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Working Paper

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IFRO Working Paper, No. 2013/10

Provided in Cooperation with:

Department of Food and Resource Economics (IFRO), University of Copenhagen

Suggested Citation: Rasmussen, Svend (2013) : A model for the optimal risk management of (farm) firms, IFRO Working Paper, No. 2013/10, University of Copenhagen, Department of Food and Resource Economics (IFRO), Copenhagen

This Version is available at:

<https://hdl.handle.net/10419/204357>

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Abstract

Current methods of risk management focus on efficiency and do not provide operational answers to the basic question of how to optimise and balance the two objectives, maximisation of expected income and minimisation of risk. This paper uses the Capital Asset Pricing Model (CAPM) to derive an operational criterion for the optimal risk management of firms. The criterion assumes that the objective of the firm manager is to maximise the market value of the firm and is based on the condition that the application of risk management tools has a symmetric effect on the variability of income around the mean. The criterion is based on the expected consequences of risk management on relative changes in the variance of return on equity and expected income. The paper demonstrates how the criterion may be used to evaluate and compare the effect of different risk management tools, and it illustrates how the criterion should be applied to integrate risk management at the strategic, tactical and operational level. The paper concludes that the derived criterion for optimal risk management provides a valuable theoretical tool for the economic evaluation of the consequences of risk management.

Key words: firm value, CAPM, optimal risk management, return on equity, risk, expected income

1. Introduction

Risk management is an integrated part of business or firm management which deals with the problem of how to minimise the risk of economic losses when the objective is to maximise expected profit. Historically, the methods used for risk management have to a large extent been based on the

Expected Utility model (von Neumann and Morgenstern, 1947) and the Modern Portfolio Theory based on the work of Markowitz (1952). Since World War II, along with the development of computer technology, Monte Carlo simulation has become an important tool for (numerical) risk analysis. Another important contribution to the development of tools for risk management is the concept of stochastic dominance, which is based on the work of Hadar and Russell (1969) and Hanoch and Levy, and Option Theory which is based on the work of Black and Scholes (1973). Additional methods such as Value-at-Risk (Jorion, 1997) have evolved from Modern Portfolio Theory, Option Theory, and the concept of stochastic dominance, but according to Hubbard (2009), none of these methods represent a significant improvement on the original earlier methods.

Despite the comprehensive work, none of the methods which have been developed so far actually provide an operational answer to the basic question of how the individual manager should behave (carry out risk management) so as to *optimise* the balance between expected profit/income and risk. Current methods for risk management are typically based on numerical analysis and the concept of *efficiency*, and operational criteria for *optimal* risk management do not exist.

In this paper we use the theory of financial economics to derive and demonstrate the use of a new criterion for the *optimal* risk management of firms. The criterion is derived using the Capital Asset Pricing Model (CAPM) and is based on the premise that the objective of risk management is to maximise the value of the firm. With this criterion in place, the paper demonstrates the use of the criterion for risk management at the strategic, the tactical and the operational level and it illustrates the interaction between the applications of risk management tools at the three management levels, with risk management in agriculture used as the case.

Before the model is presented in Section 3, a short review of the concepts and problems related to risk management is presented in Section 2. The application of the model is illustrated in Section 4, which also derives general rules of thumb for risk management. The application of the model is further demonstrated in Section 5, which focuses on model application within the context of strategic, tactical and operational risk management. The challenges faced when applying the model are further discussed in Section 6, and a summary and conclusion is presented in Section 7.

2. Background

Risk management is an important part of business management. The last five years of development with the global financial crisis has increased the focus on corporate risk management. Furthermore, in agriculture, highly volatile product prices and a tendency to reorientate agricultural policy so that farmers to a higher degree bear the consequences of uncertain prices and production conditions have also increased interest in risk management.

The literature reveals that the objective of risk management is difficult to describe in operational terms. Efficiency seems to be the key term, and the objective of risk management is often - implicitly or explicitly - stated as a way to manage the business in an *efficient way*, i.e. to minimise risk for a given level of the other objectives (profit/income) or to maximise the achievement of other objectives (profit/income) for a given level of risk. Hubbard (2009) provides a good review of the theoretical and methodological development of risk management since World War II. He also provides a formal definition of risk management as: ‘The identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimise, monitor, and control the probability and/or impact of unfortunate events’ (ibid, p. 10). Chapman (2011) provides an extensive introduction to the state-of-the-art regarding applied business risk management based on the ISO 31000 standards published by the International Organization for Standardization (ISO, 2009). According to the ISO 31000 standards, risk management “...aids decision making by taking account of uncertainty and its effect on achieving objectives and assessing the need for any actions” (p. V), and risk management “... should thus help avoid ineffective and inefficient responses to risk that can unnecessarily prevent legitimate activities and/or distort resource allocation” (p. VI).

In agricultural economics (farm management), risk management has historically been based on the Expected Utility (EU) model and various ad hoc procedures based on stochastic simulation, the concept of stochastic dominance and Value-at-Risk.¹ However, none of the methods used so far actually take the normative approach in the sense that they describe how to actually balance the cost of implementing each risk management option against the benefits derived from it. For instance, threshold levels of probabilities of loss used in the Value-at-Risk method do not provide an

¹ Hardaker (2006) provides a good review of the approaches to risk management in agriculture and the challenges ahead.

objective measurement which is commensurate with the cost of implementing risk management tools. In general, the cost of managing risks needs to be commensurate with the benefits obtained.

Although the maximisation of expected utility (EU) has become the standard paradigm for risk analysis, the basic problem faced by the researcher, namely that the decision maker's utility function is normally unknown, still exists. Under certain assumptions it is possible to quantify expected utility U as: $U = E(w) - 0.5 \cdot R_A \cdot V(w)$, where w is wealth, $V(w)$ is variance of wealth, R_A is absolute risk aversion coefficient $R_A = -v''(w)/v'(w)$, v is the Von Neumann-Morgenstern utility function, and $E(w)$ is the expected value of wealth (see Freund, 1956 and Hardaker et al., 2004). An alternative commonly used version of the utility function is $U = 1 - \exp(-R_A \cdot w)$ (Hardaker et al., 2004). These functional forms of the utility function are the closest the theory of risk management has come to an operational model on which to objectively base risk management decisions (OECD, 2009, p. 47 and Nartea and Webster, 2009). However, even though these models seem operational, they are not, because the risk aversion coefficient R_A is normally unknown. One way to circumvent this has been to apply alternative values of R_A (Bielza, Garrido and Sumpsi, 2007) using values of $R_A \cdot w$ from 0.5 to 4.0 as proposed by Anderson and Dillon (1992).

In its report on international standards for risk management (ISO, 2009), the International Organization for Standardization mentions as the first of its suggested principles for risk management, that risk management *creates value*. This statement generated our idea, that instead of focussing on the maximisation of expected utility, we could change the approach and instead maximise the value of the firm. By changing the focus to maximising the value of the firm, the theory of financial economics can be used as the theoretical framework, and the Capital Asset Pricing Model (CAPM) can be used as the operational tool on which to base the analysis of (optimal) risk management. This is the background for the new approach to risk management presented in the following.

3. The model

The model is based on the classical assumption that the goal of firm management is to maximise profit measured as the market value of the firm's equity capital. For a shareholder company this means that the goal is to maximise the market value of the company's shares. For the individually owned firms this means that the goal is to maximise the net value of the firm (i.e. market value of

the firm minus the market value of debt). This is in accordance with Chapman (2011, p. 10) who defines (enterprise) risk management as “a comprehensive and integrated framework for managing company-wide risk in order to maximise a company’s value”.

The relationship between the *expected return* on equity, the *risk on return* on equity and the *market value* of equity can be analysed within the framework of the Capital Asset Pricing Model (CAPM) (Sharpe (1964), Lintner (1965), Mossin (1966)). According to the CAPM, a firm/company that still wants to attract equity to finance its business must yield an expected return $E(R)$ on equity of at least $R_F + \beta(E(R_M) - R_F)$, where E is the expected value operator, R_F is the risk-free rate of interest, R_M is the interest on the market portfolio (i.e. the rate of return on a well-diversified portfolio of assets in which the potential investor would alternatively invest her/his money) and β is a risk factor defined as:

$$\beta = \frac{C(R, R_M)}{V(R_M)} \quad (1)$$

where $C(R, R_M)$ is the covariance between R and R_M , and $V(R_M)$ is the variance of R_M .

This means that in equilibrium the firm must yield an expected return on equity of:

$$E(R) = R_F + \beta(E(R_M) - R_F) \quad (2)$$

Equation (2) may be used to determine the market value of the equity of the firm. If the firm has an expected earnings before interest of $E(a)$, a total debt of B on which it pays a fixed interest R_B , then the *expected earnings to equity* (in the following called *expected profit*) is $E(c) = E(a) - R_B B$. To provide an expected return on equity of $E(R)$, the market value (S) of equity has to be:²

$$S = \frac{E(c)}{E(R)} \quad (3)$$

In the CAPM, the relationship between returns to equity (R) and the *risk* of returns to equity, quantified as the *variance* of returns to equity ($V(R)$), is determined through the risk factor β . The

² We implicitly assume that the exact amount of earnings to equity (c) is always fully paid out as dividend to the owner(s) of the firm each year. In the following we will refer to the firm/company owner as “shareholder(s)” or simply as “owner(s)” indiscriminately.

first step therefore is to consider the relationship between the variance of returns to equity $V(R)$ and the covariance $C(R, R_M)$. To do this, the following result will prove useful:

Proposition 1: If increasing (decreasing) variance is interpreted as being a proportional increase (decrease) in the deviations from the mean, then:

$$\frac{dC(R, R_M)}{dV(R)} = \frac{C(R, R_M)}{2V(R)} \quad (4)$$

which means that the change in the covariance between R and R_M , when the variance of R changes, is equal to the covariance between R and R_M divided by two times the variance of R .

Proof of Proposition 1: First multiply all deviations $(R_i - E(R))$ ($i = 1 \dots T$) by a constant λ after which the variance of R can be expressed as:

$$V(R) = V(\lambda R) \Big|_{\lambda=1} = \lambda^2 V(R) \Big|_{\lambda=1} \quad (5)$$

In a similar way, the covariance between R and R_M can be expressed as:

$$C(R, R_M) = C(\lambda R, R_M) \Big|_{\lambda=1} = \lambda C(R, R_M) \Big|_{\lambda=1} \quad (6)$$

The derivative of $C(R, R_M)$ with respect to λ (at the point $\lambda=1$) is therefore equal to $C(R, R_M)$, and the derivative of $V(R)$ with respect to λ is $2V(R)$. Dividing the derivative of $C(R, R_M)$ with respect to λ by the derivative of $V(R)$ with respect to λ produces the result in Equation (4).

Using Proposition 1, it is possible to show that the change of the risk factor β in Equation (1) caused by a change of $V(R)$ is:

$$d\beta = \frac{\partial \frac{C(R, R_M)}{V(R_M)}}{\partial V(R)} dV(R) = \frac{1}{V(R_M)} \frac{\partial C(R, R_M)}{\partial V(R)} dV(R) = \frac{C(R, R_M)}{V(R_M) 2V(R)} dV(R) \quad (7)$$

and the corresponding change in the required expected returns to equity is therefore (see Equation (2)):

$$dE(R) = d\beta(E(R_M) - R_F) = \frac{C(R, R_M)}{V(R_M)2V(R)} dV(R)(E(R_M) - R_F) \quad (8)$$

The change of the variance of the return on equity ($dV(R)$) on the right-hand side of Equation (8)) thus changes the demand for the return on equity ($dE(R)$) on the left hand side of Equation (8)), which - according to Equation (3) - implies a change in the market value of equity (S). At the same time, the change of the variance of return on equity (accomplished through risk management) implies change in costs, so that the expected profit $E(c)$ changes. This will also - according to Equation (3) - imply changes in the market value of equity (S).

The optimal effort of risk management is the effort that maximises the market value of equity. This is the case when the two contributions ($dV(R)$ and $dE(c)$) balance each other, i.e. have the same (but opposite) influence on the market value of equity, so that the change in the value of equity for marginal changes in risk management is zero. The formal derivation is the following:

The market value of equity (S) is (see Equation (3)) a function of $E(c)$ and $E(R)$. The total differential of this function is:

$$dS = \frac{\partial S}{\partial E(c)} dE(c) + \frac{\partial S}{\partial E(R)} dE(R) \quad (9)$$

The optimal effort of risk management is accomplished by maximising the equity S . The necessary condition for this is that dS in Equation (9) is zero. Setting the right-hand side of Equation (9) equal to zero and solving for $dE(c)$ yields:

$$dE(c) = - \frac{\frac{\partial S}{\partial E(R)}}{\frac{\partial S}{\partial E(c)}} dE(R) \quad (10)$$

If the differentiation of S (see Equation (3)) is performed, the equation becomes:

$$dE(c) = \frac{E(c)}{E(R)} dE(R) \quad (11)$$

Now insert the expressions of $E(R)$ (from Equation (2)) and $dE(R)$ (from Equation (8)) in Equation (11), then the condition in Equation (11) may be expressed as:

$$\frac{dE(c)}{dV(R)} = \left(\frac{E(c)}{R_F + \frac{C(R, R_M)}{V(R_M)} (E(R_M) - R_F)} \right) \frac{C(R, R_M)}{2V(R)V(R_M)} (E(R_M) - R_F) \quad (12)$$

which is *the necessary condition for optimal risk management*.

The condition in Equation (12) may also be written in elasticity form as:

$$\frac{dE(c)}{dV(R)} \frac{V(R)}{E(c)} = \frac{1}{2} \left(\frac{\beta(E(R_M) - R_F)}{R_F + \beta(E(R_M) - R_F)} \right) \quad (13)$$

where the left hand side term $(dE(c)/dV(R)) * (V(R)/E(c))$ is denoted *variance elasticity* in the following.

The optimal level of risk management is therefore determined by the condition that the variance elasticity (the relative change of the expected profit $E(c)$) divided by the relative change in the variance of the return on equity $V(R)$ is equal to the value of the term on the right-hand side of Equation (13).

It is reasonable to assume that the feasible combinations of $E(c)$ and $V(R)$ are a convex set as illustrated by the set P in Figure 1. As the market value of equity increases in $E(c)$ and decreases in $V(R)$, the optimal solution will be at the efficient frontier of P . If the present position of the firm is for instance the efficient point A and this point is *not* optimal because the left-hand side of Equation (13) is different from the right-hand side of Equation (13), then the relationship between expected earnings and return on equity should be adjusted through risk management. If the left-hand side of Equation (13) is *larger than* the right-hand side, then the firm should *accept greater risk* by increasing expected earnings (by relaxing risk management). If the left-hand side of Equation (13) is *less than* the right-hand side, then the firm should reduce risk and *accept less expected earnings*.

Further risk management is carried out as long as the left-hand side in Equation (13) is different from the right-hand side. Assume that the value on the right and left-hand side of Equation (13) become equal at the point O , then the optimal combination of expected earnings and variance is $E^*(c)$ and $V^*(R)$.

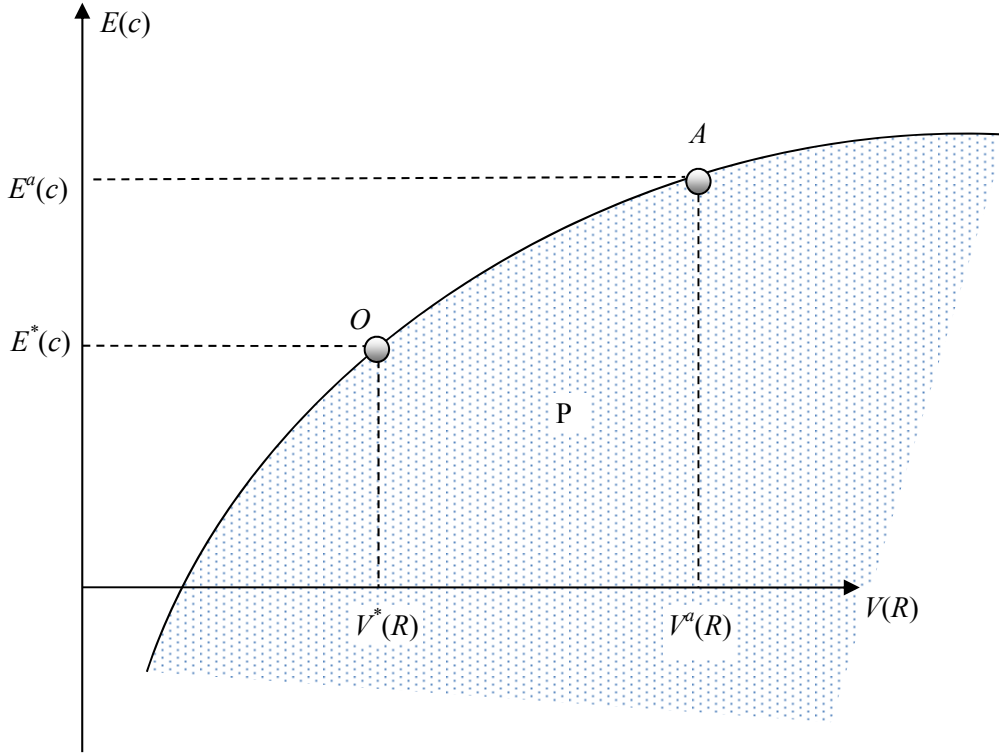


Figure 1. Risk management from A to O

4. Illustration of model application

Equation (13) only provides the necessary conditions for optimal risk management. The further condition is that risk management is carried out efficiently, i.e. that risk is reduced with a minimum reduction in expected income, and that increases in expected income are achieved with a minimum increase in risk.

The following example illustrates the application of the model: Assume that $E(R_M)$ is equal to 0.15, $V(R_M)$ is equal to 0.04, $V(R)$ equals 0.02 and $C(R, R_M)$ is equal to 0.008. According to Equation (2), β is then equal to 0.20. Furthermore, assume that the risk-free interest rate R_F is equal to 0.03 and

the expected annual profit $E(c)$ is equal to \$ 1 million. In this case, the right-hand side of Equation (13) is equal to 0.222. This means that implementing a risk management tool which reduces the variance of return on equity by, e.g. 0.005 ($dV(R) = -0.005$) will be advantageous if $dE(c)$ is greater than $0.222 * (-0.005/0.02) * 1,000,000 = \$ -55,556$, i.e. if the annual cost of implementing this risk management tool is less than \$ 55,556 per year.

The condition expressed by Equation (13) shows that if β is zero (i.e. the company's return on equity varies independently of the return on the market portfolio), then the right-hand side of Equation (13) is also zero. In this case, it is not worthwhile spending money on reducing the risk. Potential buyers of company shares are completely indifferent to the variation in returns, because the variation can be totally eliminated by diversification within the market portfolio. If instead β is 1 (i.e. return on equity varies exactly in line with the interest on the market portfolio), then the situation is different. Assuming in the example above that $C(R, R_M)$ is equal to 0.04, and all other parameters remain unchanged. In this case β is equal to 1. The right-hand side of Equation (13) then becomes equal to 0.4, and a risk management approach which aims at reducing the variance of return on its equity by, e.g. 0.005 ($dV(R) = -0.005$) would be advantageous if $dE(c)$ were greater than $0.4 * (-0.005/0.02) * 1,000,000 = \$ -100,000$, i.e. if the annual cost of implementing this risk management tool were less than \$ 100,000 per year.

The model may also be used the other way around. Assume, for example, a change in production, which implies an increase in the expected annual profit after interest of \$ 150,000 per year ($dE(c) = \$ 150,000$). How great an increase in risk (variance of return on equity) can be permitted? Using again the first example with a value of the right-hand side of Equation (13) equal to 0.222, then the project would be attractive if the increase in the variance of return on equity ($dV(R)$) were less than $(150.000/1.000.000)/(0.222/0.02) = 0.0135$.

Notice that for a risk-free rate of interest larger than or equal to zero ($R_F \geq 0$), the right-hand side of the condition in Equation (13) is less than or equal to $\frac{1}{2}$. This means that the optimal solution is characterised by the relative change in the expected profit $E(c)$ being less than half the corresponding relative change in the variance of return on equity $V(R)$. From this we derive the following rules of thumb for risk management:

Rule of thumb no. 1: Reduce risk when the relative change in the variance of return on equity $V(R)$ is at least twice the corresponding relative change in the expected profit $E(c)$.

Rule of thumb no. 2: Accept greater risk if the corresponding relative increase in expected profit, $E(c)$, is at least half the corresponding relative increase in the variance of return on equity, $V(R)$.

5. Application to risk management

The model is general in the sense that it applies to all firms where the goal is to maximise the market value of equity and where equity is part of a well-diversified portfolio of assets. In the following we demonstrate the application of the model to a simple example of risk management in agriculture.³

Hardaker et al. (2004), EU (2005), Hardaker (2006), OECD (2009) and Schaffnit-Chatterjee (2010) present the mainstream views on risk and contemporary methods for risk management in agriculture. The major sources of risk in agriculture are *production risks*, which are due to unpredictable weather, diseases and other unpredictable production conditions, *price and market risks* associated with the variability in prices and access to markets, *financial risks* resulting from changes in financing conditions and interest rates, *institutional risks*, which embody changes in government policy and regulation and *technological risks*, which are risk factors related to the introduction of new technology. Farm managers may use four main strategies to manage risk. The first, *retaining the risk*, is a strategy of self-protection where farm managers cope with the consequences of bad outcome by taking ex-post actions based on the use of financial reserves, such as savings, or of credit, which allows them to smooth consumption in the face of varying income. *Avoiding the risk* is a strategy where risk is evaded by not taking (too) risky actions or by eliminating their negative effects (choice of products and technologies). *Reducing the risk* can be achieved through flexibility and through diversification by engaging in various uncorrelated risky activities. Finally, *transferring the risk*, is based on classical risk management tools like insurance,

³ In agriculture, the condition that equity is part of a well-diversified portfolio of assets does not typically apply, because ordinary agricultural firms are typically family farms. However, company ownership is becoming more common in agriculture, and to farm managers, who want to attract equity from external capital investors, the risk management approach described here is certainly relevant.

but also future contracts, weather derivatives and financial derivatives like options and futures. Each of these strategies is based on a number of *risk management tools*.

Using the derived model requires – according to Equation (13) - that one calculates the company's expected profit, $E(c)$, the variance on its return on equity ($V(R)$) and the covariance between the firm's return on equity and the return on the market portfolio ($C(R, R_M)$), as well as the external parameter values, i.e. the expected return on the market portfolio, $E(R_M)$, the variance of return on the market portfolio, $V(R_M)$, and the risk-free rate of interest, R_F .

The choice of the market portfolio is not a trivial task. In principle, the market portfolio includes shares of all risky assets in the market. In practice one often chooses a general financial index such as the FTSE 100, the Dow Jones Euro Stoxx 50 or Standard & Poor's 500 (Hillier et al., 2010), or other financial indices.⁴

When estimating $C(R, R_M)$, one should be aware that the degree of correlation depends on the company's financial leverage, since the variation on the company's return on equity increases with increasing financial leverage. Instead of estimating $C(R, R_M)$ and $V(R_M)$, it is possible to simplify the calculations by estimating β directly by using linear regression based on Equation (2) (Hillier et al., 2010).

The choice and intensity of risk management tool(s) determine the value of $dE(c)$ and $dV(R)$ on the left-hand side of Equation (13). $dE(c)$ is the change in the firm's expected profit by changing the use of the risk management tool in question, and $dV(R)$ is the corresponding change in the variance of the firm's return on equity. Calculating $dE(c)$ and $dV(R)$ for risk management tools that focus on income smoothing (tools that focus on avoiding risk, reducing risk or transferring risk), does not involve any methodological problems as long as we keep the assumption that the dividend paid out to the owner(s) is exactly equal to the profit, c . However, calculating $dV(R)$ for risk management tools that focus on retaining risk by using financial reserves to smooth the dividend paid out to the owner(s) calls for a change of calculation. The reason is that the dividend paid out may no longer be equal to the profit c . When the company faces low profit (low c), implementation of the risk management tool, use of financial reserves, may involve using financial reserves to add to the profit so that dividend paid out to shareholders is higher than c . When the company faces high profit (high c), the risk management tool may involve the decision to add to the financial reserves by paying out

⁴ This subject will not be dealt with here See, e.g. Hillier et al. (2010))

less than c as dividend to the shareholders.⁵ The consequence is that the variance in the return on equity (measured as the amount of money actually paid out to the shareholders) decreases compared to the variance in the return on equity based on the pure profit measure, c . Thus the costs of increasing financial reserves and the resulting lower than expected profit ($dE(c)$) is compensated for by a lower variance in return on equity ($dV(R)$).⁶

Before considering risk management in general, it is important to consider the time horizon. In this context, the risk management process may be divided into three levels: 1) The *strategic level*, where the optimal long-term relation between expected profit ($E(c)$) and the variance in return on equity ($V(R)$) is determined. 2) The *tactical level*, with a planning horizon of one year and where the objective is to plan and implement risk management during the coming year so that the firm moves in the direction of the optimal long-term state determined at the strategic level, and 3) the *operational level* which refers to hands-on decision making concerning how to cover potential economic losses compared to the budget generated at the tactical level.⁷

5.1 Strategic Risk Management

The first step involves the determination of where the company - in the long term - should position itself in terms of risk profile. At this general level, all risk management tools are available, including the building of financial reserves, changes in production, diversification, etc. The procedure is as follows:

- 1) Draw up a budget for the company based on the objective of maximising the expected profit, $E^o(c)$. Estimate also the corresponding variance in return on equity, $V^o(R)$.
- 2) Calculate the value of the right-hand side of Equation (13).
- 3) Calculate, for each relevant risk management tool, the variance elasticity (the value on the left-hand side of Equation (13)).

⁵ We assume that the financial reserves are unchanged in the long run.

⁶ In the following we assume that the estimation of the variance in return on equity is always based on a return on equity measure where profit is replaced by the actual pay out of dividend to the owner(s).

⁷ The decisions at the operational level may also include decisions on how to allocate income over and above the income according to the one year budget generated at the tactical level.

4) The risk management tools that provide lower variance elasticity than the value on the right-hand side of Equation (13) should be implemented in the budget. The risk management tools are implemented in order of efficiency meaning that the tools with the lowest variance elasticity are implemented first, and Equation (13) is re-calculated after adding each risk management tool. Implementing risk management tools in the budget continues as long as the variance elasticity is increasing and is lower than the value of the right-hand side of Equation (13).

5) After implementation of risk management according to item 4), if any risk management tools have a higher variance elasticity than the value on the right-hand side of Equation (13), then the risk management tools in question should be relaxed until the point, where all risk management tools have a variance elasticity which corresponds to the value on the right-hand side of Equation (13).

6) The optimal application of risk management tools and the optimal long term risk profile of the company have now been determined. The optimal long term combination of expected profit ($E^*(c)$) and variance in return on equity ($V^*(R)$) is recorded as the optimal long-term risk profile.

By comparing the actual ($(E^a(c), V^a(R))$) and the optimal ($(E^*(c), V^*(R))$) risk profile of the firm, it is now possible to identify how far the company is from the optimal (long run) risk profile. This information is used in the second step of risk management described next.

5.2 Tactical Risk Management

The purpose of the second step is to determine the risk management tools to be launched in the coming year with the purpose of steering the company in the direction of the (long term) optimal risk profile identified during the strategic level. Notice that the range of risk management tools available at this (tactical) level is different from the range of tools available at the strategic level, because only the short term tools (works within a year) can be used. The procedure is as follows:

1) Draw up a budget for the company based on the current state of the firm (current

production system, contracts, etc.). Record the corresponding expected profit, $E^a(c)$, and the variance in return on equity, $V^a(R)$.

2) Compare the current risk profile ($E^a(c)$, $V^a(R)$) and the optimal risk profile ($E^*(c)$, $V^*(R)$) in an E - V diagram similar to the one shown in Figure 1. If the risk profile ($E^a(c)$, $V^a(R)$) is located farther north-east than ($E^*(c)$, $V^*(R)$), then the focus should be on reducing the variance in return on equity (see following item 3)). If the risk profile ($E^a(c)$, $V^a(R)$) is located farther south-west than ($E^*(c)$, $V^*(R)$), then the focus should be on increasing expected returns on equity by relaxing (part of) the current risk management (see following item 4)).

3) *Reduction of variance*: Whether there are any available risk management tools which fulfil the condition that $dE(c)/dV(R)$ is less than or equal to $(E^*(c) - E^a(c))/(V^*(R) - V^a(R))$ should be examined. If this is the case, the tool with the lowest value of $dE(c)/dV(R)$ should be applied first. Then new values of $E^a(c)$ and $V^a(R)$ are calculated. The procedure is repeated iteratively until the condition $dE(c)/dV(R) \leq (E^*(c) - E^a(c))/(V^*(R) - V^a(R))$ is no longer met.

4) *Increase in expected profit*: Whether the current management of the company involves the use of risk management tools where $dE(c)/dV(R)$ is greater than or equal to $(E^*(c) - E^a(c))/(V^*(R) - V^a(R))$ should be examined. If this is the case, the current risk management should be relaxed by relaxing the use of the tool with the highest value of $dE(c)/dV(R)$ first. Then new values of $V^a(R)$ and $E^a(c)$ are should be calculated. The procedure is repeated iteratively until the condition $dE(c)/dV(R) \geq (E^*(c) - E^a(c))/(V^*(R) - V^a(R))$ is no longer met.

The plan for the optimal use of risk management tools in the next year has now been determined. The final level of risk management is to prepare for the condition that the actual in and outgoing payments deviate from the budget.

5.3 Operational Risk Management

The purpose of this last step of the risk management process is to consider how possible losses

(compared to the budget established at the tactical level) should be covered. One can distinguish between two types of losses:

- 1) Losses in the form of lower net payments than expected (liquidity).
- 2) Losses in the form of larger changes in the value of assets and liabilities than originally expected (value).

Only the losses in the form of liquidity have to be dealt with at this level. Changes in the value of assets and liabilities do not require immediate action, but the changes will materialise when the tactical risk management is carried out at the beginning of the following year.

There are three options when facing liquidity losses:

- Deposits of equity
- Proceeds from loans
- Realisation of assets

The choice depends on which option best supports the strategy identified during the strategic and tactical level. This implies the following rules:

- If the decision at the strategic and tactical level was to reduce the risk (variance), then loss of liquidity should be covered by the realisation (sale) of assets, and by first selling the assets that have the lowest value of $dE(c)/dV(R)$.
- If the decision at the strategic and tactical level was to increase income (by relaxing risk management), then loss of liquidity should be covered by raising/increasing loans.
- If the decision at the strategic and tactical level was to continue the current risk profile, then loss of liquidity should be covered by deposit of equity.

5.4 Numerical Illustration

To demonstrate, consider a farm which produces grain and piglets. The budget based on the present production system shows an expected profit ($E(c)$) of \$ 108,680, and based on historical data, it has been estimated that the variance in the return on equity ($V(R)$) is 0.0043. The farm has no irrigation system and piglets are sold at spot prices.

Yields and prices are stochastic variables. The expected values and variances are shown in Table 1 together with other parameter values.⁸

Table 1. Farm budget data for numerical example

	E(value)	Variance
Land, hectares	130	
Sows, number	500	
Piglets, number per sow and year	25	4
Grain yield, kg per hectare	8,500	765,625
Price of grain, \$ per kg	0.256	0.002
Price per piglet, \$	76	100
Feed price, \$ per kg	0.30	0.002
	Value	
Market portfolio interest, R_M	0.09	
Risk free rate of interest, R_F	0.03	
Beta, β	0.20	

Assume that there are two possible tools for risk management: 1. The establishment of an irrigation system for the irrigation of grain and 2. Price contracts for the sale of piglets. Investment in and use of an irrigation system involves an additional annual cost (capital costs and energy) of \$ 6,800, an expected increase in grain yield of 200 kg per ha and a reduction in the variance in grain yields from 765,625 to 15,625. Price contracts on piglets imply a fixed price of \$ 74 per piglet, which means that the price variance decreases to zero.

First consider risk management at the *strategic level* at which all (here both) risk management tools are available. Using the data mentioned above, the irrigation system has a variance elasticity of 0.0918 and the price contract has a variance elasticity of 0.3553. Since the irrigation system has the

⁸ Notice that only some of the normal budget variables are included

lowest variance elasticity and since it is lower than 0.1429, which is the value on the right-hand side of Equation (13), irrigation should be implemented in the budget first. Expected profit thereby decreases from \$ 108,680 to \$ 108,536, and the variance in return on equity decreases from 0.0043 to 0.0042.

Then we calculate if it *also* pays to sign price contracts on piglets. Based on the budget *after* the establishment of the irrigation system, it is calculated that the variance elasticity for price contracts is 0.3507, which is higher than 0.1429 (the right-hand side of Equation (13)). Therefore, price contracts on piglets do not pay (do not improve the value of the firm); a reduction of the variance would imply too great a decline in expected profit.

This completes risk management at the strategic level. The result is that with the available risk management tools it is optimal for the company in question to have a risk profile with an expected profit of \$ 108,536 and a variance in return on equity of 0.0042.

We then turn to the *tactical level*. At the tactical level (one year time horizon) only the price per contract for piglets is available as a risk management tool. The establishment of an irrigation system belongs to the strategic level, requiring a longer time horizon than one year.

The initial situation is as before (see Table 1). But the long-term optimal system is - as just shown - a farm which also has an irrigation system. The difference between the actual condition and the optimal condition is therefore $(E^*(c)-E^a(c))/(V^*(R)-V^a(R))=(108,536-108,680)/(0.0042-0.0043) = 1,440,000$. This measure should be compared with the effect of the price contract (which is the only risk management tool available at the tactical level), which is $dE(c)/dV(R) = (83,536-108,536)/(0.0015-0.0042) = 9,259,259$.⁹ Since $dE(c)/dV(R)$ for price contracts is greater than $(E^*(c)-E^a(c))/(V^*(R)-V^a(R))$, it is not advantageous to take out price contracts for piglets.

Finally we turn to the *operational level*. As shown in the previous calculations, the variance of the current economic system (0.0043) is higher than the long-term optimal variance (0.0042). Therefore, losses at the operational level should be covered by the realisation of assets.

⁹ 83,536 is the expected profit when both irrigation and contract on piglets are implemented and 0.0015 is the variance of return on equity when both irrigation and contract on piglets are implemented.

6. Discussion

The model is valuable in the sense that it provides a relatively *simple, objective* and *operational method* for evaluating risk management effort. Its weakness is that it is based on the assumption that all relevant information is available in the trade-off between expected income and the variance in return on equity. However, this simplification is also present in the CAPM model which has been an extremely valuable tool in empirical work and practice (Brounen, de Jong and Joedijk, 2006).

Another key assumption is that risk management has a proportional and symmetric impact on the stochastic return on equity, i.e. it does not affect the covariance between the return to equity and the return on the market portfolio. The significance of this assumption is hardly greater than the assumption of normally distributed profits, behind the often routine use of the classical EU model in empirical work.

Only the necessary conditions have been derived. The further condition is that risk management is carried out efficiently, i.e. that risk is reduced with a minimum reduction in expected income, and that an increase in expected income is achieved with a minimum increase in risk. However, this may be insufficient. The condition in Equation 13 may, in practice, have no solution or possibly as many as two depending on the available risk management tools. In the case of two solutions, the sufficient condition is that the left-hand side of Equation 13 is decreasing in $V(R)$.

Concerning the model application, the critical point is the choice of market portfolio and the estimation of β . The market portfolio is ideally a portfolio consisting of all securities in the market where the proportion invested in each security corresponds to its relative market value. It is a theoretical concept, and in practice one often chooses a general (local) financial index. For large stockholder companies with shares traded on the open market, β may be readily available from the current financial statistics. However, companies with shares not traded on the open market, and farm firms owned by the individual farmer, the β -values are certainly not available. In this case, the estimation of β has to be carried out using for instance linear regression. Although this may involve empirical problems, they are far less significant than the empirical problems involved when facing the alternative, which would be the classical approach, i.e. the estimation of the von Neumann-Morgenstern utility function.

The estimation of the economic consequences of the various risk management tools involves all the well-known problems related to the budgeting of profit. Notice in this context that the risk management tool “use of financial reserves” is quantified in the form of planning when to apply the financial reserves (determine the profit limits beyond which to apply reserves) and by how much. This determines the change in variance of owner income, while the foregone interest (by keeping financial reserves) determines the cost. An alternative way of estimating the economic consequences of applying this tool is to estimate how much changes in the financial reserves would change the probability of the company going bankrupt, where bankrupt means the total loss of equity.

7. Conclusion

Risk management is an important part of business management. The historical approach to risk management has typically been to cope with risk, i.e. to avoid risk or to reduce the consequences of risk. However, fighting risk typically implies costs, and none of the methods for risk management developed so far actually provide an operational answer to the question of how to balance the two objectives; the maximisation of expected income and the minimisation of risk. In the cases where normative approaches have been applied, they have been based on subjective assessments of the mathematical form of the utility function and the degree of risk aversion.

The approach presented in this paper is based on the classical assumption that the objective of firm owners is to maximise the value of the firm. As the value of the firm can be directly linked to the expected profit and the variance (risk) through the Capital Asset Pricing Model, it has been possible to derive operational criteria for the optimal risk management of firms. The derived model is attractive in the sense that it provides a relatively *simple, objective and operational method* for evaluating risk management effort. Its weakness is that it is based on the same assumptions as the CAPM itself, and the condition that risk management results in symmetric changes in the deviation of income around the expected mean. However, the alternative is that decisions concerning risk management are based on subjective assessment of the balance between expected profit and risk without a real theoretical foundation.

The derived criterion for optimal risk management makes it possible to actually calculate whether a specific risk reducing effort is worth pursuing, or whether a specific risk management effort should

be relaxed to save costs and thereby increase expected profit. The criterion also provides the opportunity to compare the value of alternative risk management tools. In this context, it is stressed that timing is important, because some risk management tools (e.g. a change of production system) are only applicable in the long run, while others (e.g. trading futures) may be applied in the short run. The paper demonstrates how the model may be used to determine the optimal long run risk profile of the firm, and how to relate risk management at the tactical and operational level to the optimal risk profile determined at the strategic level.

The overall conclusion is that the criterion for optimal risk management derived in this paper provides a valuable tool for evaluating the economic consequences of implementing risk management tools both in the short and in the long run.

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