(a + be - q_{21} - q_{22}) \text{ and} \quad (27)$$

$$\max_{q_{22}} \pi_{22}^{lr} = (q_{21} + s \cdot q_{22})(a + be - q_{21} - q_{22}) \quad (28)$$

lr denotes that both firms choose renting under the licensing. Solving the maximization

problems, $q_{2i}^{lr} = (a + be)/(3 - s)$.

The maximization problem for both firms in the first period as follows:

$$\max_{q_{11}} \Pi_1^{lr} = (1 - s)(q_{12}(a + be - q_{11} - q_{12}) + (a + be)^2/(3 - s)^2) \text{ and} \quad (29)$$

$$\max_{q_{12}} \Pi_2^{lr} = (q_{11} + s \cdot q_{12})(a + be - q_{11} - q_{12}) + (a + be)^2/(3 - s)^2 - fe^2/2. \quad (30)$$

4.3: Firm 1 prefers selling, and Firm 2 prefers renting

The maximization problems for firms in the second period are

$$\max_{q_{21}} \pi_{21}^{las} = q_{21}(1 - s)(a + be - q_{11} - q_{21} - q_{22}) \text{ and} \quad (31)$$

$$\max_{q_{22}} \pi_{22}^{lar} = (q_{22} + s \cdot q_{21})(a + be - q_{11} - q_{21} - q_{22}). \quad (32)$$

las denotes that one firm chooses selling, and lar denotes that the other firm chooses renting

under the licensing. From $p_{21}^{las*} = (a + be - q_{11})/(3 - s) = p_{22}^{lar*}$ and $p_{11}^{las} = a + b - q_{11} - q_{12} +$

\hat{p}_{21} .

Thus, the maximization problems for firms in the first period are

$$\max_{q_{11}} \Pi_1^{las} = (1 - s)[q_{11}(a + be - q_{12} + (a + be - q_{11})/(3 - s)) + (a + be - q_{11})^2/(3 - s)^2] \text{ and}$$

(33)

$$\max_{q_{12}} \Pi_2^{lar} = q_{12}(a + be - q_{11} - q_{12}) + s \cdot q_{11}(a + be - q_{12} + (a + be - q_{11})/(3 - s))$$

$$+ (a + be - q_{11})^2/(3 - s)^2 - fe^2/2. \quad (34)$$

4.4 Firm 1 chooses renting and Firm 2 choose selling

The maximization problems for firms in the second period are

$$\max_{q_{21}} \pi_{21}^{lar} = q_{21}(1 - s)(a + be - q_{12} - q_{21} - q_{22}) \text{ and} \quad (37)$$

$$\max_{q_{22}} \pi_{22}^{las} = (q_{22} + s \cdot q_{21})(a + be - q_{12} - q_{21} - q_{22}). \quad (38)$$

The maximization problems for firms in the first period are

$$\max_{q_{11}} \Pi_1^{lar} = (1 - s)[q_{11}(a + be - q_{11} - q_{12}) + (a + be - q_{12})^2/(3 - s)^2] \text{ and} \quad (39)$$

$$\max_{q_{12}} \Pi_2^{las} = q_{12}(a + be - q_{12} + (a + be - q_{12})/(3 - s)) + s \cdot q_{11}(a + be - q_{11} - q_{12}) \\ + (a + be - q_{12})^2/(3 - s)^2 - fe^2. \quad (40)$$

The above equilibrium values are mentioned in Table 2.

TABLE 2

Equilibrium Values Duopoly market in the four cases

Ad valorem	royalty licensing			
Firm 1	q_{11}	p_{11}	q_{21}	p_{21}
Selling(Selling)	$(a+be)(5-s)(2-s)/D_1$	$(a+be)(4-s)^2/D_1$	$(a+be)(4-s)/D_1$	$(a+be)(4-s)/D_1$
Selling (Renting)	$(a+be)(11-8s+8s^2)/D_2$	$(a+be)(20-11s+s^2)/D_2$	$(a+be)(8-5s+s^2)/D_2$	$(a+be)(8-5s+s^2)/D_2$
Renting (Selling)	$(a+be)(12-7s+s^2)/D_2$	$(a+be)(12-7s+s^2)/D_2$	$2(a+be)(4-s)/D_2$	$2(a+be)(4-s)/D_2$
Renting (Renting)	$(a+be)/(3-s)$	$(a+be)/(3-s)$	$(a+e)/(3-s)$	$(a+e)/(3-s)$
Firm 2	q_{12}	p_{12}	q_{22}	p_{22}
Selling (Renting)	$(a+be)(11-6s+s^2)(1-s)/D_2$	$(a+be)(20-9s+s^2)/D_2$	$2(a+be)(4-s)(1-s)/D_2$	$2(a+be)(4-s)/D_1$
Renting (Renting)	$(a+be)(1-s)/(3-s)$	$(a+be)/(3-s)$	$(a+be)(1-s)/(3-s)$	$(a+be)/(3-s)$

$$\begin{array}{cccc}
\Pi_1^{ls} = (a+be)^2(4-s)^2(1-s) & \Pi_1^{las} = (a+be)^2(1-s) & \Pi_1^{lar} = (a+be)^2(1-s) & \Pi_1^{lr} = 2(a+be)^2 \\
\times(11-(7-s)s)/(D_1)^2 & \times(284-361s+160s^2 & \times(4-s)(13-6s+s^2) & \times(1-s)/(3-s)^2 \\
& -29s^3+2s^4)/(D_2)^2 & /(D_2)^2 & \\
\Pi_1^{ls} = (a+be)^2(4-s)^2 & \Pi_2^{lar} = (a+be)^2(2-s) & \Pi_2^{las} = (a+be)^2 & \Pi_2^{lr} = 2(a+be)^2 \\
\times(11-(7-s)s)/(D_1)^2 - e^2f & \times(104-16s-29s^2 & \times(4-s)(71-64s & /(3-s)^2 - e^2f/2 \\
& + 10s^3 - s^4)/(D_2)^2 - e^2f/2 & + 19s^2 - 2s^3)/(D_2)^2 & \\
& & - e^2f &
\end{array}$$

$$D_1 = 32 - 31s + 10s^2 - s^3, D_2 = 35 - 31s + 9s^2 - s^3$$

Second stage: Firm 1's choice

Firm 1's choice depends on royalty s . We set $\hat{s}(b, e) \equiv_{\text{def}} \{s : \Pi_1^{lar*}(b, e) = \Pi_1^{ls*}(b, e)\}$ and $\tilde{s}(b, e) \equiv_{\text{def}} \{s : \Pi_1^{las*}(b, e) = \Pi_1^{lr*}(b, e)\}$. If s is high, Firm 1's profit is controlled by s . In such a case, Firm 1 prefers renting to increase the price. If s is low, the competition between both firms becomes intense. In such a case, Firm 1 chooses selling to gain market share. When Firm 2 chooses renting, the condition that Firm 1 chooses renting is $\tilde{s} \leq s$, and $\Pi_1^{lr*} \geq \Pi_1^{nas*}$ or $\Pi_1^{lr*} \geq \Pi_1^{ns*}$. However, the condition shows $\tilde{s} > s$. Thus, Firm 1 chooses selling. When Firm 2 chooses selling, the licensing condition Firm 1 chooses renting is $\hat{s} \leq s$, and $\Pi_1^{lar*} \geq \Pi_1^{nas*}$ or $\Pi_1^{lar*} \geq \Pi_1^{ns*}$. As the profit condition shows $\hat{s} > s$, Firm 1 chooses selling. Thus, in royalty licensing contract, Firm 1 always prefers selling.

First stage: Firm 2's decision

Firm 2's choice depends on the cost saving effect. When Firm 1 choose selling, $\Pi_2^{lar} > \Pi_2^{ls}$, Firm 2 chooses renting. In such a case, the royalty Firm 2 offers is $s_1 \equiv_{\text{def}} \{s : \Pi_1^{las*}(b, e, s) = \Pi_1^{nas*}\}$. The royalty shows $s_1 < \hat{s}$ and $\Pi_2^{lar*}(s_1) \geq \Pi_2^{nar*}$. When $0 < e < e^*(b_1, b_2)$, $\Pi_2^{lar} > \Pi_2^{ls}$, Firm 2 chooses renting. The royalty Firm 2 offers is $s_2 \equiv_{\text{def}} \{s : \Pi_1^{las*}(s) = \Pi_1^{ns*}\}$. The

royalty shows $s_2 < \hat{s}$. However, when the cost saving effect is low, $\Pi_2^{lar*}(s_2) < \Pi_2^{ns*}$, licensing reduces Firm 2's advantage. When we set $\bar{e}(s_2) \equiv_{\text{def}} \{e : \Pi_2^{lar*}(s_2) = \Pi_2^{ns*}(b_1, b_2, e)\}$, we have the following Lemma.

Proposition 1. In royalty licensing, when $\bar{e} < e < \hat{e}$, Firm 2 offers s_1 . Firm 1 chooses selling, and Firm 2 chooses renting.

According to proposition 1, Firm 2 chooses licensing to control competition and investment, but the licensing effect is limited by the degree of the cost-saving effect. Thus, as in royalty licensing, Firm 2 cannot offer high royalty, s is low, Firm 1 prefers selling, and Firm 2 prefers renting.

5. Discussion

In this section, we consider a two-part tariff (F, s) , where F is the fixed fee charged upfront and $s \in (0,1)$ is ad valorem royalty based on the licensee's revenue. Firm 2's profit is a

monotonically increasing sequence with s . When we set $\bar{s} \equiv_{\text{def}} \{s : \Pi_1^{ns*}(b_1, b_2, e) = \Pi_1^{lr*}(b, e)\}$,

$\bar{s} > \hat{s}$ and $\bar{s} > \tilde{s}$. Firm 2 compensates lost revenue with fixed fee F . In such condition, Firm 1

prefers renting. We set $\bar{e}(b, s) \equiv_{\text{def}} \{e : \Pi_2^{tas*}(b, e, s) = \Pi_2^{tr*}(b, e, s)\}$. That is, when $\bar{e} < e <$

\hat{e} , $\bar{F} \equiv \Pi_1^{lr}(\bar{s}) - \Pi_1^{nas}$. Under a two-part tariff (\bar{F}, \bar{s}) , Firm 2's profit is $\Pi_2^{lr}(s_1) \leq \Pi_2^{tr*}(\bar{F},$

$\bar{s})$, and Firm 1's profit is $\Pi_1^{nas} = \Pi_1^{tr*}(\bar{F}, \bar{s})$. When $0 < e < \bar{e}$ and Firm 2 prefers selling, we

set $\bar{F} \equiv \Pi_1^{lar}(\bar{s}) - \Pi_1^{ns}$. Under a two-part tariff (\bar{F}, \bar{s}) , profit for both firms are $\Pi_2^{nas} <$

$\Pi_2^{tas}(\bar{F}, \bar{s})$ and $\Pi_1^{tar}(\bar{F}, \bar{s}) = \Pi_1^{ns}$. Thus, Firm 1 chooses renting. Superscript *tar* and *tas*

denote Firm 1's renting and Firm 2's selling respectively, and *tr* denotes both firms' renting in a

two-part tariff. Comparing the two conditions, if $e = 0.05$, $b_1 = 0.6$, $b_2 = 0.65$, $f = 1.5$ and $b = 0.7$, as $\bar{s} = 0.451$, $\Pi_2^{tas*}(\bar{F}, \bar{s}) = 0.302 > 0.180 = \Pi_2^{ns}$. If $e = 0.3$, as $\bar{s} = 0.514$, $\Pi_2^{tr*}(\bar{F}, \bar{s}) = 0.322 > 0.220 = \Pi_2^{lar}(s_1)$. Thus, in a two-part tariff, when $0 < e < \ddot{e}$, two-part tariff (\bar{F}, \bar{s}) provides better revenue for Firm 2 with $(\Pi_1^{tar}(\bar{F}, \bar{s}), \Pi_2^{tas}(\bar{F}, \bar{s}))$ than (Π_1^{ns}, Π_2^{ns}) . When $\ddot{e} < e < \hat{e}$, two-part tariff (\bar{F}, \bar{s}) brings better revenue for Firm 2 with $(\Pi_1^{tr*}(\bar{F}, \bar{s}), \Pi_2^{tr*}(\bar{F}, \bar{s}))$ than $(\Pi_1^{lar}(s_1), \Pi_2^{las}(s_1))$. However, total output in a two-part tariff is lower, compared to royalty licensing. On the one hand, if $\underline{s} = s_1 \times 0.8$ and $\underline{F} \equiv \Pi_1^{lr}(\underline{s}) - \Pi_1^{nas}$. Under a two-part tariff $(\underline{F}, \underline{s})$, Firm 2's profit is $\Pi_2^{lar}(s_1) > \Pi_2^{tr}(\underline{F}, \underline{s})$ and Firm 1's profit is $\Pi_1^{nas} = \Pi_1^{tr*}(\underline{F}, \underline{s})$. If $s < s_1$, total production increase, and the price is low, but Firm 2's profit with a two-part tariff $(\underline{F}, \underline{s})$ is worse.

Proposition 2. *In the two-part tariff contract, if $0 < e \leq \ddot{e}$, Firm 2 offers (\bar{F}, \bar{s}) . Firm 1 chooses renting, and Firm 2 chooses selling. If $\ddot{e} < e < \hat{e}$, Firm 2 offers (\bar{F}, \bar{s}) . Both firms choose renting.*

Proposition 2 explains that if a licensor guarantees a licensee's profit with fixed fee F in a two-part tariff (F, s) , the licensor gains greater revenue than royalty licensing. Further, we find that a two-part licensing brings a cost saving effect to firms and consumers. In such a case, as firms gain high profit, other firms enter the market, but renting mitigates competition.

6. Concluding remarks

In this paper, we analyzed optimal means for firms that provide cloud infrastructure. In royalty licensing, as the licensee attempts to gain a broader market share, the licensee prefers selling. The licensor's choice depends on investment and the degree of facility and cost saving and facility for customers. In contrast, in two-part tariffs, which are high royalty and fixed fee, when the cost saving and facility effects are low, the licensee prefers renting and the licensor prefers selling. When the cost saving and facility effect is high, both firms choose renting. That is, implementing environment is an important element for providers. These results reflect the real condition of a cloud market, in which providers, such as Amazon and Microsoft, mainly rent cloud infrastructure to customers. From our examination, we realize that Microsoft's entry strategy, which attempts to sustain dominant position in the cloud market. When Windows Server's consumers migrate to cloud infrastructure market, it is necessary arrangement to prepare cloud infrastructure market as Microsoft, who has dominant position in enterprise SaaS.¹⁴ That is, it is important to enter the cloud computing market for licensor as Microsoft. However, our results depend crucially on the model specifications. Specifically, the assumption about level of investment is important. If level of investment is the same between selling and renting, our results are different. However, when we take a realistic view of the investment, the difference in investment between selling and renting seems reasonable. In future, we would like to examine a collaboration effect between PaaS or IaaS and SaaS for vender in cloud market.

¹⁴ See <https://www.srgresearch.com/articles/quarterly-saas-spending-reaches-20-billion-microsoft-extends-its-market-leadership> and <https://www.srgresearch.com/articles/2017-review-shows-180-billion-cloud-market-growing-24-annually>.

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Appendix

(1) From $\prod_2^{ns*} \geq 0$, the condition, Firm 2 gains the positive profit is

$$\hat{e} = -22a^2/(a(55b_2 - 33b_1) - 4\sqrt{11}\sqrt{a^2(16f - (b_2 - b_1)^2)}) > 0 \quad (A.1)$$

From $b_2 > b_1$, $a^2(16f - (b_2 - b_1)^2) > 0$.

And as $a(55b_i - 33b_j) < 4\sqrt{11}\sqrt{a^2(16f - (b_i - b_j)^2)}$, the denominator is a minus. Thus, $\bar{e} > 0$.

(2) From $\prod_2^{ns*}(b_1, b_2, e) = \prod_2^{nar*}(b_1, b_2, e)$, the condition, Firm 2 gains the positive profit, is

$$e^* = 326a^2/(140\sqrt{701(b_2 - b_1)^2 + 1304f} + 4073b_1 - 4399b_2) > 0 \quad (A.2)$$

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