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Leonor Modesto

May 2004

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## **ABSTRACT**

### **Unions, Firing Costs and Unemployment**

In this paper we conduct an analysis of the effects of firing costs in models that consider simultaneously worker heterogeneity, imperfect information on their productivity and union power. We consider an OLG model where heterogeneous workers participate in the labour market both when young and old. Each generation of workers is represented by its own union. Unions set wages unilaterally taking into account firm behavior. Firms are atomistic and choose employment treating wages parametrically. There is imperfect information about worker productivity.

We find that at given wages firing costs increase youth unemployment and decrease old age unemployment. However, once we take the wage response into account, we find that firing costs increase both youth and old age unemployment. This happens because unions react strategically, and respond to higher firing costs. Indeed, when firing costs increase, firms refrain from hiring youths since, if a young worker turns out to be inadequate, it will be more costly to fire him. The union, knowing this, reduces the wage of young workers in order to attempt to increase their employment prospects. However, despite this cut youth unemployment still increases with firing costs. In the second period, on the contrary, higher firing costs give the union more power. In fact, knowing that firms will be less likely to cut their labour force when firing costs are high, the union increases the wage of old workers, and, therefore, old age unemployment increases.

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# 1 Introduction

The purpose of this paper is to conduct an analysis of the effects of firing costs in models that consider simultaneously worker heterogeneity, imperfect information on their productivity, and union power.

The labour market consequences of firing costs have been analyzed in several works. See for example Bentolila and Bertola (1990), Bertola, (1990), Hopenhayn and Rogerson (1993) and Modesto and Thomas (2001). However, in all these works labour is an homogenous factor. More recently Montgomery (1999), Canziani and Petrongolo (2001), and Kugler and Saint-Paul (2003) have introduced worker heterogeneity, and analyzed the effects of turnover costs on employment, when there is imperfect information about worker productivity. In these papers, as in Gibbons and Katz (1991), the case of private information by current employers vis-a-vis the market is considered, i.e., it is assumed that current employers are better informed about the worker's ability than prospective employers. Therefore, if firms have discretion over whom to lay off, when a worker is fired the market infers that he or she is of low ability, so that past employment histories become important determinants of future hiring decisions.

Montgomery (1999) and Canziani and Petrongolo (2001) analyze respectively the effects of hiring or firing costs on the level and composition of unemployment. Montgomery (1999) solves his adverse selection, search model numerically, and finds that, when hiring costs are low the system converges to a steady state in which firms hire every period and most of the unemployed are low-ability workers. On the contrary, when hiring costs are sufficiently large employment cycles emerge, and firms alternate between hiring and not hiring. Canziani and Petrongolo (2001) use an overlapping generations model with young and old heterogenous workers. They find that firing costs reduce the option value of hiring an young worker and therefore increase youth unemployment. Moreover they also find that firing costs reduce the re-employment prospects of dismissed workers because prospective employers infer that they are very low-productivity individuals. Kugler and Saint-Paul (2003), using a matching model, explore the consequences of adverse selection on the relative job finding probability of unemployed and employed job seekers. They find that hiring and firing costs reduce the hiring of both unemployed and employed job seekers. However, the hiring of the former is more affected by turnover costs. This can explain why the ratio of employment-to-unemployment flows to unemployment-to-employment flows

tends to be higher in high-job-security economies.

Nevertheless in these models wages are exogenous or fixed at the competitive level. However, if the labour market is not competitive, but rather is unionized, then, as in Modesto and Thomas (2001), one would expect unions to behave strategically and to react to firing costs. Hence it might be expected that employment levels and the relative incidence of youth and long-term unemployment will differ between competitive and unionized models, even in the presence of imperfect information. Therefore in this paper we study the effects of firing costs in unionized economies with heterogenous workers.

We consider a simple overlapping generations model where heterogenous workers participate in the labour market for two periods. We have therefore two generations of workers: the young and the old. Each generation of workers is represented by its own union. There is imperfect information about workers productivity, and current employers are better informed than prospective employers. Firms, when hiring (firing) a worker, form expectations about the worker's profitability, treating wages parametrically. Unions announce wages unilaterally, taking into account firms behavior. We show that firing costs affect hiring and firing decisions at given wages, and also through wages, since wage setting responds to firing costs. Indeed, when firing costs increase, firms refrain from hiring youths since, if a young worker turns out to be inadequate, it will be more costly to fire him. This is essentially an option value effect.<sup>1</sup> The union, knowing this, will reduce the wage of young workers in order to increase their employment prospects. However, the effect of this reduction is not sufficiently strong to change the final result, and we still find that firing costs increase youth unemployment when we take the wage effect into account. Also, when firing costs increase, the employment prospects of old workers that were not hired when young improve at given wages. This happens because the stigma of having not been hired decreases with firing costs.<sup>2</sup> However, when we take into account the response of wages to firing costs, the previous result is reversed, i.e., firing costs decrease the employment prospects of old workers that were unemployed as youths. Indeed, we find that when firing costs increase, the union also increases the wage of old workers, so that firms will hire less, and this last effects dominates. In fact, when firing costs are higher firms refrain from

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<sup>1</sup>See on this issue Canziani and Petrongolo (2001).

<sup>2</sup>See also Canziani and Petrongolo (2001).

firing. This gives the union more power so that it increases the wage of old workers. Therefore, we find that firing costs increase old age unemployment, although at given wages firing costs increase the probability of employment of old workers.

The rest of the paper is organized as follows. In section 2 we present the model and derive the optimal hiring, firing and wage-setting policies. In section 3 we analyze the response of unemployment to wages. The effects of firing costs on wages and on the level and composition of unemployment are presented and discussed in section 4. Finally in section 5 we conclude. Proofs and simulation results are presented in the Appendix.

## 2 The Model

The model used is based on Canziani and Petrongolo (2001), where we have introduced collective bargaining to capture the phenomena discussed in the introduction. It is a simple overlapping generations model with heterogenous workers. Individuals live and participate in the labour market for two periods. Therefore, in every period there are two generations alive and participating in the labour market: the young and the old. Both generations have equal mass that we normalize to one. Workers have different productivities. Let  $a_i$  be the productivity of worker  $i$  that is constant over time. We assume that  $a_i \sim N(\bar{a}, \sigma_a^2)$ . We further assume that the productivity draws are unobserved by both workers and firms.

There are a large number of identical, (atomistic), infinitely lived firms. The output of firm  $j$ ,  $y_j$ , is linearly additive in the productivities of its workers:  $y_j = \int_0^{n_j} a_i d_i$ , where  $n_j$  denotes the measure of firm  $j$ 's labour force. Firms are forward-looking and decide whether to hire or fire a worker. For simplicity we assume that the discount rate is zero. As individual productivities are not observed, firms make their decisions, taking wages as given, and after observing the workers' age, employment history and current productivity signal. Indeed every time a firm meets a worker<sup>3</sup> it receives an imperfect signal of the workers' productivity. Figure 1 shows the signalling structure of the model.<sup>4</sup> When a firm meets a young worker it receives a signal  $s_i = a_i + \varepsilon_i$ . We assume that  $\varepsilon_i$  is independent of  $a_i$  and that  $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ . This means

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<sup>3</sup>We assume that every period all workers meet a firm with probability 1.

<sup>4</sup>Note that the signalling structure is identical to the one considered in Canziani and Petrongolo (2001).

that  $s_i \sim N(\bar{a}, \sigma_s^2)$  where  $\sigma_s^2 = \sigma_a^2 + \sigma_e^2$ . If a firm hires a young worker it gets better information (although still imperfect) of his productivity. We model this by assuming that the firm receives, at the end of the period, a second signal,<sup>5</sup>  $s'_i$ . The firm then decides on the basis of all the available information to fire or to keep the worker. If the worker is fired the firm bears a firing cost<sup>6</sup>  $f \geq 0$ . If a worker is fired or has not been hired when young, he meets again a firm at the beginning of the second period, and a new productivity signal  $s''_i$  is observed.<sup>7</sup> The firm then decides to hire the worker or not, processing optimally all the relevant information.

We assume that each generation of individuals is represented by its own union. This means that each period we have two unions and two different wages, one for each generation,  $w_1$  for the young and  $w_2$  for the old. We also assume that unions set wages unilaterally and that firms choose employment at those wages. Hence, each firm ignores any effect of its own decisions on wages.

## 2.1 Employment

A firm hires a worker when the expected profitability of the worker is non negative. Similarly it fires a worker when its expected profitability is negative and larger than the firing cost. Each firm treats wages parametrically, ignoring any effects of its behavior on wages.

### 2.1.1 Young workers

The expected profitability of a young worker is equal to the sum of the expected profitability in the current and in the next period considering the

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<sup>5</sup>Note that all signals have exactly the same structure, i.e., they have the same distribution. Also the error terms of different signals are independent.

<sup>6</sup>As in Canziani and Petrongolo (2001), Kugler and Saint-Paul (2003) and Boeri and Jimeno (2003) we model firing costs as a tax that is paid to a third party, and that cannot be internalized in the employer-employee relationship (for example red-tape costs on employers), rather than considering transfers from employers to dismissed workers. In this way we abstract from the redistributive effects of severance payments.

<sup>7</sup>Note that the distributions of  $s'_i$  and  $s''_i$  exist and are defined over the entire population of workers, i.e., they are marginal and not conditional distributions. The fact that we only observe  $s'_i$  for young workers that were hired does not mean we can not define a marginal distribution for  $s'_i$  over the entire population of workers. Indeed, if those young workers that were not hired had been hired they could have produce a signal, generated from the marginal distribution of  $s'_i$ .



possibility that the worker may be fired. Therefore to solve the firm's hiring problem we must first solve the firing problem.

**Firing Decision** At the end of each period firms must decide whether to keep or to fire their young employees. A young worker will be fired if

$$w_2 - E[a_i \setminus s_i, s'_i] > f \quad (1)$$

where  $w_2$  is the wage that a worker receives when old and  $E[a_i \setminus s_i, s'_i]$  is the mean of the posterior distribution of  $a_i$  after the two signals. Note that before meeting a worker a firm expects his productivity to be  $\bar{a}$ . However, once the firm gets to know the worker better (i.e. after observing one or more signals) it revises its expectation about the workers' productivity. We have that (see Appendix A.2):

$$E[a_i \setminus s_i, s'_i] = \frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a} + \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \tilde{s}_i \quad (2)$$

where  $\tilde{s}_i = (s_i + s'_i)/2$ . This means that the mean of the posterior distribution of  $a_i$  is a weighted average of the mean of the prior distribution ( $\bar{a}$ ) and of the signals received. The weights are inversely proportional to the respective variances.<sup>8</sup>

Using (2) we can rewrite (1) as:

$$\tilde{s}_i < \bar{s}' \quad \text{where} \quad (3)$$

$$\bar{s}' = \frac{2\sigma_a^2 + \sigma_e^2}{2\sigma_a^2} (w_2 - f) - \frac{\sigma_e^2}{2\sigma_a^2} \bar{a}. \quad (4)$$

This means that there is a cutoff level for the average of the signals,  $\bar{s}'$ , such that a worker will be kept (fired) if the average of the two signals exceeds (is below) the cutoff. Note that the firing cutoff is increasing in  $w_2$  and decreasing in  $f$  and  $\bar{a}$ . Indeed, when average productivity decreases or the wage of an old worker increases, the option value of keeping an worker decreases and therefore firms will fire more. Also when firing costs are higher, firing is more costly and less productive workers are kept, i.e., the firing cutoff decreases.

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<sup>8</sup>Note that, to ease the task of the reader, we provide in Appendix A.1 the joint distribution of  $(a_i, s_i, \tilde{s}_i, s''_i)$ .

**Hiring Decision** A young worker will be hired if his expected profitability, given the current productivity signal, is non negative, i.e. if  $E[\pi_i \setminus s_i] \geq 0$ . We have seen that the expected profitability of a young worker is obtained as the sum of his expected profitability in the current and next periods, admitting that the worker may be fired, i.e.<sup>9</sup>

$$E[\pi_i \setminus s_i] = E[a_i \setminus s_i] - w_1 - \Pr[\text{fire} \setminus s_i]f + [1 - \Pr[\text{fire} \setminus s_i]] [E[a_i \setminus \tilde{s}_i \geq \bar{s}', s_i] - w_2]. \quad (5)$$

We have that (see Appendix A.2)

$$E[a_i \setminus s_i] = \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i \quad (6)$$

i.e. the mean of the posterior distribution of  $a_i$  after signal  $s_i$  is observed is a weighted average of the mean of the prior distribution ( $\bar{a}$ ) and of the signal received. The weights are inversely proportional to their variances. We also have that

$$\Pr[\text{fire} \setminus s_i] = \Pr[\tilde{s}_i < \bar{s}' \setminus s_i] = \Phi \left( \frac{\bar{s}' - \mu_{\tilde{s} \setminus s}}{\sigma_{\tilde{s} \setminus s}} \right) \quad (7)$$

where  $\Phi(\cdot)$  stands for the standard normal cumulative distribution, and where  $\mu_{\tilde{s} \setminus s}$  and  $\sigma_{\tilde{s} \setminus s}$  (see Appendix A.3) are respectively the mean and the standard deviation of the conditional distribution of  $\tilde{s}_i$  on  $s_i$ .

Moreover we also have that  $E[a_i \setminus \tilde{s}_i \geq \bar{s}', s_i]$ , the expected productivity of a worker that is not fired ( $\tilde{s}_i \geq \bar{s}'$ ) and gives signal  $s_i$  as a youth, is given by (see Appendix A.3):

$$E[a_i \setminus \tilde{s}_i \geq \bar{s}', s_i] = \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i + \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{\tilde{s} \setminus s} \frac{\phi \left( \frac{\bar{s}' - \mu_{\tilde{s} \setminus s}}{\sigma_{\tilde{s} \setminus s}} \right)}{1 - \Phi \left( \frac{\bar{s}' - \mu_{\tilde{s} \setminus s}}{\sigma_{\tilde{s} \setminus s}} \right)} \quad (8)$$

where  $\phi(\cdot)$  is the p.d.f. of the standard normal distribution. This means that we can rewrite (5) as:

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<sup>9</sup>Note that we assume that firms have perfect foresight in what concerns aggregate variables.

$$\begin{aligned}
E[\pi_i \setminus s_i] &= \left[ 2 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right] \left[ \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i \right] - w_1 \\
&\quad - f \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) + \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{\bar{s} \setminus s} \phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \\
&\quad - \left[ 1 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right] w_2.
\end{aligned} \tag{9}$$

Equation (9) is a continuous and increasing function of  $s_i$ <sup>10</sup>. Therefore we can define  $\bar{s}$  as the value of  $s_i$  such that  $E[\pi_i \setminus s_i] = 0$ , and only young workers with a signal  $s_i \geq \bar{s}$  will be hired. This means that in this model youth unemployment (YU) is simply given by:

$$YU = \Pr[s_i < \bar{s}] = \Phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right). \tag{10}$$

Note that as  $\bar{s}$  depends on  $w_1$  and  $w_2$  (see (9)) youth unemployment is affected by the union's choices. In section 3.1 we analyze the effects of wages on youth unemployment. Youth unemployment also depends on firing costs through  $\bar{s}$ . See (9). The effects of firing costs on youth unemployment are discussed in section 4.1.

### 2.1.2 Old Workers

**Hiring Decision** An old worker may be looking for a job for two reasons. Either he was fired or he did not get a job when young. In both cases firms, when hiring old workers will, as said before, take into account all the relevant information, i.e., wages, their past history and their current signal. Firms will hire those workers whose expected profitability is non-negative. In the case of an old worker this is simply his expected current profitability.

For an old worker that did not work when young ( $s_i < \bar{s}$ ) and gives now

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<sup>10</sup>Note that as  $\partial E[\pi_i \setminus s_i] / \partial \mu_{\bar{s} \setminus s} = 0$  we have that  $\partial E[\pi_i \setminus s_i] / \partial s_i = \left[ 2 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right] \frac{\sigma_a^2}{\sigma_s^2} > 0$ .

signal  $s_i''$  his expected current profitability is given by (see Appendix A.4):

$$\begin{aligned} E[\pi_i \setminus s_i < \bar{s}, s_i''] &= E[a_i \setminus s_i < \bar{s}, s_i''] - w_2 \\ &= \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i'' - \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{s \setminus s''} \frac{\phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}{\Phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)} - w_2 \end{aligned} \quad (11)$$

where  $\mu_{s \setminus s''}$  and  $\sigma_{s \setminus s''}$  (also given in Appendix A.4) are respectively the mean and the standard deviation of the conditional distribution of  $s_i$  on  $s_i''$ .

As equation (11) is continuous and increasing in  $s_i''$  (proof in Appendix A.7) we can define a cutoff value  $\bar{s}^{nh}$  such that, for  $s_i'' = \bar{s}^{nh}$ ,  $E[\pi_i \setminus s_i < \bar{s}, s_i''] = 0$ . Therefore only old workers that did not work when young with a signal  $s_i'' \geq \bar{s}^{nh}$  will be hired. Note also that from (11) we have that  $\bar{s}^{nh}$  is a function of  $w_1$ ,  $w_2$  and  $f$ .<sup>11</sup>

The expected current profitability of an old worker that worked when young ( $s_i \geq \bar{s}$ ), was fired ( $\tilde{s}_i < \bar{s}'$ ) and gives now signal  $s_i''$  is given by (see Appendix A.5):

$$\begin{aligned} E[\pi_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i''] &= E[a_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i''] - w_2 \\ &= \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i'' + \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{s \setminus s''} \frac{\phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)} \\ &\quad - \frac{2\sigma_a^2}{3\sigma_a^2 + \sigma_e^2} \sigma_{\tilde{s} \setminus \bar{s}, s''} \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}, s''}}{\sigma_{\tilde{s} \setminus \bar{s}, s''}}\right)}{\Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}, s''}}{\sigma_{\tilde{s} \setminus \bar{s}, s''}}\right)} - w_2 \end{aligned} \quad (12)$$

where  $\mu_{\tilde{s} \setminus \bar{s}, s''} = E[\tilde{s}_i \setminus s_i \geq \bar{s}, s_i'']$  and  $\sigma_{\tilde{s} \setminus \bar{s}, s''} = [Var[\tilde{s}_i \setminus s_i \geq \bar{s}, s_i'']]^{1/2}$  are also given in Appendix A.5.

As equation (12) is continuous and increasing<sup>12</sup> in  $s_i''$  we can define a cutoff value  $\bar{s}^f$  such that, for  $s_i'' = \bar{s}^f$ ,  $E[\pi_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i''] = 0$ . Therefore for those workers that did work when young and were fired, only the ones with a signal  $s_i'' \geq \bar{s}^f$  will be hired when old. Note that from (12) we have that  $\bar{s}^f$  is a function of  $w_1$ ,  $w_2$  and  $f$ .<sup>13</sup>

<sup>11</sup>Remark that  $\bar{s}^{nh}$  is a function of  $w_1$  and  $f$  through  $\bar{s}$ .

<sup>12</sup>This result was obtained by simulation.

<sup>13</sup>Note that  $\bar{s}^f$  depends on  $w_1$  through  $\bar{s}$  and on  $f$  through  $\bar{s}$  and  $\bar{s}'$ .

It is interesting to compare the two hiring cutoffs for old-age workers, i.e.,  $\bar{s}^{nh}$  and  $\bar{s}^f$ . We have that  $E[a_i \setminus s_i < \bar{s}, s_i''] > E[a_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i'']$  so that  $\bar{s}^f > \bar{s}^{nh}$ .<sup>14</sup> This means that, conditional on a given signal  $s_i''$ , firms are more willing to hire a worker that was not hired in the past than a worker that was fired, since the expected profitability of the former is higher than the profitability of the later.

**Old age unemployment** It is now easy to see that in this model old age unemployment (OU) is simply given by:

$$\begin{aligned} OU &= \Pr[\text{Not hired when young}] \Pr[\text{Not hired when old} \setminus \text{Not} \\ &\quad \text{hired when young}] \\ &\quad + \Pr[\text{Hired when young}] \Pr[\text{fired}] \Pr[\text{Not hired when old} \setminus \text{fired}] \end{aligned}$$

that we can write as:

$$\begin{aligned} OU &= \Pr[s_i < \bar{s}] \Pr[s_i'' < \bar{s}^{nh} \setminus s_i < \bar{s}] \\ &+ [1 - \Pr[s_i < \bar{s}]] \Pr[\tilde{s}_i < \bar{s}' \setminus s_i \geq \bar{s}] \Pr[s_i'' < \bar{s}^f \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] \end{aligned} \quad (13)$$

where (see Appendix A.6):

$$\Pr[s_i'' < \bar{s}^{nh} \setminus s_i < \bar{s}] = \Phi \left( \frac{\bar{s}^{nh} - \mu_{s'' \setminus \bar{s}}}{\sigma_{s'' \setminus \bar{s}}} \right) \quad (14)$$

$$\Pr[\tilde{s}_i < \bar{s}' \setminus s_i \geq \bar{s}] = \Phi \left( \frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}} \right) \quad (15)$$

$$\Pr[s_i'' < \bar{s}^f \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] = \Phi \left( \frac{\bar{s}^f - \mu_{s'' \setminus \bar{s}', \bar{s}}}{\sigma_{s'' \setminus \bar{s}', \bar{s}}} \right). \quad (16)$$

$\mu_{s'' \setminus \bar{s}} = E[s_i'' \setminus s_i < \bar{s}]$ ,  $\mu_{\tilde{s} \setminus \bar{s}} = E[\tilde{s}_i \setminus s_i \geq \bar{s}]$ ,  $\mu_{s'' \setminus \bar{s}', \bar{s}} = E[s_i'' \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}]$ ,  $\sigma_{s'' \setminus \bar{s}} = [Var[s_i'' \setminus s_i < \bar{s}]]^{1/2}$ ,  $\sigma_{\tilde{s} \setminus \bar{s}} = [Var[\tilde{s}_i \setminus s_i \geq \bar{s}]]^{1/2}$  and  $\sigma_{s'' \setminus \bar{s}', \bar{s}} = [Var[s_i'' \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}]]^{1/2}$  are also given in Appendix A.6.

We found that (see the simulations results in Appendix A.11)  $\Pr[s_i'' < \bar{s}^{nh} \setminus s_i < \bar{s}] < \Pr[s_i'' < \bar{s}^f \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}]$  i.e., the probability of unemployment for an old worker that was hired when young and was fired, is higher than the probability of unemployment for an old worker that did not work

<sup>14</sup>This result was obtained by simulation. See Appendix A.11.

while young. Indeed we have seen that  $\bar{s}^f > \bar{s}^{nh}$ . Moreover, both the mean and the variance of the distribution of the signals of old workers that did not work when young, are higher than the mean and the variance of the distribution of the signals of old age workers that were employed as youths and fired, i.e.  $\mu_{s''\setminus\bar{s}} > \mu_{s''\setminus\bar{s}',\bar{s}}$  and  $\sigma_{s''\setminus\bar{s}} > \sigma_{s''\setminus\bar{s}',\bar{s}}$ . See also Appendix A.11. Therefore from (14) and (16) it is easy to see that old-age workers that were unemployed when young have better employment prospects than those that worked while young, but got fired. This implies that, in this model, having been fired affects more adversely future employment prospects than having failed to obtain a job previously.<sup>15</sup>

From the previous expressions one can also easily see that old age unemployment is affected by  $w_1$ ,  $w_2$  and  $f$ . In section 3.2 we discuss the response of old age unemployment to wages. The effects of firing cost on old age unemployment are analyzed in section 4.2.

## 2.2 Wages

Each generation is represented by a union which is a wage-setter, allowing firms to choose their employment levels in response. We assume that unions wish to maximize the income of their members.<sup>16</sup> Thus, the union that represents the generation born at time  $t$  maximizes the following function:

$$\Omega_t = w_{1t}(1 - YU_t) + \bar{w}YU_t + w_{2t+1}(1 - OU_{t+1}) + \bar{w}OU_{t+1} \quad (17)$$

where  $YU_t$  and  $OU_{t+1}$  are given respectively by (10) and (13), and where we have assumed that the union also has a zero discount rate and that  $\bar{w}$ , the reservation wage, is constant. For simplicity, in what follows we normalize the reservation wage to zero.

We assume that a union at time  $t$  is unable to precommit to its future wage. Instead the union announces each period a wage, knowing that in the future it will reoptimise. A key feature of this framework is time-consistency.

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<sup>15</sup>Note that in our model the productivity of worker  $i$ ,  $a_i$ , is constant in time, i.e., it does not increase with experience. Therefore, being employed while young does not increase productivity, while having been fired is perceived by potential employers as a sign of very low productivity.

<sup>16</sup>Note that this assumption is frequently used in the literature, and is equivalent to the assumption that the union is a utilitarian one with risk neutral members. See Oswald (1982).

Therefore we have to solve the union's problem backwards, starting with the determination of  $w_2$ .

In the second period the union solves the following problem:

$$\max_{w_{2t+1}} w_{2t+1} [1 - OU(w_{1t}, w_{2t+1})] \quad (18)$$

with  $OU(w_{1t}, w_{2t+1})$  given by eq.(13). The FOC is:<sup>17</sup>

$$(1 - OU(w_1, w_2)) - w_2 \frac{\partial OU(w_1, w_2)}{\partial w_2} = 0 \quad (19)$$

that is solved by  $w_{2(w_1)}$ . Note that equation (19) tell us that in the second period the union wage-setting policy is such that it selects the wage that makes the wage elasticity of old age employment,  $\eta_{(1-OU),w_2}$ , equal to -1, where  $\eta_{(1-OU),w_2} = \frac{\partial[1-OU(w_1,w_2)]/[1-OU(w_1,w_2)]}{\partial w_2/w_2}$ .

In the first period the problem of the union is the following:

$$\max_{w_{1t}} w_{1t} [1 - YU(w_{1t}, w_{2(w_1)})] + w_{2(w_1)} [1 - OU(w_{1t}, w_{2(w_1)})] \quad (20)$$

with  $YU(w_1, w_2)$  and  $OU(w_1, w_2)$  given respectively by eq.(10) and eq.(13). The FOC is:<sup>18</sup>

$$(1 - YU(w_1, w_{2(w_1)})) - w_1 \left[ \frac{\partial YU(w_1, w_{2(w_1)})}{\partial w_1} + \frac{\partial YU(w_1, w_{2(w_1)})}{\partial w_2} \frac{dw_{2(w_1)}}{dw_1} \right] - w_{2(w_1)} \frac{\partial OU(w_1, w_{2(w_1)})}{\partial w_1} = 0 \quad (21)$$

where, from (19) we have that<sup>19</sup>

$$\frac{dw_{2(w_1)}}{dw_1} = - \left[ \frac{\frac{\partial OU(w_1, w_2)}{\partial w_1} + w_2 \frac{\partial^2 OU(w_1, w_2)}{\partial w_2 \partial w_1}}{2 \frac{\partial OU(w_1, w_2)}{\partial w_2} + w_2 \frac{\partial^2 OU(w_1, w_2)}{\partial w_2^2}} \right]. \quad (22)$$

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<sup>17</sup>We assume that the second order derivative of problem (18) is negative, i.e.  $2 \frac{\partial^2 OU(w_1, w_2)}{\partial w_2^2} + w_2 \frac{\partial^3 OU(w_1, w_2)}{\partial w_2^3} > 0$ . Note that for the sake of simplicity we have omitted the time subscripts.

<sup>18</sup>We assume that the second order derivative of problem (20) (see Appendix A.9) is negative. Again, for the sake of simplicity we have omitted the time subscripts.

<sup>19</sup>Note that the denominator of expression (22) is positive. See footnote (16).

Equations (19), (21) and (22) summarize the wage-setting policy of the union. We can see that the union behaves strategically, taking into account the current and future effects of wages on employment. Also, as employment reacts to firing costs, the union, knowing this, will exploit this fact, so that wages will also respond to firing costs. In section 4.1.2 we analyze the effects of firing costs on wages.

### 3 The response of unemployment to wages

We have seen that wages affect hiring and firing decisions and therefore employment. In this section we discuss in detail the effects of wages on unemployment. We start with the case of youth unemployment.

#### 3.1 Youth unemployment

From (10) we have that

$$\frac{\partial YU}{\partial w_j} = \frac{1}{\sigma_s} \phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right) \frac{\partial \bar{s}}{\partial w_j} \quad \text{with } j = 1, 2. \quad (23)$$

Now, as  $\bar{s}$  is the value of  $s_i$  such that  $E[\pi_i \setminus s_i] = 0$ , from (9) we have that

$$\frac{\partial \bar{s}}{\partial w_j} = - \frac{\partial E[\pi_i \setminus s_i] / \partial w_j}{\partial E[\pi_i \setminus s_i] / \partial s_i}$$

which implies that:<sup>20</sup>

$$\frac{\partial \bar{s}}{\partial w_1} = \frac{1}{\left[ 2 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right]} \frac{\sigma_s^2}{\sigma_a^2} > 0 \quad (24)$$

$$\frac{\partial \bar{s}}{\partial w_2} = \frac{\left[ 1 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right]}{\left[ 2 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right]} \frac{\sigma_s^2}{\sigma_a^2} \geq 0. \quad (25)$$

Therefore we have that  $\frac{\partial YU}{\partial w_1} > 0$ ,  $\frac{\partial YU}{\partial w_2} \geq 0$  and that  $\frac{\partial YU}{\partial w_1} \geq \frac{\partial YU}{\partial w_2}$ . As expected higher wages for the young make the firm hire less youths. Also, when second period wages increase, the firm also refrains from hiring young workers

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<sup>20</sup>Note that  $\partial E[\pi_i \setminus s_i] / \partial \bar{s}' = 0$ .



today, since it anticipates that their future value will decrease. However, also as expected, the effect of current wages on current employment is higher than the effect of future wages on current employment. Indeed when  $w_2$  increases a firm can react in two ways. It can either hire less and/or fire more youths. When the  $\Pr[\text{fire} \setminus s_i = \bar{s}] = \Phi\left(\frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}}\right)\Big|_{s_i = \bar{s}} = 0$  the firm will not fire and will only react using the hiring margin. In this case the effect of  $w_2$  on youth unemployment is maximum and identical to the effect of  $w_1$  on  $YU$ . On the contrary, when the  $\Pr[\text{fire} \setminus s_i = \bar{s}] = \Phi\left(\frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}}\right)\Big|_{s_i = \bar{s}} = 1$  the firm will only react by firing youths, and will not change its hiring policy. In this case  $\partial YU / \partial w_2 = 0$ . In the intermediate cases, i.e. when  $0 < \Pr[\text{fire} \setminus s_i = \bar{s}] < 1$ , the firm will use both margins and therefore  $\frac{\partial YU}{\partial w_1} > \frac{\partial YU}{\partial w_2}$ . Note that, since  $\Phi\left(\frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}}\right)$  increases with  $\bar{s}'$ , we have that when firing costs are high the firm will use less the firing margin. Therefore the response of youth unemployment to future wages will be higher when firing costs are high.

## 3.2 Old age unemployment

We now discuss the effects of wages on old age unemployment. We start by analyzing the effects of  $w_1$ . Note that, due to the analytical complexity of our model, we decided in some cases to perform numerical simulations. In Appendix A.11.1 the reader can find some of the simulations performed.

### 3.2.1 The effects of $w_1$ on old age unemployment

In our model there are two types of old age unemployed. See (13). Those workers who were never hired, and that from now on we will call long-term unemployed (LTU), and those workers that worked when young but got fired, and were not able to find a new job. We start by analyzing the case of long-term unemployment. We have that

$$LTU = YU \cdot \Pr[\text{Not hired when old} \setminus \text{Not hired when young}]. \quad (26)$$

We have seen that the probability that a worker is not hired when young (YU) increases with  $w_1$ . We also have that the probability of unemployment for an old worker that did not get a job when young decreases with  $w_1$ .<sup>21</sup>

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<sup>21</sup>The  $\Pr[\text{Not hired when old} \setminus \text{Not hired when young}]$ , is given by (14). We have that  $\frac{\partial \bar{s}^{nh}}{\partial w_1} < 0$ ,  $\frac{\partial \mu_{\bar{s}' \setminus \bar{s}}}{\partial w_1} > 0$  and  $\frac{\partial \sigma_{\bar{s}' \setminus \bar{s}}}{\partial w_1} > 0$ . See Appendixes A.8.1-A.8.3. Indeed, when the wage

Indeed, when the wages of youths are high the stigma of having not been hired when young decreases, and therefore, the probability of employment in old-age of these workers increases. Therefore the effect of  $w_1$  on long term unemployment is a priori ambiguous, since it augments youth unemployment, but increases the employment prospects in old age of those that were unemployed as youths. In the simulations we performed (see an example in Appendix A.11.1) we found that the first effect was always dominant, and therefore that an increase in  $w_1$  increased long term unemployment.

We now address the case of those workers that were fired when young and not hired when old, i.e., we want to analyze the effects of  $w_1$  on  $\Pr[Hired\ when\ young] \Pr[fired] \Pr[Not\ hired\ when\ old \setminus fired]$ . We saw that the probability of a youth being hired decreases with  $w_1$ . The probability of a worker being fired is given by eq. (15). Although an increase in  $w_1$  does not affect the firing cutoff,  $\bar{s}'$ , (see (4)) it does increase the hiring cutoff,  $\bar{s}$ , (see (24)) so that fewer and better young workers are hired.<sup>22</sup> As they are more productive firms will fire less. Therefore the probability of a worker being fired decreases with  $w_1$ . See the simulations results in Appendix A.11.1.

The probability that a young worker that was fired does not get a job when old,  $\Pr[Not\ hired\ when\ old \setminus fired]$ , is given by eq. (16). We found (by simulation) that this probability decreased, although only very slightly, with  $w_1$ .<sup>23</sup> The rationale for this is the following. As  $w_1$  increases only the more productive youngsters are hired, and the expected productivity of both hired and non-hired workers increases. Therefore, as we have seen, less workers will be fired. What happens to the profitability of the ones that were fired is a priori ambiguous, but, in the simulations performed, we found that their productivity also increased, although not much. Therefore they will be more easily hired by firms.

All this implies that the probability that an individual works when young

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of young workers increases fewer young workers are hired. This means that the expected profitability of non-hired workers increases (and therefore  $\bar{s}^{nh}$  decreases) and that the mean ( $\mu_{s'' \setminus \bar{s}}$ ) and the variance ( $\sigma_{s'' \setminus \bar{s}}^2$ ) of the distribution of the signals of old workers that were not hired when young also increase, implying that the  $\Pr[Not\ hired\ when\ old \setminus Not\ hired\ when\ young]$  also decreases.

<sup>22</sup>This in turn implies an increase in the mean ( $\mu_{\bar{s} \setminus \bar{s}}$ ) and a decrease in the variance ( $\sigma_{\bar{s} \setminus \bar{s}}^2$ ) of the distribution of the average of signals of the workers that were hired. See Appendix A.8.4. and A.8.5.

<sup>23</sup>Note that we found that  $\frac{\partial \mu_{s'' \setminus \bar{s}' \setminus \bar{s}}}{\partial w_1} > 0$ ,  $\frac{\partial \sigma_{s'' \setminus \bar{s}' \setminus \bar{s}}}{\partial w_1} < 0$  and that  $\frac{\partial \bar{s}^f}{\partial w_1} < 0$ . However, all these effects are very small. See the simulation results in Appendix A.11.1.

and doesn't work when old decreases with  $w_1$ . Since we have seen that LTU increased with  $w_1$  we have that the total effect of  $w_1$  on old age unemployment is a priori ambiguous. However, in the simulations we performed (see an example in Appendix A.11.1) we found that the effects on long-term unemployment dominated, so that old age unemployment increases with  $w_1$ .

### 3.2.2 The effects of $w_2$ on old age unemployment

We start again by analyzing the effects of  $w_2$  on LTU. We know that YU increases with  $w_2$ . In fact when  $w_2$  increases the firm refrains from hiring youths, which implies that the productivity of non-hired workers increases. However, it increases less than  $w_2$  and therefore the expected profitability of an old worker that was not hired when young decreases. See (11). This in turn implies that the probability that a worker that was not hired when young is again not hired when old increases with  $w_2$ . See (14) and Appendix A.11.1.<sup>24</sup> Since YU also increases we have that LTU unambiguously increases with  $w_2$ .

We also have that as  $w_2$  increases firms will tend to fire more. See (15) and Appendix A.11.1. Indeed from (4) we have that the firing cutoff,  $\bar{s}'$ , increases with  $w_2$ .<sup>25</sup> Also the probability of unemployment for an old worker that was fired,  $\Pr[Not\ hired\ when\ old \setminus fired]$ , increases with  $w_2$ . See (16) and Appendix A.11.1. In fact as  $w_2$  increases the productivity of hired youths increases, and even those that are fired are more productive. However, as  $w_2$  increases more than the expected productivity, we obtain a decrease in the expected profitability of an old worker that was fired, so that the  $\Pr[Not\ hired\ when\ old \setminus fired]$  increases with  $w_2$ .<sup>26</sup>

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<sup>24</sup>Note that when the expected profitability of an old worker that was not hired when young decreases we obtain an increase in  $\bar{s}^{nh}$ . See Appendix A.8.1. Also, as we have now more productive workers among the ones that were not hired when young, we obtain an increase in the mean ( $\mu_{s'' \setminus \bar{s}}$ ) and the variance ( $\sigma_{s'' \setminus \bar{s}}^2$ ) of the distribution of signals of old workers that were not hired when young. See Appendixes A.8.2 and A.8.3. However, the global effect (obtained by simulation) is such that the probability that a worker that was not hired when young is again not hired when old increases with  $w_2$ .

<sup>25</sup>Note however that, as with a higher  $w_2$  less and better youths are hired, we observe an increase in the mean ( $\mu_{\bar{s} \setminus \bar{s}}$ ) and a decrease in the variance ( $\sigma_{\bar{s} \setminus \bar{s}}^2$ ) of the distribution of the average of signals of workers that were hired. See Appendix A.8.4. and A.8.5. However, the total effect (obtained by simulation) of  $w_2$  on the probability of a worker being fired is positive.

<sup>26</sup>Note that  $\bar{s}^f$  increases and we also observe increases in the mean ( $\mu_{s'' \setminus \bar{s}' \setminus \bar{s}}$ ) and in the

We have seen that the probability of a worker being fired and the  $\Pr[\text{Not hired when old}\backslash\text{fired}]$  increase with  $w_2$ . But, as the probability of a worker being hired when young decreases with  $w_2$  the global effect on the  $\Pr[\text{Hired when young}] \Pr[\text{fired}] \Pr[\text{Not hired when old}\backslash\text{fired}]$  is a priori uncertain. However, in the simulations performed (see an example in Appendix A.11.1) we always got a positive effect of  $w_2$  on the probability of unemployment of old workers that were hired as youths and fired. As  $LTU$  also increases with  $w_2$  we can conclude that old age unemployment increases with  $w_2$ .

In short, an increase in  $w_2$  decreases the employment prospects of both dismissed workers, and workers that were not hired when young, implying also that firms will fire more. In fact, facing higher labour costs for senior workers, firms will want to reduce the amount of senior employment. They do so by simultaneously hiring less old-age individuals and firing more youths. Therefore, old age unemployment increases with  $w_2$ .

## 4 The effects of firing costs on unemployment and wages

In this section we analyze the effects of firing costs on wages and unemployment. We start with youth unemployment. Again, due to the complexity of the model, some results were obtained by simulation. In Appendix A.11.2 and A.11.3 the reader can find some of the simulations performed.

### 4.1 Youth unemployment

Youth unemployment is affected by firing costs in two different ways. Both directly and, since wages also react to firing costs, through wages. Therefore, since we have that  $YU(w_{1(f)}, w_{2(f)}, f)$  we can write:

$$\frac{dYU}{df} = \frac{\partial YU}{\partial f} + \frac{\partial YU}{\partial w_1} \frac{dw_1}{df} + \frac{\partial YU}{\partial w_2} \frac{dw_2}{df} \quad (27)$$

where the first term on the RHS of (27),  $\partial YU/\partial f$ , corresponds to the effect of firing costs on youth unemployment at given wages, and where the two

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variance ( $\sigma_{s'\backslash\bar{s}'}^2$ ) of the distribution of signals of old workers that were fired. (Remark that these results were obtained by simulation). However, the total effect (obtained by simulation) was that the  $\Pr[\text{Not hired when old}\backslash\text{fired}]$  increases with  $w_2$ .

other terms correspond to the effects of firing costs on youth unemployment through wages.

#### 4.1.1 The effects of firing costs on youth unemployment at given wages

Using (10) we have that

$$\begin{aligned} \frac{\partial YU}{\partial f} &= \frac{1}{\sigma_s} \phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right) \frac{\partial \bar{s}}{\partial f} \\ &= \frac{1}{\sigma_s} \phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right) \frac{\sigma_s^2}{\sigma_a^2} \frac{\Phi \left( \frac{\bar{s}' - \mu_{\bar{s}|s}}{\sigma_{\bar{s}|s}} \right)}{\left[ 2 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s}|s}}{\sigma_{\bar{s}|s}} \right) \right]} > 0. \end{aligned} \quad (28)$$

Therefore, in the absence of unions, or if unions did not react strategically to firing costs, an increase in firing costs would unambiguously imply an increase in youth unemployment. Indeed, at given wages, if firing becomes more costly, firms, anticipating higher costs in the future if a youth turns out to be less productive than expected and should be fired, will refrain from hiring youths today. This is essentially an option value effect<sup>27</sup>. However, when unions set wages strategically, the final outcome depends also on union behavior towards firing costs and on the response of firms to wages. In the previous section we have analyzed the response of unemployment to wages:  $\partial YU/\partial w_1 > 0$  and  $\partial YU/\partial w_2 \geq 0$  are given by (23)-(25). We must now analyze how wages react to firing costs.

#### 4.1.2 The effects of firing costs on wages

In Appendix A.9 the reader can find the expressions for  $dw_1/df$  and  $dw_2/df$ . Since these expressions are very complex we decided to obtain by simulation the effects of firing costs on wages. (See an example in Appendix A.11.3). In the simulations performed we found that  $w_1$  decreased with firing costs while  $w_2$  increased. The rationale for this is the following. We have seen that when firing costs increase, at given wages, firms will tend to employ less youths. Anticipating this behavior, the union will respond by decreasing  $w_1$  in order to boost youth employment. In the second period, on the contrary, higher firing costs give the union more power. Indeed, knowing that firms will be

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<sup>27</sup>Note that this effect was already obtained by Canziani and Petrongolo (2001).

less likely to cut their labour force when firing costs are high, the union will increase second period wages.

### 4.1.3 The total effect of firing costs on youth unemployment

The global effect of firing costs on youth unemployment is therefore a priori ambiguous. See (27). Indeed at given wages firing costs increase youth unemployment. Also the anticipated increase in  $w_2$  will make firms hire less youths. However, these effects may be compensated by the fall in  $w_1$ . Substituting (23)-(25) and (28) in (27) it is easy to see that youth unemployment will decrease with firing costs if the response of  $w_1$  to firing costs is sufficiently strong, i.e. if  $\Phi\left(\frac{\bar{s}' - \mu_{\bar{s}\setminus s}}{\sigma_{\bar{s}\setminus s}}\right) + \left[1 - \Phi\left(\frac{\bar{s}' - \mu_{\bar{s}\setminus s}}{\sigma_{\bar{s}\setminus s}}\right)\right] dw_2/df < -dw_1/df$ . Otherwise youth unemployment will increase with firing costs. In the simulations performed we always obtained an increase in youth unemployment. See an example in Appendix A.11.3. This means that, although unions reduce the wage of youths when firing costs increase, this cut is not sufficiently strong to imply a decrease in youth unemployment.

## 4.2 Old-age unemployment

In this section we discuss the effects of firing costs on old age unemployment. Since firing costs affect old age unemployment both directly and through their effects on wages we have that  $OU(w_1(f), w_2(f), f)$ . Therefore we can write:

$$\frac{dOU}{df} = \frac{\partial OU}{\partial f} + \frac{\partial OU}{\partial w_1} \frac{dw_1}{df} + \frac{\partial OU}{\partial w_2} \frac{dw_2}{df} \quad (29)$$

where the first term on the RHS of (29),  $\partial OU/\partial f$ , corresponds to the effect of firing costs on old age unemployment at given wages, and where the two other terms correspond to the effects of firing costs on old age unemployment through wages.

### 4.2.1 The effects of firing costs on old age unemployment at given wages

We start by discussing the effects of firing costs on old age unemployment at given wages. As before, and again due to the complexity of our model, to get some results we relied on simulations. We start with the effects of firing costs on long term unemployment. See (26). We have seen that, at given wages,

youth unemployment increases with firing costs. Also, at given wages, the probability of unemployment for an old worker that did not get a job when young decreases with firing costs.<sup>28</sup> In fact, when firing costs are high fewer youngsters are hired. Therefore remaining jobless is not perceived by potential employers as a signal of low productivity. In short, higher firing costs decrease the stigma of having not been hired and increase the employment prospects of workers who were unemployed when young.<sup>29</sup> But, since youth unemployment increases with firing costs at given wages, the effect of firing costs on LTU is a priori ambiguous. In the simulations performed (see an example in Appendix A.11.2) the effect on youth unemployment dominated, and therefore we found that, at given wages, LTU increased with firing costs.

We analyze now the case of an individual that worked when young, was fired, and did not obtain a job in old age, i.e. we analyze now the effects of firing costs on  $\Pr[\text{Hired when young}] \Pr[\text{fired}] \Pr[\text{Not hired when old} \setminus \text{fired}]$  at given wages. The probability of a youth being hired decreases with firing costs at given wages. The same happens, at given wages, with the probability of an employed individual being fired. See Appendix A.11.2. Indeed, at given wages, when firing costs increase not only the firing threshold,  $\bar{s}'$ , decreases (see (4)), but we also observe an increase in the hiring threshold,  $\bar{s}$ . See (28). Therefore, as only more productive workers are hired and firing is more costly, firms will tend to fire less. When fewer workers are fired, having been fired is seen by potential employers as a sure sign of very low productivity. Therefore the employment prospects of dismissed workers decrease with firing costs at given wages.<sup>30</sup> In other words, at given wages, higher firing costs increase the stigma of having been fired and reduce the re-employment probability of dismissed workers.<sup>31</sup>

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<sup>28</sup>The probability of unemployment for an old worker that did not get a job when young is given by (14). At given wages, since firms hire less youths when firing costs increase, the expected profitability of non-hired workers increases with firing costs. Therefore, at given wages,  $\bar{s}^{nh}$  decreases, and the mean ( $\mu_{s'' \setminus \bar{s}}$ ) and the variance ( $\sigma_{s'' \setminus \bar{s}}^2$ ) of the distribution of the signals of old workers that were not hired when young increase, so that  $\Phi\left(\frac{\bar{s}^{nh} - \mu_{s'' \setminus \bar{s}}}{\sigma_{s'' \setminus \bar{s}}}\right)$  decreases. See Appendix A.10.

<sup>29</sup>The same result was obtained by Canziani and Petrongolo (2001).

<sup>30</sup>Indeed, at given wages, the expected profitability of an individual that was fired decreases with firing costs. (See (12)). Therefore  $\bar{s}^f$  increases, and the mean ( $\mu_{s'' \setminus \bar{s}', \bar{s}}$ ) and the variance ( $\sigma_{s'' \setminus \bar{s}', \bar{s}}^2$ ) of the distribution of signals of old workers that were fired decrease. See Appendix A.11.2.

<sup>31</sup>On this issue see also Canziani and Petrongolo (2001).

However, in the simulations we performed (see again an example in Appendix A.11.2) this last effect was not sufficiently strong to offset the effects of firing costs on the probability of an individual being hired when young and fired. Therefore we conclude that, at given wages, firing costs decrease the probability of an individual being employed when young and unemployed in old age. As, at given wages, firing costs increase LTU, the effects on old age unemployment are a priori ambiguous. Nevertheless we found by simulation (see an example in Appendix A.11.2) that this last effect dominated the effect on LTU so that, at given wages, old age unemployment decreased with firing costs, i.e.  $\partial OU/\partial f < 0$ .

#### 4.2.2 The total effect of firing costs on old age unemployment<sup>32</sup>

As unions behave strategically, to obtain the global effect of firing costs on old age unemployment we need to consider also the effects of firing costs on old-age unemployment through wages. See (29). We have seen in section 4.1.2 that  $dw_1/df < 0$  and  $dw_2/df > 0$ . We also have established in section 3.2 that  $\partial OU/\partial w_1 > 0$  and  $\partial OU/\partial w_2 > 0$ . Therefore, the second term in the RHS of expression (29) is negative, while the third term is positive, i.e., firing costs increase old-age unemployment through their effects in  $w_2$ , but reduce old-age unemployment through  $w_1$ . In the simulations performed (see an example in Appendix A.11.3) we found that the positive effect of firing costs on old-age unemployment via  $w_2$  dominated the other two, so that, once we take into account wage behavior, firing costs increase old age unemployment.

The total effect of firing costs on long-term unemployment is also positive. (See the simulation results in Appendix A.11.3). Indeed, when we take the wage effect into account, YU still increases with firing costs (see section 4.1.3) and the probability of unemployment for an old worker that did not get a job when young also increases with firing costs, so that LTU increases unambiguously. In fact, although at given wages, higher firing costs decrease the stigma of having not been hired, both the decrease in  $w_1$  and the increase in  $w_2$  imply an increase in the probability of unemployment for those old workers that were not hired when young<sup>33</sup> and these effects dominate.

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<sup>32</sup>Note that all the results in this section were obtained using simulations. See Appendix A.11.3.

<sup>33</sup>Note that  $\partial \Phi \left( \frac{\bar{s}^{nh} - \mu_{s''\backslash\bar{s}}}{\sigma_{s''\backslash\bar{s}}} \right) / \partial w_1 < 0$  and  $\partial \Phi \left( \frac{\bar{s}^{nh} - \mu_{s''\backslash\bar{s}}}{\sigma_{s''\backslash\bar{s}}} \right) / \partial w_2 > 0$ . See section 3.2.



The probability of an worker having a job when young, being fired and failing to obtain a job in old age, still decreases with firing costs when we take the wage effects into account. In fact, the probability of a worker being fired, taking into account the reaction of wages to firing costs, still decreases with firing costs although this probability increases both through  $w_1$  and  $w_2$  with firing costs.<sup>34</sup> We also found that the re-employment probability of dismissed workers still decreases with firing costs, when we consider the effects through wages. Indeed, the effects of firing costs on this probability through  $w_1$  and  $w_2$  are both negative<sup>35</sup> so that the total effect of firing costs on the re-employment probability of dismissed workers is still negative. As the employment probability of young workers decreases with firing costs (see section 4.1) the total effect on the  $\Pr[\textit{Hired when young}] \Pr[\textit{fired}] \Pr[\textit{Not hired when old} \setminus \textit{fired}]$  is a priori ambiguous. Nevertheless, as stated before, in the simulations performed taking also into account the effects through wages, we always found a negative effect of firing costs on the  $\Pr[\textit{Hired when young}] \Pr[\textit{fired}] \Pr[\textit{Not hired when old} \setminus \textit{fired}]$ . See an example in Appendix A.11.3. However, firing costs, as we have seen, increase LTU and this effect dominates, so that old-age unemployment, when we consider also the effects though wages, increases with firing costs. See again Appendix A.11.3.

### 4.3 Empirical evidence and further discussion on the effects of firing costs on unemployment and wages

According to the model presented above, both at given wages and when we take the wage effect into account, firing costs increase youth unemployment. Moreover, firing costs not only reduce the hiring of young job seekers, but they also reduce the firing of employed youths, i.e., firing costs reduce the flows of youths into and out of employment. Therefore, our results on the effects of firing costs on youth hiring and firing are consistent with the findings of Bentolila and Bertola (1990).

Let us now investigate whether the predictions of our model receive empirical support. The fact that firing costs reduce flows *into* unemployment is a well documented fact. For example the OECD 1999 Employment Outlook, using cross-country data, concludes that stricter employment protec-

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<sup>34</sup>Note that  $\partial\Phi\left(\frac{\bar{s}' - \mu_{\bar{s} \setminus \bar{s}}}{\sigma_{\bar{s} \setminus \bar{s}}}\right) / \partial w_1 < 0$  and  $\partial\Phi\left(\frac{\bar{s}' - \mu_{\bar{s} \setminus \bar{s}}}{\sigma_{\bar{s} \setminus \bar{s}}}\right) / \partial w_2 > 0$ . See section 3.2.

<sup>35</sup>Note that  $\partial\Phi\left(\frac{\bar{s}f - \mu_{s'' \setminus \bar{s}' \setminus \bar{s}}}{\sigma_{s'' \setminus \bar{s}' \setminus \bar{s}}}\right) / \partial w_1 < 0$  and  $\partial\Phi\left(\frac{\bar{s}f - \mu_{s'' \setminus \bar{s}' \setminus \bar{s}}}{\sigma_{s'' \setminus \bar{s}' \setminus \bar{s}}}\right) / \partial w_2 > 0$ . See section 3.2.

tion is associated with lower flows into unemployment.<sup>36</sup> Turning now to the hiring of youths we found that firing costs reduce the hiring of youths, despite the decrease in the wage of young workers induced by higher firing costs. Previous empirical works seem to support this outcome. Scarpetta (1996) concludes that youth unemployment is adversely affected by stricter employment protection legislation, particularly in the presence of wage compression. This is a very interesting finding, which is in accordance with our model. Indeed, if existing legal or other institutional arrangements prevent wage adjustment, according to our model youth unemployment would be more severely affected by firing costs.<sup>37</sup> Also, the OECD 1999 Employment Outlook, finds a positive bivariate association between stricter employment protection legislation and higher youth unemployment that is also (weakly) confirmed when multivariate regression techniques are used.

Turning now to old-age workers our model predicts, both at given wages and also when the wage effect is considered, that firing costs reduce the reemployment prospects of dismissed workers. This is the stigma effect of dismissals on the future career, that is exacerbated when firing costs are high since, in this case, only the less able are fired. Notice that Canziano and Petrongolo (2001) found, using microeconomic data for Spain, that indeed workers who lost their jobs through costly firing procedures have worse re-employment prospects.

Also, according to our model, although at given wages firing costs increase the employment prospects of old workers that were not hired as youths, the opposite happens once we take into account the wage effect. Indeed, the stigma associated with having not had a job decreases with firing costs. However, as the wage of senior works increase with firing costs, firms hire less senior workers and this last effects dominates. Therefore, the employment prospects of workers that were unemployed as youths decrease with firing costs. Note that this result is confirmed by the finding that stricter employment protection lengthens the time spent unemployed. Indeed, the OECD

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<sup>36</sup>Note that this conclusion is obtained both by examining the simple associations between several employment protection legislation indicators and measures of labour market dynamics, and also when multivariate regression techniques are used to control for other factors that might influence the result.

<sup>37</sup>Note that, in some countries, the existence of a minimum wage, that we know to be particularly binding in the case of youths, may prevent the type of wage adjustment for youths considered in this paper, neutralizing therefore the positive wage effect of firing costs on youth unemployment. See also on this issue Bertola and Rogerson (1997).

1999 Employment Outlook reports that mean unemployment durations are higher in countries with stricter employment protection.

Our model prediction that firing costs, both at given wages and considering the wage effect, increase long term unemployment is also confirmed empirically. The OECD 1999 Employment Outlook shows that the share of the unemployed who have been jobless for at least a year, i.e. long term unemployment, is higher in countries with stricter employment protection, and Machin and Manning (1999) find that long term unemployment tends to be higher in countries with higher firing costs. The OECD 1993 Employment Outlook also finds that job security is associated with higher long term unemployment rates, and that in some countries (notably in Southern Europe and Ireland) job security accounts for more than half of the long term unemployment rate.

In what concerns the effects of firing costs on wages empirical evidence is more difficult to find. Despite the widespread notion that, due to insider effects, wages will be higher in the presence of firing costs,<sup>38</sup> Bertola (1990) does not find that wages are higher in high-job security countries. However, both the OECD 1993 Employment Outlook, using data for 20 countries between 1970 and 1991, and Layard et al. (1991), using data for Britain, find that wages increase much faster when long term unemployment is important. Combining this finding with the existence of a positive relation between firing costs and long term unemployment, we obtain a positive association between firing costs and wages, in accordance with the view that higher firing costs increase the bargaining power of workers. Moreover, Elmeskov et al (1998) find, using data for 20 OECD countries between 1983 and 1995, that different collective wage bargaining arrangements influence significantly the way in which employment protection legislation affects unemployment. They show that the positive impact of employment protection legislation on unemployment is stronger and statistically significant in countries where insiders have strong bargaining power in wage determination, while employment protection legislation does not affect so significantly employment in countries with either a decentralized or highly coordinated (among employers and trade unions) wage bargaining system. This finding supports our claim that wage setting influences significantly the way firing costs affect unemployment, and confirms our model predictions in what concerns the effects of firing costs on

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<sup>38</sup>Note that this view is in accordance with our finding that, as with higher firing costs incumbent workers are more protected, unions will push the wages for old workers.

unemployment and wages of old workers.

## 5 Concluding Remarks

In this paper we have analyzed the effects of firing costs on unemployment and wages in the presence of worker heterogeneity, imperfect information on individual productivity and collective bargaining. We considered a very simple OLG model where heterogeneous workers participate in the labour market, both when young and old, and where each generation of workers is represented by its own union. Unions announce wages unilaterally and firms, which are atomistic, choose employment at those wages after observing the age, employment history and current productivity signals of workers.

In this set-up firing costs affect employment in several ways. First, there is an option value effect. When firing costs are high firms refrain from hiring young workers since, if they turn out to be of low quality, it will be very expensive to fire them. Therefore firing costs reduce both the hiring and firing of youths. Second, there is a stigma effect. In the presence of imperfect information on worker quality firms infer worker productivity observing their past history and current signals. In our model firing costs exacerbate the stigma from being fired and reduce the stigma from not being hired. This in turn implies, respectively, positive and negative effects of firing costs on old age unemployment. Finally, since unions behave strategically, firing costs affect wages, and therefore employment. We have seen that the wage of youths decrease with firing costs, while the opposite happens with the wage of senior workers. Therefore, through wages, firing costs have both positive and negative effects on unemployment.

The total effects of firing costs on unemployment are therefore, in general, a priori ambiguous, depending on the relative strength of the various effects. In the simulations performed we found that at given wages firing costs decrease the hiring and firing of youths, increase long term unemployment, decrease old age unemployment, improve the employment prospects of unemployed youths and reduce the re-employment probability of dismissed workers. When the wage effect is considered we found that some of these results were reversed. Indeed, although we still find that firing costs decrease the hiring and firing of youths, increase long term unemployment and decrease the employment prospects of dismissed workers, now we find that firing costs increase old age unemployment and reduce the probability of em-

ployment of unemployed youths. Moreover, our findings, namely that wage setting influences the way firing costs affect unemployment, receive empirical support. We believe that these results show the importance of the consideration of the wage channel on the study of the effects of turnover costs on employment. Therefore, a natural priority for further research would be the study of alternative wage setting mechanisms in order to understand if and how this changes the results.

## A Appendix

### A.1 The joint distribution of $(a_i, s_i, \tilde{s}_i, s_i'')$

The joint distribution of  $(a_i, s_i, \tilde{s}_i, s_i'')$  is given by:

$$(a_i, s_i, \tilde{s}_i, s_i'') \sim N \left( \begin{bmatrix} \bar{a} \\ \bar{a} \\ \bar{a} \\ \bar{a} \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_a^2 & \sigma_a^2 & \sigma_a^2 \\ & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 \\ & & \sigma_{\tilde{s}}^2 & \sigma_s^2 \\ & & & \sigma_s^2 \end{bmatrix} \right) \quad (30)$$

where  $\sigma_s^2 = \sigma_a^2 + \sigma_e^2$  and  $\sigma_{\tilde{s}}^2 = \sigma_a^2 + \frac{\sigma_e^2}{2}$ .

### A.2 Computation of $E[a_i \setminus s_i]$ and $E[a_i \setminus s_i, s_i']$

Given (30), after one signal we have that :

$$a_i/s_i \sim N \left( \frac{\sigma_e^2}{\sigma_a^2 + \sigma_e^2} \bar{a} + \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2} s_i, \frac{\sigma_a^2 \sigma_e^2}{\sigma_a^2 + \sigma_e^2} \right). \quad (31)$$

After two signals we have that:

$$a_i/s_i, s_i' \sim N \left( \frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a} + \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \tilde{s}_i, \frac{\sigma_a^2 \sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \right) \quad (32)$$

where  $\tilde{s}_i = (s_i + s_i')/2$ .

### A.3 Computation of $E[a_i \setminus \tilde{s}_i \geq \bar{s}', s_i]$

From (30) we have that:

$$\tilde{s}_i/s_i \sim N(\mu_{\tilde{s}/s}, \sigma_{\tilde{s}/s}^2) \quad (33)$$

where

$$\mu_{\tilde{s}\backslash s} = \frac{\sigma_e^2}{2\sigma_s^2}\bar{a} + \frac{2\sigma_a^2 + \sigma_e^2}{2\sigma_s^2}s_i \quad (34)$$

$$\sigma_{\tilde{s}\backslash s}^2 = \frac{\sigma_e^2(2\sigma_a^2 + \sigma_e^2)}{4\sigma_s^2}. \quad (35)$$

Moreover as we have that  $E[a_i\backslash\tilde{s}_i, s_i] = E[a_i\backslash\tilde{s}_i] = E[a_i\backslash s_i, s'_i]$  (see (32)) it is easy to see that

$$E[a_i\backslash\tilde{s}_i \geq \bar{s}', s_i] = \frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2}\bar{a} + \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}E[\tilde{s}_i/\tilde{s}_i \geq \bar{s}', s_i] \quad (36)$$

where

$$E[\tilde{s}_i/\tilde{s}_i \geq \bar{s}', s_i] = \mu_{\tilde{s}\backslash s} + \sigma_{\tilde{s}\backslash s} \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s}\backslash s}}{\sigma_{\tilde{s}\backslash s}}\right)}{1 - \Phi\left(\frac{\bar{s}' - \mu_{\tilde{s}\backslash s}}{\sigma_{\tilde{s}\backslash s}}\right)}.$$

We can therefore rewrite (36) as follows:

$$E[a_i\backslash\tilde{s}_i \geq \bar{s}', s_i] = \frac{\sigma_e^2}{\sigma_a^2 + \sigma_e^2}\bar{a} + \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}s_i + \sigma_{\tilde{s}\backslash s} \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s}\backslash s}}{\sigma_{\tilde{s}\backslash s}}\right)}{1 - \Phi\left(\frac{\bar{s}' - \mu_{\tilde{s}\backslash s}}{\sigma_{\tilde{s}\backslash s}}\right)}.$$

#### A.4 Computation of $E[a_i\backslash s_i < \bar{s}, s''_i]$

Using (30) we have that:

$$a_i/s_i, s''_i \sim N\left(\frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2}\bar{a} + \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}(s_i + s''_i), \frac{\sigma_a^2\sigma_e^2}{2\sigma_a^2 + \sigma_e^2}\right) \quad (37)$$

so that

$$E[a_i\backslash s_i < \bar{s}, s''_i] = \frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2}\bar{a} + \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}s''_i + \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}E[s_i\backslash s_i < \bar{s}, s''_i]. \quad (38)$$

Now

$$s_i/s''_i \sim N(\mu_{s\backslash s''}, \sigma_{s\backslash s''}^2) \quad (39)$$

where

$$\mu_{s\backslash s''} = \frac{\sigma_e^2}{\sigma_s^2}\bar{a} + \frac{\sigma_a^2}{\sigma_s^2}s''_i \quad (40)$$

$$\sigma_{s\backslash s''}^2 = \frac{\sigma_e^2(2\sigma_a^2 + \sigma_e^2)}{\sigma_s^2}. \quad (41)$$

Therefore we have that

$$E[s_i \setminus s_i < \bar{s}, s_i''] = \mu_{s \setminus s''} - \sigma_{s \setminus s''} \frac{\phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}{\Phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}. \quad (42)$$

Substituting now (42) in (38) we finally obtain:

$$E[a_i \setminus s_i < \bar{s}, s_i''] = \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i'' - \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{s \setminus s''} \frac{\phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}{\Phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}. \quad (43)$$

### A.5 Computation of $E[a_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i'']$

The distribution of  $a_i$  conditional on  $s_i$ ,  $\tilde{s}_i$ , and  $s_i''$  is given by:<sup>39</sup>

$$a_i / s_i, \tilde{s}_i, s_i'' \sim N\left(\frac{\sigma_e^2}{3\sigma_a^2 + \sigma_e^2} \bar{a} + \frac{2\sigma_a^2}{3\sigma_a^2 + \sigma_e^2} \tilde{s}_i + \frac{\sigma_a^2}{3\sigma_a^2 + \sigma_e^2} s_i'', \frac{\sigma_a^2 \sigma_e^2}{3\sigma_a^2 + \sigma_e^2}\right). \quad (44)$$

Therefore we have that:

$$E[a_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i''] = \frac{\sigma_e^2}{3\sigma_a^2 + \sigma_e^2} \bar{a} + \frac{\sigma_a^2}{3\sigma_a^2 + \sigma_e^2} s_i'' + \frac{2\sigma_a^2}{3\sigma_a^2 + \sigma_e^2} E[\tilde{s}_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i'']. \quad (45)$$

Now

$$\tilde{s}_i / s_i, s_i'' \sim N(\mu_{\tilde{s} \setminus s, s''}, \sigma_{\tilde{s} \setminus s, s''}^2) \quad (46)$$

where

$$\mu_{\tilde{s} \setminus s, s''} = \frac{\sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2} \bar{a} + \frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2} s_i + \frac{\sigma_a^2}{4\sigma_a^2 + 2\sigma_e^2} s_i'' \quad (47)$$

$$\sigma_{\tilde{s} \setminus s, s''}^2 = \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{8\sigma_a^2 + 4\sigma_e^2}. \quad (48)$$

We also have that:

$$E[\tilde{s}_i \setminus s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i''] = \mu_{\tilde{s} \setminus \bar{s}, s''} - \sigma_{\tilde{s} \setminus \bar{s}, s''} \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}, s''}}{\sigma_{\tilde{s} \setminus \bar{s}, s''}}\right)}{\Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}, s''}}{\sigma_{\tilde{s} \setminus \bar{s}, s''}}\right)} \quad (49)$$

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<sup>39</sup>Note that  $f(a_i \setminus s_i, \tilde{s}_i, s_i'') = f(a_i \setminus \tilde{s}_i, s_i'')$ .

where  $\mu_{\bar{s}\backslash\bar{s},s''} = E[\tilde{s}_i\backslash s_i \geq \bar{s}, s_i'']$  and  $\sigma_{\bar{s}\backslash\bar{s},s''}^2 = Var[\tilde{s}_i\backslash s_i \geq \bar{s}, s_i'']$ .

Now, from (47), we have that:

$$\mu_{\bar{s}\backslash\bar{s},s''} = \frac{\sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2}\bar{a} + \frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2}E[s_i\backslash s_i \geq \bar{s}, s_i''] + \frac{\sigma_a^2}{4\sigma_a^2 + 2\sigma_e^2}s_i'' \quad (50)$$

with

$$E[s_i\backslash s_i \geq \bar{s}, s_i''] = \mu_{s\backslash s''} + \sigma_{s\backslash s''} \frac{\phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}$$

so that using (40) and (41) we obtain:

$$\mu_{\bar{s}\backslash\bar{s},s''} = \frac{\sigma_e^2}{\sigma_s^2}\bar{a} + \frac{\sigma_a^2}{\sigma_s^2}s_i'' + \frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2}\sigma_{s\backslash s''} \frac{\phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}. \quad (51)$$

Moreover we have that

$$\sigma_{\bar{s}\backslash\bar{s},s''}^2 = Var[\tilde{s}_i\backslash s_i \geq \bar{s}, s_i''] = E[\tilde{s}_i^2\backslash s_i \geq \bar{s}, s_i''] - E[\tilde{s}_i\backslash s_i \geq \bar{s}, s_i'']^2. \quad (52)$$

Now, since

$$E[\tilde{s}_i^2\backslash s_i, s_i''] = Var[\tilde{s}_i\backslash s_i, s_i''] + E[\tilde{s}_i\backslash s_i, s_i'']^2$$

using (46)-(48) we obtain

$$\begin{aligned} E[\tilde{s}_i^2\backslash s_i, s_i''] &= \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{8\sigma_a^2 + 4\sigma_e^2} + \left(\frac{\sigma_e^2\bar{a} + \sigma_a^2 s_i''}{4\sigma_a^2 + 2\sigma_e^2}\right)^2 + \left(\frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2}\right)^2 s_i^2 \\ &\quad + 2\left(\frac{\sigma_e^2\bar{a} + \sigma_a^2 s_i''}{4\sigma_a^2 + 2\sigma_e^2}\right)\left(\frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2}\right) s_i \end{aligned}$$

so that

$$\begin{aligned} E[\tilde{s}_i^2\backslash s_i \geq \bar{s}, s_i''] &= \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{8\sigma_a^2 + 4\sigma_e^2} + \left(\frac{\sigma_e^2\bar{a} + \sigma_a^2 s_i''}{4\sigma_a^2 + 2\sigma_e^2}\right)^2 \\ &\quad + \left(\frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2}\right)^2 E[s_i^2\backslash s_i \geq \bar{s}, s_i''] + 2\left(\frac{\sigma_e^2\bar{a} + \sigma_a^2 s_i''}{4\sigma_a^2 + 2\sigma_e^2}\right)\left(\frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2}\right) E[s_i\backslash s_i \geq \bar{s}, s_i'']. \end{aligned} \quad (53)$$



Substituting now (53) and (51) in (52) we obtain:

$$\sigma_{\bar{s}\backslash\bar{s},s''}^2 = Var[\tilde{s}_i \backslash s_i \geq \bar{s}, s_i''] = \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{8\sigma_a^2 + 4\sigma_e^2} + \left( \frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2} \right)^2 Var[s_i \backslash s_i \geq \bar{s}, s_i''].$$

As

$$Var[s_i \backslash s_i \geq \bar{s}, s_i''] = \sigma_{s\backslash s''}^2 \left\{ 1 - \frac{\phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)} \left[ \frac{\phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)} - \left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right) \right] \right\}$$

we have that

$$\sigma_{\bar{s}\backslash\bar{s},s''}^2 = \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{2\sigma_s^2} \left\{ 1 - \frac{3\sigma_a^2 + \sigma_e^2}{4\sigma_a^2 + 2\sigma_e^2} \frac{\phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)} \left[ \frac{\phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)} - \left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right) \right] \right\} \quad (54)$$

Substituting now (51) and (54) in (49), and (49) in (45) we finally obtain:

$$\begin{aligned} E[a_i \backslash s_i \geq \bar{s}, \tilde{s}_i < \bar{s}', s_i''] &= \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i'' + \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{s\backslash s''} \frac{\phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)}{1 - \Phi\left(\frac{\bar{s} - \mu_{s\backslash s''}}{\sigma_{s\backslash s''}}\right)} \\ &\quad - \frac{2\sigma_a^2}{3\sigma_a^2 + \sigma_e^2} \sigma_{\bar{s}\backslash\bar{s},s''} \frac{\phi\left(\frac{\bar{s}' - \mu_{\bar{s}\backslash\bar{s},s''}}{\sigma_{\bar{s}\backslash\bar{s},s''}}\right)}{\Phi\left(\frac{\bar{s}' - \mu_{\bar{s}\backslash\bar{s},s''}}{\sigma_{\bar{s}\backslash\bar{s},s''}}\right)}. \end{aligned} \quad (55)$$

## A.6 Obtaining old age unemployment

### A.6.1 Computation of $\Pr[s_i'' < \bar{s}^{nh} \backslash s_i < \bar{s}]$

The distribution of  $s_i''$  conditional on  $s_i$  is given by:

$$s_i'' \backslash s_i \sim N\left(\frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} s_i, \frac{\sigma_e^2(2\sigma_a^2 + \sigma_e^2)}{\sigma_s^2}\right). \quad (56)$$

Therefore we have that

$$\mu_{s''\backslash\bar{s}} = E[s_i'' \backslash s_i < \bar{s}] = \frac{\sigma_e^2}{\sigma_s^2} \bar{a} + \frac{\sigma_a^2}{\sigma_s^2} E[s_i \backslash s_i < \bar{s}] = \bar{a} - \frac{\sigma_a^2}{\sigma_s} \frac{\phi\left(\frac{\bar{s} - \bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s} - \bar{a}}{\sigma_s}\right)} \quad (57)$$

and that

$$\sigma_{s'' \setminus \bar{s}}^2 = Var[s_i'' \setminus s_i < \bar{s}] = \sigma_s^2 \left\{ 1 - \left( \frac{\sigma_a^2}{\sigma_s^2} \right)^2 \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} + \left( \frac{\bar{s}-\bar{a}}{\sigma_s} \right) \right] \right\}. \quad (58)$$

This means that

$$\Pr[s_i'' < \bar{s}^{nh} \setminus s_i < \bar{s}] = \Phi\left(\frac{\bar{s}^{nh} - \mu_{s'' \setminus \bar{s}}}{\sigma_{s'' \setminus \bar{s}}}\right).$$

### A.6.2 Computation of $\Pr[\tilde{s}_i < \bar{s}' \setminus s_i \geq \bar{s}]$

From (33)-(35) we have that:

$$\mu_{\tilde{s} \setminus \bar{s}} = E[\tilde{s}_i \setminus s_i \geq \bar{s}] = \frac{\sigma_e^2}{2\sigma_s^2} \bar{a} + \frac{2\sigma_a^2 + \sigma_e^2}{2\sigma_s^2} E[s_i \setminus s_i \geq \bar{s}] = \bar{a} + \frac{\sigma_s^2}{\sigma_s} \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1 - \Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \quad (59)$$

and that

$$\sigma_{\tilde{s} \setminus \bar{s}}^2 = Var[\tilde{s}_i \setminus s_i \geq \bar{s}] = \sigma_s^2 \left\{ 1 - \left( \frac{\sigma_s^2}{\sigma_s^2} \right)^2 \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1 - \Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1 - \Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} - \left( \frac{\bar{s}-\bar{a}}{\sigma_s} \right) \right] \right\} \quad (60)$$

where  $\sigma_s^2 = (\sigma_a^2 + \frac{\sigma_e^2}{2})$ . Therefore we have that:

$$\Pr[\tilde{s}_i < \bar{s}' \setminus s_i \geq \bar{s}] = \Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right).$$

### A.6.3 Computation of $\Pr[s_i'' < \bar{s}^f \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}]$

The distribution of  $s_i''$  conditional on  $\tilde{s}_i$  and  $s_i$  is given by:<sup>40</sup>

$$s_i'' \setminus \tilde{s}_i, s_i \sim N\left(\frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a} + \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \tilde{s}_i, \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{2\sigma_a^2 + \sigma_e^2}\right). \quad (61)$$

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<sup>40</sup>Note that  $f(s_i'' \setminus \tilde{s}_i, s_i) = f(s_i'' \setminus \tilde{s}_i)$ .

Therefore

$$\mu_{s'' \setminus \bar{s}', \bar{s}} = E[s_i'' \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] = \frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a} + \frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} E[\tilde{s}_i \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}]$$

where

$$E[\tilde{s}_i \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] = \mu_{\tilde{s} \setminus \bar{s}} - \sigma_{\tilde{s} \setminus \bar{s}} \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)}{\Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)}$$

so that

$$\mu_{s'' \setminus \bar{s}', \bar{s}} = \bar{a} + \frac{\sigma_a^2}{\sigma_s} \frac{\phi\left(\frac{\bar{s} - \bar{a}}{\sigma_s}\right)}{1 - \Phi\left(\frac{\bar{s} - \bar{a}}{\sigma_s}\right)} - \frac{\sigma_a^2 \sigma_{\tilde{s} \setminus \bar{s}}}{\sigma_s^2} \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)}{\Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)}. \quad (62)$$

Moreover as

$$\text{Var}[s_i'' \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] = E[s_i''^2 \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] - E[s_i'' \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}]^2$$

and

$$\begin{aligned} E[s_i''^2 \setminus \tilde{s}_i, s_i] &= \text{Var}[s_i'' \setminus \tilde{s}_i, s_i] + E[s_i'' \setminus \tilde{s}_i, s_i]^2 \\ &= \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{2\sigma_a^2 + \sigma_e^2} + \left(\frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a}\right)^2 + \left(\frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}\right)^2 \tilde{s}_i^2 \\ &\quad + 2 \left(\frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a}\right) \left(\frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}\right) \tilde{s}_i \end{aligned}$$

we have that:

$$\begin{aligned} E[s_i''^2 \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] &= \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{2\sigma_a^2 + \sigma_e^2} + \left(\frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a}\right)^2 \\ &+ \left(\frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}\right)^2 E[\tilde{s}_i^2 \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] + 2 \left(\frac{\sigma_e^2}{2\sigma_a^2 + \sigma_e^2} \bar{a}\right) \left(\frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}\right) E[\tilde{s}_i \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] \end{aligned}$$

and therefore

$$\sigma_{s'' \setminus \bar{s}', \bar{s}}^2 = \text{Var}[s_i'' \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] = \frac{\sigma_e^2(3\sigma_a^2 + \sigma_e^2)}{2\sigma_a^2 + \sigma_e^2} + \left(\frac{2\sigma_a^2}{2\sigma_a^2 + \sigma_e^2}\right)^2 \text{Var}[\tilde{s}_i \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] \quad (63)$$

where:

$$Var[\tilde{s}_i \setminus \tilde{s}_i < \bar{s}', s_i \geq \bar{s}] = \sigma_{\tilde{s} \setminus \bar{s}}^2 \left\{ 1 - \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)}{\Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)} \left[ \frac{\phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)}{\Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right)} + \left(\frac{\bar{s}' - \mu_{\tilde{s} \setminus \bar{s}}}{\sigma_{\tilde{s} \setminus \bar{s}}}\right) \right] \right\} \quad (64)$$

with  $\mu_{\tilde{s} \setminus \bar{s}}$  and  $\sigma_{\tilde{s} \setminus \bar{s}}^2$  given respectively by (59) and (60).

This means that:

$$\Pr[s_i'' < \bar{s}' \setminus \tilde{s}_i < \bar{s}'', s_i \geq \bar{s}] = \Phi\left(\frac{\bar{s}' - \mu_{s'' \setminus \bar{s}'}}{\sigma_{s'' \setminus \bar{s}'}}\right).$$

## A.7 Proof that $E[\pi_i \setminus s_i < \bar{s}, s_i'']$ is increasing in $s_i''$ .

**Proof.**  $E[\pi_i \setminus s_i < \bar{s}, s_i'']$  is increasing in  $s_i''$  since

$$\begin{aligned} \frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial s_i''} &= \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2} - \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{s \setminus s''} \frac{\partial \left( \frac{\phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}{\Phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)} \right)}{\partial s_i''} \\ &= \frac{\sigma_a^2}{\sigma_s^2} \left\{ 1 - \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \frac{\phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}{\Phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)} \left[ \frac{\phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)}{\Phi\left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right)} + \left(\frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}}\right) \right] \right\} > 0 \end{aligned} \quad (65)$$

as  $\frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} < 1$  and  $0 < \frac{\phi(z)}{\Phi(z)} \left[ \frac{\phi(z)}{\Phi(z)} + (z) \right] < 1$ . ■

## A.8 The effects of wages on old-age unemployment

### A.8.1 Proof that $\frac{\partial \bar{s}^{nh}}{\partial w_1} < 0$ and $\frac{\partial \bar{s}^{nh}}{\partial w_2} > 0$ .

**Proof.** Since  $\bar{s}^{nh}$  is the value of  $s_i''$  such that  $E[\pi_i \setminus s_i < \bar{s}, s_i''] = 0$  from (11) we have that

$$\frac{\partial \bar{s}^{nh}}{\partial w_j} = - \frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial w_j}{\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial s_i''} \quad \text{with } j = 1, 2. \quad (66)$$

Now, from (11) we also have that

$$\frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial w_1} = \frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial w_1} \quad (67)$$

$$\frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial w_2} = \frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial w_2} - 1 \quad (68)$$

where

$$\begin{aligned} \frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial \bar{s}} &= -\frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \sigma_{s \setminus s''} \frac{\partial \left( \frac{\phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)}{\Phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)} \right)}{\partial \bar{s}} \quad (69) \\ &= \frac{\sigma_a^2}{2\sigma_a^2 + \sigma_e^2} \frac{\phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)}{\Phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)} \left[ \frac{\phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)}{\Phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)} + \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right) \right] > 0 \end{aligned}$$

We also have that  $\frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial s_i''} > 0$  (see (65) in Appendix A.7). Therefore, since  $\frac{\partial \bar{s}}{\partial w_1} > 0$  (see (24)), substituting (65) and (67) in (66) we have that  $\frac{\partial \bar{s}^{nh}}{\partial w_1} < 0$ .

Also using (25) and (69) we can rewrite (68) as:

$$\frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial w_2} = \frac{\sigma_a^2}{\sigma_s^2} \frac{\phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)}{\Phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)} \left[ \frac{\phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)}{\Phi \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right)} + \left( \frac{\bar{s} - \mu_{s \setminus s''}}{\sigma_{s \setminus s''}} \right) \right] \frac{\left( 1 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right)}{\left( 2 - \Phi \left( \frac{\bar{s}' - \mu_{\bar{s} \setminus s}}{\sigma_{\bar{s} \setminus s}} \right) \right)} - 1 < 0 \quad (70)$$

Therefore since  $\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial s_i'' > 0$  (see (65)) we have that  $\partial \bar{s}^{nh} / \partial w_2 > 0$ . ■

### A.8.2 Proof that $\frac{\partial \mu_{s'' \setminus \bar{s}}}{\partial w_1} > 0$ and $\frac{\partial \mu_{s'' \setminus \bar{s}}}{\partial w_2} > 0$ .

**Proof.** Differentiating (57) we have that

$$\frac{\partial \mu_{s'' \setminus \bar{s}}}{\partial w_j} = \frac{\sigma_a^2}{\sigma_s^2} \frac{\phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right)}{\Phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right)} \left[ \frac{\phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right)}{\Phi \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right)} + \left( \frac{\bar{s} - \bar{a}}{\sigma_s} \right) \right] \frac{\partial \bar{s}}{\partial w_j} \quad \text{with } j = 1, 2. \quad (71)$$

Therefore, since  $0 < \frac{\phi(z)}{\Phi(z)} \left[ \frac{\phi(z)}{\Phi(z)} + (z) \right] < 1$  and  $\frac{\partial \bar{s}}{\partial w_j} > 0$  (see (24)-(25)) we have that  $\frac{\partial \mu_{s'' \setminus \bar{s}}}{\partial w_j} > 0$ . ■

**A.8.3 Proof that  $\frac{\partial \sigma_{s'' \setminus \bar{s}}}{\partial w_1} > 0$  and that  $\frac{\partial \sigma_{s'' \setminus \bar{s}}}{\partial w_2} > 0$**

**Proof.** From (58) we have that

$$\frac{\partial \sigma_{s'' \setminus \bar{s}}}{\partial w_j} = -\frac{1}{2} \frac{\sigma_s^2}{\sigma_{s'' \setminus \bar{s}}} \left( \frac{\sigma_a^2}{\sigma_s^2} \right)^2 \frac{\partial \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} + \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \right]}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial w_j} \quad \text{with } j = 1, 2 \quad (72)$$

Therefore, since  $\partial \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} + \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \right] / \partial \bar{s} < 0$  and  $\frac{\partial \bar{s}}{\partial w_j} > 0$  (see (24)-(25)) we have that  $\frac{\partial \sigma_{s'' \setminus \bar{s}}}{\partial w_j} > 0$ . ■

**A.8.4 Proof that  $\frac{\partial \mu_{\bar{s} \setminus \bar{s}}}{\partial w_1} > 0$  and that  $\frac{\partial \mu_{\bar{s} \setminus \bar{s}}}{\partial w_2} > 0$**

**Proof.** Differentiating (59) we have that

$$\frac{\partial \mu_{\bar{s} \setminus \bar{s}}}{\partial w_j} = \frac{\sigma_s^2}{\sigma_s^2} \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1 - \Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1 - \Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} - \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \frac{\partial \bar{s}}{\partial w_j} \quad \text{with } j = 1, 2. \quad (73)$$

Therefore, since  $0 < \frac{\phi(z)}{1-\Phi(z)} \left[ \frac{\phi(z)}{1-\Phi(z)} - (z) \right] < 1$  and  $\frac{\partial \bar{s}}{\partial w_j} > 0$  (see (24)-(25)) we have that  $\frac{\partial \mu_{\bar{s} \setminus \bar{s}}}{\partial w_j} > 0$ . ■

**A.8.5 Proof that  $\frac{\partial \sigma_{\bar{s} \setminus \bar{s}}}{\partial w_1} < 0$  and that  $\frac{\partial \sigma_{\bar{s} \setminus \bar{s}}}{\partial w_2} < 0$**

**Proof.** From (60) we have that

$$\frac{\partial \sigma_{\bar{s} \setminus \bar{s}}}{\partial w_j} = -\frac{1}{2\sigma_{\bar{s} \setminus \bar{s}}} \left( \frac{\sigma_s^2}{\sigma_s^2} \right)^2 \frac{\partial \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1-\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1-\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} - \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \right]}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial w_j} \quad \text{with } j = 1, 2. \quad (74)$$

Therefore, since  $\partial \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1-\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{1-\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} - \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \right] / \partial \bar{s} > 0$  and  $\frac{\partial \bar{s}}{\partial w_j} > 0$  (see (24)-(25)) we have that  $\frac{\partial \sigma_{\bar{s} \setminus \bar{s}}}{\partial w_j} < 0$ . ■

## A.9 The effects of firing costs on wages

Using (19) we have that:

$$\frac{dw_{2(w_1)}}{df} = - \left[ \frac{\frac{\partial OU(w_1, w_{2(w_1)}, f)}{\partial f} + w_2 \frac{\partial^2 OU(w_1, w_{2(w_1)}, f)}{\partial w_2 \partial f}}{2 \frac{\partial OU(w_1, w_{2(w_1)}, f)}{\partial w_2} + w_2 \frac{\partial^2 OU(w_1, w_{2(w_1)}, f)}{\partial w_2^2}} \right]. \quad (75)$$

Note that the denominator of expression (75) is positive. See footnote 16.

Now, from (21) we obtain:

$$\begin{aligned} \frac{dw_1}{df} = & -\frac{1}{\Psi} \left\{ \frac{\partial YU(w_1, w_{2(w_1)}, f)}{\partial f} \right. \\ & + w_1 \left[ \frac{\partial^2 YU(w_1, w_{2(w_1)}, f)}{\partial w_1 \partial f} + \frac{\partial^2 YU(w_1, w_{2(w_1)}, f)}{\partial w_2 \partial f} \frac{\partial w_{2(w_1)}}{\partial w_1} \right. \\ & \left. \left. + \frac{\partial^2 w_{2(w_1)} \partial YU(w_1, w_{2(w_1)}, f)}{\partial w_1 \partial f \partial w_2} \right] \right. \\ & \left. + \frac{\partial OU(w_1, w_{2(w_1)}, f)}{\partial w_1} \frac{\partial w_{2(w_1)}}{\partial f} + w_{2(w_1)} \frac{\partial^2 OU(w_1, w_{2(w_1)}, f)}{\partial w_1 \partial f} \right\} \end{aligned} \quad (76)$$

where

$$\begin{aligned} \Psi = & 2 \left[ \frac{\partial YU(w_1, w_{2(w_1)}, f)}{\partial w_1} + \frac{\partial YU(w_1, w_{2(w_1)}, f)}{\partial w_2} \frac{\partial w_{2(w_1)}}{\partial w_1} \right] \\ & + w_1 \left[ \frac{\partial^2 YU(w_1, w_{2(w_1)}, f)}{\partial w_1^2} + 2 \frac{\partial^2 YU(w_1, w_{2(w_1)}, f)}{\partial w_1 \partial w_2} \frac{\partial w_{2(w_1)}}{\partial w_1} \right. \\ & \left. + \frac{\partial^2 YU(w_1, w_{2(w_1)}, f)}{\partial w_2^2} \left( \frac{\partial w_{2(w_1)}}{\partial w_1} \right)^2 \right] \\ & + \frac{\partial OU(w_1, w_{2(w_1)}, f)}{\partial w_1} \frac{\partial w_{2(w_1)}}{\partial w_1} \\ & + w_{2(w_1)} \left[ \frac{\partial^2 OU(w_1, w_{2(w_1)}, f)}{\partial w_1^2} + \frac{\partial^2 OU(w_1, w_{2(w_1)}, f)}{\partial w_1 \partial w_2} \frac{\partial w_{2(w_1)}}{\partial w_1} \right] \end{aligned} \quad (77)$$

Note that  $\Psi > 0$ , since  $(-\Psi)$  is the second derivative of problem (20).

Finally we have that

$$\frac{dw_2}{df} = \frac{\partial w_{2(w_1)}}{\partial w_1} \frac{dw_1}{df} + \frac{dw_{2(w_1)}}{df}. \quad (78)$$

## A.10 The effects of firing costs on old age unemployment at given wages

### A.10.1 Proof that $\frac{\partial \bar{s}^{nh}}{\partial f} < 0$ .

**Proof.** Since  $\bar{s}^{nh}$  is the value of  $s_i''$  such that  $E[\pi_i \setminus s_i < \bar{s}, s_i''] = 0$  from (11) we have that

$$\frac{\partial \bar{s}^{nh}}{\partial f} = - \frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial f}{\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial s_i''}. \quad (79)$$

where  $\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial s_i'' > 0$ . See (65).

From (11) we also have that:

$$\frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial f} = \frac{\partial E[\pi_i \setminus s_i < \bar{s}, s_i'']}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial f} \quad (80)$$

Since  $\partial \bar{s} / \partial f > 0$  (see (28)) and  $\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial \bar{s} > 0$  (see (68)) we have that  $\partial E[\pi_i \setminus s_i < \bar{s}, s_i''] / \partial f > 0$ . Therefore  $\partial \bar{s}^{nh} / \partial f < 0$ . ■

### A.10.2 Proof that $\frac{\partial \mu_{s'' \setminus \bar{s}}}{\partial f} > 0$ .

**Proof.** Differentiating (57) we have that

$$\frac{\partial \mu_{s'' \setminus \bar{s}}}{\partial f} = \frac{\sigma_a^2 \phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\sigma_s^2 \Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} + \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \frac{\partial \bar{s}}{\partial f}. \quad (81)$$

Therefore, since  $0 < \frac{\phi(z)}{\Phi(z)} \left[ \frac{\phi(z)}{\Phi(z)} + (z) \right] < 1$  and  $\frac{\partial \bar{s}}{\partial f} > 0$  (see (28)) we have that  $\frac{\partial \mu_{s'' \setminus \bar{s}}}{\partial f} > 0$ . ■

### A.10.3 Proof that $\frac{\partial \sigma_{s'' \setminus \bar{s}}}{\partial f} > 0$

**Proof.** From (58) we have that

$$\frac{\partial \sigma_{s'' \setminus \bar{s}}}{\partial f} = - \frac{1}{2} \frac{\sigma_s^2}{\sigma_{s'' \setminus \bar{s}}} \left( \frac{\sigma_a^2}{\sigma_s^2} \right)^2 \frac{\partial \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} + \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \right]}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial f} \quad (82)$$

Therefore, since  $\frac{\partial \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} \left[ \frac{\phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)}{\Phi\left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right)} + \left(\frac{\bar{s}-\bar{a}}{\sigma_s}\right) \right] \right]}{\partial \bar{s}} < 0$  and  $\frac{\partial \bar{s}}{\partial f} > 0$  (see (28))

we have that  $\frac{\partial \sigma_{s'' \setminus \bar{s}}}{\partial f} > 0$ . ■



## A.11 Simulation Results

### A.11.1 The effects of wages on unemployment

To analyze the effects of wages on unemployment we fixed the wages at different values and simulated the model. In Tables 1 and 2 we present the results obtained using the following values for the parameters of the model:  $\sigma_a^2 = 15$ ,  $\sigma_e^2 = 20$ ,  $\bar{a} = 65$ ,  $f = 3$ .

| $w_2 = 63$  | $w_1 = 58$ | $w_1 = 60$ | $w_1 = 62$ |
|---|------------|------------|------------|
| $YU$  | 0.03111    | 0.07880    | 0.16003    |
| $LTU$   | 0.02969    | 0.07416    | 0.13584    |
| $OU$  | 0.0653     | 0.09887    | 0.14999    |
| $\Phi\left(\frac{\bar{s}^{nh} - \mu_{s'' \bar{s}}}{\sigma_{s'' \bar{s}}}\right)$                | 0.98589    | 0.94100    | 0.84888    |
| $\Phi\left(\frac{\bar{s}' - \mu_{\bar{s} \bar{s}}}{\sigma_{\bar{s} \bar{s}}}\right)$            | 0.03673    | 0.02685    | 0.0168512  |
| $\Phi\left(\frac{\bar{s}^f - \mu_{s'' \bar{s}',\bar{s}}}{\sigma_{s'' \bar{s}',\bar{s}}}\right)$ | 0.99928    | 0.99923    | 0.99914    |
| $\bar{s}$   | 53.883     | 56.640     | 59.117     |
| $\bar{s}^{nh}$  | 71.142     | 68.773     | 66.782     |
| $\bar{s}'$  | 56.666     | 56.666     | 56.666     |
| $\bar{s}^f$   | 75.432     | 75.373     | 75.257     |
| $\mu_{s'' \bar{s}}$   | 59.253     | 60.271     | 61.145     |
| $\mu_{\bar{s} \bar{s}}$   | 65.297     | 65.674     | 66.224     |
| $\mu_{s'' \bar{s}',\bar{s}}$  | 58.847     | 58.931     | 59.029     |
| $\sigma_{s'' \bar{s}}^2$  | 29.354     | 29.581     | 29.856     |
| $\sigma_{\bar{s} \bar{s}}^2$  | 23.250     | 21.799     | 20.255     |
| $\sigma_{s'' \bar{s}',\bar{s}}^2$   | 27.069     | 26.931     | 26.7819    |

**Table 1: The effects of  $w_1$  on unemployment**

| $w_1 = 60$  | $w_2 = 61$ | $w_2 = 63$ | $w_2 = 65$ |
|---|------------|------------|------------|
| $YU$  | 0.03728    | 0.07880    | 0.13779    |
| $LTU$   | 0.03027    | 0.07416    | 0.13632    |
| $OU$  | 0.03611    | 0.09887    | 0.21415    |
| $\Phi\left(\frac{\bar{s}^{nh} - \mu_{s'' \bar{s}}}{\sigma_{s'' \bar{s}}}\right)$                | 0.81192    | 0.94100    | 0.98935    |
| $\Phi\left(\frac{\bar{s}' - \mu_{\bar{s} \bar{s}}}{\sigma_{\bar{s} \bar{s}}}\right)$            | 0.00607    | 0.02685    | 0.0903     |
| $\Phi\left(\frac{\bar{s}^f - \mu_{s'' \bar{s}',\bar{s}}}{\sigma_{s'' \bar{s}',\bar{s}}}\right)$ | 0.99904    | 0.99923    | 0.99943    |
| $\bar{s}$   | 54.451     | 56.640     | 58.550     |
| $\bar{s}^{nh}$  | 64.264     | 68.773     | 73.515     |
| $\bar{s}'$  | 53.333     | 56.666     | 60.000     |
| $\bar{s}^f$   | 73.112     | 75.373     | 77.701     |
| $\mu_{s'' \bar{s}}$   | 59.466     | 60.271     | 60.949     |
| $\mu_{\bar{s} \bar{s}}$   | 65.357     | 65.674     | 66.079     |
| $\mu_{s'' \bar{s}',\bar{s}}$  | 57.074     | 58.931     | 60.736     |
| $\sigma_{s'' \bar{s}}^2$  | 29.395     | 29.581     | 29.786     |
| $\sigma_{\bar{s} \bar{s}}^2$  | 22.987     | 21.799     | 20.617     |
| $\sigma_{s'' \bar{s}',\bar{s}}^2$   | 26.734     | 26.931     | 27.215     |

**Table 2: The effects of  $w_2$  on unemployment**

### A.11.2 The effects of firing costs on unemployment at given wages

To analyze the effects of firing costs on unemployment at given wages we fixed  $w_1 = 62.02$  and  $w_2 = 58.76$ , and simulated the model for different values of  $f$ . In Table 3 we present the results obtained considering the following values for the parameters of the model:  $\sigma_a^2 = 15$ ,  $\sigma_e^2 = 20$ ,  $\bar{a} = 65$ .

| $w_1 = 62.02; w_2 = 58.76$  | $f = 0$ | $f = 1$ | $f = 1.5$ | $f = 3$ | $f = 3.25$ |
|---|---------|---------|-----------|---------|------------|
| $YU$  | 0.03245 | 0.03400 | 0.03428   | 0.03450 | 0.03451    |
| $LTU$   | 0.01197 | 0.01220 | 0.01224   | 0.01227 | 0.01227    |
| $OU$  | 0.02163 | 0.01665 | 0.01500   | 0.01278 | 0.01265    |
| $\Phi\left(\frac{\bar{s}^{nh} - \mu_{s'' \bar{s}}}{\sigma_{s'' \bar{s}}}\right)$                    | 0.36886 | 0.35889 | 0.35711   | 0.35573 | 0.35570    |
| $\Phi\left(\frac{\bar{s}^f - \mu_{\bar{s} \bar{s}}}{\sigma_{\bar{s} \bar{s}}}\right)$               | 0.01298 | 0.00495 | 0.00295   | 0.00053 | 0.00039    |
| $\Phi\left(\frac{\bar{s}^f - \mu_{s'' \bar{s}^f, \bar{s}}}{\sigma_{s'' \bar{s}^f, \bar{s}}}\right)$ | 0.76951 | 0.93041 | 0.96854   | 0.99877 | 0.99937    |
| $\bar{s}$   | 54.079  | 54.203  | 54.225    | 54.242  | 54.243     |
| $\bar{s}^{nh}$  | 57.512  | 57.414  | 57.397    | 57.383  | 57.383     |
| $\bar{s}^f$   | 54.600  | 52.933  | 52.1      | 49.6    | 49.183     |
| $\bar{s}^f$   | 61.573  | 64.493  | 65.992    | 70.593  | 71.371     |
| $\mu_{s'' \bar{s}}$   | 59.327  | 59.373  | 59.382    | 59.388  | 59.388     |
| $\mu_{\bar{s} \bar{s}}$   | 65.317  | 65.330  | 65.332    | 65.334  | 65.334     |
| $\mu_{s'' \bar{s}^f, \bar{s}}$  | 57.753  | 56.849  | 56.391    | 54.997  | 54.762     |
| $\sigma_{s'' \bar{s}}^2$  | 29.368  | 29.377  | 29.379    | 29.380  | 29.380     |
| $\sigma_{\bar{s} \bar{s}}^2$  | 23.161  | 23.104  | 23.094    | 23.086  | 23.086     |
| $\sigma_{s'' \bar{s}^f, \bar{s}}^2$   | 26.849  | 26.713  | 26.656    | 26.520  | 26.501     |

**Table 3: The effects of firing costs on unemployment at given wages**

### A.11.3 The total effects of firing costs on unemployment and wages

To analyze the effects of firing costs on unemployment and wages we simulated the model, letting wages be determined endogenously, for different values of  $f$ . In Table 4 we present the results obtained considering the following values for the parameters of the model:  $\sigma_a^2 = 15$ ,  $\sigma_e^2 = 20$ ,  $\bar{a} = 65$ .

|   | $f = 0$ | $f = 1$ | $f = 1.5$ | $f = 3$ | $f = 3.25$ |
|---|---------|---------|-----------|---------|------------|
| $w_1$   | 62.02   | 61.73   | 61.57     | 61.50   | 61.46      |
| $w_2$   | 58.76   | 59.17   | 59.38     | 59.76   | 59.81      |
| $YU$  | 0.03245 | 0.03546 | 0.03657   | 0.04232 | 0.04252    |
| $LTU$   | 0.01197 | 0.01563 | 0.01762   | 0.02263 | 0.02318    |
| $OU$  | 0.02163 | 0.02220 | 0.02270   | 0.02411 | 0.02435    |
| $\Phi\left(\frac{\bar{s}^{nh} - \mu_{s'' \bar{s}}}{\sigma_{s'' \bar{s}}}\right)$                | 0.36886 | 0.44074 | 0.48172   | 0.53479 | 0.54511    |
| $\Phi\left(\frac{\bar{s}' - \mu_{\tilde{s} \bar{s}}}{\sigma_{\tilde{s} \bar{s}}}\right)$        | 0.01298 | 0.00730 | 0.00543   | 0.00155 | 0.00123    |
| $\Phi\left(\frac{\bar{s}^f - \mu_{s'' \bar{s}',\bar{s}}}{\sigma_{s'' \bar{s}',\bar{s}}}\right)$ | 0.76951 | 0.93308 | 0.97039   | 0.99888 | 0.99943    |
| $\bar{s}$   | 54.079  | 54.315  | 54.399    | 54.798  | 54.812     |
| $\bar{s}^{nh}$  | 57.512  | 58.607  | 59.198    | 60.069  | 60.215     |
| $\bar{s}'$  | 54.600  | 53.617  | 53.133    | 51.267  | 50.933     |
| $\bar{s}^f$   | 61.573  | 64.980  | 66.715    | 71.699  | 72.527     |
| $\mu_{s'' \bar{s}}$   | 59.327  | 59.415  | 59.447    | 59.596  | 59.601     |
| $\mu_{\tilde{s} \bar{s}}$   | 65.317  | 65.342  | 65.351    | 65.398  | 65.400     |
| $\mu_{s'' \bar{s}',\bar{s}}$  | 57.753  | 57.225  | 56.963    | 55.940  | 55.755     |
| $\sigma_{s'' \bar{s}}^2$  | 29.368  | 29.385  | 29.391    | 29.422  | 29.423     |
| $\sigma_{\tilde{s} \bar{s}}^2$  | 23.161  | 23.051  | 23.011    | 22.815  | 22.809     |
| $\sigma_{s'' \bar{s}',\bar{s}}^2$   | 26.849  | 26.759  | 26.721    | 26.590  | 26.572     |

Table 4: The effects of firing costs on unemployment and wages

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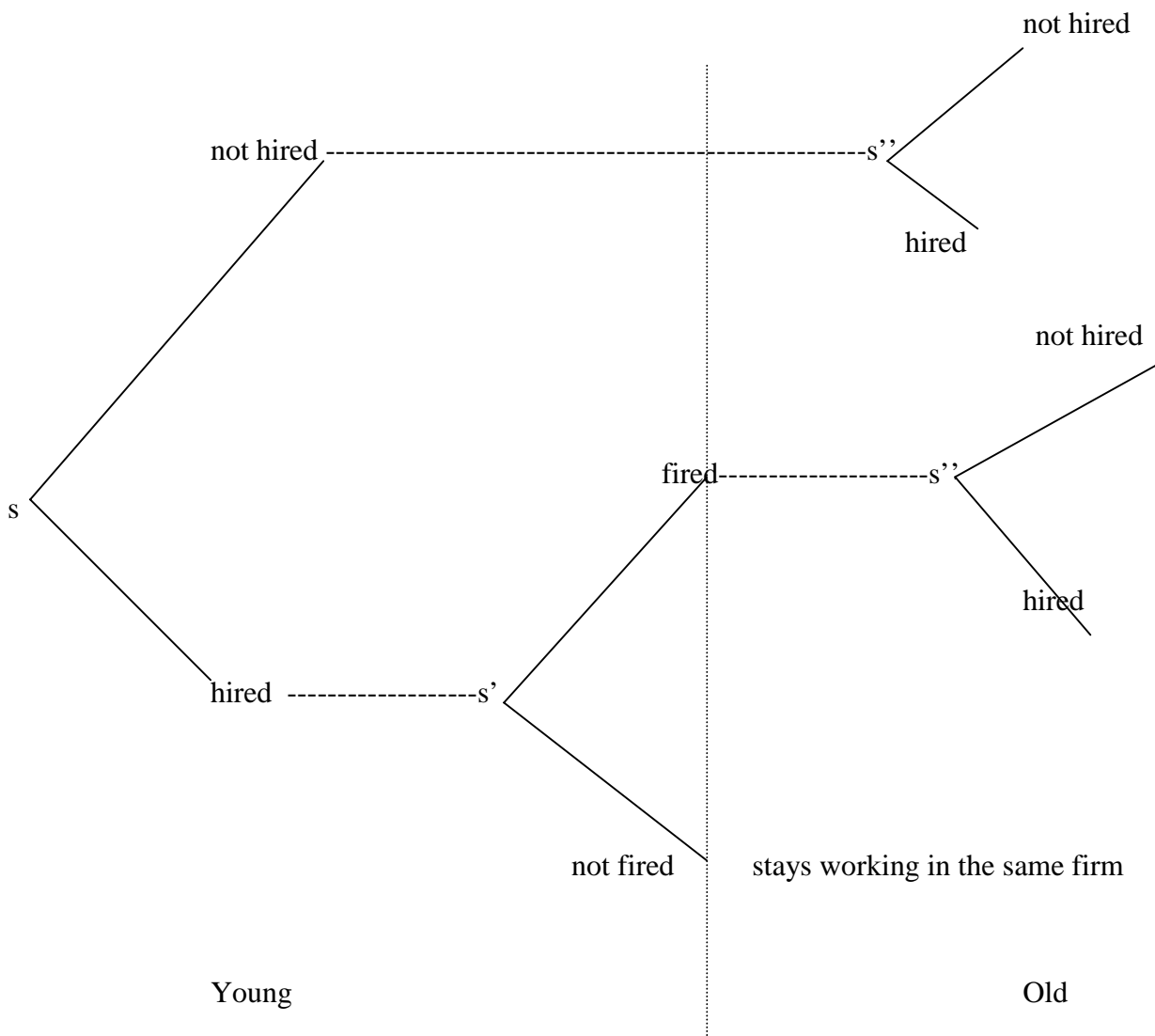


Figure 1