

Steger, Thomas; Grossmann, Volker; Larin, Benjamin; Löfflad, Hans Torben

Conference Paper

Distributional Effects of Surging Housing Costs under Schwabe's Law of Rent

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Macroeconomics IV, No. E23-V3

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Steger, Thomas; Grossmann, Volker; Larin, Benjamin; Löfflad, Hans Torben (2019) : Distributional Effects of Surging Housing Costs under Schwabe's Law of Rent, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Macroeconomics IV, No. E23-V3, ZBW - Leibniz-Informationszentrum Wirtschaft, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/203613>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Distributional Effects of Surging Housing Costs under Schwabe’s Law of Rent*

Volker Grossmann[†], Benjamin Larin[‡], Hans Torben Löfflad[§], Thomas Steger[¶]

March 1, 2019

Abstract

The upward sloping trend of rents and house prices has initiated a debate on the consequences of surging housing costs for wealth inequality and welfare. We employ a frictionless two-sectoral macroeconomic model with a housing sector to investigate the dynamics of wealth inequality and the determinants of welfare. Households have non-homothetic preferences, implying that the poor choose a higher housing expenditure share, being compatible with Schwabe’s Law of Rent. We examine at first the isolated effects of increasing housing costs in partial equilibrium. The model is closed by introducing a production sector that enables us to analyze the general equilibrium consequences of a widely discussed policy option that aims at dampening the growth of housing costs. Abolishing zoning regulations triggers a slower rent growth and reduces wealth inequality by about 0.7 percentage points (measured by top 10 percent share). Average welfare increases by about 0.5 percent. However, the household-specific welfare effects are clearly asymmetric. The poor benefit more than the rich. The richest wealth decile is even worse off.

Key words: Macroeconomics and Housing; Long-Term Growth; Schwabe’s Law of Rent; Wealth Inequality; Welfare.

JEL classification: E10, E20, O40.

*We are grateful to Christian Bayer, Philip Bunn, Michael Burda, Matthias Meier, Dirk Krueger, Michael Kumhof, David Miles, Thomas Piketty, Edward Prescott, Moritz Schularick, Sjak Smulders, Uwe Sunde and Klaus Waelde for valuable comments and suggestions. We also thank seminar participants at the Bank of England, Paris School of Economics, the Universities of Bonn, Konstanz, Muenich, Pennsylvania, Zuerich, the 6th Lindau Nobel Laureate Meeting on Economic Sciences, and the First WID.world conference 2017 in Paris for helpful discussions.

[†]University of Fribourg; CESifo, Munich; Institute for the Study of Labor (IZA), Bonn; Centre for Research and Analysis of Migration (CReAM), University College London. Email: volker.grossmann@unifr.ch.

[‡]Leipzig University; Halle Institute for Economic Research (IWH). Email: larin@wifa.uni-leipzig.de.

[§]Leipzig University. Email: loefflad@wifa.uni-leipzig.de.

[¶]Leipzig University; CESifo, Munich; Halle Institute for Economic Research (IWH). Email: steger@wifa.uni-leipzig.de.

1 Introduction

The real housing rent and the real house price have risen, on average, since WW2 in most industrialized economies (Knoll, 2017; Knoll, Schularick & Steger, 2017). At the same time, housing expenditures exhibit a striking (and time-invariant) pattern. The aggregate housing expenditure share, being the largest single expenditure category, in the US economy appears largely constant over time at about 19 percent, despite rising real per capita incomes (BLS, 2016; Piazzesi & Schneider, 2017). However, at a given point in time, the percentage of total expenditures spent on housing varies inversely with income. For instance, US households in the first income quintile devoted about 25 percent of total expenditures to housing in 2015, whereas this number was only 18 percent for the fifth income quintile.¹ This observation is closely related to a pattern that is extremely robust across space and time: Schwabe’s Law of Rent. Indeed, Stigler (1954, p. 100) characterized Schwabe’s Law of Rent as second fundamental law of consumer behavior.²

Hermann Schwabe, the director of the Berlin statistical bureau, proposed a second "law" in 1868. He had salary and rent data for 4,281 public employees receiving less than 1,000 thaler a year, and income and rent data for 9,741 families with incomes in excess of 1,000 thaler. For each group he found the percentage of income (or salary) spent on rent declined as income rose, and proposed the law: ‘The poorer any one is, the greater the amount relative to his income that he must spend for housing.’ The law seemed to contemporaries less obviously true than Engel’s, and a considerable literature arose about it. Ernst Hasse found that it held for Leipzig in 1875, and E. Laspeyres confirmed it for Hamburg. Engel also accepted Schwabe’s law.

Surging housing costs under asymmetric spending patterns for housing across income groups have initiated a debate on the implications for wealth inequality and welfare (Summers, 2014; Albouy, Ehrlich & Liu, 2016; Albouy & Ehrlich, 2018; Dustmann, Fitzenberger & Zimmermann, 2018). However, a dynamic macro approach to address the issue has been lacking. This paper aims to fill the void. We investigate the dynamics of wealth inequality and the determinants of welfare in a growing economy that experiences surging housing costs. At first we examine the isolated dynamic effects of

¹The cross-sectional variation of housing expenditure shares is even more pronounced in other advanced economies, such as France, Germany, and the UK (Section 6.1).

²The first law of consumer behavior is the well-known Engel’s law (Sigler, 1954).

exogenously increasing housing costs in partial equilibrium. This step is helpful for our general equilibrium analysis that features endogenous evolution of rents over time. As a natural candidate for an (exogenous) event that triggers changes in rents, we consider the (counterfactual) abolishment of zoning regulations. In fact, zoning regulations are widely recognized as an important amplifier for surging housing costs in a growing economy (Glaeser, Gyourko, & Saks, 2005; Saiz, 2010; Albouy & Ehrlich, 2018). Our general equilibrium analysis addresses two research questions. *First*, how are rents and wealth inequality associated in general equilibrium provided that the government abolishes residential zoning regulations? As we are ultimately interested in life-time utility (being a function of initial endowments and the entire time path of future prices), we turn to the welfare effects by asking: *Second*, how does heterogeneity of housing expenditure shares across income groups affect the change in household-specific welfare in response to the abolishment of zoning regulations?³

We employ a frictionless two-sectoral macroeconomic model with a housing sector. Abstracting from financial frictions enables us to derive analytical insights into the dynamics of wealth inequality and the determinants of welfare. The general equilibrium model captures endogenously rising rents in a growing economy. This is important as future expected rent growth exerts a systematic impact on the saving decisions if households are forward-looking (saving for future rent burden to smooth housing consumption). The supply side of the model, that is introduced to endogenize rents, follows the long-term macro and housing model by Grossmann & Steger (2017). It distinguishes between the extensive margin (the number of houses) and the intensive margin (the size of the average house) of the housing stock. This enables us to analyze in a meaningful way the general equilibrium consequences of the abolishment of residential zoning regulations that target the extensive margin of the housing stock. There is exogenous household heterogeneity with respect to initial wealth and labor income (Chatterjee, 1994; Caselli & Ventura, 2000). The demand side features non-homothetic preferences to replicate the inverse variation of housing expenditure shares across income groups. Specifically, we assume that households have status concerns for housing, being in line with empirical evidence

³Notice that heterogeneity of housing expenditure shares across income groups, which will turn out to be directly relevant for welfare, is closely related to Schwabe's law of rent, as explained in Section 2.

(Leguizamón & Ross, 2012; Bellet, 2017). Stronger status concerns for housing, as will be shown below, amplify existing welfare differences among poor and rich households by enlarging the variation in household specific price indices.

The analysis proceeds in two steps. In a first step, we investigate the dynamics of wealth inequality and the determinants of welfare in partial equilibrium. It is shown that stronger rent growth produces *less* wealth inequality in partial equilibrium, provided that the utility function is sufficiently concave. The reason is that the differences in the saving rates across wealth groups (a force contributing to diverging wealth holdings in the population) is shrinking in response to stronger rent growth. This counterintuitive result appears robust across a large set of models, as it hinges merely on the widely accepted assumptions of forward-looking and optimizing households. The analysis also clarifies, somewhat surprisingly at first glance, that Schwabe’s Law of Rent is not important with regard to the dynamics of wealth inequality. This insight is in striking contrast to the welfare implications. Stronger status concerns for housing induce more heterogeneity in housing expenditure shares. This amplifies the welfare differences by enlarging the heterogeneity in household-specific price indices. That is, Schwabe’s Law of Rent is important with regard to inequality of welfare. The underlying mechanism is empirically supported. For instance, Albouy, Ehrlich & Liu (2016) show that real income inequality in the US increased 25 percent more since 1970 when deflated with household specific price indices.

In a second step, we analyze a growing economy in general equilibrium. It is shown that the wealth distribution is stationary in a steady state, despite continuously rising rents and house prices. However, any policy change that induces transitional dynamics triggers a permanent change in the wealth distribution. We show that removing residential zoning regulations leads to a temporarily slower rent growth (relative to the baseline scenario), which is associated with a reduction in wealth inequality (measured by the top 10 percent wealth share) by about 0.7 percentage points over time. That is, in contrast to the partial equilibrium result, rent growth and wealth inequality are *positively* associated, i.e. slower rent growth induced by abolishing zoning regulations goes hand in hand with a reduction in wealth inequality. Average welfare increases by about 0.5 percent. However,

the household-specific welfare effects are clearly asymmetric. The poor benefit more than the rich. The richest wealth decile is even worse off. The important lesson to draw from this policy experiment is twofold. First, despite a potentially negative causal effect of surging rents on wealth inequality, a policy measure that slows down rent growth may nevertheless lower welfare inequality through its additional general equilibrium effects. Second, under Schwabe’s law of rent, surging rents are unambiguously and positively associated with inequality of welfare and harmful for the poor (but not necessarily for the rich).

There are three strands of related literature. The first strand addresses the importance of the housing sector in macro models. Many models are designed to discuss business cycle phenomena, such as Davis and Heathcote (2005), Iacoviello (2005), Iacoviello & Neri (2010), Kiyotaki, Michaelides & Nikolov (2011), Favilukis, Ludvigson & Van Nieuwerburgh (2017), and Kydland, Rupert & Sustek (2017).⁴ More recently, a literature has emerged that focusses on the long term, such as Grossmann & Steger (2017), Miles & Sefton (2017) and Borri & Reichlin (2018). Our research questions necessarily require a long-term perspective. The second strand of literature analyzes one-sector economies under household heterogeneity with the representative household property (Chatterjee 1994; Caselli & Ventura, 2000; Álvarez-Peláez & Diaz, 2005; Garcia-Penalosa & Turnovsky; 2006; Krusell & Rios Rull, 1999). We add to this literature by analyzing a two-sectoral model (with a continuously changing relative price) under household heterogeneity with non-homothetic preferences and the representative household property. A third strand examines the savings behavior and wealth inequality by employing stochastic overlapping generations models under incomplete markets. These contributions typically focus on alternative mechanisms that shape the wealth distribution in steady state.⁵ We explore mechanisms that shape the dynamics of wealth inequality and welfare differences apart from borrowing constraints and provide analytical insights that apply equally to transitional dynamics and the steady state. Our analysis rests on fundamental market forces that would equally be present in an economy with borrowing constraints from which we

⁴Piazzesi and Schneider (2017) provide an excellent survey.

⁵Some exceptions analyze the wealth distribution over time by employing numerical techniques, such as Gabaix, Lasry, Lions & Moll (2016), Kaymak & Poschke (2016), Hubmer, Krusell & Smith (2016), and Wälde (2016). De Nardi & Fella (2017) provide an excellent survey.

abstract for the sake of simplicity.

The plan of this paper is as follows. Section 2 describes the household side. Section 3 provides partial equilibrium results. Section 4 discusses analytical insights into the dynamics of wealth inequality and the determinants of household-specific welfare. Section 5 introduces the production side and characterizes the steady state. Section 6 investigates the consequences of removing zoning regulations numerically in general equilibrium. Section 7 discusses whether the results depend on modelling households as renters or as homeowners. Section 8 concludes.

2 Households

Consider a set of infinitely lived households in a perfectly competitive economic environment. There are $J \in \mathbb{N}$ groups of households (or individuals) indexed by $j \in \{1, 2, \dots, J\}$. Every group j has measure $n_j \in \mathbb{R}^+$ of households. Each household of group j has time-invariant labor endowment, denoted as l_j , that is supplied inelastically to the labor market.⁶ Aggregate labor supply accordingly reads as $L \equiv \sum_j n_j l_j$. The total amount of households is $\mathcal{L} \equiv \sum_j n_j$. Time is continuous and indexed by $t \geq 0$. Households are allowed to differ in their labor endowment, l_j , and their initial wealth holding, $W_j(0)$.

Let c_j describe consumption of the numeraire good of household j , s_j consumption of housing services, and $\bar{s} \equiv \frac{1}{\mathcal{L}} \sum_j n_j s_j$ the average amount of housing services across all households, respectively. Preferences of household j are described by the intertemporal utility function

$$U_j(0) = \int_0^\infty u(c_j(t), s_j(t)) e^{-\rho t} dt \quad \text{with} \quad (1)$$

$$u(c_j, s_j) = \frac{[(c_j)^{1-\theta} (s_j - \phi \bar{s})^\theta]^{1-\sigma} - 1}{1-\sigma}, \quad (2)$$

where $\sigma > 0$ denotes a concavity parameter of the outer instantaneous utility function, $\theta \in (0, 1)$ a concavity parameter of the inner utility function, $\rho > 0$ the subjective discount rate, and $\phi \in [0, 1)$, respectively.⁷ For $\phi > 0$ the instantaneous utility function

⁶In what follows, we employ the short formulation “household j ” instead of “household of group j ”.

⁷The time index t is often suppressed provided that this does not lead to ambiguity.

(2) captures status concerns for housing services consumption and implies that preferences are non-homothetic. The importance of status concerns is increasing in ϕ . (It will turn out below that an increase in ϕ amplifies existing welfare differences among poor and rich households.)

There are two reasons why we capture status concerns for housing services. First, the importance of status concerns for housing is widely recognized. For instance, by employing US microdata, Bellet (2017) shows that suburban homeowners who experienced a relative downscaling of their homes due to the building of larger units in their suburb record lower satisfaction and house values.⁸ Status concerns for housing actually represent an old topic that has already been discussed by Karl Marx (1847):

A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirement for a residence. But let there arise next to the little house a palace, and the little house shrinks to a hut. The little house now makes it clear that its inmate has no social position at all to maintain, or but a very insignificant one; and however high it may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.

Second, the assumption of status preferences for housing implies, as shown below, that housing expenditure shares, $e_j \equiv \frac{ps_j}{c_j + ps_j}$, differ across households, where p is the price for residential services (i.e., the rent). The employed model can thus replicate the pattern that the expenditure share for housing, at a given point in time, declines with income. It is closely related to *Schwabe's Law of Rent*, which states that, in the cross section, the percentage of income spent on housing, $\frac{ps_j}{y_j}$, declines with income, y_j (see introduction). The utility function (2) implies that the percentage of total expenditures spent on housing, e_j , declines with income (overall wealth).⁹ The two statements are not identical but, of course, are related. This becomes obvious by writing $\frac{c_j + ps_j}{y_j} \cdot e_j = \frac{ps_j}{y_j}$. If the poor choose a higher consumption rate ($\frac{c_j + ps_j}{y_j}$), as is empirically plausible and in line with our calibration, then the negative variation of $\frac{ps_j}{y_j}$ with income is even more

⁸See also Frank (2005), Turnbull, Dombrow & Sirmans (2006), and Leguizamon & Ross (2012).

⁹In what follows we focus on e_j because this variable determines differences in household specific price indices (Section 3).

pronounced than the negative variation of e_j with income.¹⁰

Each household j chooses consumption paths $\{c_j(t), s_j(t)\}_{t=0}^{\infty}$ by maximizing U_j subject to the standard No-Ponzi game condition, non-negativity constraints, and the intertemporal budget constraint¹¹

$$\dot{W}_j = rW_j + wl_j - c_j - ps_j, \quad (3)$$

where r denotes the interest rate and w the wage rate per unit of labor, respectively. As individuals have mass zero, they take factor prices, the rental rate p , and average consumption of housing services, \bar{s} , as given.¹²

Despite non-homothetic preferences a representative household exists. The following remark makes this property explicit.

Remark 1 (Representative household). *An economy populated by a set of households whose preferences are described by (1) together with (2) and whose intertemporal budget constraint is given by (3) admits a (positive) representative household. That is, the demand side can be described as if there were a single household who owns the entire endowments and makes the aggregate consumption and saving decisions. As a consequence, the distribution of labor endowment, l_j , and wealth, W_j , does not play a role for the evolution of aggregate variables. This does also apply for the case of non-homothetic preferences ($\phi > 0$).*

The validity can be proven by showing that the aggregation of the household's first-order conditions yields the same set of first-order conditions that result from the problem of a single household who owns the entire endowments ($\sum_j l_j$ and $\sum_j W_j$) and is making the aggregate consumption and saving decisions. All proofs are relegated to the Appendix.

The representative household setup surely has pros and cons. On the one hand, it may be considered as implausible that the distribution does not feed back on aggregate

¹⁰In Online-Appendix 10.7 we discuss alternative utility specifications. It is shown that the results are robust to assuming status concerns for both goods, provided they are stronger for housing. We also show that alternative formulations (CES utility and multiplicative reference level) are inconsistent with major empirical observations.

¹¹A dot above a variable denotes the partial derivative with respect to time.

¹²The most straightforward interpretation is that households are renters. In Section 7 we show that the results do not change if we model households as homeowners.

variables and that there is no change in the consumption distribution over time, as far as one abstracts from idiosyncratic shocks. On the other hand, this setup enables us to gain analytical insights into the dynamics of wealth inequality and the determinants of welfare that operate even in the absence of financial market frictions. Moreover, we add to the theoretical literature on dynamic macro models with a representative household by analyzing a two-sectoral model under heterogeneity and non-homothetic preferences.¹³

3 Partial Equilibrium Results

This section provides partial equilibrium results that are based on household optimization. All prices $\{r, w, p\}$ are taken as given at this layer of analysis.

3.1 Housing Expenditure Shares

Define total wealth, \mathcal{W}_j , as the sum of non-human wealth, W_j , and human wealth, $\tilde{w}l_j$, i.e.

$$\mathcal{W}_j(t) \equiv W_j(t) + \tilde{w}(t)l_j \quad \text{with} \quad (4)$$

$$\tilde{w}(t) \equiv \int_t^{\infty} w(\tau) e^{\int_t^{\tau} -r(v)dv} d\tau. \quad (5)$$

Let \bar{W} denote average non-human wealth in the economy and define the average labor supply by $\bar{l} \equiv L/\mathcal{L}$. Average total wealth is thus given by $\bar{\mathcal{W}} \equiv \bar{W} + \tilde{w}\bar{l}$. Also define the relative to average total wealth level of household j by $\Omega_j \equiv \mathcal{W}_j/\bar{\mathcal{W}}$.

Proposition 1 (Housing expenditure shares) *The expenditure share for housing services of household j , defined as $e_j \equiv \frac{ps_j}{c_j + ps_j}$, is time-invariant and given by*

$$e_j = \theta \left(1 + \frac{(1 - \theta)\phi}{[1 - (1 - \theta)\phi] \Omega_j(0)} \right). \quad (6)$$

¹³Previous macro models under household heterogeneity with the representative household property are mostly, if not exclusively, one-sectoral models (Chatterjee, 1994; Caselli & Ventura, 2000; Álvarez-Peláez & Diaz, 2005; Garcia-Penalosa & Turnovsky, 2006; and Krusell & Rios Rull, 1999).

For $\phi > 0$, there is a negative relationship between e_j and the relative total wealth level of household j in the initial period, $\Omega_j(0)$.

The housing expenditure share of the representative household, with $\Omega_j = 1$, is given by $\bar{e} \equiv \frac{\theta}{1-(1-\theta)\phi}$. Most importantly, the housing expenditure share is decreasing in total relative wealth at time $t = 0$, whenever individuals have status preferences ($\phi > 0$).

3.2 Ideal Price Indices

Instantaneous utility may be written as $u(\mathcal{C}_j) \equiv \frac{(\mathcal{C}_j)^{1-\sigma}-1}{1-\sigma}$ with consumption index $\mathcal{C}_j \equiv (c_j)^{1-\theta}(s_j - \phi\bar{s})^\theta$. The associated overall consumption expenditure is denoted as $\mathcal{E}_j \equiv c_j + ps_j$. Hence, the price index of a household j is given by $\mathcal{P}_j \equiv \mathcal{E}_j/\mathcal{C}_j$. When \mathcal{C}_j and \mathcal{P}_j are evaluated at equilibrium quantities and prices, we refer to \mathcal{C}_j and \mathcal{P}_j as ideal consumption index and ideal price index of j .

Proposition 2 (Ideal price indices) *The ideal price index of household j in period t is given by*

$$\mathcal{P}_j(t) = \frac{p(t)^\theta}{\theta^\theta(1-\theta)^{-\theta}} \frac{1}{1-e_j} \equiv \tilde{\mathcal{P}}(p(t), e_j). \quad (7)$$

The ideal price index of the representative household (equal to the aggregate price index) is obtained for $e_j = \bar{e}$. If $\phi = 0$ (homothetic preferences), i.e. $e_j = \theta$ for all j , the price index is the same for all households and given by $\tilde{\mathcal{P}}(p, \theta) = \frac{p^\theta}{\theta^\theta(1-\theta)^{1-\theta}}$. For $\phi > 0$, we obtain partial derivatives $\frac{\partial \tilde{\mathcal{P}}(p, e)}{\partial e} > 0$ and $\frac{\partial \tilde{\mathcal{P}}(p, e)}{\partial p} > 0$. That is, the poorer a household is in terms of total wealth, i.e. the higher the housing expenditure share (Proposition 1), the more \mathcal{P}_j is affected by an increase in the price for housing services, p .¹⁴

3.3 Saving Rates

Let $y_j \equiv rW_j + wl_j$ denote income and let $\mu_j \equiv \mathcal{E}_j/\mathcal{W}_j$ denote the (average) propensity to consume out of total wealth.¹⁵ The saving rate $sav_j = 1 - \frac{\mathcal{E}_j}{y_j}$ will turn out helpful

¹⁴Albouy, Ehrlich & Liu (2016) construct an ideal cost-of-living index that varies with income and prices. They show that, based on US microdata, real income inequality, measured by the 90 percentile / 10 percentile ratio, rose by 10 percentage points more when income is deflated by their individual cost-of-living index.

¹⁵In our model, the average propensity to consume equals the marginal propensity to consume.

in the subsequent analysis. To discuss the properties of this saving function, we turn at first to the propensity to consume.

Proposition 3 (Propensity to consume) *The propensity to consume is at any time t identical for all households and given by*

$$\mu(t) = \left(\int_t^\infty \left(\frac{\bar{p}(\tau, t)^\theta}{\exp [\bar{r}(\tau, t) + \frac{\rho}{\sigma-1}(\tau - t)]} \right)^{\frac{\sigma-1}{\sigma}} d\tau \right)^{-1}, \quad (8)$$

where $\bar{r}(\tau, t) \equiv \int_t^\tau r(v)dv$ is the cumulative interest rate and $\bar{p}(\tau, t) \equiv p(\tau)/p(t)$ is the growth factor of the housing rent between τ and t .

That all households choose the same propensity to consume reflects the absence of heterogeneity in terms of preferences and the absence of borrowing constraints. We turn at next to the saving function.

Proposition 4 (Saving rates) *Let $\omega_j = W_j/l_j$ denote the wealth-to-labor ratio. The saving rate at any time t may be expressed as*

$$sav_j(t) = 1 - \frac{\mu(t) \cdot [\omega_j(t) + \tilde{w}(t)]}{r(t)\omega_j(t) + w(t)} \equiv Sav(\omega_j, \cdot). \quad (9)$$

The notation $sav_j = Sav(\omega_j, \cdot)$ highlights that the saving rate is a function of the wealth-to-labor ratio, ω_j . An implication of the preceding proposition is given by

Corollary 1 (Saving rate differentials). *The saving rate changes with the wealth-to-labor ratio according to $\frac{\partial Sav_j}{\partial \omega_j} = \frac{\mu(r\tilde{w}-w)}{(\omega_j r + w)^2}$. That is, the saving rate increases with ω_j , $\frac{\partial Sav_j}{\partial \omega_j} > 0$, provided that $r\tilde{w} > w$.*

Why does the saving rate increase in the wealth-to-labor ratio, provided that $r\tilde{w} > w$? To see this, notice that the total-wealth-to-income ratio, $\frac{\mathcal{W}_j}{y_j}$, can be expressed as a weighted average according to

$$\frac{\mathcal{W}_j}{y_j} = \frac{\omega_j + \tilde{w}}{r\omega_j + w} = \frac{\omega_j}{r\omega_j + w} + \frac{\tilde{w}}{r\omega_j + w} = \frac{1}{r} \frac{r\omega_j}{y_j/l_j} + \frac{\tilde{w}}{w} \frac{w}{y_j/l_j}.$$

The term $\frac{1}{r}$ represents the total-wealth-to-income ratio of a pure capitalist ($\omega_j > 0$, $l_j w = 0$), while the term $\frac{\tilde{w}}{w}$ represents the total-wealth-to-income ratio of a pure worker ($\omega_j = 0$, $l_j w > 0$). The condition $\frac{\tilde{w}}{w} > \frac{1}{r}$ (or $r\tilde{w} > w$) is equivalent to assuming that the total-wealth-to-income ratio of a pure worker exceeds that of a pure capitalist such that the total-wealth-to-income ratio is decreasing in ω_j . Hence, households characterized by a low ω_j choose a high level of consumption relative to income, $\frac{\mu W_j}{y_j}$, implying that the saving rate, $sav_j = 1 - \mu \frac{W_j}{y_j}$, is low, and vice versa.

Notice also that the condition $r\tilde{w} > w$ is satisfied in any steady state with positive wage growth. Assuming $w(\tau) = w(t)e^{g(\tau-t)}$ with $g > 0$, we have $\tilde{w}(t) = \frac{w(t)}{r-g}$. Plugging this into $r\tilde{w}(t) > w(t)$ boils down to $g > 0$.

4 Wealth Inequality and Welfare

The dynamics of wealth inequality and the determinants of welfare are discussed analytically, based on the partial equilibrium results reported above. The mechanisms discussed below still hold in general equilibrium, i.e., when prices are fully endogenous.

4.1 Surging Rents and Wealth Inequality

4.1.1 Wealth Divergence and Wealth Convergence

We start out with some preliminary considerations that are important for the subsequent analysis. Let the growth rate of (non-human) wealth of household j be defined as $\hat{W}_j \equiv \dot{W}_j/W_j$ and express the saving rate as $sav_j \equiv \dot{W}_j/y_j$, where $y_j \equiv rW_j + wl_j$ denotes income. The growth rate of household's j wealth may then be expressed as follows:¹⁶

$$\hat{W}_j \equiv \frac{\dot{W}_j}{W_j} = sav_j \frac{y_j}{W_j} = Sav(\omega_j, \cdot) \left(r + \frac{w}{\omega_j} \right) \equiv G(\omega_j, \cdot). \quad (10)$$

To simplify the analysis, we assume that the following assumption holds.

¹⁶The notation $G(\omega_j, \cdot) \equiv Sav(\omega_j, \cdot) \left(r + \frac{w}{\omega_j} \right)$ highlights that the wealth growth rate is a function of $\omega_j = W_j/l_j$. The function $G(\omega_j, \cdot)$ is well defined for positive and negative wealth. It is not defined for $W_j = 0$. However, the limits are defined: $\lim_{W_j \rightarrow 0^-} \hat{W}_j = -\infty$ and $\lim_{W_j \rightarrow 0^+} \hat{W}_j = +\infty$.

Assumption 1. *If $W_j \geq W_{j'}$ for any j and j' , then it also holds true that $\omega_j \geq \omega_{j'}$.*

Notice that $\omega_j \geq \omega_{j'}$ implies $\frac{W_j}{W_{j'}} \geq \frac{l_j}{l_{j'}}$. That is, the relative distance in wealth ($\frac{W_j}{W_{j'}}$) has to be larger than the relative distance in earnings ($\frac{l_j w}{l_{j'} w}$). This assumption does not seem too restrictive, given that wealth is more unequally distributed than earnings (Kuhn & Ríos-Rull, 2015). With Assumption 1 at hand, we define wealth divergence and wealth convergence as follows. Consider two individuals $j, j' \in [1, \dots, J]$ with $\omega_j > \omega_{j'}$. Wealth levels diverge (converge) between j and j' in a small time interval $[t, t + dt]$ if $\hat{W}_j(t) > (<) \hat{W}_{j'}(t)$.¹⁷ If this is true for any two households, there is *global wealth divergence (convergence)*. Equation (10) is instructive for two reasons. First, it represents a general statement that is not conditional on any specific model. Second, it indicates that there are two opposing forces at work. On the one hand, wealth-rich households choose a higher saving rate compared to the wealth poor, provided that $r\tilde{w} > w$ (Corollary 1). This represents a divergence mechanism. On the other hand, the income-to-wealth ratio, $\frac{y_j}{W_j} = r + \frac{w}{\omega_j}$, is unambiguously decreasing in ω_j . This represents a convergence mechanism.

An overall measure for the change of the wealth distribution, summarizing the net effect of the divergence and the convergence mechanism, is given by $\frac{\partial G(\omega_j(t), \cdot)}{\partial \omega_j(t)}$. This measure is readily obtained by forming the derivative of $G(\omega_j, \cdot)$ with respect to ω_j , noting (9) and (10), to get

$$\frac{\partial G(\omega_j(t), \cdot)}{\partial \omega_j(t)} = \frac{\mu(t)\tilde{w}(t) - w(t)}{[\omega_j(t)]^2}. \quad (11)$$

Equation (11) shows that the wealth distribution is stationary in any period t provided that $\mu(t)\tilde{w}(t) = w(t)$ holds. Moreover, there is wealth divergence (convergence) in period t for $\mu(t)\tilde{w}(t) > (<) w(t)$.¹⁸

¹⁷In the following, rather than referring to a change within time interval $[t, t + dt]$ we will refer to a change at point in time t .

¹⁸The expression on the RHS of (11) exploits that aggregate quantities and prices affect the distribution of wealth, but the distribution of wealth (holding the mean constant) does not affect aggregate quantities and prices (Remark 1). We will return to the importance of (11) in Sections 4.1.2 and 5.3.

4.1.2 The Rent Channel

We are now ready to answer how rising real rents, holding $\{r, w\}$ constant, affect wealth inequality.

Proposition 5 (Rent channel) *An increase (decrease) in the growth of real rents between the current period t and some future period τ , measured by $\bar{p}(\tau, t)$, contributes to less (more) wealth inequality in the current period t , measured by $\frac{\partial G(\omega_j(t), \cdot)}{\partial \omega_j(t)}$, provided that $\sigma > 1$ ($\sigma < 1$).*

Let us focus on the empirically relevant case of a sufficiently concave utility function ($\sigma > 1$). Stronger rent growth, measured by an increase in $\bar{p}(\tau, t)$, induces less wealth inequality. The economic intuition behind this result can be explained in two steps. First, all households choose a higher saving rate (by reducing the propensity to consume) to provide for the future rent burden in order to smooth housing consumption over time. This results immediately from Propositions 3 and 4. Second, this increase in the saving rates is asymmetric across households. It is stronger for the wealth poor than for the wealth rich. To see this, notice that the saving rate, $sav_j = 1 - \mu \frac{w_j}{y_j}$, increases as μ is being reduced. This effect is amplified by the total-wealth-to-income ratio, provided that $r\tilde{w} > w$. This condition, again, implies that the wealth poor exhibit a comparably high total-wealth-to-income ratio, $\frac{w_j}{y_j}$, such that a reduction in the propensity to consume, $\frac{\partial \mu}{\partial \bar{p}(\tau, t)} < 0$, implies a comparably strong increase in the saving rate. As a result, the differences in the saving rates across wealth groups are being reduced and the divergence mechanism is weakened.

This dampening effect of rising rents on wealth inequality depends on the assumptions of forward-looking and optimizing households together with an empirically plausible intertemporal elasticity of substitution ($\frac{1}{\sigma} < 1$). It can, therefore, be expected to be robust across a large set of different models.¹⁹ The analysis clarifies that accounting for

¹⁹ As a caveat, notice that things may change under borrowing constraints. If the poor cannot finance going consumption expenditures by running into debt, their propensity to consume is lower compared to the unconstrained case. Stronger rent growth may then not induce these households to lower their propensity to consume and increase their saving rate.

Schwabe's Law of Rent does not matter in this context.²⁰ Moreover, Section 7 shows that the mechanisms discussed above do not depend on whether households are modelled as renters or homeowners. Any rise in rents is reflected, in equilibrium, by an equiproportional increase in the user cost of housing. That is, in an economy with homeowners, the rent channel is still operative, but shall be relabelled to the user-cost-of-housing channel.

4.2 Status Concerns, Price Indices, and Welfare

How does the status-induced heterogeneity of housing expenditure shares affect household-specific welfare in a growing economy? To discuss this question, we consider the welfare position of household j relative to the representative household. Specifically, measure $\psi_j(t)$ shows by how much household j is better off, in terms of consumption-equivalent variations, relative to the representative household.²¹ For the representative household $\psi_j(t) = 1$ by definition. If $\psi_j(t) > (<)1$, then household j is better (worse) off than the representative household.

Proposition 6 (Welfare) *The welfare of a household j relative to the representative household, at any time t , is given by*

$$\psi_j(t) = \frac{\mathcal{W}_j(t)}{\overline{\mathcal{W}}(t)} \left(\frac{\mathcal{P}_j(t)}{\overline{\mathcal{P}}(t)} \right)^{-1} - 1 = \frac{\Omega_j(t)}{\frac{\mathcal{P}_j(t)}{\overline{\mathcal{P}}(t)}} - 1. \quad (12)$$

Relative welfare of household j , measured by $\psi_j(t)$, depends positively on relative overall wealth, $\Omega_j(t) = \frac{\mathcal{W}_j(t)}{\overline{\mathcal{W}}(t)}$, and negatively on the relative household-specific price index, $\frac{\mathcal{P}_j(t)}{\overline{\mathcal{P}}(t)}$.²² The first term, $\Omega_j(t) = \frac{\mathcal{W}_j(t)}{\overline{\mathcal{W}}(t)}$, is standard. The second term, $\left(\frac{\mathcal{P}_j(t)}{\overline{\mathcal{P}}(t)} \right)^{-1}$, adds a new channel. The relative price index enters because of the two-sectoral structure together with non-homothetic preferences, as can be seen as follows. First, the ideal

²⁰This appears consistent with a two-stage logic. First, households maximize life time utility w.r.t. \mathcal{C}_j (intertemporal problem). Second, households maximize instantaneous utility w.r.t. c_j and s_j (intratemporal problem). Within the setup at hand the decisions on both stages are separable.

²¹For details see Appendix 9.3.

²²If the propensities to consume are not the same across agents, then welfare of agent j is equal to welfare of agent j' if we multiply, for all $\tau \geq t$, ideal consumption $\mathcal{C}_{j'}(\tau)$ by $\frac{\mu_j(t)}{\mu_{j'}(t)} \frac{\mathcal{W}_j(t)}{\mathcal{W}_{j'}(t)} \left(\frac{\mathcal{P}_j(t)}{\mathcal{P}_{j'}(t)} \right)^{-1}$. This expression points to an additional channel in models with, say, borrowing constraints that operates via differences in the propensities to consume.

price index is trivially equal to unity in a one-sectoral model. Second, in a two-sectoral model under homothetic preferences ($\phi = 0$) the ideal price index, $\mathcal{P} = \frac{p^\theta}{\theta^\theta(1-\theta)^{1-\theta}}$, is identical across households. It is the combination of a two-sectoral model structure and non-homothetic preferences that gives rise to household specific price indices as an independent source for welfare differences.

Consider the general case of a two-sectoral economy under non-homothetic preferences. By substituting (6) into (7), one gets

$$\mathcal{P}_j(t) = \frac{p(t)^\theta}{\theta^\theta(1-\theta)^{1-\theta}} \frac{1 - (1-\theta)\phi}{1 - (1-\theta)\phi - \frac{\theta\phi}{\Omega_j(0)}} \quad (13)$$

for all $t \geq 0$. Thus, for $\phi > 0$, the price index of household j is decreasing with relative total wealth in the initial period, $\Omega_j(0)$. That is, the household-specific price index is relatively large for wealth-poor households. As a consequence, the weak welfare position of a wealth-poor household ($\Omega_j < 1$) for a given price index (like in a one-sector economy) is being further worsened by a price index above average $\mathcal{P}_j(t) > \bar{\mathcal{P}}(t)$.

An important implication of Proposition 6 together with (13), noting that overall wealth Ω_j is time invariant, is given by

Corollary 2 (Amplification of welfare differences). *Stronger status concerns for housing amplify, at any t , the welfare differences, measured by $\psi_j(t)$, i.e.*

$$\frac{\partial \psi_j(t)}{\partial \phi} = \frac{\theta [\Omega_j(t) - 1]}{(\phi - 1)^2} \begin{cases} > 0 & \text{for } \Omega_j(t) > 1 \\ < 0 & \text{for } \Omega_j(t) < 1 \end{cases}. \quad (14)$$

Stronger status concerns for housing improve the relative welfare position $\psi_j(t)$ of wealth rich households, $\Omega_j(t) > 1$, and worsen the relative welfare position $\psi_j(t)$ of wealth poor households, $\Omega_j(t) < 1$.²³ As a result, stronger status concerns amplify the welfare differences across households. The intuition is simple. Stronger status concerns magnify the (endogenous) heterogeneity in housing expenditure share. Hence, the dispersion of household-specific price indices and the welfare distribution is getting more unequal.

²³Strictly speaking, Corollary 2 focuses on the first-order effect of a change in ϕ , neglecting possible feedback effects due to changes of Ω_j that may occur in general equilibrium.

Accounting for Schwabe’s Law of Rent has first order welfare implications.

5 General Equilibrium

So far, prices $\{r, w, p\}$ have been taken as given as given. To endogenize prices, we close the model by introducing the production sector and define aggregate wealth. The definition of the general equilibrium is relegated to Appendix 9.1.²⁴

5.1 Firms

We employ the two-sectoral macro model with a housing sector of Grossmann & Steger (2017) on the production. This model is designed to think long term and distinguishes between the extensive margin of the housing stock (number of houses) and the intensive margin of the housing stock (size of the average house). It is, therefore, well suited to investigate the macroeconomic implications of residential zoning regulations.

5.1.1 Numeraire Good Sector

The *non-residential sector* produces a final good, Y , chosen as numeraire, according to a Cobb-Douglas production function:

$$Y = (K^Y)^\alpha (B^Y L^Y)^\beta (B^Y Z^Y)^{1-\alpha-\beta}, \quad (15)$$

where K^Y , L^Y and Z^Y denote physical capital, labor and land devoted to the Y sector, respectively. The productivity parameter, $B^Y > 0$, may change over time. The technology parameters $\alpha, \beta > 0$ satisfy $\alpha + \beta < 1$. The capital resource constraint reads $K^Y \leq K$, where $K = \sum_j n_j K_j$ denotes the total supply of physical capital. In equilibrium $K^Y = K$ will turn out to hold. Capital depreciates at rate $\delta^K \geq 0$ such that gross physical capital investment reads $I^K \equiv \dot{K} + \delta^K K$. $K(0)$ is given.

²⁴The reduced-form, dynamic system is stated in Online-Appendix 10.1.

5.1.2 Housing Sector

There are three types of firms in the housing sector. *Real estate development firms* invest in infrastructure and transform non-residential land into developed real estates (residential land). Real estate development diminishes the amount of land that can be employed in the Y sector. Overall land supply, Z , is exogenous and assumed to be fixed. *Housing services firms* combine a developed real estate with residential buildings to produce housing services. *Construction firms* manufacture residential buildings by employing materials and labor.

Real Estate Development The amount of houses is denoted by N , a real number. This variable captures the extensive margin of the housing stock. Real estate development firms transform one unit of non-residential land into one unit of residential land. Total land usage in the housing sector is given by $N \leq \kappa Z$, where $0 < \kappa < 1$ denotes a policy parameter that may constrain the amount of residential land and Z describes the total amount of economically usable land. Let P^Z denote the price per unit of land. The costs $\mathfrak{C}(\tilde{I}^N, P^Z)$ of increasing the number of developed real estates by $\dot{N} = \tilde{I}^N$ amounts to

$$\mathfrak{C}(\tilde{I}^N, P^Z) = P^Z \tilde{I}^N + P^Z \frac{\xi}{2} (\tilde{I}^N)^2, \quad (16)$$

$\xi > 0$. The first cost component, $P^Z \tilde{I}^N$, shows the costs associated with the purchase of \dot{N} units of land. The second component, $P^Z \frac{\xi}{2} (\tilde{I}^N)^2$, captures the transformation costs that result from the private infrastructure investment (in terms of the numeraire). These adjustment costs are convex in the number of newly developed real estates per unit of time, $\dot{N} = \tilde{I}^N$. $N(0)$ is given.

Housing Services Producing housing services requires to purchase a developed real estate (the fixed input at the level of housing services firm) and combine it with structures (the variable input). The amount of housing services per house produced increases with the amount of residential structures employed per house. However, because a developed real estate serves as fixed input, it increases less than proportionate with the amount of residential structure. That is, the production of housing services per house is character-

ized by decreasing returns to scale. Formally, let x denote the amount of structures per housing project. An amount x produces housing services h per house according to

$$h = x^\gamma, \quad (17)$$

$0 < \gamma < 1$. Total consumption of housing services cannot exceed total supply, i.e. $\sum_j n_j s_j \leq Nh$. Denoting the rental rate of structures by R^X , profits (residual income) from supplying housing services are given by $\pi \equiv ph - R^X x$ per house. Thus, in equilibrium, $R^X = p\gamma x^{\gamma-1}$ and $\pi = (1 - \gamma)ph$.

Construction The construction firm produces structures, that are rented out to housing services producers, by combining labor, L^X , and construction materials, M . The production of one unit of materials requires one unit of the numeraire good. Gross investment in structures are produced according to $I^X = M^\eta (B^X L^X)^{1-\eta}$, $0 < \eta < 1$, where $B^X > 0$ is a (possibly time-variant) productivity parameter. The total stock of structure, X , evolves according to

$$\dot{X} = \underbrace{M^\eta (B^X L^X)^{1-\eta}}_{I^X} - \delta^X X, \quad (18)$$

where $\delta^X > 0$ denotes the depreciation rate of residential structure and $X(0)$ is given. The amount of residential buildings that is employed by all housing services firms must satisfy $Nx \leq X$. Construction labor is limited by $L^X \leq L - L^Y$.

5.2 Wealth

Aggregate (non-human) wealth is defined by $W \equiv \sum_j n_j W_j$, where W_j denotes asset holdings by household j . The portfolio of any household j comprises ownership claims on physical capital (K_j), the property of housing units (N_j), and non-residential land (Z_j^Y). Total asset holdings per household j therefore reads as

$$W_j \equiv \underbrace{P^H N_j}_{\text{housing wealth}} + \underbrace{\underbrace{K_j}_{\text{physical capital}} + \underbrace{P^Z Z_j^Y}_{\text{non-residential land wealth}}}_{\text{non-residential wealth}}, \quad (19)$$

where P^H denotes the price of houses and P^Z the price of non-residential land, respectively. The house price, $P^H \equiv q^N + q^X x$, is the sum of the value of a developed real estate (q^N) and the value of the employed structure ($q^X x$), where q^X denotes the value per unit of structure.

5.3 Steady State

Assume that TFP in the numeraire sector (B^Y) and TFP in the construction sector (B^X) evolve according to $B^Y(t) = B^Y(0) \exp(g^Y t)$ and $B^X(t) = B^X(0) \exp(g^X t)$ with possibly positive rates of exponential growth, $g^Y, g^X \geq 0$. Given that the model at hand is a two-sectoral Ramsey growth model, it is unsurprising that the steady state growth rates are linear transforms of $g^Y, g^X \geq 0$. The steady state growth rates are stated in Appendix 9.2. Here we focus on two price variables. The steady state growth rate of the rent is $\hat{p} = (1 - \gamma\eta) g^Y + \gamma(1 - \eta) g^X$, while the steady state growth rate of the house price reads $\hat{P}^H = g^Y$.²⁵

Given that the rent and the house price grow exponentially, one would like to know how the wealth distribution behaves in a steady state. The subsequent proposition clarifies this aspect.

Proposition 7 (Stationary wealth distribution) *(i) The condition $\mu(t)\tilde{w}(t) = w(t)$ is satisfied in any steady state. This implies that the wealth distribution is stationary in the sense that, for any two households j and j' , the relative wealth position $W_j/W_{j'}$ does not change over time; see (11). (ii) This applies for a steady state with growth rates equal to zero (due to $g^Y, g^X = 0$) as well as for a steady state with positive long run growth rates (due to $g^Y, g^X > 0$).*

As a result, a change in wealth inequality over time requires some form of transitional dynamics. Hence, the implementation of any policy measure (as the one analyzed in Section 6.2) that induces transitional dynamics triggers a change in wealth inequality over time.

²⁵The price-to-rent ratio, $\frac{P^H}{ph}$, is constant in a steady state.

Why does $\mu(t)\tilde{w}(t) = w(t)$ imply a stationary wealth distribution? This can be seen by rewriting the growth rate of wealth of household j as follows

$$\hat{W}_j = r - \mu + \frac{l_j(w - \mu\tilde{w})}{W_j}.$$

Provided that $\mu\tilde{w} = w$, consumption out of human wealth equals the going wage such that the third term is zero. As a result, the growth rate of wealth is the same across households and equal to $\hat{W}_j = r - \mu$. Viewed from another perspective, one can easily show that, in a steady state, $\hat{W}_j = Sav(\omega_j, \cdot) \left(r + \frac{w}{\omega_j} \right) = g^Y$ for all j . That is, the condition $\mu\tilde{w} = w$ ensures that the divergence mechanism, $\frac{\partial Sav(\omega_j, \cdot)}{\partial \omega_j} > 0$, compensates the convergence mechanism, $\frac{\partial \left(r + \frac{w}{\omega_j} \right)}{\partial \omega_j} < 0$.

6 Numerical Analysis

The consequences of abolishing residential zoning regulations on wealth inequality and welfare in general equilibrium are investigated by comparing two scenarios. In the baseline scenario (constrained steady state under zoning) the rent grows at a constant growth rate and the wealth distribution is stationary. In the counterfactual scenario (transitional dynamics in response to abolishment of zoning) the rent grows temporarily at a lower pace. We are interested in the differential effects on wealth inequality and welfare.

6.1 Calibration

We calibrate the model economy's steady state to the postwar US economy at an annual frequency. This implies a stationary wealth distribution, which is roughly in line with recent data on the wealth distribution (World Inequality Database, 2018; Kuhn, Schularick & Steins, 2018).

6.1.1 Externally Calibrated Parameters

Household sector Total population is normalized to unity, implying an aggregate labor supply $L = 1$. We calibrate the joint distribution of initial wealth and labor

productivity by matching wealth deciles and average earnings of the age group 33 – 55 from the 2013 SCF.²⁶ Similar to Kuhn & Rios-Rull (2016) and Krusell & Rios-Rull (1999), we focus on this age-group to calibrate a dynastic model by abstracting from life-cycle effects. We consider $J = 10$ wealth groups that correspond to the observed wealth deciles and the average earnings within each decile.²⁷ Moreover, Havránek (2015) shows that the majority of studies find an intertemporal elasticity of substitution (IES) below 0.8. We set $\sigma = 2$, implying an IES of 0.5. It is further assumed that every household holds the same portfolio as the representative agent.

Nominaire sector The total amount of land that can be used economically, Z , is normalized to one. The annual depreciation rate of capital, δ^K , is set to 5.6 percent (Davis & Heathcote, 2005). The concavity parameters of the production function, α and β , are set to match the expenditure shares for labor, $\beta = wL/Y$, and land, $1 - \alpha - \beta = R^Z Z^Y / Y$, respectively. Grossmann & Steger (2017) compute $\beta = 0.69$ and $1 - \alpha - \beta = 0.03$, implying $\alpha = 0.28$. GDP grows at the rate g^Y in the model economy and therefore g^Y is set equal to the average annual growth rate of real US GDP per capita of 2.0 percent between 1950 and 2017.²⁸

Housing sector The annual depreciation rate of structures, δ^X , is set to 1.5 percent (Hornstein, 2009, p. 13). The labor expenditure share in the construction sector, $1 - \eta$, amounts to 62 percent on average in the postwar US economy, implying $\eta = 0.38$ (Grossmann & Steger, 2017). The growth rate of TFP in the construction sector, g^X , is chosen such that we match the empirical growth differential between housing at the intensive margin (X) and the extensive margin (N) of 1.3 percent (Davis & Heathcote, 2007). The growth differential between X and N is given by $\eta g^Y + (1 - \eta)g^X$ in the model. Applying the values for η and g^Y yields $g^X = 0.009$.²⁹ We calibrate κ to match the observed allocation of land in the residential sector. According to geographic land-use

²⁶We are grateful to Moritz Kuhn for providing the data.

²⁷This implies also that each group is of the same size, $n_j = 1/J = 0.1$.

²⁸The data is obtained from FRED (<https://fred.stlouisfed.org/>), series *A93RX0Q048SBEA_P* (accessed on November 23, 2018).

²⁹The low value for g^X in comparison to g^Y is supported by evidence of low, sometimes even negative, productivity growth in the construction sector (Davis & Heathcote, 2005).

data provided by Falcone (2015), 30.2 percent of the total US surface is used economically and 16.9 percent of this land is used as residential land. This implies binding zoning regulations, measured by κ , according to $\kappa = 0.169$.³⁰

6.1.2 Internally Calibrated Parameters

The remaining set of parameters $\{\gamma, \rho, \theta, \phi, \xi\}$ is calibrated internally by solving the model numerically. The preference parameters ϕ and θ are set to match two key moments of the expenditure share distribution in the US in 2015, as displayed in Table 1: (i) An aggregate housing expenditure share of 19 percent and (ii) a difference between the expenditure shares of the first and fifth income quintiles of 7 percentage points. This results in $\phi = 0.104$ and $\theta = 0.174$.

quintile	aggregate	1st	2nd	3rd	4th	5th
US data	19	25	21	20	19	18
Model: baseline calibration	19	25	22	20	19	18
UK data (normalized)	19	33	23	19	16	15
Model: alternative calibration	19	34	26	23	20	16

Table 1: Housing expenditure shares by income quintiles in percent.

Notes: (a) Housing expenditure share is defined as the ratio of expenditures on housing services (including imputed rent) and total consumption expenditures. (b) First row "U.S. data" shows the empirical values for the US in 2015. Data source: www.bls.gov/cex/data.htm (accessed June 19, 2017). (c) Second row "Model: baseline calibrated" shows the model based expenditure shares such that $\bar{e} = 0.19$ and the difference between first and fifth income quintile according to U.S. data (7 percentage points) is matched. (d) Third row "Model: alternative calibrated" shows the model based expenditure shares such that $\bar{e} = 0.19$ and the difference between first and fifth income quintile of the (to $\bar{e} = 0.19$) normalized distribution according to UK data published by Office for National Statistics (2015) is matched.

To study sensitivity, we do also consider two alternative values for ϕ and θ . When changing ϕ , we adjust θ such that the aggregate housing expenditure share, \bar{e} , remains

³⁰Without zoning regulations the model would imply a steady state share, N/Z , equal to 62 percent. Land use regulations started to play a major role in the residential sector in the 1970s (Gyourko & Molly, 2015). Consistent with this observation, average annual growth of residential land was about 3 – 5 percent during 1945 to 1975 and is almost zero since then (Davis & Heathcote, 2007).

at 19 percent. For a given ϕ this implies, together with the FOC (24), that θ is obtained from

$$\theta = \frac{\bar{e}(1 - \phi)}{1 - \phi\bar{e}}. \quad (20)$$

First, we consider $\phi = 0$ (no status preferences), implying that housing expenditure shares are homogeneous. This leads to $\theta = 0.19$. Second, the case of strong status preferences is motivated by the case of UK, where the heterogeneity in housing expenditures is considerably higher than in the US.³¹ We normalize the UK data to the average US housing expenditure share as can be seen in Table 1. Matching the difference between housing expenditure shares at the first and fifth income quintiles of 18 percentage points yields $\phi = 0.26$ and $\theta = 0.148$.

In a two-sectoral model the steady state growth rate of consumption, as implied by the Euler equation, is not only a function of the interest rate, but also of the growth rate of the relative price. Hence, the time preference rate, ρ , has to be calibrated jointly with θ and γ . We match the average rate of return on wealth for the postwar US of 5.77 percent (Jordà, Knoll, Kuvshinov, Schularick & Taylor, 2018, Table 12). This yields $\rho = 0.019$.

We choose γ to match the share of residential land value in total housing value ($1 - \gamma$). Using time series of the aggregate residential land value and the total value of housing from Davis & Heathcote (2007) reveals that the share of residential land in total housing value has been increasing from 10 percent in 1950 to around 30 percent in 1975. Since then it has been fluctuating between 25 and 40 percent. We target an average value of one third, implying a value of γ equal to 0.78.

The parameter ξ captures the importance of adjustment costs associated with land reallocations between the housing and the numeraire sector. This parameter is difficult to calibrate, as it does not affect the steady state and has an impact only along the transition. We calibrate ξ such that the (average) speed of convergence of residential

³¹The aggregate expenditure share in the UK amounts to 17 percent, while the expenditure shares in income quintiles 1-5 read as {29, 20, 17, 14, 14} (Office for National Statistics, 2015). For Germany (2013) the aggregate expenditure share amounts to 27 percent, while the expenditure shares in income quintiles 1-5 read {37, 33, 29, 27, 24} (Statistisches Bundesamt, 2015). The aggregate housing expenditure share in France is 22 percent (averaged over 2011 to 2015), while the expenditure shares of income quintiles 1-5 read {26, 24, 24, 23, 18} (Accardo et al., 2017).

land, N , computed in Section 6.2, is identical to the speed of convergence implied by the empirical data for the period 1945 to 1975. This yields $\xi = 745$.³²

6.1.3 Model Fit: Rent Growth and Saving Rates

The calibration strategy does not target the rent growth rate and the average saving rate. Nonetheless, the calibrated model matches these two variables quite accurately. The implied annual growth rate of rents equals $\hat{p}^* = 0.99$ percent. This value lies within the range of empirical values observed in the postwar US of 0.8 – 1.5 percent (Knoll, 2017; Albouy, Ehrlich & Liu, 2016). The saving rate of the representative consumer (equal to the aggregate saving rate) $\overline{sav} \equiv \dot{\bar{W}}/\bar{y}$ with income $\bar{y} \equiv r\bar{W} + w\bar{l}$ equals 11.8 percent. This value is in line with the US aggregate saving rate of 9 percent on average from 1950 to 2010 (Piketty & Zucman, 2014, Table A86). The saving rates of the 1st to 5th income quintiles are 0.9, 1.8, 4.7, 8.5 and 17.1 percent. These values are in the range of the estimated saving rates by Dynan, Skinner & Zeldes (2004).³³

6.2 Abolishing Zoning Regulations

Residential zoning regulations are widely considered as an important amplifier for surging housing costs in a growing economy (Glaeser, Gyourko, & Saks, 2005; Saiz, 2010). For instance, Albouy & Ehrlich (2018) find that, based on data for 230 metropolitan areas in the US from 2005 to 2010, observed land-use restrictions do substantially increase housing costs. Moreover, Gyourko and Molly (2015) argue that zoning regulations were effectively introduced in the US during the 1970s. This is consistent with the data provided by Davis & Heathcote (2007) showing that residential land grew by an average annual growth rate of 5 percent during 1945-1975 and grew merely by an average annual growth rate of 0.7 percent during 1976-2016.

³²We assume that the long-run dynamics in N came to a halt by the introduction of zoning regulations in the 1970s (Gyourko & Molly, 2015). We then combine the steady state from the model with the observed data between 1945 to 1974 (Davis & Heathcote, 2007) to determine the average speed of convergence. This shows that after 30 years about 31 percent of the gap between the initial N and the steady state has been closed. Hence, we set ξ such that residential land, N , has closed 31 percent of the gap between start value and steady state after 30 years of the transition in the experiment of Section 6.2.

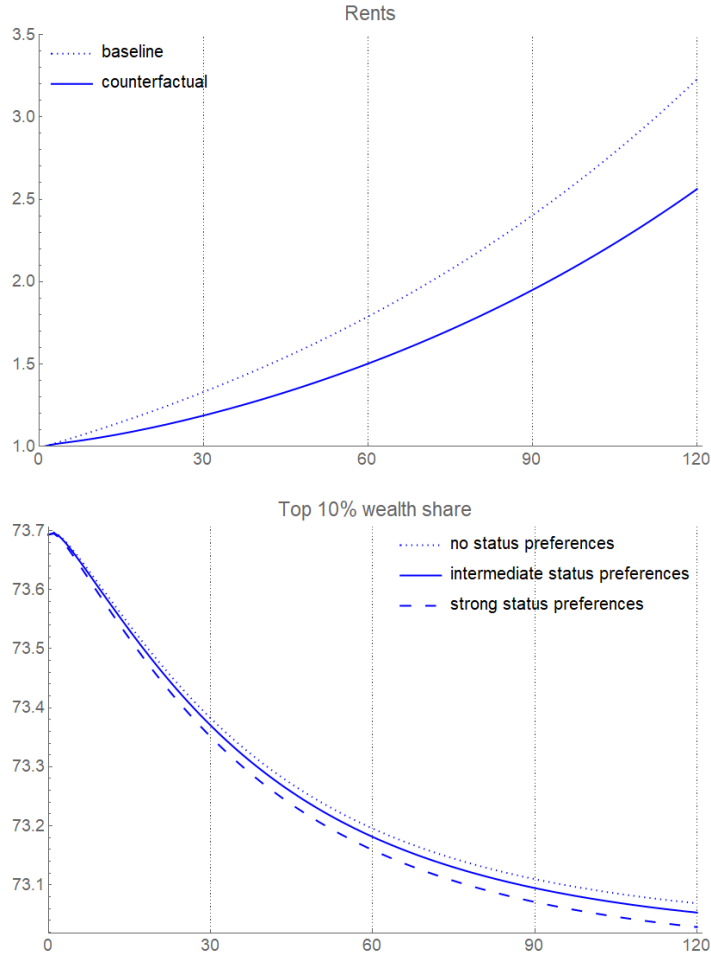
³³Moreover, also the implied wealth-to-NDP ratio and the implied house-price-to-rent ratio are empirically highly plausible.

To address the two research questions set up in the introduction, we compare two scenarios. In the baseline scenario (zoning) the economy is in a steady state, conditional on the binding zoning regulation $N = \kappa Z$ with $\kappa = 0.16$. In the counterfactual scenario (no zoning), residential zoning regulations are abolished completely. That is, we set $\kappa = 1$ such that the zoning constraint $N \leq \kappa Z$ is not binding anymore. The counterfactual scenario exhibits transitional dynamics, starting from the constrained steady state of the baseline scenario and converging to an unconstrained steady state. The analysis captures all general equilibrium effects. That is, all prices $\{w, r, p\}$ are fully endogenous and change in response to an exogenous policy trigger.³⁴

Rents and wealth inequality (RQ #1) Figure 1 (a) displays the time path of rents in the baseline scenario (zoning) and in the counterfactual scenario (no zoning). It can be seen that rents grow temporarily at a slower pace in the counterfactual scenario (solid curve) compared to the baseline scenario (dashed curve). This is quite intuitive as the economy extends the supply of housing along the extensive margin in response to the abolishment of zoning. (Notice, however, that this is a temporary effect. The steady state growth rate of rents is unaffected.) Figure 1 (b) shows that wealth inequality (measured by the top 10 percent wealth share) declines by about 0.7 percentage points (from 73.7 percent to 73 percent) over time. That is, rent growth and wealth inequality are positively associated in this general equilibrium experiment (slower rent growth goes hand in hand with declining wealth inequality). This observation is obviously in contrast to the rent channel in partial equilibrium (Section 4.1.2). This seemingly contradiction can be easily explained. The policy experiment under study gives rise to a set of price changes that accompanies a process of sectoral change. The abolishment of zoning regulations triggers a substitution of structures, X , by residential land, N , in the aggregate production of housing services, $S = X^\gamma N^{1-\gamma}$. That is, residential development is taking place at the cost of less construction activity. This process goes hand in hand with a reallocation of input factors. Labor is being reallocated instantaneously from the construction sector to

³⁴Favilukis, Mabillez & Van Nieuwerburg (2018) consider a similar policy experiment in a model that is calibrated to New York. They model zoning regulations to exert an effect on labor productivity in the construction sector. In our model, relaxing zoning regulation is captured by an increase in κ , which constrains the amount of land allocated to the housing sector.

the final output sector.³⁵ In addition, land is being reallocated over time from the final output sector to the housing sector. As a consequence, the wage drops on impact (due to the reallocation of labor) and further declines along the transition due to the decline of complementary factors (Z^Y and K). Lower future wages exert a convergence force as poorer households increase their saving rate by relatively more in order to smooth consumption over time.³⁶ Hence, the divergence mechanism described in Section 4.1.1 (higher saving rates for the rich) is weakened. This effect dominates the reinforcement of the divergence mechanism due to lower rent growth, as described in Section 4.1.2. See Online-Appendix 10.7.4 for details.



³⁵Recall that final output is an input in real estate development or, equivalently, that real estate development uses the same technology as the final output sector.

³⁶The initial drop in the wage exerts a divergence mechanism. Households smooth consumption by saving less today. This effect is stronger for the poorer households such that saving rates of poorer households decrease by relatively more. (Rich households do not care about the wage drop because the main income source is capital income.) This effect is quantitatively weak because the initial drop in wages is small.

Figure 1: Rent and wealth dynamics in response to abolishment of zoning regulation.

Notes. Panel (a): Evolution of the housing rent in the baseline scenario (zoning) and the counterfactual scenario (no zoning), holding the housing expenditure share of the representative consumer constant (at $\bar{e} = 0.19$) and recalibrating θ according to (20). Panel (b): Evolution of the top 10 percent wealth share in response to the abolishment of the zoning regulation for $\phi = 0$ (no status concerns), $\phi = 0.104$ (intermediate status concerns), $\phi = 0.26$ (strong status concerns), holding the housing expenditure share of the representative consumer constant (at $\bar{e} = 0.19$) and recalibrating θ according to (20). Calibration otherwise as described in Section 6.1.

Welfare (RQ #2) Figure 2 displays the welfare results for the policy experiment under consideration. The analysis distinguishes, first, between the partial equilibrium effects and the general equilibrium effects and, second, between the case of no status preferences ($\phi = 0$) and an intermediate level of status preferences ($\phi = 0.104$).

Consider at first an intermediate level of status preferences ($\phi = 0.104$). Figure 2 shows the differential welfare effect between the counterfactual and the baseline scenario, measured by consumption equivalent variations, for any household j and denoted as $\tilde{\psi}_j$. Welfare of the household with average wealth ($\frac{W_j}{\bar{W}} = 1$) increases by almost 0.5 percent in general equilibrium (see the lowest curve marked by squares). However, the welfare effects are clearly asymmetric. The poor benefit by more than the rich. The slower rent growth in the counterfactual scenario relative to the baseline scenario affects welfare of the poor stronger because they devote a higher expenditure share on housing. Hence, the poor benefit the most from a slower rent growth as they experience the strongest favorable price index effect (Proposition 2: $\frac{\partial \tilde{P}(p,e)}{\partial e \partial p} > 0$). Notice that the differential welfare effect is even negative for the richest decile, as these households are strongly affected by a negative wealth effect due to dropping house and land prices. The partial welfare effect is displayed by the highest curve (marked by triangles), assuming that the rent grows slower in the counterfactual scenario (as shown in Figure 1) but the wage and the interest rate grow according to the baseline scenario. That is, a slower rent growth, everything else the same, produces an average welfare gain of about 1.8 percent.³⁷ (The peak at $\frac{W_j}{\bar{W}} = 0$ is due to the non-monotonicity in earnings implying that these households possess the

³⁷In the counterfactual scenario the wage grows at a slower pace and the interest rate is higher along the transition. Both effects exert a negative effect on overall wealth.

lowest overall wealth.)

The picture changes if we set $\phi = 0$ (no status preferences). The partial welfare effect is now symmetric across wealth groups (horizontal line marked by circles). Moreover, the differential welfare effect in general equilibrium is stronger under no status preferences (as indicated by the curve marked by diamonds). Abolishing the zoning regulation has a stronger positive welfare effect under no status preferences compared to the case of status preferences. In the latter case there is overconsumption of housing services, due to the negative externality associated with housing, such that a zoning constraint does, in principle, address an inefficiency as a second best instrument (Schünemann & Trimborn, 2017).³⁸

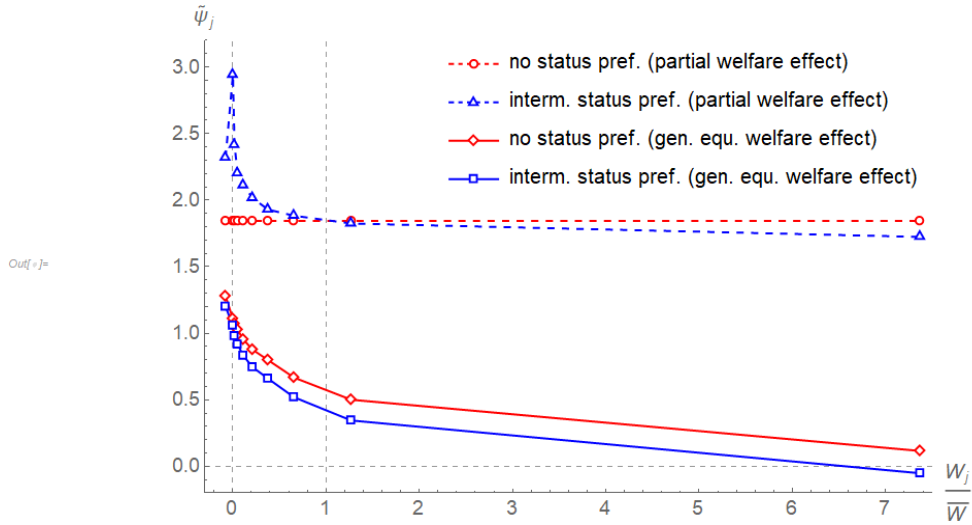


Figure 2: Percentage change of household-specific welfare, in response to the abolishment of the zoning regulation, as a function of relative (non-human) wealth, $\frac{W_j}{\bar{W}}$, for $\phi = 0$ (no status concerns) and for $\phi = 0.104$ (intermediate status concerns).

Note: Welfare measure $\tilde{\psi}_j$ is implicitly defined by $U_j^{BS}[t, (1 + \tilde{\psi}_j)C_j^{BS}(\tau)] = U_j^{CF}[t, C_j^{CF}(\tau)]$, where $U_j^{BS}(t, \cdot)$ denotes life time utility of household j at t in the baseline scenario, $U_j^{CF}(t, \cdot)$ life time utility of household j at t in the counterfactual scenario, $C_j^{BS}(\tau)$ the time path of ideal consumption for $\tau \in [t, \infty]$ of household j in the baseline scenario, and $C_j^{CF}(\tau)$ the time path of ideal consumption for $\tau \in [t, \infty]$ of household j in the counterfactual scenario, respectively. Calibration as described in Section 6.1.

³⁸The second-best optimal zoning constraint, assuming $\phi = 0.104$, amounts to $\kappa = 0.49$.

7 Renters vs. Homeowners

Two thirds of US households are homeowners while only one third are renters.³⁹ Does this mean that the mechanisms discussed above apply merely to one third of the US population, the renters? We show that the model can equivalently be interpreted as an economy of homeowners and that all results still hold true, independently of whether we call households renters or homeowners.

Assume that all housing is owner-occupied such that $s_j = N_j h$, where N_j is the amount of housing owned by group j and h is the flow of housing services derived from one unit of housing.⁴⁰ Instead of choosing the flow of housing services (s_j) when being a renter, a homeowner chooses the stock of housing (N_j) that she owns. The household problem for homeowners then modifies to read

$$\begin{aligned} & \max_{\{c_j(t), N_j(t)\}_{t=0}^{\infty}} \int_0^{\infty} u(c_j(t), N_j(t)h(t)) e^{-\rho t} dt \\ \text{s.t. } \dot{W}_j(t) &= r(t)A_j(t) - ucost(t)N_j(t) + w(t)l_j - c_j(t) \\ A_j(t) &= W_j(t) - P^H(t)N_j(t), \end{aligned}$$

where $ucost \equiv rP^H + \delta^X q^X x + q^X \dot{x} - \dot{P}^H$ denotes the user cost of housing. It consists of the sum of foregone interest payments (rP^H) and expenditures for maintenance ($\delta^X q^X x + q^X \dot{x}$) net appreciation gains (\dot{P}^H). The FOCs of this problem are identical to the FOCs of the problem for renters, described in Section 2.⁴¹ To transform the renter's FOC into the homeowner's FOC, one merely has to replace s_j with $N_j h$ and p with $\frac{ucost}{h}$. The latter results from perfect arbitrage on capital markets, implying that $ph = ucost$.⁴² Since homeowners' and renters' FOCs are identical, all results presented above hold true if one models households as homeowners instead of renters. The difference is merely in the interpretation. Instead of rent it is the user-cost of housing that affects the distribution

³⁹US Bureau of the Census, Homeownership Rate for the United States, retrieved from FRED, Federal Reserve Bank of St. Louis (<https://fred.stlouisfed.org/series/RHORUSQ156N>), series RHORUSQ156N (accessed December 21, 2018).

⁴⁰The assumption that all housing is owner-occupied is common in the macro-housing literature (Iacoviello, 2005).

⁴¹The proof of this statement is available upon request.

⁴²This is implied by the definition of the house price, $P^H \equiv q^N + q^X x$, together with the capital market no arbitrage conditions (cf. Appendix 9.1, Definition of General Equilibrium, Condition 5).

of wealth and welfare. For example, the result that a rising rent reduce wealth inequality in partial equilibrium (Proposition 5) is equivalent to stating that a rising user cost of housing reduce wealth inequality.

Under financial frictions additional mechanisms start playing a role. For instance, the rent and the user cost per unit of housing services may diverge implying that renters pay a higher price for housing services. Similarly, if houses pay a rate of return that differs from the rate of return paid by other assets, the portfolio structure plays a role for the wealth effects of surging house prices (Kuhn, Schularick & Steins, 2018).

8 Summary

We employ a frictionless two-sectoral macroeconomic model with a housing sector to investigate the dynamics of wealth inequality and the determinants of welfare. Households have non-homothetic preferences, implying that the poor choose a higher housing expenditure share, being compatible with Schwabe’s Law of Rent. The model is employed to analyze the general equilibrium consequences of a widely discussed policy option that aims at dampening the growth of housing costs. Abolishing zoning regulations triggers a slower rent growth and reduces wealth inequality by about 0.7 percentage points (measured by top 10 percent share). Average welfare increases by about 0.5 percent. However, the household-specific welfare effects are clearly asymmetric. The poor benefit more than the rich. The richest wealth decile is even worse off.

9 Appendix

9.1 Definition of General Equilibrium

Definition A.1. *A general equilibrium is a sequence of quantities, a sequence of prices, and a sequence of operating profits of housing services producers*

$$\begin{aligned} & \{Y(t), K(t), X(t), N(t), x(t), h(t), M(t), L^Y(t), L^X(t), Z^Y(t)\}_{t=0}^{\infty}, \\ & \{\{c_j(t), s_j(t), W_j(t), K_j(t), Z_j^Y(t), N_j(t)\}_{j=1}^J\}_{t=0}^{\infty} \\ & \{p(t), P^Z(t), q^N(t), q^X(t), w(t), r(t), R^Z(t), R^X(t)\}_{t=0}^{\infty}, \quad \{\pi_t\}_{t=0}^{\infty} \end{aligned}$$

for initial distributions $\{K_j(0), Z_j^Y(0), N_j(0)\}_{j=1}^J$ and given time paths $\{B^X(t), B^Y(t)\}_{t=0}^{\infty}$ such that

1. individuals maximize lifetime utilities; and for all t
2. the representative firms in X sector and Y sector, the representative real estate developer, and the representative housing services firm maximize the PDV of their respective infinite profit stream, taking prices as given;
3. labor markets clear: $L^X(t) + L^Y(t) = L$ with $L = \sum_j n_j l_j$;
4. all asset markets clear: $K(t) = \sum_j n_j K_j(t)$, $N(t) = \sum_j n_j N_j(t)$, $Z^Y(t) = \sum_j n_j Z_j^Y(t) = Z - N(t)$;
5. perfect arbitrage across all assets classes holds: $\frac{\dot{q}^N}{q^N} + \frac{\pi}{q^N} = \frac{\dot{q}^X}{q^X} + \frac{R^X}{q^X} - \delta^X = \frac{\dot{P}^Z}{P^Z} + \frac{R^Z}{P^Z} = r$;
6. the market for housing services clears: $\sum_j n_j s_j(t) = N(t)h(t)$;
7. the market for the numeraire good clears: $Y(t) = C(t) + I^K(t) + I^N(t) + M(t)$;⁴³

It is assumed that appropriate terminal conditions on asset prices hold (i.e. no asset price bubbles). In equilibrium, $q^N(t) = \int_t^{\infty} \pi(\tau) e^{\int_t^{\tau} -r(v)dv} d\tau$, $q^X(t) = \int_t^{\infty} R^X(\tau) e^{\int_t^{\tau} -(r(v)+\delta^X)dv} d\tau$, and $P^Z = \int_t^{\infty} R^Z(\tau) e^{\int_t^{\tau} -(r(v)+\delta^X)dv} d\tau$.

⁴³The goods market clearing condition is redundant, according to Walras' law. To exclude conceptual or calculation errors, we analytically checked that the long run equilibrium derived from conditions 1-6 fulfills condition 7.

9.2 Steady State

Proposition A.1. *Assume that TFP in the numeraire sector (B^Y) and TFP in the construction sector (B^X) grow at constant exponential growth rates, i.e. $B^Y(t) = B^Y(0)e^{g^Y \cdot t}$ and $B^X(t) = B^X(0)e^{g^X \cdot t}$ with $g^Y, g^X \geq 0$. The unique steady state growth rates then read as follows.*

- (i) Variables $\{K, W, C, M, q^N, P^Z, R^Z, P^H, w\}$ grow at the rate g^Y
- (ii) Variables $\{X, x\}$ grow at the rate $\eta g^Y + (1 - \eta) g^X$
- (iii) Variables $\{\hat{p}\}$ grow at the rate $(1 - \gamma\eta) g^Y + \gamma(1 - \eta) g^X$
- (iv) Variables $\{q^X, R^X\}$ grow at the rate $(1 - \eta)(g^Y - g^X)$
- (v) Variables $\{h, S\}$ grow at the rate $\gamma(\eta g^Y + (1 - \eta) g^X)$
- (vi) Variables $\{N, Z^Y, L^X, L^Y, r\}$ remain constant.

The proof can be found in Section 9.3. Notice also that Proposition A.1 implies that the steady state growth rate of $GDP = Y + pNh + wL^X$ equals g^Y .⁴⁴

9.3 Proofs

Proof of Proposition 1 (Housing expenditure shares). The dynamic optimization problem of any agent j reads as follows

$$\max_{\{c_j(t), s_j(t)\}_{t=0}^{\infty}} \int_0^{\infty} \frac{[(c_j(t))^{1-\theta}(s_j(t) - \phi\bar{s}(t))^{\theta}]^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt \quad \text{s.t.} \quad (21)$$

$$\dot{W}_j(t) = r(t)W_j(t) + w(t)l_j - c_j(t) - p(t)s_j(t), \quad (22)$$

$\lim_{t \rightarrow \infty} W_j(t)e^{-\int_0^t r(v)dv} \geq 0$, $W_j(0)$ given. The associated current-value Hamiltonian reads as

$$\mathcal{H}_j = \frac{[(c_j)^{1-\theta}(s_j - \phi\bar{s})^{\theta}]^{1-\sigma} - 1}{1 - \sigma} + \lambda_j[rW_j + wl_j - c_j - ps_j]. \quad (23)$$

⁴⁴The steady state growth rate of each component (Y, pNh, wL^X) equals g^Y .

The first-order optimality conditions can be written as

$$\frac{\theta}{1-\theta} \frac{c_j}{s_j - \phi \bar{s}} = p, \text{ i.e., } c_j = p \frac{1-\theta}{\theta} (s_j - \phi \bar{s}), \quad (24)$$

$$(1-\theta)(c_j)^{-\theta-\sigma(1-\theta)}(s_j - \phi \bar{s})^{\theta(1-\sigma)} = \lambda_j, \quad (25)$$

$$-\frac{\dot{\lambda}_j}{\lambda_j} = r - \rho \quad (26)$$

$$\lim_{t \rightarrow \infty} W_j(t) \lambda_j(t) e^{-\rho t} = 0. \quad (27)$$

(24) confirms that expansion paths are co-linear. Summing over all j , (24) implies that average consumption levels, \bar{c} and \bar{s} , are related according to

$$\bar{c} = \frac{(1-\phi)(1-\theta)}{\theta} p \bar{s}. \quad (28)$$

Eq. (28) describes, for constant p , the expansion path of the representative consumer, who owns the average wealth and articulates the average demand (e.g. Mas-Colell, Whinston and Green 1995, Chapter 4). Using (24) in (25) we also obtain

$$\lambda_j = (1-\theta)^{1+(\sigma-1)\theta} \theta^{(1-\sigma)\theta} (c_j)^{-\sigma} p^{(\sigma-1)\theta}, \text{ i.e. } -\frac{\dot{\lambda}_j}{\lambda_j} = \sigma \frac{\dot{c}_j}{c_j} + (1-\sigma)\theta \frac{\dot{p}}{p}. \quad (29)$$

Combining (26) and (29) we have

$$\frac{\dot{c}_j}{c_j} = \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}}{p} \equiv g_c. \quad (30)$$

Denoting average housing expenditure by $E \equiv p \bar{s}$ and its growth rate by g_E , we obtain

$$g_E = g_c = \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}}{p}, \quad (31)$$

according to (28) and (30).

Now define $\mathfrak{s}_j \equiv s_j/\bar{s}$ (consumption of housing services of agent j relative to the

average) and use (24) to write

$$e_j = \frac{ps_j}{c_j + ps_j} = \frac{1}{\frac{c_j}{ps_j} + 1} = \frac{1}{\frac{1-\theta}{\theta} \frac{s_j - \phi \bar{s}}{s_j} + 1} = \frac{\theta}{1 - \frac{(1-\theta)\phi}{\mathfrak{s}_j}}. \quad (32)$$

According to (24), $E = p\bar{s}$ and $\mathfrak{s}_j = s_j/\bar{s}$, we also have

$$c_j = \frac{1-\theta}{\theta}(\mathfrak{s}_j - \phi)E, \text{ i.e.} \quad (33)$$

$$\log c_j = \log \left[\frac{1-\theta}{\theta} E \cdot (\mathfrak{s}_j - \phi) \right] = \log \frac{1-\theta}{\theta} + \log E + \log(\mathfrak{s}_j - \phi). \quad (34)$$

Taking the derivative with respect to time τ , we obtain

$$g_c = g_E + \frac{d \log(\mathfrak{s}_j - \phi)}{d\tau}, \text{ i.e., } \frac{d \log(\mathfrak{s}_j - \phi)}{d\tau} = 0, \quad (35)$$

according to (31). Thus, \mathfrak{s}_j is time-invariant.

Next, define $\mathcal{C}_j \equiv (c_j)^{1-\theta}(s_j - \phi \bar{s})^\theta$ (inner instantaneous utility) and using (24) to obtain

$$\mathcal{C}_j = \theta^{\theta-1}(1-\theta)^{1-\theta} p^{1-\theta} \bar{s}(\mathfrak{s}_j - \phi). \quad (36)$$

Using $E = p\bar{s}$, we can rewrite (36) as

$$\mathcal{C}_j = \theta^{\theta-1}(1-\theta)^{1-\theta} E p^{-\theta}(\mathfrak{s}_j - \phi). \quad (37)$$

Taking logs on both sides of (37) and the derivative with respect to time τ , we obtain

$$\frac{\dot{\mathcal{C}}_j}{\mathcal{C}_j} = g_E - \theta \frac{\dot{p}}{p} = \frac{r - \rho}{\sigma} - \frac{\theta}{\sigma} \frac{\dot{p}}{p} \equiv g_c, \quad (38)$$

where we used (30). Also define consumption expenditure of agent j as $\mathcal{E}_j \equiv c_j + ps_j$ and $\mathcal{P}_j \equiv \mathcal{E}_j/\mathcal{C}_j$. According to (3), we can then write (with period index τ)

$$\dot{W}_j(\tau) = r(\tau)W_j(\tau) + w(\tau)l_j - \mathcal{P}_j(\tau)\mathcal{C}_j(\tau). \quad (39)$$

Multiplying both sides of (39) by $e^{-\int_t^\tau r(v)dv}$ and integrating from period t forward yields

$$\int_t^\infty \dot{W}_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau = \int_t^\infty r(\tau) W_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau + \int_t^\infty [w(\tau) l_j - \mathcal{P}_j(\tau) \mathcal{C}_j(\tau)] e^{-\int_t^\tau r(v)dv} d\tau, \quad (40)$$

Integrating by parts implies that

$$\begin{aligned} \int_t^\infty \dot{W}_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau &= \lim_{T \rightarrow \infty} \left[W_j(\tau) e^{-\int_t^\tau r(v)dv} \right]_t^T + \int_t^\infty r(\tau) W_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau \\ &= -W_j(t) + \int_t^\infty r(\tau) W_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau, \end{aligned} \quad (41)$$

where the latter equation uses the transversality condition $\lim_{T \rightarrow \infty} W_j(\tau) e^{-\int_t^T r(v)dv} d\tau = 0$. Using (41), $\tilde{w}(t) = \int_t^\infty w(\tau) e^{-\int_t^\tau r(v)dv} d\tau$ and $\mathcal{W}_j(t) = W_j(t) + \tilde{w}(t) l_j$ as defined in (5) and (4), respectively, in (40) implies

$$\int_t^\infty \mathcal{P}_j(\tau) \mathcal{C}_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau = \mathcal{W}_j(t). \quad (42)$$

The solution of differential equation (38) is

$$\mathcal{C}_j(\tau) = \mathcal{C}_j(t) e^{\frac{1}{\sigma} \int_t^\tau [r(v) - \rho - \theta \frac{\dot{p}(v)}{p(v)}] dv}. \quad (43)$$

Substituting (43) into (42) and multiplying both sides with $\mathcal{P}_j(t)$ gives us

$$[\mathcal{E}_j(t) =] \mathcal{P}_j(t) \mathcal{C}_j(t) = \frac{\mathcal{W}_j(t)}{\int_t^\infty \frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)} e^{\frac{1}{\sigma} \int_t^\tau [(1-\sigma)r(v) - \rho - \theta \frac{\dot{p}(v)}{p(v)}] dv} d\tau}. \quad (44)$$

Next, using $\mathfrak{s}_j = s_j/\bar{s}$ in (24), group-specific consumption expenditures, $\mathcal{E}_j = c_j + p s_j$, read as

$$\mathcal{E}_j = \frac{p\bar{s}}{\theta} [\mathfrak{s}_j - (1 - \theta)\phi]. \quad (45)$$

Using (36) and (45) in $\mathcal{P}_j = \mathcal{E}_j/\mathcal{C}_j$ implies

$$\mathcal{P}_j = \frac{p^\theta}{\theta^\theta(1-\theta)^{1-\theta}} \frac{\mathfrak{s}_j - (1-\theta)\phi}{\mathfrak{s}_j - \phi}. \quad (46)$$

According to (46) and the fact that \mathfrak{s}_j is time-invariant, we have

$$\frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)} = \left(\frac{p(\tau)}{p(t)} \right)^\theta. \quad (47)$$

Using (47) in (44) we find that

$$\mathcal{E}_j(t) = \frac{\mathcal{W}_j(t)}{\int_t^\infty \left(\frac{p(\tau)}{p(t)} \right)^\theta e^{-\frac{1}{\sigma} \int_t^\tau [\rho + \theta \frac{\dot{p}(v)}{p(v)} + (\sigma-1)r(v)] dv} d\tau}. \quad (48)$$

According to (45), for two different agents, j and k , we have

$$\frac{\mathcal{E}_j}{\mathcal{E}_k} \left[= \frac{\mathcal{P}_j \mathcal{C}_j}{\mathcal{P}_k \mathcal{C}_k} \right] = \frac{\mathfrak{s}_j - \phi(1-\theta)}{\mathfrak{s}_k - \phi(1-\theta)}, \quad (49)$$

Using (48) and recalling that a household k with $\mathfrak{s}_k = 1$ has average total wealth $\bar{\mathcal{W}}(t)$, (49) can be written as

$$\frac{\mathfrak{s}_j - (1-\theta)\phi}{1 - (1-\theta)\phi} = \frac{\mathcal{W}_j(t)}{\bar{\mathcal{W}}(t)} \equiv \Omega_j(t). \quad (50)$$

Since \mathfrak{s}_j is time-invariant, according to (35), (50) implies that total wealth must grow at the same rate for all j . Using this fact in (50) and solving for \mathfrak{s}_j implies

$$\mathfrak{s}_j = (1-\theta)\phi + [1 - (1-\theta)\phi] \Omega_j(0). \quad (51)$$

Substituting (51) into (32) confirms (6). This concludes the proof. ■

Proof of Remark 1 (Representative Household). The equations of interest are (22), (24) and (30). Summing the left and right hand sides of these equation over groups

j and dividing by J yields:

$$\frac{C}{1-\theta} = \frac{p(1-\phi)}{\theta} S \quad (52)$$

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma} - \frac{\theta(1-\sigma)}{\sigma} \frac{\dot{p}}{p} \quad (53)$$

$$\dot{W} = rW + wL - C - pS \quad (54)$$

These FOC are identical to the FOC that result from the problem of a single household who owns the entire endowments ($\sum_j l_j$ and $\sum_j W_j$) and makes the aggregate consumption and saving decisions by taking the reference level of housing consumption, $\bar{s}(t)$, as exogenous. ■

Proof of Proposition 2 (Ideal price indices). We can rearrange (32) to obtain

$$\mathfrak{s}_j = \frac{(1-\theta)\phi}{1 - \frac{\theta}{e_j}} \quad (55)$$

Substituting (55) into (46) confirms (7). ■

Proof of Proposition 3 (Propensity to consume). According to (48) and definition $\mu_j = \mathcal{E}_j/\mathcal{W}_j$, we find

$$\mu_j(t) = \frac{1}{\int_t^\infty \left(\frac{p(\tau)}{p(t)} \right)^\theta e^{-\frac{1}{\sigma} \int_t^\tau [\rho + \theta \frac{\dot{p}(v)}{p(v)} + (\sigma-1)r(v)] dv} d\tau}. \quad (56)$$

Using $\bar{r}(\tau, t) = \int_t^\tau r(v) dv$ and rearranging terms in (56) by using

$$\exp \left(-\frac{\theta}{\sigma} \int_t^\tau \left[\frac{\dot{p}(v)}{p(v)} \right] dv \right) = \left(\frac{p(\tau)}{p(t)} \right)^{-\frac{\theta}{\sigma}} = \bar{p}(\tau, t)^{-\frac{\theta}{\sigma}} \quad (57)$$

confirms (8). ■

Proof of Proposition 4 (Saving rates). Plug $\mathcal{E}_j = \mu_j \mathcal{W}_j$, $\mathcal{W}_j = W_j + \tilde{w}l_j$, $y_j = rW_j + wl_j$, and $\omega_j = W_j/l_j$ into the definition of the saving rate, given by $sav_j = 1 - \frac{\mathcal{E}_j}{y_j}$. ■

Proof of Proposition 5 (Rent channel). Forming the derivative of the propensity to consume, noting (8), with respect to $p(\tau)$ yields

$$\frac{\partial \mu(t)}{\partial p(\tau)} = - \frac{\int_t^\infty \frac{\frac{\sigma-1}{\sigma} \theta \left(\frac{p(\tau)}{p(t)} \right)^{\theta-1} \left(\frac{\left(\frac{p(\tau)}{p(t)} \right)^\theta}{\exp \left[\bar{r}(\tau, t) + \frac{\rho}{\sigma-1} (\tau-t) \right]} \right)^{-\frac{1}{\sigma}}}{\exp \left[\bar{r}(\tau, t) + \frac{\rho}{\sigma-1} (\tau-t) \right] p(t)} d\tau}{\mu(t)^2}.$$

Noting that $\theta \left(\frac{p(\tau)}{p(t)} \right)^{\theta-1}$, $\left(\frac{\left(\frac{p(\tau)}{p(t)} \right)^\theta}{\exp \left[\bar{r}(\tau, t) + \frac{\rho}{\sigma-1} (\tau-t) \right]} \right)^{-\frac{1}{\sigma}}$, $\exp \left[\bar{r}(\tau, t) + \frac{\rho}{\sigma-1} (\tau-t) \right] p(t)$, and $\mu(t)$ are positive, one obtains

$$\frac{\partial \mu(t)}{\partial p(\tau)} \begin{cases} < 0 & \text{for } \sigma > 1 \\ = 0 & \text{for } \sigma = 1 \\ > 0 & \text{for } \sigma < 1 \end{cases}.$$

From (11) one gets

$$\frac{\partial^2 G(\omega_j(t), \cdot)}{\partial \omega_j(t) \partial p(\tau)} = \frac{\tilde{w}(t)}{\omega_j(t)^2} \frac{\partial \mu(t)}{\partial p(\tau)} \begin{cases} < 0 & \text{for } \sigma > 1 \\ = 0 & \text{for } \sigma = 1 \\ > 0 & \text{for } \sigma < 1 \end{cases} \quad \blacksquare$$

Proof of Proposition 6 (Welfare). Let $\mathcal{C}_j(\tau)$ denote the ideal consumption index of agent j at time τ . Define, as usual, life-time utility of agent j as

$$U_j(t) \equiv \int_t^\infty \frac{\mathcal{C}_j(\tau)^{1-\sigma} - 1}{1-\sigma} e^{-\rho(\tau-t)} d\tau. \quad (58)$$

Moreover, let $\bar{\mathcal{C}}(\tau)$ denote ideal consumption of the representative agent at time τ . Life-time utility of agent j at time t may alternatively be expressed as

$$V_j(\psi_j, t) \equiv \int_t^\infty \frac{[(1 + \psi_j) \bar{\mathcal{C}}(\tau)]^{1-\sigma} - 1}{1-\sigma} e^{-\rho(\tau-t)} d\tau. \quad (59)$$

The consumption equivalent variation that measures the welfare of agent j relative to

the welfare of the representative agent at time t is then given by $\psi_j(t)$ that solves

$$V_j(\psi_j, t) = U_j(t). \quad (60)$$

Using $\bar{r}(\tau, t) = \int_t^\tau r(v)dv$ and (57), (43) can be written as

$$\mathcal{C}_j(\tau) = \mathcal{C}_j(t) \bar{p}(\tau, t)^{-\frac{\theta}{\sigma}} e^{\frac{\bar{r}(\tau, t) - \rho(\tau - t)}{\sigma}}. \quad (61)$$

Substituting (61) into (58) and (59), we see that (60) implies $1 + \psi_j(t) = \mathcal{C}_j(t)/\bar{\mathcal{C}}(t)$.

Using $\mathcal{E}_j = \mathcal{P}_j \mathcal{C}_j = \mu \mathcal{W}_j$, according to the definition of $\mu_j = \mathcal{E}_j/\mathcal{W}_j$ and result $\mu_j = \mu$ for all j (Proposition 3) confirms (12). ■

Proof of Proposition 7 (Stationary wealth distribution).

To show that $\mu(t)\tilde{w}(t) = w(t)$ holds in any steady state, we determine the steady state solution of each term (μ, \tilde{w}, w) and then verify that the preceding equation holds.

In this section a hat above a variable denotes the steady state growth rate.

Using $w(\tau) = w(t)e^{\hat{w}(\tau-t)}$, the PDV of wages, $\tilde{w}(t) \equiv \int_t^\infty w(\tau)e^{\int_t^\tau -r(v)dv}d\tau$, may be written as⁴⁵

$$\tilde{w}(t) \equiv \int_t^\infty w(t)e^{\hat{w}(\tau-t)}e^{-r(\tau-t)}d\tau \stackrel{w(t)=const.}{=} \frac{w(t)}{r - \hat{w}} = \frac{w(t)}{\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} - \hat{w}},$$

where $\hat{\mathcal{C}}$ is steady state growth rate of ideal consumption, $\hat{\mathcal{P}}$ the steady state growth rate of the ideal price index, and \hat{w} the steady state growth rate of the wage, respectively.

Consider next the propensity to consume, as given by (8). Using $\bar{p}(\tau, t) \equiv e^{\hat{p}(\tau-t)}$ and

⁴⁵Notice that $w(t)$ is the level of the wage at $\tau = t$, which is not changing as the running time index τ moves forward.

$\bar{r}(\tau, t) \equiv \int_t^\tau r(v)dv = (\tau - t)(\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}})$ one obtains

$$\mu(t) = \left(\int_t^\infty \left(\frac{e^{\theta\hat{p}(\tau-t)}}{\exp \left[(\tau-t)(\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}}) + \frac{\rho}{\sigma-1}(\tau-t) \right]} \right)^{\frac{\sigma-1}{\sigma}} d\tau \right)^{-1} \quad (62)$$

$$= \left(\int_t^\infty \left(\frac{e^{\theta\hat{p}(\tau-t)}}{\exp \left[(\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} + \frac{\rho}{\sigma-1})(\tau-t) \right]} \right)^{\frac{\sigma-1}{\sigma}} d\tau \right)^{-1} \quad (63)$$

$$= \left(\int_t^\infty \left(e^{[\theta\hat{p} - (\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} + \frac{\rho}{\sigma-1})](\tau-t)} \right)^{\frac{\sigma-1}{\sigma}} d\tau \right)^{-1} \quad (64)$$

$$= \left(\int_t^\infty e^{[\theta\hat{p} - (\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} + \frac{\rho}{\sigma-1})] \frac{\sigma-1}{\sigma} (\tau-t)} d\tau \right)^{-1} \quad (65)$$

$$= \left(\int_t^\infty e^{a(\tau-t)} d\tau \right)^{-1} = \left(-\frac{1}{a} \right)^{-1} = -a \quad (66)$$

with $a = \left[\theta\hat{p} - \left(\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} + \frac{\rho}{\sigma-1} \right) \right] \frac{\sigma-1}{\sigma}$. Simplifying the RHS, noting $\hat{\mathcal{P}} = \theta\hat{p}$, yields

$$\begin{aligned} a &= \left[\theta\hat{p} - \left(\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} + \frac{\rho}{\sigma-1} \right) \right] \frac{\sigma-1}{\sigma} \\ &= (1-\sigma)\hat{\mathcal{C}} - \rho - \frac{\sigma-1}{\sigma}\hat{\mathcal{P}} + \frac{\sigma-1}{\sigma}\theta\hat{p} \\ &= -(\sigma-1)\hat{\mathcal{C}} - \rho. \end{aligned}$$

Putting things together, the condition $\mu(t)\tilde{w}(t) = w(t)$ implies

$$\begin{aligned} - \left[-(\sigma-1)\hat{\mathcal{C}} - \rho \right] \frac{w(t)}{\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} - \hat{w}} &= w(t) \\ \frac{(\sigma-1)\hat{\mathcal{C}} + \rho}{\rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}} - \hat{w}} &= 1 \\ \hat{\mathcal{P}} + \hat{\mathcal{C}} &= \hat{w}. \end{aligned}$$

From Proposition A.1 and $\hat{\mathcal{P}} = \theta\hat{p}$, which is implied by $\mathcal{P} = \frac{p^\theta}{\theta^\theta(1-\theta)^{1-\theta}} \frac{1}{1-\bar{e}}$, one gets

$$\hat{\mathcal{P}} = \theta \left(g^Y - \gamma \left(\eta g^Y + (1-\eta)g^X \right) \right).$$

Moreover, from $\mathcal{C} = C^{1-\theta}(S - \phi S)^\theta = C^{1-\theta}((1 - \phi)S)^\theta$ and Proposition A.1 we get

$$\hat{\mathcal{C}} = (1 - \theta)\hat{C} + \theta\hat{S} = (1 - \theta)g^Y + \theta\gamma(\eta g^Y + (1 - \eta)g^X)$$

Taken together one obtains

$$\begin{aligned}\hat{\mathcal{P}} + \hat{\mathcal{C}} &= \theta(g^Y - \gamma(\eta g^Y + (1 - \eta)g^X)) + (1 - \theta)g^Y + \theta\gamma(\eta g^Y + (1 - \eta)g^X) \\ &= g^Y.\end{aligned}$$

In a steady state we also have $\hat{w} = g^Y$ (Proposition A.1). ■

Proof of Proposition A.1. (Steady state).

Assume that B^Y and B^X grow at constant exponential growth rates, i.e. $B^Y(t) = B^Y(0)\exp(g^Y t)$ and $B^X(t) = B^X(0)\exp(g^X t)$ with $g^Y, g^X \geq 0$. *In this section a hat above a variable denotes the steady state growth rate.* From the Keynes Ramsey rule, one gets the steady state interest rate r^* to read

$$\hat{\mathcal{C}} = \frac{1}{\sigma} \left(r - \rho - \hat{\mathcal{P}} \right) \Leftrightarrow r^* = \rho + \sigma\hat{\mathcal{C}} + \hat{\mathcal{P}}.$$

From the first order conditions (profit maximization in the numeraire sector) one gets

$$\begin{aligned}r^* + \delta^K &= \alpha \frac{Y}{K} & \Rightarrow \hat{Y} &= \hat{K} \\ w &= \beta \frac{Y}{L^Y} & \Rightarrow \hat{Y} &= \hat{w} \\ R^Z &= (1 - \alpha - \beta) \frac{Y}{Z^Y} & \Rightarrow \hat{Y} &= \hat{R}^Z.\end{aligned}$$

Notice that allocation variables L^Y and Z^Y are constant in a steady state (the same applies to L^X and $Z^X = N$). Writing the production function $Y = K^\alpha (B^Y L^Y)^\beta (B^Y Z^Y)^{1-\alpha-\beta}$

in growth rates then yields

$$\hat{Y} = \alpha \underbrace{\hat{K}}_{\hat{Y}} + \beta \left(\underbrace{\hat{B}^Y}_{g^Y} + \underbrace{\hat{L}^Y}_0 \right) + (1 - \alpha - \beta) \left(\underbrace{\hat{B}^Y}_{g^Y} + \underbrace{\hat{Z}^Y}_0 \right) \quad (67)$$

$$\hat{Y} = g^Y. \quad (68)$$

From the capital market no arbitrage conditions for land in the numeraire and housing sector, the equation of motion for residential land and the growth rate of wealth one gets

$$\begin{aligned} \frac{\dot{P}^Z}{P^Z} &= r - \frac{R^Z}{P^Z} & \Rightarrow \hat{P}^Z &= \hat{R}^Z (= g^Y) \\ \frac{\dot{q}^N}{q^N} &= r - \frac{\pi}{q^N} & \Rightarrow \hat{\pi} &= \hat{q}^N (= g^Y) \\ 0 &= \frac{\dot{N}}{N} = \frac{q^N - P^Z}{\xi N} \Leftrightarrow q^N = P^Z & \Rightarrow \hat{q}^N &= \hat{P}^Z (= g^Y) \\ \frac{\dot{W}}{W} &= sav \frac{wl + r^* W}{W} = sav \left(\frac{w}{W} l + r^* \right) & \Rightarrow \hat{W} &= \hat{w} (= g^Y). \end{aligned}$$

In equilibrium wages in the numeraire and construction sector are identical. This allows to determine the growth rate of materials \hat{M} and the growth rate of the price of structures \hat{q}^X

$$\begin{aligned} w &= \frac{1-\eta}{\eta} \frac{1}{B^M} M (L^X)^{-1} & \Rightarrow \hat{M} &= \hat{w} \\ q^X &= \frac{1}{\eta B^M (B^X L^X)^{1-\eta} M^{\eta-1}} & \Rightarrow \hat{q}^X &= -(1-\eta)\hat{B}^X - (\eta-1)\hat{M} = (1-\eta)(g^Y - g^X). \end{aligned}$$

Combining the growth rate of structures with the no-arbitrage condition for residential structure yields

$$\frac{\dot{R}^X}{R^X} = r + \delta^X - \frac{R^X}{q^X} \Rightarrow \hat{R}^X = \hat{q}^X.$$

The equation of motion of structures implies

$$\frac{\dot{X}}{X} = \frac{I^X}{X} - \delta^X \Rightarrow \hat{I}^X = \hat{X}$$

From the production function for gross construction investment one gets

$$I^X = M^\eta (B^X L^X)^{1-\eta} \Rightarrow \hat{I}^X = \eta \hat{M} + (1-\eta) (\hat{B}^X + \hat{L}^X) = \eta g^Y + (1-\eta) g^X = \hat{X}.$$

Turning to $x = \frac{X}{N}$, $h = x^\gamma$, and $S = Nh$, noting that $N = \text{const.}$, we obtain⁴⁶

$$\begin{aligned} x \equiv \frac{X}{N} &\Rightarrow \hat{x} = \hat{X} \\ h = x^\gamma &\Rightarrow \hat{h} = \gamma \hat{x} \\ S \equiv Nh &\Rightarrow \hat{S} = \hat{h}. \end{aligned}$$

The growth rate of rents can be obtained from the profit function of housing services firms to read

$$\pi = (1 - \gamma)px^\gamma \Rightarrow \hat{p} = \hat{\pi} - \gamma \hat{x}.$$

Given the growth rates of housing services \hat{S} and the growth rate of rents \hat{p} , the growth rate of numeraire consumption is given by the following first order condition

$$C = \frac{(1-\phi)(1-\theta)}{\theta} pS \Rightarrow \hat{C} = \hat{p} + \hat{S}(= \hat{Y}).$$

Finally, we turn to the growth rate of the house price, $P^H \equiv q^N + q^X x$. Notice that the two terms on the RHS of the preceding definition, q^N and $q^X x$, grow at the same rate, i.e. $\hat{q}^N = g^Y$ and $\hat{q}^X + \hat{x} = (1 - \eta)(g^Y - g^X) + \eta g^Y + (1 - \eta)g^X = g^Y$. Hence, one obtains

$$\hat{P}^H = g^Y.$$

⁴⁶This is also true in an economy with zoning regulations, where $Z^X = \kappa Z$, with $\kappa \in (0, 1)$.

9.4 Calibration

Parameter	Value	Explanation/Target
L	1	Normalization
J	10	Match deciles
$\{W_j(0)/\bar{W}(0)\}_{j=1}^J$	see text	Wealth deciles, US, 2013 SCF
$\{l_j(0)/\bar{l}(0)\}_{j=1}^J$	see text	average earnings within wealth percentile, US, 2013 SCF
σ	2	$IES = 0.5$ (Havránek, 2015)
Z	1	Normalization
δ^K	0.056	Davis and Heathcote (2005)
α	0.28	Land income share in Y sector (Grossmann and Steger, 2017)
β	0.69	Labor expenditure share Y sector (Grossmann and Steger, 2017))
g^Y	0.02	Growth rate GDP per capita (FRED)
δ^X	0.015	Hornstein (2009)
η	0.38	Labor expenditure share X sector (Grossmann and Steger, 2017)
g^X	0.009	Growth rate $\frac{X}{N}$ (Davis and Heathcote, 2007, JME)
κ	0.169	Share of residential land: 16.9 percent (Falcone 2015)
θ	$\{0.19, \mathbf{0.17}, 0.15\}$	Average housing expenditure share: 0.19 (CEX, 2015)
ϕ	$\{0.000, \mathbf{0.104}, 0.260\}$	Difference between bottom and top income quintiles' housing expenditure share: $\{0, .07, .18\}$ (CEX, 2015; UK)
ρ	0.019	Real interest rate: 0.0577 (Jordà et al., 2018)
γ	0.78	Land's share in housing wealth: $1/3$
ξ	745	Transition speed in N : 31 percent in 30 years (Davis and Heathcote, 2007)

Table A.1: Set of parameters for the calibrated model.

Note. Calibration strategy is discussed in Section 6.1.

References

- [1] Albouy, David, Gabriel Ehrlich and Yingyi Liu (2016), Housing Demand, Cost-of-Living Inequality, and the Affordability Crisis, *NBER Working Paper* No. 22816.
- [2] Albouy, David, Gabriel Ehrlich (2018), Housing Productivity and the Social Cost of Land-Use Restrictions, *Journal of Urban Economics*, Forthcoming.
- [3] Accardo, Jérôme, Sylvain Billot, Maël-Luc Buron (2017), Household income, consumption and savings per broad category between 2011 and 2015, French economy - Accounts and files 2017 Edition, INSEE.
- [4] Álvarez-Peláez, Maria J and Antonia Díaz (2005). Minimum consumption and transitional dynamics in wealth distribution. *Journal of Monetary Economics* 52 (3), 633–667.
- [5] Bellet, Clement (2017). The Paradox of the Joneses Superstar Houses and Mortgage Frenzy in Suburban America, *CEP Discussion Paper* No 1462.
- [6] Borri, Nicola and Pietro Reichlin (2018). The housing cost disease , *Journal of Economic Dynamics and Control* 87, 106-123.
- [7] Caselli, Francesco and Jaume Ventura (2000). A Representative Consumer Theory of Distribution, *American Economic Review*, 90 (4), 909–926.
- [8] Chatterjee, Satyajit (1994). Transitional dynamics and the distribution of wealth in a neoclassical growth model, *Journal of Public Economics* 54 97–119.
- [9] Davis, Morris A. and Jonathan Heathcote (2005). Housing and the Business Cycle. *International Economic Review*, Vol. 46, Issue 3, 751-784.
- [10] Davis, Morris A. and Jonathan Heathcote (2007). The Price and Quantity of Residential Land in the United States, *Journal of Monetary Economics* 54, 2595–2620.

- [11] Dustmann, C, B Fitzenberger and M Zimmermann (2018), “Housing Expenditures and Income Inequality”, CReAM Discussion Paper.
- [12] Dynan, Karin E., Jonathan Skinner and Stephen P. Zeldes (2004). Do the Rich Save More?, *Journal of Political Economy* 112 (2), 397-444.
- [13] Falcone, James A. (2015). U.S. conterminous wall-to-wall anthropogenic land use trends (NWALT), 1974–2012: U.S. Geological Survey Data Series 948.
- [14] Favilukis, Jack, Sydney C Ludvigson, and Stijn Van Nieuwerburgh, (2017). The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium, *Journal of Political Economy*, Vol. 125, 140–223.
- [15] Favilukis, Jack, Pierre Mabillez and Stijn Van Nieuwerburgh, Affordable Housing and City Welfare, Working Paper, November 2018.
- [16] Frank, Robert H. (2005). Positional externalities cause large and preventable welfare losses. *American Economic Review Papers & Proceedings* 95 (2), 137–141.
- [17] Gabaix, X. , Lasry, J. , Lions, P. and Moll, B. (2016). The Dynamics of Inequality. *Econometrica*, 84: 2071-2111.
- [18] Garcia-Penalosa, Cecilia and Stephen J. Turnovsky (2006). Growth and Income Inequality: A Canonical Model, *Economic Theory*, 28, 25–49
- [19] Glaeser, E. L., Gyourko, J., and Saks, R. (2005). Why is manhattan so expensive? regulation and the rise in housing prices. *The Journal of Law and Economics*, 48(2):331–369
- [20] Grossmann, Volker and Thomas Steger (2017). Das House-Kapital: A Long Term Housing & Macro Model, *IMF Working Paper* No. 17/80.
- [21] Gyourko, Joseph and Raven Molloy (2015). Regulation and Housing Supply, *Handbook of Regional and Urban Economics*, Vol. 5A, Chapter 19, 1289–1338.

- [22] Havránek, Tomas (2015). Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting, *Journal of the European Economic Association*, 13: 1180-1204.
- [23] Hornstein, Andreas (2009). Problems for a Fundamental Theory of House Prices. *Economic Quarterly*, Vol. 95, 1-24.
- [24] Hubmer, J., P. Krusell, and J. Anthony A. Smith (2016). The Historical Evolution of the Wealth Distribution: A Quantitative-Theoretic Investigation, NBER Working Paper 23011.
- [25] Iacoviello, Matteo (2005). House prices, Borrowing Constraints and Monetary Policy in the Business Cycle. *American Economic Review*, 95, 3, 739-764.
- [26] Iacoviello, Matteo and Stefano Neri (2010). Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics*, 125-164.
- [27] Jordà, Oscar, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan Taylor (2018). The rate of return on everything. CEPR Discussion Paper No. 12509.
- [28] Kaymak, B., and M. Poschke (2016). The evolution of wealth inequality over half a century: The role of taxes, transfers and technology, *Journal of Monetary Economics* 77, 1-25.
- [29] Knoll, Katharina, Our Home in Days Gone By: Housing Markets in Advanced Economies in Historical Perspective, PhD Thesis, Freie University Berlin, 2017.
- [30] Knoll, Katharina, Moritz Schularick and Thomas Steger (2017). No Price Like Home: Global House Prices, 1870–2012, *American Economic Review* 105, 331–353.
- [31] Krusell, Per, and Jose-Victor Rios-Rull (1999). On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model. *American Economic Review*, 89 (5): 1156-1181.

- [32] Kuhn, Moritz, Moritz Schularick and Ulrike Steins (2018). Income and Wealth Inequality in America, 1949-2016 *CEPR Working Paper*.
- [33] Kuhn, Moritz and José-Víctor Ríos-Rull (2016). 2013 Update on the U.S. Earnings, Income, and Wealth Distributional Facts: A View from Macroeconomic Modelers. *Federal Reserve Bank of Minneapolis Quarterly Review*, 37 (1).
- [34] Kiyotaki, N. , Michaelides, A. and Nikolov, K. (2011), Winners and Losers in Housing Markets. *Journal of Money, Credit and Banking*, 43: 255-296.
- [35] Kydland, Finn, Peter Rupert and Roman Sustek (2017). Housing Dynamics over the Business Cycle, *International Economic Review*, forthcoming.
- [36] Leguizamon, Susane J. and Justin M. Ross (2012). Revealed preference for relative status: Evidence from the housing market, *Journal of Housing Economics* 21(1), 55–65.
- [37] Mas-Colell, Andreu, Michael D. Whinston and Jerry R. Green (1995). *Microeconomic Theory*, Oxford, Oxford University Press.
- [38] Marx, Karl (1847). *Wage Labour and Capital*, edited by Sankar Srinivasan, Independently Published, 2017.
- [39] Miles, David K. and James Sefton (2017), Houses across time and across place, *CEPR Working Paper* DP12103.
- [40] De Nardi, M. and G. Fella (2017), Saving and Wealth Inequality, *Review of Economic Dynamics* 26, 280-300.
- [41] Office for National Statistics (2015), Family Spending 2014, Table 3.1. URL: <https://www.ons.gov.uk/peoplepopulationandcommunity/personalandhouseholdfinances/incomeandwealth/compendium/familyspending/2015/familyspending> (accessed on October, 02 2017)

- [42] Piazzesi, Monica and Martin Schneider (2016). Housing and Macroeconomics, Elsevier, *Handbook of Macroeconomics*, John B. Taylor and Harald Uhlig (eds.), Vol. 2.
- [43] Piketty, Thomas and Gabriel Zucman (2014). Capital is Back: Wealth-Income Ratios in Rich Countries, 1700–2010, *Quarterly Journal of Economics* 129 (3), 1255–1310.
- [44] Saiz, Albert (2010). The Geographic Determinants of Housing Supply. *Quarterly Journal of Economics*, Vol. 125, Issue 3, 1253-1296
- [45] Summers, L. H. (2014). The inequality puzzle. Democracy: A Journal of Ideas, Issue #32, Spring, 65–73.
- [46] Schünemann, Johannes, Timo Trimborn (2017). Boosting taxes for boasting about houses: Status concerns in the housing market, ECON WPS - Vienna University of Technology Working Papers in Economic Theory and Policy, No. 05/2017.
- [47] Statistisches Bundesamt. 2015. Einkommens- und Verbrauchsstichprobe 2013 - Aufwendungen privater Haushalte für den Konsum. Fachserie 15, Heft 5. Wiesbaden. URL: https://www.destatis.de/DE/Publikationen/Thematisch/EinkommenKonsumLebensbedingungen/Konsumausgaben/EVS_AufwendungprivaterHaushalte.htm (accessed September 1, 2017).
- [48] Stigler, George J. (1954), The Early History of Empirical Studies of Consumer Behavior, *Journal of Political Economy* 62 (2), 95-113.
- [49] Turnbull, Geoffrey K, Jonathan Dombrow and Clemon F Sirmans (2006). Big house, little house: relative size and value. *Real Estate Economics*, 34 (3), 439–456.
- [50] U.S. Bureau of the Census, Homeownership Rate for the United States, retrieved from FRED, Federal Reserve Bank of St. Louis, URL: <https://fred.stlouisfed.org/series/RHORUSQ156N> (accessed December 21, 2018).

- [51] U.S. Bureau of Labor Statistics, Consumer Expenditure Survey (2016), URL: <https://www.bls.gov/cex/data.htm> (accessed October 6, 2016).
- [52] Wälde, Klaus (2016). Pareto-Improving Redistribution of Wealth - The Case of the NLSY 1979 Cohort, University of Mainz (mimeo).
- [53] World Income & Wealth Data Base (2017), URL: http://wid.world/data/#countrytimeseries/shweal_p0p1_992_i/US/1930/2015/eu/k/p/yearly/s (accessed on February, 20 2017).

10 Online Appendix

10.1 Reduced Form Dynamic System

The macroeconomic model with housing is fully described by seven differential equations plus a set of static equations. The dynamic system is derived in Grossmann and Steger (2017) for the case $\sigma = 1$ and $\phi = 0$ and can be readily extended to allow for $\sigma \neq 1$ and $\phi > 0$. In contrast to Grossmann and Steger (2017), we abstract from capital income taxation and normalize the land requirement per house to unity ($\psi = 1$ according to the notation in Grossmann and Steger, 2017).

$$\dot{X} = M^\eta (B^X L^X)^{1-\eta} - \delta^X X \quad (69)$$

$$\dot{N} = \frac{q^N - P^Z}{\xi} \quad (70)$$

$$\dot{W} = rW + wL - c - ps \quad (71)$$

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma} + \frac{(\sigma - 1)\theta}{\sigma} \frac{\dot{p}}{p} \quad (72)$$

$$\dot{q}^X = -R^X + (r + \delta^X)q^X \quad (73)$$

$$\dot{q}^N = -\pi + rq^N \quad (74)$$

$$\dot{P}^Z = -R^Z + rP^Z \quad (75)$$

$$K = W - (q^N N + q^X X + P^Z Z^Y) \quad (76)$$

$$p = \frac{\theta}{(1 - \theta)(1 - \phi)} \frac{C}{S} \quad (77)$$

$$R^X = \gamma p \left(\frac{X}{N} \right)^{\gamma-1} \quad (78)$$

$$\pi = (1 - \gamma) p \left(\frac{X}{N} \right)^\gamma \quad (79)$$

$$S = X^\gamma N^{1-\gamma} \quad (80)$$

$$L^Y + L^X = L, \quad Z^Y + N = Z. \quad (81)$$

where $K(0)$, $N(0)$, $X(0)$ are given. In total, there are 21 equations and 21 endogenous variables: X , x , N , W , C , π , q^N , P^Z , q^X , K , Z^Y , L^X , L^Y , M , S , h , r , w , R^X , R^Z , p . For

details see Grossmann and Steger (2017).

10.2 Computation of transitional dynamics for all agents

The computation of time paths for all J type of agents takes series for (normalized) prices and aggregate quantities – obtained from the solution of the representative agent economy in the first step – as given and derives time paths for each agent $j \in \{1, 2, \dots, J\}$ by exploiting the recursive structure of the household problem. It is not necessary to employ numerical techniques like solving non-linear equation systems, interpolation, or numerical integration. Given the minor approximation error in the solution of the representative agent economy, the computed time paths for all J type of agents are hence exact to machine precision.

10.3 Discretization

In order to solve the model numerically we have to discretize the differential equation system that describes the economy. For a differential equation $\dot{x}(t) = f(x(t), y(t))$ we discretize according to $x_{t+1} - x_t = f(x(t), y(t))$.⁴⁷ The growth-adjusted first-order conditions read

$$\frac{\tilde{c}_j}{1 - \theta} = \frac{\tilde{p}}{\theta}(\tilde{s}_j - \phi\tilde{s}) \quad (82)$$

$$\frac{\dot{\tilde{c}}_j}{\tilde{c}_j} = \frac{(1 - \tau_r)r - \rho}{\sigma} + \frac{\sigma - 1}{\sigma}\theta \left(\frac{\dot{\tilde{p}}}{\tilde{p}} + g^p \right) - g^c \quad (83)$$

$$\dot{\tilde{W}} = (r - g_c)\tilde{W}_j + \tilde{w}l_j - \tilde{c}_j - \tilde{p}\tilde{s}_j \quad (84)$$

$$0 = \lim_{t \rightarrow \infty} e^{-\tilde{\rho}t} \tilde{W}_{jt} \tilde{p}_t^{\theta(\sigma-1)} (\tilde{c}_{jt})^{-\sigma} \quad (85)$$

$$\tilde{W}_{j0} = \text{given}, \quad (86)$$

where g_c is the exogenous growth rate of consumption (the numeraire) in the steady state, g^p is the steady state growth rate of rents, and $\tilde{\rho} \equiv \rho + (\sigma - 1)(g^c - \theta g^p)$. The discretized

⁴⁷We explored also different approximations, e.g. $x_{t+1} - x_t = f(\frac{x_{t+1} - x_t}{x_t}, \frac{y_{t+1} - y_t}{y_t})$ and the differences in the results are negligible.

version hence reads (the \sim above variables is suppressed)

$$\frac{c_{jt}}{1-\theta} = \frac{p_t(s_{jt} - \phi \bar{s}_t)}{\theta} \quad (87)$$

$$c_{jt+1} - c_{jt} = \frac{(1-\tau_r)r_t - \rho}{\sigma} c_{jt} - \frac{\theta(1-\sigma)}{\sigma} \left(\frac{p_{t+1} - p_t}{p_t} + g_p \right) c_{jt} - g^c c_{jt} \quad (88)$$

$$W_{jt+1} - W_{jt} = (r_t - g^c)W_{jt} + w_t l_j - c_{jt} - p_t s_{jt} \quad (89)$$

$$0 = \lim_{t \rightarrow \infty} e^{-\tilde{\rho}t} \tilde{W}_{jt} \tilde{p}_t^{\theta(\sigma-1)} (\tilde{c}_{jt})^{-\sigma} \quad (90)$$

$$\tilde{W}_{j0} = \text{given}, \quad (91)$$

where the subscript “ jt ” now denotes the group j and (discrete) time t . This constitutes a linear, non-homogeneous system of first-order difference equations with time-variant coefficients and two boundary conditions. Rearranging yields

$$c_{jt+1} = \underbrace{\frac{(1-g^c)\sigma + (1-\tau_r)r_t - \rho + \theta(\sigma-1) \left(\frac{p_{t+1}-p_t}{p_t} + g^p \right)}{\sigma}}_{\equiv f_t} c_{jt} \quad (92)$$

$$W_{jt+1} = \underbrace{(1+r_t-g^c)}_{\equiv g_t} W_{jt} - \underbrace{\frac{1}{1-\theta}}_{\equiv h} c_{jt} + \underbrace{[w_t l_j - \phi p_t \bar{s}_t]}_{\equiv l_t^j} \quad (93)$$

$$0 = \lim_{t \rightarrow \infty} e^{-\rho t} W_{jt} p_t^{\theta(\sigma-1)} (c_{jt})^{-\sigma} \quad (94)$$

$$W_{j0} = \text{given}. \quad (95)$$

The solution is

$$c_{jt} = c_{j0} \prod_{s=0}^{t-1} f_s \quad (96)$$

$$W_{jt} = W_{j0} \prod_{s=0}^{t-1} g_s - h \sum_{k=0}^{t-1} c_{jk} \prod_{s=k+1}^{t-1} g_s + \sum_{k=0}^{t-1} l_k^j \prod_{s=k+1}^{t-1} g_s. \quad (97)$$

10.4 Initial consumption

One obtains c_{j0} by applying the transversality condition (TVC) to (97) and plugging the solution for c_{jt} – as given by (96) – into the result. Define $\Theta_t^1 \equiv W_{j0} \prod_{s=0}^{t-1} g_s$,

$\Theta_t^2 \equiv h \sum_{k=0}^{t-1} c_{jk} \prod_{s=k+1}^{t-1} g_s$ and $\Theta_t^3 \equiv \sum_{k=0}^{t-1} l_k^j \prod_{s=k+1}^{t-1} g_s$ and write (97) as

$$W_{jt} = \Theta_t^1 - \Theta_t^2 + \Theta_t^3. \quad (98)$$

We know that $\lim_{t \rightarrow \infty} W_{jt} \Xi_t = 0$, where $\Xi_t \equiv e^{-\rho t} p_t^{\theta(\sigma-1)} (c_{jt})^{-\sigma}$, such that

$$\lim_{t \rightarrow \infty} W_{jt}^j \Xi_t = 0 = \lim_{t \rightarrow \infty} \Xi_t (\Theta_t^1 + \Theta_t^3) - \lim_{t \rightarrow \infty} \Xi_t \Theta_t^2 \quad (99)$$

$$\Leftrightarrow 1 = \frac{\lim_{t \rightarrow \infty} \Xi_t (\Theta_t^1 + \Theta_t^3)}{\lim_{t \rightarrow \infty} \Xi_t \Theta_t^2} \quad (100)$$

$$= \lim_{t \rightarrow \infty} \frac{\Xi_t (\Theta_t^1 + \Theta_t^3)}{\Xi_t \Theta_t^2} \quad (101)$$

$$= \lim_{t \rightarrow \infty} \frac{\Theta_t^1 + \Theta_t^3}{\Theta_t^2}. \quad (102)$$

Replacing Θ_t^1 , Θ_t^2 , and Θ_t^3 by their respective expressions yields

$$1 = \frac{W_{j0} \prod_{s=0}^{\infty} g_s + \sum_{k=0}^{\infty} l_k^j \prod_{s=k+1}^{\infty} g_s}{h \sum_{k=0}^{\infty} c_{jk} \prod_{s=k+1}^{\infty} g_s} = \frac{W_{j0} + \sum_{k=0}^{\infty} l_k^j \prod_{s=0}^k (g_s)^{-1}}{h \sum_{k=0}^{\infty} c_{jk} \prod_{s=0}^k (g_s)^{-1}}.$$

Inserting the solution for c_{jk} as given by (96) gives

$$c_{j0} = \frac{W_{j0} + \sum_{k=0}^{\infty} l_k^j \prod_{s=0}^k (g_s)^{-1}}{\sum_{k=0}^{\infty} \frac{h}{f_k} \prod_{s=0}^k \frac{f_s}{g_s}} \quad (103)$$

10.5 How to deal with infinity

In the computation we have to assume that the dynamic system is in its steady state after a some period T , where the number of transition periods, T , is chosen sufficiently large. Then, sums and products can be modified to $\sum_{s=0}^{\infty} x_t = \sum_{s=0}^{T-1} x_t + \sum_{s=T}^{\infty} x$ and $\prod_{s=0}^{\infty} x_t = (\lim_{t \rightarrow \infty} x^t) \prod_{s=1}^{T-1} x_t$, where x denotes the respective steady state of x_t . The

steady states for the time-dependent parameters are

$$f = 1 \quad (104)$$

$$g = 1 + r - g_c \quad (105)$$

$$l^j = wl^j - \phi p \bar{s}. \quad (106)$$

Accordingly, the denominator of (103) becomes

$$\sum_{k=0}^{\infty} \frac{h}{f_k} \prod_{s=0}^k \frac{f_s}{g_s} = \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^k \frac{f_s}{g_s} + \frac{1}{1-\theta} \sum_{k=T}^{\infty} f_k^{-1} \prod_{s=0}^{T-1} \frac{f_s}{g_s} \prod_{s=T}^k \frac{f}{g} \quad (107)$$

$$= \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^k \frac{f_s}{g_s} + \frac{1}{1-\theta} \left(\prod_{s=0}^{T-1} \frac{f_s}{g_s} \right) \sum_{k=T}^{\infty} [1+r-g_c]^{T-k-1} \quad (108)$$

$$= \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^k \frac{f_s}{g_s} + \frac{1}{(1-\theta)(r-g_c)} \prod_{s=0}^{T-1} \frac{f_s}{g_s}. \quad (109)$$

Similarly, the second term in the numerator of (103) can be written as

$$\sum_{k=1}^{\infty} l_k^j \prod_{s=1}^k g_s^{-1} = \sum_{k=0}^{T-1} l_k^j \prod_{s=0}^k g_s^{-1} + \frac{l^j}{r-g_c} \prod_{s=0}^{T-1} g_s^{-1}.$$

Putting all together gives

$$c_{j0} = \frac{W_{j0} + \sum_{k=0}^{T-1} l_k^j \prod_{s=0}^k g_s^{-1} + \frac{l^j}{r-g_c} \prod_{s=0}^{T-1} g_s^{-1}}{\frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^k \frac{f_s}{g_s} + \frac{1}{(1-\theta)(r-g_c)} \prod_{s=0}^{T-1} \frac{f_s}{g_s}}. \quad (110)$$

10.6 Solution algorithm

For each $j \in \{1, 2, \dots, J\}$:

1. Obtain initial consumption c_{j0} with (110).
2. Obtain individual consumption levels $\{c_{jt}\}_{t=0}^T$ from (96) or by iterating over the discretized Euler equation.
3. Obtain individual wealth levels $\{W_{jt}\}_{t=0}^T$ by making use of (97) or by iterating over

the discretized budget constraint.

4. Obtain individual housing consumption $\{s_{jt}\}_{t=0}^T$ from the intra-temporal optimality condition.

10.7 Robustness

10.7.1 Status Preferences for Both Goods

If we replaced instantaneous utility (2) by

$$u(c_j, s_j) = \frac{[(c_j - \phi_c \bar{c})^{1-\theta} (s_j - \phi_s \bar{s})^\theta]^{1-\sigma} - 1}{1 - \sigma}, \quad (111)$$

with $\phi_c, \phi_s \geq 0$, where \bar{c} is average consumption of the numeraire good, then the housing expenditure share would still read as (6), with $\phi \equiv \frac{\phi_s - \phi_c}{1 - \phi_c}$. Since $\phi > 0$ iff $\phi_s > \phi_c$, assuming status concerns for housing services only ($\phi_c = 0$) captures, without loss of generality, that status concerns are higher for housing than for non-housing consumption.

10.7.2 CES Utility

Consider the following utility specification

$$u(c_j, s_j) = \frac{(\mathcal{C}_j)^{1-\sigma} - 1}{1 - \sigma} \quad \text{with} \quad \mathcal{C}_j = \left[\theta (s_j - \phi \bar{s})^{1-\frac{1}{\kappa}} + (1 - \theta) c_j^{1-\frac{1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}},$$

where $\kappa > 0$. The housing expenditure share of agent j (e_j) and the aggregate housing expenditure share (\bar{e}) are then given by

$$e_j = \frac{\theta^\kappa p^{1-\kappa}}{\theta^\kappa p^{1-\kappa} + (1 - \theta)^\kappa \left(1 - \frac{\phi}{\mathfrak{s}_j}\right)}$$

$$\bar{e} = \frac{\theta^\kappa p^{1-\kappa}}{\theta^\kappa p^{1-\kappa} + (1 - \phi)(1 - \theta)^\kappa}, \quad (112)$$

where $\mathfrak{s}_j \equiv \frac{s_j}{\bar{s}}$. The aggregate housing expenditure share (\bar{e}) is only constant, given that rents (p) may grow, provided that $\kappa = 1$ (Piazzesi and Schneider, 2016). Notice that

the utility specification in the main text, given by (2), is the limiting case of the above stated CES utility function for $\kappa \rightarrow 1$.

10.7.3 Status Preferences: Multiplicative Reference Level

Status preferences are often also captured as ratios instead of differences (Clark, Frijters & Shields, 2008; Schünemann and Trimborn, 2018). A typical formulation looks like this

$$v(c_j, s_j) = \frac{\left[s_j^\theta \left(\frac{s_j}{s} \right)^\phi (c_j)^{1-\theta} \right]^{1-\sigma} - 1}{1 - \sigma},$$

where $\theta \in (0, 1)$, $\phi \in [0, 1)$, and $\sigma > 0$. In this case, the housing expenditure share of agent j is given by

$$e_j = \frac{\theta + \phi}{1 + \phi}.$$

Hence, this preference specification is not compatible with heterogenous housing expenditure shares that vary systematically with income.

10.7.4 Decomposing Wealth Convergence: Zoning Experiment

Why do we observe wealth convergence in the zoning experiment? To shed light on this question, we start by noting that the distribution of wealth converges (diverges) when the following expression is negative (positive) for all j ⁴⁸

$$\frac{\partial \hat{W}_j(t)}{\partial W_j(t)} = \frac{\mu(t)\tilde{w}(t) - w(t)}{W_j(t)^2}. \quad (113)$$

When comparing wealth dynamics across different scenarios, one must distinguish between i) relative wealth and ii) the level of wealth. Define relative wealth by $\zeta_j \equiv W_j/\bar{W}$. To differentiate between the two effects, we take the derivative of the growth rate of

⁴⁸Here we set $l_j = 1$ for all j which is inconsequential for the developed insights.

individual wealth w.r.t. the relative wealth position

$$\frac{\partial \hat{W}_j}{\partial \zeta_j} = \frac{\partial \hat{W}_j}{\partial W_j} \frac{\partial W_j}{\partial \zeta_j} = \frac{\mu \tilde{w} - w}{W_j^2} \bar{W} = \frac{\mu \tilde{w} - w}{\zeta_j^2 \bar{W}}. \quad (114)$$

The relative wealth position of each household at period zero, $\zeta_j(t=0)$, does not change across two scenarios. However, \bar{W} might change due to valuation effects operating via changes in house and land prices. It is therefore sufficient to study how changes in aggregate wealth \bar{W} affect the dynamics of wealth inequality.

The aggregate variables affecting the dynamics of the wealth distribution are the i) rent p , ii) wage w , iii) interest rate r , and iv) aggregate wealth \bar{W} . Totally differentiating (114) w.r.t. all four aggregate variables for some future date $l \geq t$ yields⁴⁹

$$\begin{aligned} d \frac{\partial \hat{W}_j(t)}{\partial \zeta_j(t)} &= \frac{1}{\zeta_j(t)^2 \bar{W}} \left[\underbrace{\tilde{w}(t) \frac{\partial \mu(t)}{\partial p(l)} dp(l)}_{\text{rent channel}} + \underbrace{\frac{\partial \mu(t) \tilde{w}(t)}{\partial r(l)} dr(l)}_{\text{interest rate channel}} \right] \\ &+ \frac{1}{\zeta_j(t)^2 \bar{W}} \left[\underbrace{\frac{\partial [\mu(t) \tilde{w}(t) - w(t)]}{\partial w(l)} dw(l)}_{\text{wage channel}} - \underbrace{[\mu(t) \tilde{w}(t) - w(t)] d\bar{W}(l)}_{\text{valuation channel}} \right]. \end{aligned}$$

This expression shows how, for example, a change in the rent at date $l = 20$ (for a transition starting in $t = 0$) affects wealth divergence in period $t = 0$. If the rent does not jump across two scenarios, then it is sufficient to consider the above expression for any $l > t = 0$. (In the case of a jump, one also needs to evaluate the preceding expression for $l = t = 0$.) In the following we discuss each partial price effect in turn.

⁴⁹This decomposition was inspired by Moll, Kaplan and Violante (2018).

Rising rent is a convergence channel For $l > t$ the expression for the rent channel can be expressed as

$$\begin{aligned}\tilde{w}(t) \frac{\partial \mu(t)}{\partial p(l)} dp(l) &= \tilde{w}(t) \left\{ -\frac{\sigma-1}{\sigma} \theta \mu(t)^2 \left[\bar{p}(l, t)^\theta e^{-\bar{r}(l, t) - \frac{\rho}{\sigma-1}(l-t)} \right]^{-\frac{1}{\sigma}} \bar{p}(l, t)^{\theta-1} e^{-\bar{r}(l, t) - \frac{\rho}{\sigma-1}(l-t)} \frac{1}{p(t)} \right\} dp(l) \\ &= -\tilde{w}(t) \frac{\sigma-1}{\sigma} \theta \mu(t)^2 \left[\bar{p}(l, t)^\theta e^{-\bar{r}(l, t) - \frac{\rho}{\sigma-1}(l-t)} \right]^{\frac{\sigma-1}{\sigma}} \frac{1}{p(l)} dp(l).\end{aligned}$$

Given that the rent increases at some future point $l > t$, i.e. $dp(l) > 0$, this derivative is negative if $\sigma > 1$ (otherwise it is positive). Notice that θ determines the strength of this effect.

For $l = t$, i.e. today's rents increase, the expression for the rent channel reads

$$\begin{aligned}\tilde{w}(t) \frac{\partial \mu(t)}{\partial p(t)} dp(t) &= \tilde{w}(t) \left\{ -\mu(t)^2 \int_t^\infty \frac{\sigma-1}{\sigma} \left[\bar{p}(\tau, t)^\theta e^{-\bar{r}(\tau, t) - \frac{\rho}{\sigma-1}(\tau-t)} \right]^{-\frac{1}{\sigma}} \theta \bar{p}(\tau, t)^{\theta-1} e^{-\bar{r}(\tau, t) - \frac{\rho}{\sigma-1}(\tau-t)} (-1) \bar{p}(\tau, t) \frac{1}{p(t)} d\tau \right\} dp(t) \\ &= \frac{\sigma-1}{\sigma} \theta \frac{\tilde{w}(t) \mu(t)}{p(t)} dp(t)\end{aligned}$$

If today's rents increase, i.e. $dp(t) > 0$, then this derivative is positive if $\sigma > 1$ (otherwise it is negative). Hence, an increase in today's rents only implies that wealth inequality diverges stronger.

Interest rates exert an ambiguous effect For $l \geq t$ the expression for the interest rate channel reads

$$\begin{aligned}\frac{\partial \mu(t) \tilde{w}(t)}{\partial r(l)} dr(l) &= \frac{\partial}{\partial r(l)} \frac{\int_t^\infty w(\tau) e^{-\bar{r}(\tau, t)} d\tau}{\int_t^\infty \left[\bar{p}(\tau, t)^\theta e^{-\bar{r}(\tau, t) - \frac{\rho}{\sigma-1}(\tau-t)} \right]^{\frac{\sigma-1}{\sigma}} d\tau} dr(l) \\ &= \frac{-w(l) e^{-\bar{r}(l, t)} \frac{1}{\mu(t)} - \tilde{w}(t) \frac{\sigma-1}{\sigma} \left[\bar{p}(l, t)^\theta e^{-\bar{r}(l, t) - \frac{\rho}{\sigma-1}(l-t)} \right]^{\frac{-1}{\sigma}} \bar{p}(l, t)^\theta e^{-\bar{r}(l, t) - \frac{\rho}{\sigma-1}(l-t)} (-1)}{1/\mu(t)^2} dr(l) \\ &= \mu(t) e^{-\bar{r}(l, t)} \left[\mu(t) \tilde{w}(t) \frac{\sigma-1}{\sigma} \left[\bar{p}(l, t)^\theta e^{-\bar{r}(l, t) - \frac{\rho}{\sigma-1}(l-t)} \right]^{\frac{\sigma-1}{\sigma}} e^{\frac{1}{\sigma} \bar{r}(l, t)} - w(l) \right] dr(l)\end{aligned}$$

If $\sigma < 1$, this expression is negative (wealth convergence channel). Now assume $\sigma > 1$.

If $\mu\tilde{w} - w > 0$, then this expression is positive iff

$$\begin{aligned} & \frac{\sigma - 1}{\sigma} \left[\bar{p}(l, t)^\theta e^{-\frac{\rho}{\sigma-1}(l-t)} \right]^{\frac{\sigma-1}{\sigma}} e^{\frac{1}{\sigma}\bar{r}(l,t)} > 1 \\ \Leftrightarrow & \bar{p}(l, t) > \left(\frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma}{\theta(\sigma-1)}} e^{\frac{1}{\theta(\sigma-1)}[\rho(l-t) - \bar{r}(l,t)]}. \end{aligned}$$

To sum up, the interest rate channel is ambiguous under $\sigma > 1$. If rents increase sufficiently, as defined by the last inequality, then the interest rate channel is positive, i.e. higher rents (at some future point in time) imply more wealth divergence.

Higher future wages are a divergence and higher current wages a convergence channel For $l > t$ the expression for the wage channel reads

$$\mu(t) \frac{\partial \tilde{w}(t)}{\partial w(l)} dw(l) = \mu(t) e^{-\bar{r}(l,t)} dw(l) \begin{cases} \geq 0 & \text{for } dw(l) \geq 0 \\ < 0 & \text{for } dw(l) < 0 \end{cases}.$$

Hence, if at any point in the future the wage increases, this implies that the wealth distribution is getting more unequal today.

For $l = t$ the expression for the wage channel reads

$$\frac{\partial \mu(t) \tilde{w}(t) - w(t)}{\partial w(t)} dw(t) = (\mu(t) - 1) dw(t) < 0.$$

If only today's wages increase this leads to wealth convergence.

Higher aggregate wealth is a convergence channel Changes in aggregate wealth act as an amplification or dampening mechanism. Higher aggregate wealth amplifies both wealth convergence and divergence, while lower aggregate wealth dampens wealth convergence or divergence. Notice that this depends on the sign of $\mu\tilde{w} - w$. If $\mu\tilde{w} - w < 0$ (wealth convergence), then $d\bar{W} > 0 (< 0)$ increases (decreases) wealth convergence. If $\mu\tilde{w} - w > 0$ (wealth divergence), then $d\bar{W} > 0$ increases (decreases) wealth

divergence.

Numerical Illustration To shed further light on the evolution of the top 10 percent wealth share, displayed in Fig. 1 (b), we consider the following decomposition $\frac{\hat{W}_{10}}{\hat{W}} = \frac{sav_{10}}{\bar{s}av} \frac{y_{10}/W_{10}}{\bar{y}/\bar{W}}$, which follows from (10).

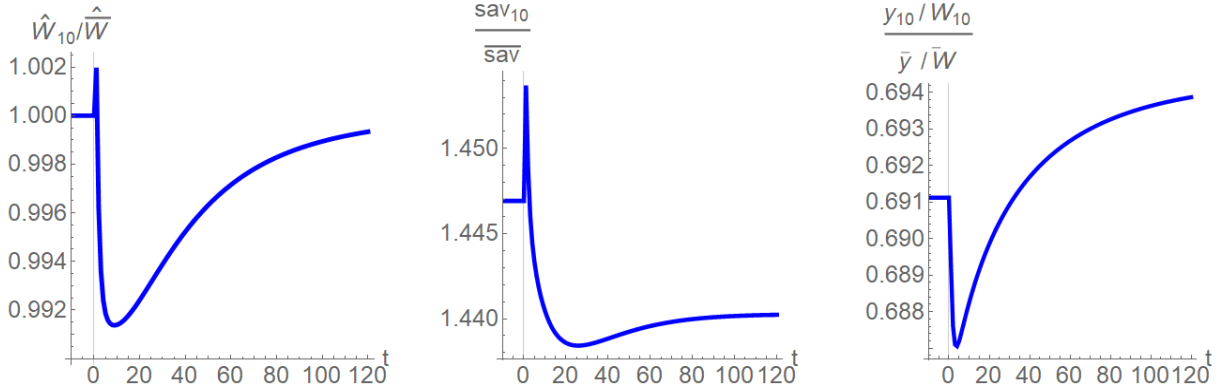


Figure OA.1: Wealth growth rate of the top percentile relative to the average wealth growth rate.

Notes: As we focus on the evolution of the top 10 percent wealth share, displayed in Fig. 1 (b), we consider $\frac{\hat{W}_{10}}{\hat{W}} = \frac{sav_{10}}{\bar{s}av} \frac{y_{10}/W_{10}}{\bar{y}/\bar{W}}$.

References

- [1] Clark, Andrew E., Paul Frijters, and Michael A. Shields (2008). Relative Income, Happiness, and Utility: An Explanation for the Easterlin Paradox and Other Puzzles. *Journal of Economic Literature*, 46 (1), 95-144.
- [2] Grossmann, Volker and Thomas Steger (2017). Das House-Kapital: A Long Term Housing & Macro Model, *IMF Working Paper* No. 17/80.
- [3] Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante (2018). Monetary Policy According to HANK. *American Economic Review*, 108 (3): 697-743.

- [4] Trimborn, Timo, Karl-Josef Koch and Thomas Steger (2008). Multi-Dimensional Transitional Dynamics: A Simple Numerical Procedure, *Macroeconomic Dynamics* 12, 301–319.