

Kalamov, Zarko; Runkel, Marco

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Sin Taxes, Insurance and the Correction of Internalities¹

Zarko Y. Kalamov

University of Technology Berlin, CESifo

Marco Runkel

University of Technology Berlin, CESifo, NoCeT

PRELIMINARY VERSION

Abstract

We analyze individuals with heterogeneous time-inconsistent preferences that consume sin goods and make a savings decision. A government may tax the sin good and provide mandatory health insurance. Due to time-inconsistency, the individual sin good and savings choices inflict internalities. Due to the *ex-ante* moral hazard of health insurance, sin good consumption also causes an externality. If the individuals' utility is such that savings and sin good demand decisions are decoupled, the government can achieve the first-best outcome using a uniform tax rate and uniform health insurance. Moreover, in the optimum, the tax rate internalizes only the externality and the government provides full insurance. When the savings and sin good consumption choices are interrelated, the government can still achieve the first-best outcome by additionally using Social Security to stipulate minimum savings requirements.

Key Words: sin tax, health insurance, moral hazard, hyperbolic discounting, inter-nality

JEL Codes: D11, D15, H21, H31, I12, I13, I18

¹Kalamov: Faculty of Economics and Management, University of Technology Berlin, Straße des 17. Juni 135, H51, 10623 Berlin, Germany, phone: +49 (0) 30 314 29528, email: zarko.y.kalamov@tu-berlin.de; Runkel: Faculty of Economics and Management, University of Technology Berlin, Straße des 17. Juni 135, H51, 10623 Berlin, Germany, phone: +49 (0) 30 314 22940, email: marco.runkel@tu-berlin.de.

1 Introduction

When making consumption choices, individuals often exhibit different forms of behavioral biases and inflict internalities on their future selves. For example, a person who engages in risky health behavior (such as, e.g., smoking, drinking alcohol, eating unhealthy food) may be present-biased and neglect the future health costs of this behavior. If, furthermore, individuals have health insurance, they do not fully internalize the monetary treatment costs associated with possible future behavior-related illnesses. In such cases, individuals exert both an internality on their future selves and an externality on society.

The behavioral public finance literature’s standard approach to address such situations is a tax that internalizes both the internality and the externality (Bernheim and Taubinsky 2018, Mullainathan et al. 2012, Farhi and Gabaix, 2015). Because different agents may be biased at different degrees, a government that is constrained to use uniform taxation can only achieve a second-best outcome where it corrects the average internality.

We consider a situation where individuals consume a sin good that is detrimental to health and make savings decisions. The agents exhibit different levels of present-bias in the form of quasi-hyperbolic discounting. We allow a benevolent social planner to use both uniform taxation of the sin good and uniform health insurance as policy instruments. We show that when the sin good and savings decisions are decoupled, the government chooses to provide full health insurance and sets the tax such that it internalizes *only* the externality of the sin good consumption. Moreover, the government achieves the first-best outcome in this situation. The reason for this result is that, under full insurance, the internality vanishes. This is true irrespective of the degree of present-bias of consumers. Hence, a uniform tax rate that internalizes the externality of the moral hazard associated with full insurance is sufficient to achieve the first-best.

If the sin good and savings decisions are interrelated, there are interactions between the internalities in these two dimensions. However, a social planner can still achieve the first-best outcome by additionally choosing minimum savings requirements through mandatory schemes such as Social Security. In this case, first-best is again attainable with the same insurance and tax levels as in the previous situation.

This article is related to the literature that studies the ex-ante moral hazard in health insurance and to the modern behavioral public finance literature.

The seminal work on the ex-ante moral hazard is from Ehrlich and Becker (1972). They show that health insurance incentivizes individuals to engage in unhealthy behavior (or neglect prevention). When taxation is not available as a policy instrument, the insurer finds it optimal to offer only partial insurance (Zweifel et al., 2009). Arnott and Stiglitz (1986) are the first to consider taxation as a policy instrument to correct the moral hazard problem. While Arnott and Stiglitz (1986) consider only fully rational agents, we focus on the case of a population with heterogeneous degrees of present-bias.

The use of taxation to address internalities was pioneered by O’Donoghue and Rabin (2003, 2006). They show that a small sin tax has first-order benefits to consumers with self-control problems and imposes only a second-order cost to time-consistent agents. Therefore, it is optimal to tax sin goods to internalize internalities even when a large proportion of the population is rational. When sin goods consumption exerts both an externality and an internality, the optimal tax internalizes both (see, e.g., Lockwood and Taubinsky, 2017).

We contribute to the behavioral public finance literature by deriving two main results. First, in the presence of health insurance, the optimal tax internalizes only the externality and not the internality. The rationale behind this result is that the internality vanishes under full insurance, which is the government’s optimal choice. Second, we show that the combination of uniform tax and insurance policy results in a first-best outcome. Thus, it is better than taxation policy only, which corrects the average externality and internality in the population and achieves only a second-best.

The rest of the paper is structured as follows. In Section 2, we present the model. Section 3 solves for the first-best outcome. Section 4 analyzes the equilibrium sin good consumption and government policy. Section 5 concludes.

2 The Model

Consider a population of individuals with mass one who live for two periods $t = 1, 2$. Each individual starts period 1 with an exogenous income Y_1 . In the same period, she consumes a numéraire good X in the amount X_1 and a sin good V . The sin good may represent any good with negative long-term health effects, such as unhealthy food, alcohol, cigarettes.

We use the terms sin good consumption and risky health behavior interchangeably. The government taxes V at a rate τ and returns the tax revenues to the individuals in the form of a uniform lump-sum transfer T . Denote the average consumption of V in the population as $E[V]$, where $E[\cdot]$ denotes the expectation operator. The uniform lump-sum transfer is, thus, $T = \tau E[V]$. The individuals also finance mandatory health insurance at a uniform premium P and save an amount S . Assuming, without loss of generality, that the net prices of X and V are unity, the period 1 budget constraint is

$$Y_1 + T = X_1 + (1 + \tau)V + P + S. \quad (1)$$

Each individual invests her savings S and earns an exogenous interest rS . Hence, in period 2 she derives income from the previous period savings equal to $(1 + r)S$. She uses the income to purchase the numéraire good in quantity X_2^i and health services L in the case of getting sick, where $i = s, h$ denotes the consumption of the numéraire good in the sick and healthy states, respectively. The probability of illness is $\pi(V) \in]0, 1[$ where $\pi_V > 0, \pi_{VV} > 0$ (we use a subscript to denote partial derivatives). Thus, risky health behavior increases the probability of illness at an increasing rate. In the case of illness, the insurance pays an indemnity I . Thus, the second period budget constraint is

$$(1 + r)S = X_2^s + L - I, \quad \text{if sick,} \quad (2)$$

$$(1 + r)S = X_2^h, \quad \text{if healthy.} \quad (3)$$

The insurer also invests the premium P and has available funds in period 2 equal to $(1 + r)P$. The insurance premium is actuarially fair and, thus, is determined by $(1 + r)P = E[\pi(V)]I$, where $E[\pi(V)]I$ gives the expected expenditures of the insurer in period 2.

Period 1 utility is given by

$$W(X_1, V), \quad (4)$$

where W is strictly concave in both arguments, i.e., $W_j > 0 > W_{jj}, j = X_1, V$ and $W_{X_1 X_1} W_{VV} - W_{X_1 V}^2 > 0$. In period 2, utility in state i is $U(X_2^i)$ with $U_{X_2} > 0 > U_{X_2 X_2}$.

If a person suffers from present-bias, she does not fully take into account the impact of behavior in period 1 on her own welfare in period 2. This effect is referred to as an internality. To consider such behavior, we follow the standard approach in the literature

and assume the individual is a quasi-hyperbolic discounter with a discount rate $\delta\beta$, where $\beta \in]0, 1]$ (following Laibson (1997) and subsequent literature). If $\beta = 1$, the individual discounts exponentially at the rate δ and behaves rationally. In the case $\beta \in]0, 1[$, the individual discounts hyperbolically, meaning that she exhibits present-bias, and does not fully take the effect of sin good consumption on the probability of being sick in the future. Thus, the individual's expected utility can be written as

$$EU = W(X_1, V) + \delta\beta [\pi(V)U(X_2^s) + (1 - \pi(V))U(X_2^h)]. \quad (5)$$

We follow the literature on self-control problems and suppose the social planner is paternalistic and views the expected utility (5) as the short-term utility of the individual (O'Donoghue and Rabin, 2003, 2006; DellaVigna and Malmendier, 2004), whereas the true long-term utility is given by

$$\widehat{EU} = W(X_1, V) + \delta [\pi(V)U(X_2^s) + (1 - \pi(V))U(X_2^h)]. \quad (6)$$

The present-bias β is distributed according to a cumulative distribution function $F(\beta)$. Hence, for some individuals (5) and (6) coincide (those with $\beta = 1$), while other individuals exhibit present-bias ($\beta < 1$) such that (5) and (6) differ. Thus, each individual chooses savings S and unhealthy behavior V to maximize (5). The social planner puts equal weight to each person and maximizes a utilitarian welfare function $E[\widehat{EU}]$ over the tax rate τ and indemnity I .

We solve the model in the following way. First, we derive in the next section the first-best levels of sin good consumption, insurance, taxation and savings that a social planner would choose. In the social optimum, there is full insurance and zero taxation. Subsequently, we derive the equilibrium that emerges when risky health behavior is unobservable for the social planner and all individuals have rational preferences with $\beta = 1$. In this case, a moral hazard problem emerges and each individual's choice of the sin good intake exerts an externality through health insurance. The government can, however, implement the first-best outcome, characterized by full insurance and a tax that exactly internalizes the externality. Next, we consider the case where some individuals have $\beta < 1$. These agents impose both an externality on society and an internality on their selves in period 2. We show that, under certain restrictions on the utility function,

the government can again impement the social optimum with uniform taxation and health insurance, where the tax internalizes only the externality associated with moral hazard and the internality vanishes under the optimal insurance contract. Lastly, we show how appropriate mandatory pension insurance can be used to achieve a first-best without any restrictions on the utility function.

3 Benchmark

Suppose that a social planner chooses S, V, τ and I to maximize the individuals' expected long-term utility (6). As β is the only heterogeneous parameter in the population, all individuals have the same long-term utility and $E[\widehat{EU}] = \widehat{EU}$. Insert the budget constraints (1)-(3) and the fair insurance premium level in Equation (6). The social planner solves

$$\begin{aligned} \max_{S, V, \tau, I} \widehat{EU} = & W \left(Y_1 + \tau V - (1 + \tau)V - \frac{\pi(V)I}{1 + r} - S, V \right) \\ & + \delta [\pi(V)U((1 + r)S - L + I) + (1 - \pi(V))U((1 + r)S)]. \end{aligned} \quad (7)$$

We solve the maximization problem (7) in Appendix A. Using a superscript $^\circ$ to denote the first-best, we derive the following results:

Lemma 1. *In the case of observable sin good consumption, the social planner chooses full insurance $I^\circ = L$ and zero taxation $\tau^\circ = 0$. Furthermore, the optimal sin good consumption and savings solve $W_V(X_1^\circ, V^\circ) = W_{X_1}(X_1^\circ, V^\circ)[1 + \pi_V(V^\circ)L/(1 + r)]$ and $W_{X_1}(X_1^\circ, V^\circ) = \delta(1 + r)U_{X_2}((1 + r)S^\circ)$, respectively.*

Proof: See Appendix A. □

When V is not observable, however, the first-best cannot be directly implemented. In the next section, we analyze the case of unobservable individual risky health behavior. We will compare our results to the benchmark from Lemma 1.

4 Risky Health Behavior is Unobservable

In this section, we assume the individual decides on the degree of risky behavior V and savings S , while the social planner chooses the policy variables of insurance coverage

I and tax rate τ . Due to unobservability, the insurer cannot set the level of V in the insurance contract. The equilibrium may differ from the first-best outcome because the individual takes the premium P as given and does not take into account that her own behavior affects it. This is the *ex-ante* moral hazard problem (Ehrlich and Becker, 1972). It constitutes an externality, as the marginal and social costs of unhealthy behavior differ (Arnott and Stiglitz, 1986).

The timing of the game is as follows. There are two stages in period 1. In stage 1, the government decides on the policy parameters I and τ , taking into account that its choices affect the individual behavior in stage 2. In stage 2, the individuals decide on how much of the sin good V to consume and the amount of savings. In the beginning of period 2, nature chooses the state $i = s, h$ for each agent, the insurer pays the indemnity I in case of sickness ($i = s$), and the individuals consume.

4.1 No Internality ($\beta = 1$)

Suppose initially that all individuals are rational and discount exponentially (i.e., $\beta = 1$ holds for everyone).

Consider first the problem of the individual in stage 2. She maximizes her expected utility EU , taking as given the premium P and lump-sum transfer T . Therefore, she chooses V and S so as to

$$\begin{aligned} \max_{S,V} EU &= W(Y_1 + T - (1 + \tau)V - P - S, V) \\ &\quad + \delta [\pi(V)U((1 + r)S - L + I) + (1 - \pi(V))U((1 + r)S)]. \end{aligned} \quad (8)$$

The first-order conditions are given by

$$EU_S = -W_{X_1}(X_1, V) + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)](1 + r) = 0, \quad (9)$$

$$EU_V = -W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)] = 0. \quad (10)$$

The individual first-order conditions determine, V and S as functions of both the indemnity I and the tax rate τ given by $\tilde{V}(I, \tau), \tilde{S}(I, \tau)$. The government chooses the two policy instruments to maximize the individual utility \widehat{EU} :

$$\max_{I, \tau} \widehat{EU} = W \left(Y_1 - (1 + \tau)\tilde{V}(I, \tau) + \tau\tilde{V}(I, \tau) - \frac{\pi(\tilde{V}(I, \tau))I}{1 + r} - \tilde{S}(I, \tau), \tilde{V}(I, \tau) \right)$$

$$+\delta \left[\pi(\tilde{V}(I, \tau))U((1+r)\tilde{S}(I, \tau) - L + I) + (1 - \pi(\tilde{V}(I, \tau)))U((1+r)\tilde{S}(I, \tau)) \right]. \quad (11)$$

We solve the maximization problem (11) in Appendix B. Denote the equilibrium values with an asterisk *. We prove the following results:

Lemma 2. *In the absence of internalities ($\beta = 1$ for everyone in the population), the government sets the tax rate according to*

$$\tau^* = \frac{\pi_V(V^\circ)L}{1+r}. \quad (12)$$

The indemnity, sin good level and savings equal their first-best values: $I^ = L = I^\circ$, $V^* = V^\circ$, $S^* = S^\circ$.*

Proof: See Appendix B. □

When taxation and health insurance are available, the government can use the tax to solve the moral hazard problem. This is the result of Arnott and Stiglitz (1986). The optimal tax rate τ^* exactly equals the effect of sin good consumption on the premium P that the representative individual fails to take into account. Hence, when the tax internalizes the externality, the government can choose full insurance without invoking unhealthy behavior ($I^* = L$). Consequently, full insurance and an appropriate tax rate induce the individual to choose the first-best level of the sin good $V^* = V^\circ$. Lastly, full insurance is sufficient for the individual to smooth consumption as in the first-best case and set $S^* = S^\circ$.

4.2 Self-Control Problems

We now relax the assumption of time-consistent preferences throughout the population. When some individuals exhibit $\beta < 1$, unhealthy behavior and savings are chosen so as to maximize Equation (5). The first-order conditions are similar to Equations (9) and (10) and are given by

$$EU_S = -W_{X_1}(X_1, V) + \delta\beta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)](1+r) = 0, \quad (13)$$

$$EU_V = -W_{X_1}(X_1, V)(1+\tau) + W_V(X_1, V) + \delta\beta\pi_V [U(X_2^s) - U(X_2^h)] = 0. \quad (14)$$

The only difference between Equations (13) and (14) and Equations (9) and (10), respectively, is the presence of hyperbolic discounting $\beta < 1$ in the former. As in the model with exponential discounting only, the first-order conditions determine the sin good level and savings as functions of the policy parameters $\tilde{V}(I, \tau)$ and $\tilde{S}(I, \tau)$.

The social planner maximizes the welfare function $E[\widehat{EU}]$ taking into account that $\tilde{V}(I, \tau)$ and $\tilde{S}(I, \tau)$ are determined by the first-order conditions (13) and (14). Denote again the equilibrium variables with an asterisk. We characterize the equilibrium in the following Proposition:

Proposition 1. *Suppose that some individuals exert present-bias ($\beta < 1$) and period 1 utility takes the form*

$$W(X_1, V) = W(X_1 + Z(V)), \quad (15)$$

where $W' > 0 \geq W''$ and $Z' > 0 > Z''$.

Then, a paternalistic government sets the tax rate to internalize the externality only:

$$\tau^* = \frac{\pi_V(V^\circ)L}{1+r}. \quad (16)$$

There is full insurance ($I^* = L = I^\circ$) and each individual chooses the first-best sin good consumption level $V^* = V^\circ$ irrespective of their degree of self-control problems β . Individuals with present-bias save less than in the social planner's optimum ($S^*(\beta < 1) < S^\circ$). Individuals without present bias choose the first-best level of savings: $S^*(\beta = 1) = S^\circ$.

If the period 1 utility does not satisfy (15), then the tax rate internalizes both the internality and externality and second-best levels of risky health behavior, insurance and savings are reached.

Proof: See Appendix C. □

According to Proposition 1, the social planner can achieve the first-best sin good consumption and insurance even with uniform taxation and insurance, when utility takes the special form, specified in Equation (15). The intuition is the following. When there is full insurance, the health internality, given by $\delta(1 - \beta)\pi_V[U(X_2^s) - U(X_2^h)]$ vanishes. Moreover, under the functional form (15), the sin good demand and savings are decoupled under full insurance and the sin tax does not affect S^* . Thus, the tax is chosen such that

it internalizes the externality only. Furthermore, full insurance is indeed optimal, as it maximizes the expected utility of all individuals, irrespective of their degree of self-control problems.

However, when the utility function is not of the form given in Equation (15), the decisions on V and S are interrelated even with full insurance. In this situation, the first-best is not implementable due to the time-inconsistent savings decisions of present-biased individuals. In the next section, we analyze how the government can achieve the social optimum even when utility is not specified by Equation (15).

4.3 Social Security

In this section, we assume the government is able to specify a minimum amount of savings through mandatory retirement savings, similarly to Fadlon and Laibson (2017). Such systems exist in many countries in the form of Social Security (or pension insurance). Suppose the government specifies uniform mandatory minimum savings S^F in period 1 and pays the individuals $(1+r)S^F$ in period 2. Thus, the individual budget constraints become

$$Y_1 + T = X_1 + (1+\tau)V + P + S + S^F, \quad (17)$$

$$(1+r)(S + S^F) = X_2^s + L - I, \quad \text{if sick}, \quad (18)$$

$$(1+r)(S + S^F) = X_2^h, \quad \text{if healthy}. \quad (19)$$

Note that, in this case, the first-best outcome is again given by Lemma 1 with the difference that the total amount of savings must be optimal, i.e., $S + S^F = S^\circ$. Consider now the individual first-order conditions, given by Equations (13) and (14) in Section 4.2. While Equation (14) remains unchanged, we need to take into account that (13) may have a corner solution, such that it changes to

$$\begin{aligned} EU_S &= -W_{X_1}(X_1, V) + \delta\beta [\pi(V)U_{X_2}(X_2^s) + (1-\pi(V))U_{X_2}(X_2^h)](1+r) \leq 0, \\ S &\geq 0, \quad S \times EU_S = 0. \end{aligned} \quad (20)$$

We now look at the government. It decides on τ , I and S^F to maximize the welfare function $E[\widehat{EU}]$, taking into account that the first-order conditions (14) and (20) determine the sin

good demand and savings as functions of the policy instruments, i.e, $V = \tilde{V}(\tau, I, S^F), S = \tilde{S}(\tau, I, S^F)$. In Appendix D, we derive the following results:

Proposition 2. *Suppose that some individuals exhibit present-bias ($\beta < 1$). Then, a paternalistic government sets the tax rate to internalize the externality only:*

$$\tau^* = \frac{\pi_V(V^\circ)L}{1+r}. \quad (21)$$

There is full insurance ($I^ = L = I^\circ$) and each individual chooses the first-best sin good consumption level $V^* = V^\circ$ irrespective of their degree of self-control problems β . The government sets the mandatory retirement savings at $S^{F*} = S^\circ$ and all individuals choose zero voluntary savings $S^* = 0$.*

Proof: See Appendix D. □

According to Proposition 2, the equilibrium is characterized by the first-best outcome for all variables. The intuition regarding the optimal tax rate and indemnity is the same as in Proposition 1. These results emerge irrespective of the form of the utility function because the government uses the pension insurance to solve the problem of time-inconsistent savings. Consequently, it can solve the health externality by choosing full insurance and internalize the externality by setting the tax rate according to (21).

At this point, we note that the first-best is achieved in Proposition 2 because savings are not time-inconsistent. However, a government mandated minimum savings program is not the only instrument that can achieve this goal. Suppose that individuals with present-bias are sophisticated and are aware of their self-control problems. Such individuals are aware that their true utility is given by Equation (6) and that they will, however, maximize Equation (5) in period 1. Assume furthermore, as in Amador et al. (2006), that a commitment device is available to the individuals prior to the beginning of period 1. Such a commitment device is, for instance, a minimum-savings rule, which restricts the individuals to save at least a given level S^M . This commitment device may be in the form of voluntary pension insurance. A sophisticated person with self-control problems can choose whether and to what extent to commit to minimum savings S^M prior to period 1 by maximizing the true long-term utility (6). Since the problem of a sophisticated individual who chooses S^M prior to period 1 is identical to the problem of the government that chooses mandatory savings S^F , we can immediately write the following Corollary:

Corollary 1. *Suppose that all time-inconsistent individuals in the population are sophisticated. Then, a commitment device offering a minimum-savings rule prior to period 1 leads to the same equilibrium as a mandatory pension insurance.*

Therefore, the first-best outcome is achievable with taxation and health insurance as the only government instruments when both utility is of the special form from Equation (15) and when the time-inconsistent individuals are sophisticated and the market provides minimum savings rules. Otherwise, a mandatory minimum savings rule, such as Social Security, is necessary to arrive at the social optimum.

5 Conclusions

The standard behavioral public finance approach to addressing sin goods consumption of present-biased consumers is the use of taxation. Because taxes are usually constrained to be uniform, the standard approach achieves only a second-best result with heterogeneous agents (O'Donoghue and Rabin, 2006). We propose the use of insurance and taxation. Full health insurance can solve the externality irrespective of the degree of present-bias of consumers. Hence, this approach can achieve a first-best outcome. If savings decisions create an externality that interrelates with the health externality, the use of pension insurance in conjunction with health insurance and taxation achieves the same outcome.

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A Proof of Lemma 1

To prove Lemma 1, we take the first-order condition of (6) with respect to S, V, τ and I . They are given by

$$\widehat{EU}_S = -W_{X_1}(X_1, V) + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r) = 0, \quad (\text{A.1})$$

$$\widehat{EU}_V = -W_{X_1}(X_1, V) \left[1 + \frac{\pi_V I}{1 + r} \right] + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)] = 0, \quad (\text{A.2})$$

$$\widehat{EU}_\tau = W_{X_1}(X_1, V) [-V + V] = 0, \quad (\text{A.3})$$

$$\widehat{EU}_I = W_{X_1}(X_1, V) \left[-\frac{\pi(V)}{1 + r} \right] + \delta \pi(V)U_{X_2}(X_2^s) = 0. \quad (\text{A.4})$$

Equation (A.3) immediately gives $\tau^\circ = 0$. Inserting (A.1) in (A.4), we get

$$\delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r) \left[-\frac{\pi(V)}{1 + r} \right] + \delta \pi(V)U_{X_2}(X_2^s) = 0. \quad (\text{A.5})$$

Rearranging (A.5), we get

$$\delta \pi(V)(1 - \pi(V)) [U_{X_2}(X_2^s) - U_{X_2}(X_2^h)] = 0. \quad (\text{A.6})$$

Equation (A.6) is fulfilled only for $X_2^s = X_2^h$, i.e., when $I^\circ = L$. Using $X_2^s = X_2^h$ in Equations (A.1) and (A.2), we immediately get $W_V(X_1^\circ, V^\circ) = W_{X_1}(X_1^\circ, V^\circ)[1 + \pi_V(V^\circ)L/(1 + r)]$ and $W_{X_1}(X_1^\circ, V^\circ) = \delta(1 + r)U_{X_2}((1 + r)S^\circ)$. \square

B Proof of Lemma 2

The government solves the maximization problem (11). Its first-order conditions with respect to τ and I are given by

$$\begin{aligned} \widehat{EU}_\tau &= [-W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)]] \frac{d\tilde{V}}{d\tau} \\ &\quad + W_{X_1}(X_1, V) \left[\tau \frac{d\tilde{V}}{d\tau} - \tilde{V} + \tilde{V} - \frac{\pi_V \frac{d\tilde{V}}{d\tau} I}{1 + r} \right] \\ &\quad + [-W_{X_1}(X_1, V) + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r)] \frac{d\tilde{S}}{d\tau} = 0, \end{aligned} \quad (\text{B.1})$$

$$\widehat{EU}_I = [-W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)]] \frac{d\tilde{V}}{dI} \quad (\text{B.2})$$

$$\begin{aligned}
& + W_{X_1}(X_1, V) \left[\tau \frac{d\tilde{V}}{dI} - \frac{\pi(V) - \pi_V \frac{d\tilde{V}}{dI} I}{1+r} \right] + \delta \pi(V) U_{X_2}(X_2^s) \\
& + [-W_{X_1}(X_1, V) + \delta [\pi(V) U_{X_2}(X_2^s) + (1 - \pi(V)) U_{X_2}(X_2^h)] (1+r)] \frac{d\tilde{S}}{dI} = 0.
\end{aligned}$$

According to the first-order conditions of the individual (9) and (10), the first and third rows of (B.1) and (B.2) equal zero. Thus, we get

$$\widehat{EU}_\tau = W_{X_1}(X_1, V) \left[\tau - \frac{\pi_V I}{1+r} \right] \frac{d\tilde{V}}{d\tau} = 0, \quad (\text{B.3})$$

$$\widehat{EU}_I = W_{X_1}(X_1, V) \left[\tau \frac{d\tilde{V}}{dI} - \frac{\pi(V) - \pi_V \frac{d\tilde{V}}{dI} I}{1+r} \right] + \delta \pi(V) U_{X_2}(X_2^s) = 0. \quad (\text{B.4})$$

Equation (B.3) gives $\tau^* = \pi_V(V^*) I^* / (1+r)$. Inserting $\tau = \tau^*$ in Equation (B.4), we get

$$\widehat{EU}_I = -\frac{\pi(V)}{1+r} W_{X_1}(X_1, V) + \delta \pi(V) U_{X_2}(X_2^s) = 0. \quad (\text{B.5})$$

Equation (B.5) is identical to the first-order condition of the social planner (A.4) from Appendix A. Using Equation (9) to substitute for $W_{X_1}(X_1, V)$ in (B.5), we immediately find that the solution to (B.5) must satisfy $X_2^s = X_2^h$. Thus, $I^* = L = I^\circ$. Therefore, Equations (10) and (B.3) together determine $V^* = V^\circ$ and $\tau^* = \pi_V(V^\circ) L / (1+r)$. Lastly, the equilibrium savings are determined by the first-order condition (9) which is identical to the social planner's first-order condition (A.1) from Appendix A. As the tax revenues are returned in lump-sum fashion to the individual, the solutions to (9) and (A.1) are identical such that $S^* = S^\circ$. \square

C Proof of Proposition 1

The maximization problem of the government is given by

$$\begin{aligned}
\max_{I, \tau} E[\widehat{EU}] &= E \left[W \left(Y_1 - (1+\tau) \tilde{V}(I, \tau) + \tau E[\tilde{V}(I, \tau)] - \frac{E[\pi(\tilde{V}(I, \tau))] I}{1+r} - \tilde{S}(I, \tau), \tilde{V}(I, \tau) \right) \right. \\
&\quad \left. + \delta \left[\pi(\tilde{V}(I, \tau)) U((1+r) \tilde{S}(I, \tau) - L + I) + (1 - \pi(\tilde{V}(I, \tau))) U((1+r) \tilde{S}(I, \tau)) \right] \right].
\end{aligned} \quad (\text{C.1})$$

The first-order conditions with respect to τ and I are

$$E[\widehat{EU}]_\tau = E \left[[-W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)]] \frac{d\tilde{V}}{d\tau} \right. \\ \left. + W_{X_1}(X_1, V) \left[\tau \frac{dE[\tilde{V}]}{d\tau} - \tilde{V} + E[\tilde{V}] - \frac{E \left[\pi_V \frac{d\tilde{V}}{d\tau} \right] I}{1 + r} \right] \right. \\ \left. + [-W_{X_1}(X_1, V) + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r)] \frac{d\tilde{S}}{d\tau} \right] = 0, \quad (C.2)$$

$$E[\widehat{EU}]_I = E \left[[-W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)]] \frac{d\tilde{V}}{dI} \right. \\ \left. + W_{X_1}(X_1, V) \left(\tau \frac{dE[\tilde{V}]}{dI} - \frac{E \left[\pi(V) + \pi_V \frac{d\tilde{V}}{dI} I \right]}{1 + r} \right) + \delta \pi(V)U_{X_2}(X_2^s) \right. \\ \left. + [-W_{X_1}(X_1, V) + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r)] \frac{d\tilde{S}}{dI} \right] = 0. \quad (C.3)$$

We use the individual's first-order conditions (13) and (14) to simplify the first and third rows of (C.2) and (C.3). We get

$$E[\widehat{EU}]_\tau = E \left[\delta(1 - \beta) \left\{ \pi_V [U(X_2^s) - U(X_2^h)] \frac{d\tilde{V}}{d\tau} \right. \right. \\ \left. \left. + [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r) \frac{d\tilde{S}}{d\tau} \right\} \right. \\ \left. + W_{X_1}(X_1, V) \left(\tau \frac{dE[\tilde{V}]}{d\tau} - \tilde{V} + E[\tilde{V}] - \frac{E \left[\pi_V \frac{d\tilde{V}}{d\tau} \right] I}{1 + r} \right) \right] = 0, \quad (C.4)$$

$$E[\widehat{EU}]_I = E \left[\delta(1 - \beta) \left\{ \pi_V [U(X_2^s) - U(X_2^h)] \frac{d\tilde{V}}{dI} \right. \right. \\ \left. \left. + [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r) \frac{d\tilde{S}}{dI} \right\} \right. \\ \left. + W_{X_1}(X_1, V) \left(\tau \frac{dE[\tilde{V}]}{dI} - \frac{E \left[\pi(V) + \pi_V \frac{d\tilde{V}}{dI} I \right]}{1 + r} \right) + \delta \pi(V)U_{X_2}(X_2^s) \right] = 0. \quad (C.5)$$

Suppose now that period 1 utility takes the form

$$W(X_1, Z) = W(X_1 + Z(V)), \quad (C.6)$$

with $W' > 0 \geq W''$ and $Z' > 0 > Z''$, where the $'$ denotes a partial derivative. In this case, the individual first-order conditions (13) and (14) become

$$EU_V = W' \left(Y_1 - \tilde{V} - \frac{\pi(\tilde{V})I}{1+r} - \tilde{S} + Z(\tilde{V}) \right) \left[-(1+\tau) + Z'(\tilde{V}) \right] \quad (\text{C.7})$$

$$+\delta\beta\pi_V [U(X_2^s) - U(X_2^h)] = 0,$$

$$EU_S = -W' \left(Y_1 - \tilde{V} - \frac{\pi(\tilde{V})I}{1+r} - \tilde{S} + Z(\tilde{V}) \right) \quad (\text{C.8})$$

$$+\delta\beta \left[\pi(\tilde{V})U_{X_2}(X_2^s) + (1 - \pi(\tilde{V}))U_{X_2}(X_2^h) \right] (1+r) = 0.$$

Now suppose that the equilibrium is characterized by full insurance $I^* = L$, i.e. $X_2^s = X_2^h \equiv X_2$. Suppose furthermore that $\tau^* = \pi_V(V^\circ)L/(1+r)$. In this case, the individual first-order condition (C.7) gives $Z'(V^*) = 1 + \tau^* = 1 + \pi_V(V^\circ)L/(1+r)$. Thus, each individual chooses the same sin good consumption level, irrespective of their β values. Moreover, applying the utility function from (C.6) to Lemma 1, we see that V° is characterized by $Z'(V^\circ) = 1 + \pi_V(V^\circ)L/(1+r)$. Hence, in this situation, each individual chooses $V^* = V^\circ$ and we have $E[V^*] = E[V^\circ] = V^\circ$. We can, therefore, rewrite the government's first-order conditions (C.4) and (C.5) as

$$E[\widehat{EU}]_\tau \Big|_{\{\tau = \frac{\pi_V(V^\circ)L}{1+r}, I=L\}} = E \left[\delta(1-\beta)U_{X_2}(X_2)(1+r) \frac{d\tilde{S}}{d\tau} \right], \quad (\text{C.9})$$

$$E[\widehat{EU}]_I \Big|_{\{\tau = \frac{\pi_V(V^\circ)L}{1+r}, I=L\}} = E \left[\delta(1-\beta)U_{X_2}(X_2)(1+r) \frac{d\tilde{S}}{dI} \right. \quad (\text{C.10})$$

$$\left. + W'(\cdot) \left(-\frac{\pi(V^\circ)}{1+r} \right) + \delta\pi(V^\circ)U_{X_2}(X_2) \right].$$

Next, we prove that both (C.9) and (C.10) equal zero and, thus $\tau = \frac{\pi_V(V^\circ)L}{1+r}, I = L$ constitute an optimum. To do so, we must derive $d\tilde{S}/d\tau$ and $d\tilde{S}/dI$. To derive these terms, we totally differentiate the individual first-order conditions (C.7) and (C.8) with respect to V, S, τ and I to get

$$\underbrace{\begin{pmatrix} E_{VV} & E_{VS} \\ E_{SV} & E_{SS} \end{pmatrix}}_{=J} \begin{pmatrix} d\tilde{V} \\ d\tilde{S} \end{pmatrix} = - \begin{pmatrix} E_{V\tau} \\ E_{S\tau} \end{pmatrix} d\tau - \begin{pmatrix} E_{VI} \\ E_{SI} \end{pmatrix} dI, \quad (\text{C.11})$$

where the terms in the matrices are defined as follows (in the following equations, we drop the \sim for simplicity):

$$EU_{VV} = W''(\cdot) [-(1 + \tau) + Z'(V)] \left[-1 - \frac{\pi_V I}{1 + r} + Z'(V) \right] + W'(\cdot) Z''(V) + \delta\beta\pi_{VV} [U(X_2^s) - U(X_2^h)], \quad (\text{C.12a})$$

$$EU_{VS} = -W''(\cdot) [-(1 + \tau) + Z'(V)] + \delta\beta\pi_V [U_{X_2}(X_2^s) - U_{X_2}(X_2^h)] (1 + r), \quad (\text{C.12b})$$

$$EU_{SV} = -W''(\cdot) \left[-1 - \frac{\pi_V I}{1 + r} + Z'(V) \right] + \delta\beta\pi_V [U_{X_2}(X_2^s) - U_{X_2}(X_2^h)] (1 + r), \quad (\text{C.12c})$$

$$EU_{SS} = W''(\cdot) + \delta\beta(1 + r)^2 [\pi U_{X_2 X_2}(X_2^s) + (1 - \pi) U_{X_2 X_2}(X_2^h)], \quad (\text{C.12d})$$

$$EU_{V\tau} = -W'(\cdot), \quad (\text{C.12e})$$

$$EU_{S\tau} = 0, \quad (\text{C.12f})$$

$$EU_{VI} = W'' [-1 + Z'(V)] \left[-\frac{\pi}{1 + r} \right] + \delta\beta\pi_V U_{X_2}(X_2^s), \quad (\text{C.12g})$$

$$EU_{SI} = W'' \frac{\pi}{1 + r} + \delta\beta\pi(1 + r) U_{X_2 X_2}(X_2^s). \quad (\text{C.12h})$$

The second-order condition requires $|J| = E_{VV}E_{SS} - E_{VS}E_{SV} > 0$. We use Cramer's rule to derive $d\tilde{S}/dI$ and $d\tilde{S}/d\tau$:

$$\frac{d\tilde{S}}{d\tau} = \frac{1}{|J|} \begin{vmatrix} E_{VV} & -E_{V\tau} \\ E_{SV} & 0 \end{vmatrix} = -\frac{W'(\cdot)E_{SV}}{|J|}, \quad (\text{C.13})$$

$$\frac{d\tilde{S}}{dI} = \frac{1}{|J|} \begin{vmatrix} E_{VV} & -E_{VI} \\ E_{SV} & -E_{SI} \end{vmatrix} = \frac{-E_{VV}E_{SI} + E_{SV}E_{VI}}{|J|}. \quad (\text{C.14})$$

Now, evaluate the expressions (C.13) and (C.14) at $I = L$ and $\tau = \pi_V(V^\circ)L/(1 + r)$. These values of I and τ give $X_2^s = X_2^h = X_2$, $Z'(V) = 1 + \tau$ and $V = V^\circ$ (from Equation (C.7) and Lemma 1). Thus, we have

$$EU_{SV} \Big|_{\{\tau = \frac{\pi_V(V^\circ)L}{1+r}, I=L\}} = 0. \quad (\text{C.15})$$

Inserting (C.15) in (C.13) and (C.14), we get

$$\frac{d\tilde{S}}{d\tau} \Big|_{\{\tau = \frac{\pi_V(V^\circ)L}{1+r}, I=L\}} = 0, \quad (\text{C.16})$$

$$\left. \frac{d\tilde{S}}{dI} \right|_{\{\tau = \frac{\pi_V(V^\circ)L}{1+r}, I=L\}} = -\frac{\pi(V^\circ)}{1+r}. \quad (\text{C.17})$$

Using Equation (C.16), one can immediately see that the government's first-order condition (C.9) equals zero at $\tau = \pi_V(V^\circ)L/(1+r)$, $I = L$. To see that (C.10) also equals zero, rewrite it as

$$\begin{aligned} E[\widehat{EU}]_I \Big|_{\{\tau = \frac{\pi_V(V^\circ)L}{1+r}, I=L\}} &= E \left[\delta(1-\beta)U_{X_2}(X_2)(1+r)\frac{d\tilde{S}}{dI} \right. \\ &\quad \left. + \delta\beta(1+r)U_{X_2}(X_2) \left(-\frac{\pi(V^\circ)}{1+r} \right) + \delta\pi(V^\circ)U_{X_2}(X_2) \right], \end{aligned} \quad (\text{C.18})$$

where we used the individual first-order conditions (C.8) to substitute $\delta\beta(1+r)U_{X_2}(X_2)$ for $W'(\cdot)$. Use now (C.17) to substitute for $d\tilde{S}/dI$. We immediately see that (C.18) equals zero. Thus, we proved that $\tau^* = \pi_V(V^\circ)L/(1+r)$, $I^* = L$ and $V^* = V^\circ$ satisfy the government's first-order conditions when utility is given by (C.6).

Lastly, we have to prove $S^* < S^\circ$ for $\beta < 1$ and $S^* = S^\circ$ for $\beta = 1$. To prove that result, note that S^* is determined separately from V^* in the equilibrium and can be evaluated from the first-order condition (C.8) only. Evaluate (C.8) at $S = S^\circ$. We get

$$\begin{aligned} EU_S \Big|_{S=S^\circ} &= -W'(S = S^\circ) + \delta\beta(1+r)U'((1+r)S^\circ) \\ &= \delta(\beta - 1)(1+r)U'((1+r)S^\circ) \leq 0 \quad \Leftrightarrow \quad \beta \leq 1. \end{aligned} \quad (\text{C.19})$$

Hence, the first-order condition with respect to S is negative at $S = S^\circ$ for $\beta < 1$ and due to $EU_{SS} < 0$ (see Equation (C.12d)), we can conclude $S^* < S^\circ$ in this case. In the case $\beta = 1$, we have $S^* = S^\circ$.

Consider now the second part of Proposition 1. If the utility function $W(X_1, V)$ does not satisfy (15), then τ^* , $I^* = L$ and $V^* = V^\circ$ for all individuals cannot be optimal. To see this, note that in this case, the first-order condition (C.4) becomes

$$E[\widehat{EU}]_\tau = E \left[U_{X_2}(X_2)(1+r)\frac{d\tilde{S}}{d\tau} \right] \gtrless 0. \quad (\text{C.20})$$

Because for general utility functions $d\tilde{S}/d\tau$ can take any value, the government cannot implement τ^* , $I^* = L$ and $V^* = V^\circ$. Therefore, the terms in the first and second rows of the first-order conditions (C.4) and (C.5) do not vanish in equilibrium and τ^* , I^* depend on the distribution of β . \square

D Proof of Proposition 2

The maximization problem of the government is given by

$$\begin{aligned}
\max_{I, \tau, S^F} E[\widehat{EU}] &= E \left[W \left(Y_1 - (1 + \tau) \tilde{V}(I, \tau, S^F) + \tau E[\tilde{V}(I, \tau, S^F)] - \frac{E[\pi(\tilde{V}(I, \tau, S^F))]I}{1 + r} \right. \right. \\
&\quad \left. \left. - \tilde{S}(I, \tau, S^F) - S^F, \tilde{V}(I, \tau, S^F) \right) \right. \\
&\quad \left. + \delta \left[\pi(\tilde{V}(I, \tau, S^F))U((1 + r)(\tilde{S}(I, \tau, S^F) + S^F) - L + I) \right. \right. \\
&\quad \left. \left. + (1 - \pi(\tilde{V}(I, \tau, S^F)))U((1 + r)(\tilde{S}(I, \tau, S^F) + S^F)) \right] \right]. \tag{D.1}
\end{aligned}$$

The first-order conditions with respect to τ, I and S^F are

$$\begin{aligned}
E[\widehat{EU}]_\tau &= E \left[[-W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)]] \frac{d\tilde{V}}{d\tau} \right. \\
&\quad \left. + W_{X_1}(X_1, V) \left[\tau \frac{dE[\tilde{V}]}{d\tau} - \tilde{V} + E[\tilde{V}] - \frac{E \left[\pi_V \frac{d\tilde{V}}{d\tau} \right] I}{1 + r} \right] \right. \\
&\quad \left. + [-W_{X_1}(X_1, V) + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r)] \frac{d\tilde{S}}{d\tau} \right] = 0, \tag{D.2}
\end{aligned}$$

$$\begin{aligned}
E[\widehat{EU}]_I &= E \left[[-W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)]] \frac{d\tilde{V}}{dI} \right. \\
&\quad \left. + W_{X_1}(X_1, V) \left(\tau \frac{dE[\tilde{V}]}{dI} - \frac{E \left[\pi(V) + \pi_V \frac{d\tilde{V}}{dI} I \right]}{1 + r} \right) + \delta \pi(V)U_{X_2}(X_2^s) \right. \\
&\quad \left. + [-W_{X_1}(X_1, V) + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r)] \frac{d\tilde{S}}{dI} \right] = 0, \tag{D.3}
\end{aligned}$$

$$\begin{aligned}
E[\widehat{EU}]_{S^F} &= E \left[[-W_{X_1}(X_1, V)(1 + \tau) + W_V(X_1, V) + \delta \pi_V [U(X_2^s) - U(X_2^h)]] \frac{d\tilde{V}}{dS^F} \right. \\
&\quad \left. + W_{X_1}(X_1, V) \left(\tau \frac{dE[\tilde{V}]}{dS^F} - \frac{E \left[\pi_V \frac{d\tilde{V}}{dS^F} I \right]}{1 + r} \right) + [-W_{X_1}(X_1, V) \right. \\
&\quad \left. + \delta [\pi(V)U_{X_2}(X_2^s) + (1 - \pi(V))U_{X_2}(X_2^h)] (1 + r)] \left(1 + \frac{d\tilde{S}}{dS^F} \right) \right] = 0. \tag{D.4}
\end{aligned}$$

We now show that the individual first-order conditions (14) and (20) and the government's first-order conditions (D.2)-(D.4) have the following solutions: $S^{F*} = S^\circ, S^* = 0, V^* = V^\circ, I^* = L = I^\circ, \tau^* = \pi_V(V^\circ)L/(1+r)$. To do so, note that at $I = L$, there is full insurance and (14) gives $W_V = W_{X_1}(1+\tau)$. This expression results in a uniform demand for V for uniform savings, which is the case for $S^F = S^\circ, S = 0$. Thus, the government's first-order condition with respect to S^F , evaluated at $S^F = S^\circ, S = 0, I = L$ is given by

$$E[\widehat{EU}]_{S^F} = E \left[[-W_{X_1}(X_1, V) + \delta [U_{X_2}(X_2)](1+r)] \left(1 + \frac{d\tilde{S}}{dS^F} \right) \right] = 0. \quad (\text{D.5})$$

The solution to (D.5) is $W_{X_1} = \delta(1+r)U_{X_2}$. This is the condition that determines S° in Lemma 1. Hence, we have $S + S^F = S^\circ$. Moreover, the individual first-order condition with respect to S (Equation (20)), evaluated at $W_{X_1} = \delta(1+r)U_{X_2}, I = L$, gives

$$EU_S = \delta(\beta - 1)U_{X_2}(X_2)(1+r) \leq 0, \quad S \geq 0, \quad S \times EU_S = 0. \quad (\text{D.6})$$

Time consistent individuals with $\beta = 1$ choose $S = 0$ as the utility-maximizing private savings, while time-inconsistent individuals ($\beta < 1$) have a corner solution $S = 0$. Thus, all individuals choose $S^* = 0$ and the government sets $S^{F*} = S^\circ$.

Note furthermore that at $\tau = \tau^* = \pi_V(V^\circ)L/(1+r)$ and $I = L$, the first-order condition (14)'s solution is $V^* = V^\circ$, i.e., all individuals choose the efficient sin good level, irrespective of their β parameters.

We can now insert $I = L, \tau = \pi_V(V^\circ)L/(1+r)$, the solution to (14) ($W_V = W_{X_1}(1+\tau)$), $V^* = V^\circ$ and the solution to (D.4) ($W_{X_1} = \delta(1+r)U_{X_2}$) in the government's remaining first-order conditions (D.2) and (D.3). One can immediately see that they are both satisfied. \square