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**Working Paper**

## Technical Change, Effective Demand and Economic Growth

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# **WIIW Working Papers**

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Robert Stehrer*

**Technical Change,  
Effective Demand and  
Economic Growth**

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## Abstract

The model developed in this paper has distinctly classical, but also Schumpeterian and Keynesian features. The main analysis is explored in an aggregate (one goods) setting, but many of the results carry over to a multi-sectoral setting. Sections 2 and 3 present the main components of the model in its equilibrium setting. Here we show the classical wage-profits and consumption vs. investment/growth trade-offs. We also show the conditions which have to be satisfied for full employment growth. Important are distributive requirements - particularly the determination of the real wage - which allow the economy to achieve full employment in the face of (positive) productivity shocks and changes in the available labour force. This issue is further explored in section 4 in the context of changing (exogenous) growth rates of productivity and the labour force. Real wage adjustments are also required if the economy is to remain on a full employment path and profit-receivers change their behaviour with respect to investment vs. consumption spending. Section 5 moves to discuss off-steady state analysis and introduces further components into the model. In 'disequilibrium' there is no immediate price-to-cost adjustment; thus Schumpeterian rents emerge and we allow for some bargaining between workers and capitalists over these rents. In the wage equation features also the impact of the unemployment rate. Hence, as the economy absorbs a (positive) productivity shock, there is a shift of income shares towards 'rents', and different distributive scenarios of such rents can emerge. The next important ingredient to understand the growth implications is to look at the 'spending' pattern out of these rents: these can go into consumption spending, investment spending or they 'leak out' of the real system (e.g. into liquid or financial assets). We show that the classical results obtained earlier with regard to wage-profit consumption-growth trade-offs can be seriously modified when such 'off-equilibrium' dynamics are explored. Here, both Schumpeterian and Keynesian features emerge. Section 6 models the interaction between the financial and the real sector of the economy. We show that the financial sector can contribute through 'leakages' to a fall in output growth (Keynesian perspective) or contributes positively to growth through the pre-financing of investment (Schumpeterian perspective). In simulations we also introduce an 'endogenous growth' element into our model. One of the motivations behind the study is to arrive at an understanding of the so-called 'Solow Paradox', i.e. that there might be evidence of a strong positive technology boost which, however, does not translate - at least for some time - into higher economic growth.

*JEL-Classification: E11, E12, O40, O41*

*Keywords: technical change, effective demand, economic growth, distribution, classical and Schumpeterian dynamics, technological rents, endogenous growth, dynamic economic modelling*

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# TECHNICAL CHANGE, EFFECTIVE DEMAND AND ECONOMIC GROWTH

Michael A. Landesmann and Robert Stehrer <sup>1</sup>

## 1 Introduction

The model developed in this paper has distinctly classical, but also Schumpeterian and Keynesian features. The analysis is explored in an aggregate (one good) setting, but many of the results carry over to a multi-sectoral setting (for the latter see Landesmann and Stehrer, 2000; Stehrer, 2001). Sections 2 and 3 present the main components of the model in its equilibrium setting. Here we show the classical wage-profits and consumption vs. investment/growth trade-offs. We also show the conditions which have to be satisfied for full employment growth. Important are distributive requirements - particularly the determination of the real wage - which allow the economy to achieve full employment in the face of (positive) productivity shocks and changes in the available labour force. This issue is further explored in section 4 in the context of changing (exogenous) growth rates of productivity and labour force growth. Real wage adjustments are also required if the economy is to remain on a full employment path and profit-receivers change their behaviour with respect to investment vs. consumption spending. Section 5 moves to discuss off-steady state analysis and introduces further components into the model. In 'disequilibrium' there is no immediate price-to-cost adjustment; thus Schumpeterian rents emerge and we allow for some bargaining between workers and capitalists over these rents. In the wage equation features also the impact of the unemployment rate. Hence, as the economy absorbs a (positive) productivity shock, there is a shift of income shares towards 'rents', and different distributive scenarios of such rents can emerge. The next important ingredient to understand the growth implications is to look at the 'spending' pattern out of these rents: these can go into consumption spending, investment spending or they 'leak out' of the real system (e.g. into liquid or financial assets). We show that the classical results obtained earlier with regard to wage-profit consumption-growth trade-offs can be seriously modified when such 'off-equilibrium' dynamics are explored. Here, both Schumpeterian and Keynesian features emerge. Section 6 attempts to model the interaction between the financial and the real sector of the economy more fully. We discuss both the 'leakage' effects but also the role which pre-financing of investments by financial institutions plays in absorbing productivity shocks and affecting economic growth. Finally, we introduce endogenous (productivity) growth features into the model. Section 7 illustrates further some of the features of the model through simulations. One of the motivations behind the study is to arrive at an understanding of the so-called 'Solow

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Paradox', i.e. that there might be evidence of a strong positive technology boost which, however, does not translate - at least for some time - into higher economic growth.

There exists of course an immense literature dealing with the questions which are discussed within the modelling framework of this paper. These questions are at the heart of economic theory since the beginning. There are a number of recent publications applying a classical framework and using different closures of the model (for some of the principal contributions, see e.g. Marglin, 1984; Dutt, 1990; Taylor, 1991). The difference of the analysis presented in this paper and most of the earlier contributions lies in the focus on disequilibrium (or transitory) dynamics. This is where we demonstrate Schumpeterian and Keynesian features of the model. The model is robustly anchored in steady-state properties but the emphasis lies in the reactive processes which emerge when there are 'shocks' (particularly productivity shocks) to which the steady-state structures are exposed. Previous contributions by the authors (see Landesmann and Stehrer, 2000; Stehrer, 2001) have shown the usefulness of this type of approach to study the ways how structural change can impinge upon the growth features of the system. Thus the model has proved useful when studying the interaction between macroeconomic dynamics and structural change within the context of a multisectoral model (for earlier contributions see Goodwin, 1986; Pasinetti, 1981; Goodwin and Punzo, 1987).

In this paper in contrast we follow up a number of features of the macroeconomic dynamics in more detail in the context of an aggregate model. The paper extensively discusses the properties of the model in the steady-state (stationary or growing) with the aim to set up a dynamic framework which is also applied for disequilibrium states and transitory (i.e. between steady-states) dynamics. As will be seen, we first analyse the model as far as possible in the case of steady-state growth. Although some analytical results can be derived for transition paths we also use simulations to show the dynamics of the model in disequilibrium states. The study of the steady-state properties of the model (i.e. the existence of solutions and the stability properties) is important for a proper understanding of the transitory and out-of-equilibrium dynamics of the model. We are able to show that both the long-run outcome of output and employment paths as well as the transitory dynamics depends heavily on behavioural adjustments in the transitory phases, particularly in phases of rapid technological progress. This is where we think the contribution of this paper lies in relation to the other literature in this field.

As mentioned above, in sections 6 and 7 we discuss the integration of a financial sector on the performance of the real sector. Although we do not set up a complete model of the financial sector the effects on real growth and distribution are emphasized. This part thus contributes to the literature on interactions between the real and the financial sector. For important recent statements of the financial-real interaction in a Keynesian framework see e.g. Flaschel et al. (1997) and Foley (1986).

## 2 The model

In this paper we explore a one good model. Most results can be extended to the multisectoral case (see Landesmann and Stehrer, 2000; Stehrer, 2001). In this section we present

the main accountancy relationships of the model.

## 2.1 Technology

Technology is denoted by a pair of input coefficients  $a$  and  $b$  where  $a$  denotes the input coefficient of intermediate goods and  $b$  the input coefficient for labour. The standard relationships are:

1.  $aq(t)$  is the intermediate demand for production of output  $q(t)$
2.  $(1 - a)q(t)$  is the amount of output  $q$  not used as intermediate input
3.  $(1 - a)^{-1}f(t)$  gives the necessary gross production for a final demand  $f(t)$

The labour input coefficient  $b$  is assumed to be strictly positive (thus we assume that some labour is always used in production). Labour productivity is then  $b^{-1}$ . As indicated by the time indexing, we shall take account of changes in labour productivity  $b^{-1}(t)$ , but assume that the input coefficient  $a$  remains fixed.

## 2.2 Wages, unit labour costs, costs and prices

The (nominal) wage rate is denoted by  $w(t)$  and unit labour costs are defined as  $b(t)w(t)$ .

Total unit costs are then  $c(t) = p(t)a + w(t)b(t)$  and prices are given by  $p(t) = (1 + \pi)c(t)$ . where  $\pi$  refers to a long-run mark-up ratio. This formulation implies that wages are paid at the beginning of the period, i.e. *ante factum*.

The equilibrium of the price system can easily be determined in the case where the coefficient  $a$ , and  $w$ ,  $b$  and thus the unit labour costs  $wb$  and the mark-up ratio  $\pi$  are exogenously given and constant. The equilibrium price is

$$p = (1 + \pi)wb[1 - (1 + \pi)a]^{-1}$$

Given the price one may express the wage rate as  $w = \frac{p[1 - (1 + \pi)a]}{(1 + \pi)b}$ . The real wage in equilibrium is

$$\frac{w}{p} = \frac{[1 - (1 + \pi)a]}{(1 + \pi)b(t)}$$

and the ratio of unit labour costs to prices is

$$\frac{w}{p}b = \frac{1 - (1 + \pi)a}{1 + \pi}$$

In the special case where  $\frac{w}{p} = 0$  we obtain the maximum mark-up rate  $\pi^{max} = \frac{1}{a} - 1$ .

## 2.3 Profits and rents

The per unit (equilibrium) profit is defined as a mark-up on costs

$$r(t) = \pi c(t) = \pi(p(t)a + w(t)b(t))$$

In the case that the price is not at the equilibrium level there arise per unit-rents which are defined as:

$$s(t) = p(t) - (1 + \pi)c(t) = p(t)[1 - (1 + \pi)a] - (1 + \pi)w(t)b(t)$$

In equilibrium these rents are zero as one can show by inserting the equilibrium price

$$\begin{aligned} s &= (1 + \pi)wb[1 - (1 + \pi)a]^{-1}[1 - (1 + \pi)a] - (1 + \pi)wb \\ &= (1 + \pi)wb - (1 + \pi)wb \\ &= 0 \end{aligned}$$

In the following we define a scalar  $m(t)$  which adds up profits  $r(t)$  and rents  $s(t)$ , which we refer to as 'total profits',

$$m(t) = r(t) + s(t) = p(t) - c(t) = p(t)(1 - a) - wb(t)$$

In equilibrium total profits per unit in real terms are

$$\begin{aligned} \frac{m}{p} = \frac{r}{p} &= (1 - a) - \frac{w}{p}b \\ &= (1 - a) - \frac{[1 - (1 + \pi)a]}{(1 + \pi)} \\ &= \frac{\pi}{1 + \pi} \end{aligned}$$

## 2.4 Demand components and total demand

Next we turn to the quantity system. Total supply/production is denoted by  $q(t)$ . Demand consists of three different components. Demand for (1) intermediate goods, (2) consumption goods, and (3) investment goods. We shall discuss them in detail below.

### 2.4.1 Demand for intermediate products

Demand for intermediate goods used in production is  $aq(t)$ . Intermediate goods can be interpreted as capital stock, which in this model consists only of circulating capital.

### 2.4.2 Wage income

Workers earn a nominal wage rate  $w(t)$  and thus total nominal wage income is  $w(t)b(t)q(t)$  as  $l(t) = b(t)q(t)$  is labour demand. In real terms this is

$$f_w(t) = \frac{w(t)}{p(t)}b(t)q(t) = \frac{w(t)}{p(t)}l(t)$$

With constant  $b, w, \pi, a, p$  demand out of wage income depends only on the equilibrium real wage and the quantity of labour demanded

$$f_w(t) = \frac{w}{p}bq(t) = \frac{w}{p}l(t)$$

We shall henceforth assume in classical fashion that all labour income is spent on consumption. By this assumption,  $f_w(t)$  also denotes the amount of goods demanded by workers.

### 2.4.3 Income out of profits

The nominal total profit income is  $m(t)q(t)$ . Expressed in real terms this is

$$\begin{aligned} f_m(t) &= \frac{m(t)}{p(t)}q(t) \\ &= \frac{p(t)(1-a) - w(t)b(t)}{p(t)}q(t) \\ &= \left( (1-a) - \frac{w(t)}{p(t)}b(t) \right) q(t) \end{aligned}$$

In fact, this is the residual real income, i.e. total output  $q(t)$  minus demand for intermediate inputs and real wage income. How income from profits is turned into spending on investment goods and thus affects the growth of output is specified below (section 3.2). Until section 3.5 we shall (again in classical fashion) assume that all profits are spent on investment. In the case that  $m(t) = 0$  one gets  $(1-a) = \frac{w(t)}{p(t)}b(t)$  which means that the workers in the economy attain the maximum amount of consumption.

### 2.4.4 Total demand

If all incomes are spent then total demand  $d(t)$  is the sum of the three components

$$d(t) = aq(t) + f_m(t) + f_w(t)$$

In equilibrium total demand equals total supply

$$\begin{aligned} q(t) &= aq(t) + f_m(t) + f_w(t) \\ &= aq(t) + \left[ (1-a) - \frac{w(t)}{p(t)}b(t) \right] q(t) + \frac{w(t)}{p(t)}b(t)q(t) \end{aligned}$$

The types of demand and shares of income (intermediate, demand from profit receivers and demand from workers) in real terms are given by

$$1 = a + \left[ (1-a) - \frac{w(t)}{p(t)}b(t) \right] + \frac{w(t)}{p(t)}b(t)$$

Before presenting the solutions to this model we specify labour demand and supply and hence unemployment.

## 2.5 Labour demand and supply

As already mentioned above, labour demand is given by  $l(t) = b(t)q(t)$ . We denote labour supply by  $k(t)$ . We define the unemployment rate as  $u(t) = \frac{k(t)-l(t)}{k(t)}$ .

## 3 Equilibrium solutions

### 3.1 Existence

In equilibrium total supply must equal total demand,  $q(t) = d(t)$ . The condition stated above may be rewritten as

$$q(t) = d(t) = \left( a + \frac{m(t)}{p(t)} + \frac{w(t)b(t)}{p(t)} \right) q(t)$$

Rearranging this condition yields

$$\left( (a - 1) + \frac{m(t)}{p(t)} + \frac{w(t)}{p(t)}b(t) \right) q(t) = 0$$

In the case that all income is spent this condition holds at each point in time  $t$  (i.e. even in the case that the price system is not in equilibrium) as

$$\begin{aligned} 0 &= (a - 1) + \frac{w(t)}{p(t)}b(t) + \frac{p(t)(1 - a) - w(t)b(t)}{p(t)} \\ &= (a - 1) + \frac{w(t)}{p(t)}b(t) + (1 - a) - \frac{w(t)}{p(t)}b(t) \end{aligned}$$

It is clear that the level of output cannot be determined by this condition alone.

### 3.2 Steady-state (classical) growth

In order to examine the steady-state we take the input coefficients  $a$  and  $b$  as constant. Wage rates and prices are at their equilibrium values respectively. Further, rents  $s$  are equal to zero and real per-unit profit is  $\frac{\pi}{1+\pi}$  as shown above, hence  $m = \pi c$ . Further we assume that all incomes are spent and that there is no shortage of labour. The demand for labour which equals supply is given by

$$k(t) = l(t) = bq(t)$$

Returning to demand out of profits and assuming that  $\frac{m}{p}q(t)$  is fully reinvested, the demand and supply for inputs will be growing at a rate

$$\frac{f_m(t)}{aq(t)} = \frac{1}{a} \frac{m}{p} = \frac{1}{a} \frac{\pi}{1 + \pi} \equiv \gamma_{aq} = \gamma_q$$

In this model with circulating capital this represents the (gross and net) accumulation rate (a positive depreciation rate can be easily inserted). The growth path of the economy can then be written as<sup>2</sup>

$$\begin{aligned}\dot{q}(t) &= (1 + \gamma_q)(1 - a)^{-1}(f_m(t) + f_w(t)) - q(t) \\ &= [1 - (1 + \gamma_q)a]^{-1} (1 + \gamma_q)f_w(t) - q(t)\end{aligned}$$

By using  $(f_m(t) + f_w(t)) = (1 - a)$  this yields

$$\frac{\dot{q}(t)}{q(t)} = \gamma_q = \frac{1}{a} \frac{\pi}{1 + \pi}$$

This dynamic equation shows that profit receivers finance accumulation out of the current income. If  $\pi = 0$  the economy remains stationary (all surplus  $(1 - a)q(t)$  being consumed). Note that for  $\pi = \pi^{max} = \frac{1}{a} - 1$ , i.e the maximum mark-up rate, this reduces to

$$\gamma_q^{max} = \frac{1}{a} \frac{\frac{1}{a} - 1}{1 + \frac{1}{a} - 1} = \frac{1}{a} - 1 = \pi^{max}$$

In this case the economy grows at the maximum rate which is equal to the profit rate.<sup>3</sup>

### 3.3 Full-employment solution

In the steady-state full employment is guaranteed when labour supply equals demand at the initial point of time  $k(0) = l(0) = bq(0)$  and there are neither labour supply restrictions nor insufficient labour demand in the course of growth, i.e.

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{l}(t)}{l(t)} = \frac{\dot{q}(t)}{q(t)}$$

---

<sup>2</sup>The second formulation is analogous to the solution of a dynamic Leontief model with only circulating capital (see e.g. Pasinetti, 1977). This can be shown in the following way:

$$\begin{aligned}\frac{\dot{q}}{q} = \gamma_q &= (1 + \gamma_q)(1 - a)^{-1}(1 - a) - 1 = (1 + \gamma_q)(1 - a)^{-1}(f_m + f_w) - 1 \\ &= [1 - (1 + \gamma_q)a]^{-1} (1 + \gamma_q) [1 - (1 + \gamma_q)a] - 1 \\ &= [1 - (1 + \gamma_q)a]^{-1} (1 + \gamma_q) ((1 - a) - a\gamma_q) - 1 \\ &= [1 - (1 + \gamma_q)a]^{-1} (1 + \gamma_q) \left( (1 - a) - \frac{m}{p} \right) - 1 \\ &= [1 - (1 + \gamma_q)a]^{-1} (1 + \gamma_q) \frac{f_w}{q} - 1\end{aligned}$$

<sup>3</sup>This result also holds if wages are paid *post factum*. In this case the price has to be formulated as  $p(t) = (1 + \pi)p(t)a + w(t)b(t)$  and the equilibrium price is  $p = wb[1 - (1 + \pi)a]^{-1}$ . Demand out of profits and rents which equals investment by assumption then is  $f_m = \frac{m}{p}q = \frac{\pi pa}{p} = \pi a$ . The growth rate then turns out to be  $\gamma_q = \frac{1}{a}\pi a = \pi$ . In the case that  $\pi = \pi^*$  the economy grows at the maximum growth rate  $\gamma_q^* = \pi^* = \frac{1}{a} - 1$ .

### 3.4 Exogenous technical progress

We first introduce a growth of labour productivity at a constant exogenously given rate  $\gamma_b$  which implies that the input coefficient is decreasing

$$\dot{b} = -\gamma_b b$$

Under the assumptions that  $\pi = 0$  and that the labour supply is constant  $\dot{k} = 0$ , using the full-employment assumption<sup>4</sup> implies that

$$\frac{\dot{l}}{l} = \frac{\dot{b}}{b} + \frac{\dot{q}}{q} = 0$$

and thus

$$-\frac{\dot{b}}{b} = \frac{\dot{q}}{q} \quad \text{or} \quad \gamma_b = \bar{\gamma}_q$$

This means, that for keeping employment at the initial full-employment level, output has to grow with the same rate as labour productivity ( $\bar{\gamma}_q$  refers to 'full employment output growth'). To guarantee that output grows at this rate two conditions must be met: First, enough demand has to be created and, second, enough resources have to be reinvested. As shown above, the condition

$$\bar{\gamma}_q = \frac{1}{a} \frac{m(t)}{p(t)}$$

has to be satisfied, but now with  $\bar{\gamma}_q$  given (rather than  $\pi$  given). Rearranging gives

$$\bar{\gamma}_q = \frac{1}{a} \frac{m(t)}{p(t)} = \frac{1}{a} \frac{p(t) - p(t)a - w(t)b(t)}{p(t)} = \frac{1}{a} \left[ (1 - a) - \frac{w(t)b(t)}{p(t)} \right]$$

As  $\gamma_q$  and  $(1 - a)$  are constant,  $\frac{w(t)b(t)}{p(t)}$  also has to be constant, thus

$$\begin{aligned} 0 &= \frac{\dot{w}}{w} + \frac{\dot{b}}{b} - \frac{\dot{p}}{p} \\ \gamma_w - \gamma_p &= \gamma_b \end{aligned}$$

This says, that the real wage has to grow with the rate of productivity  $\gamma_b$ . This condition assures that consumption demand rises in line with productivity growth. If it would rise beyond productivity growth there would not be enough profits to allow the required net accumulation to take place to assure full employment. If it would grow below that rate a mismatch between net accumulation (i.e. increase in productive capacity) and final consumption demand would take place.

We can also see that the ratio of unit labour costs to price has to be lower than it would be if the economy would not undergo positive productivity growth. We get

$$\bar{\gamma}_q = \frac{1}{a} \left[ (1 - a) - \frac{w(t)}{p(t)} b(t) \right]$$

---

<sup>4</sup>A growing labour force is discussed below.

Rearranging this expression yields

$$\frac{w(t)}{p(t)}b(t) = (1 - a) - a\bar{\gamma}_q = (1 - a) - a\gamma_b$$

This condition assures that enough resources are available for keeping the economy growing at the full employment path.

### 3.5 Exogenous population growth

Next, we turn to a situation where the labour force is growing at an exogenously given and constant rate,  $\gamma_k$ . For keeping the economy at the full employment level, this means that labour demand must rise with the (exogenously given) rate of the labour supply

$$\gamma_k = \gamma_l$$

and thus the growth rate of output must be

$$\bar{\gamma}_q = \gamma_k + \gamma_b$$

Again, as  $\gamma_b$  and  $(1 - a)$  are assumed to be constant, the term  $\frac{w(t)}{p(t)}b(t)$  has to be constant which analogously to above yields

$$\gamma_w - \gamma_p = \gamma_b$$

i.e. the real wage has to grow at the rate of productivity. But, different from above, the ratio of unit labour costs to price will have to be even lower, as

$$\gamma_k + \gamma_b = \frac{1}{a} \left[ (1 - a) + \frac{w(t)}{p(t)}b(t) \right]$$

or equivalently

$$\frac{w(t)}{p(t)}b(t) = (1 - a) - a(\gamma_k + \gamma_b)$$

This implies that the consumption level must be lower to 'save' relatively more output for reinvestment and hence for an expansion of capacity. Output must be growing not only to keep employment at a constant level (thus only capturing the labour saving effects of technical progress) but also to create employment for the growing labour force (population). The balance between necessary total demand and the growth of productive capacity is created by adjustments in the real wage and the demand from new workers.

### 3.6 Endogenous technical progress

In the simulations below we also introduce endogenous technical progress along the Kaldor-Verdoorn mechanism. Labour input coefficients are assumed to be falling as a



function of the growth rate of output (or of intermediate inputs, i.e. investments; within the specification of the model, the two cases have the same dynamic implications):

$$\dot{b} = -\kappa_b a \frac{\dot{q}}{q} b = -\kappa_b a \gamma_q b$$

where  $\kappa_b \geq 0$  denotes the 'Kaldor-Verdoorn parameter'.

To simplify we also assume that profit margins are constant. In equilibrium, with  $\frac{m}{p}$  constant, the growth rate of the economy is given by  $\gamma_q = \frac{1}{a} \frac{m}{p}$  and productivity growth is given by  $\frac{\dot{b}}{b} = -\kappa_b \frac{m}{p}$ . For profit per unit  $\frac{m}{p} = (1 - a) - \frac{wb}{p}$  to be constant, the labour share has to be constant too, i.e.

$$\frac{\dot{w}}{w} + \frac{\dot{b}}{b} - \frac{\dot{p}}{p} = 0 \quad \text{or} \quad \frac{\dot{w}}{w} - \frac{\dot{p}}{p} = \kappa_b a \gamma_q$$

i.e. the real wage rate grows in line with productivity. In equilibrium, labour demand grows at a rate determined by the difference of the growth of output and the growth of productivity. Assuming full employment und using the (dynamic) full-employment condition yields

$$\frac{\dot{l}}{l} = \frac{\dot{b}}{b} + \frac{\dot{q}}{q} = -\kappa_b a \gamma_q + \gamma_q = (1 - a \kappa_b) \gamma_q = \frac{\dot{k}}{k}$$

Thus for  $\kappa_b = 0$  labour demand (supply) must rise at the same rate as output, as shown in section 3.3. With  $\kappa_b > 0$  this condition is relaxed as part of the additional labour necessary for keeping the economy growing at a rate  $\gamma_q$  comes from labour-saving (endogenous) technical progress. For  $\kappa_b = \frac{1}{a}$  the growth rate of employment is zero and the result is similar to the case of exogenous productivity growth in section 3.4 (the labour cost share becomes  $\frac{wb}{p} = (1 - a) - \frac{m}{p}$ ).

The demand condition is satisfied as there are additional workers and as real wages are rising, the relative weights of these factors being determined by the value of  $\kappa_b$ . Note that in this case, output, employment and the real wage grow at different rates.

### 3.7 Consumption out of profits

So far we have assumed that workers income is spent on consumption and income out of profits is spent on investment. We now relax this assumption and consider the case in which profit earners spend part of their income on consumption and the other part on investment. The part of income spent on consumption is denoted by  $\kappa_m$  with  $0 \leq \kappa_m \leq 1$ .

In steady-state growth the demand and supply for inputs will in this case be growing at the rate

$$(1 - \kappa_m) \frac{f_m(t)}{aq(t)} = (1 - \kappa_m) \frac{1}{a} \frac{m}{p} = (1 - \kappa_m) \frac{1}{a} \frac{\pi}{1 + \pi} \equiv (1 - \kappa_m) \gamma_q$$

where  $\gamma_q = \frac{1}{a} \frac{1}{1-a}$ . Thus the growth rate of the economy is reduced relative to the case in which all profits were reinvested. The growth path of the economy is then

$$\dot{q}(t) = (1 + (1 - \kappa_m)\gamma_q)(1 - a)^{-1} \left( \underbrace{(1 - \kappa_m)f_m(t)}_{\text{Investment demand}} + \underbrace{\kappa_m f_m(t) + f_w(t)}_{\text{Consumption demand}} \right) - q(t)$$

Also in the case of exogenous technological progress and population growth one has to take into account that part of total profit income is spent on consumption. The condition for full-employment remains

$$\bar{\gamma}_q = \gamma_k + \gamma_b$$

For the economy to provide enough resources to grow at the rate  $\bar{\gamma}_q$  the condition becomes

$$\bar{\gamma}_q = (1 - \kappa_m) \frac{1}{a} \frac{m(t)}{p(t)} = (1 - \kappa_m) \frac{1}{a} \left( (1 - a) - \frac{w(t)}{p(t)} b(t) \right)$$

which yields a per unit real wage

$$\frac{wb}{p} = (1 - a) - a \frac{\gamma_k + \gamma_b}{1 - \kappa_m}$$

where

$$(1 - a) \geq a \frac{\gamma_k + \gamma_b}{1 - \kappa_m}$$

has to be satisfied. This condition says that given that there is consumption out of profits, some the required extension of capacities has to be financed by lower real wage income. (In the case that  $\kappa_m = 1$  there would be no possibility for the economy to grow whatsoever.)

## 4 Underemployment equilibria

### 4.1 Stationary economy

In this section we return to an economy in which all profits are invested. In such an economy, with no technical progress, no exogenous population growth and  $\pi = 0$ , the equilibrium of the economy is given by  $\gamma_q = 0$  and  $(1 - a) = \frac{w}{p}b$ . The output is allocated to (intermediate) investment and consumption demand so that the economy exactly reproduces itself over time. If  $(1 - a) < \frac{w}{p}b$  the economy would contract and if  $(1 - a) > \frac{w}{p}b$  the economy would be able to grow.

What happens if the economy starts in an unemployment situation  $u(t) > 0$ . The answer is quite simple: nothing, if the factum of unemployment would have no impact on the other variables. One way out of the unemployment situation will be (this is shown below in the disequilibrium dynamic model), that the wage rate falls, rents emerge and these rents enable the economy to grow until it reaches the full-employment equilibrium. The same situation is reached if prices are growing which again leads to emerging rents via falling real wages. These two processes both have in common that the real wage has

to be lower (at least temporarily) to reach the full-employment situation. An analogous analysis can be derived for an influx of new labour (migration).

Another possibility to reach full-employment seems to be to switch to more labour-intensive production techniques, which implies a rising  $b$ . But this alone does not do the job! With sticky real wages this measure would only raise the ratio  $\frac{w}{p}b$  which would lead to a contraction of the economy in the long run. Only if the real wage rate is adjusted for lower productivity so that  $\frac{w}{p}b$  remains constant unemployment may diminish. Note that this solution implies that the real wage is lower for all time.

## 4.2 Constant growth economy

Above we have seen that - given initial conditions - the model exhibits a full-employment growth path at a growth rate

$$\gamma_q = \gamma_b + \gamma_k$$

Further the condition

$$\frac{w}{p}b = (1 - a) - a(\gamma_b + \gamma_k)$$

has to be satisfied. Analogously to the constant economy case we can state that the economy is growing at a lower rate if

$$(1 - a) - a(\gamma_b + \gamma_q) < \frac{w}{p}b$$

and the economy has the potential to grow faster if

$$(1 - a) - a(\gamma_b + \gamma_q) > \frac{w}{p}b$$

In the case that the economy is starting at the full-employment level this means, in the first case, that the actual growth rate is lower than the necessary growth rate to keep the economy at the full-employment level (the economy is constrained by the available intermediate inputs). In the second case the potential growth rate of the economy is higher than the necessary growth rate to keep the economy at the full-employment level. In this situation the economy is in a sense constrained by a labour (and consumption) shortage. Profit-receivers can react to this by investing less than is available as retained earnings (rents) and the difference between profits and required investments may be consumed, exported, or saved in some form which does not lead directly to additional productive investment.

## 4.3 Technology shocks and emerging unemployment equilibria

### 4.3.1 Stationary economy

We first discuss the case with  $\gamma_k = \gamma_b = 0$  and with the economy at the full-employment level. What happens if there is a sudden and once for all rise in the productivity level to  $b_1 < b_0$ ? First of all  $\frac{w}{p}b$  would fall and thus  $(1 - a) > \frac{w}{p}b_1$ . Thus there would be a

potential for growth as inputs are saved. If these are actually reinvested the economy may expand and come back to full employment. Now assume that the nominal wage rate is adjusted to this increase in labour productivity such that  $\frac{w_1}{p}b_1 = \frac{w_0}{p}b_0$ . In this case the economy would produce exactly the same amount as before the technological shock but there are less workers employed. If these unemployed workers have no influence on wage setting they will remain unemployed forever.

### 4.3.2 Growing economy

In a growing economy the implications are similar. A sudden rise of the productivity level would imply that the economy may expand faster because of the emergence of rents. If real wages adjust immediately to the increase in labour productivity (and hence no rents emerge) then the economy would continue to grow at the same (long-term) rate of growth as before but at a lower (but not constant) level of employment.<sup>5</sup>

## 4.4 Conclusions

What can be learned from this? To achieve full-employment in an economy with has to change its growth path (due to increased productivity or population growth) two conditions have to be satisfied:

1. Rents have to emerge (in our model by falling real per unit wages) to enable the economy to grow at a higher rate.
2. These rents have to be reinvested despite (temporarily) falling real wages and thus (in the short-run) lower consumption demand. Higher effective demand then comes from investment which widens the productive capacities and thus creates the condition for employment absorption.

## 5 Out-of-equilibrium dynamics

In this section we shall first restate the dynamic full employment conditions when productivity and labour force growth change over time and then introduce some further behavioural ingredients into the model which characterise out-of-equilibrium behaviour.

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<sup>5</sup>A referee pointed out that a full employment situation could be reached without emerging rents if the capital coefficient  $a$  adjusts accordingly, i.e. no rents have to emerge if  $a$  is falling when compared to the case that  $a$  is constant. But it is still the case that the workers have either a lower income share if  $a$  is constant (with a lower real wage) or the workers have a lower income share given the adjusted coefficient  $a'$  although the real wage might be constant which would, however, still be lower than the long-run real wage and would imply a lower than the long-run income distribution implied by the new  $a'$ . If the labour income share remains constant rents would emerge by a higher productivity of the system which could be accumulated such that the economy returns to the long run coefficient  $a$ .

## 5.1 Full-employment path

Above we have seen that keeping the economy at the full employment level the (dynamic) condition  $\bar{\gamma}_q = \gamma_b + \gamma_k$  has to be satisfied. In this section we discuss the implications for the wage movements in the case that technical progress and labour force growth is not constant but that the growth rates change over time. As we are focusing on transitory dynamics in the following we simplify the analysis by setting the long-run mark-up to zero, i.e.  $\pi = 0$ . In a situation where all investment spending occurs from (transitory) profits  $s(t)$ , the dynamic full employment condition requires that

$$\bar{\gamma}_q(t) = \frac{1}{a} \frac{m(t)}{p(t)} = \frac{1}{a} \frac{(1-a)p(t) - w(t)b(t)}{p(t)} = \frac{1}{a} \left( (1-a) - \frac{w(t)}{p(t)} b(t) \right)$$

has to be satisfied. In the rest of this section we assume that prices are fixed, i.e.  $\dot{p} = 0$  to study the implications for wage setting. For fixed prices this can also be seen as real wage policy. Thus the condition above reduces to

$$\bar{\gamma}_q(t) = \frac{1}{a} \left( (1-a) - \frac{w(t)}{p} b(t) \right)$$

and thus

$$w(t) = \frac{1}{b(t)} p \left( (1-a) - a \bar{\gamma}_q(t) \right)$$

i.e. the wage rate at time  $t$  is determined by the productivity level and the growth rates  $\bar{\gamma}_q(t) = \gamma_b(t) + \gamma_k(t)$ . Differentiating with respect to time gives

$$\begin{aligned} \dot{w}(t) &= -p \frac{1}{b(t)^2} \left( a(\dot{\gamma}_b + \dot{\gamma}_k) b(t) - a \dot{b}(t) (\gamma_b + \gamma_k) + \dot{b}(t) (1-a) \right) \\ &= -p \frac{1}{b(t)} \left( a(\dot{\gamma}_b + \dot{\gamma}_k) - a \gamma_b^2 - a \gamma_b \gamma_k + \gamma_b (1-a) \right) \end{aligned}$$

Rearranging and using the condition  $\frac{w(t)}{p} b(t) = (1-a) - a(\gamma_b + \gamma_k)$  yields<sup>6</sup>

$$\frac{\dot{w}}{w} = -\gamma_b - a(\dot{\gamma}_b + \dot{\gamma}_k) \left( (1-a) - a(\gamma_b + \gamma_k) \right)^{-1}$$

Note, that for  $\dot{\gamma}_b = \dot{\gamma}_k = 0$  these conditions are the same as stated in sections 3.4 and 3.5, respectively.

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<sup>6</sup>A similar condition holds for fixed wage rates and flexible prices

$$\frac{\dot{p}}{p} = \gamma_b + a(\dot{\gamma}_b + \dot{\gamma}_k) \left( (1-a) - a(\gamma_b + \gamma_k) \right)^{-1}$$

If both, wage rate and price, are flexible the condition is

$$\frac{\dot{w}}{w} - \frac{\dot{p}}{p} = -\gamma_b - a(\dot{\gamma}_b + \dot{\gamma}_k) \left( (1-a) - a(\gamma_b + \gamma_k) \right)^{-1}$$

## 5.2 Dynamic formulation of prices and the emergence of rents

Price adjustment can be modeled as a price to cost (plus normal mark-up) adjustment

$$\dot{p}(t) = \delta_p [(1 + \pi)(p(t)a + w(t)b(t)) - p(t)]$$

with  $0 < \delta_p \leq 1$  being the adjustment parameter. With  $\delta_p < 1$  and a positive technology shock ( $a$  or  $b(t)$  falling) rents emerge in addition to profits.<sup>7</sup>

We now model the distribution of rents. As rents emerge, a certain proportion of such rents gets distributed to workers; this proportion depends on the bargaining strength of workers. In that case the portion of rents at the disposal of capital owners is

$$(1 - \kappa_s)s(t) = (1 - \kappa_s)[p(t) - (1 + \pi)c(t)]$$

where  $0 \leq \kappa_s \leq 1$  denotes the share of unit rents which goes to workers (see also the wage equation below). Total profits then have a normal mark-up and a rent component

$$m(t) = r(t) + (1 - \kappa_s)s(t)$$

to which we refer as 'retained profits'. Out of these,  $\kappa_m m(t)q(t)$  is spent for consumption and  $(1 - \kappa_m)m(t)q(t)$  for investment.

## 5.3 Labour market dynamics

The (out-of-equilibrium) dynamics of the wage rate is now modeled as follows:

$$\dot{w} = \kappa_s \frac{s(t)}{b(t)} + \kappa_u u(t)w(t)$$

where  $u(t)$  denotes the unemployment rate  $u(t) = \frac{k(t) - l(t)}{k(t)}$ . We assume that  $0 \leq \kappa_s \leq 1$  and  $\kappa_u \leq 0$ . The first term means that part of transitory rents are distributed to workers (e.g. for compensating them for the increases in productivity) and the second term imposes a negative effect of unemployment on the growth of the nominal wage rate.

For reasons which will become clear below we assume that population  $n(t)$  grows at a constant rate  $\gamma_n$ , i.e.  $\frac{\dot{n}(t)}{n(t)} = \gamma_n$ . Then labour supply is modeled as

$$\dot{k}(t) = \delta_k (l(t) - k(t)) + \delta_n (k(t) - \zeta n(t))$$

where  $\zeta$  is the long-run participation rate.<sup>8</sup> We assume that

$$\delta_k = \begin{cases} \delta_{k,IN} > 0 & \text{if } l(t) - k(t) \geq 0 \\ \delta_{k,OUT} \geq 0 & \text{if } l(t) - k(t) < 0 \end{cases}$$

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<sup>7</sup>This rather mechanic specification represents the Schumpeterian insight that technological innovations happen through transitory changes in the market structures which accompany the introduction and diffusion of new technologies. The specification thus models the macro-dynamics of technical change and does not model the individuals firms' pricing behaviour.

<sup>8</sup>The model could also allow the inclusion of a changing participation rate, i.e.  $\zeta(t)$ , but we shall not introduce this in this paper.

This formulation allows for an asymmetric adjustment of the actual participation rate to positive or negative excess demand for labour (in general, one would expect a faster adjustment to positive excess demand and a relatively slow adjustment to negative excess demand). The actual participation rate may differ in the short to medium run from the long-term rate  $\zeta$  and is defined as

$$\frac{k(t)}{n(t)} = \frac{l(t) + u(t)}{n(t)}$$

Of course, the absolute constraint of labour supply is  $k(t) \leq n(t)$ .

## 5.4 Quantity dynamics

We now come to the dynamics of the quantity system outside the steady-state.

With the assumption that all incomes are spent the growth path is determined as before, i.e.

$$\dot{q}(t) = (1 + \gamma_q)(1 - a)^{-1}((1 - \kappa_m)f_m(t) + \kappa_m f_m(t) + f_w(t)) - q(t)$$

where the modification lies in the determination of spending on accumulation and the part of rents distributed to wages:

$$\gamma_q = (1 - \kappa_m) \frac{1}{a} \frac{f_m(t)}{q(t)} = (1 - \kappa_m) \frac{1}{a} \frac{m(t)}{p(t)} = (1 - \kappa_m) \frac{1}{a} \frac{r(t) + (1 - \kappa_s)s(t)}{p(t)}$$

This expression can be further rearranged to

$$\gamma_q = (1 - \kappa_m) \frac{1}{a} \left[ (1 - a) - \frac{w(t)b(t)}{p(t)} - \kappa_s(1 + \pi) \left( \frac{1 - (1 + \pi)a}{1 + \pi} - \frac{w(t)b(t)}{p(t)} \right) \right]$$

In equilibrium, i.e.  $\frac{wb}{p} = \frac{1 - (1 + \pi)a}{1 + \pi}$ , the economy is growing at the rate  $\bar{\gamma}_q = (1 - \kappa_m) \frac{1}{a} \frac{\pi}{1 + \pi}$ . This can be verified by inserting the equilibrium real per unit wage in the equation above. In general, the emergence of rents  $s(t)$  allows the economy to grow faster as more intermediate goods are available for investment allowing a faster growth rate. On the other hand the real wage will be lower than the equilibrium real wage. Note that the (supply equal) demand condition is satisfied in any case as it does not matter whether it is demand for consumption or for investment.

In disequilibrium with  $s(t) > 0$  and thus with actual real wages below the equilibrium level, i.e.  $\frac{w(t)b(t)}{p(t)} < \frac{1 - (1 + \pi)a}{1 + \pi}$ , the economy is able to grow faster. However, if part of the rents are (immediately) redistributed to workers as  $\kappa_s \geq 0$ , this lowers the growth rate as can be seen in the equation above. For  $\kappa_s = 0$  the equation reduces to the cases discussed above. For  $\kappa_s = 1$ , which means that all rents are distributed to the workers, the equation reduces to

$$\gamma_q = (1 - \kappa_m) \pi \frac{1}{a} \frac{p(t)a + w(t)b(t)}{p(t)} = (1 - \kappa_m) \pi \left( 1 + \frac{1}{a} \frac{wb}{p} \right)$$

In equilibrium this reduces to the growth rate  $\gamma_q = (1 - \kappa_m) \frac{1}{a} \frac{\pi}{1 + \pi}$ .

## 6 Financial assets, distributional dynamics and effective demand

In this section we continue with transitional dynamics where we again focus on the impact of technical change. We shall show that the impact of technical change on the growth path of an economy depends on the parameters which determine the distribution of rents, through price adjustments and adjustments in the labour market. We shall then show that the distribution of rent income between workers and capitalists has further repercussions on the growth path if the assumption is dropped that all income is being spent. We introduce a 'leakage-factor' which reflects the relative attractiveness of investing into the real or financial assets and/or the hesitancy of capitalists to proceed with ('real') investment spending when the growth of demand is not assured. Dropping the assumption that all incomes are being spent on goods leads us to introduce the possibility of saving in the form of the acquisition of financial assets.

### 6.1 Saving in financial assets out of retained earnings

We now allow for the possibility that capitalists are not spending all their income on goods purchases. So far, we have discussed that earnings from total profits are either reinvested or consumed. Both have exerted a demand effect (either for investment goods or consumption goods). The effect was that consumption out of profits reduces the growth rate of the economy as less is reinvested. We now introduce the possibility that part of the earnings are invested in financial assets and thus do not have a direct demand effect. This represents a 'leakage' from the real part of the economy. It also expresses the potential instability of investment demand, i.e. the hesitancy of ploughing back income into productive investment when there is no assurance of rising demand. This further contributes to the possibility that actual growth falls short of potential growth.

The retained earnings are distributed across different uses in the following way:

1. Spending for consumption:  $\kappa_m \left( (r(t) + (1 - \kappa_s)s(t))q \right)$
2. Spending for investment:  $(1 - \kappa_m) \left( (r(t) + (1 - \kappa_s)s(t))q \right)$

Investment is either in productive capacities or in financial assets:

- (a) Productive capacities:  $(1 - \eta^+)(1 - \kappa_m) \left( (r(t) + (1 - \kappa_s)s(t))q \right)$
- (b) Financial assets:  $\eta^+(1 - \kappa_m) \left( (r(t) + (1 - \kappa_s)s(t))q \right)$

$\eta^+ \geq 0$  denotes the share of investments in financial assets.

To concentrate on the effects of the factor  $\eta^+$  we assume that  $\kappa_m = 0$  and  $\kappa_s = 0$ . Demand for investment goods in the real sector reduces to  $(1 - \eta^+) \frac{m(t)}{p(t)} q(t)$  and the investment ratio which determines the growth rate is  $(1 - \eta^+) \frac{1}{a} \frac{m(t)}{p(t)}$ .



**Growth effects of investments in liquid assets** We now examine the impact of 'leakage' on the growth path of the economy. The growth path of the economy (with  $b$  constant) becomes

$$\dot{q}(t) = \left(1 + (1 - \eta^+) \frac{1}{a} \frac{m}{p}\right) (1 - a)^{-1} \left((1 - \eta^+) \frac{m}{p} + \frac{w}{p} b\right) q - q$$

With  $\frac{m}{p} > 0$  and constant real unit labour costs this can be written as

$$\frac{w}{p} b = (1 - a) - \frac{m}{p}$$

Inserting into the growth equation and rearranging yields

$$\begin{aligned} \frac{\dot{q}}{q} &= \left(1 + (1 - \eta^+) \frac{1}{a} \frac{m}{p}\right) (1 - a)^{-1} \left((1 - a) - \eta^+ \frac{m}{p}\right) - 1 \\ &= \frac{1 - a - \eta^+ \frac{m}{p}}{a(1 - a)} - \frac{\eta^+(1 - \eta^+)}{a(1 - a)} \left(\frac{m}{p}\right)^2 \end{aligned}$$

This implies an inverse U-shaped relationship between  $\eta^+$  and retained (profit) earnings  $\frac{m}{p}$  (see figure 6.1). Setting  $\frac{\dot{q}}{q} = 0$  gives the critical value of real retained earnings: larger

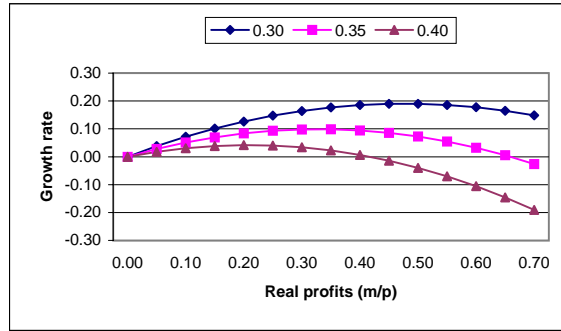


Figure 6.1: Relationship between (per unit) profit share and growth rate at different values of  $\eta^+$

retained earnings would mean a negative growth rate. The critical value is given by

$$\left(\frac{m}{p}\right)^{cr} = \frac{(1 - a) - \eta^+}{\eta^+(1 - \eta^+)}$$

(for  $\eta^+ > 0$ ). Positive growth of output requires that  $\eta^+ < 1 - a$ . Differentiating the growth rate with respect to  $\frac{m}{p}$  and setting to zero yields the growth maximising (the second derivative is negative) real retained earnings:

$$\left(\frac{m}{p}\right)^{max} = \frac{(1 - a) - \eta^+}{2\eta^+(1 - \eta^+)}$$

The reason for an inverse U-shaped relationship between  $\frac{m}{p}$  and  $\gamma_q$  is the following: On the one hand we know that, given the classical assumption that all investment spending is made out of profits, there can be no positive growth without profit income. Consequently, initially, a redistribution from wages to profits must generate growth if all profits are reinvested. A positive value of  $\eta^+$  reduces the growth rate, because less is invested which implies that less capacities are being built up and less workers are employed which means that the growth of the volume of wage income is reduced. At positive values of  $\eta^+$  the 'leakage' out of profits reduces the level of 'effective demand'. On the other hand, wage income which in the above formulation is specified as the residual income always has a real demand effect albeit being spent only on consumption. The consequence is that with a given 'leakage' factor  $\eta^+$  there will be positive net accumulation and hence growth only when profits are positive, but - on the other hand - a too high distributive share of profits depresses 'effective demand' (because of the leakage) and hence becomes counter-productive for growth.

With leakage there is excess supply of goods given by  $\eta^+ \frac{m}{p} q$ . We assume that these goods vanish immediately. Furthermore, we have so far not discussed that ownership claims on actual output are emerging from the investments into financial assets, which may also bear interest rate payments. We shall consider this issue below.

## 6.2 Schumpeterian (ex-ante) financing of innovative investment

In this section we look at the opposite of 'leakage', i.e. investors raising loans from banks.

Investments of these loans raise the rate of investment (and hence adds to the existing stock of intermediate goods  $aq$ ) and enables the economy to grow faster. Income will also be rising as more workers are employed and the volume of retained earnings  $mq$  is rising; consequently, demand is increasing as well.

There is however a physical constraint: In equilibrium we have  $q = aq + \frac{m}{p}q + \frac{wb}{p}q$  which implies a growth rate of  $\frac{1}{a} \frac{m}{p}$ . The necessary additional resources for investment to enable the economy to grow faster can therefore only come from

- workers incomes (forced saving)
- exogenous resources (e.g. foreign countries or unused inventories)
- a higher productivity of the system (i.e. a lower coefficient  $a$ )

The last point would mean that from the stock of inputs available for production at time  $t$  a higher output can be produced. This could be justified by the assumption that 'normal' productivity is lower than the (technically) feasible productivity of the system, i.e.  $a^{max} < a$ .

In either case there would be additional physical resources available which can be used by entrepreneurs. Financing the mobilization of these resources has to occur via credits. The additional investment goods enable the economy to grow at a faster rate which (in the continuous time model) immediately raises the volume of profits  $mq$ ; by the assumption that labour supply is infinitely elastic with respect to labour demand the demand for

consumption  $wbq = wl$  will also growing. This ensures that the demand condition is satisfied.

**Growth effects of credits for investment** With a pre-financing of productive investment through credits from the banking sector the total nominal sum available for buying investment goods including credit financing now becomes

$$mq + \eta^- pq$$

where the additional finance for investment is modeled as a share  $\eta^-$  of the nominal value of production  $pq$ . In this case the growth path of the economy becomes<sup>9</sup>

$$\dot{q} = \left[ 1 + \frac{1}{a} \left( \frac{m}{p} + \eta^- \right) \right] (1-a)^{-1} \underbrace{\left( \frac{m}{p} + \frac{wb}{p} \right)}_{1-a} q - q$$

which yields a growth rate of the economy

$$\frac{\dot{q}}{q} = \frac{1}{a} \left( \frac{m}{p} + \eta^- \right)$$

We can see that pre-financing of productive investment can boost economic growth (an important role emphasised by Schumpeter). This is quite important in the case that the economy operates below its full capacity level or credits are financing productivity enhancing investments.

If the economy, however, operates at the full capacity level there are constraints on mobilizing the additional investment goods. First, there could be a constraint from the financial sector which may not be willing to extend finance much in such circumstances.<sup>10</sup> Second, there are constraints from the physical side of the economy. This physical constraint requires an explicit consideration concerning the the specific source of the additional physical goods. If these are coming via forced savings of workers the natural constraint is given by  $\frac{wb}{p}q$ . In the case that there are exogenous resources (imports, inventories) the amount of these imposes a natural constraint. In the third case, finally, the physical constraint is the maximum productivity of the system given by  $\frac{1}{a^{max}}$ .

### 6.3 Accumulation of assets and debts

So far we have neglected that assets and debts accumulate. The growth of financial assets amounts to

$$\dot{h}^+ = \eta^+ mq + (\rho^+ - \vartheta^+) h^+$$

---

<sup>9</sup>In this specification it is assumed that additionally required investment goods (see discussion in section 6.2) are made available from outside the system. Alternatively, there could be an adjustment in the productivity parameter  $a$  or in the wage rate, as discussed above, but this would have implications for the price equations and the distribution of income.

<sup>10</sup>E.g. out of fear that the high pressure on capacities might lead to a lowering of returns due to a Phillips curve effect.

$\rho^+$  denotes the interest rate on assets and  $\vartheta^+ h^+$  are the flows from the financial sector back to the real sector which shall be discussed below in more detail. In real terms this amounts to a long-term claim of  $h^+ \frac{p_{h^+}}{p}$  where  $p_{h^+}$  refers to the relative price of financial ( $h^+$ ) versus productive asset holdings ( $pq$ ). Over time we assume that the financial asset holders will be realising their claims (i.e. demand in the real sector) which implies a transversality condition  $\lim_{t \rightarrow \infty} p_{h^+} h^+ = 0$ . Thus if  $h^+$  remains positive for whatever reason, the price of the asset must approach zero. We shall, however, assume subsequently that the stock of financial assets will tend towards zero in the long run.

We now turn symmetrically to the accumulation and repayment of loans: So far we have assumed that the investor can borrow without paying interest and without having to pay back the loans. Taking these into consideration, the outstanding claim of the financier (the debt of the investor) accumulates over time with

$$\dot{h}^- = \eta^- pq + (\rho^- - \vartheta^-) h^-$$

where  $\rho^-$  is the interest on loans and  $\vartheta^- h^-$  are repayments of the principal. The claim in real terms of the financiers (e.g. banks) are given by  $h^- \frac{p_{h^-}}{p}$  where  $p_{h^-}$  refers to the relative value of (one unit of) credit claims. To avoid unlimited indebtedness we again introduce a transversality condition as  $\lim_{t \rightarrow \infty} p_{h^-} h^- = 0$  where  $p_{h^-}$  denotes the unit value of the loan. As before, we assume that the transversality condition is satisfied with  $\lim_{t \rightarrow \infty} p_{h^-} h^- = 0$ .<sup>11</sup>

## 6.4 Growth effects of asset decumulation and loan repayments

Let us now focus on financial asset decumulation, i.e.  $\dot{h}^+ < 0$ . For simplicity we also assume that no further financial asset accumulation takes place, i.e.  $\eta^+ = 0$ . These assumptions imply that  $\rho^+ - \vartheta^+ < 0$ . We have to consider various possibilities.

As pointed out above, the realization of ownership claims from financial assets depends upon the valuation of financial assets as defined by the relative price  $p_{h^+}$ . The realization of the ownership claims at any point of time, given by  $p_{h^+} h^+$ , may be introduced through an adjustment of either wage income or profit income or both. We assume that the economy operates at the full capacity level. Other mechanisms discussed above in the case of credit financing would be possible as well (e.g. exogenous resources, unused capacities, higher productivity of the system). These would have partly different implications for growth and consumption. The impact on economic growth will depend on whether the liquidation of financial assets adds to consumption or investment:

1. (a) Let us start with the case that forced saving is imposed on the workers and that the claims are used for consumption. The growth path of the economy then becomes

$$\dot{q} = \left(1 + \frac{1}{a} \frac{m}{p}\right) (1 - a)^{-1} \left(\frac{m}{p} + \frac{\tilde{w}b}{p} + \frac{\vartheta^+(p_{h^+} h^+)}{pq}\right) q - q$$

---

<sup>11</sup>The role of  $p_{h^+}$  and  $p_{h^-}$  will only emerge in a more elaborate version of this paper when dynamics of financial asset valuation will be explicitly considered.

where  $\tilde{w} = w - \frac{\vartheta^+ p_h + h^+}{bq} = w - \frac{\vartheta^+ p_h + h^+}{l}$ . Note that in this case the growth rate of the system is not reduced. Workers, however, get a lower actual real income. We can see here that workers actual real income falls by  $\frac{\vartheta^+ p_h + h^+}{l}$  (per worker), i.e. the amount required to satisfy the real claims of financial asset holders who wish to realize at a point of time  $\vartheta^+ h^+$  portion of their financial asset.

- (b) The second case is that the claims are financed by forced savings of workers but are used for investment.

$$\dot{q} = \left(1 + \frac{1}{a} \frac{m}{p} + \frac{\vartheta^+ h^+}{paq}\right) (1 - a)^{-1} \left(\frac{m}{p} + \frac{\tilde{w}b}{p} + \frac{\vartheta^+ p_h + h^+}{pq}\right) q - q$$

where again  $\tilde{w} = w - \frac{\vartheta^+ p_h + h^+}{l}$ . In this case there is a higher growth rate financed by the workers.

2. In the second case the claims of the financial asset holders are financed from the entrepreneurs incomes. Here again there are two subcases depending on whether the financial asset holders spend the realization from these claims on consumption or investment.

If they spend it on consumption the growth effects are equivalent to the earlier case where profit receivers spend a proportion of their income on consumption rather than on investment (i.e.  $\kappa_m > 0$ ) while in the second case no change in growth takes place.

Of course, various combinations of the above possibilities can also be considered.

The issue of loan repayments and its impact on growth can be dealt with in the same or symmetric manner as in the cases discussed above (financial asset accumulation or decumulation) depending on the expenditure/savings behaviour of the creditors (which are being repaid). We do not treat this any further.

## 7 Simulation studies

In this section we explore the quantity, price, wage and employment dynamics through simulations of the model. The purpose of these simulations is to show the sensitivity of transition paths with respect to particular behavioural parameters in the model especially with respect to the investment behaviour in liquid or financial assets and the financing of investment via credits. In these simulations we abstract from exogenous growth of population and do not restrict the dynamics to a given labour supply. Further we assume that  $\delta_{k,OUT} = 0$ , which means that there are no effects of unemployment on the labour supply. This assumes that participation increases in line with labour demand, i.e.  $\delta_{k,IN} = 1$ , and does not converge to an exogenous level. We set the long-run mark-up rate  $\pi = 0$ . This implies that in equilibrium the economy is stationary. (The simulations below could then also be interpreted as deviations from a long-term growth path.) The parameters and

Parameter	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
$a$	0.500	0.500	0.500	0.500	0.500
$\pi$	0.000	0.000	0.000	0.000	0.000
$\delta_p$	0.250	0.250	0.250	0.250	0.250
$\kappa_s$	0.000	0.000	0.000	0.000	0.000
$\kappa_m$	0.000	0.000	0.000	0.000	0.000
$\delta_{k,IN}$	1.000	1.000	1.000	1.000	1.000
$\delta_{k,OUT}$	0.000	0.000	0.000	0.000	0.000
$\kappa_u$	-0.010	-0.010	-0.010	-0.010	-0.010
$\eta^+$	0.000	<b>0.485</b>	<b>0.485</b>	0.000	<b>0.485</b>
$\eta^-$	0.000	0.000	<b>0.300</b>	0.000	<b>0.300</b>
$\kappa_b$	0.000	0.000	0.000	<b>0.050</b>	<b>0.050</b>

Table 7.1: Parameter values used in simulations

Variable	Values
$b$	1.000
$w$	1.000
$v$	1.000
$p$	2.000
$c$	2.000
$r$	0.000
$s$	0.000
$q^I$	0.000
$q^C$	0.500
$q$	1.000
$l$	1.000
$k$	1.000
$u$	0.000
$\gamma_q$	0.000
$n$	2.000

Table 7.2: Starting values used in simulations

starting values of the scenarios are reported in tables 7.1 and 7.2, respectively. The starting values are equilibrium values for the given parameters. We model the technological shock as a logistic pattern which implies a time path of the labour input coefficient as shown in figure 7.2.

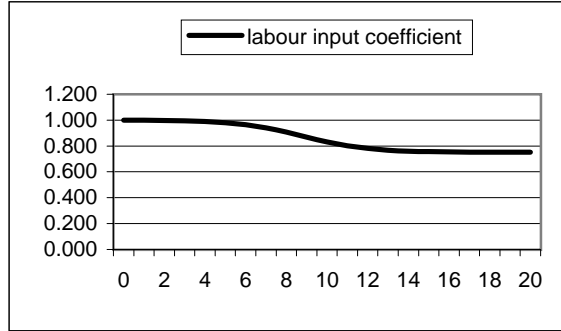


Figure 7.2: Labour input coefficient

## 7.1 Technical progress and effective demand

In the first scenario we assume that all emerging rents are reinvested, whereas in the second scenario a large part of the emerging rents is invested in financial assets,  $\eta^+ = 0.485$ . In the third scenario, however, there is additionally to the investment into the financial sector an active banking sector supports investments via credit finance, i.e.  $\eta^- = 0.300$ .

Figure 7.3 shows the trajectories of several variables. In the first and the third scenario there is no unemployment (or even slightly excess demand for labour) and the nominal wage rate is almost stable. In the second scenario, however, the wage rate is falling due to high unemployment. The reason for the behaviour of the unemployment rate can be found in the output dynamics (as the effect of technological progress on output per worker is the same in all three scenarios). In the first scenario (with no investment into liquid assets) output growth is highest as all emerging rents are reinvested which also boosts demand for labour with no unemployment arising. In the second scenario unemployment is rising as output is not growing (output growth is even slightly negative at the beginning) but output per worker is rising due to the exogenous technical progress. Finally, in the third scenario the inflow of financial capital used for additional investment helps to keep unemployment at a very low level. In all three scenarios there are positive rents emerging. These are highest in the second scenario due to a depressing effect of unemployment on the wage rate and almost identical in the first and the third scenario.

There is a small albeit important difference in real wages in the three scenarios. Although the parameter of the Phillips-curve effect is small in the simulations,  $\kappa_u = -0.01$ , together with the bargaining parameter set to zero,  $\kappa_s = 0$ , the high unemployment rate in the second scenario leads to lower real wages which feeds back into higher real rents.

These simulations, which are in accordance with the analytical results, but derived from a richer model with wage, rent, and output dynamics may give a first understanding

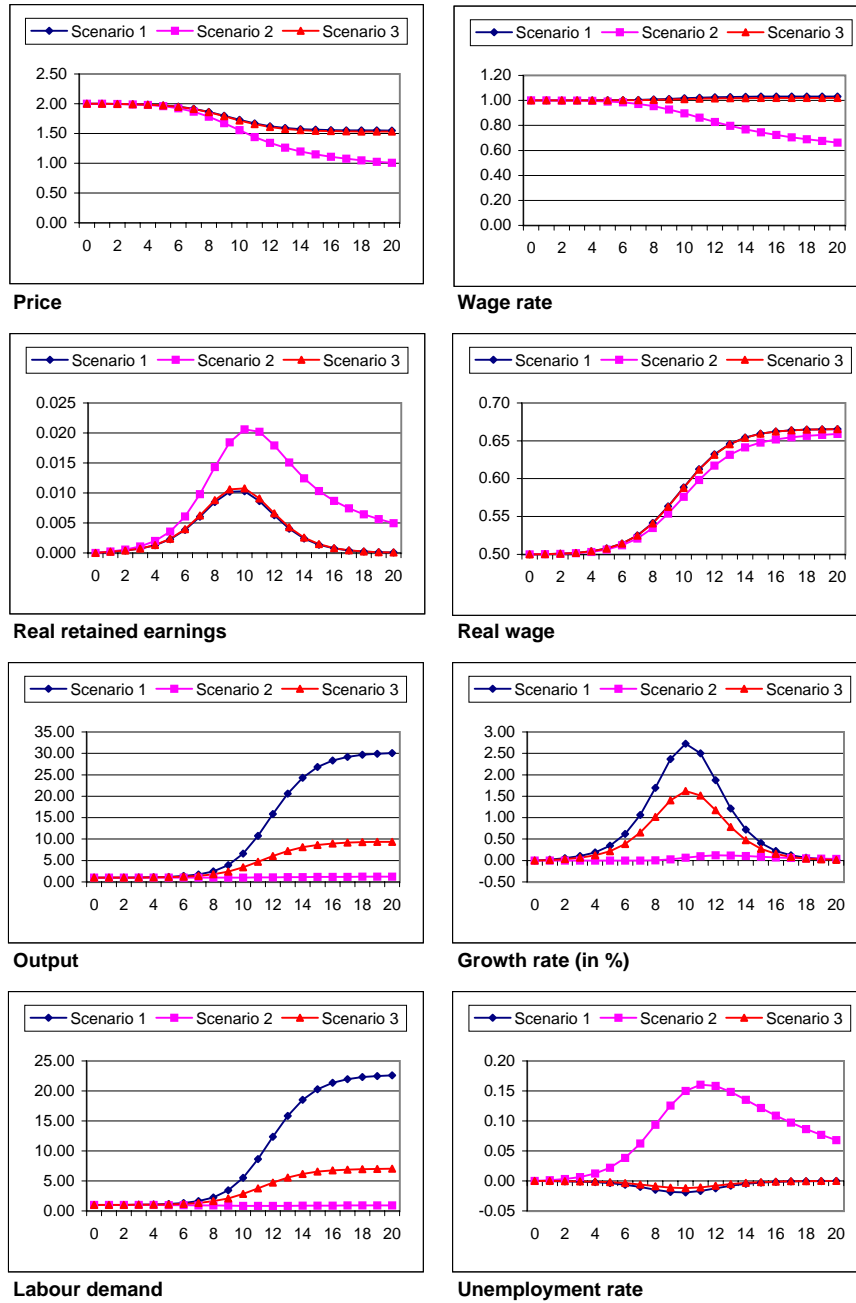


Figure 7.3: Simulations with leakages and injections to/from the financial sector



of the 'Solow-paradoxon': Even if there are high rents due to technical progress these need not lead immediately to a higher growth rate of the economy. A pre-financing of investments may, however, help to contribute to higher output growth.

## 7.2 Endogenous productivity growth and the Solow paradoxon

So far we have shown that growth paths of an economy exposed to an exogenous technology shock are a function of distributional and expenditure dynamics induced by such a shock. However, in all the scenarios described in the previous section, productivity growth remained unaffected, as it was exogenously determined. We now introduce the simple specification of endogenous productivity growth following a Kaldor-Verdoorn formulation in which productivity growth is a function of output growth (see also section 3.6 above). The dynamics of the labour input coefficient then becomes  $\frac{\dot{b}}{b} = -\kappa_b a \frac{\dot{q}}{q}$  where  $\kappa_b > 0$  can be referred to as the Kaldor-Verdoorn parameter, translating the effect of output growth (or, more precisely, the growth rate of working capital or investment) into (labour) productivity growth. Differences in scenarios regarding output growth will therefore further affect productivity growth and through this again distributional rent dynamics and expenditure growth.

It is in this context that we can show that technology shocks affect not only output dynamics - depending upon the effective demand issues discussed in the previous section - but also productivity dynamics. The latter was the focus of Solow's remarks about developments in the 1980s.

Figure 7.4 shows the results of two simulations with endogenous productivity growth in which scenario 1 shows a simulation without any leakage (i.e. all income is spent on goods) and scenario 3 shows a simulation with leakage (i.e. some investment into a non-interest bearing liquid asset). We can see - as before - that leakage has an impact on output growth, but in this case with endogenous productivity growth also a (negative) impact on productivity development. Notice that such detrimental effect on productivity growth takes place in spite of the simulation runs 'with leakage' showing up higher per unit profits than the simulation without leakage. The reason for this is of course the wage-depressing effect of higher unemployment levels.

Hence this version of the model tracks well the 'stylised facts' of the 1980s: a positive shift in GDP towards profits, but a relatively low<sup>12</sup> output and productivity performance in a period in which the economy is exposed to a (positive) technology shock. We can also easily show that the presence of venture capital (such as in the US) which amounts to a scenario with injections from the banking sector stimulating real investments can turn the situation towards a better exploitation of the productivity potential; this is the story of the US in the 1990s.

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<sup>12</sup>'Relatively low' can be interpreted here in relation to a scenario without leakage.

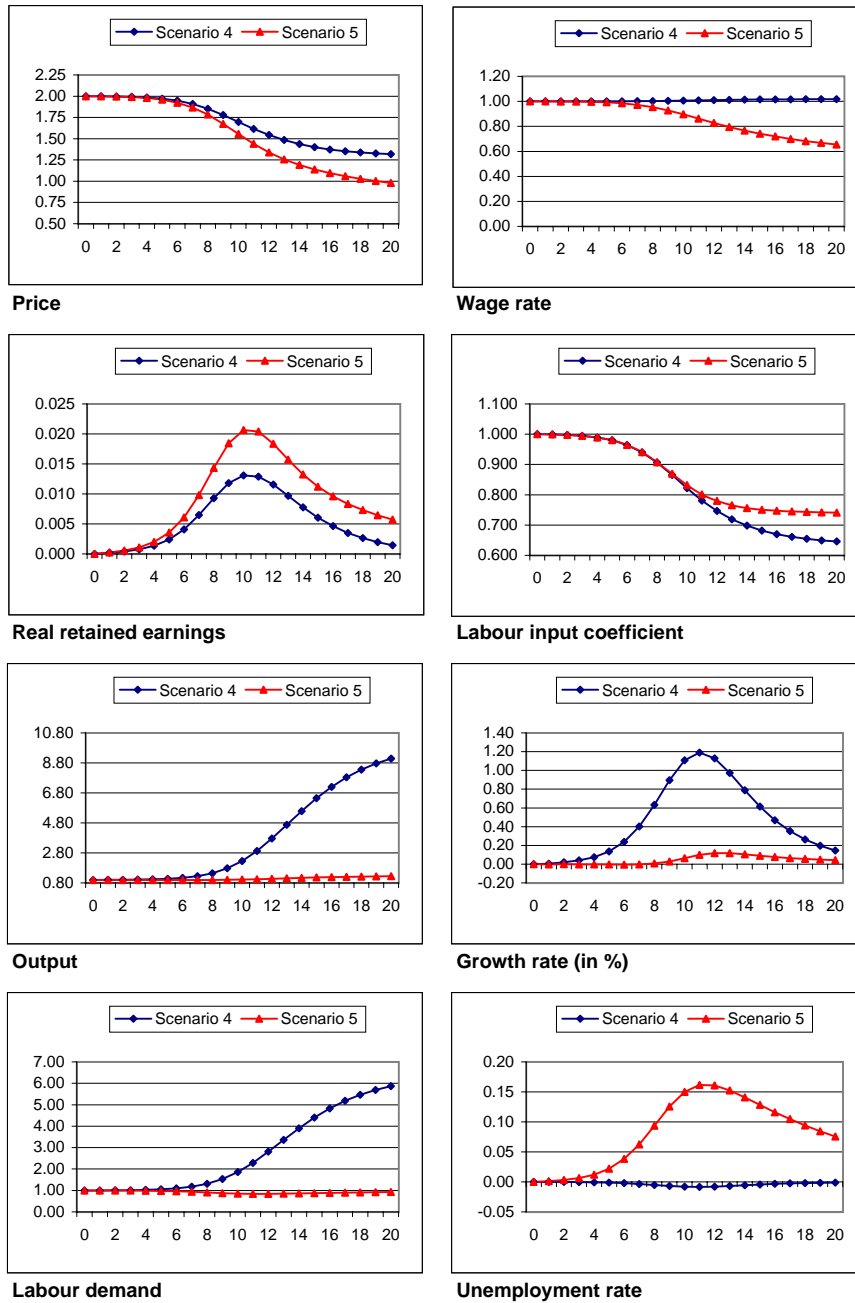


Figure 7.4: Simulations with endogenous productivity growth

## 8 Conclusions

We have shown the following in this paper: We started with a classical model in which the well-known classical results concerning a wage-profit consumption-investment/growth trade-off emerged. We showed that the full employment condition required an increase in the profit share when there was a higher rate of (labour-saving) technical progress or an increase in the (actual) labour force. Also an increase in the share of profits spent on consumption had to be accomodated by a fall in the wage share if full employment was to be obtained. We then introduced some Keynesian elements into the analysis to show that these 'classical' results get modified when a 'whimp' factor gets introduced in terms of a hesitancy to spend on productive investment on the part of profit receivers. In this case we could show that an increase in the profit share above a certain level depresses economic growth. Next we modelled more extensively the relationships between the financial and the real sector of the economy. We showed the impact on economic growth both of 'leakage' effects from shifting investment spending towards financial assets as well as the role which financial institutions can play in pre-financing productive investments in the real sector (a role which J. Schumpeter emphasised). In a number of simulations we show the qualitative behaviour of the model as it absorbs 'technology shocks'. We show that the features of the model are quite able to explain how a positive 'technology shock' could lead to effective demand problems which keep the actual growth rate substantially below the potential growth rate. The model thus serves as a way to understand the underlying factors which could explain the so-called 'Solow paradoxon' much discussed in the 1970s and 1980s.

## A Stability analysis

The model can be summarized with the following system of differential equations:

$$\begin{aligned}\dot{p} &= \delta_p p(t)(a - 1) + \delta_p w(t)b \\ \dot{q} &= \frac{1}{a} \underbrace{\left( (1 - a) - \frac{w(t)}{p(t)}b \right)}_{f_m(t)} q(t) \\ \dot{w} &= \kappa_u \left( 1 - \frac{bq(t)}{k} \right) w(t)\end{aligned}$$

As we have seen above the dynamics of the model depends on the behaviour of the real wage  $\omega(t) = \frac{w(t)}{p(t)}$ . We discuss the following simplified version of the model:

$$\begin{aligned}\dot{\omega} &= \delta_\omega \left( \frac{1 - a}{b} - \omega(t) \right) + \kappa_u \left( 1 - \frac{b}{k} q(t) \right) \\ \dot{q} &= \frac{1}{a} ((1 - a) - \omega(t)b) q(t)\end{aligned}$$

The first equation gives the dynamic of the real wage  $\omega$ . Here, the first term means that the real wage converges to the equilibrium value  $\frac{1-a}{b}$  whereas the second term reflects the labour market effect. In the case of unemployment  $1 - \frac{b}{k} q(t) > 0$  there is a pressure on the real wage as  $\kappa_u < 0$ . The system is non-linear because of the term  $\omega(t)q(t)$  in the quantity equation. Setting  $\dot{\omega}$  and  $\dot{q}$  equal to zero yields the fix-point

$$\begin{aligned}\omega^* &= \frac{1 - a}{b} \\ q^* &= \frac{k}{b}\end{aligned}$$

Linearizing the system and evaluating at the fix-point gives the Jacobian matrix  $\mathbf{J}^*$ .

$$\mathbf{J}^* = \begin{pmatrix} -\delta_\omega & -\kappa_u \frac{b}{k} \\ -\frac{k}{a} & 0 \end{pmatrix}$$

The eigenvalues of the system can be calculated as

$$\lambda_{1,2} = \frac{1}{2} \left( \text{tr } \mathbf{J}^* \pm \sqrt{(\text{tr } \mathbf{J}^*)^2 - 4 \det \mathbf{J}^*} \right)$$

where  $\text{tr } \mathbf{J}^* = -\delta_\omega \leq 0$  denotes the trace of the matrix and  $\det \mathbf{J}^* = -\kappa_u \frac{b}{a} \geq 0$  is the determinant.

The structure of the eigenvalues can be used to examine the stability of the model. Table A.3 shows the trace, the determinant, and the discriminant of the Jacobian  $\mathbf{J}^*$  for different sets of the adjustment parameters  $\delta$  and  $\kappa$ . Let us start with the case in which

	$\delta\omega > 0$	$\delta\omega = 0$	$\delta\omega < 0$
$\kappa_u > 0$	$\text{tr} < 0$ $\det < 0$ $\Delta > 0$ $\lambda_1 > 0$ $\lambda_2 < 0$	$\text{tr} = 0$ $\det < 0$ $\Delta > 0$ $\lambda_1 > 0$ $\lambda_2 < 0$	$\text{tr} > 0$ $\det < 0$ $\Delta > 0$ $\lambda_1 > 0$ $\lambda_2 > 0$
$\kappa_u = 0$	$\text{tr} < 0$ $\det = 0$ $\Delta > 0$ $\lambda_1 = 0$ $\lambda_2 < 0$	$\text{tr} = 0$ $\det = 0$ $\Delta = 0$ $\lambda_i = 0$	$\text{tr} > 0$ $\det = 0$ $\Delta > 0$ $\lambda_1 > 0$ $\lambda_2 = 0$
$\kappa_u < 0$	$\text{tr} < 0$ $\det > 0$ (1) $\Delta > 0$ $\lambda_1 < 0$ $\lambda_2 < 0$ (2) $\Delta = 0$ $\lambda_i < 0$ (3) $\Delta < 0$ $\text{Re } \lambda_i < 0$	$\text{tr} = 0$ $\det > 0$ $\Delta < 0$ $\text{Re } \lambda_i = 0$	$\text{tr} > 0$ $\det > 0$ (1) $\Delta > 0$ $\lambda_1 > 0$ $\lambda_2 > 0$ (2) $\Delta = 0$ $\lambda_i > 0$ (3) $\Delta < 0$ $\text{Re } \lambda_i > 0$

Table A.3: Stability analysis

$\kappa > 0$  which would mean that in the case of unemployment the real wage would even rise (or vice versa). If additionally the real wage-to-rent adjustment parameter is less than zero, i.e.  $\delta_\omega < 0$  (which would mean that prices diverge from costs) the system is clearly unstable as both eigenvalues become positive. For  $\delta = 0$  or  $\delta < 0$  the model is saddle-point stable (both real eigenvalues have opposite signs).

If  $\delta_\omega < 0$ , then irrespective of the value of the sign of  $\kappa_u$  the system is unstable as the eigenvalues or the real part of the complex eigenvalues are positive.

For  $\delta = 0$  and  $\kappa = 0$  both eigenvalues are zero, which means that the system just remains at the starting values and there are no adjustments at all.

For  $\kappa = 0$ , i.e. with no effects of unemployment on wages, but  $\delta_\omega < 0$ , i.e. a perverse real wage-to-rent adjustment, one can see that the first eigenvalue is zero and the second eigenvalue is negative. In economic terms this means that prices adjust to costs but there is no 'mechanism' which assures full employment. The system converges to a steady state but without assuring full employment.

In the other case with  $\kappa_u < 0$  and  $\delta_\omega = 0$  we find complex eigenvalues with real parts equal zero. Thus in this case we can observe stable oscillations. However it is structurally unstable as changes of  $\delta_\omega$  shift the regime either into the unstable or the stable region.

Finally, we discuss the economic meaningful cases with positive real wage-to-rent adjustment a negative effect effect of unemployment on the real wage. In this case the trace is negative and the determinant is positive which implies stability of the system. We have to distinguish three cases. First, the discriminant is positive and thus both eigenvalues are negative and different from each other. In the second case we have equal eigenvalues with negative signs. Finally, the third case in which the discriminant is negative and the eigenvalues are complex with negative real parts. In this case we can observe dampened oscillations to the equilibrium path.

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