

Hott, Christian

**Conference Paper**

## Save or pay-as-you-go: The effects of ageing on optimal retirement funding

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Public Economics - Pensions and Savings, No. B10-V3

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Hott, Christian (2019) : Save or pay-as-you-go: The effects of ageing on optimal retirement funding, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Public Economics - Pensions and Savings, No. B10-V3, ZBW - Leibniz-Informationszentrum Wirtschaft, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/203642>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Save or Pay-As-You-Go

## The Effects of Ageing on Optimal Retirement Funding

Christian Hott<sup>\*†</sup>

### Abstract

Ageing of most societies is driven by two factors: (1) birth rates are declining and (2) people are living longer. These developments have substantial effects on economies and, in particular, on the funding of our living standards in retirement. We develop an overlapping generation model in order to analyse the effects of ageing on the efficiency of retirement savings relative to pay-as-you go systems. Our results indicate that more wealth should be transferred from the young to the old generation and that ageing makes a transfer via pay-as-you-go pension relatively more attractive. In optimum, the implicit return on a pay-as-you-go system and the interest rate on savings should be equal to the growth rate of the economy. If people would start to save more, there would be an oversupply of capital and interest rates would decrease. A higher pay-as-you-go tax, however, would not affect its implicit return as it is always equal to the growth rate of the economy.

*Keywords:* Ageing, Savings, Pay-as-you-go.

*JEL-Classifications:* E21, J11, J26.

## 1 Introduction

Ageing is one of the most important drivers of social and economic developments. It is the result of decreasing birth rates and increasing life expectan-

---

<sup>\*</sup>Helmut-Schmidt-University Hamburg, Holstenhofweg 85, 22043 Hamburg, Germany

<sup>†</sup>e-mail: hottc@hsu-hh.de

cies and affects economic output as well as interest rates, it has distributional effects and creates a major challenge for the funding of retirement income.

In many countries, retirement funding is built on three pillars: first, a public pay-as-you-go system, second, an occupational pension scheme and third, private retirement savings. The second pillar (occupational) is, however, either based on a pay-as-you-go system<sup>1</sup> or the accumulation and investment of savings.<sup>2</sup> Hence, even though there are three pillars, there are basically only two ways to fund retirement: savings and pay-as-you-go systems.

Numerous retirement savings products as well as pay-as-you-go systems currently suffer and appear unsustainable when we look into the future. A main reason for this is that these systems make promises (i.e. guaranteed interest rates on retirement saving and defined benefit pay-as-you-go-systems) which are difficult to meet, not at least due to ageing. We abstract from such promises and assume that the interest rate on savings is given by the market and that the benefits in a pay-as-you-go system are driven by the contributions.

With some delay, decreasing fertility rates lead to a decreasing growth rate of the working age population and hence of the labour force. Figure 1 displays the development of the working age population in different countries. In Germany and Japan, for example, the working age population has peaked already about 15 years ago and is expected to continue declining. China's working age population is currently at its peak. Supported by high migration rates, the labour force of the US is expected to continue growing, although at a much lower rate than in the past.

The second demographic factor is the increasing life expectancy which will lead to an increasing number of retired people. In combination with the decreasing growth rate of the working age population, this development results in an increasing ratio of the retired to the working age population (see Figure 2). This development can not only be observed in advanced economies but also in many developing economies.

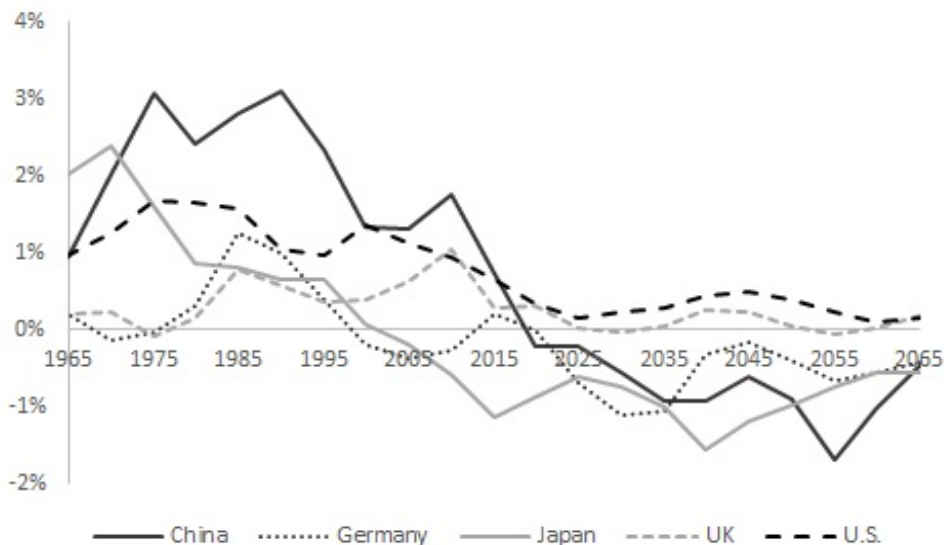
The increasing ratio of the retired to the working age population is a major challenge for public pay-as-you-go retirement funding systems. In pay-as-you-go systems the current working age population transfers a part of its income via a tax to the current retired population. Hence, as long as the

---

<sup>1</sup>Yet based on a smaller and therefore more vulnerable pool of individuals than the public scheme.

<sup>2</sup>Occupational pension schemes, in particular opt-out systems, are often superior to private savings when it comes to mobilizing savings.

Figure 1: Annual Growth Rates of Working Age Population (aged 20 to 64, sources: UN estimates and medium projections as well as own calculations)



retirement age is not increased or labour participation increases sufficiently, there are only two ways to react on ageing within the pay-as-you-go system: increase the contributions (or tax) or lower the benefits. Both measures are politically difficult to introduce and, hence, there have been many political efforts to fill the emerging retirement funding gap by increasing private (or occupational) retirement savings.<sup>3</sup>

The transformation of a pension system from a pay-as-you-go system to a pre-funded system that is based on retirement savings is far from trivial. In the transition phase, the young suffer as they have to pay for the old and their own retirement savings and the retired generation suffers as they cannot rely on past private savings.<sup>4</sup> In addition, Sinn and Uebelmesser (2002) highlight that a pension reform can only be successful if the age cohorts that would profit from the reform form a democratic majority.<sup>5</sup>

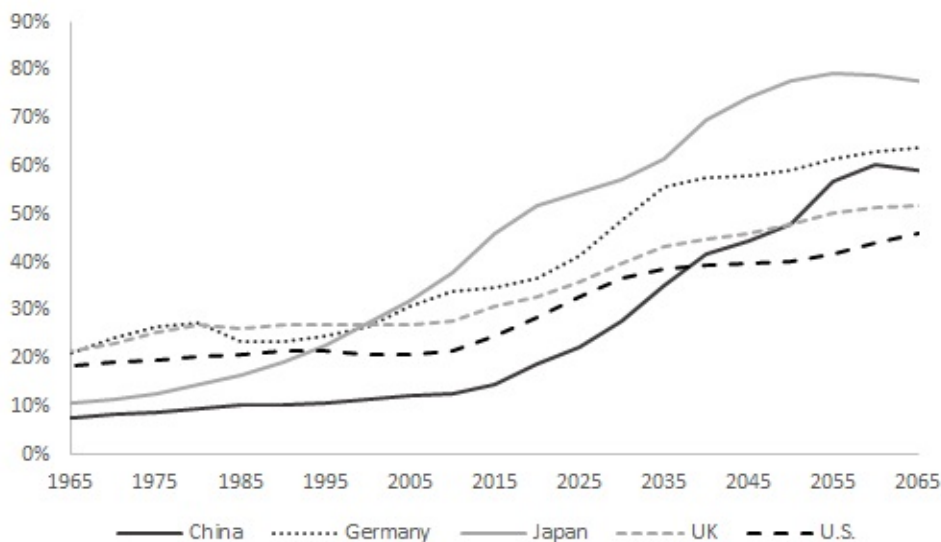
Saving for retirement is, however, also challenged by the demographic de-

<sup>3</sup>Especially until the mid-2000s. See European Commission (2015) or Andersen and Bhattacharya (2016).

<sup>4</sup>See also Brunner (1996).

<sup>5</sup>See also Boeri et al. (2002).

Figure 2: Retired (aged 65+) to Working Age (20 to 64) Population (sources: UN estimates and medium projections as well as own calculations)



velopments as the decreasing growth rate of the labour force reduce economic growth and likely also interest rates.<sup>6</sup> Samuelson (1958) develops a three generation overlapping generation model. The two younger generations produce a consumption good which cannot be stored (depreciates at a rate of 100%). Without technical progress, the interest rate should be equal to the growth rate of the population. Samuelson argues that this "biological" interest rate can only be achieved with help of a social contract. However, this is large extent a result of the assumed production technology and the nature of the consumption good which turns savings into a kind of pay-as-you-go system.

Gonzalez-Elias and Niepelt (2012) analyse the effect of ageing on government budgets and per-capita growth. The authors introduce ageing and policy choices in a standard overlapping generation model and analyse the direct effect of ageing on savings and labour-supply as well as the indirect effect via changing political preferences. The model predicts higher public spend-

<sup>6</sup>Galor and Weil (1999) look at the historical link between population growth and economic growth. Carvalho et al. (2016) or Gagnon et al. (2016) show that demographic factors can explain the drop in interest rate over the past decades to a large extent.

ing, an increasing retirement age and higher per-capita income growth. In a similar setting Cipriani (2016) show that ageing should lead to an increase in the retirement age and the capital level. Both effects, in turn, reduce the pressure on the public pay-as-you-go system. Cutler et al. (1990), however, conclude that demographics will cause a reduction of the savings rate as slower population growth will reduce investment needs. Due to an increasing dependency ratio, per capita income will decrease if not accompanied by a sufficiently increasing productivity. Boersch-Supan et al. (2014) argue that a shift from a defined benefit pay-as-you-go system to a defined contribution system in combination with a ageing society should increase retirement savings and should, hence, lead to a higher capital stock in the economy.

In general, it is important to note that if the government wants avoid an increase in the pay-as-you-go tax, it indirectly forces people to save more for retirement. This leads to two additional questions: First, do people really save the money they are not spending on the public pension system? Second, do people profit from saving the money instead of paying the public tax? Or in other words, is the return on savings higher than the return on the pay-as-you-go-system? Aaron (1966, 374) provides a specific answer to the latter question:

”If the rate of interest exceeds the sum of the rate of growth of real wages and the rate of growth of population, then introduction of social insurance either on a pay-as-you-go or a funded basis will reduce welfare...”

Our paper adds to the existing literature by explicitly analysing the effect of ageing on the relative efficiency of pay-as-you-go systems vs. retirement savings and the question whether additional retirement funding needs should increasingly rely on pay-as-you-go or savings. Based on an overlapping generation model which explicitly considers the growth rate of the working age population and the life expectancy, we analyse the optimal savings behaviour of the working age population for a given life expectancy and a given public pay-as-you-go pension tax. As already mentioned, we assume that the pay-as-you-go system as well as the savings products do not do not make unsustainable promises with regard to the their (implicit) return (i.e. guarantee interest rates or defined benefits).

Our results indicate that ageing will lead *ceteris paribus* to lower interest rates and higher wages. This has distributional effects leading to higher

consumption and thus utility for the young and lower consumption and utility for the old. These effects could be mitigated by increasing public pay-as-you-go pensions. In fact, we show that if it is optimal to have a pay-as-you-go pension, only the pay-as-you-go pension tax should react to ageing and not the savings rate. This optimal pension policy also assures that implicit return on the pay-as-you-go pension tax is equal to the return on the savings and both are equal to the growth rate of the economy. If the interest rate is higher than the growth rate of the economy - as it has been the case in the past for several countries - it would be optimal not to have a pay-as-you-go system. Nevertheless, ageing makes the pay-as-you-go system at least less inefficient.

The next chapter presents the analytical framework and the resulting theoretical effects of ageing on economic growth, interest rates and optimal retirement funding. Chapter 3 discusses the economic impact of ageing. Chapter 4 looks at the corresponding empirical evidence as well as limitations and chapter 5 offers some concluding remarks.

## 2 Analytical Framework

The starting point of our analysis of the social and economic effects of ageing is a standard macroeconomic production function. The GDP  $Y_t$  of an economy is produced by the labour force  $L_t$ , the capital stock  $K_t$  and the production technology (or total factor productivity)  $A_t$ :

$$Y_t = A_t L_t^\alpha K_t^{1-\alpha}, \quad (1)$$

where  $\alpha$  is the output elasticity of labour and  $(1 - \alpha)$  the output elasticity of capital. In line with empirical evidence, we assume  $0.5 < \alpha < 1$ . As the prices of input factors should be equal to their marginal contribution to the output, the wage  $w_t$  and the interest rate  $r_t$  should be:

$$w_t = \frac{\partial Y_t}{\partial L_t} = \alpha A_t \left( \frac{K_t}{L_t} \right)^{1-\alpha} \quad (2)$$

$$r_t = \frac{\partial Y_t}{\partial K_t} = (1 - \alpha) A_t \left( \frac{L_t}{K_t} \right)^\alpha. \quad (3)$$

Hence, the wage depends positively on the total factor productivity and the capital stock and negatively on the size of the labour force. The interest rate

depends positively on the total factor productivity and the size of the labour force and negatively on the capital stock.

The development of the total factor productivity is driven by technical progress and is closely linked with the development of human capital and, hence, education and research. This development is the main driver of per capita income, wages and investment returns. However, as this is not the focus of our paper, we assume a constant productivity growth rate  $\gamma$  and, hence that  $A_t = (1 + \gamma)A_{t-1}$ .

The development of the labour force is driven to a large extent by birth rates but also by the length of the working life (retirement age), the labour participation rate and migration. We assume that the growth rate of the labour force is  $g$  and, hence that  $L_t = (1 + g)L_{t-1}$ .

The development of the capital stock is driven by depreciation and investments. We assume a depreciation rate of 100%. Investments depend on the savings behaviour of the population. To assess this savings behaviour, we assume that the fraction  $0 < \Omega_j$  of the young generation (or labour force)  $L_t$  lives a second period in retirement. Hence, in period  $t$  there are  $\Omega L_{t-1}$  retired people. The older generation lives on a public pay-as-you-go pension and its savings during working live. In addition, in line with Gonzalez-Eiras and Niepelt (2012), we assume that the savings of the young are insured against longevity risk. As a result, the savings of the individuals that do not live a second period will be distributed among the survivors. Hence, in period  $t$  we have the following consumption structure:

$$L_t \text{ young : } c_t^y = w_t(1 - \tau) - s_t \quad (4)$$

$$\Omega L_{t-1} = (\Omega L_t)/(1 + g) \text{ old : } c_t^o = w_t \tau_t (1 + g)/\Omega + s_{t-1} r_t / \Omega, \quad (5)$$

where  $\tau$  is the tax which finances the public pensions. As we can see, the implicit return on this public pay-as-you-go pension contribution is equal to the growth rate of the wage ( $w_{t+1}/w_t$ ) multiplied with the growth rate of the labour force ( $1 + g$ ).

## 2.1 Saving Decision

The saving decision is made by the young. We assume that they maximize the following inter-temporal utility function:



$$\begin{aligned}
V_t &= \ln(c_t^y) + \Omega \ln(c_{t+1}^o) \\
&= \ln[w_t(1 - \tau) - s_t] + \Omega \ln \left[ w_{t+1} \tau \frac{1 + g}{\Omega} + s_t \frac{r_{t+1}}{\Omega} \right]. \tag{6}
\end{aligned}$$

For simplicity, we assume that people are indifferent between consumption in the first and in the second period, meaning they discount future utility by one. Maximization leads to the following savings function:

$$s_t = w_t \frac{\Omega(1 - \tau)}{1 + \Omega} - \frac{w_{t+1} \tau(1 + g)}{r_{t+1} \frac{1 + \Omega}{\Omega}}. \tag{7}$$

Using equation (2) and (3) as well as  $L_{t+1} = L_t(1 + g)$ , we get:

$$s_t = w_t \frac{\Omega}{1 + \Omega} \left[ 1 - \tau \left( 1 + \frac{1}{r_t \Omega} \frac{K_{t+1}}{K_t} \right) \right]. \tag{8}$$

Hence, savings decrease with increasing public pensions (as it is a substitute for retirement savings), they increase with  $\Omega$  (as the pensions per retired person gets lower), they increase with the interest rate (as savings get more attractive) and they decrease with the growth rate of the capital stock (as we will see, the growth rate is strongly linked to the growth rate of the labour force and hence a positive indicator of the public pension). If we assume a world without public pension ( $\tau = 0$ ) and where everyone lives two periods ( $\Omega = 1$ ), people would save exactly half of their income for retirement.

By aggregating these savings ( $K_{t+1} = L_t s_t$ ) and using the equations for the wage (2) and interest rate (3), we get the following capital stock:

$$K_t = \left[ \frac{\Omega(1 - \tau)\alpha(1 - \alpha)}{(1 + \Omega)(1 - \alpha) + \tau\alpha} \frac{K_t}{K_{t+1}} A_t \right]^{\frac{1}{\alpha}} L_t. \tag{9}$$

As we can see, the capital stock increases with the labour force ( $L_t$ ) and the total factor productivity ( $A_t^{1/\alpha}$ ). Hence, we get  $K_{t+1} = (1 + g)(1 + \gamma)^{1/\alpha} K_t$  and can rewrite our equation for the capital stock to:

$$K_t = \left[ \frac{\Omega(1 - \tau)\alpha(1 - \alpha)}{(1 + \Omega)(1 - \alpha) + \tau\alpha} \frac{A_t}{(1 + g)(1 + \gamma)^{1/\alpha}} \right]^{\frac{1}{\alpha}} L_t. \tag{10}$$

Using this capital stock we can also calculate the wage and the interest rate:

$$w_t = (\alpha A_t)^{\frac{1}{\alpha}} \left[ \frac{\Omega(1-\tau)(1-\alpha)}{(1+\Omega)(1-\alpha) + \tau\alpha} \frac{1}{(1+g)(1+\gamma)^{1/\alpha}} \right]^{\frac{1-\alpha}{\alpha}}. \quad (11)$$

and

$$r_t = r = \frac{(1+\Omega)(1-\alpha) + \tau\alpha}{\Omega(1-\tau)\alpha} (1+g)(1+\gamma)^{1/\alpha}. \quad (12)$$

Hence, while the interest rate is a constant, the wage is constantly increasing with productivity. However, productivity growth has a negative effect on the wage level and a positive effect on the interest rate level. Also ageing has an opposite effect on the two variables: Both, the increasing life expectancy ( $\Omega$ ) as well as the decreasing fertility ( $g$ ) have a positive effect on wages and a negative effect on interest rates. The pay-as-you-go pension tax ( $\tau$ ) has a negative effect on wages and a positive effect on interest rates.

Given that we now know that the wage increases at the rate  $(1+\gamma)^{\frac{1}{\alpha}}$ , we can see that the implicit return on the pay-as-you-go pension is given by the growth rate:  $\Delta = (1+g)(1+\gamma)^{1/\alpha}$ .<sup>7</sup> Whether this return is higher or lower than the return on the retirement savings ( $r$ ) depends on whether the fraction before this growth rate in equation (12) is higher or lower than one.

Using  $s_t = K_{t+1}/L_t$  and  $\Delta = (1+g)(1+\gamma)^{1/\alpha}$ , we can rewrite optimal savings to:

$$s_t = (\alpha A_t)^{\frac{1}{\alpha}} \left[ \frac{1}{\Delta} \right]^{\frac{1-\alpha}{\alpha}} \left[ \frac{\Omega(1-\tau)(1-\alpha)}{(1+\Omega)(1-\alpha) + \tau\alpha} \right]^{\frac{1}{\alpha}}. \quad (13)$$

As a result we get:

$$\frac{s_t}{w_t} = \frac{\Omega(1-\tau)(1-\alpha)}{(1+\Omega)(1-\alpha) + \tau\alpha}. \quad (14)$$

Hence, the optimal savings rate is a constant which is independent of the growth of the economy  $(1+g)(1+\gamma)^{1/\alpha}$ . It depends positively on life expectancy ( $\Omega$ ) and negatively on the pay-as-you-go pension tax ( $\tau$ ).

## 2.2 Optimal Pay-As-You-Go Tax

A social planner has to decide on the optimal level of the pay-as-you-go pension. We assume that the planner chooses the tax  $\tau$  that maximizes the lifetime

---

<sup>7</sup>This result is in line with Aaron (1966).

utility of the young households. A more democratic policy goal would seem to be to maximize lifetime utility of the entire population. However, this would imply that utility of retirees is counted twice (of the old and young) and that of the utility in working age only once (the young). Hence we assume that social planner maximizes the inter-temporal utility function (6) while knowing the optimal reaction of the people which leads to (11), (12), and (14):

$$V_t = (1 + \Omega) \ln(w_t) + \ln \left[ \frac{(1-\tau)(1-\alpha+\tau\alpha)}{(1+\Omega)(1-\alpha)+\tau\alpha} \right] + \Omega \ln \left[ \Delta \frac{(1-\alpha+\tau\alpha)}{\Omega\alpha} \right]. \quad (15)$$

or, with (13):

$$V_t = \frac{1+\Omega(1-\alpha)}{\alpha} \ln \left[ \frac{(1-\tau)}{(1+\Omega)(1-\alpha)+\tau\alpha} \right] + (1 + \Omega) \ln[1 - \alpha + \tau\alpha] + \frac{1+\Omega}{\alpha} \ln(\alpha A_t) - \frac{1+\Omega-\alpha-2\Omega\alpha}{\alpha} \ln[\Delta] + \frac{1+\Omega(1-\alpha)}{\alpha} \ln[\Omega(1-\alpha)] + \Omega \ln(\Omega\alpha). \quad (16)$$

The utility maximizing pension tax  $\tau$  is given by:

$$\tau = \frac{\Omega}{1 + \Omega} - \frac{1 - \alpha}{\alpha}. \quad (17)$$

It is important to note that  $\tau$  cannot be negative. For  $\Omega/(1+\Omega) < (1-\alpha)/\alpha$ , the optimal pension tax would be zero. Given that  $1 > \Omega$ , this can only be the case if  $\alpha > 2/3$ . Hence, it is not clear if it is optimal to have a pay-as-you-go system at all.<sup>8</sup>

For  $\tau > 0$ , this optimal pension tax has some interesting features. First, as the optimal savings rate, it is independent of the growth rate  $\Delta$  and therefore of the growth rate of the labour force  $(1 + g)$ . Second, it depends positively on  $\alpha$  which implies that the pay-as-you-go tax should be higher if elasticity of the output is high with regard to labour and low with regard to capital. This makes sense as the tax can be interpreted as an "investment" in labour income which depends positively on  $\alpha$ . Third, the pay-as-you-go tax depends positively on  $\Omega$  and should therefore increase with a growing live expectancy. This is not surprising as the consumption of more retirees needs to be financed.

For  $\tau > 0$ , we can replace  $\tau$  in equation (14) with equation (17) and get the following optimal savings rate:

---

<sup>8</sup>This result is in line with Aaron (1966) and Andersen and Bhattacharya (2016).

$$\frac{s_t}{w_t} = \frac{1 - \alpha}{\alpha}. \quad (18)$$

As we can see, this optimal savings rate only depends on the output elasticity of capital relative to the output elasticity of labour. This implies that the more the output of the economy depends on capital, the higher should be the savings rate and, when we look at equation (17), the higher should be the relevance of savings relative to the pay-as-you-go tax. However, the savings rate is not only independent of economic growth  $\Delta$ , but also independent of the life expectancy  $\Omega$ . This implies that ageing is completely compensated by the optimal pay-as-you-go tax and that a growing life expectancy should lead to a higher relevance of the public pay-as-you-go pension. The resulting total fraction of the wage that is used for old age provision is given by:

$$\tau + \frac{s_t}{w_t} = \frac{\Omega}{(1 + \Omega)} \quad (19)$$

and depends only on the life expectancy. This would also be the optimal savings rate if  $\tau = 0$ .

### 3 Economic Impact of Ageing

The optimal pension tax has also some interesting implications for the interest rate, the savings rate, the wage, income and utility. For  $\tau > 0$ , we can replace  $\tau$  in equation (12) by its optimal value given by equation (17) and get:

$$r = (1 + g)(1 + \gamma)^{1/\alpha} = \Delta. \quad (20)$$

Hence, the interest rate is equal to the growth rate of the economy which in turn is influenced only by technical progress and population growth. This interest rate is therefore independent of the life expectancy  $\Omega$ . Furthermore, looking at equation (6) we can see the interest rate is equal to the implicit return on an investment in the pay-as-you-go tax. Hence, the optimal pension policy would make sure that the pay-as-you-go pension is as efficient as capital based retirement saving.

For  $\tau = 0$  we would get the following interest rate:

$$r = \frac{(1 + \Omega)(1 - \alpha)}{\Omega\alpha} (1 + g)(1 + \gamma)^{1/\alpha}. \quad (21)$$

Since  $\tau = 0$  implies that  $\Omega/(1 + \Omega) \leq (1 - \alpha)/\alpha$ , this interest rate is higher than the growth rate of the economy  $\Delta$ . This result is in line with Aaron (1966). However, if it is not optimal to have a public pension ( $\tau = 0$ ), ageing in form of an increasing life expectancy (increasing  $\Omega$ ) would bring the interest rate closer to the the growth rate of the economy  $\Delta$  and would, therefore, make a pay-as-you-go pension relatively more attractive.

Now we have a look at the effect of the optimal pension tax on wages. By pugging (17) into (11) we get:

$$w_t = (\alpha A_t)^{\frac{1}{\alpha}} \left[ \frac{1 - \alpha}{\alpha} \frac{1}{\Delta} \right]^{\frac{1-\alpha}{\alpha}}. \quad (22)$$

The growth rate  $\Delta$  still has a negative impact on the wage which implies that the decreasing labour force should lead to higher wages. However, with the optimal pension tax, the wage is independent of the life expectancy  $\Omega$ . This result changes, of course, if  $\tau = 0$ . Then the wage would be:

$$w_t = (\alpha A_t)^{\frac{1}{\alpha}} \left[ \frac{\Omega}{1 + \Omega} \frac{1}{\Delta} \right]^{\frac{1-\alpha}{\alpha}}. \quad (23)$$

Now the wage is increased due to the increasing life expectancy.

For the output  $Y_t$  of the economy we get similar results. If  $\tau > 0$ ,

$$Y_t = (A_t)^{\frac{1}{\alpha}} L_t \left[ \frac{1 - \alpha}{\Delta} \right]^{\frac{1-\alpha}{\alpha}}, \quad (24)$$

which is growing at  $\Delta$  but which level depends negatively to  $\Delta$  and it is independent of the live expectancy  $\Omega$ . Since with the optimal pension tax the life expectancy of the labour force has no impact on its savings rate and no impact on its wage, it is not surprising to see that the life expectancy has also no impact on the capital stock of the economy and therefore no impact on the total output of the economy. Nevertheless, it has an impact on per capita income  $y_t$ :

$$y_t = \frac{(1 + g)^{\frac{2\alpha-1}{\alpha}}}{1 + g + \Omega} (A_t)^{\frac{1}{\alpha}} \left[ \frac{1 - \alpha}{(1 + \gamma)^{1/\alpha}} \right]^{\frac{1-\alpha}{\alpha}}. \quad (25)$$

As a higher life expectancy leads *ceteris paribus* to a higher number of people without increasing the output of the economy, it has a negative impact on per capita income. The growth rate of the population  $g$  has two opposite effects on per capita income. On the one hand, according to (24) it has a

negative impact on the output level. On the other hand it has a negative impact on the fraction of not productive old people in the society. Therefore, the overall effect is not clear and depends on the parameters  $\alpha$ ,  $g$ , and  $\Omega$ . Hence, also the overall effect of ageing (increasing  $\Omega$  and decreasing  $g$ ) on per capita income is not clear.

Now we look at the case where  $\tau = 0$ . In this case per capita income is:

$$y_t = \frac{(1+g)^{\frac{2\alpha-1}{\alpha}}}{1+g+\Omega} (A_t)^{\frac{1}{\alpha}} \left[ \frac{\Omega}{1+\Omega} \frac{\alpha}{(1+\gamma)^{1/\alpha}} \right]^{\frac{1-\alpha}{\alpha}}. \quad (26)$$

The impact of the population growth rate  $g$  is the same as under a positive pension tax. However, the impact of the higher life expectancy is now less negative or even positive as it leads to a higher capital stock.

## 4 Empirical Evidence and Limitations

According to equation (24), GDP is growing with growth of the working age population  $g$  and growth of the total factor productivity  $\gamma$  (or more precisely with  $(1+\gamma)^{1/\alpha}$ ). Figure 3 provides a simple breakdown of GDP growth into the two different sources of GDP growth.<sup>9</sup> The development of the working age population is provided by the UN in five years steps. As we are also looking at five year steps of GDP growth, the shorter business fluctuations should be eliminated. However, GDP figures are still influenced by the longer cycles (e.g. based on construction cycles). As the result, the correlations between factor productivity and GDP appears larger than it actually is. Nevertheless, we can see that the decline of GDP growth in many advanced economies is driven by declining growth rate of the working age population and a declining growth rate of the total factor productivity.<sup>10</sup>

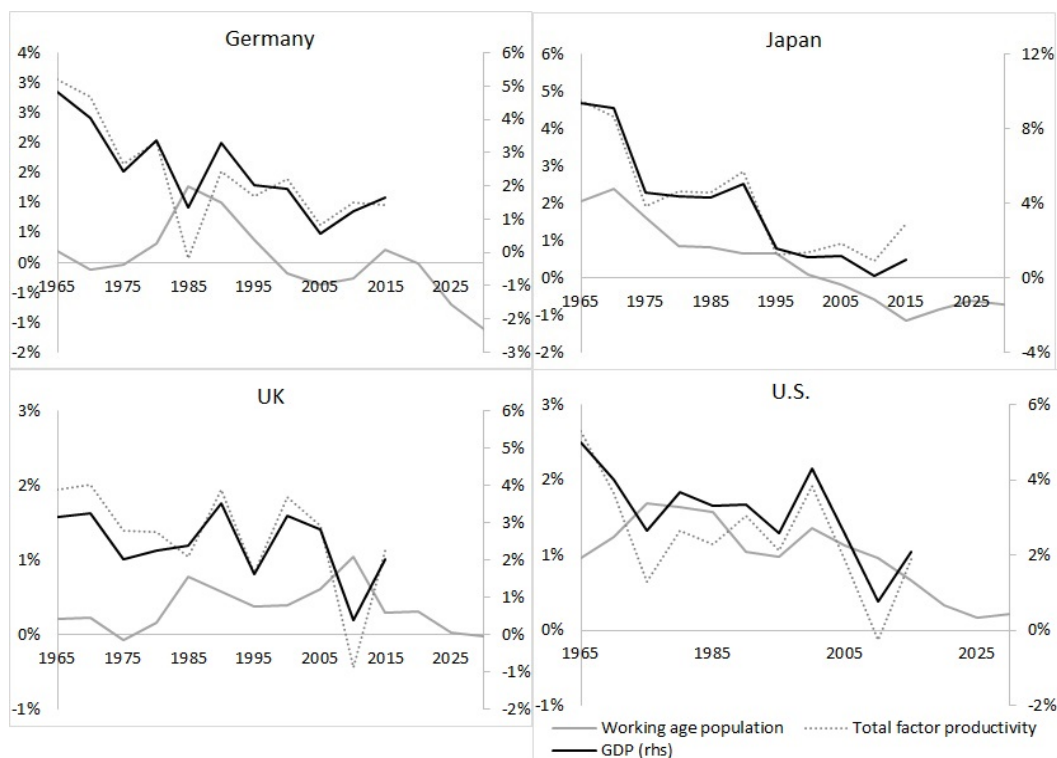
Following our model the decline in the growth rates of the working age population and the total factor productivity should be a main driver of the decline in real interest rates. As displayed in Figure 4, real government bond yields and GDP growth rates ( $\Delta$ ) declined over the past three decades. However, especially in Germany real interest rates used to be much higher than GDP growth rates and have only recently fallen below them. According

---

<sup>9</sup>GDP Growth for Germany before 1990 only West Germany.

<sup>10</sup>See also Adler et al. (2017).

Figure 3: Real GDP, Working Age Population, and Total Factor Productivity, Annual Growth, 5y Geometric Average, sources: Statistisches Bundesamt, UN, World Bank, and own calculations)

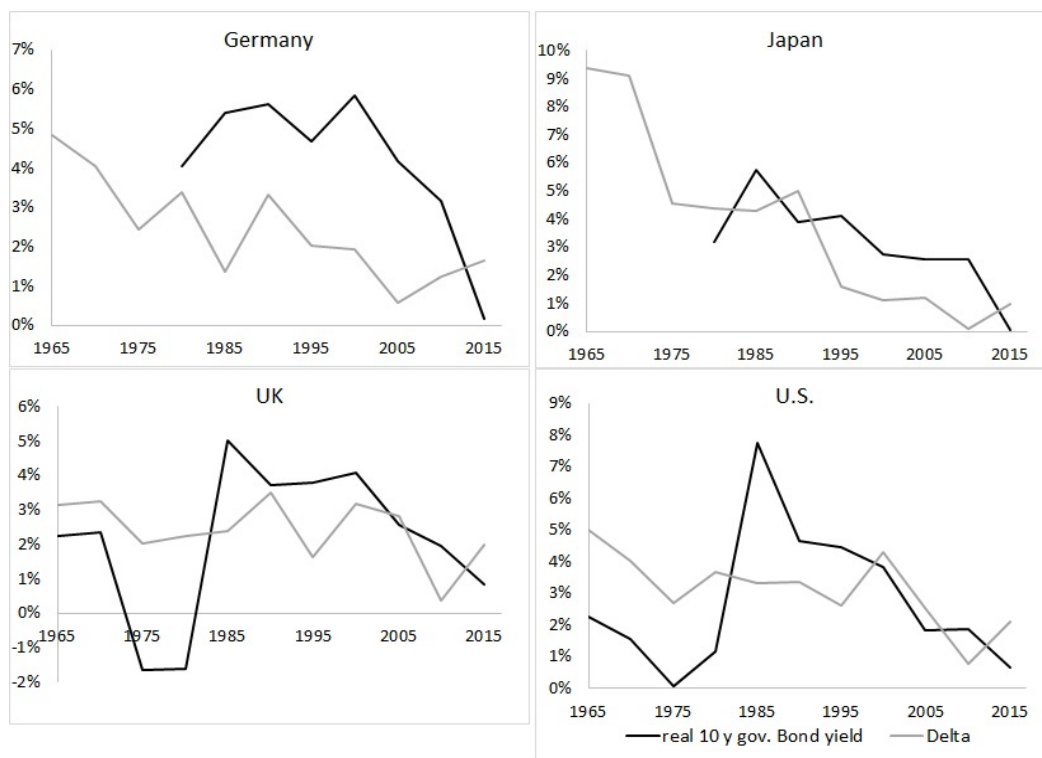


to our model, this indicates that there should have been either no pay-as-you-go tax or it used to be too high. Hence, recent reform efforts to increase retirement savings (e.g. Riester Rente) have reinforced the decline in interest rates<sup>11</sup> and have erased the overweight on pay-as-you-go pensions. In the 1970s UK and U.S. real interest rates were much lower than the actual real GDP growth. On the one hand this indicates that pay-as-you-go taxes were too low. On the other hand, this also reflects that inflation rates were higher than expected and we are not using expected inflation rates to calculate real rates but use the GDP deflator.

Hence, the compensation of a formally too high weight on pay-as-you-go

<sup>11</sup>This result is in line with e.g. Boersch-Supan et al. (2005).

Figure 4: Government Bond Yield vs. GDP Growth ( $\Delta$ ), in Real Terms, 5y Geometric Average, sources: Statistisches Bundesamt, UN, World Bank, and own calculations)



pension counteracts the positive effect of the increasing life expectancy on the optimal weight of the pay-as-you-go pension. However, since at the end of our data sample in 2015 real interest rates were below GDP growth rates, retirement saving is no longer more profitable than pay-as-you-go pension taxes and the compensation should come to an end. As a result, we should expect an increasing relevance of the pay-as-you-go pension in the future.

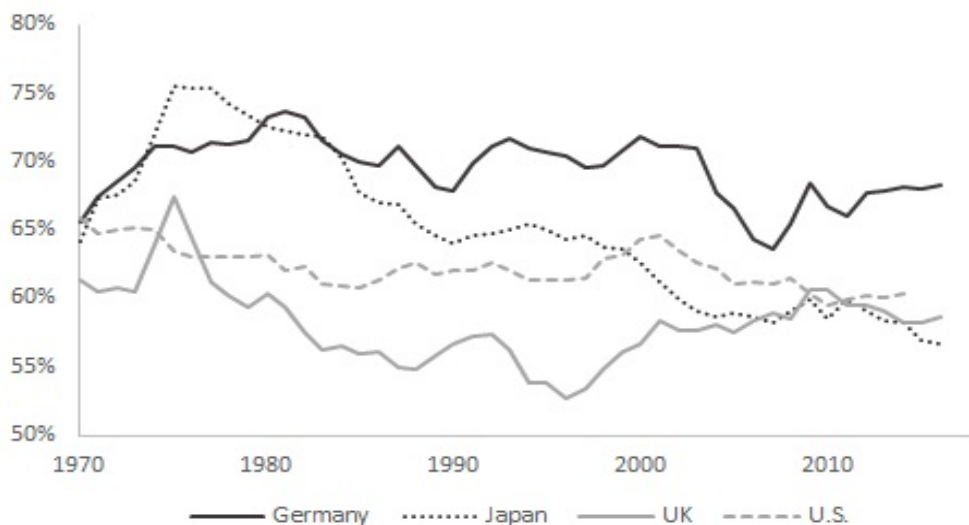
Besides the compensation effect, there is an additional effect that might have led to an increasing weight on retirement savings: an increasing output elasticity of capital. According to (17) and (18) an increasing output elasticity of capital relative to the output elasticity of labour (a decreasing  $\alpha$ ) should lead to a lower optimal pay-as-you-go tax and a higher optimal savings



rate. Standard macroeconomic theory assumes that  $\alpha$  is a constant. There is, however, empirical evidence that  $\alpha$ , represented by the labour share in total income, has declined over the past decades, especially in Japan and the U.S. (see Figure 5).<sup>12</sup> According to our model, this should have supported an increasing relevance of retirement savings. Nevertheless, the changes seems relatively moderate in comparison with the changes in life expectancy indicated in Figure 2.

Monetary policy also affects the relationship between interest rates and GDP growth rates. Given that we are looking at long-term yields, the effect of relatively tight or expansive monetary policy should be moderate in general. However, since the recent financial crisis extraordinary monetary policy measures also affected the longer end of the yield curve. This might be part of the explanation why at the end of our time sample interest rates are below GDP growth rates in all four countries. Nevertheless, it makes savings less efficient relative to pay-as-you-go pensions.

Figure 5: Share of Labour Compensation in GDP ( $\alpha$ ), sources: St.Louise Fed and Statistisches Bundesamt)



<sup>12</sup>See also e.g. Karabarbounis and Neiman (2013).

## 5 Conclusions

In this paper we have analysed the effects of ageing on optimal retirement funding. In particular, we were looking at the question whether a decreasing growth rate of the working age population and an increasing life expectancy should lead to a higher or lower weight on the public pay-as-you-go pension relative to retirement savings. Our results indicate that the decreasing growth rate of the working age population should not have an impact on optimal combination of the two sources of retirement funding. However, the increasing life expectancy should lead to a higher relevance of the pay-as-you-go pension.

The rationale behind this result is the following: In optimum, the transfer of wealth into retirement via a pay-as-you-go system or savings should be equally efficient and the return on both options should be equal to the growth rate of the economy. The declining growth rate of the working age population lowers (potential) economic growth and, therefore, affects both ways of retirement funding to the same degree. Hence, the relative weights of saving and pay-as-you-go does not change in optimum.

As long as we are not in a Keynesian unemployment regime, an increasing life expectancy should not directly affect the growth rate of the economy. However, it affects the need to transfer funds into retirement. If people would start to save more, there would be an oversupply of capital and the interest rate (return on savings) would decrease. A higher pay-as-you-go tax, however, would not affect its implicit return as it is always equal to the growth rate of the economy. Hence, it would be optimal to compensate the increasing life expectancy entirely by an increase in the pay-as-you-go pension tax.

A limitation of the validity of the theoretical results of our model is the assumption that we are coming from an optimal equilibrium. Over the past years, however the theoretical effect on the relevance of pay-as-you-go vs. retirement saving was counteracted by the compensation of a formally more than optimal reliance on public pay-as-you-go pensions and an increasing output elasticity of capital.

Another limitation is that the model is the assumption, that neither of the ways to fund retirement income includes any kinds of (financial) guarantees. While such savings products and pay-as-you-go systems are always sustainable, they leave these risks with the risk averse individuals. While we have abstracted from these risks, future research on optimal retirement

funding should also look at the welfare implications of the distribution of risks.

## References

- [1] **Aaron, H. (1966)** The Social Insurance Paradox. *Canadian Journal of Economics and Political Science*, 32, 371-374.
- [2] **Adler, G., R. Duval, D. Furceri, S. Celik, K. Koloskova and M. Poplaski-Ribeiro (2017)** Gone with the Headwinds: Global Productivity. *IMF Staff Discussion Note 17/04*, available at <http://www.imf.org/en/Publications/Staff-Discussion-Notes/Issues/2017/04/03/Gone-with-the-Headwinds-Global-Productivity-44758>.
- [3] **Andersen, T.M. and J. Bhattacharya (2016)** The Intergenerational Welfare State and the Rise and Fall of Pay-As-You-Go Pensions. *The Economic Journal* , 127, 896-923.
- [4] **Boeri, T., A. Boersch-Supan and G. Tabellini (2002)** Pension Reforms and the Opinions of European Citizens. *The American Economic Review*, Vol. 92, No. 2, *Papers and Proceedings of the 114th Annual Meeting of the American Economic Association*, 396-401.
- [5] **Boersch-Supan, A., A. Ludwig and J. Winter (2005)** Aging, Pension Reform, and Capital Flows: A Multi-Country Simulation Model. *NBER Working Paper* No. 11850.
- [6] **Boersch-Supan, A., K. Haertl and A. Ludwig (2014)** Aging in Europe: Reforms, International Diversification, and Behavioral Reactionst. *American Economic Review: Papers & Proceedings* 104(5): 224-229.
- [7] **Brunner, J. (1996)** Transition from a Pay-As-You-Go to a Fully Funded Pension System: The case of differing individuals and intra-generational fairness. *Journal of Public Economics*, 60 (1),131-146.
- [8] **Cipriani, G.P. (2016)** Aging, Retirement, and Pay-As-You-Go Pensions. *Macroeconomic Dynamics* , 1-11.

- [9] **Carvalho, C., A. Ferrero and F. Nechio (2016)** Demographics and Real Interest Rates: Inspecting the Mechanism. *Federal Reserve Bank of San Francisco Working Paper* 2016-05. <http://www.frbsf.org/economic-research/publications/working-papers/wp2016-05.pdf>
- [10] **Cutler, D., J. Poterba, L. Sheiner and L. Summers (1990)** An Aging Society: Opportunity or Challenge? *Brookings Papers on Economic Activity*, 1190:1, 1-73.
- [11] **European Commission (2015)** The 2015 Pension Adequacy Report: current and future income adequacy in old age in the EU. *Joint Report prepared by the Social Protection Committee (SPC) and the European Commission (DG EMPL)*, <https://publications.europa.eu/en/publication-detail/-/publication/2a4451ef-6d06-11e5-9317-01aa75ed71a1/language-en>.
- [12] **Gagnon, E., B. Johannsen and D. Lopez-Salido (2016)**. Understanding the New Normal: The Role of Demographics. *Finance and Economics Discussion Series* 2016-080. Washington: Board of Governors of the Federal Reserve System, <http://dx.doi.org/10.17016/FEDS.2016.080>.
- [13] **Galor, O. and D. Weil (1999)** From Malthusian Stagnation to Modern Growth. *American Economic Review*, 89(2),150–154.
- [14] **Gonzales-Eiras, M. and D. Niepelt (2012)** Aging, Government Budgets, Retirement, and Growth. *European Economic Review*, 56, 97-115.
- [15] **Karabarbounis, L. and B. Neiman (2013)** The Global Decline of the Labor Share. *NBER Working Paper* 19136 <http://www.nber.org/papers/w19136>.
- [16] **Samuelson, P. (1958)** An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *The Journal of Political Economy*, 66 (6), 467-482.
- [17] **Sinn, M. and S. Uebelmesser (2002)** Pensions and the Path to Gerontocracy in Germany. *European Journal of Political Economy*, 19, 153-158.