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House Price Expectations and Housing Choice

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House Price Expectations and Housing Choice

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Preliminary and Incomplete. Comments welcome!

Abstract

What is the role of heterogeneous house-price expectations for boom-bust cycles in the housing market? We exploit a unique Dutch panel data set on households' house price expectations and their consumption, savings and housing choices for the period 2003-2016. This period was characterized by a pronounced boom-bust cycle in the housing market. Conditioning the sample on household heads who report non-zero house price expectations, we find that expectations closely track realized house prices. We next develop a structural life-cycle model of the Dutch housing market where we distinguish household types according to their house price expectations. We employ a calibrated model variant to test if observed variations in expectations can account for the housing boom-bust cycle. First results show that our model closely matches the observed fluctuations of the rent-to-price ratio in the data but overshoots the size of the housing boom.

JEL Classification: D14,D84,D31,E21,E30,G21, R21

Keywords: Housing, survey expectations, house price cycles, life-cycle model

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1 Introduction

Much of the recent literature on macroeconomics and housing is motivated by the international housing boom-bust episode of the early 2000's and the ensuing economic recession. One of the key questions is to identify the forces behind the increase and subsequent collapse in housing. The existing literature provides two alternative views on this question. While Justiniano et al. (2015) and Favilukis et al. (2017) find that shocks to credit conditions are the determining factor in explaining the housing boom and bust, Kaplan et al. (2017) and Landvoigt (2017) stress the role played by expectations/beliefs about future house prices. Thus, in these papers, as in Bailey et al. (2017), expectations or beliefs about future prices developments are central.

Our paper contributes to this literature by first providing direct empirical evidence on the evolution of house price expectations using data of the Dutch housing market. We directly test the potential for observed variations in house-price expectations to drive a boom-bust cycle in house prices. In a nutshell, we feed the observed expectations into a structural macroeconomic model of the Dutch housing market to quantitatively assess the role of expectations for observed house price movements. As the main difference to the existing literature, we thereby explore explicit measures of expectations and do not rely on implicit measures derived from a particular economic model.

As a first step, we document the key characteristics of house price fluctuations and house price expectations in the data. We show that average house price expectations feature the same timing of the boom-bust cycle as the nation wide house price index but with a much smaller amplitude. We next note that a significant number of household heads reports zero house price expectations. Conditioning the sample on household heads who report non-zero expectations we show that their expectations track both the realized own house price growth and the nationwide house price growth very well. It is important to note that we observe the realized house price growth from period $t - 1$ to t and expectations from t to $t + 1$.

Based on this observation we define as *attentive* those households who report non-zero house price growth expectations and accordingly as *inattentive* households those who report zero house price growth expectations. With this definition we investigate how heterogeneity in beliefs affects the moving decision. We find that attentive households are more likely to move than inattentive households when they expect positive house price growth and that they are less likely to move when they expect negative house price growth. Thus transaction decisions are closely linked to expectations. These observations lead us to conjecture that observed heterogeneity in expectations may play a key role for actual house price movements.

To investigate the quantitative role of house price expectations for observed movements in house prices, we proceed by employing a structural macroeconomic models of

the Dutch housing market. Our structural model features many elements that are standard in the quantitative macroeconomics housing literature (Berger et al., 2015; Kaplan et al., 2017) such as a home ownership and a rental markets, idiosyncratic income shocks, a warm glow bequest motive, and long-term mortgage contracts. We abstract from default as this option is essentially not observed in the Netherlands. The main feature of the model is heterogeneity in expectations according to household types with a fixed and exogenous expectations process.

We calibrate the model to the Dutch housing market. Specifically, we adopt our definition of attentive and inattentive households, and calibrate the respective fraction of household types. As their expectation formation we assume that attentive households expect house price growth as measured in the data. That is, from period t to $t + 1$ they expect the observed house price growth from period $t - 1$ to t . In contrast, inattentive households assume zero house price growth. We then adopt the concept of a *temporary equilibrium* approach suggested by Piazzesi and Schneider (2016) to simulate the model forward in time.¹ Accordingly, going from period $t - 1$ to t we take expectations as exogenous, compute decisions, aggregate and clear the housing market in period t . We then move on to the next period, again taking expectations as exogenous. More precisely, in our baseline experiment, we feed into the model the expectations observed in the data. Accordingly, attentive households assume from any period t to $t + 1$ that house prices grow at the rate observed from $t - 1$ to t , whereas inattentive households assume zero house price growth. It is important to note that households act fully rationally based on their expectations.

Our first experiment shows that the price-rent ratio simulated in the model tracks quite well the observed price-rent ratio, with slightly higher fluctuations than in the data. At the same time it overshoots the observed house price boom. To reconcile these two observations, note that our model also roughly matches the observed upward trend of the rental rate until 2013, but again features stronger fluctuations of this data.

In the future, we plan to explore calibrated versions of a rational expectations equilibrium as well and compare these model variants to quantify the role of expectation formation for actual house price movements.

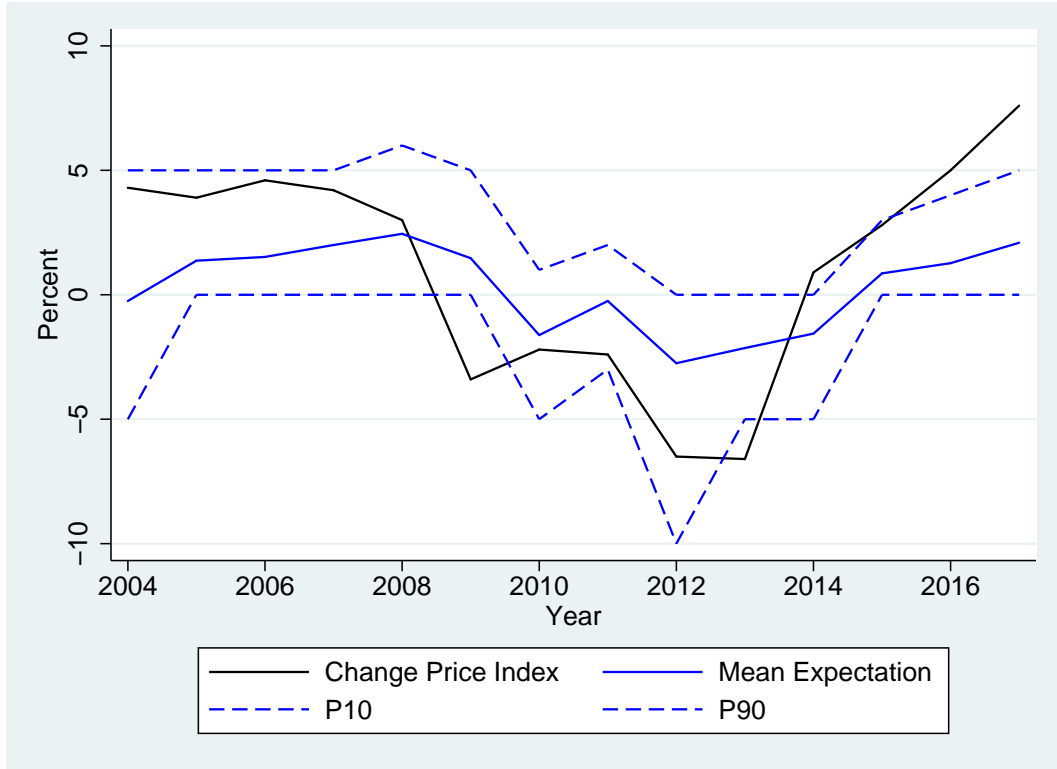
The remainder of this paper is structured as follows. Section 2 describes our data and summarizes the main results of our data analysis. We proceed in Section 3 by developing the quantitative life-cycle model of the Dutch housing market. Section 4 defines the temporary equilibrium approach and describes our choices of functional forms and our calibration. Section 5 documents the results from our first experiments and provides an outlook on future experiments we aim at conducting in the future. Finally, Section 6 concludes the paper. A separate appendix provides more details on our computational procedures.

¹Piazzesi and Schneider (2016) base this notion on early work by Grandmont (1977).

2 Data: Heterogeneity in Expectations

In this section we explore the panel data on household house-price expectations and house-adjustment decisions. In Figure 1 we plot the time series for house price growth and expectations of house price growth. The Netherlands had very fast house price growth around 2000, monotonically declining but positive house price growth until 2008, negative house price growth between 2009 and 2013, and positive house price growth since 2014. House prices increased by about 7% per year on average over the entire period. The average forecast of house price growth is fairly stable around zero (literally zero in 2004, slightly positive 2005-2009, slightly negative 2010-2014, slightly positive since 2015). This average forecast masks a lot of heterogeneity; about 50% of the respondents expect zero house price growth. The forecasts of the remaining 50% are dispersed and move with realized aggregate house price growth.

Figure 1: House Prices and House Price Expectations

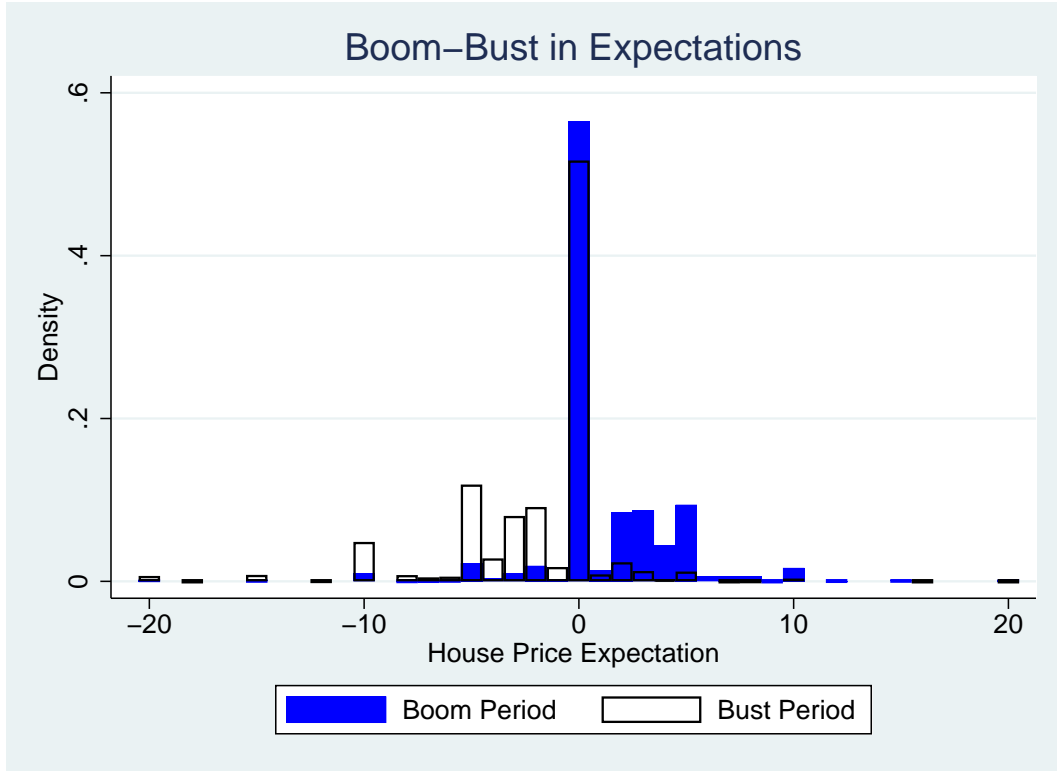


In Figure 2 below we plot the distribution of the forecast of aggregate house price growth for the two phases of the boom-bust episode. The data are generated based on households' response to the two following questions: "What kind of price movement do you expect on the housing market **in the next two years**? Will the housing prices increase, decrease or remain about the same?"² and "How much percentage points a year will they increase/decrease on average?"³

²Bold in the survey.

³We have restricted the data to be within $\pm 20\%$.

Figure 2: Distribution of Expectations

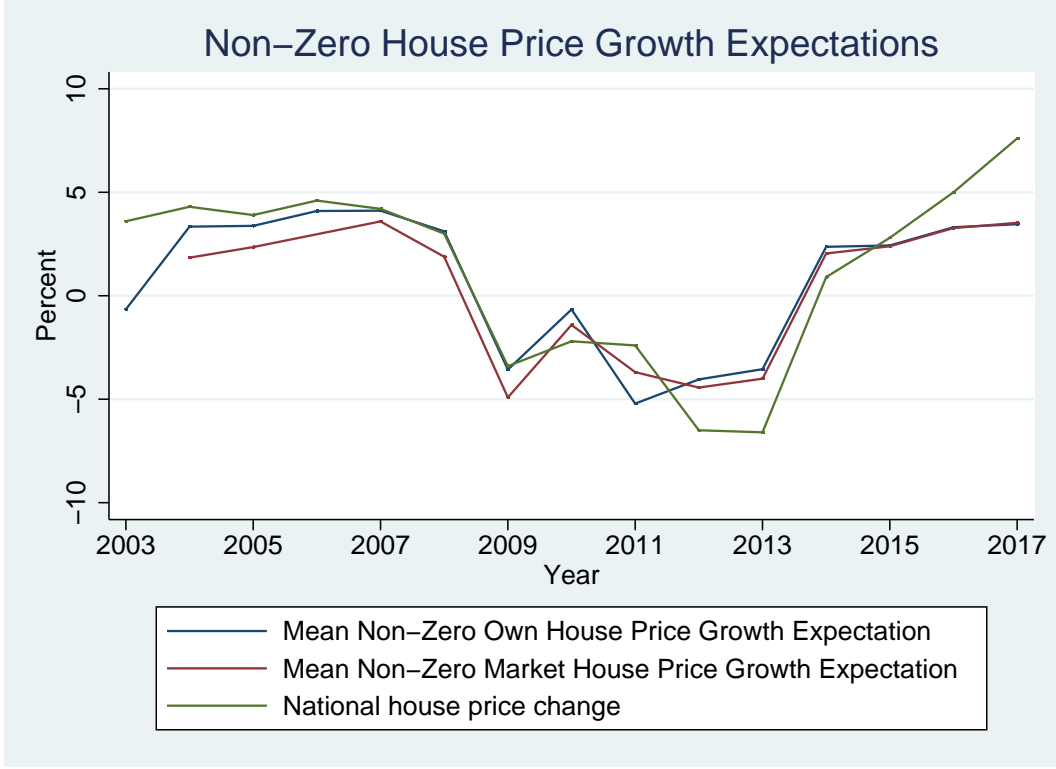


The graph reveals that there is a significant number of households that register an expectation of zero growth in house prices. This is the case both during the boom and bust phases of the housing market cycle. There is also a large number of households that report non-zero house price growth. The distribution of these households is shifted to the right, relative to zero, during the boom phase and to the left during the bust phase.

Focusing on households that register non-zero house price growth, in Figure 3 we show the mean expectation for own and market house price growth, as well as the realized change in house prices. In reading the graph, it is important to recall the timing of the data: at any year t , the data points that are graphed are (i) households' average expected *own home* house price inflation for the next year⁴ reported in year t , (ii) households' average expected *aggregate* house price inflation for the next year reported in year t , and, if applicable, (iii) realized *aggregate* house price inflation between year $t - 1$ and year t ; which is in the household's information set when they register their expectation.

⁴More precisely, their average expected house price growth for the next two years.

Figure 3: House Price Expectations and Market Prices



We see that households who register non-zero house price growth expectations track the market quite closely and expect realized house price changes to continue in the current period. Hence, we posit the following two types of expectations, which we will refer to as *attentive* and *inattentive* for reasons to be explained below:

$$E^i(\pi_{t+1}^H | \pi_t^H) = \begin{cases} 0 & \text{if } i = \text{Inattentive} \\ \pi_t^H & \text{if } i = \text{Attentive} \end{cases} \quad (1)$$

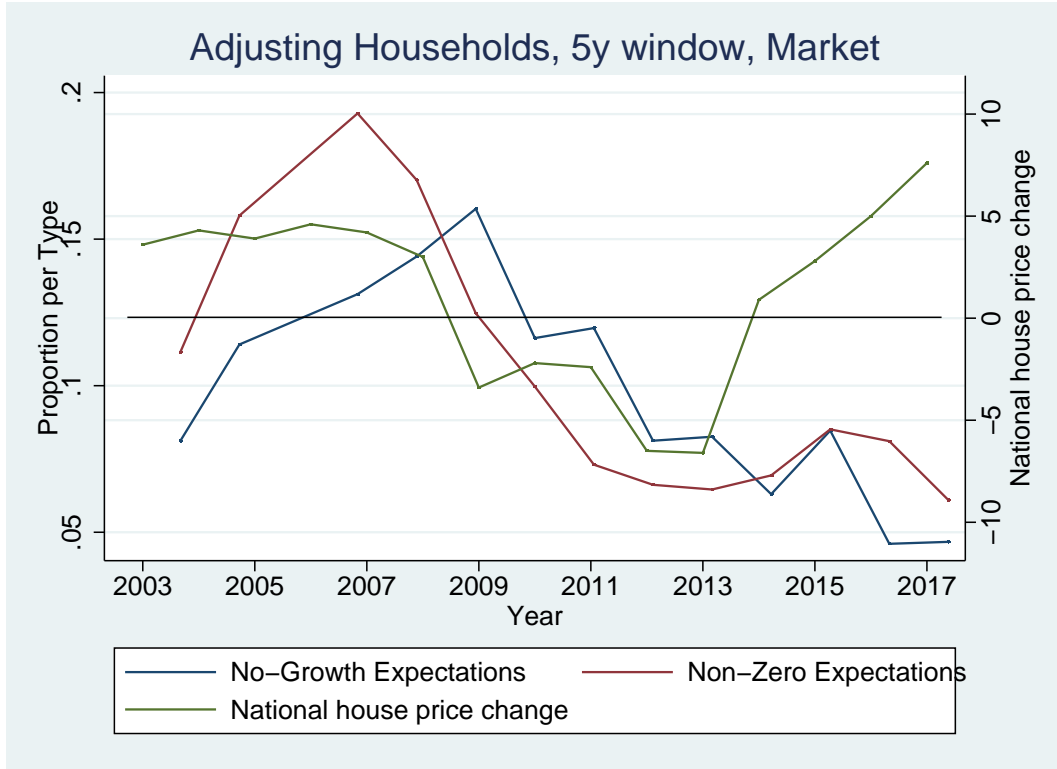
Hence, we summarize that there are broadly two types of agents, based on the expectations they report. But do these expectations vary systematically with their consumption and savings behavior? Lets focus on household adjustment.

Figure 4 plots the time series for the *proportion* of home-adjusting agents, per type of expectations. We construct a *house-adjustment indicator variable* for a window of five years. The *number of respondents* is the cross-sectional sum of the relevant indicator variable by type: (i) No-growth expectations and (ii) Non-zero expectations. This is done using expectations on market prices, and agents are categorized based on the answer they give in the current year.⁵ There is no data on expectations of market prices for 2003 or

⁵More specifically, the *indicator variables* are constructed as follows: set 1 if agent purchased its house two years ago, last year, in the current year, next year, or in two years from the current year; set 0 if agent did not purchase its house two years ago, last year, in the current year, next year, or in two years from the current year. The *proportion per type* for type i at period t is calculated by summing cross-sectionally within type i the relevant indicator variable and dividing by the number of type- i agents in period- t .

2006.

Figure 4: Housing Adjustment and Expectations



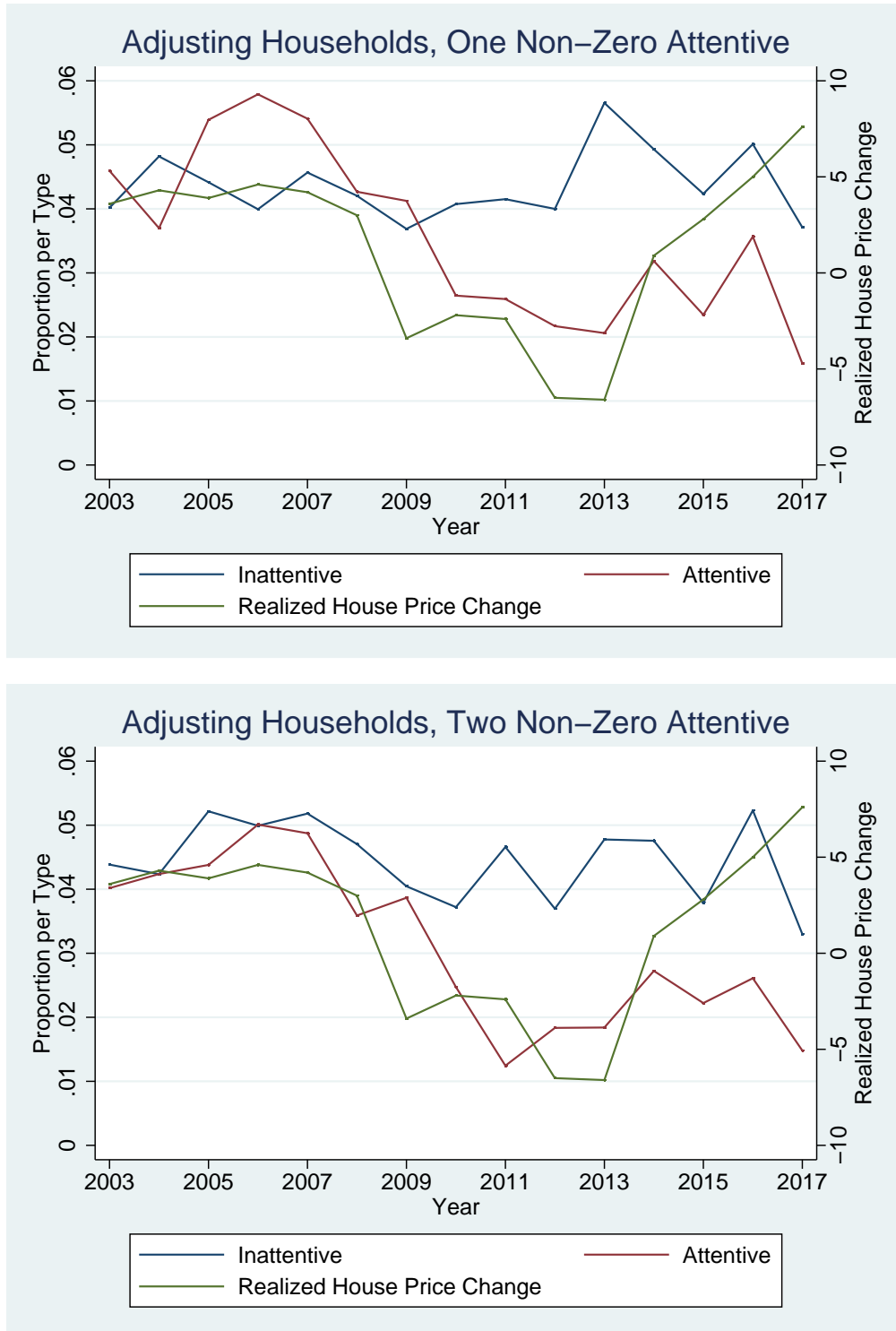
The figure reveals a clear above-below-above pattern. When house price growth is positive, non-zero-growth-expectation households are more likely to move than zero-growth-expectations households. When realized house price growth is negative, the opposite is true: non-zero-growth-expectation households are less likely to adjust their housing consumption than zero-growth-expectations households.

There two natural interpretations of this figure. The first interpretation is that households with higher house price growth expectations are more likely to move. In the boom years (pre-2009 and post-2013), those are on average the non-zero-growth-expectation households. In the bust years (2009-2013), those are on average the zero-growth-expectations households. The second interpretation is that the non-zero-growth-expectation, or attentive, households respond more with their moving decision to realized house price growth. Therefore, in the boom years they move more, and in the bust year they move less.

To discriminate between these two interpretations, in Figure 5 we focus on the zero-growth-expectation households. Some households who report a zero expectation may track house prices. Most households who report a zero expectation probably do not track house prices. To distinguish between the two, we look at consecutive answers for the same household and categorize them as follows. We label a household as *attentive* if it reports (i) at least one non-zero expectation, and (ii) at least two *consecutive* non-zero

expectations. Every other household is classified as *inattentive*.⁶ Note that this implies that a household's type is fixed. For reference, we include the time series for realized market house price changes.

Figure 5: Proportion of Adjusting Households



⁶Note that missing expectations are *de facto* treated as zero-growth expectations with this classification scheme.

Thus, we see that while inattentive households are largely insensitive in their housing choices to changes in aggregate house prices, attentive households respond markedly to movements in house prices by adjusting less their consumption of housing when prices fall. In the quantitative analysis below, we focus on this dimension of heterogeneity.

3 A Structural Housing Model

In order to study the potential for observed variations in house-price expectations – along with fluctuations in the distribution of households over states – to account for the boom-bust cycle in house prices, we turn to a structural model. The model features idiosyncratic income shocks, warm glow bequests, home-ownership and rental markets for housing services, and long-term mortgage contracts. We abstract from default as this option is essentially not observed in the Dutch data set. The model is standard and very close to those of Kaplan et al. (2017) and Berger et al. (2015), but we allow for a small degree of heterogeneity in expectations.

3.1 Households

The household sector is composed of a continuum of households that live for a fixed number of discrete-time periods, $j = \{1, 2, \dots, J\}$, and are indexed at any period $t = \{0, 1, \dots\}$ by the index i . They derive utility from non-durable consumption, C_{it} , and housing services, $S(H_{it}, \tilde{H}_{it})$, according to the instant utility function $U(C_{it}, S(H_{it}, \tilde{H}_{it}))$. Each household receives during its working life a stochastic income, and a deterministic pension after retirement at age J^{ret} . We denote by Y_{it} net earnings from either labor or pension payments. Subject to a no short-selling constraint, households can save in risk-free bonds that pay a net return, r_b , and in discrete housing units, $H_{it} \in \mathcal{H}$ where $\mathcal{H} = \{h^0, \dots, h^{N_h}\}$ and $0 < h^0 < \dots < h^{N_h}$, that sell at unit price P_t . In purchasing housing units, households have the option to finance part of the purchase through a long-term mortgage contract at a fixed rate, r_m , that is subject to an intermediation spread such that $r_m = r_b + \zeta$, where $\zeta > 0$ denotes the spread. Finally, households can rent discrete housing, $\tilde{H}_{it} \in \tilde{\mathcal{H}}$ where $\tilde{\mathcal{H}} = \{\tilde{h}^0, \dots, \tilde{h}^{N_{\tilde{h}}}\}$ and $0 < \tilde{h}^0 < \dots < \tilde{h}^{N_{\tilde{h}}}$, at rental price \tilde{P}_t .

It is understood that the elements in \mathcal{H} and $\tilde{\mathcal{H}}$ represent the size and quality of houses. Further, if a household decides to adjust the size of the house it owns or move into rental housing, it must incur a house-sales transaction cost linked to the size of the house being sold, $\kappa(H_{t-1}) = \theta(1 - \delta)P_t H_{t-1}$. Housing units are subject to a per-period depreciation rate, δ . Households cannot simultaneously own and rent housing.

A household i born in period t wishes to maximize its expected lifetime utility:

$$E_t^i \left[\sum_{j=1}^J \beta^j U \left(C_{it+j}, S \left(H_{it+j}, \tilde{H}_{it+j} \right) \right) + \beta^{J+1} B \left(\tilde{W}_{it+J+1} \right) \right]$$

where $\beta \in (0, 1)$ is the household's subjective discount factor and $B \left(\tilde{W}_{it+J+1} \right)$ represents utility received from the wealth, \tilde{W}_{it+J+1} , it bequeaths to younger generations.

Following Landvoigt (2017), note that given $\zeta > 0$, households will never choose to take out a mortgage and save in bonds at the same time. Therefore, we need only keep track of households' net non-housing (liquid) asset position, which we denote by A_{it} . Mortgage contracts are such that at origination, a *house-adjusting household* is subject to the following borrowing constraint:

$$-A_{it} \leq \lambda_m P_t H_{it}$$

After the mortgage contract has been entered into, a *non-adjusting household* must only make a *minimum* payment while the mortgage is outstanding, according to the constraint

$$A_{it} \geq [1 + r_M - \phi(r_M, j)] A_{it-1}$$

The function $\phi(r_M, j)$ is be defined, following Kaplan et al. (2017), by the constant-amortization formula

$$\phi(r_M, j) = \frac{r_M (1 + r_M)^{J+1-j}}{(1 + r_M)^{(J+1-j)} - 1}$$

Renting households are subject to a no-borrowing constraint $A_{it} \geq 0$. Hence, there are three types of households: (i) adjusting households, (ii) non-adjusting households, and (iii) renting households.

Idiosyncratic labor income, Y_{it} , is given by

$$\begin{aligned} Y_{it} &= g(j_{it}) \chi_{it} \varepsilon_{it} \\ \chi_{it} &= \chi_{it-1}^\rho \nu_{it} \end{aligned}$$

where $\rho \in [0, 1)$, $g(j_{it})$ is a deterministic age-dependent parameter, ε_{it} is a transitory income shock, and ν_{it} is a shock to the persistent component of income, χ_{it} .

The general budget constraint for households is given by

$$\begin{aligned} C_{it} + A_{it} + P_{it} H_{it} + \{H_{it} \neq H_{it-1}\} \theta (1 - \delta) P_t H_{it-1} + \{\tilde{H}_{it} > 0\} \tilde{P}_t \tilde{H}_{it} &= W_{it} \\ W_{it} &= Y_{it} + (1 + r_b + \{A_{it-1} < 0\} \zeta) A_{it-1} + (1 - \delta) P_t H_{it-1} \\ -A_{it} &\leq \begin{cases} \lambda_m P_t H_{it} & \text{if } H_{it-1} \neq H_{it} > 0 \\ -[1 + r_M - \phi(r_M, j)] A_{it-1} & \text{if } H_{it-1} = H_{it} > 0 \text{ and } A_{it-1} < 0 \\ 0 & \text{else} \end{cases} \end{aligned}$$

Where W_{it} denotes the household's beginning-of-period liquid resources and $\{\cdot\}$ denotes the indicator function.

The Housing Capital Gains Mechanism In line with the stylized fact presented in section 2, we assume two types of households: attentive and inattentive. Households' type is fixed throughout their life. They differ in their forecasts of future house-price growth according to equation (1).

Further, we assume that households abstract from aggregate uncertainty (or are risk-neutral with regard to it), in the sense that they take their central forecast for house price growth to be a certain outcome. Also, income risk is assumed to be orthogonal to house price changes. Hence, suppressing the subscript i when unnecessary and letting the household's state vector be given by $X_t = (Y_t, A_{t-1}, H_{t-1}, j_t)$, we can sum up the household's problem as any date t as follows:

$$\begin{aligned} V_t^i(X_t) &= \max_{\{C_t, A_t, d_t\}} \{U(C_t, S(d_t)) + \beta E^i[V_{t+1}^i(X_{t+1}) \mid Y_t, \pi_t^H, A_t, d_t]\} \\ &= \max_{\{C_t, A_t, d_t\}} \{U(C_t, S(d_t)) + \beta E[V_{t+1}^i(E^i(X_{t+1} \mid \pi_t^H, A_t, d_t)) \mid Y_t]\} \end{aligned}$$

where the superscript i denotes the household's expectation type, $i = \{\text{Attentive}, \text{Inattentive}\}$, d_t is the household's housing choice and π_t^H is the change in house prices from period $t - 1$ to period t .⁷

So, how do house price changes affect the household's state vector, X_t ? Recall that the budget constraint reads as follows:

$$C_t + A_t + F(d_t) = W_t \quad (2)$$

where W_t are the household's beginning-of-period liquid resources and $F(d_t)$ denotes housing-related expenses, given by

$$F(d_t) = \begin{cases} P_t H_t + \theta(1 - \delta) P_t H_{t-1} & \text{if } H_{t-1} \neq H_t > 0 \\ P_t H_t & \text{if } H_{t-1} = H_t > 0 \\ \tilde{P}_t \tilde{H}_t + \theta(1 - \delta) P_t H_{t-1} & \text{else} \end{cases} \quad (3)$$

In the presence of movements in the house price, the household must account for potential housing capital gains since the law of motion of W_t is given by

$$\begin{aligned} W_{t+1} &= Y_{t+1} + (1 + r_b + \{A_t < 0\}\zeta) A_t + (1 - \delta) P_{t+1} H_t \\ &= Y_{t+1} + (1 + r_b + \{A_t < 0\}\zeta) A_t + (1 - \delta) P_t H_t (1 + \pi_{t+1}^H) \\ &= Y_{t+1} + (1 + r_b + \{A_t < 0\}\zeta) A_t + (1 - \delta) P_t H_t + \underbrace{(1 - \delta) P_t H_t \pi_{t+1}^H}_{\text{Housing Capital Gains}} \end{aligned} \quad (4)$$

⁷Throughout we will assume that household's expectations of the rental rate obey the user cost of housing formula, and so are implicitly defined by their expectations of house-price growth.

Hence, *conditional on a current-period housing and saving choices*, the household will forecast its future beginning-of-period resources as:

$$\begin{aligned} E^i (W_{t+1} | Y_t, \pi_t^H, A_t, H_t) &= E(Y_{t+1} | Y_t) + (1 + r_b + \mathbb{1}_{\{A_t < 0\}} \zeta) A_t \\ &+ (1 - \delta) P_t H_t + (1 - \delta) P_t H_t E^i (\pi_{t+1}^H | \pi_t^H) \end{aligned} \quad (5)$$

where expected housing capital gains – i.e., the last term of equation (5) – varies by expectation type, i , according to equation (1). Importantly, whether or not the household receives next-period housing capital gains depends on its *current period housing choice*. Hence, the mechanism through which differing expectations about future house price growth influence consumption and savings decisions works through a wealth/endowment effect due to future capital gains.⁸

This implies that the household's problem in recursive form – following Berger et al. (2015) – is as follows; again, suppressing the subscript i when it is not essential.

Definition 1 *Given the environment and vector of state variables defined above, X_t , the household's dynamic programming problem is given by:*

$$V_t^i(X_t) = \max \left\{ V_t^{i,adj}(X_t), V_t^{i,noadj}(X_t), V_t^{i,rent}(X_t) \right\}$$

where

$$\begin{aligned} V_t^{i,adj}(X_t) &= \max_{\{C_t, A_t, H_t\}} \left\{ U(C_t, S(H_t)) + \beta E^i [V_{t+1}(X_{t+1}) | Y_t, \pi_t^H] \right\} \\ s.t. \quad C_t + A_t + P_t H_t &= Y_t + (1 + r_b + \mathbb{1}_{\{A_{t-1} < 0\}} \zeta) A_{t-1} + (1 - \theta)(1 - \delta) P_t H_{t-1} \\ A_t &\geq -\lambda_m P_t H_t \end{aligned}$$

$$\begin{aligned} V_t^{i,noadj}(X_t) &= \max_{\{C_t, A_t\}} \left\{ U(C_t, S(H_{t-1})) + \beta E^i [V_{t+1}(X_{t+1}) | Y_t, \pi_t^H] \right\} \\ s.t. \quad C_t + A_t + \delta P_t H_{t-1} &= Y_t + (1 + r_b + \mathbb{1}_{\{A_{t-1} < 0\}} \zeta) A_{t-1} \\ A_t &\geq \begin{cases} [1 + r_M - \phi(r_M, j)] A_{t-1} & \text{if } A_{t-1} < 0 \\ 0 & \text{if } A_{t-1} \geq 0 \end{cases} \end{aligned}$$

$$\begin{aligned} V_t^{i,rent}(X_t) &= \max_{\{C_t, A_t, \tilde{H}_t\}} \left\{ U(C_t, S(\tilde{H}_t)) + \beta E^i [V_{t+1}(X_{t+1}) | Y_t, \pi_t^H] \right\} \\ s.t. \quad C_t + A_t + \tilde{P}_t \tilde{H}_t &= Y_t + (1 + r_b + \mathbb{1}_{\{A_{t-1} < 0\}} \zeta) A_{t-1} + (1 - \theta)(1 - \delta) P_t H_{t-1} \\ A_t &\geq 0 \end{aligned}$$

⁸Recall that in solving its dynamic programming problem, the household will compare choice-specific (with regard to housing) value functions in order to select its optimal discrete choice. Thus, the expected capital gains, being a function of the current discrete choice (H_t), directly affect the household's current decision on housing.

with the terminal value of the household's value function (at period $T + 1$) given by:

$$V_{T+1}(X_{T+1}) = B\left(\tilde{W}_{T+1}\right)$$

where

$$\tilde{W}_{T+1} = (1 + r_b) A_T + (1 - \delta) P_{T+1} H_T$$

since at T the relevant borrowing constraint is $A_T \geq 0$ and where income follows the process specified above.

The model is solved using the discrete-continuous endogenous grid method (DC-EGM) as in Iskakov et al. (2017). This procedure builds on the EGM of Carroll (2006) and consists in using an exogenous end-of-period (i.e., post-decision) savings grid and the household's smoothed Euler equation to back out an endogenous grid for beginning-of-period financial wealth. The addition of taste shocks to the household's problem helps smooth out kinks in the Euler equation that arise from the presence of discrete choices and, by assuming an i.i.d. extreme value distribution, speeds up computation due to the availability of an analytical expression for the discrete choice probabilities. Lastly, secondary kinds in choice-specific value functions are handled by eliminating segments that fall below the upper envelope of the correspondence. This approach helps eliminate discontinuities in the aggregate demand of housing, which facilitates the computation of equilibrium prices.

4 Temporary Equilibria and Price Dynamics

In order to study the implications of household expectations heterogeneity for aggregate price dynamics, we use the structural housing model to look at the sequence of temporary equilibria generated by the observed distribution of household income, wealth, demographics and expectations. A temporary equilibrium for date t , following Piazzesi and Schneider (2016), is defined as “a collection of prices and allocations such that markets clear given beliefs and agents' preferences and endowments” (p. 1587). That is, a temporary equilibrium is a static concept where the cross-sectional distribution of expectations and wealth is kept exogenous, and market-clearing is imposed. In this sense, a *full-information rational expectations equilibrium* is as sequence of temporary equilibria such that agents' price expectations coincide with their physical measure. Further, again following Piazzesi and Schneider (2016), “a sequence of temporary equilibria is a collection of date t temporary equilibria that are connected via the updating of endowments” (p 1589).

By modeling the dynamic fluctuations of aggregate prices as a sequence of temporary equilibria, we are able to account for the effect of distributional changes (including in expectations) within the household sector on aggregate prices, while remaining agnostic

about the source of such changes. Thus, we incorporate the influence of shifting heterogeneous expectations on house-price growth without taking a stand on the specific expectation-formation process that is behind the observed joint distribution of households. Further, by taking the supply of assets directly from the data, we do not need to explicitly model the supply side of the economy. And given that our results arise from an exogenous sequence of joint distributions, they will continue to hold for any model that delivers an identical sequence of equilibrium distributions – regardless of the source of fluctuations and supply-side dynamics. Finally, note that we do not need to treat the distribution of households as a state variable in the household’s dynamic programming problem since this is only relevant (in the presence of aggregate risk) when it informs the household’s price expectations, which we already directly observe.

Next, we provide a formal definition of the temporary equilibrium in the environment we study and assuming a small open economy. Let \mathcal{E} be the set of possible income realizations, $\mathcal{A} = \mathbb{R}$ the set of possible non-housing assets held by the household, \mathcal{H} the set of possible housing assets, as defined above, and \mathcal{J} the set of possible ages. Let $X = (Y, A, H, j)$ and $\mathcal{X} = \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{J}$, where it is understood that timing convention for X is as described in the recursive formulation of the household’s problem. Further, let $\mathcal{P}(\iota)$ and $\mathcal{B}(\iota)$ denote the power set and the Borel σ -algebra of ι , respectively. Finally, let \mathcal{M} be the set of all probability measures on the measurable space $(\mathcal{X}, \mathcal{B}(\mathcal{X}))$.⁹

Definition 2 *Given interest rates r_b and r_m , a fixed supply of owner-occupied and rental housing – \bar{H}_t and $\tilde{\bar{H}}_t$, respectively –, and a joint distribution of endowments, assets, housing, ages and expectation types, Φ_t , a temporary equilibrium is a set of value and policy functions $V_t^i : \mathcal{X} \rightarrow \mathbb{R}$, $h_t^i : \mathcal{X} \rightarrow \mathcal{H}$, and $\tilde{h}_t^i : \mathcal{X} \rightarrow \mathcal{H}$, where the superscript i denotes agents’ expectations-type, and prices P and \tilde{P} such that*

1. *The functions V_t^i , h_t^i , and \tilde{h}_t^i are measurable with respect to $\mathcal{B}(\mathcal{X})$, the function V_t^i satisfies the household’s Bellman equation and functions h_t^i and \tilde{h}_t^i are the associated policy functions; given P , \tilde{P} , $E^i(P)$ and $E^i(\tilde{P})$.*

2. *Markets clear*

$$\begin{aligned}\bar{H}_t &= \int h_t^i d\Phi_t \\ \tilde{\bar{H}}_t &= \int \tilde{h}_t^i d\Phi_t\end{aligned}$$

Further, again following Piazzesi and Schneider (2016), “a sequence of temporary equilibria is a collection of date t temporary equilibria that are connected via the updating of endowments” (p 1589). Additional discussion is provided in appendix A.

⁹Where $\mathcal{B}(\mathcal{X}) = \mathcal{P}(\mathcal{E}) \times \mathcal{B}(\mathcal{A}) \times \mathcal{P}(\mathcal{H}) \times \mathcal{P}(\mathcal{J})$.

Computationally, the problem of finding the date t temporary equilibrium is as follows. (i) From the data set construct a matrix where each row represents a household and columns register the year, assets, owned housing value, rental housing value, income state, assets state, housing state, age and expectation type. (ii) Define a function that computes for a given price vector, (P_t, \tilde{P}_t) , a vector of excess demand, $(R^H, R^{\tilde{H}})$, as follows:

$$\begin{aligned} R^H &= P_t \sum_{n=1}^N \left[h(Y_t^n, A_{t-1}^n, H_{t-1}^n, j_t^n, E(\pi_{t+1}^H | \pi_t^H)^n) \right] - \sum_{n=1}^N (PH)_t^n \\ R^{\tilde{H}} &= \tilde{P}_t \sum_{n=1}^N \left[\underbrace{\tilde{h}}_{\text{Model}} \left(\underbrace{Y_t^n, A_{t-1}^n, H_{t-1}^n, j_t^n, E(\pi_{t+1}^H | \pi_t^H)^n}_{\text{Data}} \right) \right] - \sum_{n=1}^N \underbrace{(\tilde{P}\tilde{H})_t^n}_{\text{Data}} \end{aligned}$$

where $n \in N$ denotes a household's number in the sample. (iii) Finally, use a root-finding algorithm to derive the price vector, (P_t, \tilde{P}_t) , such that markets clear, i.e., $(R^H, R^{\tilde{H}}) = (0, 0)$.

In this way, the sequence of temporary equilibria generated by the model allows us to map the *observed* sequence of distributions over expectation types and states, $\{\Phi_t\}_{t=2003}^{t=2016}$, to a sequence of price vectors, $\{(P_t, \tilde{P}_t)\}_{t=2003}^{t=2016}$, which includes the boom-bust cycle in the housing sector in the Netherlands.

4.1 Functional Forms and Calibration

In this section, we specify the functional forms relating to households' preferences and discuss calibration.

Functional Forms Households' instantaneous utility function, following Landvoigt (2017) and Berger et al. (2015) is given by:

$$U(C_t, S(H_t, \tilde{H}_t)) = \frac{[C_t^{1-\sigma} S(H_t, \tilde{H}_t)^\sigma]^{1-\gamma} - 1}{1-\gamma}$$

where $S(\cdot)$ is linear in its first two arguments and given by (where ϖ captures social housing):

$$S(H_t, \tilde{H}_t) = \omega H_t + \tilde{H}_t + \varpi \quad \text{where} \quad \varpi \geq 0 \quad \text{and} \quad \omega \geq 1$$

The specification for utility due to bequests is taken from Kaplan et al. (2017), who follow De Nardi (2004),

$$B(\tilde{W}) = \vartheta_1 \frac{(\tilde{W} + \vartheta_2)^{1-\gamma} - 1}{1-\gamma}$$

Calibration The parameter values that are calculated directly from the data are reported in Table 2 and the targeted moments for the parameters chosen within the model are reported in Table 3 below. Further, we take the distribution over assets and house holdings, and the demographic distribution from the data.

Table 2: Parameters Fixed to Data-Counterparts

Parameter	Interpretation	Value in Data
Demographics		
j	Period length in years	1
J	Length of life	80
J^{ret}	Retirement age	65
J^{born}	Age of newborns	25
Income Process		
$\{g(j_t)\}$	Deterministic age profile polynomial order	4
rr	Replacement rate	0.80
ρ	Autocorrelation of persistent component	0.9669
σ_χ^2	Variance of persistent shock	0.0146
σ_ε^2	Variance of transitory shock	0.2908
Financial Instruments		
r_b	Risk-free rate	0.03
ζ	Mortgage loan markup	0.01
λ_m	Maximum LTV ratio on mortgage loans	0.90

As shown in table 2, the model is specified at an annual frequency, with households starting their working life at age 25, retiring at age 65 and dying at age 80. The replacement rate is set to 0.80,¹⁰ in accordance with the data. The estimation of the income process follows Storesletten et al. (2000, 2004). Finally, the share of housing services is set to 0.15 following Kaplan et al. (2017). The risk-free rate is set to 3% p.a. The mortgage loan mark-up is set to the period average of 1% p.a., and the maximum LTV ratio is set to 0.90 equal to the estimated average in the data.

The parameters to be chosen within the model are (i) the discount factor, (ii) additional utility from home-ownership, (iii) the strength of the bequest motive, (iv) the degree of luxuriousness of bequests, and (v) the rental and owner-occupied housing grids. We assume a small open economy and estimate the temporary equilibrium of the model while targeting the distributional moments obtained from the average over the entire period of analysis, from 2003 to 2016. The parameter values and targeted moments are reported in tables 3 and 4, respectively. The algorithm employed for calibration of the model is detailed in appendix B.

The housing sector supply side is approximated by discrete housing bins for both rental and owner-occupied housing. Following Kaplan et al. (2017), the owner-occupied housing set is constructed through a log-linear grid where the minimum grid point, the number of

¹⁰Recall that this does not refer to income received from a government-managed pension system, but captures all of the non-interest income households receive after retirement.

Table 3: Calibrated Parameter Values

Parameter	Interpretation	Value
β	Discount factor	1.015
ω	Additional utility from owning	0.747
ϑ_1	Strength of bequest motive	1582.326
ϑ_2	Luxuriousness of bequests	43.063
\mathcal{H}	Owned Housing Bins	in text
$\tilde{\mathcal{H}}$	Rental Housing Bins	in text

grid points and the (fixed) difference between grid points are chosen by targeting the 10th percentile (69.10%), the mean (90.94%) and the 90th percentile (100.00%) percentiles of the distribution of housing net wealth to total net wealth. For rental units of housing, we follow an analogous strategy but target the distribution of rental housing expenditures to net income; where the corresponding percentiles (now using the 50th percentile instead of the mean) are 17.99%, 37.38% and 55.37%, respectively. Additionally, we take from the literature the house depreciation rate, $\delta = 0.03$, and the house sales transaction cost, $\theta = 0.07$, from studies of the Dutch housing sector (see Van Ommeren and Van Leuvensteijn (2005) and Sánchez and Andrews (2011)). Finally, following Fernandez-Villaverde and Krueger (2011), the parameter governing the value of social housing is set low enough for it to be irrelevant for the quantitative properties of the model.¹¹

Table 4: Target Data Moments

Targeted Moment	Data	Model
Average financial assets	-0.454	-0.248
Home-ownership rate	0.748	0.773
Median $NW_{j=75}$ / Median $NW_{j=50}$	1.444	1.330
Percent of bequest HHs in bottom half of NW dist.	0.112	0.080
Housing/Net Worth 10th percentile	0.700	0.797
Housing/Net Worth mean	0.916	0.948
Housing/Net Worth 90th percentile	1.000	1.000
Rent / Income 10th percentile	0.178	0.054
Rent / Income 50th percentile	0.316	0.133
Rent / Income 90th percentile	0.554	0.277

The calibration implies a rent-to-price ratio of 0.12. Even though the rent-price ratio is not targeted in our estimation procedure, its value in the model, 0.12, is not that far from the value in the data, 0.07.

5 Results

In this section we study the role of household house-price expectations in the housing boom-bust cycle.

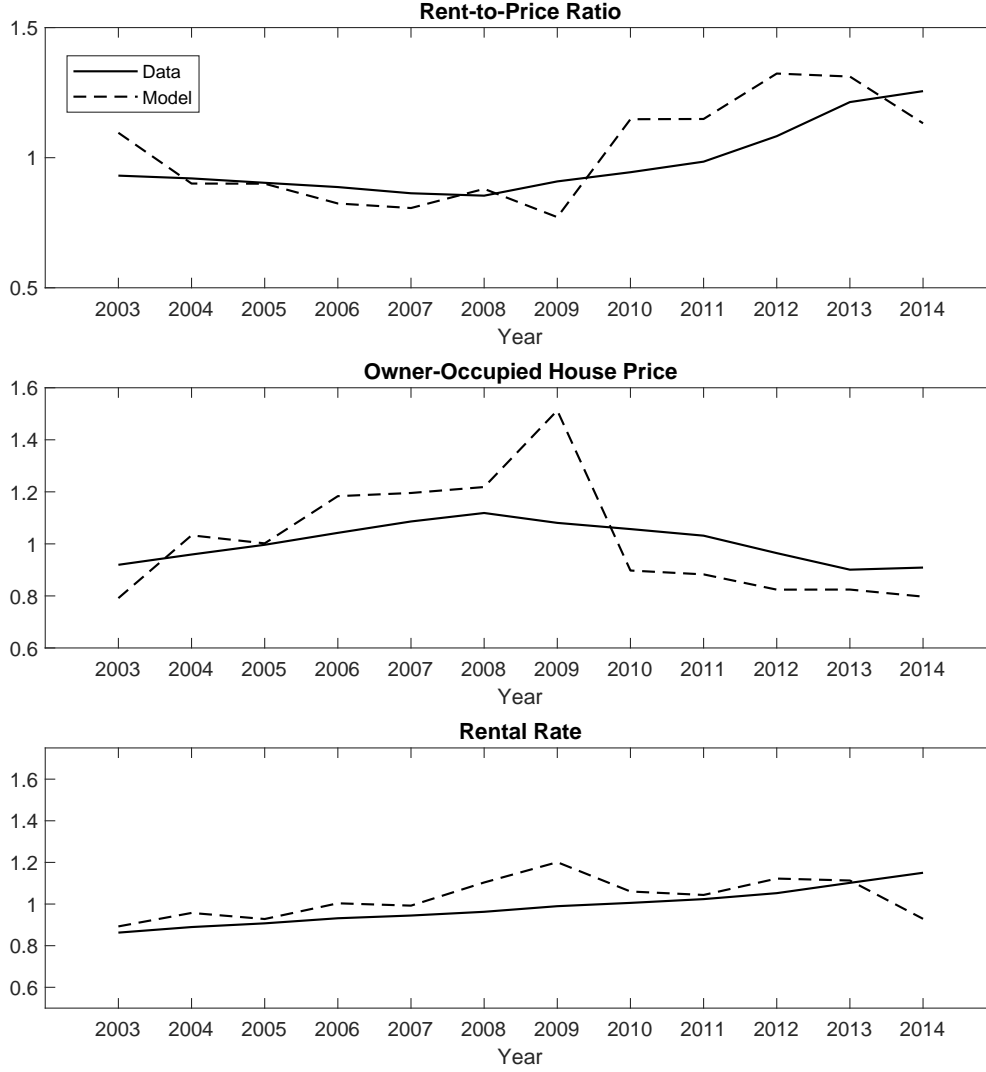
¹¹In practice, this is achieved by setting $\varpi = 0.000001$, implying that only about 1% of households choose to not consume positive levels of rental or owner-occupied housing. Additionally, we smooth out the household dynamic programming problem by including taste shocks as in Iskhakov et al. (2017).

5.1 Temporary Equilibria Sequence with Extrapolative Expectations

In this section we present the sequence of temporary equilibria generated by the model and compare it to the data. This exercise is reported in figure 6 below. Specifically, the figure plots the time series for the aggregate house price and rental rate in the Netherlands, along with the sequence of temporary equilibria generated by the model. For every period, we employ the corresponding cross-sectional distribution over states and expectation types, Φ_t , and set attentive households' expectations according to equation (1). All series are normalized to their mean. Note that the aggregate series is taken directly from Statistics Netherlands, while the sequence of cross-sectional distributions generating the model prices are constructed from the DNB Household Panel Survey; and recall that none of the plotted series are targeted in calibration.

The model does a fairly good job of matching the general pattern in house prices and rental rates, and hence also in the rent-to-price ratio. This serves as a direct validation of the potential for household expectations to play a key role in driving boom-bust cycles in house prices. However, the model overshoots both the boom and bust phases of the episode. This could be because (i) extrapolative expectations lead to too strong an expected capital gains mechanism, that is households expect prices to rise in the future, therefore demand a lot of housing in the current period, thereby driving up current prices or (ii) the absence of aggregate risk in the model leads households to underestimate the risk from fluctuating house prices.

Figure 6: Sequence of Temporary Equilibria



5.2 Prices and Expectation Types

In this section we carry out a series of counter-factual experiments to better understand the contribution of different expectation types on house-price fluctuations. We begin by comparing the series computed above with the cases where (i) all households are attentive, (ii) all houses are inattentive, (iii) the average expectation is held constant, but there are two types of households, and (iv) all households hold the same expectation.

TBC

5.3 Prices and Expectations

In this section we look at the relationship between equilibrium prices and house-price expectations, while holding constant the distribution over states. Specifically, we plot the equilibrium rent-to-price ratio, the price of owner-occupied housing units and the rental rate against the expected house-price growth of attentive households. Note that each households' type is unchanged across equilibria. We employ the distribution over the entire period, that is, we pool all observations. In this way, we isolate the effect of house-price expectations on equilibrium prices.

TBC

6 Conclusion

TBC

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Appendix

A A Note on Equilibrium Concepts

A.1 Two Related Equilibrium Concepts

There are three equilibrium concepts:

1. Temporary equilibrium
2. Partial equilibrium
3. Modified temporary equilibrium

and two types of agents

1. Inattentive agents
2. Attentive agents

A.1.1 Expectations by Agent Type

Before turning to the equilibrium concepts, let's define expectations of the two types of agents formed in any period t . Inattentive agents have static expectations on house price dynamics whereas attentive agents form expectations on the basis of house price growth. We define expectations on the basis of some current period price \tilde{p}_t . The equilibrium concepts are distinguished by a definition of \tilde{p}_t (to be made precise below) and an according cross-sectional distribution $\tilde{\Phi}_t$.

Using \tilde{p}_t , expectations are as follows:

1. Inattentive agents:

$$Ep_{t+1} = \tilde{p}_t$$

2. Attentive agents:

$$\begin{aligned} E\pi_{t+1} &= \rho\pi_t \\ \pi_t &= \ln \tilde{p}_t - \ln \tilde{p}_{t-1} \\ \Leftrightarrow Ep_{t+1} &= \tilde{p}_t + E\pi_{t+1} = \tilde{p}_t + \rho\pi_t \end{aligned}$$

A.1.2 Equilibrium Concepts

To compute the equilibrium, we assume that agents act under point expectations, i.e., we solve the model under the assumption that $p_{t+1} = Ep_{t+1}$. This enables us to compute the equilibrium using methods for models without any aggregate risk.

The three equilibrium concepts are as follows:

1. Temporary equilibrium.

We compute a temporary equilibrium taking as given the data observations on (i) the equilibrium house prices and (ii) the cross-sectional distribution, thus

$$\tilde{p}_t = p_t^d \quad \text{and} \quad \tilde{\Phi}_t = \Phi_t^d$$

where superscript d denotes a data object.

Market clearing in this model gives an equilibrium price \hat{p}_t which does almost surely not coincide with the data equilibrium price p_t^d . This implicitly defines a shock ϵ_t . Assuming that the post-shock equilibrium price coincides with the data price $p_t = p_t^d$, this implies that $p_t = \hat{p}_t + \epsilon_t = p_t^d + \epsilon_t$, i.e., $\epsilon_t = p_t^d - \hat{p}_t$, but this shock does play no further role in the model. The reason is that the shock would affect the cross-sectional distribution but since we aggregate with the data distribution Φ_t^d , the implicit shock realization does not appear explicitly in the model computations.

Thus:

- (a) Given the price expectations of the two types we compute policy functions.
- (b) Given the policy functions we aggregate and compute the equilibrium price in the housing market \hat{p}_t by aggregating with the data distribution Φ_t^d .
- (c) Upon market clearing (=convergence on \hat{p}_t), the MIT shock hits the economy such that the post-shock equilibrium price is $p_t = p_t^d$, which implies $p_t = \hat{p}_t + \epsilon_t$, i.e., $\epsilon_t = p_t - \hat{p}_t = p_t^d - \hat{p}_t$.
- (d) However, the shock does not play an explicit role in the computations because $\Phi_t = \Phi_t^d$.

Thus, in the temporary equilibrium there is no link between time periods t and $t+s$, for $s \geq 1$.

2. Partial equilibrium.

We compute a partial equilibrium, i.e., a market clearing price p_t in any period t and continue to base expectations on the data price p_t^d . We aggregate with the cross-sectional distribution that is implied by the policy functions and the exogenous laws

of motion (shock processes) in the respective period t , thus

$$\tilde{p}_t = p_t^d \quad \text{and} \quad \tilde{\Phi}_t = \hat{\Phi}_t$$

In this variant, a crucial distinction is between a beginning of period and an end of period cross-sectional distribution. The MIT shock $\epsilon_t = p_t^d - \hat{p}_t$ now plays a crucial role, where again \hat{p}_t is the pre-shock equilibrium price:

- (a) We enter period t with a cross-sectional distribution Φ_{t-1} , which is the end of period cross-sectional distribution from period $t - 1$.
- (b) Given the price expectations of the two types we compute policy functions.
- (c) Given the policy functions we aggregate and compute the equilibrium price in the housing market \hat{p}_t and associated cross-sectional distribution $\hat{\Phi}_t$. Observe that $\hat{\Phi}_t = G(\Phi_{t-1}, \eta_{t-1}, \eta_t)$, where $G(\cdot)$ maps Φ_{t-1} into $\hat{\Phi}_t$ through policy functions (endogenous laws of motion) and income shocks η_t (exogenous laws of motion).
- (d) Upon market clearing (=convergence on \hat{p}_t implying the equilibrium cross-sectional distribution $\hat{\Phi}_t$), the MIT shock hits the economy such that the post-shock equilibrium price is $p_t = p_t^d$, which implies $p_t = \hat{p}_t + \epsilon_t$, i.e., $\epsilon_t = p_t - \hat{p}_t = p_t^d - \hat{p}_t$.
- (e) This gives rise to a post-shock end of period cross-sectional distribution, which is implied by the distribution $\hat{\Phi}_t$ and the shock, thus $\Phi_t = H(\hat{\Phi}_t, \epsilon_t)$.

Thus, in the partial equilibrium time periods t and $t+s$, for $s \geq 1$ are linked. Specifically, the cross-sectional distribution Φ_t is $\Phi_t = H(\hat{\Phi}_t, \epsilon_t) = H(G(\Phi_{t-1}, \eta_{t-1}, \eta_t), \epsilon_t)$.

3. Modified temporary equilibrium.

In the modified temporary equilibrium we base expectations on the current equilibrium price \hat{p}_t , i.e.,

$$\tilde{p}_t = \hat{p}_t \quad \text{and} \quad \tilde{\Phi}_t = \Phi_t^d$$

B Calibration Algorithm

In this section, we describe the calibration algorithm.

Algorithm 1 *This algorithm calibrates the model. Note that the ratio of equilibrium prices is not a targeted object.*

1. Start with an initial guess for the parameter values, ϕ^0 .

2. Given current guess for parameters, ϕ^k , compute the price vector of the temporary equilibrium, \bar{P}^k ; where k denotes the iteration step.
 - Note that this step implies finding the price vector that clears the owner-occupied and rental housing markets.
3. Compute the relevant moments implied by the equilibrium, $\{m(\bar{P}^k)\}$.
4. Check the value for the pre-specified metric, $\|m(\bar{P}^k) - m^d\|$, where m^d denotes the targeted data moments.
 - i. If the metric is lower than the tolerance level, $\|m(\bar{P}^k) - m^d\| < \varepsilon$, **stop**.
 - ii. Otherwise, $\|m(\bar{P}^k) - m^d\| \geq \varepsilon$; **update guess** as follows.
5. Given the current iterate price of housing, \bar{P}^k , compute ϕ^{k+1} s.t. $\|m(\bar{P}^k) - m^d\| < \varepsilon$.
6. Given the new guess ϕ^{k+1} , go back to Step 2.