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Income Distribution and Shock Transmission

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Income Distribution and Shock Transmission

A Simple Heterogeneous Agent New Keynesian Perspective

March 1, 2019

Our very simple two agent New Keynesian model is highly stylised. It consists of an entrepreneur, who owns the economies' firms, consumes and saves, but does not work (or does not receive wage income), while the worker consumes and works, but cannot save. The allocation of the ability to save only to entrepreneurs instead of only workers differentiates our model from the only similarly simple model we are aware of Broer *et al.* (2016). As opposed to Broer *et al.* (2016), who additionally require the introduction of sticky wages in their saving-worker-non-saving-entrepreneur-model we find that our heterogeneous agent version gives qualitatively quite similar impulse responses to a monetary policy shocks already in the baseline version with sticky-prices only. Quantitatively, the response of monetary policy is weaker in the heterogeneous agent model, hinting at the importance of (the correct representation of) heterogeneity for the transmission of monetary policy. Additionally, the model allows to consider distributional effects in the wake of a shock. They appear to be in line with empirics for monetary and preference shocks.

keywords: *(functional) income distribution, monetary policy transmission, heterogeneous agents, two agents new keynesian model*

1 Introduction

The need to better understand monetary policy transmission in the wake of the financial crisis but also a revived interest in distributional aspects of monetary policies and events has led to the development of various types of heterogeneous agent models. Not least of all, only heterogeneous agent models allow to analyse the impact of idiosyncratic shocks on the aggregate economy (see, e.g. Heathcote *et al.* (2009) and Ahn *et al.* (2017)).

We aim at developing an “as simple as possible” model which allows to analyse the transmission of shocks in the presence of inequality while at the same time allowing to derive the functional income distribution. To the best of our knowledge, this has not been analysed before.

We are primarily interested in how heterogeneous ownership of productive capital influences the response to monetary policy and preference shocks to analyse the model response to financial crisis type shocks. In the baseline version, our model does not even include productive capital explicitly, but only entrepreneurs own firms (and benefit from firms’ profits).

Numerous New Keynesian type models have been developed, often integrating more than the standard “safe” financial asset to transfer consumption over time and often limiting the access of a fraction of the households or firms to financial markets. When heterogeneity is not expressed by unequal access to credit markets, it is expressed by different degrees of “patience”, i.e. differing discount factors between entrepreneurs and the more patient, non-entrepreneurial, households.

Our baseline model is the most simple model with two representative agents and limited participation in financial markets imaginable (to us): a worker, who does not save intertemporally, i.e. a hand-to-mouth consumer, and an entrepreneur, who has access to credit markets. The worker derives income from labour only while the entrepreneur does not supply labour on the labour market but derives income from profits. Most other two agent New Keynesian (TANK¹) models we are aware of are considerably more complex (two early ones are Galí *et al.* (2007) and Bilbiie (2008)) and thus make it harder to identify the relevant transmission channels.

A related simple TANK model, where, however, the workers are able to save and the entrepreneurs (“capitalists”) are not,² is analysed in Broer

¹ According to Debortoli and Gali (2018) this acronym was introduced by Kaplan *et al.* (2016)

² They decide on this initially surprising assumption, because the model then faces indeterminacy under a standard Taylor rule. We circumvent this problem by permitting $\sigma \neq 1$, i.e. abandoning the log-utility function.

et al. (2016), who find that their model output is unresponsive to monetary policy shocks as hours worked do not change with changes in wages. Hence, they introduce sticky wages to arrive at almost identical equilibrium responses to an interest rate shock as the baseline sticky-price representative agent New Keynesian (RANK) model.

The relatively large difference in the strength of model responses between these two simple models illustrates how important it is to pay attention to who actually saves (in terms of the functional income distribution). In a simple New Keynesian two agent model (with sticky prices only), having only the worker save does not change labour supply and actually increases profits in response to a contractionary monetary policy shock (see Broer *et al.* (2016), fig. 1) while having only the entrepreneur save slightly reduces profits and more strongly reduces employment in response to an interest rate rise.

A related, already slightly more complicated, model is Bilbiie (2008), whose agents are all identical (workers and entrepreneurs at the same time as in a baseline representative agent model) except for their access to financial markets. A share of the agents cannot save. Bilbiie offers an analysis of the dynamics after a cost-push shock, but unfortunately not for a monetary policy or preference shock.³

We find that in our worker-entrepreneur-model, most responses are qualitatively similar to the single representative agent New Keynesian (RANK) model. The negative response of output and consumption to a contractionary monetary policy shock is stronger in comparison to the RANK model. The inflation rate reacts very timidly. Hence, distribution of ownership of income sources (production factors), appears of considerable importance for the transmission. We find that unlike empirically found⁴ the worker's consumption⁵ increases in the wake of a monetary policy shock. We also note that for standard calibration parameters the share of the worker's consumption in total consumption is unrealistically low, below 1% of total consumption - which is why we developed a second (still comparatively simple) model, that includes capital on the entrepreneurs side. This model allows for the functional income distribution derived from a Cobb-Douglas technology and for a negative impact of interest rate increases on entrepreneurs consumption.

We are interested in the impact of a monetary policy shock on a very

³ This could be a medium run extension for this paper.

⁴ An overview is given in Deutsche Bundesbank (2016). See also Furceri *et al.* (2016), who find contractionary monetary policy to increase income inequality.

⁵ We equate the worker with the lower end of the income distribution because the worker is defined as the agent with no access to capital markets, which could be explained by low income - although it is not explicitly modelled. Empirics tend to find that lower income groups tend to suffer from interest rate hikes while higher income groups tend to benefit.

simple heterogeneous agent model. The focus is to illustrate the impact of limiting (in our extreme case “forbidding”) access to financial markets to a share of economic agents could impact on the transmission mechanism in the simplest possible model while at the same time considering the functional income distribution.

How does the model react compared to a baseline Representative Agent New Keynesian model and how has the monetary transmission mechanism changed in comparison to a Rank model? Besides, we are interested in the distributional impact of a monetary shock in this simple model.

As a type of robustness analysis and to see what this simple model might predict for the last financial crisis, a preference shock is also analysed in the model. We find that changes in the saving behaviour of the entrepreneur due to a preference shock actually affect the worker’s consumption relatively more than the entrepreneurs consumption.

Although the preference shock is an aggregate one, we can as well interpret it as idiosyncratic, because it directly impacts only on the entrepreneur’s decision rule.

In the next section we will present the model and compare it to a baseline representative agent model before presenting some impulse responses and discussing the transmission of the shock.

2 Simple Two Agent New Keynesian Model

In our Simple Two Agent New Keynesian Model we have the extreme case of an entrepreneur, who do not work (at least this is not modelled in the equations) and a worker, who is a hand-to-mouth consumer and does not save.

Although those two types appear to behave differently, their decisions are basically due to identical utility functions, i.e. their different behaviour is solely due to differing budget constraints.

It should be possible to skip the model derivation and go directly to the description of the impulse responses if one is less interested in the underlying equations.

The worker’s variables are indexed by w in the following. The variables referring solely to the entrepreneur are indexed by e . An exception is labour, N , which is only provided by workers and thus requires no indexation.

Both agents’ behaviour is based on the period utility function

$$U(C_{i,t}, N_{i,t}; Z_t) = Z_t \left(\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \eta \frac{N_{i,t}^{1+\phi}}{1+\phi} \right) \quad \text{with } i \in \{e, w\} \quad . \quad (1)$$

where C represents consumption, N labour, σ is the inverse of the intertemporal elasticity of substitution and ϕ the inverse of the Frisch elasticity of labour. Period utility is subject to an aggregate preference shock $\log(Z_t) = \rho_z \log(Z_{t-1}) + \epsilon_z$ with $\epsilon_z \sim N(0, \sigma_z)$. Hence, an aggregate preference shock impacts only on the entrepreneur's choices. It cancels out in the worker's decision rule. We could, therefore, also take the preference shock as an "idiosyncratic" one that hits only the entrepreneur's utility function.

Since the entrepreneur does not work by definition, $N_{e,t} = 0, \forall t$, total labour supply equals the worker's labour supply $N_{w,t} = N_t, \forall t$.

The worker's budget constraint is the most simple imaginable. Workers do not save. This could be due to a lack of access to credit markets, extreme impatience or an income at the subsistence level of consumption (which is not explicitly modelled, however). This modelling of a worker who does not save is rather extreme, of course, but then this is an extremely simplified model.

$$P_t C_{w,t} = W_t N_t \quad (2)$$

Workers are thus faced with the relatively simple optimisation of

$$\max_{C_{w,t}, N_t} \mathcal{L}_w = Z_t \left(\frac{C_{w,t}^{1-\sigma}}{1-\sigma} - \eta \frac{N_t^{1+\phi}}{1+\phi} \right) + \lambda_{w,t} (P_t C_{w,t} - W_t N_t) \quad (3)$$

which leads to the standard first order condition

$$\frac{\eta N_t^\phi}{C_{w,t}^{-\sigma}} = \frac{W_t}{P_t} \quad . \quad (4)$$

The entrepreneur maximises the expected stream of (infinite) lifetime consumption. Future periods are discounted by the constant per-period discount factor β . Owing to the intertemporal maximisation, the preference shock does not cancel out for the entrepreneur.

$$\max_{C_{e,t}, B_t, \lambda_t} \mathcal{L}_e = E_t \sum_{j=0}^{\infty} \beta^j Z_{t+1} \left(\frac{C_{e,t+1}^{1-\sigma}}{1-\sigma} \right) + \lambda_{e,t+j} (P_{t+j} C_{e,t+j} + Q_{t+j} B_{t+j} - B_{t+j-1} - D_t) \quad (5)$$

Entrepreneurs are assumed to finance their consumption simply by savings in riskless bonds B , acquired in period $t-1$ and "windfall" prof-

its from firms, D_t . Possibly somewhat surprisingly, we do not assume that entrepreneurs have any influence on firms' profits (by putting in a higher effort or more time, for example), but remember, this is a simple model. And if we see entrepreneurs as investors instead of managing entrepreneurs, this could be approximately true, assuming investors invest in some market portfolio. What is not consumed in the present period is again invested in riskless bonds that will pay B_t in the future period. The current price of these bonds is Q_t .

$$\frac{\partial \mathcal{L}_e}{\partial C_{e,t+j}} = \beta^j (E_t Z_t C_{e,t+j}^{-\sigma} + \lambda_{e,t+j} P_{t+j}) \stackrel{!}{=} 0 \quad (6)$$

$$\frac{\partial \mathcal{L}_e}{\partial B_{e,t+j}} = \beta^j \lambda_{e,t+j} E_t Q_{t+j} - \beta^{j+1} \lambda_{e,t+j+1} \stackrel{!}{=} 0 \quad (7)$$

From the first order conditions (6) and (7), we can derive the Euler condition of the entrepreneur (8).

$$E_t \beta \left(\frac{Z_t}{Z_{t+1}} \right) \left(\frac{C_{e,t}}{C_{e,t+1}} \right)^\sigma = E_t \frac{1 + \pi_{t+1}}{1 + i_t} \quad \text{where} \quad E_t \log(Z_{t+1}) = \rho_z \log(Z_t) \quad (8)$$

with Q_t being the price of bonds equal to the nominal discount factor $\frac{1}{1+i_t}$ and $1 + \pi_{t+1} = \frac{P_{t+1}}{P_t}$. Note that the right hand side of equation (8) corresponds to the real discount factor. We will later employ the left hand side, when we require the stochastic discount factor in the derivation of the ‘‘Phillips curve’’.

The infinite number of individual monopolistically competitive firms that belong to the entrepreneur produce according to the standard model technology

$$Y_{e,t} = A_t N_t^{1-\alpha} \quad \text{with} \quad 0 \leq \alpha < 1 \quad (9)$$

A_t gives the level of technology. For the time being, it is normalised to 1.

The single consumption good of the economy is then aggregated from the individual firms' output by Dixit-Stiglitz-technology:

$$Y_t = \left(\int_0^1 Y_{e,t}^{\frac{\epsilon-1}{\epsilon}} de \right)^{\frac{\epsilon}{\epsilon-1}} \quad (10)$$

The price for the individual goods produced under monopolistic competition is determined via the maximisation of the (expected) discounted stream of real profits (real revenues minus real costs of production). As a firm will change its price with a probability of $1 - \theta$ each period, the

probability of a price remaining constant, θ , resembles an additional discount factor. $\beta = \frac{1}{1+r_t}$ is the standard real discount factor. As in the baseline model, the entrepreneur discounts future profits by the same factor as future utility.

The demand for each individual good can then be represented by

$$Y_{e,t} = \left(\frac{P_{e,t}}{P_t} \right)^{-\epsilon} Y_t \quad (11)$$

where $P_{e,t}$ is the price for the individual good $Y_{e,t}$ and P_t the aggregate price level and the price for the composite good Y_t .

$$\max_{P_e} \sum_{j=0}^{\infty} \theta^j \prod_{k=0}^j \frac{1}{1+r_{t+k}} \left[\left(\frac{P_e}{P_{t+j}} \right)^{1-\epsilon} Y_{t+j} - \frac{W_{t+j}}{P_{t+j}} \left(\frac{P_e}{P_{t+j}} \right)^{\frac{-\epsilon}{1-\alpha}} Y^{\frac{1}{1-\alpha}} \right] \quad (12)$$

Deriving the New Keynesian Phillips curve in the standard fashion (details can be found in the appendix) gives

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{\mathcal{B} - \mathcal{A}}{1 - \epsilon + \frac{\epsilon}{1-\alpha}} \hat{y}_t + \beta E_t \pi_{t+1} \quad (13)$$

The terms $\mathcal{B} - \mathcal{A}$ represent a collection of parameters that are basically some combinations of the different consumption levels of worker and entrepreneur, which end up in the New Keynesian Phillips curve in different ways because the subjective discount factor stems solely from the entrepreneur while the labour supply stems solely from the worker.

The IS curve is again derived according to standard procedure, with the exception that the Euler equation corresponds only to the entrepreneur's Euler equation here (more details are again found in the appendix), as the worker does not save. The IS-curve is basically identical to the homogenous agent basic model, only the response of output to the real rate gap and the shock is influenced by an additional factor now that is due to the fact that only a part of the population maximises intertemporally.

$$\hat{y}_t = \hat{y}_{t+1} - (1 - AY^{A-1})^{-1} \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t) + (1 - AY^{A-1})^{-1} \frac{1}{\sigma} (1 - \rho_z) z_t \quad (14)$$

where z_t is the logarithm of the preference shock.

To complete the model, we employ a standard Taylor-rule that reacts to the deviation of inflation from a zero-inflation target and the deviation of output from its natural level.

$$i_t = r_t + \phi_\pi \pi_t + \phi_y \hat{y}_t \quad (15)$$

Overall, the worker suffers considerably more from a preference shock (when we compare consumption levels). The reaction of aggregate variables to a preference shock is similar to the basic (homogenous agent) NKM only when the disutility of labour, ϕ , is not too strong. Some impulse responses are given in the next chapter.

2.1 Impulse Responses to a Monetary Policy Shock

To calibrate the model we chose the following values: $\alpha = 0.333, \beta = 0.99, \sigma = 1.5, \phi = 0.575, \epsilon = 10, \theta = 0.75, \phi_\pi = 1.5, \phi_y = 0.5/4, \rho_z = 0.5, \epsilon_z = -0.25$. Most of these values are quite common. As already mentioned $\sigma = 1$, i.e. the log-utility specification, does not allow for a determinate solution. The shock to the nominal interest rate is a 0.25 percent as in Broer *et al.* (2016).

We plot the same impulse responses as given in the saving-worker-model by Broer *et al.* (2016), adding only the total impulse response of consumption to the picture. Total output and consumption react a little more strongly than in the baseline RANK, while inflation reacts hardly at all (although qualitatively similar). As opposed to Broer *et al.*, we find a positive worker's consumption to the interest rate hike, likely due to the fact that the non-saving workers cannot smooth consumption over time while saving entrepreneurs decide to consume less today, thus reducing overall demand, which increases worker productivity and real wage, finally increasing worker consumption (it might be possibly to find some parameter combinations for which the somewhat counterintuitive consumption response goes into another directions, we did not find any across several standard parameter calibrations, however). In our model, as opposed to the saving-worker-model but in line with a baseline model, employment is reduced in response to a monetary policy shock.

Unlike our two benchmark-models, we do not observe an increase in profits, but rather a decrease, after the interest rate hike.⁶ This fact together with the different real wage response and the stronger response of consumption and output to a monetary policy shock appear to be the main differences to a baseline representative agent model with sticky prices. Hence, removing the ability to save from all workers, which is an extreme case, of course, appears to increase the ability of monetary policy to influence real variables, which appears somewhat surprising and

⁶ Some preliminary sensitivity analysis hints at possible positive profit gaps when the parameter in front of the disutility of labour part, η , changes.

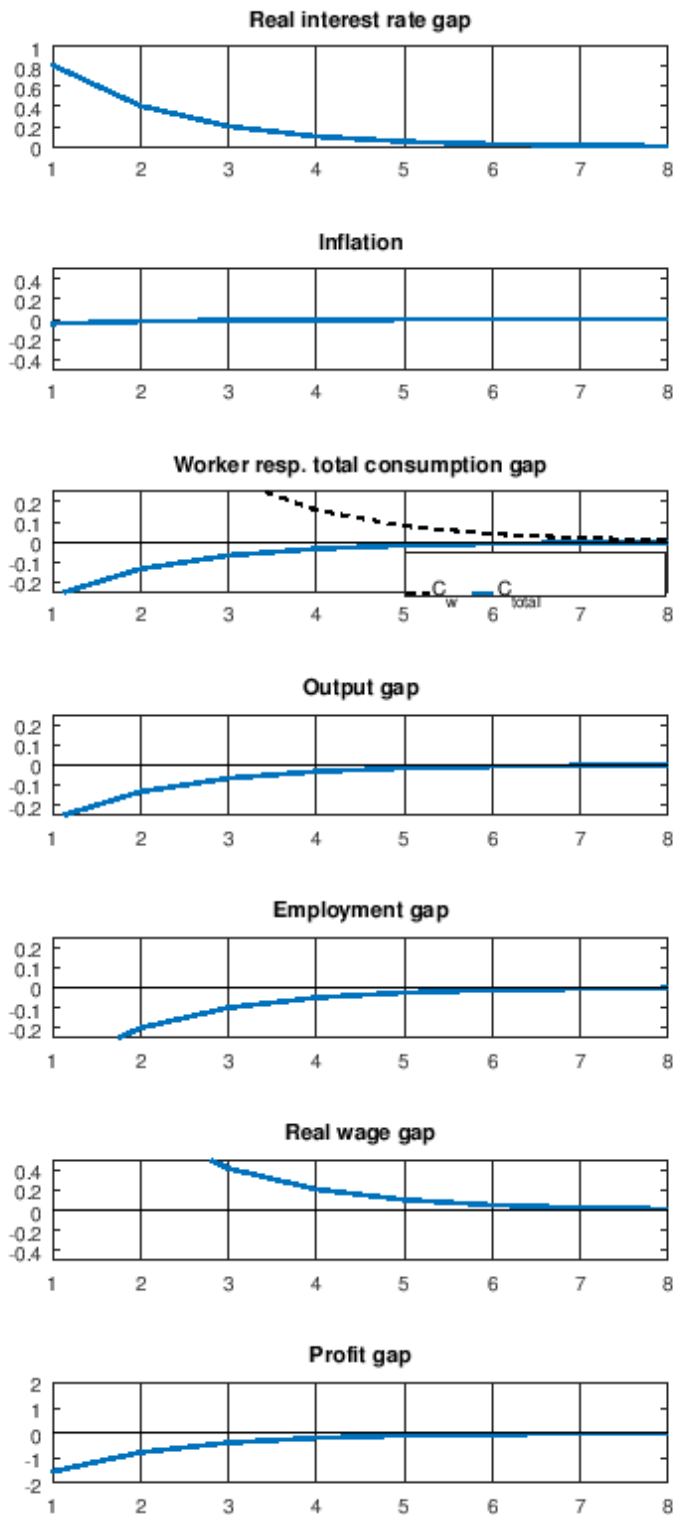


Figure 1: Impulse responses to a monetary policy shock (0.25% interest rate hike) - same axis-scale is in Broer *et al.* (2016)

is likely due to the model being “too simple”. We introduce a model with productive capital below, that should also remedy the counterintuitive consumption response of workers in the case of a monetary policy shock.⁷

2.2 Impulse Responses to a Preference Shock

The preference shock calibration follows Gali (2015).

We find that the aggregate model reacts qualitatively similar to a baseline model when the negative preference shock hits, at least with respect to the inflation and output gap response (compare, e.g. Gali (2015)). The reactions are quantitatively considerable smaller (well below 50%).

The worker suffers a much larger reduction in consumption, which agrees with the empirical and model findings of Krueger *et al.* (2016), but both agents’ consumption is lower after the negative preference shock hits.

As the entrepreneur’s consumption is somewhat higher than the worker’s consumption in steady-state, the preference shock causes consumption levels to diverge additionally.

While the worker’s consumption responds more strongly to the preference shock, the worker’s share in total income increases. If we assume that our entrepreneur-savers can be roughly equated with higher income earners and our hand-to-mouth workers with lower income earners, the relatively higher income losses of entrepreneur-savers appear in line with observations in the wake of the financial crisis, which is often modelled as a preference shock in simple models. Due to the ability to smooth consumption over time, the entrepreneur still does not suffer an as large consumption decline as the hand-to-mouth-worker.

⁷The model should likewise reduce the surprising disparity in consumption distribution: The worker’s consumption amount to less than 1% of total consumption.

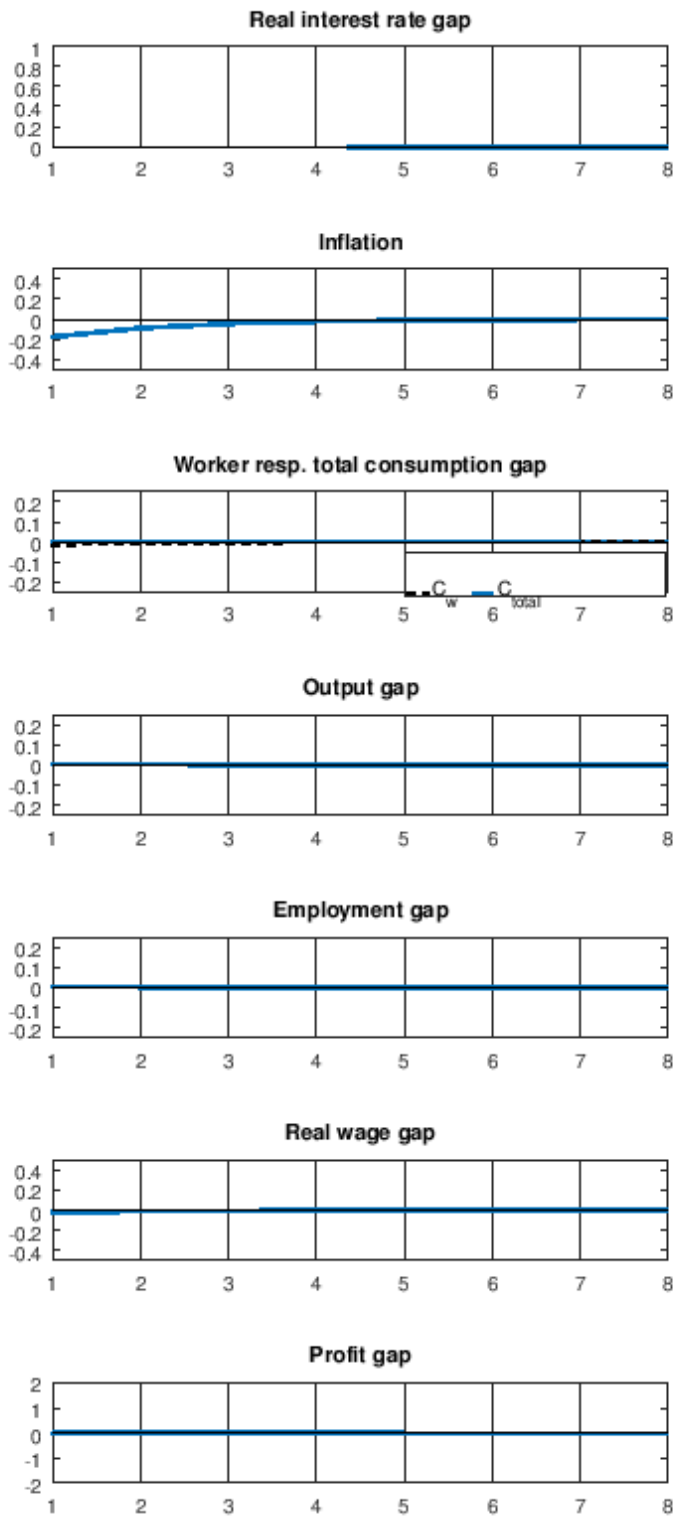


Figure 2: Impulse responses to a preference shock

3 Improved (slightly less simple) model

In order to obtain the stylised reaction of entrepreneurs and workers income to a monetary policy shock, we add productive capital to the model in two shapes. Entrepreneurs are endowed with a fixed amount of durable capital, K , that does not depreciate over time, and they can decide to acquire a share of the output, Y_t , which then enters next period's consumption as the input good I_{t-1} . The input good is fully used up in the production process. Production now takes place according to a Cobb-Douglas function $Y_t = (K + I_{t-1})^\alpha N_t^{1-\alpha}$. With respect to all other characteristics, the model remains as before.

This model is comparable to Basu (1995), who, however, allows for the input good to be used for production in the same period in which it was produced. Our formulation is more comparable to a financing-in-advance approach as discussed, e.g. in Christiano *et al.* (2010).

For this model we can derive a very useful functional income distribution.

The entrepreneur receives a fixed share of income

$$\frac{1 - \alpha}{\alpha} \frac{I_{t-1} + K}{\frac{Y_t - \rho_{t-1}(I_{t-1} + K)}{\omega_t}} = \frac{\omega_t}{\rho_{t-1}} \quad (16)$$

as well as the worker

$$\alpha Y_t = \rho_{t-1}(I_{t-1} + K). \quad (17)$$

Apart from this very convenient formulation of the functional income distribution, we can also derive a negative response of the entrepreneur's consumption from our extended model:

$$C_e = \alpha Y_t - \frac{\alpha Y_{t+1}}{\rho_t} + K \quad (18)$$

Unfortunately, at the time of writing, the model simulation was not finished, so we cannot yet gauge the quantitative impact.

4 Conclusion

Our version of the simplest two agent New Keynesian model was constructed to analyse distributional impacts of shocks. While the aggregate response appears to be in roughly line with a standard baseline model, we can also investigate the worker's and entrepreneur's consumption levels separately. However, some work is still required to bring aggregate labour and consumption share closer to aggregate shocks. The intro-

duction of productive capital owned solely by entrepreneurs appears to remedy this.

We find that when a negative preference shock hits, which is one popular simple way to represent the occurrence of the financial crisis, the worker's consumption decreases by more than the entrepreneur's consumption, although the model does not include a direct impact of the preference shock on the worker's behavioural equations. The entrepreneur, who is directly hit by a reduced discount rate shock, will reduce consumption by less. The entrepreneurs share in total income, however, declines temporarily due to a preference shock, which is in line with empirical observations, if we equate the entrepreneurs with higher income earners and workers with lower income earners, which holds true in our stylised model. The model appears to be well suited for a first stylised analysis of shock impact. It does require some help in reproducing the steady-state capital and labour share in line with stylised facts, however.

With respect to a monetary policy shock our simple entrepreneur-saver and worker-non-saver model (without productive capital) produces a surprising positive effect of contractionary monetary policy on worker (non-saver) consumption. It might be worth to investigate this in the context of Neo-Fisher theories.

Appendix

The first order condition resulting from (12) multiplied by P^* gives

$$\sum_{j=0}^{\infty} \theta^j \prod_{k=0}^j \frac{1}{1+r_{t+k}} \left[(1-\epsilon) \left(\frac{P^*}{P_{t+j}} \right)^{1-\epsilon} Y_{t+j} + \frac{\epsilon}{1-\alpha} \frac{W_{t+j}}{P_{t+j}} \left(\frac{P^*}{P_{t+j}} \right)^{\frac{-\epsilon}{1-\alpha}} Y_{t+j}^{\frac{1}{1-\alpha}} \right] \stackrel{!}{=} 0 \quad (19)$$

(with expectation operators dropped for brevity).

We substitute the subjective discount rate from the entrepreneurs Euler equation (8) and the real wage from the workers labour supply decision (4) and budget constraint (2).

Substituting for the real wage in (4) by the budget constraint (2) gives the link between labour supplied and consumption demanded (time-index omitted as all variables refer to the same period):

$$\begin{aligned} \frac{\eta N^\phi}{C_w^{-\sigma}} &= \frac{C_w}{N} && \text{or} \\ N &= \left(\frac{1}{\eta} \right)^{\frac{1}{1+\phi}} C_w^{\frac{1-\sigma}{1+\phi}} && \text{or} \\ \eta^{\frac{1}{1-\sigma}} N^{\frac{1+\phi}{1-\sigma}} &= C_w && (20) \end{aligned}$$

The real wage (4) can be expressed in terms of labour only by replacing worker consumption by labour via the budget constraint ($PC_w = WN$).

$$\frac{W}{P} = \frac{\eta N^\phi}{\left(\frac{W}{P} N \right)^{-\sigma}} \Leftrightarrow \frac{W}{P} = \eta^{\frac{1}{1-\sigma}} N^{\frac{\phi+\sigma}{1-\sigma}}. \quad (21)$$

From the consumption Euler equation of the entrepreneur, (8), the market clearing condition $Y_t = C_{s,t} + C_{w,t}$ and the inverse of the labour supply equation (20) combined with the labour demand derived from the production function (9), where we assume $A_t = 1$ from now on, we can write the subjective discount factor as

$$\begin{aligned}
\beta E_t \left(\frac{Z_t}{Z_{t+1}} \right)^\sigma \left(\frac{C_{e,t}}{C_{e,t+1}} \right)^\sigma &= E_t \frac{1}{1+r_t} \\
&= \beta E_t \left(\frac{Z_t}{Z_{t+1}} \right)^\sigma \left(\frac{Y_t - C_{w,t}}{Y_{t+1} - C_{w,t+1}} \right)^\sigma \\
&= \beta E_t \left(\frac{Z_t}{Z_{t+1}} \right)^\sigma \left(\frac{Y_t - \eta^{\frac{1}{1-\sigma}} Y_t^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}}}{Y_{t+1} - \eta^{\frac{1}{1-\sigma}} Y_{t+1}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}}} \right)^\sigma
\end{aligned} \tag{22}$$

with $C_{w,t} = \eta^{\frac{1}{1-\sigma}} N^{\frac{1+\phi}{1-\sigma}}$ from (20) and via the production function (labour demand):
 $C_{w,t} = \eta^{\frac{1}{1-\sigma}} \left(Y^{\frac{1}{1-\alpha}} \right)^{\frac{1+\phi}{1-\sigma}}$

with

$$C_{w,t} = \eta^{\frac{1}{1-\sigma}} N_t^{\frac{1+\phi}{1-\sigma}} = \eta^{\frac{1}{1-\sigma}} \left[\left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_{e,t}}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} de \right]^{\frac{1+\phi}{1-\sigma}} \approx \eta^{\frac{1}{1-\sigma}} \left[\left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \right]^{\frac{1+\phi}{1-\sigma}}. \tag{23}$$

Gali (2015) shows that the linear approximation of the integral $\int_0^1 \left(\frac{P_{e,t}}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} de$, which represents price dispersion, approximately equals one (i.e. we have zero price dispersion) in the neighbourhood of zero inflation (where $\frac{P_{e,t}}{P_t} = 1$). Hence we set the integral equal to one in equation (23).

To unclutter the equations, we drop the expectational operator in the derivation below. All future variables are expected values. Additionally, we assume $A_t + i = 1 \forall i$ from here on (or redefine $Y_t := Y_t/A_t$).

The New Keynesian Phillips curve (NKPC) is derived following standard procedure and results in a similar equation to the baseline model. We start by the entrepreneur's profit maximising equation, setting the price as to maximise expected future profits (see (19)), resulting in

$$\sum_{j=0}^{\infty} \theta^j \prod_{k=0}^j \frac{1}{1+r_{t+k}} \left[\left((1-\epsilon) \frac{P^*}{P_{t+k}} \right)^{1-\epsilon} Y_{t+k} \right] = - \sum_{j=0}^{\infty} \theta^j \prod_{k=0}^j \frac{1}{1+r_{t+k}} \left[\frac{\epsilon}{1-\alpha} \frac{W_{t+k}}{P_{t+k}} \left(\frac{P^*}{P_{t+k}} \right)^{-\frac{\epsilon}{1-\alpha}} Y_{t+k}^{\frac{1}{1-\alpha}} \right] \tag{24}$$

Substituting the equations for the subjective real interest factor (22) and the equation for the real wage $\frac{W_t}{P_t} = \eta^{\frac{1}{1-\sigma}} \left[(Y_t)^{\frac{1}{1-\alpha}} \right]^{\frac{\sigma+\phi}{1-\sigma}}$ (via (21) and (23)) we obtain.

$$\begin{aligned}
(1 - \epsilon) \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{Z_t}{Z_{t+j}} \right)^\sigma \left(\frac{Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}}}{Y_t - \eta^{\frac{1}{1-\sigma}} Y_t^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}}} \right)^{-\sigma} \left[\left(\frac{P^*}{P_{t+j}} \right)^{1-\epsilon} Y_{t+j} \right] \\
= \frac{\epsilon}{\alpha - 1} \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{Z_t}{Z_{t+j}} \right)^\sigma \left(\frac{Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}}}{Y_t - \eta^{\frac{1}{1-\sigma}} Y_t^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}}} \right)^{-\sigma} \left[\eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{\sigma+\phi}{1-\sigma}} \left(\frac{P^*}{P_{t+j}} \right)^{-\frac{\epsilon}{1-\alpha}} Y_{t+j}^{\frac{1}{1-\alpha}} \right]
\end{aligned} \tag{25}$$

Now we multiply both sides of the equation by the denominator $\left(Y_t - \eta^{\frac{1}{1-\sigma}} Y_t^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma}$ and by $Z_t^{-\sigma}$:

$$\begin{aligned}
(1 - \epsilon) \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[\left(\frac{P^*}{P_{t+j}} \right)^{1-\epsilon} Y_{t+j} \right] \\
= \frac{\epsilon}{\alpha - 1} \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[\eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \left(\frac{P^*}{P_{t+j}} \right)^{-\frac{\epsilon}{1-\alpha}} \right]
\end{aligned} \tag{26}$$

and substitute $X_t := \frac{P^*}{P_t}$ and $\Pi_{t+k} := \frac{P_{t+k}}{P_{t+k-1}}$ to obtain

$$\begin{aligned}
(1 - \epsilon) \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[X_t^{1-\epsilon} \prod_{k=0}^j \Pi_{t+k}^{\epsilon-1} Y_{t+j} \right] \\
= \frac{\epsilon}{\alpha - 1} \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[\eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} X_t^{-\frac{\epsilon}{1-\alpha}} \prod_{k=0}^j \Pi_{t+k}^{\frac{\epsilon}{1-\alpha}} \right]
\end{aligned} \tag{27}$$

and solve for X_t

$$X_t^{1-\epsilon+\frac{\epsilon}{1-\alpha}} = \frac{\frac{\epsilon}{\alpha-1} \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[\eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \prod_{k=0}^j \Pi_{t+k}^{\frac{\epsilon}{1-\alpha}} \right]}{(1 - \epsilon) \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[\prod_{k=0}^j \Pi_{t+k}^{\epsilon-1} Y_{t+j} \right]} \tag{28}$$

We can write equation (28) as

$$\begin{aligned}
X_t^{1-\epsilon+\frac{\epsilon}{1-\alpha}}(1-\epsilon) \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[\prod_{k=0}^j \Pi_{t+k}^{\epsilon-1} Y_{t+k} \right] \\
= \frac{\epsilon}{\alpha-1} \sum_{j=0}^{\infty} \theta^j \beta^j \left(\frac{1}{Z_{t+j}} \right)^\sigma \left(Y_{t+j} - \eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \right)^{-\sigma} \left[\eta^{\frac{1}{1-\sigma}} Y_{t+j}^{\frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}} \prod_{k=0}^j \Pi_{t+k}^{\frac{\epsilon}{1-\alpha}} \right]
\end{aligned} \tag{29}$$

For reasons of readability, we will treat the two sides of equation (29) separately in the derivation of the Phillips curve. The upper line will henceforth be represented by LHS (Left Hand Side) and the lower line by RHS.

The zero-inflation deterministic steady states are denoted by the subscript ss .

$$LHS_{ss} = 1 \cdot (1-\epsilon) \frac{1}{1-\theta\beta} \left(Y - \eta^{\frac{1}{1-\sigma}} Y^A \right)^{-\sigma} Y \cdot 1 = \frac{1-\epsilon}{1-\theta\beta} \left(Y^{1-\frac{1}{\sigma}} - \eta^{\frac{1}{1-\sigma}} Y^{A-\frac{1}{\sigma}} \right)^{-\sigma} \tag{30}$$

$$\text{with } A := \frac{1}{1-\alpha} \frac{1+\phi}{1-\sigma}$$

$$RHS_{ss} = \frac{\epsilon}{\alpha-1} \frac{1}{1-\theta\beta} \left(Y - \eta^{\frac{1}{1-\sigma}} Y^A \right)^{-\sigma} \eta^{\frac{1}{1-\sigma}} Y^A \cdot 1 = \frac{\epsilon}{1-\alpha} \frac{1}{1-\theta\beta} \left(\eta^{-\frac{1}{\sigma(1-\sigma)}} Y^{1-\frac{A}{\sigma}} - \eta^{-\frac{1}{\sigma}} Y^{A-\frac{A}{\sigma}} \right)^{-\sigma} \tag{31}$$

Now we approximate each side of equation (28) by a first order Taylor approximation ($f(x_t) \approx f(x) + f'(x)(x_t - x)$) around the respective steady state, x .

$$\begin{aligned}
LHS \approx LHS_{ss} + \left(1 - \epsilon + \frac{\epsilon}{1-\alpha} \right) LHS_{ss} \frac{X_t - X}{X} \\
+ LHS_{ss} (1 - \theta\beta) \sum_{j=0}^{\infty} \theta^j \beta^j \left[\underbrace{(-\sigma) \left(\frac{Y^{1-\frac{1}{\sigma}} - A \eta^{\frac{1}{1-\sigma}} Y^{A-\frac{1}{\sigma}}}{Y^{1-\frac{1}{\sigma}} - \eta^{\frac{1}{1-\sigma}} Y^{A-\frac{1}{\sigma}}} - \frac{1}{\sigma} \right)}_A \frac{Y_{t+j} - Y}{Y} \right. \\
\left. + (\epsilon - 1) \left(\frac{P_{t+j} - P}{P} - \frac{P_t - P}{P} \right) - \sigma \frac{Z_{t+j} - Z}{Z} \right]
\end{aligned} \tag{32}$$

$RHS \approx RHS_{ss}$

$$\begin{aligned}
& +RHS_{ss}(1-\theta\beta) \sum_{j=0}^{\infty} \theta^j \beta^j \left[\underbrace{(-\sigma) \left(\frac{\eta^{-\frac{1}{\sigma(1-\sigma)}} Y^{1-\frac{A}{\sigma}} - A\eta^{-\frac{1}{\sigma}} Y^{A-\frac{A}{\sigma}}}{\eta^{-\frac{1}{\sigma(1-\sigma)}} Y^{1-\frac{A}{\sigma}} - \eta^{-\frac{1}{\sigma}} Y^{A-\frac{A}{\sigma}}} - \frac{A}{\sigma} \right)}_{\mathcal{B}} \frac{Y_{t+j} - Y}{Y} \right. \\
& \left. + \frac{\epsilon}{1-\alpha} \left(\frac{P_{t+j} - P}{P} - \frac{P_t - P}{P} \right) - \sigma \frac{Z_{t+j} - Z}{Z} \right] \quad (33)
\end{aligned}$$

Setting $RHS = LHS$, dividing by the steady state and by $(1-\theta\beta)(1-\epsilon + \frac{\epsilon}{1-\alpha})$ and rearrangeing $\hat{x}_t + \hat{p}_t$ as a first order dynamic equation, we obtain

$$\begin{aligned}
\frac{1}{1-\theta\beta} \hat{x}_t &= \sum_{j=0}^{\infty} \theta^j \beta^j \left[\frac{\mathcal{B} - \mathcal{A}}{1-\epsilon + \frac{\epsilon}{1-\alpha}} \hat{y}_{t+j} + \hat{p}_{t+j} - \hat{p}_t \right] \\
&= \sum_{j=0}^{\infty} \theta^j \beta^j \left[\frac{\mathcal{B} - \mathcal{A}}{1-\epsilon + \frac{\epsilon}{1-\alpha}} \hat{y}_{t+j} + \hat{p}_{t+j} \right] - \frac{1}{1-\theta\beta} \hat{p}_t \\
\hat{x}_t + \hat{p}_t &= (1-\theta\beta) \left(\frac{\mathcal{B} - \mathcal{A}}{1-\epsilon + \frac{\epsilon}{1-\alpha}} \hat{y}_t + \hat{p}_t + 0 \right) + \theta\beta (\hat{x}_{t+1} + \hat{p}_{t+1}) \quad (34)
\end{aligned}$$

(Hatted variables are deviations from steady-states: $\hat{x} = \frac{X_t - X}{X}$)

With $\hat{x}_t = \frac{\theta}{1-\theta} \pi_t$ from the definition of the price level via the Dixit-Stiglitz aggregator and the Calvo-pricing mechanism we can derive an equation quite close to the standard New Keynesian Phillips curve, again following standard procedure.

— Excursion - aggregate price level —

To understand why $\hat{x}_t = \frac{\theta}{1-\theta} \pi_t$, the following equations might be helpful. Aggregate Price level expressed considering the Calvo price updating mechanism:

$$P_t = (\theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon})^{\frac{1}{1-\epsilon}} \quad (35)$$

We have $\theta \Pi_t^{\epsilon-1} = 1 - (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{1-\epsilon}$ and $X_t = \left(\frac{1-\theta \Pi_t^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}}$. A Taylor approximation of the latter equation leads to the desired expression, which we substitute for \hat{x}_t below.

Around the zero inflation steady state we have $\hat{\pi}_t = \pi_t$ as $\pi = 0$.

— End excursion —

We substitute the optimal relative price

$$\frac{\theta}{1-\theta}\pi_t = (1-\theta\beta)\frac{\mathcal{B}-\mathcal{A}}{1-\epsilon+\frac{\epsilon}{1-\alpha}}\hat{y}_t + (1-\theta\beta)\hat{p}_t - \hat{p}_t + \theta\beta\hat{p}_{t+1} + \theta\beta\frac{\theta}{1-\theta}E_t\pi_{t+1} \quad (13)$$

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{\mathcal{B}-\mathcal{A}}{1-\epsilon+\frac{\epsilon}{1-\alpha}}\hat{y}_t + \beta E_t\pi_{t+1} \quad (36)$$

which is quite similar to the standard NKPC, with the exception of an additional multiplier $(\mathcal{B}-\mathcal{A})$ (see p. 17) in front of the output gap, which is a collection of some model parameters, including steady-state output, and is related to the share of the entrepreneurs consumption via some complicated terms.

(Some steps of the derivation are similar to those in Walsh (2017).)

4.1 IS curve

The model IS curve is derived from the entrepreneurs Euler equation, which is repeated here for convenience

$$E_t\left(\frac{Z_t}{Z_{t+1}}\right)\left(\frac{C_{e,t}}{C_{e,t+1}}\right)^\sigma = E_t\frac{1+\pi_{t+1}}{1+i_t}\beta^{-1}$$

Following the derivation of the IS curve in Gali (2015), we obtain

$$(1-A\eta Y^{A-1})\hat{y}_t = (1-A\eta Y^{A-1})\hat{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - r_t) + \frac{1}{\sigma}(1-\rho_z)z_t \quad (37)$$

So the model can be completed by a standard Taylor-rule

$$i_t = r_t + \phi_\pi\pi_t + \phi_y\hat{y}_t \quad (38)$$

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