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Introducing a “green” good: Implications for environmental quality and social welfare

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Abstract

This paper investigates the effects on environmental quality and social welfare if an environmentally friendly – or “green” – good is introduced on a market. The green good has been modeled as an impure public good such that its jointly produced characteristics are both available separately in terms of a private consumption good and donations to an environmental organization as a pure public good. The situations before and after the launch of the green good are quantified explicitly. Conclusions on environmental quality are drawn in terms of an Index of Easy Riding, and the social welfare gap is measured as an equivalent variation.

1. Introduction

In recent years an increasing number of “green” goods has entered supermarket shelves. New brands and labels emerge, promising more environmentally friendly products that have not been available before. The market for organic groceries in Europe has, for example, grown by more than 400 % between the years 2000 and 2015 (Willer, Schaack, & Lernoud, 2017, p. 228). A similar development has been observed in the USA where retail sales of organic food even rose by almost 500 % between 2002 and 2015 (Haumann, 2017, p. 259). However, can we be sure that the increasing consumption of green goods will actually improve environmental quality and overall social welfare?

From an economist’s point of view, environmental quality can be modeled as a public good (see, e.g., Barrett, 2012). It is, however, well known that pure public goods, such as direct donations to non-governmental organizations that aim at protecting the environment, are prone to easy riding (Cornes & Sandler, 1996, p. 30), i.e. their voluntary provision level – as given by the Nash equilibrium – usually falls short of the Pareto-optimal level. Therefore, the coupling of the public good with private co-benefits has been proposed as a potential means to mitigate underprovision (Olson, 1971; Cornes & Sandler, 1984; Posnett & Sandler, 1986; Sandler,

1996). One way to achieve this coupling is the introduction of impure public goods that are characterized by the joint production of public and private benefits (“characteristics”), whereby the latter can create an incentive for increased private provision of the public characteristic. Hence, since green goods can be modeled as impure public goods, combining private consumption with, e.g., preservation of biodiversity (Kotchen, 2005), at first sight it can be suspected that their rising market penetration will have a positive impact on both environmental quality and social welfare.

Yet, some empirical results are pointing in the direction that the availability of green consumption goods may decrease overall environmental quality (Munro & Valente, 2016; Engelmann, Munro, & Valente, 2017). There are similar concerns in the theoretical literature. Kotchen (2005, p. 283) states that “increased consumption of a green product can crowd out direct donations, with the net effect being a reduction in environmental quality”. In a follow-up paper with a related model, Kotchen (2006) demonstrates that the introduction of an impure public good can diminish environmental quality and yield a negative welfare effect if the joint characteristics are gross substitutes. Kotchen (2009) analyzes a model where the joint public characteristic of the impure public good is an adverse effect on environmental quality, i.e. the impure public good causes pollution, and consumers can voluntarily offset this effect by providing direct donations, so the overall public good provision level is the balance of both. In this framework, he shows that environmental quality and social welfare may both decrease as a result of the polluting good becoming less harmful. Similarly, Vicary (2011) develops a commons model with an environmentally neutral and an environmentally harmful consumption good and infers that environmental quality will decline in a large economy if the adverse effect of the latter good is lessened, which may cause individual utility to fall. Finus and Rübbelke (2013, p. 221) conclude, with respect to non-cooperative coalition formation in the context of climate change, that “ancillary benefits have a neutral or negative impact on the size of stable coalitions and the relative success of coalition formation measured in welfare terms”. Overall, there is a consensus in the theoretical literature that negative effects on environmental quality and social welfare are possible if private and public characteristics are substitutes whereas the beneficial privatizing impact of impure public goods is tied to complementarity of the joint characteristics (see also, e.g., Murdoch & Sandler, 1984). If multiple impure public goods with an arbitrary number of characteristics are available, however, the complementarity condition does not suffice to guarantee a rise in environmental quality due to an improvement in the joint production technology (Chan & Kotchen, 2014).

In general, the theoretical literature rarely accounts for social welfare impacts of impure public goods explicitly. Among the works evaluating the welfare implications of the introduction of a new good, a prominent example is Morey (1985) who employs the compensating variation/equivalent variation (CV/EV) method. Auld and Eden (1990) also use EV to measure the wedge between the Pareto optimum and the private optimum in an impure public good model. Cornes and Sandler (1996, pp. 159–161) suggest a similar approach: they first introduce an Index of Easy Riding (IER) composed of the share of the public good provision level in the Nash equilibrium over the Pareto-optimal level. This share serves as a measure of the gap w.r.t. public good provision, but does not allow for an overall welfare assessment. Hence, as a second step, Cornes and Sandler propose a welfare measure in the spirit of EV that determines the amount of income that consumers in the Pareto optimum have to give up or receive in order to obtain the same utility as in the Nash equilibrium. Another measure of public good provision is put forward by Sandler (2003) who proposes an Index of Optimality (IO) that is the ratio of excludable to total benefits, ranging from zero to one. A smaller value of the IO points to a larger share of public benefits and thus to a larger extent of potential underprovision. Eyckmans and Finus (2006) discuss coalition formation in a global warming game and suggest a Welfare Closing the Gap Index (CGX) to quantify the ratio of deviation from the non-cooperative payoff for partial and full cooperation, respectively.

However, a comprehensive discussion of both environmental and welfare effects that arise due to the introduction of a green good is still lacking. The aim of this paper is therefore to develop a model to investigate the effects on public good provision and welfare if an impure public good is launched on a market where only a private and a pure public good have been available before. For that purpose, the IER and the EV will be used. The virtue of the IER is that it lends itself to a straightforward assessment of environmental quality, directly contrasting public good provision in the Nash equilibrium and in the Pareto optimum, and allows for the derivation of a related welfare measure (EV). Kotchen (2009) already uses the IER to illustrate the efficiency gap for an exemplary CES utility function, but he does not explicitly compare the pure and impure public good frameworks. Moreover, his analysis is restricted to the calculation of environmental consequences; he does not deal with social welfare implications.

The remainder of the paper is organized as follows. In Section 2 I establish the impure public good model and derive the Nash equilibrium and Pareto-optimal conditions for the situations before and after the introduction of the green good. Implications for environmental quality and social welfare are derived in Section 3. Section 4 concludes.

2. Modeling the introduction of the green good

The initial (benchmark) situation before the introduction of the green good can be represented in a pure public good model where consumers can choose between a private good and direct donations to some environmental organization. As it will turn out that the pure public good model results as a special case from the impure public good model (including the green good), it is sufficient to develop the impure public good model here that is based on the one by Kotchen (2006).

2.1. The basic impure public good model

I assume, for simplicity, that two individuals $i = 1, 2$ are present on the market. They are endowed with monetary income $w_i > 0$ that they can spend on three goods: a private consumption good c_i , donations to an environmental organization, where the overall amount of donations D is the sum of individual contributions d_i , and the green good q_i . The prices of all three goods are normalized to unity. Utility is derived from the characteristics of the goods, i.e. their relevant properties.¹ The three goods are assumed to generate two characteristics: one unit of the green good produces α units of the private characteristic y_i and β units of the public characteristic Z ($0 < \alpha < 1$, $0 < \beta < 1$). The private consumption good c_i and the public good D produce the private and public characteristic in a one-to-one relationship, respectively. For the public characteristic again a summation technology applies.

Preferences are assumed to be of Cobb-Douglas type, $u_i(y_i, Z) = y_i^\gamma Z^{1-\gamma}$ with $0 < \gamma < 1$, and therefore strictly quasiconcave, strictly increasing in both arguments and everywhere twice partially differentiable. Hence, individual i 's choice problem can be modeled in the following framework:

$$\begin{aligned} \max_{c_i, d_i, q_i} u_i(y_i, Z) &= y_i^\gamma Z^{1-\gamma} \\ \text{s.t. } w_i &= c_i + d_i + q_i, \\ y_i &= c_i + \alpha q_i, \\ Z &= \sum_{i=1}^2 z_i = \sum_{i=1}^2 d_i + \beta \sum_{i=1}^2 q_i = D + \beta Q. \end{aligned} \tag{1}$$

¹ The roots of the characteristics approach are the seminal works by Muth (1966) and Lancaster (1971). The first reference with respect to its application to public goods is Sandmo (1973); important contributions to the theory of impure public goods have then been provided by, e.g., Cornes and Sandler (1984; 1994; 1996).

With respect to the technology relating goods to characteristics, as given by the second and third constraint in (1), it is important to think about the implications of the values that the parameters α and β can take. In particular, their sum is relevant for the result of the model. Two main cases have to be distinguished:

Case (a): $\alpha + \beta \leq 1$

This parameter constellation implies that the private good and the pure public good have an advantage over the green good (or are at least not worse, if the condition holds with equality) in the production of both characteristics y_i and Z (recall that the prices of all goods have been set to unity). Consequently, the green good will not be consumed ($q_i = 0$) and the model is reduced to the pure public good framework.² This case serves as the benchmark since it is equivalent to a market without the green good.

Case (b): $\alpha + \beta > 1$

If the sum of the technology parameters is strictly greater than unity, the green good will be the cheapest option to attain both characteristics jointly, so it will definitely be consumed as long as the individual wishes to obtain both characteristics. However, since the two characteristics are generated in fixed proportions by q_i , there will probably be “residual demand” for either characteristic that is covered by consumption of c_i or d_i . Note that the green good will be consumed together with either c_i or d_i (or on its own if q_i generates the two characteristics exactly in the same proportion as desired), but not with both at the same time. Hence, three sub-cases can be defined:

Sub-case (b1): $z_i < \beta q_i$

Individual i has a relatively strong preference for the private characteristic and decides to consume the green good together with the private good, i.e. $d_i = 0$ and $c_i, q_i > 0$.

Sub-case (b2): $z_i > \beta q_i$

Individual i has a relatively strong preference for the public characteristic and decides to consume the green good and provide direct donations, i.e. $c_i = 0$ and $d_i, q_i > 0$.

Sub-case (b3): $z_i = \beta q_i$

² It is assumed that the green good is not consumed if individuals are indifferent between obtaining the characteristics they value via the green good or a combination of the private and pure public good, i.e. if $\alpha + \beta = 1$. In other words, the green good is only chosen if it is strictly better than the other two goods in generating the two characteristics.

Individual i prefers to provide the public characteristic exactly at the level generated by the green good and decides to consume the green good exclusively, i.e. $c_i = d_i = 0$ and $q_i > 0$.

These cases are the foundation for the subsequent analysis. Interestingly, this case differentiation is independent of the public-characteristic level provided by the other individual, z_j , and of the utility function.

As at least one of the three goods is eliminated in every setting, problem (1) can be solved more easily. Section 2.2 deals with the benchmark case (a) and covers the situation before the introduction of the green good, which becomes viable in Section 2.3 where cases (b1) to (b3) are dealt with.

2.2. Case (a): private consumption good and direct donations

Returning to problem (1), it is stated that the consumer maximizes his utility, which depends on characteristics, over the choice of goods (since characteristics cannot be bought directly on the market). Hence, it is necessary to transfer the maximization problem to one dimension. The important point here is to transfer problem (1) either to goods or characteristics space – both options are feasible (see Cornes & Sandler, 1996, 256 f.). As the interest of this paper lies in environmental quality, i.e. the level of characteristic Z , irrespective of its origin in terms of marketed goods, the problem is reformulated in characteristics space by rearranging the second and third constraint and inserting them in the budget constraint. Taking account of the elimination of q_i , the maximization problem results in a pure public good framework and reads

$$\begin{aligned} \max_{y_i, z_i} u_i(y_i, Z) &= y_i^\gamma Z^{1-\gamma} \\ \text{s.t. } w_i &= y_i + z_i, \\ Z &= \sum_{i=1}^2 z_i. \end{aligned} \tag{2}$$

Some transformations of the first-order conditions yield the reaction functions for public good provision

$$z_i = (1 - \gamma)w_i - \gamma z_{j \neq i}. \tag{3}$$

The resulting Nash equilibrium levels of private consumption, individual donations to the public good and overall public good provision are

$$y_i^* = \frac{\gamma(1-\gamma)}{1-\gamma^2} \sum_{i=1}^2 w_i, \quad (4)$$

$$z_i^* = \frac{1-\gamma}{1-\gamma^2} (w_i - \gamma w_j), \quad (5)$$

$$Z^* = \sum_{i=1}^2 z_i^* = \frac{(1-\gamma)^2}{1-\gamma^2} \sum_{i=1}^2 w_i. \quad (6)$$

Note that equations (4), (5) and (6) must have nonnegative results. In equation (4) both numerator and denominator of the first term on the right-hand side are positive (as $0 < \gamma < 1$), so the same applies to the overall expression. For equation (5) strict positivity holds if $\gamma w_j < w_i$, which is plausible if γ is sufficiently small and the monetary income levels of the two individuals do not differ too much; otherwise, individual i will become a free rider, i.e. $z_i^* = d_i^* = 0$. In the following it is assumed that both individuals make a strictly positive contribution to the public good.³ Be aware that, although complete free riding at the extensive margin is excluded, there will still be easy riding in the sense of a gap between voluntary private provision and optimal provision at the intensive margin since the reaction functions given by equation (3) are downward-sloping.

Interestingly, the level of private good consumption of the two individuals in the Nash equilibrium is identical as equation (4) only depends on the sum of monetary incomes. Income differences merely play a role in determining public good contributions (see equation (5)). As utility is derived from own private good consumption and the aggregate of public good provision (which is of course identical for both individuals), utility levels of the two agents coincide as well and are given by

$$u_i^a(y_i^*, Z^*) = \left(\frac{\gamma}{1-\gamma}\right)^\gamma \frac{(1-\gamma)^2}{1-\gamma^2} \sum_{i=1}^2 w_i, \quad (7)$$

which clearly fulfils the condition for a Nash equilibrium that the marginal rate of substitution of agent i equals the marginal rate of transformation:

³ In the case of identical individuals with $w_i = w_j = w$ this assumption is always fulfilled because $0 < \gamma < 1$ and hence, there will only be interior solutions.

$$\frac{\frac{\partial u_i^a(y_i^*, Z^*)}{\partial Z^*}}{\frac{\partial u_i^a(y_i^*, Z^*)}{\partial y_i^*}} = 1. \quad (8)$$

The social optimum is the solution to the general problem

$$\begin{aligned} \max_{c_i, c_j, D, q_i, q_j} \quad & u_i(y_i, Z) = y_i^\gamma Z^{1-\gamma} \\ \text{s.t.} \quad & \sum_{i=1}^2 w_i = \sum_{i=1}^2 c_i + D + \sum_{i=1}^2 q_i, \\ & y_i = c_i + \alpha q_i, \\ & Z = D + \beta \sum_{i=1}^2 q_i, \\ & u_j(y_j, Z) = y_j^\gamma Z^{1-\gamma} = \bar{u}_j. \end{aligned} \quad (9)$$

Again setting $q_i = q_j = 0$ for case (a), the problem can be simplified to

$$\begin{aligned} \max_{y_i, y_j, Z} \quad & u_i(y_i, Z) = y_i^\gamma Z^{1-\gamma} \\ \text{s.t.} \quad & \sum_{i=1}^2 w_i = \sum_{i=1}^2 y_i + Z, \\ & u_j(y_j, Z) = y_j^\gamma Z^{1-\gamma} = \bar{u}_j, \end{aligned} \quad (10)$$

from which the Pareto-optimal levels of aggregate private and public good provision are obtained:

$$\sum_{i=1}^2 y_i^{**} = \gamma \sum_{i=1}^2 w_i, \quad (11)$$

$$Z^{**} = (1 - \gamma) \sum_{i=1}^2 w_i. \quad (12)$$

In order to be able to calculate individual utilities, a specific Pareto optimum has to be chosen from the set implied by equations (11) and (12).⁴ I therefore assume, as suggested by Cornes and Sandler (1996, p. 159), that public good provision is shared according to the same ratio as in the Nash case,⁵ i.e.

⁴ Although the IER can be computed on the basis of aggregate public characteristic levels, the selection of a specific Pareto optimum is necessary for the calculation of utility levels that, in turn, are needed for determining the social welfare gap.

⁵ Again $\gamma w_j < w_i$ has to be assumed to ensure strictly positive public good contributions z_i^{**} .

$$z_i^{**} = \frac{z_i^* Z^{**}}{Z^*} = w_i - \gamma w_j. \quad (13)$$

Given z_i^{**} , the individual level of private consumption y_i^{**} can be derived with the help of the consumer's budget constraint, i.e. the first constraint in problem (2):

$$y_i^{**} = w_i - z_i^{**} = \gamma w_j. \quad (14)$$

In contrast to the Nash equilibrium values, now both private consumption y_i^{**} and public good provision z_i^{**} differ across individuals if $w_i \neq w_j$. From (5) and (13) it immediately follows that both agents increase their public good contributions in the Pareto optimum as opposed to the Nash equilibrium and consequently, overall public good provision rises as well (see equations (6) and (12)). Since monetary incomes remain unchanged, both individuals have to give up some private consumption y_i to finance the additional public good contribution z_i .⁶ The associated utility level of agent i can be determined by inserting equations (12) and (14) in the utility function:

$$u_i^a(y_i^{**}, Z^{**}) = (\gamma w_j)^\gamma \left((1 - \gamma) \sum_{i=1}^2 w_i \right)^{1-\gamma}. \quad (15)$$

Again, it can be demonstrated that the general condition for a Pareto optimum (Samuelson, 1954) holds:

$$\sum_{i=1}^2 \frac{\frac{\partial u_i^a(y_i^{**}, Z^{**})}{\partial Z^{**}}}{\frac{\partial u_i^a(y_i^{**}, Z^{**})}{\partial y_i^{**}}} = 1. \quad (16)$$

2.3. Case (b): introducing the green good

Having calculated the benchmark case where the green good is not consumed, I now turn to the case where $\alpha + \beta > 1$, i.e. a green good is introduced and its technology in generating both characteristics is favorable to a combination of the private good and the pure public good. Consequently, the green good will definitely be purchased if both characteristics are normal

⁶ The assertion that $y_i^{**} < y_i^*$ can also be proven analytically: contrasting equations (4) and (14) yields the condition $\gamma w_j < w_i$ which holds by assumption.

(which is guaranteed by the presumed utility function). As described above, three sub-cases have to be discussed:

(b1): $z_i < \beta q_i$, which implies $d_i = 0$ and $c_i, q_i > 0$;

(b2): $z_i > \beta q_i$, which implies $c_i = 0$ and $d_i, q_i > 0$;

(b3): $z_i = \beta q_i$, which implies $c_i = d_i = 0$ and $q_i > 0$.

For determining voluntary provision levels, the respective manipulations of the general problem (1) yield the following maximization problems:

$$\max_{y_i, z_i} u_i(y_i, Z) = y_i^\gamma Z^{1-\gamma} \quad \text{s. t. } w_i + \frac{1-\alpha}{\beta} z_{j \neq i} = y_i + \frac{1-\alpha}{\beta} Z \quad \text{in case (b1),} \quad (17a)$$

$$\text{s. t. } w_i + z_{j \neq i} = \frac{1-\beta}{\alpha} y_i + Z \quad \text{in case (b2),} \quad (17b)$$

$$\text{s. t. } w_i + \frac{1}{\beta} z_{j \neq i} = \frac{1}{\beta} Z = \frac{1}{\alpha} y_i + \frac{1}{\beta} z_{j \neq i} \quad \text{in case (b3).} \quad (17c)$$

Since all expressions in the maximization problems (17a), (17b) and (17c) are formulated in characteristics space, prices and incomes are virtual magnitudes (Cornes & Sandler, 1994). Specifically, the virtual income (or full income) on the left-hand side of each virtual budget constraint includes the value of spill-ins of the public characteristic provided by the other individual in the two-person economy.

This setting implies that the virtual budget constraint in characteristics space is kinked (see also Kotchen, 2006), with the kink being the point where only the green good is consumed (case (b3)). Figure 1 illustrates the shape of the virtual budget constraint. Along the lower part, $d_i = 0$ (case (b1)) and the slope of the virtual budget constraint is $-\frac{\beta}{1-\alpha}$; along the upper part, $c_i = 0$ (case (b2)) and the slope is $-\frac{1-\beta}{\alpha}$.⁷

⁷ The shape of the virtual budget constraint shown in Figure 1, with the absolute value of the slope in the upper part being smaller than in the lower part, requires $\alpha + \beta > 1$, which is fulfilled by assumption.

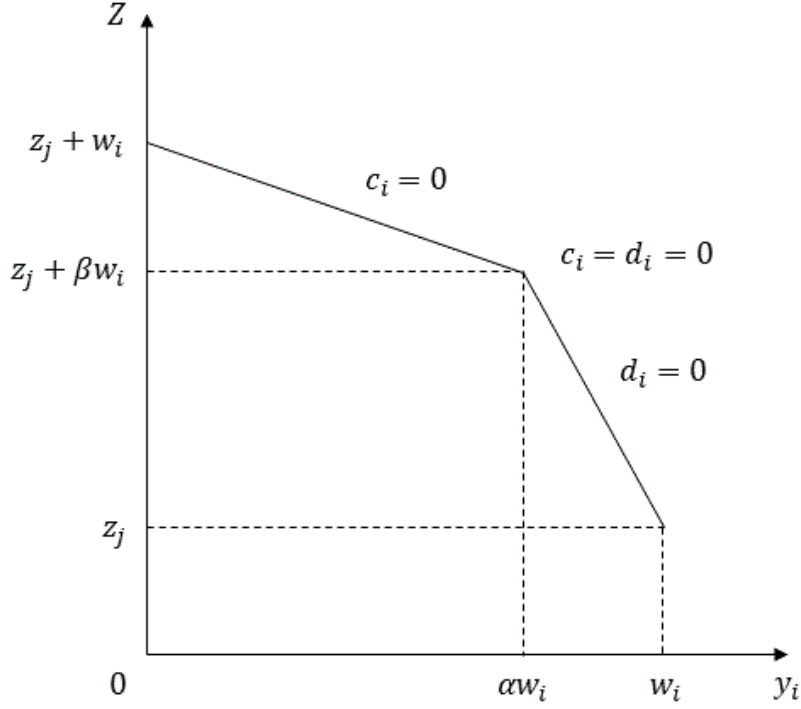


Figure 1: Virtual budget constraint in the impure public good model.

The maximization problems (17a), (17b) and (17c) yield the following reaction functions for the public characteristic z_i :

$$z_i = \begin{cases} \frac{\beta(1-\gamma)}{1-\alpha}w_i - \gamma z_{j \neq i} & \text{in case (b1),} & (18a) \\ (1-\gamma)w_i - \gamma z_{j \neq i} & \text{in case (b2),} & (18b) \\ \beta w_i & \text{in case (b3).} & (18c) \end{cases}$$

Note that equation (18b) is equivalent to the reaction function (3) in the pure public good model. (18a) differs only by the additional expression $\frac{\beta}{1-\alpha}$ in the first term. (18c) is independent of z_j because in the special case that the income is exclusively spent on the green good, the private and public characteristics are generated in fixed proportions determined by the parameters α and β . Hence, the consumer has no “choice” with respect to the amount of the two characteristics he consumes; he will in equilibrium consume exactly $y_i^* = \alpha w_i$ of the private characteristic and provide $z_i^* = \beta w_i$ of the public characteristic, irrespective of the spill-ins received from the other agent.

In any of the three sub-cases a unique Nash equilibrium will result. In cases (b1) and (b2) this is guaranteed by the strictly downward-sloping reaction functions with the absolute value of the

slope between 0 and 1. In case (b3) the reaction functions are horizontal lines which, if plotted in a z_j - z_i diagram, also have a unique point of intersection. The Nash equilibrium values of the two characteristics are given in Table 1 where the results from the pure public good model are also recalled to ease the comparison.

Note that the solution for y_i^* in case (b1) is equivalent to the Nash equilibrium value in the pure public good framework, and in case (b2) the solutions for z_i^* and Z^* are equivalent to those in the pure public good framework.

The next step is again to insert the solutions for y_i^* and Z^* in the utility function to determine the individual's utility level attained at the Nash equilibrium in the impure public good framework. The results are summarized in Table 1.

Due to the modified market structure with the newly introduced green good, the Pareto optimum in the impure public good model will differ from the Pareto optimum in the pure public good model. The general expression of maximization problem (9) now has to be adapted to cases (b1) to (b3). The analysis is simplified by the assumption of Cobb-Douglas preferences that are identical for the two agents considered here. However, the agents are allowed to differ in their monetary income levels. This situation is comparable to having only one agent and varying his income level, as illustrated in Figure 2. As the Cobb-Douglas preferences are homogeneous, the income expansion path (in the case of a linear budget constraint) is a straight line through the origin, as depicted by the ray IEP . The piecewise linear budget frontier will be shifted outwards in a parallel way if monetary income w_i increases, so the locus of all kinks, indicated by the dotted line in Figure 2, is a straight line through the origin as well. Consequently, two agents with the same preferences, but different levels of income will always be located at the same segment of the piecewise linear budget constraint (or at the kink if the two rays through the origin coincide). An example is provided in Figure 2.

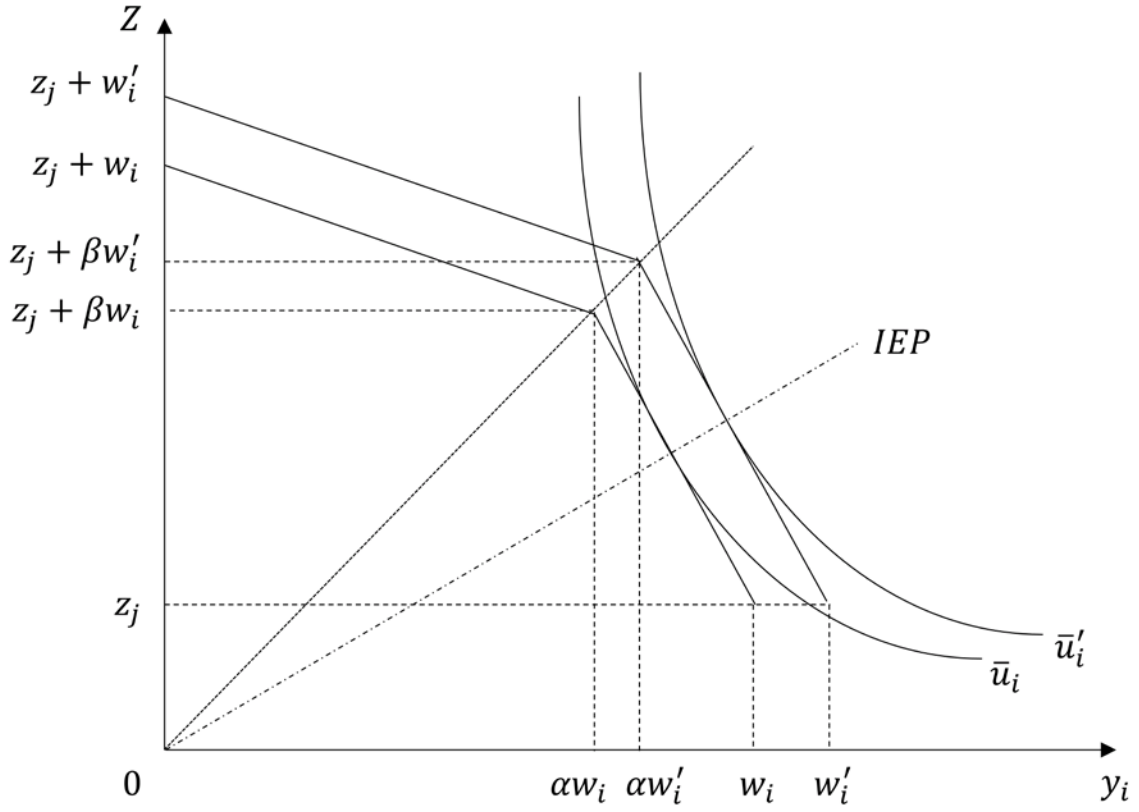


Figure 2: Piecewise linear virtual budget constraint for different levels of monetary income and income expansion path for Cobb-Douglas preferences.

Each of the three cases (b1) to (b3) therefore applies to both agents $i = 1, 2$. The maximization problems, transferred to characteristics space, now read:

$$\max_{y_i, y_j, Z} u_i(y_i, Z) = y_i^\gamma Z^{1-\gamma} \quad \text{s. t.} \quad \sum_{i=1}^2 w_i = \sum_{i=1}^2 y_i + \frac{1-\alpha}{\beta} Z, \quad \text{in case (b1),} \quad (19a)$$

$$u_j(y_j, Z) = y_j^\gamma Z^{1-\gamma} = \bar{u}_j$$

$$\text{s. t.} \quad \sum_{i=1}^2 w_i = \frac{1-\beta}{\alpha} \sum_{i=1}^2 y_i + Z, \quad \text{in case (b2),} \quad (19b)$$

$$u_j(y_j, Z) = y_j^\gamma Z^{1-\gamma} = \bar{u}_j$$

$$\text{s. t.} \quad \sum_{i=1}^2 w_i = \frac{1}{\alpha} \sum_{i=1}^2 y_i = \frac{1}{\beta} Z, \quad \text{in case (b3).} \quad (19c)$$

$$u_j(y_j, Z) = y_j^\gamma Z^{1-\gamma} = \bar{u}_j$$

The Pareto-optimal levels of aggregate private and public characteristic provision are given in Table 2. For selecting a specific Pareto optimum in order to determine individual contributions, the procedure from Section 2.2 is applied once again, i.e. it is assumed that the shares of public characteristic provision by individuals 1 and 2 are the same as in the Nash equilibrium. However, the individual budget constraint must now be expressed in characteristics space, which yields, for example, in case (b1), $w_i = c_i + q_i = y_i^{**} + \frac{1-\alpha}{\beta} z_i^{**}$ and hence, $y_i^{**} = w_i - \frac{1-\alpha}{\beta} z_i^{**} = w_i - \frac{1-\alpha}{\beta} \frac{\beta}{1-\alpha} (w_i - \gamma w_j) = \gamma w_j$. As in the pure public good framework, it can be demonstrated that individual contributions to the public characteristic z_i are larger in the Pareto optimum than in the Nash equilibrium in cases (b1) and (b2) of the impure public good framework. In case (b3) where the agents restrict their demand to the green good, the results for voluntary private provision and socially optimal provision are identical. The utility levels attained in each case can be derived by inserting the solutions given in Table 2 in the utility function; they are provided in Table 3.

3. Discussion

Having developed the solutions for the Nash equilibria and Pareto optima in the pure and impure public good frameworks, respectively, as well as the associated utility levels, I now proceed to the interpretation and discussion of these results. At first, the implications of introducing the green good for environmental quality will be assessed before turning to a measure of social welfare effects.

3.1. Implications for environmental quality

The Index of Easy Riding (IER) suggested by Cornes and Sandler (1996, pp. 159–161) lends itself to a straightforward assessment of environmental quality as it quantifies the ratio of voluntary private provision of the public characteristic over the social optimum, e.g., for the pure public good benchmark case it can immediately be computed from equations (6) and (12):

$$\text{IER}^a = \frac{Z^*}{Z^{**}} = \frac{\frac{(1-\gamma)^2}{1-\gamma^2} \sum_{i=1}^2 w_i}{(1-\gamma) \sum_{i=1}^2 w_i} = \frac{1-\gamma}{1-\gamma^2}. \quad (20)$$

Voluntary private provision will therefore be suboptimally low if $1 - \gamma < 1 - \gamma^2$, which is always fulfilled due to the assumption that $0 < \gamma < 1$. This result is also in line with the well-known divergence of the Nash equilibrium condition (8) and the Samuelson condition (16) for the pure public good model.

As Table 1 illustrates, the introduction of the green good does not change the Nash equilibrium level Z^* if the consumer has a relatively strong preference for the public characteristic, i.e. the green good is purchased together with a donation and private good consumption is zero (case (b2)). This is actually not surprising: demand for the private characteristic is satisfied by consumption of the green good⁸ and the “residual demand” for the public characteristic is met by the provision of direct donations, taking into account the existing level of the public characteristic. The situation is therefore comparable to that of the pure public good model. If, in contrast, the consumer has a relatively strong preference for the private characteristic, i.e. the green good is consumed together with the private good and donations are zero (case (b1)), voluntary private provision of environmental quality increases by the factor $\frac{\beta}{1-\alpha}$, which is strictly positive by definition and, moreover, greater than unity as $\alpha + \beta > 1$ has been assumed.⁹ This is because the demand for the private characteristic, y_i^* , is the same as in the pure public good framework, but part of the private characteristic consumption (that has been exclusively generated by the private good before) is now generated by the green good. Finally, if the agent prefers to consume the public characteristic exactly at the level provided by the green good, i.e. the green good is exclusively consumed (case (b3)), the result is ambiguous: voluntary provision of environmental quality will only rise beyond the pure public good level if $\beta > \frac{(1-\gamma)^2}{1-\gamma^2}$.

Similarly, the socially optimal levels of environmental quality coincide in the pure public good framework and in the impure public good framework if both agents have a relatively strong preference for the public characteristic (case (b2) in Table 2). If both agents have a relatively strong preference for the private characteristic (case (b1)), the expression for Z^{**} is augmented

⁸ Note that, although Z_i^* and Z^* are identical in the pure public good framework and in case (b2) of the impure public good framework, demand for the private characteristic y_i^* rises by $\frac{\alpha}{1-\beta} > 1$ if the green good is available. The given utility function then implies that individual utility also increases due to the introduction of the green good as y_i^* rises, Z^* remains unchanged and $\frac{\partial u_i}{\partial y_i} > 0$ holds (as well as $\frac{\partial u_i}{\partial Z} > 0$).

⁹ Again this result implies that utility increases due to the introduction of the green good: Table 1 reveals that demand for the private characteristic y_i^* remains unchanged whereas the provision of the public characteristic Z^* increases.

by the factor $\frac{\beta}{1-\alpha}$, as in the Nash equilibrium for the respective case. In the special case that both agents demand only the green good (case (b3)), the relationship between the Pareto optima in the pure and impure public good frameworks is again ambiguous and depends on the ratio of β and $1 - \gamma$.

However, case (b3) yields identical results for the Nash equilibrium and the Pareto optimum level of Z . Hence, there will be no underprovision of environmental quality if the green good is exclusively consumed. It then generates the private and public characteristics exactly in the same proportion as desired by the two agents. This is reflected by an IER value of one. All results for the pure public good framework and the different cases in the impure public good framework are summarized in Table 3. Note that in cases (b1) and (b2) the IER is equivalent to the outcome of the pure public good framework and hence, underprovision will definitely be an issue even though an impure public good is now traded on the market. This can be explained by the impure public good raising both Nash equilibrium and Pareto-optimal levels of public characteristic provision by the same amount in case (b1); in other words, the availability of the green good increases voluntary private provision of environmental quality, but the social optimum increases as well, so the gap remains unchanged. In case (b2) the introduction of the impure public good has no effect on overall environmental quality.

3.2. Social welfare

As the IER is not a suitable measure for welfare evaluation, the equivalent variation (EV) is now used to determine the amount of income that consumers in the Pareto optimum have to give up or receive in order to obtain the same utility as in the Nash equilibrium. Individual utility in the pure public good framework (case (a)) and in cases (b1) and (b2) of the impure public good framework can either increase or decrease when the agent moves from the Nash equilibrium to the Pareto-optimal outcome. By contrasting the utility levels given in the last column of Table 1 and the second column of Table 3 it becomes obvious that $u_i(y_i^{**}, Z^{**}) > u_i(y_i^*, Z^*)$ only if $\frac{1-\gamma}{1-\gamma^2} < \left(\frac{w_j}{\sum_{i=1}^2 w_i}\right)^\gamma$. Only in case (b3) that covers the exclusive consumption of the green good the result is unambiguous as the two utility levels $u_i(y_i^*, Z^*)$ and $u_i(y_i^{**}, Z^{**})$ are identical. Therefore, the sign of the social welfare measure can in general be positive or negative.

It has been shown previously that the Pareto-optimal overall level of the public characteristic is greater than the voluntarily provided amount in cases (a), (b1) and (b2), i.e. $Z^{**} > Z^*$. A comparison of Tables 1 and 2 reveals that the same holds at the individual level: both agents increase their individual contributions to the public characteristic in the Pareto optimum as opposed to the Nash equilibrium in these three cases ($z_i^{**} > z_i^*$ follows from $0 < \frac{1-\gamma}{1-\gamma^2} < 1$, which holds because $0 < \gamma < 1$ by assumption). This means that from the perspective of agent i , full income increases if the other agent j provides the Pareto-optimal amount of the public characteristic as i now receives more public-characteristic spill-ins. Consequently, in the pure public good framework, the virtual budget constraint (which is linear in this case) is shifted upwards by $z_j^{**} - z_j^* = 1 - \frac{1-\gamma}{1-\gamma^2} = \frac{\gamma(1-\gamma)}{1-\gamma^2} > 0$.¹⁰ This situation is illustrated in Figure 3. However, individual optimization under the new budget constraint – along i 's income expansion path IEP – would yield a public characteristic level of Z^1 , which is not Pareto-optimal since $Z^1 - Z^* < z_j^{**} - z_j^*$, i.e. individual i is tempted to take an easy ride on the contribution of j and reduce his own contribution z_i .

The Pareto-optimal level of the public characteristic will therefore be located to the north-west of Z^1 along the new virtual budget constraint, e.g., at Z^{**} in Figure 3. The Pareto optimum is not a point of tangency between i 's virtual budget constraint and an indifference curve, but is situated at the intersection of an indifference curve \bar{u}_i^{**} and the new virtual budget constraint, as depicted in Figure 4. Since utility in the Pareto optimum can either exceed or fall short of (or be identical to) \bar{u}_i^* as discussed earlier in this section, the precise location of (y_i^{**}, Z^{**}) can be either to the right of the intersection of the indifference curve belonging to \bar{u}_i^* and the new virtual budget constraint (as in Figures 3 and 4) or to the left (or at the same point).

¹⁰ This holds because the prices of all goods have been set to unity and hence, the budget constraint in problem (2) can be expressed in full-income form as $w_i + z_j = y_i + Z$. In other words, the virtual price of Z in the pure public good framework is unity as well. The value of public-characteristic spill-ins as part of the full income is therefore equal to their absolute amount.

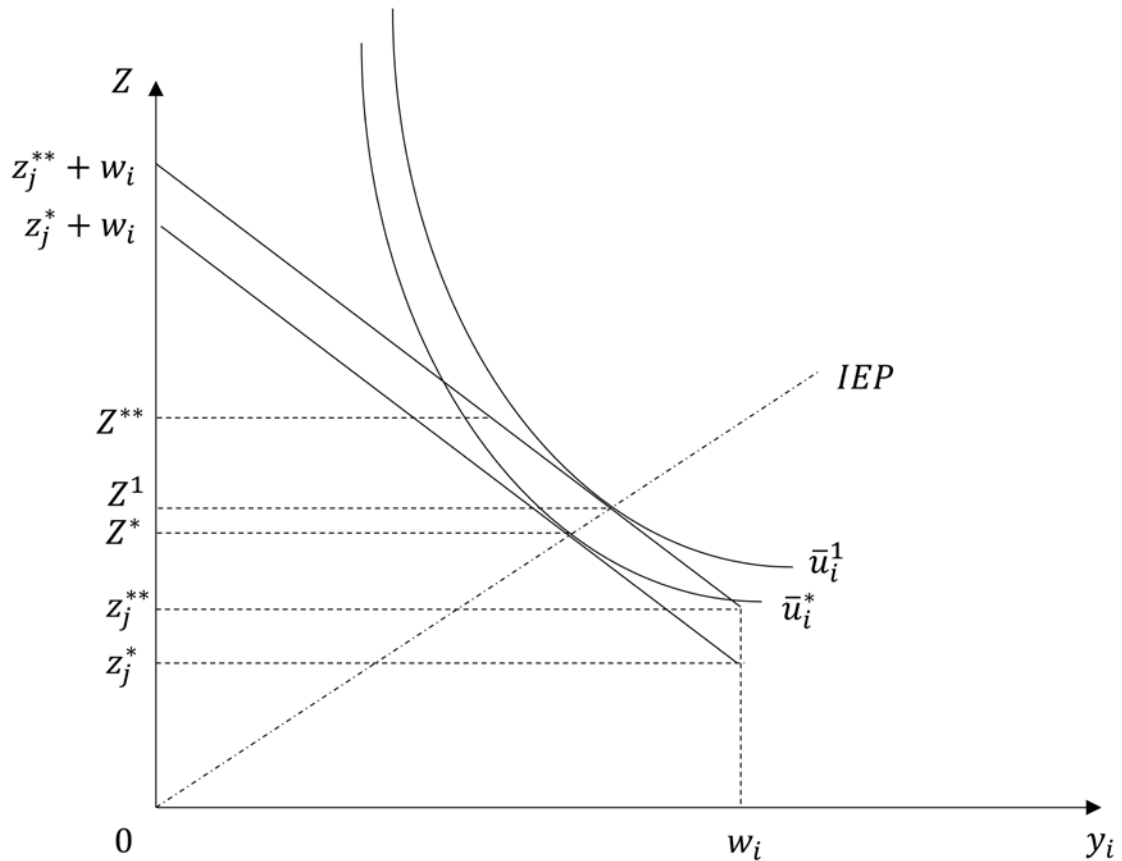


Figure 3: The effect of an increase in public-characteristic spill-ins from agent j on agent i in the pure public good framework.

Following Auld and Eden (1990), the social welfare gap EV can be computed as the vertical distance between the virtual budget constraint that is tangent to \bar{u}_i^{**} and the original virtual budget constraint that is tangent to \bar{u}_i^* . I therefore make use of the indirect utility functions given in equations (7) and (15) for voluntary private provision and socially optimal provision in the pure public good framework. Inversion of the indirect utility functions yields the expenditure functions (Deaton & Muellbauer, 1991, p. 38) from which EV can be calculated.

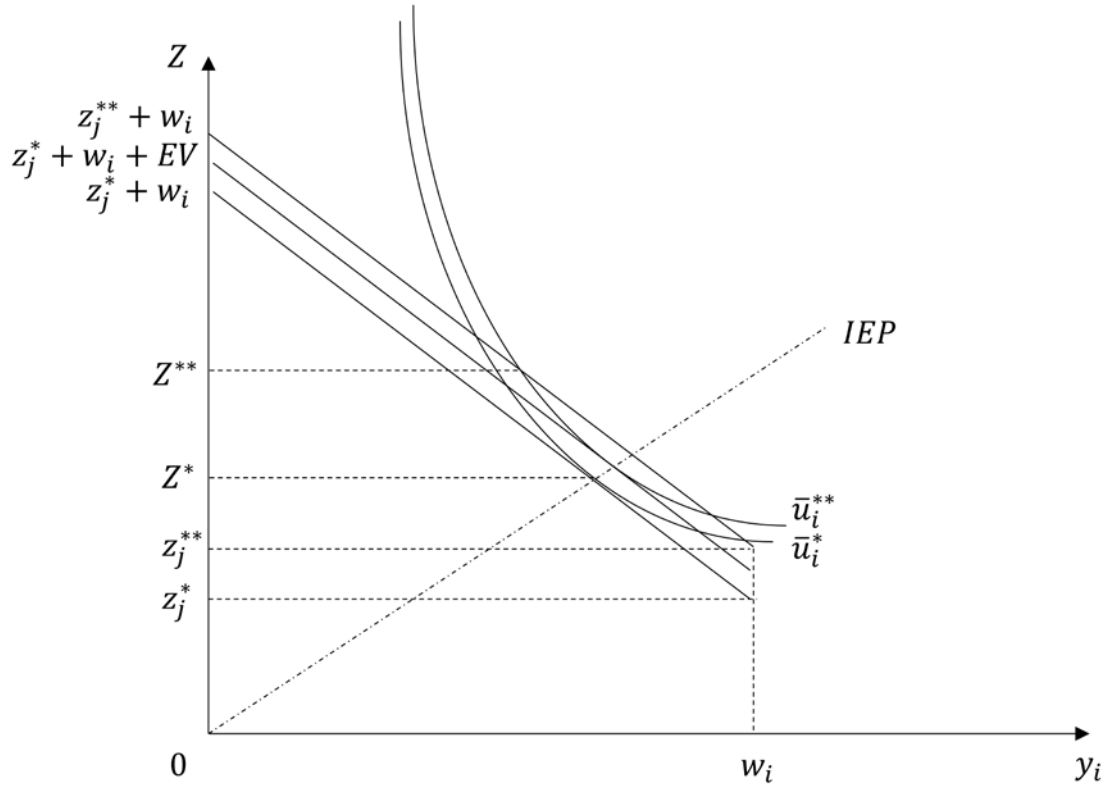


Figure 4: Determination of the social welfare measure EV.

The expenditure function for the Pareto optimum in the pure public good framework resulting from rearranging equation (15) is

$$w_i^{**} = \frac{\bar{u}_i^{** \frac{1}{1-\gamma}}}{(1-\gamma) \left(\frac{\gamma w_j}{p_c}\right)^{\frac{\gamma}{1-\gamma}}} - w_j, \quad (21)$$

and the expenditure function for the Nash equilibrium in the pure public good framework resulting from equation (7) reads

$$w_i^* = \frac{\bar{u}_i^*}{\left(\frac{\gamma}{(1-\gamma)p_c}\right)^\gamma \frac{(1-\gamma)^2}{1-\gamma^2}} - w_j. \quad (22)$$

Subtracting (22) from (21), EV can be computed:

$$EV = w_i^{**} - w_i^* = \frac{\bar{u}_i^{** \frac{1}{1-\gamma}}}{(1-\gamma) \left(\frac{\gamma w_j}{p_c}\right)^{\frac{\gamma}{1-\gamma}}} - \frac{\bar{u}_i^*}{\left(\frac{\gamma}{(1-\gamma)p_c}\right)^\gamma \frac{(1-\gamma)^2}{1-\gamma^2}}. \quad (23)$$

Given that \bar{u}_i^{**} can be larger, smaller or equal to \bar{u}_i^* and that both characteristics are normal, EV can take either sign. A positive value implies that i has to give up a certain amount of income in the Pareto optimum to remain at his Nash utility level and vice versa.

As the impact of the introduction of the green good is to be analyzed, the result from equation (23) has to be opposed to the respective social welfare gaps for cases (b1) to (b3). These are determined in a completely analogous way as in case (a) and can be found in the last column of Table 3. Starting with (b3) as the simplest possibility, it can immediately be observed that the indirect utility function in the Nash equilibrium as given in Table 1 is identical to its counterpart in the Pareto optimum (see Table 3). Hence, the expenditure functions are also identical and EV will take a value of zero. The introduction of the green good will therefore completely close the welfare gap in the special case that consumption is limited to that newly launched good.

However, in reality it is not very likely that consumers will exclusively buy the green good. The remaining cases (b1) and (b2) are probably more relevant. Inspection of the last column of Table 3 reveals that in case (b1) the first term of EV differs by $\frac{1-\alpha}{\beta} < 1$ from the same term in case (a) whereas the second term differs by $\left(\frac{1-\alpha}{\beta}\right)^{1-\gamma}$, which is also smaller than unity, but greater than $\frac{1-\alpha}{\beta}$. It follows that both terms decrease, but the decrease of the second term is numerically smaller, i.e. EV decreases due to the introduction of the green good. This implies that in the case of a positive EV the absolute value of the welfare gap becomes smaller whereas for a negative EV it increases. In other words, if the agent initially (i.e. before the introduction of the green good) benefits when moving from the Nash equilibrium to the Pareto optimum ($u_i^* < u_i^{**}$), this additional benefit is reduced after the introduction of the green good, the difference between Nash equilibrium and Pareto-optimal outcome gets smaller and the gap narrows. If, in contrast, the considered agent initially loses when moving from the Nash equilibrium to the Pareto optimum ($u_i^* > u_i^{**}$), the absolute value of this loss becomes greater, i.e. the welfare gap increases in absolute terms. If the welfare gap in the pure public good benchmark situation is zero (i.e. $u_i^* = u_i^{**}$ because $\frac{1-\gamma}{1-\gamma^2} = \left(\frac{w_j}{\sum_{i=1}^2 w_i}\right)^\gamma$ holds), it remains zero after the introduction of the green good.

Case (b2) looks more intricate as the first term of EV differs from its case (a) counterpart by the additional expression $\left(\frac{1-\beta}{\alpha}\right)^{\frac{\gamma}{1-\gamma}} < 1$, but lacks $\frac{1}{1-\gamma} > 1$. Consequently, that first term is smaller in case (b2). The second term has $\left(\frac{1-\beta}{\alpha}\right)^{\gamma} < 1$ as an additional factor and is smaller as well. Since $\left(\frac{1-\beta}{\alpha}\right)^{\gamma} > \left(\frac{1-\beta}{\alpha}\right)^{\frac{\gamma}{1-\gamma}}$, the second term decreases by less than the first one (which is additionally reduced by the missing expression $\frac{1}{1-\gamma}$). Again EV decreases due to the introduction of the green good. The interpretation of the results is completely analogous to case (b1).

4. Conclusion

The purpose of this paper was to investigate the effects on environmental quality and social welfare if an impure public good – the green good – is introduced on a market. The green good has been modeled such that its jointly produced characteristics are both available separately in terms of a private consumption good and donations to an environmental organization as a pure public good. The Nash equilibria and Pareto optima have been derived for the situations before and after the launch of the green good.

The impact of the green good on environmental quality has been assessed by means of the Index of Easy Riding (IER) that is the fraction of the Nash equilibrium quantity of the public characteristic over its Pareto-optimal quantity. Interestingly, the IER value remains the same as before the introduction of the green good if the two agents consume the green good together with either the private good or direct donations, indicating that underprovision of the public characteristic persists. In the special case where only the green good is consumed by both agents, the voluntarily provided level of environmental quality exactly matches the socially optimal level; this is the only case where underprovision can be overcome.

The social welfare gap can take a positive, negative or zero value in the pure public good framework and in the impure public good framework if in the latter the green good is consumed together with either the private good or direct donations. However, a value of zero can only occur if $\frac{1-\gamma}{1-\gamma^2} = \left(\frac{w_j}{\sum_{i=1}^2 w_i}\right)^{\gamma}$, which is a very restrictive condition and hence rather unlikely, although in principle possible. Only if the two agents restrict their consumption to the impure public good, the welfare gap definitely becomes zero. A general statement about the effect of

the introduction of the green good cannot be made as the change of the absolute value of EV depends on its sign. If the welfare gap is positive, indicating that an agent gains utility by moving from the Nash solution to the Pareto optimum, it will become smaller after the introduction of the green good. If the welfare gap is negative, i.e. an agent loses utility by moving from the Nash solution to the Pareto optimum, it will increase in absolute terms. Yet, the overall results of this paper are in line with previous findings that the introduction of a green good does not unambiguously yield an improvement of environmental quality and social welfare.

Some points remain open for further research. The model presented here could be modified such that there are no private or pure public good substitutes for the jointly produced characteristics of the impure public good, or a different type of utility function could be employed. The number of agents could also be increased to $n > 2$. Moreover, the model could be extended to the introduction of multiple impure public goods with different, possibly adverse effects on the environment.

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References

- Auld, D. A. L., & Eden, L. (1990). Public Characteristics of Non-Public Goods. *Public Finance*, 45(3), 378–391.
- Barrett, S. (2012). *Environment and Statecraft: The Strategy of Environmental Treaty-Making*. Oxford: Oxford University Press.
- Chan, N. W., & Kotchen, M. J. (2014). A Generalized Impure Public Good and Linear Characteristics Model of Green Consumption. *Resource and Energy Economics*, 37, 1–16.
- Cornes, R., & Sandler, T. (1984). Easy Riders, Joint Production, and Public Goods. *The Economic Journal*, 94(375), 580–598.
- Cornes, R., & Sandler, T. (1994). The Comparative Static Properties of the Impure Public Good Model. *Journal of Public Economics*, 54(3), 403–421.
- Cornes, R., & Sandler, T. (1996). *The Theory of Externalities, Public Goods and Club Goods* (2nd ed.). New York: Cambridge University Press.
- Deaton, A., & Muellbauer, J. (1991). *Economics and Consumer Behavior*. Cambridge, Mass., New York, Melbourne, Port Chester, Sydney: Cambridge University Press.
- Engelmann, D., Munro, A., & Valente, M. (2017). On the Behavioural Relevance of Optional and Mandatory Impure Public Goods. *Journal of Economic Psychology*, 61, 134–144.

- Eyckmans, J., & Finus, M. (2006). Coalition Formation in a Global Warming Game: How the Design of Protocols Affects the Success of Environmental Treaty-Making. *Natural Resource Modeling*, 19(3), 323–358.
- Finus, M., & Rübhelke, D. (2013). Public Good Provision and Ancillary Benefits: The Case of Climate Agreements. *Environmental and Resource Economics*, 56(2), 211–226.
- Haumann, B. F. (2017). Organic Continues to Set Records in the United States. In *The World of Organic Agriculture: Statistics & Emerging Trends 2017* (pp. 258–263). Bonn: Research Institute of Organic Agriculture FiBL; IFOAM - Organics International.
- Kotchen, M. J. (2005). Impure Public Goods and the Comparative Statics of Environmentally Friendly Consumption. *Journal of Environmental Economics and Management*, 49(2), 281–300.
- Kotchen, M. J. (2006). Green Markets and Private Provision of Public Goods. *Journal of Political Economy*, 114(4), 816–834.
- Kotchen, M. J. (2009). Voluntary Provision of Public Goods for Bads: A Theory of Environmental Offsets. *The Economic Journal*, 119, 883–899.
- Lancaster, K. (1971). *Consumer Demand: A New Approach*. *Columbia Studies in Economics: Vol. 5*. New York, London: Columbia University Press.
- Morey, E. R. (1985). Characteristics, Consumer Surplus, and New Activities: A Proposed Ski Area. *Journal of Public Economics*, 26(2), 221–236.
- Munro, A., & Valente, M. (2016). Green Goods: Are They Good or Bad News for the Environment? Evidence from a Laboratory Experiment on Impure Public Goods. *Environmental and Resource Economics*, 65(2), 317–335.
- Murdoch, J. C., & Sandler, T. (1984). Complementarity, Free Riding, and the Military Expenditures of NATO Allies. *Journal of Public Economics*, 25, 83–101.
- Muth, R. F. (1966). Household Production and Consumer Demand Functions. *Econometrica*, 34(3), 699–708.
- Olson, M. (1971). *The Logic of Collective Action*. Cambridge, Mass.: Harvard University Press.
- Posnett, J., & Sandler, T. (1986). Joint Supply and the Finance of Charitable Activity. *Public Finance Quarterly*, 14(2), 209–222.
- Samuelson, P. A. (1954). The Pure Theory of Public Expenditure. *The Review of Economics and Statistics*, 36(4), 387–389.
- Sandler, T. (1996). A Game-Theoretic Analysis of Carbon Emissions. In R. D. Congleton (Ed.), *The Political Economy of Environmental Protection: Analysis and Evidence* (pp. 251–272). Ann Arbor: University of Michigan Press.
- Sandler, T. (2003). Assessing the Optimal Provision of Public Goods: In Search of the Holy Grail. In I. Kaul, P. Conceição, K. Le Goulven, & R. U. Mendoza (Eds.), *Providing Global Public Goods: Managing Globalization* (pp. 131–151). New York, Oxford: Oxford University Press.
- Sandmo, A. (1973). Public Goods and the Technology of Consumption. *Review of Economic Studies*, 40(4), 517–528.
- Vicary, S. (2011). Public Goods and the Commons: A Common Framework. *Journal of Public Economic Theory*, 13(1), 47–69.
- Willer, H., Schaack, D., & Lernoud, J. (2017). Organic Farming and Market Development in Europe and the European Union. In *The World of Organic Agriculture: Statistics &*

Emerging Trends 2017 (pp. 207–243). Bonn: Research Institute of Organic Agriculture FiBL; IFOAM - Organics International.

	y_i^*	z_i^*	Z^*	$u_i(y_i^*, Z^*)$	
(a)	$\frac{\gamma(1-\gamma)}{1-\gamma^2} \sum_{i=1}^2 w_i$	$\frac{1-\gamma}{1-\gamma^2} (w_i - \gamma w_j)$	$\frac{(1-\gamma)^2}{1-\gamma^2} \sum_{i=1}^2 w_i$	$\left(\frac{\gamma}{1-\gamma}\right)^\gamma \frac{(1-\gamma)^2}{1-\gamma^2} \sum_{i=1}^2 w_i$	
(b1)	$z_i < \beta q_i$	$\frac{\gamma(1-\gamma)}{1-\gamma^2} \sum_{i=1}^2 w_i$	$\frac{\beta}{1-\alpha} \frac{1-\gamma}{1-\gamma^2} (w_i - \gamma w_j)$	$\frac{\beta}{1-\alpha} \frac{(1-\gamma)^2}{1-\gamma^2} \sum_{i=1}^2 w_i$	$\frac{\gamma^\gamma(1-\gamma)}{1-\gamma^2} \left(\frac{\beta}{1-\alpha}(1-\gamma)\right)^{1-\gamma} \sum_{i=1}^2 w_i$
(b2)	$z_i > \beta q_i$	$\frac{\alpha}{1-\beta} \frac{\gamma(1-\gamma)}{1-\gamma^2} \sum_{i=1}^2 w_i$	$\frac{1-\gamma}{1-\gamma^2} (w_i - \gamma w_j)$	$\frac{(1-\gamma)^2}{1-\gamma^2} \sum_{i=1}^2 w_i$	$\frac{(1-\gamma)^2}{1-\gamma^2} \left(\frac{\alpha}{1-\beta} \frac{\gamma}{1-\gamma}\right)^\gamma \sum_{i=1}^2 w_i$
(b3)	$z_i = \beta q_i$	αw_i	βw_i	$\beta \sum_{i=1}^2 w_i$	$(\alpha w_i)^\gamma \left(\beta \sum_{i=1}^2 w_i\right)^{1-\gamma}$

Table 1: Nash equilibrium levels of the private and public characteristics and Nash utility levels in the pure public good (case (a)) and impure public good (cases (b1) to (b3)) frameworks.

		$\sum_{i=1}^2 y_i^{**}$	y_i^{**}	z_i^{**}	Z^{**}
(a)		$\gamma \sum_{i=1}^2 w_i$	γw_j	$w_i - \gamma w_j$	$(1 - \gamma) \sum_{i=1}^2 w_i$
(b1)	$z_i < \beta q_i$ for $i = 1, 2$	$\gamma \sum_{i=1}^2 w_i$	γw_j	$\frac{\beta}{1 - \alpha} (w_i - \gamma w_j)$	$\frac{\beta}{1 - \alpha} (1 - \gamma) \sum_{i=1}^2 w_i$
(b2)	$z_i > \beta q_i$ for $i = 1, 2$	$\frac{\alpha \gamma}{1 - \beta} \sum_{i=1}^2 w_i$	$\frac{\alpha \gamma}{1 - \beta} w_j$	$w_i - \gamma w_j$	$(1 - \gamma) \sum_{i=1}^2 w_i$
(b3)	$z_i = \beta q_i$ for $i = 1, 2$	$\alpha \sum_{i=1}^2 w_i$	αw_i	βw_i	$\beta \sum_{i=1}^2 w_i$

Table 2: Pareto-optimal levels of the private and public characteristics in the pure public good (case (a)) and impure public good (cases (b1) to (b3)) frameworks.

	$u_i(y_i^{**}, Z^{**})$	IER	EV
(a)	$(\gamma w_j)^\gamma \left((1-\gamma) \sum_{i=1}^2 w_i \right)^{1-\gamma}$	$\frac{1-\gamma}{1-\gamma^2}$	$\frac{\bar{u}_i^{** \frac{1}{1-\gamma}}}{(1-\gamma)(\gamma w_j)^{\frac{\gamma}{1-\gamma}}} - \frac{\bar{u}_i^*}{\left(\frac{\gamma}{1-\gamma}\right)^\gamma \frac{(1-\gamma)^2}{1-\gamma^2}}$
(b1)	$(\gamma w_j)^\gamma \left(\frac{\beta}{1-\alpha} (1-\gamma) \sum_{i=1}^2 w_i \right)^{1-\gamma}$	$\frac{1-\gamma}{1-\gamma^2}$	$\frac{\bar{u}_i^{** \frac{1}{1-\gamma}}}{\frac{\beta(1-\gamma)}{1-\alpha} (\gamma w_j)^{\frac{\gamma}{1-\gamma}}} - \frac{\bar{u}_i^*}{\left(\frac{\beta}{1-\alpha}\right)^{1-\gamma} \left(\frac{\gamma}{1-\gamma}\right)^\gamma \frac{(1-\gamma)^2}{1-\gamma^2}}$
(b2)	$\left(\frac{\alpha\gamma}{1-\beta} w_j\right)^\gamma \left((1-\gamma) \sum_{i=1}^2 w_i \right)^{1-\gamma}$	$\frac{1-\gamma}{1-\gamma^2}$	$\frac{\bar{u}_i^{** \frac{1}{1-\gamma}}}{\left(\frac{\alpha\gamma}{1-\beta} w_j\right)^{\frac{\gamma}{1-\gamma}}} - \frac{\bar{u}_i^*}{\left(\frac{\alpha}{1-\beta} \frac{\gamma}{1-\gamma}\right)^\gamma \frac{(1-\gamma)^2}{1-\gamma^2}}$
(b3)	$(\alpha w_i)^\gamma \left(\beta \sum_{i=1}^2 w_i \right)^{1-\gamma}$	1	0

Table 3: Pareto utility levels, IER values and social welfare gaps in the pure public good (case (a)) and impure public good (cases (b1) to (b3)) frameworks.