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**Conference Paper**

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Public Economics - Pensions and Savings, No. B10-V2

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Schön, Matthias (2019) : Demographic change and the German current account surplus, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Public Economics - Pensions and Savings, No. B10-V2, ZBW - Leibniz-Informationszentrum Wirtschaft, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/203527>

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# Demographic Change and the German Current Account Surplus\*

Matthias Schön<sup>†</sup>

February 18, 2019  
Preliminary Version

## Abstract

This paper analyses whether the severe demographic change in Germany causes its high current account surpluses. An ageing population both increases the supply and lowers demand of capital in an economy. Due to a longer life span individuals save more. Fewer workers reduce the optimal capital stock. In addition, there are positive or negative effects on (net) foreign assets depending on how existing public pay as you go pension systems adjust in an ageing society. According to a two region model with endogenous savings, labour supply and a bequest motive that is augmented with actual demographic data projections for OECD countries, the demographic change is a key determinant of the current account. However, it cannot fully account for the magnitude of the recent German surplus. The simulation results further indicate that both a higher retirement age and a fixed pension level with a rising contribution rate reduce foreign assets. If the contribution rate is fixed and the pension level lowered to accommodate this, foreign assets increase.

*JEL Classification:* E27, E62, F21, H55, J11.

*Keywords:* demographic change, current account, capital markets, pension system, OLG models

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\*I thank for helpful comments from conversation with Josef Hollmayr, Alexander Ludwig, Nikolai Stähler, Jens Weidmann, Karsten Wendorff, and seminar participants at Deutsche Bundesbank.

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# 1 Introduction

Germany's current account has come under greater scrutiny in recent years, with the high surpluses coming in for criticism from many quarters, cf. (?), (?). However, their causes are not fully understood.

This paper examines the question of how Germany's ageing population affects the current account, the balance of which is defined here as the change in a country's foreign assets. In turn, foreign assets are the difference between domestic demand for capital and the domestic supply of capital. Demographic change has an impact on both sides of the capital market.

The domestic supply of capital equates to the population's total savings. Demographic change affects this in two ways. First, it alters the savings behaviour of individual households. Given a longer life expectancy (assuming for now an unchanged retirement age), the period over which a household draws the comparatively low income from a pension is extended. In order to smooth the consumption level even over a longer pension-drawing period in old age, the household must accumulate more wealth up until it enters retirement. Higher individual savings increase the total assets of the economy. Second, ageing also changes the compositional structure of the population. The bulk of the population shifts from asset-poor young households towards asset-rich old households. Aggregated across all households, this increases the economy's total assets.

Demand for capital falls in an ageing society. Due to the declining working-age population, the number of people in employment goes down. Fewer people in employment need less capital. As a result, a greater supply of capital and lower demand for capital domestically lead to capital exports and growing current account balances.

The effect on the current account is not caused by absolute ageing in an economy, however. The deciding factor is ageing relative to other countries it shares a common capital market. If the foreign demographic structure is constant or if the population is actually becoming younger, the domestic economy has strong capital exports. If, however, foreign demographic developments are similar to those domestically, the effects are mild.

As the following analysis will show, in addition to direct effects ageing can have indirect ones. Demographic change places a considerable strain on the pension systems, often dominated by pay-as-you-go schemes. Foreign assets can be affected positively or negatively depending on how these systems are adapted to the demographic challenges. The results of the quantitative overlapping generations (OLG) model demonstrate that both a higher retirement age and a limit on the pension level (with a rising contribution rate) reduce foreign assets. If the contribution rate is fixed and the pension level thus lowered, however, foreign assets increase.

There is a well-established strand of the academic literature that focuses on the pressure induced by demographic change on social security systems that can be quantified with the use of large scale OLG models (?), (?)) of the Auerbach-Kotlikoff type (?), (?)). A far from exhaustive list includes (?), (?), (?), (?). Other authors have argued, within structural models,

that unsynchronised demographic trends can shape the dynamics of current accounts, e.g., (1), (2), (3), (4), (5), (6), (7), (8)). This paper combines both strands of the literature and shows how social security reforms affect the international capital flows.

This paper is structured as follows. Section ?? gives a description of the OLG model used. Section ?? describes the calibration of the model. Section ?? explains the theoretical transmission channels. Section ?? describes the simulation results for three scenarios in which the regions have differing demographics and pension systems. Section ?? simulates various pension reforms and analyses the ways in which they affect foreign assets.

## 2 Model

This paper uses an OLG model with explicit age cohorts comprising two regions. These two regions I will call "Home" and "Foreign". The model builds on the basic structure of ?) and its adoption by ?). As the purpose of this paper is the analysis of the German current account, Home will refer to Germany. In this model Foreign will be an aggregate of OECD countries excluding Germany.<sup>1</sup> The model contains a single frictionless capital market. This means that a country's capital stock is not determined by its own savings. And that the rate of return on capital is identical in both countries. However, the labour markets in Home and foreign are completely separate from one another. This means that there is no endogenous labour migration. Both regions produce a homogeneous consumption and investment good, which can be traded without friction. Each region has three sectors: (i.) a household sector with a large number of utility-maximising households, (ii.) a government sector and (iii.) a corporate sector comprised of one representative profit-maximising enterprise.

## 2.1 Demographics

Demography in this model is exogenous and represents the main driving force. Households begin their (economic) lives at the age of 20 (model age  $j = 1$ ) and live to a maximum of 100 years (model age  $j_T = 80$ ). Within each period, however, households die earlier with an exogenously determined probability. These mortality probabilities rise with the individual age of the household. However, they decline over time leading to a rise in life expectancy.

The size of the population in region  $i$  of age  $j$  in period  $t$  is given recursively

$$N_{t,j,i} = N_{t-1,j-1,i} \pi_{t-1,j-1,i} + Z_{t,j,i}, \quad (1)$$

where  $\pi_{j,t,i}$  denotes the age and time specific conditional survival rate.  $Z_{t,j,i}$  is the net migra-

<sup>1</sup>The foreign region in this model consists of the following 27 countries: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Lithuania, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Republic of Korea, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, UK, USA.

tion flow into region  $i$ .<sup>2</sup> In combination with the assumed (and also time-varying) number of newborns  $N_{1,t,i}$ , this creates demographic change, which is the fundamental driver of the model.

## 2.2 Pension system

Each region  $i$  has a Pay As You Go (PAYG) pension system characterized by a contribution rate  $\phi_{t,i}$  and a replacement rate  $\gamma_{t,i}$ . The replacement rate is the ratio of the gross standard pension,  $b_{t,i}^g$ , to the gross wage income of a full time employed,  $w_{t,i}^g$ , before taxation but after deducting pension contributions.<sup>3</sup> The budget of the PAYG pension systems has to be balanced at any time  $t$ ,

$$\phi_{t,i} w_{t,i}^g \left( \sum_{j=1}^J (\varepsilon_{t,j,i} l_{t,j,i} N_{t,j,i}) \right) = \sum_{j=1}^J b_{t,j,i}^g \frac{p_{t,j,i}}{45} (1 - l_{t,j,i}) N_{t,j,i} \quad (2)$$

with

$$b_{t,j,i}^g = \begin{cases} 0 & \text{if } j < J_t^R \\ b_{t,i}^g = \gamma_{t,i} (1 - \frac{1}{2} \phi_{t,i}) w_{t,i}^g & \text{if } j \geq J_t^R \end{cases} \quad (3)$$

On the revenue side  $l_{t,j,i}$  denotes the age specific labour supply resulting from optimal household decisions and  $\varepsilon_{t,j,i}$  represents the age specific productivity parameter of the household.

On the opposite side of the pension budget equation are the aggregate expenditures. Each retiree receives the gross pension value times her earned pension points,  $p_{t,j,i}$ , divided by 45<sup>4</sup>.  $p_{t,j,i}$  is the number of pension point acquired in the working life by the household. In each period of its working life a fully working household earns one pension point.

$$p_{t+1,j+1,i} = \varepsilon_{t,j,i} l_{t,j,i} + p_{t,j,i} \quad (4)$$

Rewriting the budget constraint of the pension system gives

$$\phi_{t,i} w_{t,i}^g \mathcal{L}_{t,i} = \gamma_{t,i} (1 - \frac{1}{2} \phi_{t,i}) w_{t,i}^g \mathcal{P}_{t,i} \quad (5)$$

with  $\mathcal{L}_{t,i} = \sum_{j=1}^J \varepsilon_{t,j,i} l_{t,j,i} N_{t,j,i}$  denotes the number of contributors and  $\mathcal{P}_{t,i} = \sum_{j=J_R}^J \frac{p_{t,j,i}}{45} (1 - l_{t,j,i}) N_{t,j,i}$  denotes the number of retirees.

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<sup>2</sup>Due to computational reasons I assume that migrants enter with the exact amounts of assets and pension points as households with the same age that already live in the respective region.

<sup>3</sup>The replacement rate formula is  $\gamma_{t,i} = \frac{b_{t,i}^g}{(1-0.5\phi_{t,i})w_{t,i}^g}$ . Retirees do not have to pay pension contributions. Therefore the replacement rate is higher than the replacement rate before social insurances. Compared with other models in the literature, the modelling of the German pension system is very detailed. In addition to contribution receipts, the pension budget also receives a tax-financed central government grant. This central government grant to the pension insurance scheme is funded by taxes on employment/pension income, investment income and consumption. The pension insurance scheme is required to have a balanced budget in every period.

<sup>4</sup>The German pension system defines a standard pension as the amount an employee receives after working 45 years full-time and retires at the statutory retirement age.

The law of motion of the replacement rate in Home follows approximately the German pension formula.<sup>5</sup>

$$\gamma_{t,1} = \gamma_{t-1,1} \frac{1 - \phi_{t-1,1}}{1 - \phi_{t-2,1}} \left( \left( 1 - \frac{\frac{\mathcal{P}_{t-1,1}}{\mathcal{L}_{t-1,1}}}{\frac{\mathcal{P}_{t-2,1}}{\mathcal{L}_{t-2,1}}} \right) * 0.25 + 1 \right). \quad (6)$$

The ratio of pensioners to contributors is the main driving force for the pension replacement rate. If for example fewer households pay contributions or more households claim pension the replacement rate decreases. A decrease in the replacement rate due to rising expenditures, however, does not fully offset the increase in pension expenditures. Consequently, the contribution rate has to increase to keep the pension budget in balance. The German pension formula splits the economic burden of the demographic change between pension contributors and retirees. So it is neither a defined benefit nor a defined contribution scheme.

The PAYG system in Foreign is in the first two scenarios the same as in Home. In the third scenario it is modelled as a defined benefit scheme. In this case, the pension replacement rate is constant over time with

$$\gamma_{t,2} = \gamma_2. \quad (7)$$

Consequently, an increase in the number of pensioners and fewer contributors to the pension system do not reduce the replacement rate. All additional expenditures and the shrinking contribution base are offset by an increase in the contribution rate. The contributors bear the complete economic burden of the demographic change.

The contribution rate in both countries endogenously adjusts to equalize the pension system budget.

$$\phi_{t,i} = \left( \frac{1}{\gamma_{t,i}} \frac{\mathcal{L}_{t,i}}{\mathcal{P}_{t,i}} + \frac{1}{2} \right)^{-1} \quad (8)$$

## 2.3 Firms

Firms produce in each region with a Cobb-Douglas production function employing capital and labour.

$$Y_{t,i} = \Omega_{t,i} K_{t,i}^\alpha L_{t,i}^{1-\alpha} \quad (9)$$

where  $K_{t,i}$  denotes the aggregate capital stock,  $L_{t,i}$  the aggregate labour supply, and  $\alpha$  the capital share.  $\Omega_{t,i}$  denotes the total factor productivity (TFP) that grows with a constant rate. This TFP growth rate is identical in both regions,  $\mu_i = \mu$ .

The labour productivity of a household changes over the life-cycle. The parameter  $\varepsilon_{t,j,i}$  denotes age specific productivity. The aggregate labour supply is then the sum over all age

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<sup>5</sup>In the original pension formula exists an additional term that accounts for private old age provision.

groups of the fraction of households working times the age specific labour productivity.

$$L_{t,i} = \sum_{j=1}^{J_t} \varepsilon_{t,j,i} l_{t,j,i} N_{t,j,i} \quad (10)$$

Capital adjustment costs in the firm sector are not considered. The only constraint to the firm maximization is given by the capital accumulation condition

$$K_{t+1,i} = K_{t,i} + I_{t,i} - D_{t,i} = (1 - \delta) K_{t,i} + I_{t,i} \quad (11)$$

where  $I_{t,i}$  is gross investment,  $D_{t,i}$  is aggregate depreciation and  $\delta$  is the depreciation rate.

The firm problem is static and the profit function

$$\Pi_{t,i} = Y_{t,i} - r_{t,i}^g K_{t,i} - w_{t,i}^g \left(1 + \frac{1}{2} \phi_{t,i}\right) L_{t,i}. \quad (12)$$

The first order conditions from profit maximizing give standard expressions for equilibrium gross returns to capital

$$r_{t,i}^g = r_{t,i} + \delta = \alpha \Omega_{t,i} \left(\frac{K_{t,i}}{L_{t,i}}\right)^{\alpha-1} = \alpha \frac{Y_{t,i}}{K_{t,i}} \quad (13)$$

and gross wages

$$w_{t,i}^g \left(1 + \frac{1}{2} \phi_{t,i}\right) = (1 - \alpha) \Omega_{t,i} \left(\frac{K_{t,i}}{L_{t,i}}\right)^{\alpha} = (1 - \alpha) \frac{Y_{t,i}}{L_{t,i}} \quad (14)$$

Half of the pension system contributions are paid by the employer and the other half by the employee. Net wages are then given by  $w_{t,i}^n = (1 - \tau^y) \left(1 - \frac{1}{2} \phi_{t,i}\right) w_{t,i}^g$ .

## 2.4 Households

By choosing an optimal consumption path, each cohort born in period  $t$  maximizes at any point in time  $t + j$  and age  $j$  the sum of discounted future utility. The within period utility function exhibits constant relative risk aversion and preferences are additive and separable over time. Cohort  $t$ 's maximization problem at  $j = 1$  is given by

$$\max_{\{c_{t,j,i}, l_{t,j,i}\}_{j=1}^J} \sum_{j=1}^J \beta_i^j (s_{t,j,i} U(c_{t,j,i}, 1 - l_{t,j,i}) + (1 - s_{t+1,j+1,i}) B(a_{t,j,i})) \quad (15)$$

where  $\beta_i$  is the pure time discount factor. In addition to pure time discounting households discount future utility with their unconditional survival probability,  $s_{j,t+j,i} = \prod_{m=1}^j \pi_{t,m-1,i}$ . Households die with certainty at age  $J$ . Household gain utility from consumption,  $c_{t,j,i}$ , leisure,  $1 - l_{t,j,i}$ , and from bequeathing assets to the next generation  $B(a_{t,j})$ .

Denoting total assets by  $a_{t,j,i}$ , maximization of the household's inter-temporal utility is subject to a dynamic budget constraint given by

$$a_{t+1,j+1,i} = (1 + (1 - \tau^k) r_{t+1,i}) (a_{t,j,i} + (1 - \tau_t^y) y_{t,j,i} - (1 + \tau^c) c_{t,j,i}) \quad (16)$$

The bequests of the households are distributed equally among the newborn generation.

$$a_{t+1,0,i} N_{t+1,0,i} = \sum_{j=1}^J (1 - \pi_{t,j,i}) (1 + r_{t,i}) a_{t,j,i} N_{t,j,i} \quad (17)$$

Income consists of asset income and net wages during the working period and pension income during retirement.

$$y_{t,j,i} = (1 - \frac{1}{2} \phi_{t,i}) w_{t,i}^g \varepsilon_j l_{t,j,i} + b_{t,j,i} \frac{p_{t,j,i}}{45} (1 - l_{t,j,i}) \quad (18)$$

Unemployment protection systems are ignored. Maximization is also subject to the constraint that leisure may not exceed 1.

$$0 \leq l_{t,j,i} \leq 1 \quad (19)$$

## 2.5 Government

The government can tax labour income, pension income<sup>6</sup>, consumption and capital income in order to finance government consumption,  $G_{t,i}$ . The government budget reads as

$$G_{t,i} = \tau_{t,i}^y \sum_{j=1}^J y_{t,j,i} N_{t,j,i} + \tau^c \sum_{j=1}^J c_{t,j,i} N_{t,j,i} + \tau^k r_{t,i} \sum_{j=1}^J a_{t,j,i} N_{t,j,i} \quad (20)$$

## 2.6 Definition of equilibrium

A competitive equilibrium of the economy is defined as a sequence of dis-aggregated variables,  $\{c_{t,j,i}, l_{t,j,i}, a_{t,j,i}\}$ , aggregate variables,  $C_{t,i}, L_{t,i}, K_{t,i}$ , a wage rate  $w_{t,i}$ , government policies,  $\phi_{t,i}, \gamma_{t,i}$ , in each region  $i$  and a common world interest rate,  $r_t$  such that

1. The allocations are feasible, i.e.

$$Y_{t,i} + r_t F_{t,i} = S_{t,i}^n + C_{t,i} + D_{t,i} + G_{t,i} = S_{t,i}^g + C_{t,i} + G_{t,i} \quad (21)$$

where  $F_{t,i}$  is the amount of net foreign assets,  $D_{t,i}$  is depreciation of capital and  $S_{t,i}^n (S_{t,i}^g)$  is net (gross) savings.

2. Factor prices equal their marginal products, i.e. interest rates and wages satisfy (??) and (??).

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<sup>6</sup>Hereby assume that pension and labour income is taxed with the same tax rate.



3. Given prices and initial conditions households maximize life time utility given in equation (??) subject to constraints in equation (??) and (??). Firms maximize profits subject to the capital accumulation constraint given in equation (??).
4. Government policies satisfy equation (??) in every period
5. Markets clear. Market clearing on national markets requires that

$$S_{t,i}^n = \sum_{j=1}^J S_{t,j,i}^n, \quad C_{t,i} = \sum_{j=1}^J C_{t,j,i}, \quad A_{t,i} = \sum_{j=1}^J a_{t+1,j+1} N_{t,j,i}, \quad L_{t,j} = \sum_{j=1}^J \varepsilon_{t,j,i} l_{t,j,i} N_{t,j,i}$$

Market clearing on the international capital market and the assumption of perfect capital mobility across regions requires that the rate of return on financial investment is equalized across all regions,

$$r_t = r_{t,i} \quad (22)$$

and that the sum of all foreign assets, defined as the difference between home assets and the home capital stock,  $F_{t,i} = A_{t,i} - K_{t,i}$ , across all world regions equals zero, i.e.

$$F_{t,1} + F_{t,2} = 0. \quad (23)$$

International capital flows are defined by the difference between foreign assets in two successive periods,

$$CA_{t,i} = F_{t+1,i} - F_{t,i} = S_{t,i}^g - I_{t,i} \quad (24)$$

## 2.7 Mechanisms and experiments

Demographic trends in Germany have been identified as one of the main drivers of Germany's large current account surpluses. In the model introduced here, the current account balance is the change in external assets. In turn, the level of domestic foreign assets is the difference between domestic supply of assets and domestic demand for capital.

Capital demand is dependent on the number of employed persons for whom capital is required. If this number decreases due to demographic trends, the demand for capital also falls. All other things being equal, external assets would have to rise and the current account balance would be positive.

Demographic change in Germany affects the supply of assets primarily via two channels. First, households (on average) adapt their savings behaviour so that total assets ultimately rise. Younger individuals save in order to prepare themselves for losses of (wage) income in old age (precautionary saving). If individuals' life expectancy rises, they must then make provisions to cover a longer period of time (assuming the retirement age remains constant). Moreover, in light of lower birth rates, households are anticipating that an ever smaller number of young people will have to finance the pensions of an ever greater number of retirees. The pressure

on the pension system arising from this could, depending on the characteristics of the pension insurance scheme, lead to falling pensions (and/or rising contribution rates). The income gap in old age would become wider, and the optimal amount of assets required to smooth consumption would be higher.

Second, demographic change is altering the relative sizes of age groups. In an ageing society such as Germany, the population weight is shifting from young, asset-poor households towards old, asset-rich households. Total wealth rises as a result. This compositional effect is not dependent on the changed savings behaviour described above.

### 3 Calibration

The aim of the calibration is to match the German economy. Specifically, I match its (foreign) asset position and its pension system in order to have a good baseline model for policy evaluation. Calibration of the model requires (i) determination of values for several structural model parameters and (ii) data for the exogenous demographic processes and pension system replacement rate and/or contribution rates. Table ?? provides an overview of the parameters used.

The model period is one year. I assume that the world economy (Home & Foreign) was in a steady state in 1960. I then take the year 2015 to calibrate our model. The main focus of this model is then the forecast period of 2016 to 2080 where I assume that the structural parameters are the same as in the calibration period and only demographic and policy parameters change. I additionally assume that after 2080 the demographic and pension policy parameters are constant. A new steady state is approximately reached in 2500.

#### 3.1 Demographics

For the demographic process I use historical data and projections from the OECD. These data sets include very detailed information about the age structure of Germany and the rest of the OECD countries at national levels. Additionally, I use net migration data and projections for Germany and the OECD to derive survival rates that are consistent with population age structure. In the baseline scenario the population size and structure of Foreign is set constant at the 2015 level.

In the first scenario I focus on the domestic demographic change. Therefore the size and the structure of the population as well as the survival rate of Foreign are constant over time (and not following equation (??)).

#### 3.2 Technology

The technology parameter to be determined are  $(\alpha, \delta, A_1, A_2, \mu)$ . The capital share is set to  $1/3$ . Furthermore I target the aggregated depreciation to output ratio in Germany,  $\frac{\delta K}{Y} = 18\%$ ,

and a capital-output ration of  $\frac{K}{Y} = 2.8$ . This implies a yearly rate of depreciation of  $\delta = 6.46\%$ . The capital output ratio (equivalent the real interest rate) will be attained by appropriate calibration of the preference parameters (especially the time discount factors  $\beta_i$ ).

The parameters  $A_i$  define the level of the production in each country. They are calibrated to match gross value added in 2015 (Home: €3.407 trillion and Foreign: €42.609 trillion). The growth rate of TFP is constant and set to  $\mu = 0.7\%$ . This is the forecast value used by the Bundesbank for GDP projection.

### 3.3 Labour Market & Productivity

I observe that the age structure of the employment rate is hump shaped. In the early years of the household (probably due to ongoing education) it starts low. It reaches a maximum at the age of 30 (=biological age of 50) and then remains. At the statutory retirement age the employment rate drops sharply. To capture this pattern I introduce an age dependent labour productivity,  $\varepsilon$ . It increases for the first 30 years of the household and remains constant until the retirement age. After the statutory retirement age the labour productivity is zero.<sup>7</sup>

$$\varepsilon_{t,j+1,i} = \begin{cases} \varepsilon_{t,j,i} (1 + \hat{\varepsilon}) & \text{if } 1 \leq j \leq 30 \\ \varepsilon_{t,j,i} & \text{if } 30 < j < J_{t,i}^R \\ 0 & \text{if } j \geq J_{t,i}^R \end{cases} \quad (25)$$

with

$$\sum_{j=1}^{45} \varepsilon_{t,j,i} = 1 \quad (26)$$

### 3.4 Government Policy / Pension System

The policy parameters that need to be determined are  $(\tau^c, \tau^k, \tau^w, \gamma_{1,1}, \gamma_2)$ . Consumption tax rate, I set to constant level over time to  $\tau^c = 19\%$ . The capital tax rate is set to  $\tau^k = 25\%$ . Labour and pension income tax is endogenously calibrated to match the German tax levies burden of 39.8% in 2015. The tax rates are the same in both regions.

The pension system in Home (Germany) is closely tied to the German pension system. For the calibration period, I know the pension replacement from the data and set  $\gamma_{1,1}$  accordingly. In the forecast period the pension replacement rate follows equation (??). In the first two scenarios I tie the replacement rate in Foreign to the replacement rate in in Home. In the third scenario, the pension system in Foreign is defined benefit system and I set the replacement rate for Foreign is constant to 63%. This value is difficult to calibrate as there is not only one pension system and replacement rate in the OECD countries. I take the average replacement rate described in the recent OECD report, see ?).

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<sup>7</sup>This labour productivity profile resembles the findings in ?).

### 3.5 Preferences

It is assumed that the period specific utility function is of the standard Cobb Douglas form given by

$$U(c_{t,j,i}, 1 - l_{t,j,i}) = \begin{cases} \frac{1}{1-\theta} \left( c_{t,j,i}^{\xi_i} (1 - l_{t,j,i})^{1-\xi_i} \right)^{1-\theta} & \text{if } \theta \neq 1 \\ \ln \left( c_{t,j,i}^{\xi_i} (1 - l_{t,j,i})^{1-\xi_i} \right) & \text{if } \theta = 1 \end{cases} \quad (27)$$

where  $\theta$  is the coefficient of relative risk aversion and  $\xi_i$  is the consumption share parameter, i.e., the weight of consumption relative to leisure in household's utility.

Utility from bequests is

$$B(a_{t,j,i}) = \begin{cases} \left( \frac{v}{1-v} \right)^\theta \frac{(a_{t,j,i})^{1-\theta}}{1-\theta} & \text{if } \theta \neq 1 \\ \left( \frac{v}{1-v} \right) \ln(a_{t,j,i}) & \text{if } \theta = 1 \end{cases} \quad (28)$$

with  $v \in (0, 1)$  and where  $B(a_{t,j,i}) = 0$  if  $v = 0$ . This is a re-parametrized version of a commonly used functional form (e.g.,  $\frac{v}{1-v}$ ,  $\frac{(a_{t,j,i})^{1-\theta}}{1-\theta}$ ,  $\ln(a_{t,j,i})$ ).<sup>8</sup> In this setting preferences over consumption and bequests are homothetic and households are equally risk averse over consumption and bequests. The parameter  $v$  is the marginal propensity to bequeath in a one period problem of allocating wealth between consumption and an immediate bequest. Larger values of  $v$  mean that people leave a larger share of wealth. The bequest parameter  $v$  is calibrated to match the aggregated bequest in Germany of roughly €310 billion in 2015.<sup>9</sup> I a priori choose the  $\sigma = 1$ . Then I determine the time discount factor  $\beta_1$  such that in the baseline model the capital output ratio match the German data of  $\frac{K}{Y} = 2.8$ . This capital output ratio together with the capital share and the depreciation rate implies a net return to capital of

$$r_t = \alpha \frac{Y}{K} - \delta = 5.4\% \quad . \quad (29)$$

The total number of workers in each region in 2015 determines the utility weight of consumption,  $\xi_i$ . For Home (Germany) this is 40.2 million workers and for Foreign (OECD) I assume 534 million workers in 2015.<sup>10</sup>

## 4 Simulations

In this section, I analyse three scenarios regarding the impact of demographic change on Germany's current account below. In the first scenario, demographic change occurs only domes-

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<sup>8</sup>Bequests in this model are not modelled as luxury goods as there is no heterogeneity within cohorts where this would play a role.

<sup>9</sup>Without the inheritance motive, optimising households would use up their wealth entirely until they reach the maximum age. This cannot be observed in the data, however. The model calibrates the strength of the inheritance motive such that aggregate inheritances in the model match the data. It should be noted that the inheritance motive is not contingent on the size of the next generation (number of descendants).

<sup>10</sup>I take these employment estimates from OECD database.

Table 1: Parameter Summary

Parameter	Interpretation	Value
<b>Set parameter</b>		
$\sigma$	Relative risk aversion	1
$\tau^c$	Consumption Tax	19%
$\tau^k$	Capital Tax	25%
$J^R$	Statutory Retirement Age	65–69
$\gamma_2$	Foreign replacement rate (Scenario 3)	63%
$\alpha$	Capital Share	1/3
$\delta$	Depreciation rate	6.46%
$\mu$	Total factor productivity growth	0.7%
<b>Estimated directly from the data</b>		
$N_{t,0,i}$	Newborns	OECD
$Z_{t,i}$	Net migration	OECD
$\pi_{t,j,i}$	Survival probabilities	OECD
<b>Parameter calibrated in equilibrium – Scenario 3 (targets in brackets)</b>		
$A_1$	Total factor productivity - Home (GDP Germany 2015)	1368 ( $\text{€}3.407 * 10^{12}$ )
$A_2$	Total factor productivity - Foreign (GDP ratio Germany/(OECD-Germany) 2015)	1373 (12.5)
$\hat{\varepsilon}$	Labour productivity growth (Employment rate difference over age)	5.3% (20%)
$v$	Bequest motive utility weight (Aggregated bequests Germany 2015)	1.22 ( $\text{€}0.315 * 10^{12}$ )
$\beta_1$	Time discount factor Home (Capital Output Ratio in Germany)	0.9884 (2.8)
$\beta_2$	Time discount factor Foreign (Net foreign assets of Germany 2015)	0.9906 ( $\text{€}1.246 * 10^{12}$ )
$\xi_1$	Consumption utility weight Home (Total employment Germany 2015)	0.805 ( $40.2 * 10^6$ )
$\xi_2$	Consumption utility weight Foreign (Total employment OECD(-Germany) 2015)	0.7479 ( $534.3 * 10^6$ )
$\tau^w$	Income and pension tax (Tax burden Germany 2015)	15.2% (39.8%)

*Note:* Summary of parameter values and targets.

tically (in Germany). The population of Home is based on the OECD population forecast for Germany. The forecast period spans from 2016 to 2081. The foreign demographic structure (OECD excluding Germany) remains constant and unchanged from its state as at 2015. This highly stylised scenario highlights how domestic populating ageing affects the current account. In the second scenario, also the population forecast of the OECD (excluding Germany) is incorporated into the model. The size and the age structure of the OECD as well as the survival rates change over time. In addition, the third scenario differentiates between pension systems across regions. For Home, the German pension system with the specific pension adjustment formula is used. For Foreign, a pension system that ensures a constant 63% pension level is used.

## 4.1 Domestic demographic change

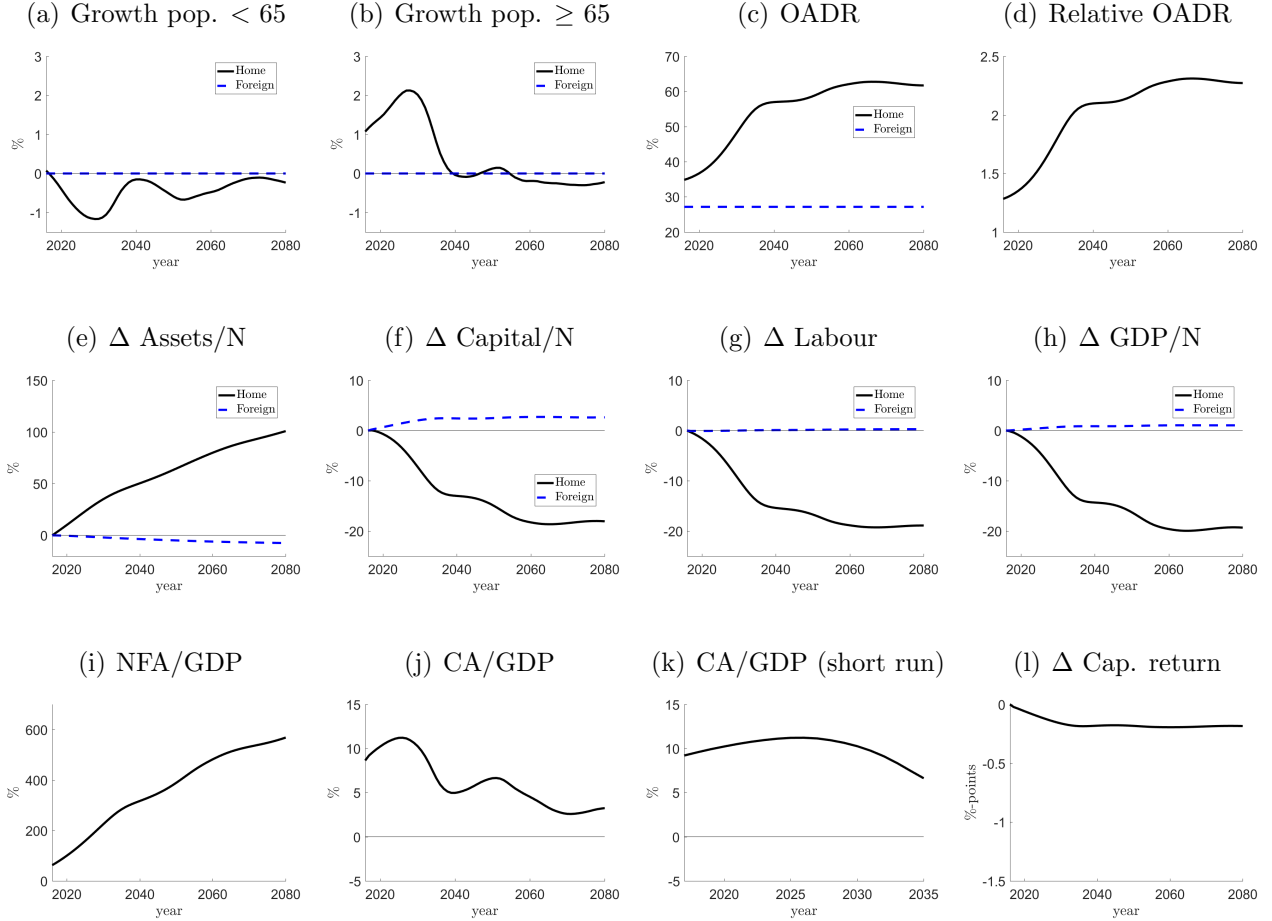
In this section I show what effects the isolated demographic change in Germany has on the German current account. In panel (a) of Figure ??, the growth rate of the Home population younger than 65 is constantly negative in the future. It reaches its minimum of  $-1.2\%$  at 2029. This downturn in the next 10 years is being driven chiefly by the entering of the baby boomer generation into retirement age. This pattern is also reflected by the growth rate of the population older than 65 (panel (b) of Figure ??). Consequently, the ratio of the population older than 65 to the population younger than 65 - the old age dependency ratio (OADR) - rises. According to the OECD forecast, the OADR in Germany will increase from just under 35% (in 2016) to 63% (in 2066) (panel (c) in Figure ??). This ratio sees significant fluctuations over time caused by disproportionately large generational cohorts. So the baby boomers account for the strong increase between 2020 and 2040. The second surge in the OADR between 2050 and 2060 results from the also relatively large generation of the baby boomers' children.

Demographic change in the Home region has an obvious impact on the domestic macroeconomic variables (panel (e)-(h) of Figure ??). Asset holdings of domestic households rise sharply. As mentioned above, this increase is a result of the longer life expectancies and a changing demographic structure. On the other side of the capital market, the declining number of employed persons in the Home region leads to reduced demand for capital. Less capital and a lower number of employed persons lead to a reduction in aggregate GDP (and GDP per capita).

Although the Foreign region is not subject to demographic change in the baseline specification, the macroeconomic variables change there, too. The reason for this is that the Home region has excess capital which it relocates abroad. This occurs because the continually high number of employed persons abroad allows for greater return on assets than in Home. The imported capital allows foreign wages to grow, and foreign households slightly increase their endogenous labour supply. In turn, the return on capital decreases (panel (l) in Figure ??). Domestic asset supply affects the yields because the Home region is not atomistically small

relative to the Foreign region. Home (Germany) constitutes approximately 8% of the total economy (OECD). However, the falling return on capital has a negative effect on the demand of foreign households for assets. In other words, domestic households asset supply crowds out asset holding of foreign households.

Figure 1: Domestic demographic change



*Notes:* Black solid lines show simulation results for Home variables. Blue dashed lines show simulation results for Foreign variables. Old age dependency ratio (OADR) is defined as the ratio between the population older than 65 and the population younger than 65. Relative OADR shows the ratio between Home and Foreign OADR. Panels (e)-(h) & (l) show relative differences to 2015 level of the respective variable.

As a consequence of lower domestic demand for capital and a rising supply of assets in the Home region, foreign assets of domestic households see strong growth. It is clear to see that there is a strong correlation between net foreign assets (NFA) (panel (i) in Figure ??) and the domestic old-age dependency ratio.<sup>11</sup> In the baseline calibration, the current account (CA) balance would have to reach its peak by around 2025 (panel (k) in Figure ??). However, the balance would also remain positive beyond that time, as the domestic population would

<sup>11</sup>This translates in a strong correlation between the change in the old age dependency ratio and the domestic current account.

continue to age.<sup>12</sup> Here, the response by return on assets has a dampening effect on the current account balance.<sup>13</sup>

## 4.2 Global demographic change

In the next scenario, Foreign (OECD excluding Germany) is now also subject to demographic change in line with the OECD forecast. As in Germany the growth rate of the population younger than 65 is most of the time negative (panel (a) in Figure ??). The growth rate of the population older than 65 is declining but constantly positive (panel (b) in Figure ??). The old-age dependency ratio rises from 27% in 2016 to 56% in 2080. Admittedly, this means that the OADR in Foreign remains below that of the Home region over the long term, but sees similar growth between 2016 and 2080. The level of the relative OADR decrease slightly over time, but with fluctuations (panel (d) in Figure ??). The main reason for the fluctuation is that the baby boomer generation in the OECD is significantly smaller than in Germany.

Similar to the effect of the domestic demographic change on foreign variables, the foreign demographic change affects domestic macroeconomic variables (panel (e) -(h) in Figure ??). In this scenario, the domestic effects are considerably less pronounced compared with the baseline scenario. There is only a small increase in wealth of domestic households. In turn, domestic capital stock actually increases in the following years before turn negative in the long run. However the drop in domestic capital stock is much less pronounced than in the baseline scenario. The number of employed persons as well as output in the Home region is higher than in the baseline scenario.

There is no decline in asset holdings of foreign households in this scenario. This is connected to the fact that households' savings behaviour and the demographic structures in other countries are now changing, too. A direct effect of population ageing abroad is that the number of employed persons in other countries now also declines over time. This leads to lower output abroad, both in absolute terms and per capita.

Population ageing in Foreign is likewise of key importance for domestic foreign assets. The smaller build-up of wealth and less pronounced drop in the number of employed persons in Home lead, on average, to a balanced current account (panel (j) in Figure ??). This means that, in this scenario, demographic change cannot explain the present current account surplus that amount up to 8.9% in 2015, cf. ?). However, there are large fluctuations in the balance,

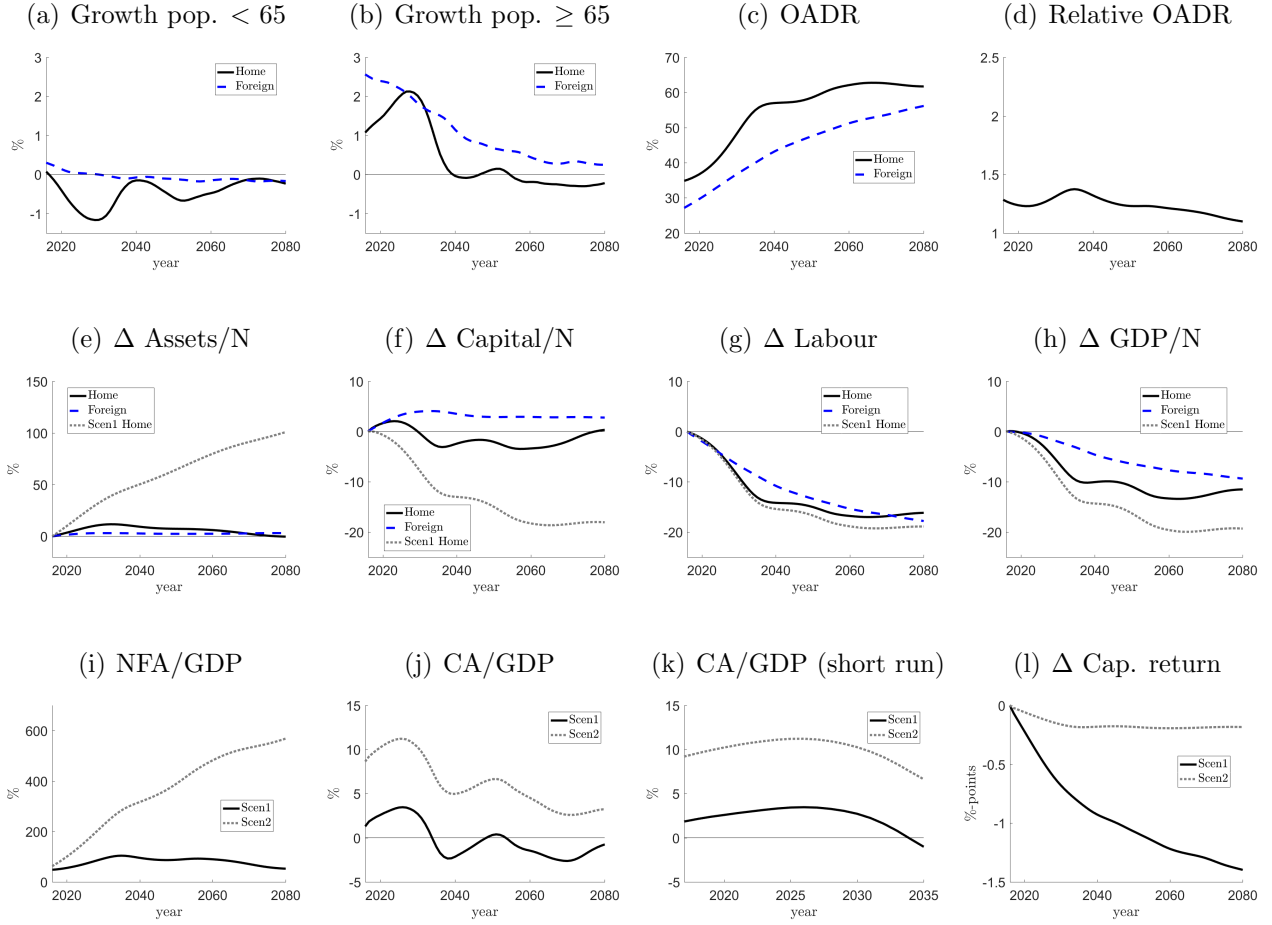
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<sup>12</sup>Analysing the present current account surpluses is not meaningful in this specification of the model, as it does not factor in German reunification. This plays a significant role in the development of German external assets, however. Between 1990 and 2000, German external assets were reimported in their entirety in order to replenish and/or expand East German capital stock. In terms of the model, German reunification would be a kind of negative capital shock that took the economy off its general path of equilibrium. The high capital surpluses in recent years would therefore be interpreted as a process of returning to the path of equilibrium.

<sup>13</sup>If it is assumed that "other countries" refers not only to the OECD but to the rest of the world, then Germany would be a small, open economy and its savings would have no impact on the global return on assets. In comparison with the described scenario, interest rates do not fall and households increase their saving as a result. The additional wealth would largely flow back out abroad, driving the current account surplus up further.



Figure 2: Global demographic change



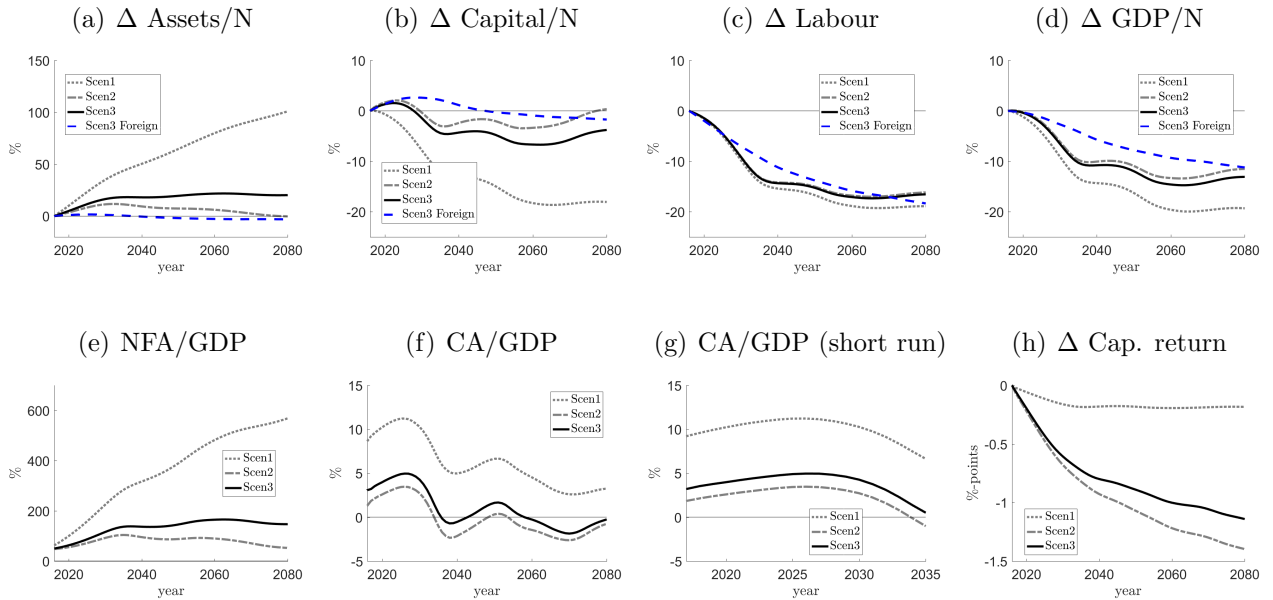
Notes: Black solid lines show simulation results for Home variables in the "Global demographic change" scenario. Blue dashed lines show simulation results for Foreign variables in the "global demographic change" scenario. Grey dotted lines show results of Home variables from previous scenario "Domestic demographic change". Old age dependency ratio (OADR) is defined as the ratio between the population older than 65 and the population younger than 65. Relative OADR shows the ratio between Home and Foreign OADR. Panels (e)-(h) & (l) show relative differences to 2015 level of respective variables.

which are due to the cohort effects mentioned above. As capital demand comes out lower in the aggregate economy (both regions combined), the return on assets also falls much more sharply than in the baseline scenario (panel (l) in Figure ??).

### 4.3 Role of the pension system

One of the main channels through which demographic change affects the current account is saving. However, this depends to a large extent on the characteristics of the pension system. In the pension system scenario, the German pension system (compared to a system with a constant level of pensions) has an effect of increasing the current account balance. The reason for this is that demographic change also leads to a reduction in pension size. In order to maintain the same level of financial security in old age, households must increase their precautionary saving.

Figure 3: Role of the pension system



*Notes:* Black solid lines show simulation results for Home variables in the "role of pension system" scenario. Blue dashed lines show simulation results for Foreign variables in the "role of pension system" scenario. Grey dotted lines show results of Home variables from previous scenario "Domestic demographic change". Grey dashed dotted lines show results of Home variables from previous scenario "Global demographic change". Panels (a)-(d) & (h) show relative differences to 2015 level of respective variables.

The domestic wealth in the economy rises (panel (a) in Figure ??). The increase is smaller compared to the baseline scenario but bigger than in the "global demographic change" scenario. The number of employed persons and output in Germany are higher than in the baseline scenario. Compared to the global demographic change scenario employees and output are almost identical. In turn, domestic capital stock does not fall as sharply as in the baseline scenario.

The negative wealth effect on other countries is also less pronounced than in the baseline

scenario, however stronger than in the global demographic change scenario. Here, it should again be noted that the long-term level of pensions abroad rises in relation to that in Germany due to the assumed differences in the pension formula. The incentive to save for old age is therefore relatively weaker in other countries.

In contrast to the scenario with identical pension systems in Germany and abroad, the domestic current account surplus comes out higher (panel (f) in Figure ??). External assets rise considerably. It will have reached 120% of domestic GDP by 2030 and 178% of domestic GDP as of 2060.<sup>14</sup> Return on assets does not fall quite as sharply as in the previous scenario as now less wealth is built up abroad. This means the price of capital does not fall as sharply as before.

## 5 Pension policy analysis

The previous section highlighted how demographic change impacts the current account and foreign assets. This section deals with reforms of the pension system in the Home region and how they affect the current account. A pension reform impacts both the demand for capital and the supply of capital in a given economy. The effects on the current account do not necessarily have to be in the same direction, however. In theory, it is not always clear which effect ultimately prevails. A quantitative economic model can answer this question, however, given the model assumptions.

In the analysis below, I simulate three different pension reforms, with current German pension legislation used as the baseline. In the first pension reform, the retirement age is gradually raised from 65 to 69 in 2055.<sup>15</sup> The second reform fixes the current pension level at 48% of the average wage. The third reform holds the pension insurance contribution rate steady at 18%.

### 5.1 Pension at 69

A gradual increase in the retirement age from 65 to 69 prolongs the phase in which households are economically active, meaning that the number of persons in work is higher.<sup>16</sup> These additional people in employment require capital. The demand for capital rises, while the current account balance falls.

A higher retirement age also reduces optimal retirement provision. For one thing, the time

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<sup>14</sup>It should be noted that there is also a decline in GDP itself.

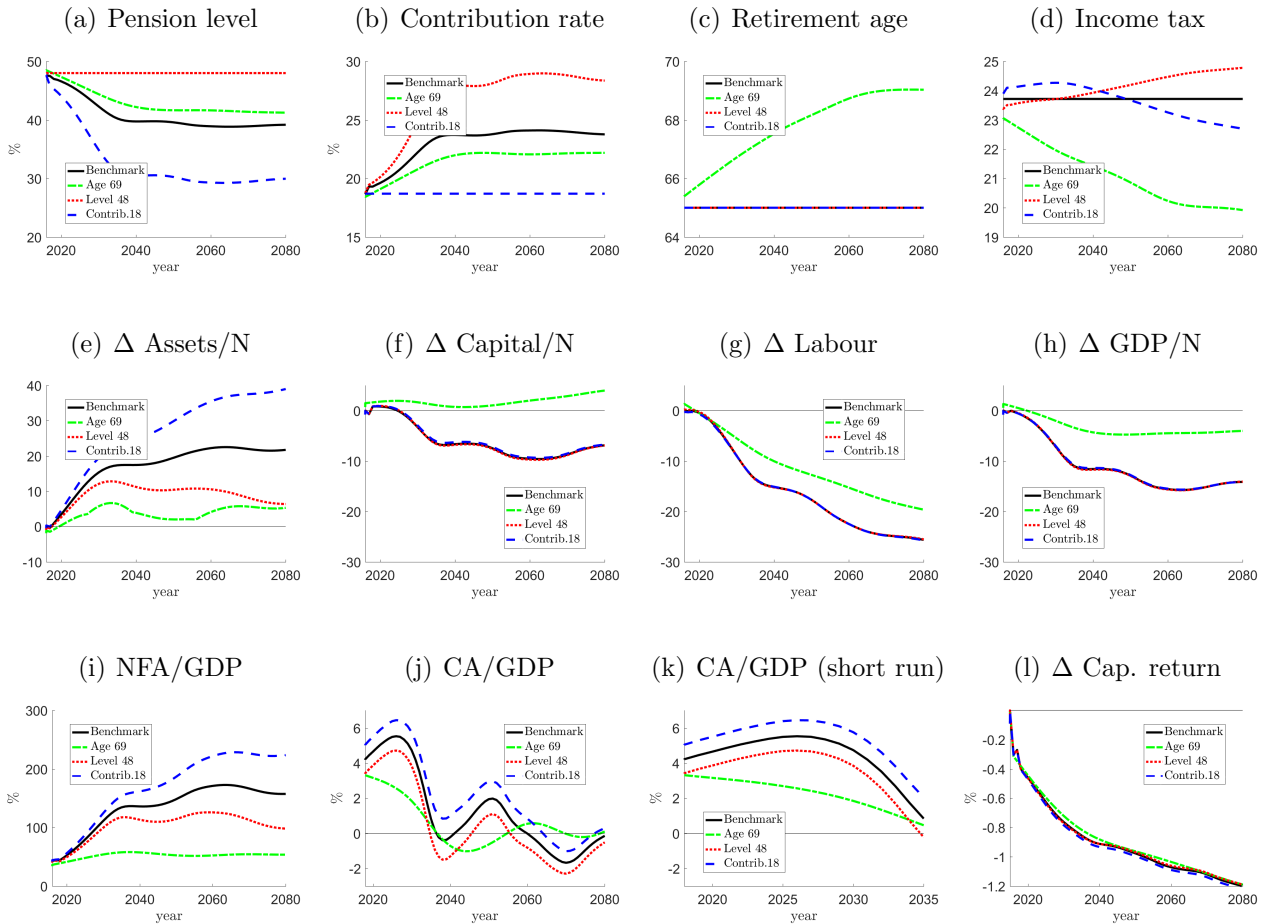
<sup>15</sup>The model period lasts one year, which means that the retirement age can only ever go up by one year. In the real world, however, the retirement age is being increased monthly and in increments. On balance, for the periods measured on a monthly basis. To avoid erratic negative fluctuations in the current account balance I smooth the time series using an HP-filter.

<sup>16</sup>In the model specification used for this paper, early retirement is not an option. This means that the retirement age has a very strong impact on the number of persons in work. This strong effect is not seen in models with flexible retirement age possibilities (early retirement, for example).

in which households draw a pension is shorter. For another, more contribution payers and fewer recipients ease the strain on the pension insurance scheme. The pension level increases relative to the baseline scenario. Both lower the optimal household wealth. Less wealth also means a lower supply of capital, and the domestic current account balance drops, too. A higher labour supply also raises tax revenue. A lower tax rate is sufficient to finance the same government consumption as in the baseline scenario. In turn, this increases the incentive to supply labour.

In theory, a higher retirement age therefore clearly reduces the current account balance. This result is also reflected in panel (i) of Figure ???. The net foreign assets to GDP ratio is 120%-points lower than in the baseline scenario by 2060.

Figure 4: Domestic pension reform



*Notes:* Black solid lines show simulation results for the baseline case. Green dashed dotted lines show simulation results for the scenario with increased retirement age. Red dotted lines show simulation results for the scenario with constant pension level. Blue dashed lines show simulation results for the scenario with constant contribution rate. Panels (e)-(h) & (l) show relative differences to 2015 level of respective variables.

## 5.2 Pension level limited to 48%

The limit on the pension level currently being called for from many quarters would be funded through higher pension contributions and increased taxes. Higher pension contributions would

mean lower net salaries, however, which would lower incentives to supply labour. A lower labour supply also reduces the demand for capital. The current account balance would therefore have to increase as capital can be invested better abroad.

The stipulated pension level would be higher than in the baseline scenario. Households would therefore have fewer incentives to plan for their retirement in this scenario too, as the loss of income in old age would be lower. All other things being equal, a smaller pension provision would lower the current account balance.

On balance, both the demand for and the supply of capital are declining in Germany. The effects on the current account therefore work in opposite directions, and it is theoretically not possible to derive the net impact. Net foreign assets are lower 45%-points lower by 2060 (panel (i) in Figure ??). Current account balance needs to fall.

### 5.3 Pension contribution rate limited to 18%

As opposed to limiting the pension level, this reform fixes the contribution rate at the current German level of 18%. Compared with the growth in the contribution rate assumed in the baseline scenario, the contribution would be lower and the net salary higher. The higher net salary raises the incentive to work. More employed persons require more capital and the current account balance drops.

Due to demographic change, stagnant contribution rates result in a lower pension level. If there is a decline in the pay-as-you-go pension, rational households save more. This increase in private precautionary saving raises the supply of capital and in turn the current account balance, too.

These effects are also theoretically working in opposite directions in this scenario. The reform results in higher demand for capital and higher supply of capital. The effect of the increasing supply of capital dominates the effect of the increasing demand for capital. The domestic current account balance would be lower. Foreign assets would rise in the long term (blue line in panel (i) in Figure ??). By 2060, they would be 50%-points higher than without a reform.

## 6 Conclusion

The OLG model simulations show that, when viewed in isolation, the demographic change in Germany leads to a positive and increasing current account balance. Various pension reforms have different effects on the current account. It turns out that a higher retirement age would reduce the current account surplus if, for this reason, households were to work longer. One explanation for this is that the higher supply of labour raises the demand for capital. Secondly, working longer reinforces financial security in old age and formation of retirement provision is thus significantly reduced.

However, demographic changes in Germany are not sufficient to explain the developments observed in the current account balance. The resulting differences between Germany and abroad are ultimately the key determinants. This concerns, for one thing, demographic trends, but also the design and, under certain circumstances, the pension reforms.

In summary, the following points can be derived from the model simulations: An ageing population affects the current account balance and, *ceteris paribus*, causes it to rise. The effect of demographics on the current account balance is small if other countries boast a similar demographic development and a similar pension system. As Germany is ageing in waves, high current account balances might be evident for a time. A rise in the current account balance should still be expected up until 2028, in particular. The pension system design has an impact on the current account balance. A reduction in pay-as-you-go pension schemes (as assumed for Germany) leads to higher current account balances. A higher retirement age has a negative effect on the current account.

## Acknowledgement

The paper presents the author's personal opinions and does not necessarily reflect the views of Deutsche Bundesbank, the Eurosystem or its staff. Any errors are ours.

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