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Micro fluidity and macro stability in inventor networks

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Abstract

From a macro perspective, inventor networks are characterized by rather stable structures. However, the high levels of fluidity of inventors and their ties found in reality contradicts this macro pattern. In order to explain these contradicting patterns, we zoom in on the intermediate group structures of co-patenting relationships found among inventors in German laser technology research over a period of 45 years. Our findings suggest that continuity of individual actors is not a key factor in maintaining structural stability of networks. Group level explorations indicate that the successor of an existing key player belonged to the exiting key player’s ego-network, indicating that the group level provides a source of stability and functionality to the system.

Keywords: Inventor network, network stability, key player analysis, innovation, laser technology

JEL classification: D22, D85, L23, O3

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1. Introduction

Over the past decades a strong growth of newly established R&D cooperation (Hagedoorn 2002; Tomasello et al. 2017) is observed, indicating that innovation processes are increasingly characterized by a division of innovative labor (Wuchty et al. 2007). The reasons for this trend are straightforward; R&D cooperation enables knowledge exchange and collective learning, and may lead to cost saving, risk sharing, as well as reduced time to market (Hamel 1991; Grant and Baden-Fuller 2004; Mowery et al. 1996; Hagedoorn 2002). The network of cooperative relationships among innovative actors constitutes an important element of the innovation system (Chaminade et al. 2019). Previous research indicates that it is not only an actor’s individual network position (Powell et al. 1996; Stuart 2000), but that the structural characteristics at the overall network level, such as core-periphery patterns (Borgatti and Everett 1999), scaling properties (Barabasi and Bonabeau 2003), and small-world characteristics (Watts and Strogatz 1998) also matter when it comes to explaining innovation outcomes.¹ This indicates that the system’s topology opens up favorable cooperation and positioning opportunities for individual actors and enables direct and indirect knowledge transfer, mutual learning, and collective innovation processes.

Previously research shows (e.g., Tomasello et al. 2017) that innovation networks tend to emerge in quite typical and structurally stable patterns at the macro level.² Contrary to the widespread notion of network stability, however, recent empirical investigations indicate a high level of fluidity at the micro level and that network actors as well as their ties are in a constant state of flux. (Fritsch and Kudic 2016; Fritsch and Zoellner 2018; 2019). This includes the entry of new actors and establishment of new ties, the discontinuation of actors and ties, as well as changes in the

¹ For instance, empirical evidence suggests a positive and significant relationship between small-world characteristics, creativity and the innovative performance of the actors involved (Uzzi and Spiro 2005; Schilling and Phelps 2007).

² We define ‘structural stability’ from a holistic perspective. It describes a network’s tendency to develop typical, non-random topologies in order to keep its functionality and its ability to retain or regenerate these characteristic patterns over time.
quality of the relationships (Graebner et al. 2018). At least at first glance, the high level of micro-level fluidity may appear to be incompatible with structural stability at the macro level.

In this paper, we seek to contribute to a better understand of the complex interplay between micro-level fluidity and structural stability at the aggregate level of inventor networks\(^3\) by addressing this question: Who or what explains the co-existence of macro-level stability and high levels of fluidity at the micro level? We base our empirical analysis on a unique dataset encompassing all patent applications in the field of laser technology in West Germany from the inception of the technology in the year 1961 until 2005. We employ this longitudinal data to reconstruct the co-patenting inventor network and its structural evolution for almost half a century.\(^4\) This exploration along different levels of aggregation provides us with important insights into the evolution of the network's structural properties. We then turn our attention to the micro level and analyze the extent to which individual actors and their connections persist over time. Finally, we employ ‘key player analyses’ (Borgatti 2006) to identify the cohesive forces that pull the overall network structure together despite the high levels of fluidity at the micro level.

Section 2 summarizes the main contributions of previous research concerning the development and underlying relationships of network structures. Section 3 introduces the data and provides a brief overview of the development of inventor networks in the West German laser industry. Next, we corroborate the macro-level stability of network structures and

\(^3\) We focus exclusively on inventor networks, which are considered to be one specific type—among many others—of innovation networks in which nodes represent individual actors (persons) and ties reflect joint R&D activities.

\(^4\) As an alternative to inventor networks, one could analyze cooperative patenting activities between organizations (e.g., public research institutes and firms). This assumes that researching organizations hold the relevant knowledge rather than the inventors. If the patent document names several organizations as applicants, identifying such cooperative relationships between organizations can be accomplished using the patent statistics. As compared to ties among inventors, co-applications of patents with several organizations are relatively rare and the construction of applicant networks is not conclusive. For example, at the onset (1961-65), we found 33 applicants while the share of isolated applicants amounts to 87%. In the middle of our observation period (1981-85), we found only 119 applicants, and again a very high share of isolates (86.3%).
show the high levels of micro-level fluidity (Section 4). The stability of network relationships at the intermediate group level is then analyzed in Section 5. Finally (Section 6) we discuss the main findings and outline some fruitful avenues for further research.

2. Previous research and own research design

2.1 The structural dimension of networks revisited

A considerable number of studies show that the emergence of typical real-world network topologies is systematically shaped by complex processes and mechanisms (see, for example, Newman et al. 2006). This clearly indicates that real-world networks are not random. The structural divergence between random and real-world networks becomes particularly evident when looking at network properties such as ‘scaling’, ‘connectedness’, ‘clustering’, and ‘core-periphery structures’.

Even though random networks typically have no direct practical relevance, they may serve as reference points or benchmarks for real-world networks. In its most basic sense, a random network (cf. Erdős–Rényi 1959) can be defined as a system that consists of a well-defined number of nodes, where each of these nodes attracts ties with the same probability.

Barabási and Albert (1999) argue that simplistic random network models typically fail to account for key features of real-world networks such as growth and preferential attachment. Accordingly, their network model incorporates the idea that actors with an above average number of ties (degree) have a higher probability of attracting additional ties than actors with fewer ties. Over time, this mechanism leads to the emergence of a ‘fat-tailed’ degree distribution that is characterized by a sharp bell shape and a high skewness. Barabási and Albert (1999, 510) showed that ‘[…] large networks tend to self-organize into a scale-free state’. The implication of such a degree distribution is straightforward: a small number

\[ \text{For an overview, see Albert and Barabási (2000, 2002).} \]
of actors attracts ties at a significantly higher rate than others. As these ‘network hubs’ (or ‘high degree’ actors) continue to expand their dominant network position, the majority of actors has only a very small number of direct ties. A broad range of studies have observed and empirically tested ‘scaling’ properties of networks. For instance, Powell et al. (2005) analyzed degree distributions for six networks (differentiated by type of partner) in the US Life Science Industry. The results suggest that a small number of actors tend to attract a significantly higher number of ties compared to the large majority of other network actors.6

Emerging real-world networks are typically characterized by a simultaneous increase in size and a decrease in density (Kudic 2015; Tomasello et al. 2017). Even more interesting, the clustering of real-world networks noticeably increases over time. Simply put, a cluster can be described as a densely interconnected sub-set of actors within a network. Typically, different clusters are either completely disconnected or only loosely coupled. Several authors have demonstrated that the co-existence of high clustering and short path length in networks (frequently referred to as ‘small-world’ phenomenon (Milgram 1967; Watts and Strogatz 1998)) affects the exchange of information, ideas and knowledge, and is positively related to creativity and innovativeness of the actors involved (Uzzi and Spiro 2005; Fleming et al. 2007; Schilling and Phelps 2007).

‘Core-periphery’ network structures can be defined as “[…] a dense, cohesive core and a sparse, loosely connected periphery” (Borgatti and Everett 1999, 375). The core is composed of key members of the community and includes actors who have developed dense connections to others and act as network coordinators. The periphery includes actors who are only loosely connected to the core, as well as to the other actors in the periphery (Cattani and Ferriani 2008, 826). It has been argued that the

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6 A fat-tailed or scale-free degree distribution of a real-world network is typically explained and interpreted as follows: “Unlike the tail of a random bell curve whose distribution thins out exponentially as it decays, a distribution generated by a popularity bias has a ‘fat’ tail for the relatively greater number of nodes that are highly connected. The fat tail contains the hubs of the network with unusually high connectivity” (Powell et al. 2005, 1151).
core of sectoral innovation networks contains essential elements of the industry’s technological knowledge (Rank et al. 2006).

In research based on publicly funded innovation networks between 1990 and 2010, Kudic et al. (2015) show the emergence and solidification of a core-periphery structure in the German laser industry. One potential explanation for the emergence of a loosely coupled and highly fragmented network periphery is based on the cooperation options available to new network entrants. Core actors prefer to connect to other well-embedded actors, while newcomers tend to enter the network by establishing partnerships with one another and only later gain access to high-degree actors. Rosenkopf and Padula (2008) confirm this line of reasoning by showing that the likelihood that an entrant attaches to a network incumbent increases with the incumbent’s ‘prominence’ in the network as proxied, e.g., by his degree centrality or his eigenvector centrality (Bonacich 1987). From a macro perspective, this is reflected in the emergence of a loosely coupled and highly fragmented network periphery.

In summary, theoretical and empirical studies suggest that real-world networks show a pronounced tendency to develop stable structural patterns at the macro level. Specifically, a fat-tail degree distribution, clustering patterns and core-periphery structures distinguish real-world networks from random networks.

2.2 Network dynamics and the fluidity of actors and ties

The micro-level dynamics of networks comprise entries and exits of actors (nodes), as well as the formation and the termination of partnerships (ties) among actors. This micro fluidity affects the structural configuration at higher levels of aggregation (cf. Kudic 2015) in nontrivial ways.

The main reason to expect high levels of stability of ties among actors is that establishing and maintaining cooperative relationships in research and development (R&D) requires considerable effort. The costs of establishing and maintaining R&D cooperation particularly include the effort of identifying a suitable cooperation partner, negotiating the terms of
the cooperation, and establishing a well-working and trust-based relationship that may require frequent face-to-face contacts (Ejermo and Karlsson 2006; Storper and Venables 2004; Gilsing and Nooteboom 2005). Moreover, the relationship may require investment in absorptive capacity, such as specific skills and equipment (e.g., Powell et al. 2005; Powell and Gianella 2010). Since these investments will be sunk if an R&D cooperation is abandoned, one may expect an incentive for actors to maintain a relationship over longer periods of time, unless continuing the relationship is more costly than establishing a more rewarding new relationship with a different actor.

There are, however, also a number of solid arguments for terminating existing partnerships and establishing new ones. The main rationale for discontinuing a cooperative relationship is that the dynamics of innovation processes require the continuous acquisition of new knowledge, and the recognition that parts of the existing knowledge may become obsolete. These changing requirements induce reorganizations of innovation activities, particularly establishing new ties to new actors while some of the established relationships are no longer needed. Similarly, the knowledge pool of inactive network incumbents erodes over time. For these reasons, one may expect high levels of fluidity of actors and their ties.

The rather few empirical studies that analyze the levels of fluidity of actors and their ties in innovation networks (Fritsch and Zoellner 2018, 2019; Greve et al 2009; Ramlogan and Consoli 2014) show surprisingly high levels of new and exiting actors, as well as newly established and terminated ties. For example, in an analysis of regional inventor networks Fritsch and Zoellner (2019) found that more than 78% of all inventors are only present in one three year period, 14.51% are active in two periods and only about 7% appear in networks for more than two successive periods. Only 9.7% of all ties between inventors can still be found in the

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7 Analyzing the effect of fluidity on the structure of the inventor networks, Fritsch and Zoellner (2019) find some statistically significant relationships with the share of the largest network component and the share of isolates. The results suggest that fluidity leads to some fragmentation of the networks, but that the effect is not very pronounced.
successive period. A study by Ramlogan and Consoli (2014) on collaborative research in medicine finds that the share of new collaborations over all collaborations is always above 70% in all years of the observation period.

2.3 What do we want to know and how can we get there?

The co-existence of macro-level stability and micro-level fluidity in innovation networks is a largely unexplored phenomenon. Our analysis of the co-existence of macro-level stability and micro-level fluidity is based on a unique dataset of co-patenting relationships among inventors in the German laser industry over a period of 45 years.

We employ a three-stage research design. The first step is analyzing the structural stability of the inventor network by exploring the structural features and pattern formation processes along different lines. We consult a set of standard metrics to gain an initial intuition of the system’s overall topology and related development patterns. Next, we test for the emergence of two exemplary structural phenomena that are frequently emphasized in the pertinent literature (scaling properties and segregation into a core-periphery structure) to check whether the system is characterized by stable pattern formation processes at the macro level. In a second step, we turn our attention to the micro level and analyze the degree of fluidity by exploring node and tie re-occurrence rates over time. Third, in order to determine who or what keeps the system together, we begin at the micro level and identify two types of key players: (1) those who warrant diffusion properties of the system, and (2) those who are responsible for the structural cohesion of the system.

Our analyses clearly show that these two types of actors are anything but persistent over time. This result raises a number of interesting questions. Who are these key players, where do they come from and why do they occupy their roles for only such a short period of time?

Relating the levels of fluidity to the performance of networks in terms of the number of patents per R&D employee (patent productivity) suggests positive effects of new actors and ties.
time? If an ‘old’ key player disappears from the network, who takes over this important role? Is a key player and his successor connected? Was there an exchange of knowledge before the role change so that the knowledge of the ‘old’ key player remains in the system? To answer at least some of these questions, we turn our attention to the intermediate network level, specify the ego networks for each of the top key players over time and analyze to what extent these highly important roles are passed over to members of the same group.

3. Technology and data

3.1 German Laser research

The acronym 'laser' was originally coined by Gould R. Gordon (1959) and stands for ‘Light Amplification by Stimulated Emission of Radiation’\(^8\) German laser research provides an ideal empirical setting for the purposes of this study for several reasons. First, laser technology can be characterized as knowledge intensive and science-driven (Bertolotti 2005; Bromberg 1991; Grupp 2000). Second, the development of laser technology requires expertise from various scientific disciplines such as optics, electronic engineering and physics (Fritsch and Medrano 2015). Thus, the creation of novel products and services in this field is often a collective process characterized by a pronounced division of labor between various actors and institutions. Third, German laser research is a well-defined and documented technological field (Albrecht 2019; Buenstorf, Fritsch, and Medrano 2015; Fritsch and Medrano 2015; Kudic 2015).

Although the roots of laser research reach back into the early 20th century (Albrecht 2019; Bromberg 1991), a research group led by Theodore H. Maiman at the Laboratories of the Hughes Aircraft Company

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\(^8\) It describes a wide range of devices for the amplification of coherent light by stimulated photon emission generated by pumping energy through an adequate medium. A laser device emits a coherent light beam, both in a spatial and a temporal sense that can be generated based on different gain media, such as solid crystals and semiconductors, for example. The coherent light beam can be modulated and amplified.
in Malibu (California, USA) realized the first laser in early 1960. The first realization of a laser in Germany occurred in the Siemens Company’s Munich laboratories in November of the same year. Over the following decade Siemens dominated German research in the field of laser technology (Albrecht 2019; Buenstorf and Fritsch 2019; Fritsch and Medrano 2015).

3.2 Data sources, co-patenting, and the construction of inventor networks

Our empirical study is based on patent applications in the field of laser technology in West Germany from 1961 to 2005, a period of nearly half a century. In order to isolate the effect of German unification in the year 1990, we exclude laser patents by inventors located in East Germany and strictly limit the analysis to inventors in West Germany. In total, we identified 4,371 laser-related patent applications between 1961 and 2005. A main benefit of our data is that it comprises the full population of all inventors active in the field of laser research over the entire observation period. The data allows us to analyze R&D cooperation activities and network entry modes for the entire population of inventors from the early emergence of this technological field onwards. The patent data provides us with information about the applicant organizations and all of the inventors, and are the basis for identifying ties between these actors.

The patent information was obtained from the database DEPATISnet (www.depatisnet.de), which is maintained by the German Patent and Trade Mark Office. From this source we selected all patent applications with priority in West Germany that were assigned to the technological field ‘devices using stimulated emission’ (IPC H01S) as either the main or secondary class. Research in this IPC class is related to laser beam sources that constitutes the basis for all kinds of applications. We account for important applications of laser technology by including those patent applications in the fields of material processing (IPC B23K), medical technology (IPC A61 without IPC A61K) and spectroscopy (IPC G01N) that mention the term ‘laser’ in the document. We found an increasing
number of patents in all these IPC classes over the entire observation period. In the early years of the technology, patenting activities reflect a clear dominance of research related to beam sources (IPC H01S). After the year 2000, we observe a clear trend shift from patenting in the basic technology of beam sources to its applications. Figure A1 and Table A1 in the Appendix provide further details on patenting and co-patenting activities in West German laser research.\(^9\)

The construction of networks based on co-patenting information requires some basic theoretical considerations and assumptions. From a graph-theoretical perspective, a network is completely defined by a well-specified set of nodes and ties among them (Newman 2010).\(^10\) A graph \((N, g)\) consists of a set of nodes \(N=\{1…n\}\) and a real-valued adjacency matrix \(g (=n \times n)\), where \(g_{ij}\) represents the relation between the node \(i\) and the node \(j\) in the network (Jackson 2008).

For the purpose of this study, we specify unvalued (i.e. the adjacency matrices that can only take the values of 0 or 1) and undirected (i.e. the adjacency matrices are symmetric) adjacency matrices. We assume that all inventors that are listed in a patent document have cooperated in R&D in order to achieve a common goal. From a methodological perspective, this is reflected by the second assumption that all partners listed in a patent document are connected to each other. Finally, we used the priority filing date of a patent application to assign a timestamp to the created network data. In order to investigate the development of inventor networks we divide the entire period of analysis into nine five-year windows of observation. Hence, we account for the limited lifetime of cooperative ties

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\(^9\) A closer look at the size distribution of inventor teams clearly indicates an increasing tendency towards co-patenting activities over time, and an increasing tendency towards larger teams over time. Of all patent applications filed between 1961 and 2005, 71% are co-patented by two or more inventors. This confirms the assessment by Wuchty at al. (2007) according to which innovation processes are increasingly characterized by a division of innovative labor for the German laser industry.

\(^10\) Nodes are frequently referred to as ‘actors’, ‘agents’, ‘players’ or ‘entities’, while ‘ties’ are frequently called ‘links’, ‘connections’ or ‘relationships’. In this study, a node represents an inventor and a tie represents a co-patent.
by assuming that co-patenting relationships persist for a limited period of
up to five years.¹¹

For benchmarking purposes, we generate *Erdös-Renyi* random
graph networks, comparable in terms of size and density to their real-world
counterparts.¹² For analytical purposes, we employ the software package
UciNet 6 (Borgatti et al. 2002; 2013).

We are well aware that patents reflect only a part of the diverse types
of formal and informal relationships among inventors. It is, however,
plausible to assume that documented co-inventorship implies other forms
of cooperation, such as co-publications and informal knowledge exchange.
A comprehensive data source that accounts for the variety of relationships
between innovating actors does not exist.¹³

4. **Macro-level stability and micro-level fluidity**

4.1 **Prevalence of macro-level stability**

To explore the structural characteristics of the German laser industry
inventor network, we draw upon basic graph-theoretical concepts and
employ social network analysis (SNA) metrics (Wasserman and Faust

We begin by focusing on a number of measures that describe the
topology of the entire system. Network size is simply defined as the
number of inventors with at least one dyadic relationship, while the total

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¹¹ This assumption is necessary since patent data provides no direct indication of tie-
duration or tie-termination dates. Although durations of projects can vary considerably,
many patent applications maybe based on joint research over a period of two to three
years (Greve et al. 2009; Phelps 2010; Ramlogan and Consoli 2014). According to Park
and Russo (1996), the average duration of a cooperative R&D project between
organizations is less than five years. We conducted several robustness checks and also
experimented with four-year and six-year windows, without significant differences in the
reported results.

¹² This procedure was always sufficiently often applied (n>30) to generated
representative random benchmarks.

¹³ A comparison of regional innovation networks constructed with different data sources
(Fritsch, Titze and Piontek 2019) finds that patent data tend to underestimate ties among
private sector firms, while universities and other public research institutions are well-
represented in patent data.
number of actors encompasses both inventors with co-inventors as well as unconnected inventors (isolates). The share of isolated inventors is expressed in percentage terms. The average degree indicates the average number of ties maintained by actors in the network. Network density is the total number of ties divided by the number of possible ties.

A component is defined as a connected sub-graph, while the component size simply reflects the number of actors involved in this component. The network diameter reflects the length of the longest geodesic path, while the measure of an average path length incorporates all geodesic distances among reachable pairs of actors, and provides an average measure at the systemic level. The clustering coefficient reflects the density of each actor’s nearer surrounding by measuring how many of the actor’s directly neighboring partners are interconnected.

Interestingly, the numbers of inventors and the number of inventors with at least one link to a co-inventor are continuously increasing, while the share of isolates decreases over time. The exception to this trend occurs during the observation window 2001-2005 (Table 1). At the same time, the number and average degree of observed R&D linkages among inventors is increasing. A network’s average density is simply the total number of actually observed ties divided by the total number of possible ties (Wasserman and Faust 1994).

The component-based network measures provide an interesting picture of the system’s overall tendency of fragmentation. We observe an increase of the number and the average size of components. A closer look at the component size distribution of the three largest components reveals particular strong growth of the main component. Finally, the network diameter and the average path length follows no clear trend. The same holds for the overall clustering coefficient. However, when using measures from our random benchmark networks (see Figures A2 and A3 in the Appendix), a comparison of the average path length with the overall clustering coefficient reveals an interesting insight. We find that German laser research networks exhibit a significantly shorter path length and
higher overall clustering coefficients, which indicates small-world properties.

Table 1: Basic network metrics, 1961-2005

<table>
<thead>
<tr>
<th>Description</th>
<th>61-65</th>
<th>66-70</th>
<th>71-75</th>
<th>76-80</th>
<th>81-85</th>
<th>86-90</th>
<th>91-95</th>
<th>96-00</th>
<th>01-05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of inventors</td>
<td>95</td>
<td>189</td>
<td>215</td>
<td>260</td>
<td>419</td>
<td>723</td>
<td>923</td>
<td>1,284</td>
<td>1,369</td>
</tr>
<tr>
<td>Number of inventors in the network</td>
<td>65</td>
<td>134</td>
<td>172</td>
<td>212</td>
<td>359</td>
<td>632</td>
<td>820</td>
<td>1,196</td>
<td>1,212</td>
</tr>
<tr>
<td>Share of isolates (%)</td>
<td>31.58</td>
<td>29.10</td>
<td>20.00</td>
<td>18.46</td>
<td>14.32</td>
<td>12.59</td>
<td>11.16</td>
<td>6.85</td>
<td>11.47</td>
</tr>
<tr>
<td>Number of ties</td>
<td>86</td>
<td>205</td>
<td>278</td>
<td>409</td>
<td>643</td>
<td>1,135</td>
<td>1,888</td>
<td>2,608</td>
<td>2,921</td>
</tr>
<tr>
<td>Average degree</td>
<td>1.969</td>
<td>2.09</td>
<td>2.36</td>
<td>2.575</td>
<td>2.735</td>
<td>2.873</td>
<td>3.249</td>
<td>3.61</td>
<td>3.893</td>
</tr>
<tr>
<td>Network density</td>
<td>0.031</td>
<td>0.016</td>
<td>0.014</td>
<td>0.012</td>
<td>0.008</td>
<td>0.005</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>Number of components</td>
<td>19</td>
<td>39</td>
<td>44</td>
<td>60</td>
<td>97</td>
<td>145</td>
<td>173</td>
<td>218</td>
<td>215</td>
</tr>
<tr>
<td>Component size distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Largest component</td>
<td>15</td>
<td>19</td>
<td>23</td>
<td>13</td>
<td>35</td>
<td>84</td>
<td>59</td>
<td>114</td>
<td>115</td>
</tr>
<tr>
<td>2nd largest component</td>
<td>5</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>11</td>
<td>22</td>
<td>28</td>
<td>59</td>
<td>72</td>
</tr>
<tr>
<td>3rd largest component</td>
<td>5</td>
<td>9</td>
<td>8</td>
<td>10</td>
<td>11</td>
<td>15</td>
<td>27</td>
<td>33</td>
<td>39</td>
</tr>
<tr>
<td>Average component size</td>
<td>3.42</td>
<td>3.44</td>
<td>3.91</td>
<td>3.53</td>
<td>3.70</td>
<td>4.36</td>
<td>4.74</td>
<td>5.49</td>
<td>5.64</td>
</tr>
<tr>
<td>Network diameter</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Average path-length</td>
<td>2.107</td>
<td>2.193</td>
<td>1.966</td>
<td>1.425</td>
<td>2.197</td>
<td>2.402</td>
<td>2.524</td>
<td>3.775</td>
<td>2.916</td>
</tr>
<tr>
<td>Overall clustering coefficient</td>
<td>1.071</td>
<td>1.316</td>
<td>1.166</td>
<td>1.417</td>
<td>1.3</td>
<td>1.169</td>
<td>1.449</td>
<td>1.163</td>
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Figure 1 illustrates the degree distribution of the German laser research network for the entire observation period (1961-2005). The graph on the left hand side provides degree distribution based on the total number of actors for the entire observation period. On the right hand side, we show the normalized numbers plotted on a log-log scale. In the case of random networks, the values on the logarithmic scale should represent a curved line (towards the upper right), while the straight line (from the upper left to the lower right) that we find indicates a fat-tailed degree distribution. Hence, the German laser research inventor network exhibits a typical scale-free degree distribution. In other words, there is a small number of actors with an extremely high number of ties (up to 40), while the majority of the actors have degrees far below ten.
Finally, we explore the core-periphery structure of the German laser inventor network by using k-core measures. A k-core measure includes all nodes (inventors) that are adjacent to at least a minimum number, k, of other nodes in the component (Wasserman and Faust 1994). In a first step, we calculate k-core measures for all inventors over time. The repeated calculation of k-core measures (for $k = 1, \ldots, n$) enables us to plot coreness layers for different k-core intensities over time, typically referred to as k-core strata. The coreness strata allows us to check for the existence and emergence of a core-periphery structure. For the sake of simplicity, we group all network actors into four categories based on their k-core values ($c_4: 12 > k \geq 10$, $c_3: 9 > k \geq 7$, $c_2: 6 > k \geq 4$ and $c_1: 3 > k \geq 1$) and plot the k-core strata over time. The dashed line (top of Figure 2) provides the total number of actors with a k-core value between one and three. These actors can be considered to be located at the network periphery. In contrast, the solid thin line depicts the number of actors with extremely high k-core measures in absolute terms. These actors can be regarded as

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14 For an overview of approaches for identifying core-periphery patterns, see Csermely et al. (2013).
constituting the very core of the network. In addition, we calculated an average k-core level, plotted in Figure 2 (top) as a solid fat line.

Figure 2: K-core strata and core-periphery structure, 1961-2005

A high spread between the low-level k-core category (i.e. dashed line) and the higher-level categories (i.e. solid thin line, dashed-dotted line, and dotted line) indicates the existence of a core-periphery structure in the German laser inventor network for the entire observation period. The bottom of Figure 2 reports the relative changes compared to the average k-core level. The k-core average is represented by 0 on the y-axis of the graph. Below average k-core values are represented by the black bar while above average k-core values are represented by the other bars in the chart. The exploration indicates a quite stable and persisting core-periphery structure over the entire observation period.

In sum, basic network metrics for the inventor networks in German laser technology indicate an increasing tendency towards a division of labor. A closer look at the connectedness and cohesiveness of the
network exhibits a remarkable degree of structural stability in terms of size, density and component size distribution. The network’s overall topology is characterized by a scale-free degree distribution. The exploration of k-core strata reveals a structurally stable and persistent pattern formation process. These results may be regarded as a stable segregation trend reflecting a core-periphery structure over the entire observation period. Overall, the reported patterns are largely in line with findings reported for other real-world networks (Powell et al. 2005; Kudic, 2015; Tomasello et al. 2017).

4.2 The various facets of micro-level fluidity

In accordance with previous studies (Fritsch and Zoellner 2018, 2019; Phelps 2010; Raamlogan and Consoli 2014), we find rather high levels of fluidity of actors and ties over time. The upper right part of Table 2 report the shares of identical inventors in the networks across the different periods of analysis. The values are the shares of identical actors (in percentage terms) for two compared time periods. For instance, the comparison between the first time period (1961-1965) and the second period (1966-1970) shows that only 8.54% of inventors remain in the network. The maximum share of identical actors in two subsequent time periods is 13.02%. This share strongly converges towards zero as the time distance between the compared sub-periods increases. This rather high fluctuation of network inventors over time indicates a low structural stability at the node level.

The numbers below the diagonal in the lower left of Table 2 shows the reoccurrence of ties between pairs of inventors across different time periods. We find that only 5.85% of all ties between inventors in the period 1966-1970 had already been established in the preceding observation period (1961-1965). For more distant periods this share also strongly converges towards zero.
The very high levels of fluidity of inventors and their ties clearly demonstrate that the German laser research network exhibits a very high level of instability at the micro-level. After only two observation periods, nearly the entire population of inventors is replaced by new actors. The fluidity of ties is even more pronounced; almost no tie between inventors lasts for more than two periods. This raises the question: How is it possible that there is so much structural stability at the macro level, when the micro level exhibits extremely fluid or unstable characteristics?

5. The co-existence of macro stability and micro fluidity

In order to shed some light on the co-existence of structural stability at the macro level and micro-level fluidity we conduct a key player analysis based on Borgatti (2003, 2006). The primary aim of a network-based key player analysis is to identify a set of inventors who either warrant the diffusion properties or stabilize the structural configuration of a given network. Hence, key player metrics go way beyond typical centrality measures, such as degree centrality, betweenness centrality, or eigenvector centrality, which are typically applied to address an actor’s position within a network.

The first identification criterion $\text{KPP}_{\text{NEG}}$ allows us to detect so-called (type-1) key players whom, if removed from the network, would cause the

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<td>0.51</td>
<td>2.33</td>
<td>6.06</td>
<td>-</td>
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most significant fragmentation of the network (Borgatti 2006). In other words, the measure allows us to identify those inventors who keep the inventor network together. The second criterion KPP_{POS} identifies those key players (type-2) who are most relevant for the diffusion knowledge in a given network structure (ibid). Hence, it enables us to determine the most significant inventors in the passing of information and knowledge through the system. The two measures are highly correlated, but they capture two qualitatively different facets of networks stability. While KPP_{NEG} measures address the physically observable link structure, KPP_{POS} measures relate directly to the functionality of the system.

Table 3 provides results of the functionally-oriented key player analysis (KPP_{POS} metrics) for each of the nine observation windows. We run the analyses for each of the nine observation periods and employ the results to find out whether the same or different actors are responsible for the structural stability at the overall network level. Table 3 reports key players metrics in the period before (t-1) and after (t+1) a given observation period (t=0).

Our findings show that only a very small share of actors occupy a stabilizing function in the network for a given observation period. This is not so surprising against the backdrop of our macro-level analyses. More interesting, we find that the key player metrics for the same inventors significantly vary over time. The changing set of top key players and the highly volatile key player metrics clearly indicate that most inventors occupy this role for only a very short duration. Thus, contrary to our initial expectations, individual inventors are not responsible for maintaining the structural stability or diffusion properties of the network over time. This observation underscores our initial finding that networks are characterized

\[15\text{ KPP}_{NEG} \text{ identifies (with very view exceptions) the same set of key players for all observation windows. This implies that the same individuals occupy key player roles according to both, the ‘diffusion’ and the ‘fragmentation’ criterion. Table A2 in the Appendix provides detailed results for the structurally-oriented key player analysis (KPP}_{NEG} \text{ metrics).}\]

\[16\text{ We used specific key player software (Borgatti 2003) to identify type-1 and type-2 key players for the main component of the inventor networks in each of the 9 sub-periods.}\]
by a high level of micro-level fluidity. At the same time, this raises the
question of what keeps the network together and ensures its functionality.

Table 3: Key player analysis, actor-specific, diffusion-based.

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<th><em>66-70</em></th>
<th><em>71-75</em></th>
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<th><em>01-05</em></th>
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<td>12.547</td>
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Based on our considerations outlined in Section 2, we have good
reasons to assume that the intermediate network level may provide some
deeper insights into the co-existence of the characteristic macro stability
and micro fluidity of innovative networks. In particular, we are curious to
see whether key player positions are passed on from a prominent inventor
in a given time period to members of his or her direct cooperation
environment, proxied by inventor-specific ego networks.¹⁷

To test this presumption, we proceed as follows. First, we explore the
component size distribution for each of the nine observation windows

¹⁷ An ego-network is defined as an actor’s (i.e., the ‘ego’) direct cooperation environment.
This environment includes the ego, all directly connected actors (so-called ‘alters’), and
all indirect ties between the alters (Ahuja 2000; Hite and Herterly 2001).
separately and sort the components by size. Second, we identify the
largest components (containing around 30%\textsuperscript{18} of all network actors) for
each observation window and calculate key player metrics for each
component separately, based on both the diffusion and the fragmentation
criterion.\textsuperscript{19} Third, we define two categories of key players for each of the
largest components over all time windows based on the KPP\textsubscript{POS} criterion.
Category I refers to the top 12.5% key players, and Category II includes
the top 25% of all identified key players. In other words, if removed from
the network, any of these top key players (ordered by his or her impact
based on the diffusion or fragmentation key player criterion used) would
hamper diffusion and cause major damage to the network structure.

Fourth, in order to identify the direct network surrounding of the top
key players, we apply an ego network approach where we treat the top
key players as focal actors and identify all of their direct ties to any other
actor in the network (alters) and all indirect ties among alters. By definition,
second tier ties are excluded from this group concept. Finally, we
construct random ego network benchmarks for the full set of the top 12.5%
(25%) key players identified and reported above.\textsuperscript{20} These random ego
network benchmarks are comparable in terms of size and structure to their
real-world ego network counterparts. The main difference between the two
networks is that alters in the benchmark networks are selected randomly.
Hence, Step 4 and 5 allow us to assign empirically observable real-world
ego networks and a randomly generated benchmark ego network to each
key player.

\textsuperscript{18} Since the size distribution is characterized by discrete size categories and varies for
each observation window. The 30% value is an approximate threshold criterion. For
instance, in the first time window (61-65), the three largest components contain 36.9 % of
all actors. In the last time window (01-05), the largest components account for 29.6 % of
all actors.

\textsuperscript{19} Since components can be interpreted as autarkic elements of an overall network, we
run the key player analysis for the entire network and identify the most dominant key
player for each component separately.

\textsuperscript{20} The random selection procedure was repeated 30 times to control for fluctuations
caused by outliers.
Figure 3: Real-world ego networks compared to random benchmarks, for top 12.5% (l.h.s.) and top 25% key players (r.h.s.)
Figure 3 shows a comparison of real-world ego networks with random benchmarks for the top key players in the German laser research network. The rationale behind our analytical design is straightforward. If top key player positions are passed on from a prominent inventor to partners located in his direct cooperation environment, the top key players of a successive period should be found (according to our considerations outlined above) at a higher rate in the ego network of the focal actor than elsewhere. The explorations on the left hand side show comparisons between the real-world and randomly generated benchmark ego networks for the top 12.5% key players. The dotted blacklines show the rate at which subsequent key player positions are filled with actors from the key players’ direct ego networks for three selected reference periods, while the grey lines represent the random ego network benchmarks. On the right hand side, we see the results of the same analysis for the top 25% key players. The results clearly show that there is a much higher probability for the successor of a top key player to be a member of the previous key player’s ego network as compared to alters from randomly generated benchmark ego networks.

6. Discussion and conclusions

We analyzed the development of the inventor network in German laser research from the inception of the technology in 1960 until 2005, a period of 45 years. From a macro perspective, the development of this network appears to be a continuous process where each step builds on the previous one. The basic properties of the network and their development are according to our expectations. In line with previous empirical findings (Wuchty et al. 2007), the propensity for co-inventorship, as well as the average size of inventor teams increased considerably over time. Finally, we identified a persistent tendency towards scaling, and observed the

21 The full set of results (for all observation periods) is provided in Figure A4 of the Appendix. Robustness checks reveal similar results and are available upon request.
emergence of a core-periphery structure. Overall, the results indicate a high level of structural stability at the macro level.

Investigating the development of the network at the micro level of actors and their ties, we found high levels of fluidity. Only a rather small share of inventors in a certain five-year period reoccurs in the subsequent period (between 8.5 and 13%). After only two periods, nearly the entire population of inventors has changed completely. According to these high levels of inventor fluidity, there is an equally pronounced degree of instability at the level of ties. Repeated and long-term ties are an exception. In sum, these findings clearly indicate rather high levels of turbulence under the surface of slowly changing macro structures.

Accordingly, our results challenge a number of theories that imply longevity and persistence of partnerships. For instance, transactions-cost arguments suggest that firms typically spend considerable time and resources to identify a suitable cooperation partner and to build up trust (Das and Teng 2000) in order to counteract opportunistic behavior and reduce the risk of terminating unsuccessful partnerships (Doz 1996). In other words, a high level of micro-level fluidity caused by the frequent termination of relationships implies considerable sunk costs for the partners involved. Similarly, principle-agent theory (Spence 1976) suggests that network actors have strong incentives to remain in the networks since it allows them to continuously improve their strategic positioning and reduce information asymmetries by sending out signals to potential partners. For instance, a high and continuously increasing number of partner (reflected by an actor’s degree) signifies a high willingness and ability of cooperation that may generate valuable opportunities for future cooperation. In a similar vein, network theorists frequently refer to the rich-get-richer argument, suggesting that actors with a high degree in a given period attract ties at a higher rate than other actors in subsequent time periods (Barabasi and Bonabeau 2003).

To explore the relationship between macro-level stability and micro-level fluidity a bit more deeply, and to gain a better understanding of who
or what keeps the network together, we conducted a key player analysis.
Our analysis reveals high levels of fluidity in even key player positions in a
network (based on both the diffusion and the fragmentation criterion),
because these positions are only rarely occupied by the same actors in
successive periods. Our explorations show, however, a pronounced
tendency for key player positions to be passed on to actors who belong to
the ego network of the previous key player. Hence, there is a tendency for
continuity at the team level that may complement our understanding of the
co-existence of macro stability and micro fluidity.

The high levels of fluidity at the micro level raise some fundamental
questions: Why do actors choose to establish a cooperative R&D
relationship? How do they select their cooperation partners? Why is an
established relationship maintained or abandoned? Our findings indicate
that there must be forces at work that are more important than the sunk
costs that occur if a relationship is abandoned. Our results could be
regarded an indication that trust may not only be relevant at the
interpersonal or interorganizational level, but also at the group level. In
such a case, individual investments in trust building and knowledge
generation may benefit a group as a whole and are, therefore, not
completely lost if a relationship between two actors is terminated. In other
words, sub-groups within inventor networks seem to play an important role
as intertemporal repositories where knowledge may persist despite entry
and exit of single members.

Any policy measure should account for the significant role of the
group level that we find. Consequently, it is important to design policy
measures that do not exclude any members of already existing groups.
For example, this may be relevant for schemes designed for a specific
region if members of the group being supported are not co-located but in
different regions.

Currently, we know very little about the dynamics of innovation
networks. Particularly, the levels of discontinuing actors and of new actors
in a network are largely unexplored. More research in different technological fields and countries is desirable to assess the levels and patterns of network dynamics, particularly the fluidity of actors and ties in different environments. In addition, the reasons for abandoning a cooperative relationship are not very clear. To the best of our knowledge, there is still no sound empirical evidence about the drivers and structural consequences of unintended tie terminations resulting from unsuccessful partnerships. Similarly, knowledge and learning-related drivers of tie terminations confront us with a number of highly interesting question. Does a new cooperation partner become more attractive primarily after the knowledge of the old partners is completely absorbed? Does the knowledge of the partner of a discontinued relationship become uninteresting or obsolete due to the general dynamics of the innovation process? Do network actors follow a long-term cooperation strategy and, if so, are these decision sequences mirrored in actor-specific network trajectories or do relevant patterns only become visible at higher aggregation levels? Do these association patterns differ between established network members and newcomers?

Finally, we know nearly nothing about the role of actor fluidity on the performance of the respective innovation system. On the one hand, one might argue that a high level of fluidity indicates an effective allocation of talent and a fast diffusion of knowledge. On the other hand, fluidity of actors and ties may involve high levels of sunk costs and loss of knowledge of discontinuing actors. Hence, it is unclear if a high level of fluidity has a positive or negative effect on system performance. To what

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22 The only comparable study of actor fluidity that we are aware of is the analysis of Fritsch and Zoellner (2018, 2019) for nine German regions over a period of 15 years. The study is based on patent data and identifies quite similar levels of actor fluidity.

23 The few available studies that consider the discontinuation of cooperative ties (e.g., Greve et al. 2009; Park and Russo 1996; Thune and Gulbrandsen 2014) name completion of the R&D project and project failure as main reasons for abandoning a cooperative relationship.

24 Belderbos et al. (2015) investigate the relationship between the dynamics of R&D cooperation and innovation performance based on a panel of Spanish firms. They conclude from their analysis that it is more the persistent collaboration that has a positive effect on firm innovativeness, while the effect of discontinued cooperation was
extent is the knowledge of discontinuing actors lost for the respective innovation system? How do new actors impact the performance of the system? Answers to such questions could considerably contribute to our understanding of collective innovation and the division of innovative labor.

insignificant. Fritsch and Zoellner (2019) measure performance based on the number of patents per R&D employee (patent productivity), and find a positive relationship between the share of new actors and ties and the performance of the respective innovation system. While there is a positive relationship between the share of discontinued inventors and patent productivity, the relationship between the share of discontinued ties and patent productivity is positive.
References


[https://doi.org/10.1038/30918](https://doi.org/10.1038/30918)

[https://doi.org/10.1126/science.1136099](https://doi.org/10.1126/science.1136099)
Appendix

Figure A1: Patent applications of laser technology in different applications: West Germany 1961-2005

Table A1: Descriptive statistics – number of single patents and co-patents over time

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Figure A2: Average path length (real-world inventor network vs. random benchmark)

Figure A3: Clustering coefficient (real-world inventor network vs. random benchmark)
### Table A2: Key player analysis, actor-specific, fragmentation-based

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Figure A4: Real-world ego networks vs. random benchmarks, for top 12.5\% (l.r.s.) and top 25\% key players (r.h.s), full observation period