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**Conference Paper**

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Labor Economics - Wages, No. C07-V1

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Kerndler, Martin (2019) : Size and persistence matters: Wage and employment insurance at the micro level, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2019: 30 Jahre Mauerfall - Demokratie und Marktwirtschaft - Session: Labor Economics - Wages, No. C07-V1, ZBW - Leibniz-Informationszentrum Wirtschaft, Kiel, Hamburg

This Version is available at:

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# Size and persistence matter: Wage and employment insurance at the micro level

Martin Kerndler\*

May 19, 2019

## Abstract

Firms provide substantial insurance against wage fluctuations and job loss. This paper studies how the interaction between shock size and persistence affects the firm's ability to insure workers against idiosyncratic firm-level shocks. Using linked employer-employee data from Germany, I find that wages respond largely symmetrically to positive and negative permanent shocks. Whereas transitory shocks lead to upward wage rigidity. Individual layoff probabilities only increase in response to negative permanent shocks. Interestingly, wage cuts and job loss after negative shocks are limited to blue-collar workers. Whereas white-collar workers are fully insured against negative shocks both in terms of wages and employment.

*Keywords:* wage insurance, layoffs, linked employer-employee data, Kalman filter

*JEL classification:* C33, D22, J33, J41

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# 1 Introduction

A common empirical finding is that wages of job stayers fluctuate relatively little with economic conditions. This applies both at the macro level with respect to business cycle indicators (Bils, 1985; Devereux, 2001; Haefke et al., 2013) and at the micro level with respect to firm-specific performance (Bronars and Famulari, 2001; Guiso et al., 2005; Card et al., 2018). Firms are therefore important providers of insurance since workers who stay with their employer can expect a relatively smooth income stream. On top of this, firms partly insure workers against job loss by hoarding labor during bad times (Fay and Medoff, 1985; Burnside et al., 1993). How much wage and employment insurance firms can provide depends on certain properties of the occurring shocks.

First, some types of risk are better insurable than others. In particular, idiosyncratic shocks can be better diversified by the firm owners than aggregate shocks, which are common to all firms. Carlsson et al. (2016) show that wages respond three times as much to productivity shocks that are shared with firms within the same sector as to purely idiosyncratic productivity shocks. Much of the literature on wage insurance has therefore focused on idiosyncratic shocks, which are also the focus of this study.

Second, the persistence of shocks matters for wage insurance. This was first documented empirically by Guiso et al. (2005). Based on a time series decomposition, the authors distinguish between temporary and permanent changes in firm performance and estimate micro wage elasticities separately for each type of shock. Applied to linked employer-employee data from Italy, the authors find that workers' wages are insensitive to temporary shocks, and that only permanent shocks are transmitted. However, the estimated wage elasticity of 0.069 indicates that even fully persistent changes in firm performance are smoothed substantially. Replication studies for Portugal (Cardoso and Portela, 2009), Germany (Guertzgen, 2014), Hungary (Kátay, 2016), and Norway (Fagereng et al., 2017) reach similar conclusions. Whereas Juhn et al. (2018) estimate a much lower wage elasticity for the United States.

Finally, the size or direction of shocks affects wage flexibility at the micro level. While downward rigidity of the aggregate wage is a long-known phenomenon, international evidence from the International Wage Flexibility Project (Dickens et al., 2007) emphasizes downward wage rigidity in the wages of job stayers. To this purpose, the observed distribution of individual wage changes is compared to a hypothetical symmetric distribution. The more right-skewed the observed distribution, the more pronounced is downward wage rigidity. Cross-country comparisons reveal that downward wage rigidity is a general property of employment relations in Europe and in the United States. Whether the rigidity applies to nominal wages or real wages depends on labor market institutions and country-specific wage setting practices (Messina et al., 2010). The extent of downward wage rigidity varies with firm and worker characteristics (Du Caju et al., 2007).

The goal of this paper is to combine the insights of the two lines of literature surveyed above, which have so far evolved in separation. It stresses how the *interaction* between persistence and size of idiosyncratic shocks shapes wage insurance at the firm level. This is a relevant question,

since the firm’s ability to spare workers from wage cuts after a negative shock may depend on the expected duration of the setback. Since firms can also adjust to negative shocks through downsizing, the analysis is extended to layoffs as well. This is one of the first papers to analyze these interaction effects (possibly alongside [Juhn et al. \(2018\)](#), see below) and it is the first study that extends the analysis to individual layoff probabilities.

The estimation strategy proposed in this paper draws on the methodology of [Guiso et al. \(2005\)](#) but allows more flexibility in modeling the functional dependence between firm-level shocks and worker-level outcomes. It follows a two step procedure. The first step is a firm-level regression that isolates idiosyncratic shocks to firm performance and identifies the stochastic process that generates these innovations, which consists of a transitory and a permanent component. The difference to [Guiso et al. \(2005\)](#) comes in the second step at which individual wage elasticities are estimated. An advantage of their approach is that the two wage elasticities with respect to transitory and permanent shocks are separately identified by the *total* shock and certain orthogonality conditions. It is therefore not necessary to decompose the total shock into its unobserved transitory and permanent component. For this reason, I refer to the method of [Guiso et al. \(2005\)](#) as the *indirect method* in the following. A caveat of this method is that identification requires wage growth to depend linearly on growth in firm performance. This implies a constant wage elasticity and therefore rules out size effects. The more flexible approach proposed in this paper is the *direct method*. The observed total shock is explicitly decomposed into its transitory and permanent component by a linear Kalman smoother. The predicted components can then be included in wage or layoff regressions in very flexible ways. Alternatively, the functional relation between firm performance and individual wages or individual layoff probabilities can be estimated itself by nonparametric methods.

Using linked employer-employee data from Germany (LIAB), I first demonstrate that the direct and the indirect method give rise to similar estimates of wage elasticity if wage responses are assumed to be linear. The data, however, indicate pronounced nonlinearities: the wage elasticity depends on the size of the shock. I detect stronger wage rigidity for tail events, i.e. shocks in the lowest and highest decile of the shock distribution. In the middle of the distribution, the sign of the shock does not matter if the shock is permanent. Between the 10th and the 90th percentile the wage elasticity is constant at 0.11 for both positive and negative permanent shocks. This is different for transitory shocks. Negative transitory shocks tend to reduce wages, while positive transitory shocks are fully reaped by the firm. While these findings indicate little downward wage rigidity in Germany, they hide substantial heterogeneity at the worker level. Downward flexibility of wages is in fact limited to blue-collar workers. Whereas wages of white-collar workers do not respond significantly to negative shocks irrespective of persistence and therefore appear perfectly downward rigid.<sup>1</sup>

Firms also adjust to shocks by dismissing workers, but only in response to negative permanent shocks. Linear probability regressions at the worker level reveal an elasticity of  $-1.44$

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<sup>1</sup>Note that the wage variable available in the LIAB data set comprises the base wage and bonus payments. Hence there can be substantial wage flexibility even if the firm cannot adjust the base wage. See also [Section 2](#) on the effect of Germany’s labor market institutions.

between layoff probability and shock size. This increase in the individual layoff probability is again limited to blue-collar workers, while white-collar workers are fully insured against job loss. The heterogeneity with respect to worker type hints at agency and turnover considerations of the employers as well as technological constraints in the production process.

The study most closely related to mine is from [Juhn et al. \(2018\)](#), who analyze wage insurance in a linked employer-employee data set of the US. They closely follow [Guiso et al. \(2005\)](#) but adopt a different identification strategy. The coefficients of interest are approximated by regressing wage growth on revenue growth for 1-, 3-, and 5-year changes. The longer the time horizon, the more of the measured variation stems from permanent shocks. In their result section, they also investigate heterogeneity by size and direction of shocks. By interacting 3-year revenue growth with dummies for positive and negative changes, the authors find that wages respond slightly more to negative shocks than to positive shocks. These OLS estimates, however, cannot clearly differentiate between transitory and permanent innovations.<sup>2</sup> Moreover, their approach only considers long-run stayers and therefore reinforces selection effects. For this reason, their approach is less suitable to analyze layoff responses at the individual level as subpopulations with high turnover would drop out of the sample.<sup>3</sup>

The paper proceeds as follows. Section 2 summarizes theoretical results on wage insurance and labor hoarding. Section 3 gives an overview of the data. The econometric analysis is conducted in Section 4. Section 5 concludes.

## 2 Theoretical considerations

This section reviews important theoretical results regarding the transmission of idiosyncratic firm-level shocks into wages and employment. In a frictionless labor market, job stayers would be perfectly insured against idiosyncratic shocks. The reason is that firms employ workers up to the point where the marginal revenue product of labor (MRPL) equals the market wage. Any deviation from the market wage leads to immediate worker relocation, such that in equilibrium MRPL is equalized across firms. Since idiosyncratic shocks neither affect the market wage nor the price on the output market, employment adjusts to keep MRPL constant. Hence even if idiosyncratic firm performance is very volatile, neither MRPL nor wages should change over time, and all adjustment is via employment. The empirical facts, in particular the observation of large and persistent fluctuations in firm-specific MRPL as documented by [Guiso et al. \(2005\)](#) and others, indicate the importance of reallocation frictions.

With search and matching frictions in the spirit of [Pissarides \(1990\)](#), wage responses to variations in firm performance depend on the particular wage-setting mechanism. In Germany, as in most continental European countries, unions play an important role in wage-setting through

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<sup>2</sup>Besides OLS, the authors obtain their baseline results under an alternative IV estimator which is able to identify the wage response to permanent shocks. This approach, however, requires a linear relation between wage growth and revenue growth, which rules out interactions with shock size. Hence the same limitation as with the original [Guiso et al. \(2005\)](#) methodology arises.

<sup>3</sup>For instance, only workers that are observed in four consecutive years enter their 3-year sample, while my methodology requires data from two consecutive years to be included in the estimation sample.

collective bargaining. Although in a strict sense, a collective bargaining agreement (CBA) only applies between members of the negotiating parties—the employer association and the labor union—, collectively bargained wages are generally extended to non-unionized labor in covered firms as well (Guertzgen, 2009). Negotiations typically take place once a year and agree on a wage floor as well as a minimum wage increase for all covered workers. Both features are likely to generate downward wage rigidity. Since collectively bargained wage increases typically compensate for (expected) inflation, Dickens et al. (2007) and Babecký et al. (2010) find that real wage rigidity is more pronounced than nominal wage rigidity in countries with more centralized bargaining. Yet, even if hourly wages are rigid downwards due to CBAs, firms can adjust their wage bill along other margins such as bonus payments or fringe benefits. Additionally, so called opening clauses allow covered firms in Germany to pay below the CBA level under certain conditions, which brings further wage flexibility and helps to avoid layoffs (Brändle and Heinbach, 2013).

Even without institutional constraints, contracted wages are likely to be rigid due to other reasons. The literature on implicit contracts evolves around the idea that risk averse workers do not have access to the capital market, and that risk neutral firms insure them against income fluctuations by paying a constant wage (Baily, 1974; Azariadis, 1975). Solvency constraints limit the degree of insurance that employers can provide, such that sufficiently large shocks may make wage adjustments necessary nonetheless (Gamber, 1988). However, since work effort and on-the-job search of employees are non-contractible and hard to observe, the feasibility of downward wage adjustments is limited by their adverse effects on motivation and quits.<sup>4</sup> A firm survey conducted by Du Caju et al. (2015) confirms that employers indeed worry about the motivational impact of wage cuts as well as their effect on quit rates of productive workers. Therefore, an optimal response to a big negative shock might combine a relatively small decrease in wages of stayers and a shrinking of the workforce through layoffs.<sup>5</sup>

But not all workers might be affected by wage cuts and layoffs to the same extent. Along the lines of Shapiro and Stiglitz (1984), lower wages are associated with a higher incentive to shirk, which is more problematic in occupations where employee effort is hard to monitor. These are typically white-collar occupations, while wages can be tied closer to actual performance in blue-collar occupations, in which some workers are even paid according to piece-rates. White-collar workers might therefore enjoy more wage insurance against negative shocks than blue-collar workers due to agency problems. Wage cuts also make quits more likely, which requires hiring and training of new workers (Stiglitz, 1974). Since training costs are usually higher for white-collar workers and it may take longer to find an adequately skilled replacement, this provides another rationale for higher downward wage rigidity among white-collar workers.

The higher downward wage rigidity of white-collar workers does not necessarily imply higher

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<sup>4</sup>Classical papers on these issues include Weiss (1980), Akerlof (1982), Shapiro and Stiglitz (1984), Lindbeck and Snower (1989), and Akerlof and Yellen (1990).

<sup>5</sup>Menzio and Moen (2010) present a model which gives rise to a similar optimal firm policy. Rather than considering unobserved work or search effort, they impose the constraint that firms should never have an incentive to replace an incumbent worker with a newly hired one. The incentive to attract new workers with high starting wages makes wages of stayers downward rigid, such that employment adjusts more strongly to negative shocks.

risk of job loss. Because employers anticipate higher recruitment costs once the economic situation improves again, white-collar workers may in fact be less likely to lose their jobs if the firm experiences trouble. Additionally, technological constraints may shift the burden of job loss on blue-collar workers. [Oi \(1962\)](#) argues that some groups of workers are more complementary in the production process than others. Employment adjusts mainly through hiring and firing of workers that are relatively easy to substitute by other fixed production factors. Along these lines, blue-collar workers performing manual tasks may be closer substitutes to physical capital than white-collar workers who perform cognitive tasks.

### 3 Data and sample selection

This study uses the longitudinal version of the linked employer-employee data of the IAB (LIAB). The data set is administered by the Institute for Employment Research (IAB) and allows for simultaneous analysis of the supply and demand side of the German labor market from 1993 until 2010. On the employer side, the LIAB uses the representative annual survey data of the IAB establishment panel. *Inter alia*, this panel contains questions on sales, investment, employment, and industrial relations. The employee information stems from the employment register, which covers all workers who pay social security contributions. Information on wages, occupation, qualification, gender, tenure, experience, and age are linked to the employer data by a common identifier. On the employer side, the unit of observation is an establishment, which mostly corresponds to a plant or a branch. Since it is unknown which establishments belong to the same firm, the econometric analysis is at the establishment level.<sup>6</sup>

The last wave considered in the econometric analysis is 2009, which contains retrospective information on investment and sales in 2008. I do not use the latest available wave as in 2009 many establishments implemented special employment and wage policies to tackle the Great Recession, such as short-time work ([Brenke et al., 2013](#)). At a smaller scale, short-time work (STW) is also used during normal times, compare [Balleer et al. \(2016\)](#). To the extent that STW provides an additional facility for troubled firms to overcome severe idiosyncratic shocks at legal conditions that vary little over time, including observations of establishments that adopt short-time work yields a more complete picture of wage and employment insurance in Germany.<sup>7</sup> To avoid bias, however, periods during which increased take-up of STW is mainly due to discretionary changes in the STW policy should be excluded. As argued by [Balleer et al. \(2016\)](#), the Great Recession was such a period.<sup>8</sup>

Only privately-owned establishments in the private, non-financial sector are included in the analysis. The financial industry has to be excluded because no sensible performance measure is available. I exclude very small establishments that in some year report less than 5 employees as

<sup>6</sup>See [Alda et al. \(2005\)](#) for more detailed information on the data set.

<sup>7</sup>Moreover, establishment-level information on STW take-up is only available in few waves of the IAB panel.

<sup>8</sup>[Balleer et al. \(2016\)](#) also report the frequent use of STW at the beginning of the 1990s in East Germany. Since East German establishments are contained in the IAB panel only from 1996 onwards, the effect on the estimation results is likely to be small. Additionally, by construction of the LIAB, the bulk of observations stems from the period 2000 to 2008, compare Table 1 in [Klosterhuber et al. \(2013\)](#).



well as establishments with consistently missing information on sales, investment, or employment. Because I perform a dynamic panel regression at the establishment level, at least three consecutive observations are required per establishment. Altogether, the establishment-level regressions are based on 2697 establishments as reported in the first column of Table A.1.

On the worker side, only male employees up to age 59 are considered due to a spike in separation rates at age 60. Women are excluded because the LIAB does not provide information about the nature of a separation (voluntary quit or involuntary layoff). As a workaround, Section 4.3 uses transitions from employment to non-employment to proxy employer-induced layoffs, drawing on Boockmann and Steffes (2010). While this appears to be a reasonable proxy for men because of their high attachment to the labor market, it is less convincing for women. Compared to men, female transitions to non-employment are more often driven by personal or family-related reasons, such as labor supply of the spouse, child care, or informal care for a relative. Since neither of these variables is observed in the LIAB, separations for family-related reasons would be incorrectly labeled as employer-induced layoffs. Including women in the wage regression is less problematic and presented as a robustness check.<sup>9</sup>

Since no information on hours worked is available, the analysis is restricted to full-time employment. This restriction could bias my estimates if in response to shocks workers switch back and forth between full-time and part-time employment. Over the whole sample period, however, less than 5% of male workers are observed to switch between these two employment states. The majority of these switches occur after age 50 and result in a permanent reduction of working time.

By nature of the data, wages are top-coded at the social security threshold. This applies to 16% of the observations. Observations with censored wages are excluded from the wage regressions but are included in the layoff regressions. The respective sample statistics can be found in Table A.1. Since all regressions are in first differences, only workers with at least two consecutive observations at the same establishment are considered. Nominal variables were deflated using the consumer price index with base year 2010.<sup>10</sup>

## 4 Econometric analysis

The econometric analysis is divided into three parts. The first part uses establishment-level data to identify idiosyncratic shocks to establishment revenue and describe their statistical properties. This closely follows Guiso et al. (2005). The second part applies wage regressions at the worker level to estimate wage elasticities. After a short review of the *indirect method* of Guiso et al. (2005), the *direct method* is introduced as a more flexible alternative. The two methods are then compared to each other assuming that revenue shocks affect wage growth linearly. Thereafter, nonparametric and piecewise linear relations are considered. In the last

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<sup>9</sup>If women are nevertheless included in the layoff regressions, coefficient estimates in Table 6 change little, while standard errors increase substantially.

<sup>10</sup>Missing wage information is sometimes imputed using a hypothetical model for wage determination, which is mostly a Mincerian wage equation. The goal of this paper, however, is exactly to come up with a model that explains wage formation by additionally taking into account firm performance.



part, the analysis is extended to layoffs.

#### 4.1 Firm performance

Firm performance is measured in terms of sales per worker  $r_{jt} = R_{jt}/L_{jt}$ , where  $R_{jt}$  refers to the value of sales (revenue) in year  $t$  and  $L_{jt}$  is the stock of employees at June 30 of year  $t$ . Both figures are taken from the IAB establishment panel. Therefore,  $L_{jt}$  measures the total workforce of an establishment and not only those workers that satisfy the sample selection criteria outlined in Section 3. From a theoretical point of view, using value added instead of sales would be preferable because it better captures establishment-level quasi-rents. The LIAB allows to construct value added by multiplying the value of sales with the reported share of material costs in total sales. However, Addison et al. (2006, p.260) argue that “unlike the sales measures, these share-in-sales values seem to be little more than ‘informed guesstimates.’” This is because the majority of values take multiples of 5 percent, and there is unrealistically high variation in these shares over time. For this reason, but also since previous studies have obtained very similar results despite using different measures of firm performance, my analysis is based on sales.<sup>11</sup>

To isolate idiosyncratic shocks, establishment sales per worker are regressed on a set of dummies that capture the aggregate cycle as well as industry- and region-specific effects. To ensure that the unexplained changes in sales per worker stem from exogenous shocks rather than variation of factor inputs, I additionally control for the capital-labor ratio  $k_{jt} = K_{jt}/L_{jt}$ . A proxy for the capital stock  $K_{jt}$  of an establishment is calculated from reported investment data as explained in Guertzen (2014). To capture predictable dynamics such as precommitted sales, the estimated model specification is autoregressive,

$$\ln r_{jt} = \rho \ln r_{j,t-1} + \alpha_K \ln k_{jt} + Z'_{jt}\gamma + \varphi_j + \varepsilon_{jt}. \quad (1)$$

where  $\varphi_j$  is an establishment-specific intercept. The matrix  $Z_{jt}$  contains linear time trends interacted with 14 year dummies, 15 industry dummies, and 18 regional dummies. These become regular dummy variables in the first differenced equation. It is important for the interpretation of the wage elasticities that the residuals of the above regression indeed capture exogenous variation in revenue. For this reason, equation (1) is grounded in theory and derived from a Cobb-Douglas production function at the establishment level. As demonstrated in Appendix B, the exact representation includes log-employment  $\ln L_{jt}$  as additional explanatory variable, unless the production function features constant returns to scale. Table B.2 shows that constant returns to scale cannot be rejected in the sample under consideration. The level of employment is therefore omitted from equation (1) altogether.

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<sup>11</sup>Guiso et al. (2005), Kátay (2016), and Fagereng et al. (2017) use value added or value added per worker, while Cardoso and Portela (2009) use sales. Guertzen (2014) conducts a similar analysis with an earlier version of the LIAB, using value added as performance indicator. The estimated variance ratio between transitory shocks and permanent shocks is about seven times higher than in the comparable studies of Guiso et al. (2005), Cardoso and Portela (2009), or Kátay (2016). Along the lines of Addison et al. (2006), most of the excess volatility in transitory shocks may be due to measurement error in the share of material costs.

	coefficient	std. err.
$\ln r_{j,t-1}$	0.2101***	0.0376
$\ln k_{jt}$	0.3173***	0.0285
	$\chi^2$ -statistic	$p$ -value
year dummies	95.72***	0.000
industry dummies	39.83***	0.000
regional dummies	10.54	0.837
	statistic	$p$ -value
AR(2) test	1.32	0.186
AR(3) test	-0.83	0.407
AR(4) test	1.11	0.267
Hansen $J$ test	39.23	0.415
establishments (observations)	2697 (17407)	

two-step difference GMM, corrected standard errors clustered at the establishment level, significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 1. GMM regression of sales per worker

Equation (1) is estimated in first differences,

$$\Delta \ln r_{jt} = \rho \Delta \ln r_{j,t-1} + \alpha_K \Delta \ln k_{jt} + \Delta Z'_{jt} \gamma + \Delta \varepsilon_{jt},$$

using the [Arellano and Bond \(1991\)](#) GMM estimator, where  $\Delta \ln r_{j,t-1}$  is instrumented with lags 2 to 4 of  $\ln r_{jt}$ . Table 1 presents the two-step GMM estimates that account for clustering at the establishment level.<sup>12</sup> The point estimate of the autoregressive coefficient is 0.21, and the coefficient of the capital-labor ratio is 0.32. Both values are highly significant and within the credible range. The AR tests and the Hansen  $J$  test confirm that the second to fourth lags are valid instruments. Furthermore, a difference-in-Hansen test (not reported) verifies that  $\ln k_{jt}$  can be treated as exogenous, which proves to be robust to different choices of instrument sets (fewer instrument lags and/or collapsed instruments) as recommended by [Roodman \(2009a\)](#). This confirms that the residuals of the GMM estimation can indeed be regarded as idiosyncratic revenue shocks that are exogenous to the establishment.

The autocovariance structure of the first differenced GMM residuals  $\Delta \hat{\varepsilon}_{jt}$  is given in Table 2. This information can be used to identify the stochastic process that generates the idiosyncratic shocks. Because the covariance estimates at lags greater than one are close to zero and statistically insignificant, Table 2 suggests that the error process consists of a random walk component and a white noise component,

$$\begin{aligned} \varepsilon_{jt} &= \zeta_{jt} + \tilde{v}_{jt}, \\ \zeta_{jt} &= \zeta_{jt-1} + \tilde{u}_{jt}, \end{aligned} \tag{2}$$

<sup>12</sup>These were obtained using the user-written `xtabond2` command in Stata ([Roodman, 2009b](#)). Reported standard errors use [Windmeijer's \(2005\)](#) correction and are clustered at the establishment level.

order ( $k$ )	$\mathbb{E}[\Delta\hat{\varepsilon}_{jt}\Delta\hat{\varepsilon}_{j,t-k}]$	std. err.
0	0.0795***	0.0038
1	-0.0344***	0.0024
2	0.0018	0.0012
3	-0.0009	0.0011
4	0.0013	0.0013
5	-0.0018	0.0018

standard errors bootstrapped with clustering at the establishment level, significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 2. Autocovariance structure of GMM residuals

where  $\tilde{v}_{jt}$  and  $\tilde{u}_{jt}$  are mutually uncorrelated white noise processes with variances  $\mathbb{E}\tilde{v}_{jt}^2 = \sigma_v^2$  and  $\mathbb{E}\tilde{u}_{jt}^2 = \sigma_u^2$ . The structural equations imply  $\mathbb{E}[\Delta\varepsilon_{jt}\Delta\varepsilon_{j,t-1}] = -\mathbb{E}\tilde{v}_{jt}^2 = -\sigma_v^2$  and  $\mathbb{E}[\Delta\varepsilon_{jt}(\Delta\varepsilon_{j,t-1} + \Delta\varepsilon_{jt} + \Delta\varepsilon_{j,t+1})] = \sigma_u^2$ . Computing the respective sample moments from the data yields variance estimates  $\hat{\sigma}_v^2 = 0.0344$  and  $\hat{\sigma}_u^2 = 0.0088$ , which are both significantly different from zero at the 1% level. In line with previous literature, shocks to establishment performance have a transitory and a permanent component.<sup>13</sup>

By virtue of (2), establishment revenue can be decomposed into a deterministic component  $D_{jt}$ , a non-stationary stochastic component  $P_{jt}$ , and a stationary stochastic component  $T_{jt}$ ,

$$\ln r_{jt} = D_{jt} + P_{jt} + T_{jt},$$

where  $D_{jt} := (1 - \rho L)^{-1}(Z'_{jt}\gamma + \varphi_j)$ ,  $P_{jt} := (1 - \rho)^{-1}\zeta_{jt}$ ,  $T_{jt} := (1 - \rho L)^{-1}[\tilde{v}_{jt} - (1 - \rho)^{-1}\rho\tilde{u}_{jt}]$ , and  $L$  denotes the lag operator. This is an application of the Granger representation theorem, compare Guiso et al. (2005). The definitions of  $P_{jt}$  and  $T_{jt}$  imply

$$\begin{aligned}\Delta P_{jt} &= (1 - \rho)^{-1}\tilde{u}_{jt} =: u_{jt}, \\ \Delta T_{jt} &= (1 - \rho L)^{-1}\Delta v_{jt},\end{aligned}\tag{3}$$

where  $u_{jt}$  and  $v_{jt} := \tilde{v}_{jt} - \rho u_{jt}$  are the innovations to the permanent and transitory component of revenue, respectively. Note that the year-on-year changes in the stochastic components are related to the total shock by  $\Delta P_{jt} + \Delta T_{jt} = (1 - \rho L)^{-1}\Delta\varepsilon_{jt}$ .

## 4.2 Wage responses

This section relates variation in wages that is unexplained by other observables to revenue shocks of the employer. Section 4.2.1 assumes that wage growth responds linearly to these shocks. After reviewing the *indirect method*, I introduce the *direct method* to estimate wage elasticities. The linearity assumption allows to compare the results of both methods to each

<sup>13</sup>Guertzen (2014) identifies the same error process and obtains the variance estimates  $\hat{\sigma}_v^2 = 0.1464$  and  $\hat{\sigma}_u^2 = 0.010$ . While the variance of the permanent shock is similar, the variance of the transitory shock is more than four times higher. This suggests that using value added instead of sales introduces substantial measurement error (compare footnote 11).

other. Section 4.2.2 generalizes the analysis to nonlinear relations between wage growth and revenue shocks.

#### 4.2.1 Linear effects on wage growth

Guiso et al. (2005) propose a wage equation of the form

$$\ln w_{ijt} = X'_{ijt}\delta + \alpha P_{jt} + \beta T_{jt} + \phi_{ij} + \psi_{ijt}, \quad (4)$$

where  $w_{ijt}$  refers to the annual average wage income that worker  $i$  earns at establishment  $j$  in year  $t$ . In the LIAB data, this income measure includes all bonus payments that a worker receives on top of the base wage. The matrix  $X_{ijt}$  contains the same dummy variables as the establishment-level regression, as well as dummies for industrial relations, educational dummies, a white-collar dummy, a cubic polynomial in age, and a cubic polynomial in tenure.<sup>14</sup> The intercept  $\phi_{ij}$  captures an unobserved fixed effect at the establishment, worker, or match level. First differencing of (4) yields

$$\Delta \ln w_{ijt} = \Delta X'_{ijt}\delta + \alpha \Delta P_{jt} + \beta \Delta T_{jt} + \Delta \psi_{ijt}, \quad (5)$$

where  $\Delta P_{jt}$  and  $\Delta T_{jt}$  are unobserved but known to satisfy the structural equations (2)–(3). Taking first differences implies that only wages of job stayers can be analyzed.

Although hours worked are not observed, the estimated wage elasticities should indeed reflect flexibility in hourly wages and not variations in hours worked. First, comparing changes in annual average wages partly washes out the effect of overtime pay. Second, the incidence of paid overtime has been decreasing in Germany, and more than 60% of the establishments in the IAB panel do not pay overtime compensation at all in any given year (Zapf, 2012). Especially large establishments, which are overrepresented in the IAB panel, increasingly use working time accounts where the annual working time is kept constant.

**The indirect method.** To identify the wage elasticities  $\alpha$  and  $\beta$  in equation (5), Guiso et al. (2005) use an approach that avoids to determine the unobserved shock components  $\Delta P_{jt}$  and  $\Delta T_{jt}$ . They proceed in two steps: First, wage changes  $\Delta \ln w_{ijt}$  are regressed on the set of observed characteristics  $\Delta X_{ijt}$ , i.e.  $\Delta \ln w_{ijt} = \Delta X'_{ijt}\delta + \Delta \omega_{ijt}$ . By equation (5), the error term in this regression satisfies  $\Delta \omega_{ijt} = \alpha \Delta P_{jt} + \beta \Delta T_{jt} + \Delta \psi_{ijt}$ . Substituting (3) and applying the operator  $(1 - \rho L)$  on both sides yields

$$\Delta \tilde{\omega}_{ijt} := (1 - \rho L)\Delta \omega_{ijt} = \alpha(1 - \rho L)u_{jt} + \beta \Delta v_{jt} + (1 - \rho L)\Delta \psi_{ijt}. \quad (6)$$

Since  $\mathbb{E}[(\Delta \tilde{\omega}_{ijt} - \beta \Delta v_{jt})\Delta \varepsilon_{j,t+1}] = 0$  and  $\mathbb{E}[\Delta \varepsilon_{jt}\Delta \varepsilon_{j,t+1}] = -\sigma_v^2$ , the wage elasticity with respect to a transitory shock,  $\beta$ , can be identified by an IV regression of  $\Delta \tilde{\omega}_{ijt}$  on  $\Delta \varepsilon_{jt}$ , using

<sup>14</sup>The capital-labor ratio and the level of employment are excluded from (4) due to endogeneity problems. However, since  $\Delta \varepsilon_{jt}$  is by construction orthogonal to these variables, their appearance in the regression has hardly any effect on the estimates of  $\alpha$  and  $\beta$ .

$\Delta\varepsilon_{j,t+1}$  as instrument. Likewise, it can be shown that IV regression of  $\Delta\tilde{\omega}_{ijt}$  on  $\Delta\varepsilon_{jt}$ , instrumented by  $\Delta\varepsilon_{j,t-1} + \Delta\varepsilon_{jt} + \Delta\varepsilon_{j,t+1}$  identifies the wage elasticity with respect to a permanent shock  $\alpha$ .

This method has two limitations. First, it is not possible to allow  $\Delta P_{jt}$  and  $\Delta T_{jt}$  to enter equation (5) nonlinearly because the exclusion restrictions of the IV no longer hold. There is no straightforward way of extending the methodology to a nonlinear setting. The second limitation is more subtle. As with any IV regression, the resulting estimates for  $\alpha$  and  $\beta$  are biased in finite samples. This bias may be substantial if instruments are weak, even for samples of the size considered in this paper (see also the discussion starting on page 15).

**The direct method.** To overcome the linearity restriction as well as the potential identification problem, I use a more direct route. Exploiting (2)–(3), the residuals of the establishment-level regression are used to predict  $\Delta T_{jt}$  and  $\Delta P_{jt}$  by a linear Kalman smoother that is applied separately for each establishment.<sup>15</sup> These predictions can be substituted into (5), from which the wage elasticities  $\alpha$  and  $\beta$  can be estimated by OLS. Moreover, once the predictions for the stochastic components of  $\Delta T_{jt}$  and  $\Delta P_{jt}$  have been obtained, any functional relation between revenue shocks and wage growth can be estimated.

To apply Kalman smoothing, the non-stationary process (2) is first differenced and written in state-space form for  $2 \leq t \leq t_j$ :

$$\begin{aligned}\Delta\varepsilon_{jt} &= \begin{pmatrix} 1 & -1 \end{pmatrix} z_{jt} + \tilde{u}_{jt}, \\ z_{jt} &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} z_{j,t-1} + \begin{pmatrix} \tilde{v}_{jt} \\ 0 \end{pmatrix},\end{aligned}$$

where  $z_{jt} := (\tilde{v}_{jt}, \tilde{v}_{j,t-1})'$  is the unobserved state vector and  $t_j$  denotes the number of years between the first and the last observation of establishment  $j$ .<sup>16</sup> If the innovation variances  $\sigma_{\tilde{u}j}^2$  and  $\sigma_{\tilde{v}j}^2$  are known, Kalman smoothing yields the best linear predictions for  $(\tilde{u}_{jt})_{t=2}^{t_j}$  and  $(z_{jt})_{t=2}^{t_j} = (\tilde{v}_{jt})_{t=1}^{t_j}$  given  $(\Delta\varepsilon_{jt})_{t=2}^{t_j}$ , irrespective of the actual error distribution (Hamilton, 1994). Feeding these predictors into (3) yields the predicted time series  $(\Delta\hat{P}_{jt})_{t=2}^{t_j}$  and  $(\Delta\hat{T}_{jt})_{t=2}^{t_j}$ .<sup>17</sup>

There are two practical complications. First, the true errors  $\Delta\varepsilon_{jt}$  are unobserved and are therefore replaced with the residuals of the establishment-level regression. (This is also necessary with the indirect method.) Second, the establishment-specific shock variances  $\sigma_{\tilde{u}j}^2$  and  $\sigma_{\tilde{v}j}^2$  are unknown and have to be estimated from the data. Hamilton (1994) suggests to estimate the variances by maximum likelihood, assuming that innovations are normally distributed. This is convenient since the Gaussian log-likelihood is easy to evaluate if a Kalman filter has already

<sup>15</sup>While the Kalman filter uses past information to form optimal predictions about the future, the Kalman smoother uses all available information to form predictions. Hamilton (1994) presents an overview of these and other state-space methods.

<sup>16</sup>Gaps can easily be handled by the Kalman smoothing algorithm. In periods with missing  $\Delta\varepsilon_{jt}$  only the state equation is used for prediction until the next observation arrives.

<sup>17</sup>Note that both with the direct and the indirect approach one observation is lost, which is due to  $\Delta T_{jt}$  being an AR(1) process.

(a) baseline								
	indirect method				direct method (ML)		direct method (MM)	
	coefficient	std. err.	$F$	$J$	coefficient	std. err.	coefficient	std. err.
$\alpha$	0.0500***	0.0173	23.14	0.747	0.0625***	0.0143	0.0617***	0.0145
$\beta$	0.0162	0.0128	28.13	0.523	0.0189*	0.0102	0.0192*	0.0105

(b) using LIAB sampling weights								
	indirect method				direct method (ML)		direct method (MM)	
	coefficient	std. err.	$F$	$J$	coefficient	std. err.	coefficient	std. err.
$\alpha$	0.1046*	0.0626	5.89	0.398	0.0507***	0.0159	0.0490***	0.0147
$\beta$	0.0000	0.0096	90.35	0.327	0.0056	0.0075	0.0068	0.0079

$F$  is the Kleibergen-Papp Wald  $F$  statistic,  $J$  is the  $p$ -value of the Hansen test, bootstrapped standard errors clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 3. Comparison of wage elasticity estimates

been computed.<sup>18</sup> The accuracy of the Kalman smoothed time series hinges on accurate variance estimates. Assuming that all establishments draw their shocks from the same distribution might be too restrictive. Guertzgen (2014) observes that the variance of permanent shocks tends to increase with establishment size, while the variance of transitory shocks decreases. I consider heteroscedasticity of the form

$$\ln \sigma_{\tilde{u}j}^2 = D_j' \lambda_{\tilde{u}}, \quad \ln \sigma_{\tilde{v}j}^2 = D_j' \lambda_{\tilde{v}}, \quad (7)$$

where  $D_j$  is a matrix of time-independent and exogenous establishment characteristics. In the baseline estimations,  $D_j$  contains dummies for the establishment size in the first period of observation. The parameter vectors  $\lambda_{\tilde{u}}$  and  $\lambda_{\tilde{v}}$  are estimated by Gaussian maximum likelihood following Hamilton (1994). As a robustness check, I obtain method-of-moments based variance estimates that do not require the normality assumption, and consider alternative choices of  $D_j$ .

**Comparison.** Panel (a) of Table 3 compares the estimates for  $\alpha$  and  $\beta$  obtained by the indirect method and the direct method. The indirect method uses the instruments described above, together with their 2nd, 3rd, and 4th power as proposed by Guiso et al. (2005). To account for heteroscedasticity at the establishment level, the coefficients were estimated by two-step efficient GMM. Two test statistics are reported in Table 3(a). Column  $F$  reports the Kleibergen-Paap Wald  $F$  statistic for detecting weak identification (Kleibergen and Paap, 2006). Column  $J$  reports the  $p$ -value of the Hansen test of overidentifying restrictions.<sup>19</sup> The estimated wage elasticity with respect to a permanent shock is 0.05 and highly significant, while a transitory shock triggers no significant wage response.<sup>20</sup> The high values of the test statistics

<sup>18</sup>Computationally, the Kalman smoother is obtained by running a Kalman filter followed by a backwards pass, see Hamilton (1994, Section 2.4).

<sup>19</sup>Estimates and test statistics were obtained using the `ivreg2` command developed by Baum et al. (2007).

<sup>20</sup>All reported standard errors are based on 1000 bootstrap replications clustered at the establishment level. The bootstrap takes into account the uncertainty at each step of the multistage estimation procedure.

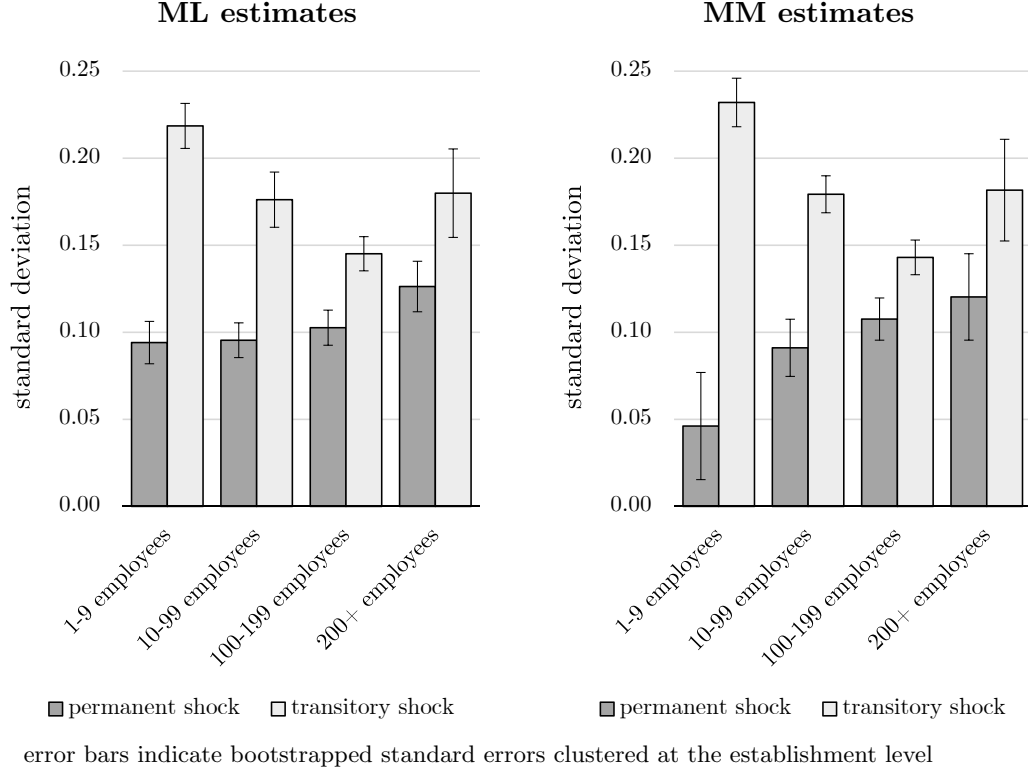


Figure 1. Estimated standard deviation of the shocks by establishment size

indicate that the instruments are valid and strong enough to identify the coefficients. The point estimates are in line with aforementioned studies that apply the same methodology in other countries.<sup>21</sup>

For the direct method, two sets of results are reported. The innovation variances (or likewise the auxiliary parameters  $\lambda_{\bar{u}}$  and  $\lambda_{\bar{v}}$ ) are once estimated by Gaussian maximum likelihood (column ML) and once by method of moments (column MM).<sup>22</sup> Figure 1 shows the standard deviations estimated by ML and MM if shocks are heteroscedastic with respect to establishment size, captured by four size categories. In both cases the standard deviation of permanent shocks increases with establishment size, while the smallest establishments experience the strongest transitory fluctuations. Apart from the smallest size category, the estimated standard deviations are virtually identical. The wage elasticity estimates for ML and MM in Table 3(a) are therefore also very close to each other. The estimate for  $\alpha$  is 0.063 and the estimate for  $\beta$  is 0.019. The point estimates are slightly higher than those obtained by the indirect method, but the difference is within one standard error. At the same time, the direct method yields smaller standard errors, which renders the coefficient estimate of  $\beta$  weakly significant. The estimates obtained by the direct method are robust to alternative variance patterns, which can be seen from Table D.4. Column (a) assumes that the variance of shocks is identical across establish-

<sup>21</sup>By contrast, [Guertzgen \(2014\)](#) estimates an insignificant wage response to permanent shocks ( $\alpha = -0.0307$ ) using an earlier version of the LIAB. Compare also footnote 11 and the discussion at the end of this subsection.

<sup>22</sup>The method of moment estimates for  $\sigma_{\bar{v}j}^2$  and  $\sigma_{\bar{u}j}^2$  are based on the theoretical moment conditions  $\sigma_{\bar{v}j}^2 = -\mathbb{E}[\Delta\varepsilon_{jt}\Delta\varepsilon_{j,t-1}]$  and  $\sigma_{\bar{u}j}^2 = \mathbb{E}[\Delta\varepsilon_{jt}(\Delta\varepsilon_{j,t-1} + \Delta\varepsilon_{jt} + \Delta\varepsilon_{j,t+1})]$ , where the expected value is replaced by the sample average of all establishments in the same size category.



ments. Column (b) allows for heteroscedasticity by establishment size and industry. In both cases, the point estimates are very similar to the baseline, while  $\alpha$  is estimated less precisely.

Panel (b) of Table 3 repeats the above analysis, taking into account the sampling weights provided with the LIAB. The indirect method yields a point estimate of  $\alpha$  that is more than twice as high as in the unweighted regression. However, this estimate bears a substantial standard error. The low  $F$  test statistic, which is well below the rule of thumb value of 10, indicates weak instruments in the IV regression. Along the lines of Wooldridge (2002, p.108), this instrument weakness seems to generate substantial finite sample bias. The point estimate for  $\alpha$  therefore bears little credibility. By contrast, the direct method continues to give plausible results. The point estimates are slightly lower compared to the unweighted estimates, while the standard errors are very similar.

That the indirect method may fail to identify the coefficients of interest has not been documented before. It is possible to investigate analytically when this is likely to happen. In general, weak identification occurs if the first stage  $F$  test statistic is small or likewise if the  $R^2$  statistic of the first stage regression is small. Appendix C analyses the two first stage regressions applied by the indirect method. It demonstrates that the population equivalents of the two first stage  $R^2$  statistics depend only on the variance ratio  $\phi := \sigma_u^2/\hat{\sigma}_v^2$ . In particular,  $R_\alpha^2 = \phi^2/[(2+\phi)(2+3\phi)]$  and  $R_\beta^2 = 1/(2+\phi)^2$ . As shown in Figure C.1,  $R_\alpha^2$  is increasing in  $\phi$  while  $R_\beta^2$  is decreasing. In the baseline case without weighting, the variance ratio estimated from the data is  $\phi = 0.79$ .<sup>23</sup> The population  $R^2$  statistics are then  $R_\alpha^2 = 0.051$  and  $R_\beta^2 = 0.129$ , which seems sufficient to identify both parameters in Table 3(a). The sampling weights provided with the LIAB correct for the oversampling of large establishments which occurs by design of the IAB establishment survey. Observations of smaller establishments receive higher weights, while observations of larger establishments are downweighted. By Figure 1, smaller establishments experience less severe permanent shocks and more severe transitory shocks. Consequently, the variance ratio drops to  $\phi = 0.29$  if weights are considered. This increases  $R_\beta^2$  to 0.191 while  $R_\alpha^2$  drops to 0.013, which is too low to identify the coefficient  $\alpha$  in Table 3(b).

Altogether, the above observations suggests that the direct method leads to similar point estimates than the indirect method, provided that the latter is able to identify the coefficients. If the ratio of variances  $\sigma_u^2/\hat{\sigma}_v^2$  is too small, however, the indirect method may fail to identify the wage response to a permanent shock, while the direct method continues to perform well. In any case, the direct method yields lower standard errors.

**Heterogeneity.** Table 3 uses all establishments from the private sector except for the financial industry. Wage elasticities may differ across industries due to different production processes, incentive structures, and industrial relations. Therefore, separate regressions are performed for four broad industry categories: manufacturing, construction, sales, and services. The results of the direct method are reported in Table D.5. No clear statistical pattern arises. Concerning

<sup>23</sup>The IV regression takes place at the worker level, where each establishment is essentially weighted by the number of its employees. The sample variances therefore do not coincide with the values reported in Section 4.1. In particular,  $\sigma_u^2 = 0.0112$  and  $\sigma_v^2 = 0.0142$ .

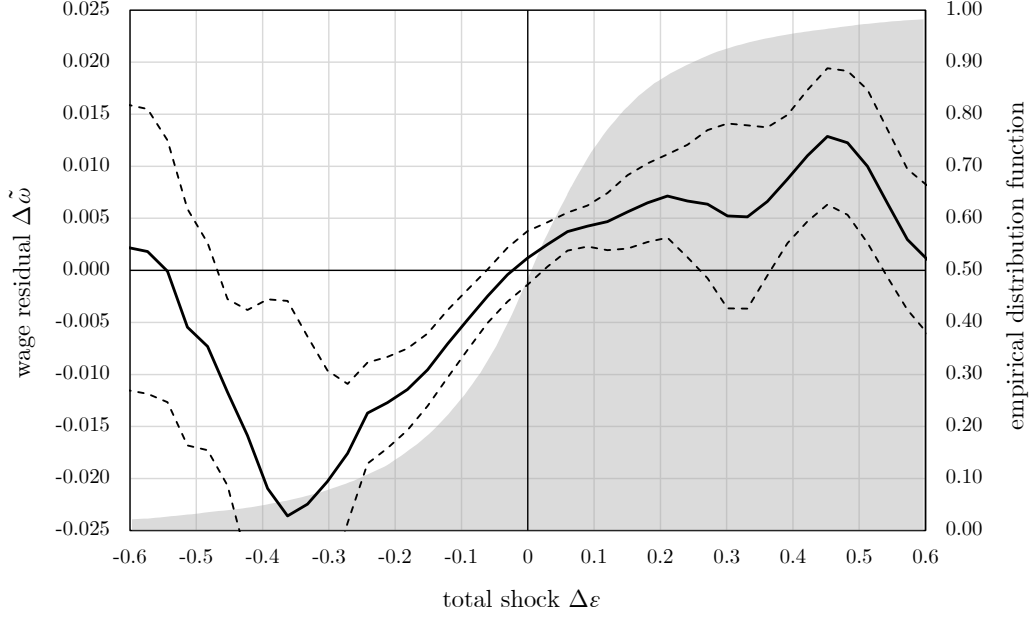
permanent shocks, wages in the construction sector seem to be somewhat more flexible, while wages in the service sector are more rigid. Yet, the standard error on both estimates is relatively large. This is because more than 80% of the workers in the sample are employed in manufacturing.

As demonstrated by Figure 1, the variance of transitory and permanent shocks differs with establishment size. This could affect the degree of wage insurance that establishments can provide. Since larger establishments have to cope with more extreme permanent shocks, they might provide less insurance. At the same time, larger establishments may have a higher financial buffer that allows them to provide more insurance. Table D.6 presents wage elasticity estimates from separate regressions in each of the four size categories. Differences across categories appear small and statistically insignificant, such that the two highlighted effects seem to cancel out on average.

Finally, I repeat the analysis of Guertzgen (2014) who highlights the role of industrial relations for wage insurance at the establishment level. A key finding of the paper is that the presence of a works council is associated with higher wage rigidity. To identify the effect of industrial relations, several interaction terms are added to equation (5). In particular, the shocks are interacted with a dummy indicating presence of a works council (WC), a dummy indicating an industry-wide collective bargaining agreement (CBA industry), and a dummy indicating whether the firm itself has negotiated a CBA with a trade union (CBA firm). The results are reported in Table D.7. Concerning permanent shocks, the estimated wage elasticity in uncovered establishments (without WC and CBA) and establishments with industry-wide CBA are similar to those of Table 8 in Guertzgen (2014). By contrast, my results cannot confirm the central finding of her study that the presence of a works council makes wages more rigid. This discrepancy remains if the estimation is performed separately for every establishment size category or if the indirect estimation method is applied (Table D.8).<sup>24</sup>

The previous results are only based on male employees. If women are included as well, the wage elasticity with respect to permanent shocks decreases. Table D.9 allows wage responses to differ by gender. To also account for level differences in wage growth, the underlying wage regression (5) is extended by gender dummies. Compared to men, wages of female employees fluctuate less irrespective of shock persistence. This also holds within industries. Disaggregated analysis suggests that the lower female wage elasticities are related to collective bargaining. Table D.10 only considers employees in the manufacturing sector. While coverage by an industry level CBA does not affect wage elasticities of men, it reduces the wage elasticity of women with respect to permanent shocks down to zero. A possible explanation is that the compensation package of women less frequently contains bonus payments, as reported by Geddes and Heywood (2003) for the US. If the base wage is rigid, as in the case of an industry-wide collective bargaining agreement, total compensation can therefore respond less to firm-specific economic conditions.

<sup>24</sup>The coefficient estimates of Guertzgen (2014) even indicate that wages do not respond to permanent shocks at all if a works council is present. Although this finding seems to be statistically robust, the author admits that “full insurance against permanent shocks under works councils is clearly at variance with other studies” (p.366).



left axis: local linear kernel regression, 95% confidence band based on bootstrapped standard errors clustered at the establishment level; right axis: empirical cdf (shaded area)

Figure 2. Nonparametric wage regression

#### 4.2.2 Nonlinear effects on wage growth

The main advantage of the direct method is the possibility to estimate nonlinear relations between revenue shocks and wage growth. Figure 2 illustrates that the linearity assumption imposed in Section 4.2.1 might indeed be too restrictive. The transformed wage residuals  $\Delta\tilde{\omega}_{ijt}$  defined in (6) are nonparametrically regressed on the revenue residuals  $\Delta\hat{\varepsilon}_{jt}$ . This implicitly assumes that permanent and transitory shocks have the same effect on wages. While this is at odds with the evidence presented above, it serves as a useful benchmark. It allows to apply a standard univariate kernel regression and does not rely on the predicted time series obtained by the Kalman smoother. The solid line in Figure 2 is obtained by local linear regression with an Epanechnikov kernel and the rule-of-thumb bandwidth estimator. The dashed lines indicate the bootstrapped 95% confidence band, accounting for clustering at the establishment level. To illustrate the support of  $\Delta\hat{\varepsilon}_{jt}$ , the shaded area illustrates the empirical distribution function of the establishment-level residuals (scale on the right axis).<sup>25</sup>

Apart from the shock realizations in the bottom tail of the distribution, the relation between revenue growth and wage growth is better described by a concave function rather than a linear one. Higher revenue growth leads to higher wage growth, but at a decreasing rate. The concave relationship vanishes for shocks below the 10th percentile. In this region, Figure 2 actually suggests that the wage elasticity (which corresponds to the slope of the curve) turns negative. The more detrimental the shock, the less it is passed on to wages. The complementary analysis

<sup>25</sup>The nonparametric estimation was done with Stata's `lpoly` command. Details on kernel and bandwidth selection can be found in the manual. The bandwidth was held constant for all bootstrap replications.

of the individual layoff probability in Figure 5 suggests that establishments facing a shock in the 10th percentile start to dismiss workers instead of cutting wages more severely. In both figures, however, the standard error gets very large at the tails, such that this observation should not be overemphasized.

To account for the nonlinearities uncovered in Figure 2, equation (5) is generalized to

$$\Delta \ln w_{ijt} = \Delta X'_{ijt} \delta + f(\Delta P_{jt}) + g(\Delta T_{jt}) + \Delta \psi_{ijt}, \quad (8)$$

where  $f$  and  $g$  can be parametric or nonparametric functions. In the estimation, the unobserved shock components are replaced by the predicted time series  $\Delta \hat{P}_{jt}$  and  $\Delta \hat{T}_{jt}$  obtained by the Kalman smoother. To find appropriate parametric forms of  $f$  and  $g$ , equation (8) is first estimated semiparametrically using an iterative backfitting procedure (Härdle et al., 2004, p.214-215). Starting with initial functional guesses  $\hat{f}_0$  and  $\hat{g}_0$ , a first estimate of the parametric part,  $\hat{\delta}_0$ , is obtained by OLS. The partial residual  $\Delta \ln w_{ijt} - \Delta X'_{ijt} \hat{\delta}_0 - \hat{f}_0(\Delta \hat{P}_{jt})$  is then regressed on  $\Delta \hat{T}_{jt}$  by local linear kernel regression to form predictions  $\hat{g}_1(\Delta \hat{T}_{jt})$ . Likewise, the partial residual  $\Delta \ln w_{ijt} - \Delta X'_{ijt} \hat{\delta}_0 - \hat{g}_1(\Delta \hat{T}_{jt})$  is nonparametrically regressed on  $\Delta \hat{P}_{jt}$  to form predictions  $\hat{f}_1(\Delta \hat{P}_{jt})$ . The procedure continues until convergence. Note that the estimated functions  $f$  and  $g$  are only identified up to an additive constant. In Figure 3 and Figure 4, the predicted functions are normalized to have zero mean.<sup>26</sup>

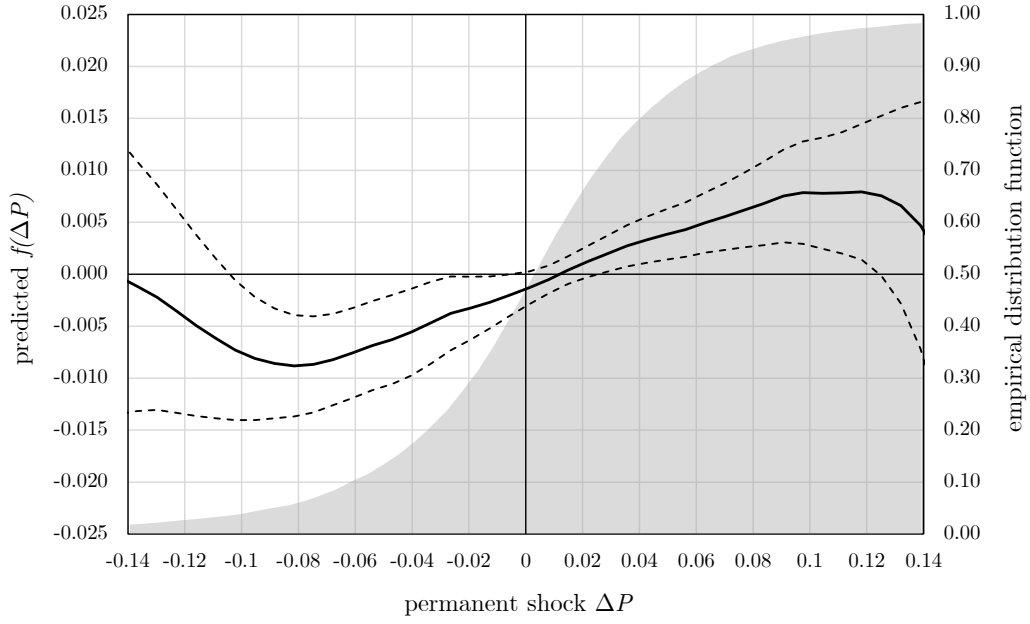
Figure 3 illustrates the predicted wage response with respect to a permanent shock,  $\hat{f}(\Delta P_{jt})$ . The wage response is approximately linear between the 5th percentile and the 95th percentile of the shock distribution. As the shock becomes more extreme, wages react less and less. The qualitative pattern of Figure 3 is strikingly similar to what Juhn et al. (2018) find for the US manufacturing sector by using a different estimation strategy.<sup>27</sup> Figure 4 illustrates the predicted wage response with respect to a transitory shock,  $\hat{g}(\Delta T_{jt})$ . It is qualitatively very similar to Figure 2, which is due to the fact that most of the variation in firm revenue is transitory. Wages hardly respond to transitory positive shocks, while negative transitory shocks above the 10th percentile are linearly passed on to wages. Below the 10th percentile, establishments again seem reluctant to further cut wages, although the confidence band becomes extremely wide.<sup>28</sup>

The above analysis yields important qualitative insights how shocks to firm revenue translate into wages. To compare to the literature, I also estimate local wage elasticities, which correspond to the slope of the wage response functions. To this purpose, I constrain  $f$  and  $g$  to linear splines and choose a set of  $K$  disjoint intervals  $\mathcal{I} = \{I_1, \dots, I_K\}$  that form a partition of the real line,

<sup>26</sup>Drawing on Figure 2, a linear spline with breakpoints at the 10th, 50th, and 90th percentile of the distributions is used as initial guess. Only one iteration of the backfitting algorithm is executed since no relevant changes in the predictions can be observed with more iterations. Using a linear starting function leads to visually identical predictions. Local linear kernel regressions use an Epanechnikov kernel with the rule-of-thumb bandwidth.

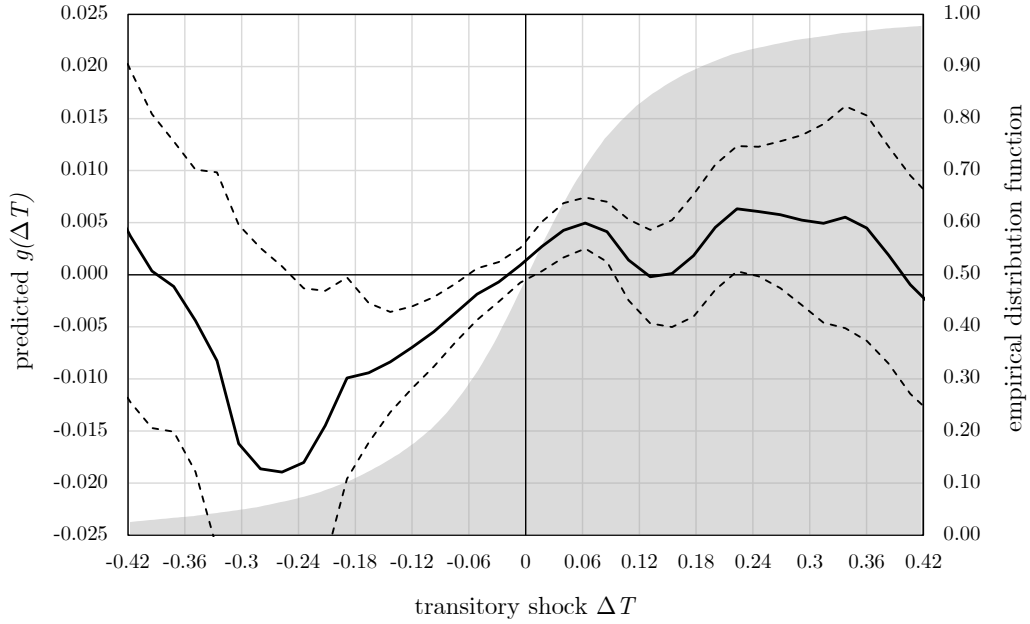
<sup>27</sup>Compare their Figure 4 as well as their Table 5 for the local wage elasticities with respect to positive and negative shocks.

<sup>28</sup>Note that the predicted transitory shocks also contain measurement error. Yet, the significant downward flexibility of wages evident from Table 4 indicates the presence of sufficiently strong fundamental shocks in the transitory component.



left axis: local linear kernel regression, 95% confidence band based on bootstrapped standard errors clustered at the establishment level; right axis: empirical cdf (shaded area)

Figure 3. Semiparametrically estimated wage response to permanent shocks



left axis: local linear kernel regression, 95% confidence band based on bootstrapped standard errors clustered at the establishment level; right axis: empirical cdf (shaded area)

Figure 4. Semiparametrically estimated wage response to transitory shocks

	interval $I_k$	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
		coefficient	std. err.	coefficient	std. err.
$\mathcal{I}_1$	$\mathbb{R}$	0.0625***	0.0143	0.0189*	0.0102
$\mathcal{I}_2$	$(-\infty, 0)$	-0.0056	0.0263	0.0462***	0.0182
	$[0, +\infty)$	0.1121***	0.0270	-0.0067	0.0098
$\mathcal{I}_3$	$(-\infty, q_{10})$	-0.0638*	0.0354	0.0003	0.0181
	$[q_{10}, q_{50})$	0.1082**	0.0524	0.0821**	0.0325
	$[q_{50}, q_{90})$	0.1149**	0.0498	0.0043	0.0220
	$[q_{90}, +\infty)$	0.0641	0.0516	-0.0285*	0.0168

$q_r$  refers to the  $r$ th percentile of the respective distribution; bootstrapped standard errors clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 4. Local wage elasticities for different partitions of  $\mathbb{R}$

$\bigcup_{k=1}^K I_k = \mathbb{R}$ . The functions  $f$  and  $g$  are assumed to be linear on each of the intervals and continuous everywhere. They satisfy

$$\begin{aligned} f'(\Delta P_{jt}) &= \alpha_k \quad \text{if } \Delta P_{jt} \in I_k, \\ g'(\Delta T_{jt}) &= \beta_k \quad \text{if } \Delta T_{jt} \in I_k, \end{aligned}$$

where the coefficients  $\alpha_k$  and  $\beta_k$  are the local wage elasticities. I consider three choices for the partition  $\mathcal{I}$ . The baseline is  $\mathcal{I}_1 = \mathbb{R}$ , which corresponds to a globally linear wage response and replicates the results from Section 4.2.1. The second partition  $\mathcal{I}_2$  features one breakpoint at  $\Delta P = \Delta T = 0$ . This allows to estimate separate wage elasticities with respect to positive and negative shocks. Finally, a third partition  $\mathcal{I}_3$  uses the qualitative insights from Figures 3 and 4 to choose the breakpoints. I set them at the 10th, 50th, and 90th percentile of the respective shock distributions. The specific values are reported in Table D.3. Table 4 reports the coefficient estimates obtained by OLS estimation of (8) with piecewise linear functions  $f$  and  $g$ . The unobserved shocks  $\Delta P_{jt}$  and  $\Delta T_{jt}$  are replaced by the predictions of the Kalman smoother.

Let me discuss the wage elasticities with respect to permanent shocks first. The first line in Table 4 assumes a globally linear relation between revenue growth and wage growth, and therefore coincides with the estimate reported in Table 3(a) in the ML column. Differentiating between positive and negative shocks suggests severe downward rigidity of real wages. While the wage elasticity with respect to a positive permanent shock is 0.1121 and highly significant, the wage response to negative permanent shocks is insignificant and close to zero. Concluding that wages are completely rigid downwards, however, is erroneous. The last block of results in Table 4 reveals that the downward elasticity of 0 hides considerable heterogeneity. Above the 10th percentile of the shock distribution, wages are actually as elastic as in the positive domain. Whereas below the 10th percentile, the wage elasticity is negative and weakly significant. The wage elasticity estimates therefore corroborate the qualitative pattern of Figure 3. There are no indications for downward wage rigidity apart from the first decile of the distribution. If

	interaction $\times$ interval $I_k$	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
		coefficient	std. err.	coefficient	std. err.
$\mathcal{I}_1$	blue-collar $\times \mathbb{R}$	0.0613***	0.0173	0.0244*	0.0134
	white-collar $\times \mathbb{R}$	0.0651***	0.0186	-0.0015	0.0088
$\mathcal{I}_2$	blue-collar $\times (-\infty, 0)$	-0.0228	0.0330	0.0634**	0.0250
	blue-collar $\times [0, +\infty)$	0.1198***	0.0331	-0.0129	0.0121
	white-collar $\times (-\infty, 0)$	-0.0007	0.0257	0.0117	0.0131
	white-collar $\times [0, +\infty)$	0.1224***	0.0277	-0.0139	0.0126
$\mathcal{I}_3$	blue-collar $\times (q_{10}, q_{50})$	0.0920	0.0611	0.1117***	0.0429
	blue-collar $\times [q_{50}, q_{90})$	0.1088**	0.0555	-0.0059	0.0285
	white-collar $\times (q_{10}, q_{50})$	0.0453	0.0555	0.0136	0.0196
	white-collar $\times [q_{50}, q_{90})$	0.1819***	0.0526	0.0054	0.0234

establishments in the manufacturing sector only; bootstrapped standard errors clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 5. Wage elasticity by worker type in the manufacturing sector

a very bad shock arrives, establishments keep wage cuts moderate and rather lay off workers (compare Table 6). In the middle of the distribution, between the 10th and 90th percentile, the estimated wage elasticity is around 0.11 and therefore almost twice as high as the estimate of 0.0625 obtained by assuming a globally linear wage response.

Let me now turn to the wage elasticities with respect to transitory shocks in Table 4. Not accounting for nonlinearities, the estimate is the same as in Table 3(a) and weakly significant. Distinguishing between positive and negative shocks reveals upwards wage rigidity. The elasticity estimate with respect to a positive transitory shock is insignificant and close to zero. Positive transitory shocks therefore purely increase firm rents. By contrast, the elasticity of 0.0462 estimated for negative shocks is highly significant. Putting additional breakpoints confirms the upwards rigidity, while downward rigidity is observed in the lower tail of the distribution. For negative transitory shocks above the 10th percentile, the wage elasticity is 0.0821 and close to the estimate for permanent shocks in the same percentile range. More detrimental shocks do not lead to further wage cuts.

The upward wage rigidity observed for transitory shocks could be rationalized by asymmetric information about the properties of shocks. Assume that workers observe the size of a shock but do not know its persistence, while firms know both. Firms have little incentive to let workers participate in unexpected temporary gains and hence refrain from wage increases. On the other hand, it may be relatively easy for firms to lower wages after a transitory shock if workers mistake them for a permanent shock. The reason is that negative permanent shocks increase layoff probabilities (compare Section 4.3). If persistence is unobserved, workers may be willing to lower wages after *any* negative shock to avoid a layoff.

**Heterogeneity.** Repeating the nonlinear analysis for particular industries or establishment size categories does not yield additional insights since standard errors become very high. Broadly, the findings of Table 4 seem to apply also on a more disaggregated level. It is feasible, how-



ever, to allow for worker heterogeneity. In particular, I interact the shocks with a dummy that indicates whether the worker is officially registered as a blue-collar or a white-collar worker. Firms may be reluctant to cut wages of white-collar workers because of agency and turnover considerations. First, their effort is more difficult to monitor such that a wage cut might result in shirking. Second, they are more expensive to replace if they shirk or decide to quit the firm voluntarily since white-collar work typically requires more firm-specific human capital. Along these lines, white-collar workers are expected to be better insured against negative shocks than blue-collar workers. The estimation results with interaction effects for worker type are reported in Table 5. To reduce selection effects, the analysis is constrained to the manufacturing sector. If linear wage responses are estimated, wages of blue-collar and white-collar workers react identically to permanent shocks. Transitory shocks, by contrast, only affect wages of blue-collar workers. Accounting for nonlinearities reveals that all the downward wage flexibility apparent in Table 4 stems from wages of blue-collar workers. Wages of white-collar workers, by contrast, do not react significantly to negative shocks, irrespective of their persistence. That downward wage rigidity is stronger for white-collar workers is in line with previous empirical evidence of Du Caju et al. (2007) for Belgium and Campbell (1997) for the US, for example.<sup>29</sup>

### 4.3 Layoff responses

Firms may adjust to negative shocks not only by lowering wages but also by dismissing workers. It is hard to statistically distinguish an employer-initiated layoff from an employee-initiated quit. Following Boockmann and Steffes (2010), I define a layoff as a transition from employment to non-employment where (a) the non-employment spell lasts for at least 60 days and (b) the next employment spell is not with the same employer.<sup>30</sup>

The layoff regressions are based on the following linear probability model at the worker level,<sup>31</sup>

$$lay_{ijt} = X'_{ijt}\delta + \alpha P_{jt} + \beta T_{jt} + \phi_{ij} + \psi_{ijt},$$

where the layoff dummy  $lay_{ijt}$  equals one if worker  $i$  is laid off by establishment  $j$  in year  $t$  and equals zero otherwise. This specification mirrors (4) and assumes a linear relationship between firm revenue and individual layoff probabilities. First differencing sweeps out the fixed effect,

$$\Delta lay_{ijt} = \Delta X'_{ijt}\delta + \alpha \Delta P_{jt} + \beta \Delta T_{jt} + \Delta \psi_{ijt}. \quad (9)$$

Replacing  $\Delta P_{jt}$  and  $\Delta T_{jt}$  with the predictions of the Kalman smoother and estimation by OLS

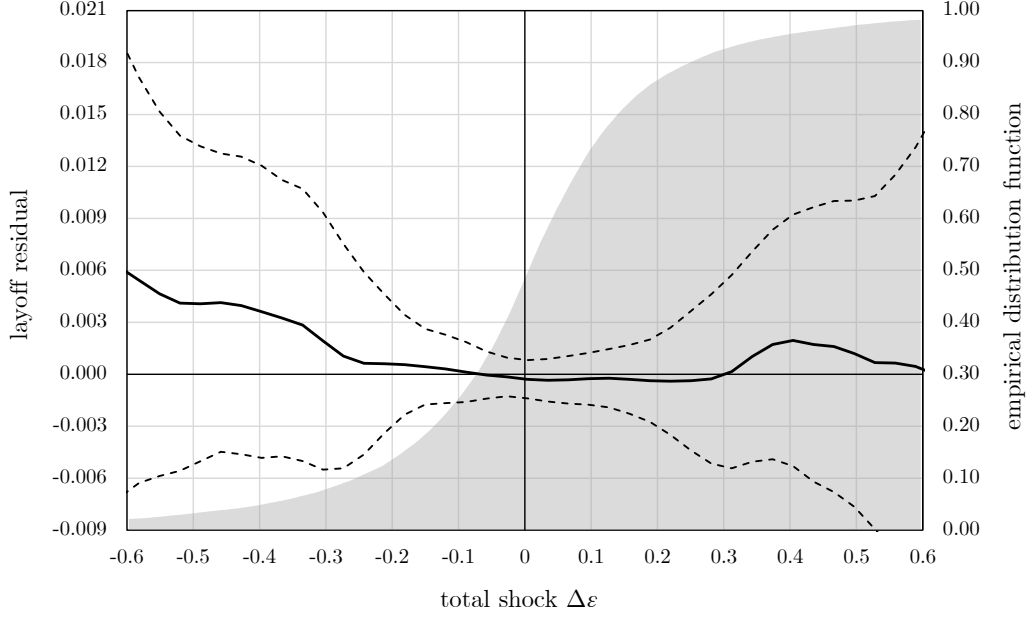
<sup>29</sup>Part of the observed downward wage flexibility of blue-collar workers might also come from a more flexible hours margin. Nevertheless, especially the wage response with respect to transitory shocks is unlikely to be explained by changes in working time alone. The asymmetric response would imply that working hours fall after a negative transitory shock, but do not increase again after the shock vanishes.

<sup>30</sup>Changing the threshold to 30 days hardly affects the results.

<sup>31</sup>Alternative approaches such as a correlated random effects probit model (Wooldridge, 2002, p.616)

$$P(lay_{ijt} = 1 | X_{ijt}, P_{jt}, T_{jt}) = \Phi(X'_{ijt}\delta + \alpha P_{jt} + \beta T_{jt} + \bar{X}'_{ij}\xi_1 + \xi_2 \bar{P}_j + \xi_3 \bar{T}_j)$$

did not lead to credible results, possibly due to remaining unobserved heterogeneity.



left axis: local linear kernel regression, 95% confidence band based on bootstrapped standard errors clustered at the establishment level; right axis: empirical cdf (shaded area)

Figure 5. Nonparametric layoff regression

gives the first line of results in Table 6. Layoff probabilities do not seem to react to shocks altogether.<sup>32</sup> Although Germany has stringent employment protection legislation that makes dismissals more complicated than in other countries, it is unlikely that troubled firms do not use this margin at all. Yet, the legal framework may lead firms to fire workers only if there is no other way to remain profitable. This is most likely to be the case after a negative permanent shock. Therefore, accounting for shock size and persistence at the same time may be crucial to obtain sensible results.

I first explore the role of nonlinearities in the relation between revenue growth and changes in the individual layoff probability by fitting a nonparametric regression. Figure 5 is generated in the same way as Figure 2. The estimated nonparametric function suggests that only shocks in the first decile of the distribution have an effect on layoffs, but this is subject to considerable uncertainty. Given an annual average layoff rate of 6.87 percent, the estimated increase in the layoff probability is quantitatively small even for the most detrimental shocks.

In a next step I distinguish again between permanent and transitory shocks,

$$\Delta \text{lay}_{ijt} = \Delta X'_{ijt} \delta + f(\Delta P_{jt}) + g(\Delta T_{jt}) + \Delta \psi_{ijt},$$

and perform a semiparametric regression. The explanatory variables are the same as in the wage regressions, and  $\Delta P_{jt}$  and  $\Delta T_{jt}$  are replaced by the predictions of the Kalman filter. Estimation uses the backfitting approach as explained in Section 4.2.2. The predictions for  $f$  and  $g$  are

<sup>32</sup>Estimating (9) with the indirect method fails to identify  $\alpha$  and  $\beta$ . The Hansen  $J$  test rejects validity of the instrument sets at the 1% level. This is most likely due to the binary nature of the dependent variable.

	interval $I_k$	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
		coefficient	std. err.	coefficient	std. err.
$\mathcal{I}_1$	$\mathbb{R}$	-0.0276	0.0213	0.0021	0.0095
$\mathcal{I}_2$	$(-\infty, 0)$	-0.0986**	0.0462	0.0048	0.0237
	$[0, +\infty)$	0.0257	0.0291	-0.0001	0.0175
$\mathcal{I}_3$	$(-\infty, q_{10})$	-0.1165	0.0696	-0.0245	0.0388
	$[q_{10}, q_{50})$	-0.0980	0.0748	0.0179	0.0337
	$[q_{50}, q_{90})$	0.0580	0.0574	-0.0115	0.0261
	$[q_{90}, +\infty)$	-0.0121	0.0681	0.0184	0.0404

$q_r$  refers to the  $r$ th percentile of the respective distribution; bootstrapped standard errors clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

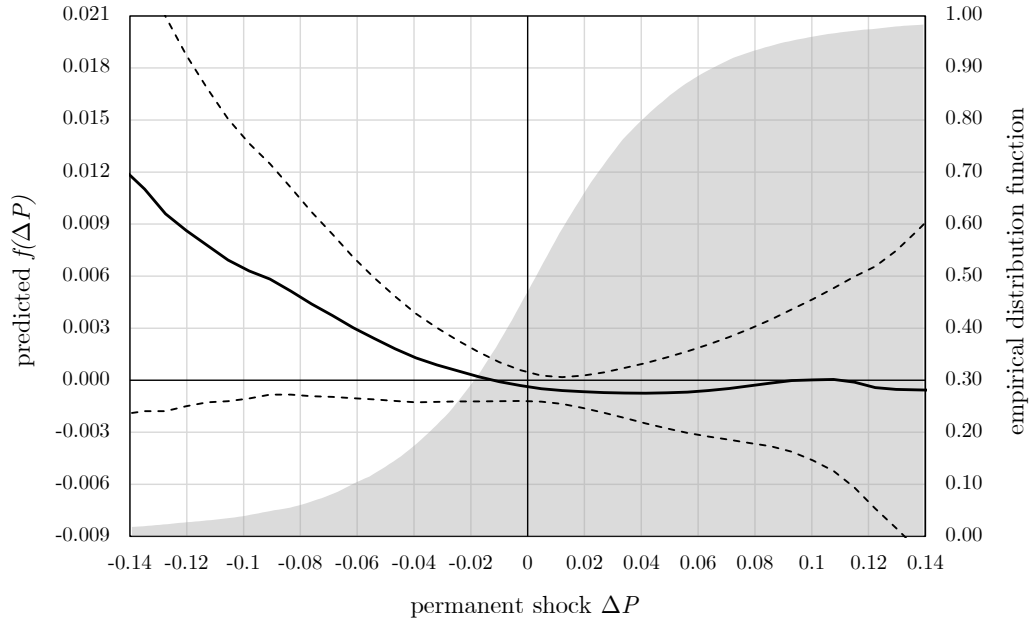
Table 6. Local semi-elasticity of the layoff probability for different partitions of  $\mathbb{R}$

depicted in Figure 6 and Figure 7, respectively. Since both functions are identified up to an additive constant, in the figures their sample mean has been normalized to zero.

Figure 6 depicts the estimated layoff response to a permanent shock. While positive shocks do not affect the layoff probability, in the negative domain the layoff probability increases with the severity of the shock. Confidence bands are still wide, but the more frequent use of layoffs in the aftermath of a bad shock appears to be at least weakly statistically significant. By contrast, Figure 7 reveals that layoff responses to transitory shocks do not follow any specific pattern. In particular, there is no evidence for an increase in layoffs following a negative transitory shock. Regarding the stringent employment protection legislation in Germany, the findings of Figures 6 and 7 are little surprising. In fact, they corroborate that the Kalman smoother is able to distinguish between the permanent and the transitory shock components with reasonable accuracy.

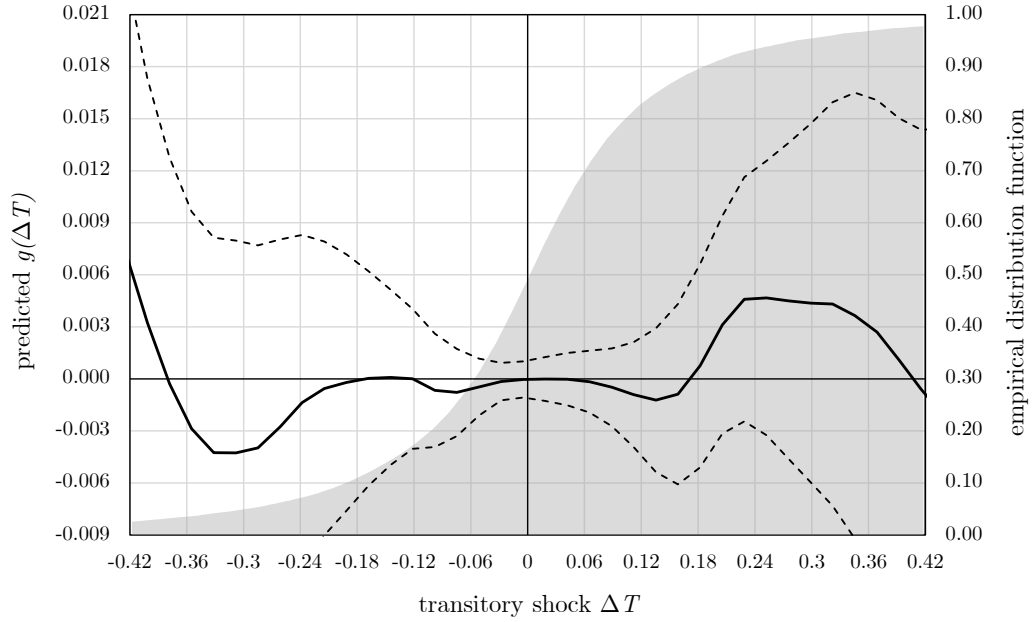
Finally, I calculate local semi-elasticities of the layoff probability using linear piecewise specifications for  $f$  and  $g$ . I use the same partitions as in Section 4.2.2, and the estimated coefficients are reported in Table 6. Differentiating positive and negative shocks confirms the graphical insight that the layoff probability only reacts to negative permanent shocks. The estimated semi-elasticity is  $-0.0986$  and significant at the 5% level. Considering that the average annual layoff rate in the sample is 6.87 percent, this implies an elasticity of  $\frac{\Delta \text{lay}_{ijt} / \overline{\text{lay}_{ijt}}}{\Delta P_{ijt}} = -\frac{0.0986}{0.0687} = -1.44$  with respect to negative permanent shocks. Adding further breakpoints suggests that the layoff elasticities above and below the 10th percentile of the shock distribution are nearly identical, such that conditioning on the sign of the shock is enough to adequately capture the nonlinearity of layoff responses.

**Heterogeneity.** Exploring heterogeneity in layoff responses by industry and establishment size does not yield any robust insights since standard errors become very large. It is feasible, however, to distinguish between white-collar and blue-collar employment as in Table 5. The observation there was that virtually all of the downward flexibility in wages is due to blue-collar workers, while wages of white-collar workers are unaffected by negative shocks. Does this



left axis: local linear kernel regression, 95% confidence band based on bootstrapped standard errors clustered at the establishment level; right axis: empirical cdf (shaded area)

Figure 6. Nonparametrically estimated layoff response to permanent shocks



left axis: local linear kernel regression, 95% confidence band based on bootstrapped standard errors clustered at the establishment level; right axis: empirical cdf (shaded area)

Figure 7. Nonparametrically estimated layoff response to transitory shocks

	interaction $\times$ interval $I_k$	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
		coefficient	std. err.	coefficient	std. err.
$\mathcal{I}_1$	blue-collar $\times \mathbb{R}$	-0.0266	0.0229	-0.0029	0.0118
	white-collar $\times \mathbb{R}$	0.0210	0.0229	-0.0003	0.0115
$\mathcal{I}_2$	blue-collar $\times (-\infty, 0)$	-0.1015*	0.0528	0.0135	0.0278
	blue-collar $\times [0, +\infty)$	0.0246	0.0321	-0.0184	0.0204
	white-collar $\times (-\infty, 0)$	-0.0087	0.0549	0.0205	0.0299
	white-collar $\times [0, +\infty)$	0.0452	0.0377	-0.0237	0.0208

establishments in the manufacturing sector only; bootstrapped standard errors clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 7. Semi-elasticity of the layoff probability by worker type in the manufacturing sector

downward wage rigidity imply that white-collar workers more often lose their job after a bad shock? Table 7 suggests the contrary. In fact, white-collar workers are perfectly insured against negative shocks. The increase in layoff probabilities after a negative permanent shock apparent from Table 6 is limited to blue-collar workers. Firms may be reluctant to fire white-collar workers as they anticipate higher hiring and training costs compared to blue-collar workers once the economic situation improves. Additionally, white-collar workers may be more complementary to other production factors such as physical capital, while blue-collar workers are easier to substitute in the production process.

## 5 Conclusion

This paper explores how the interaction between two important properties of idiosyncratic shocks, size and persistence, affect the degree of wage and employment insurance that firms provide to their employees. The econometric analysis follows a two step procedure. First, the stochastic properties of the shock process are determined along the lines of Guiso et al. (2005), and a Kalman smoother is applied at the firm level to decompose the total shock into its permanent and transitory component. In a second step, these predicted time series are included as explanatory variables in wage and layoff regressions. This approach allows to estimate arbitrary functional dependencies between wage growth and revenue growth, whereas the method proposed by Guiso et al. (2005) is confined to linear dependencies.

Using linked employer-employee data from Germany, I find that both shock persistence and shock size matters for the extent of insurance provision. The wage elasticity with respect to a permanent shock is constant between the 5th and 95th percentile of the shock distribution and becomes smaller at the tails. In response to extremely bad permanent shocks, firms seem to refrain from wage cuts and adjust via layoffs, perhaps in an effort to reduce the quitting incentive of the remaining workers. Lower wage elasticity for tail events has also been pointed out by Juhn et al. (2018). It implies that previous studies that have applied the Guiso et al. (2005) methodology have overestimated the degree of wage insurance that job stayers enjoy during normal times.

While downward wage rigidity is a widely-discussed phenomenon, the data suggest that wages are upward rigid with respect to transitory shocks. While negative transitory shocks tend to reduce wages, positive shocks are fully captured by the firm. This could be rationalized by asymmetric information about the persistence of shocks. Fearing job loss if the shock turns out to be permanent, workers may accept lower wages after *any* negative shock is observed.

Since firms can also adjust their wage bill along the extensive margin, the same methodology is used to analyze the impact of revenue shocks on individual layoff probabilities. Controlling for shock size and shock persistence at the same time turns out to be crucial. In the data, layoff probabilities only respond to negative permanent shocks. Transitory shocks do not have any significant effect. This strong result partly reflects Germany's stringent employment protection legislation that makes it difficult to dismiss workers. At the same time, this finding assures that the Kalman smoother can properly distinguish between transitory and permanent shocks.

The general patterns hide substantial heterogeneity at the worker level. Wage cuts and job loss after negative shocks are concentrated on blue-collar workers. Whereas white-collar workers enjoy full insurance against negative shocks, irrespective of their size and persistence. That the adjustment to negative shocks goes primarily at the expense of blue-collar workers hints at agency and turnover considerations of the employers: First, the effort of blue-collar workers may be easier to monitor, which allows more downward wage flexibility without spurring opportunistic behavior. Second, blue-collar workers usually require less firm-specific human capital and are therefore cheaper to replace because of lower hiring and training costs. Additionally, blue-collar employment may be easier to substitute by other production factors such as physical capital.

Overall, insurance provision by the firm is very heterogeneous, both with respect to the specific combination of shock size and shock persistence, and with respect to worker characteristics. The question of external validity necessarily arises. The qualitative pattern documented by [Juhn et al. \(2018\)](#) concerning the effect of shock size on wage insurance with respect to relatively permanent shocks is reassuring. Extensions of the analysis to the layoff margin seem to be missing to date, possibly because accounting for shock size and persistence at the same time is crucial in this regard.

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## A Sample statistics

	establishment level						worker level			
	full sample		wage sample		layoff sample		wage sample		layoff sample	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
sales per worker*	1.811	6.892	1.863	6.510	1.857	6.419	2.670	4.165	2.733	5.099
employment	181.3	772.1	204.0	828.3	198.7	813.9	2758.7	4814.4	3288.1	5477.5
capital-labor ratio*	0.947	5.913	0.950	2.751	0.952	2.808	1.409	2.172	1.426	2.182
works council	0.338		0.374		0.366		0.890		0.893	
industry CBA	0.464		0.483		0.479		0.795		0.804	
firm CBA	0.086		0.088		0.087		0.085		0.080	
1–9 employees	0.216		0.159		0.168		0.006		0.005	
10–99 employees	0.406		0.414		0.415		0.052		0.050	
100–199 employees	0.222		0.249		0.244		0.132		0.127	
200+ employees	0.156		0.177		0.173		0.810		0.818	
manufacturing	0.477		0.511		0.504		0.840		0.831	
construction	0.143		0.157		0.158		0.049		0.047	
sales	0.160		0.154		0.154		0.041		0.039	
services	0.220		0.178		0.185		0.070		0.083	
wage			85.23	25.35	87.13	29.13	107.23	27.28	116.29	39.09
tenure			9.312	4.407	8.506	4.398	12.234	7.393	11.578	7.823
age			41.98	4.974	41.93	4.771	41.459	8.702	41.817	8.762
white-collar			0.280		0.310		0.180		0.311	
no degree			0.090		0.086		0.163		0.136	
vocational degree			0.802		0.778		0.768		0.694	
high school			0.004		0.005		0.003		0.005	
voc. + high school			0.030		0.032		0.019		0.026	
applied university			0.034		0.042		0.027		0.073	
university			0.040		0.056		0.019		0.066	
establishments	2697		2531		2620					
individuals							216709		300667	

\* measured in 100000 €

Table A.1. Descriptive sample statistics

## B Microfoundation of the establishment-level regression

Equation (1) can be motivated by a Cobb-Douglas production function  $Y_{jt} = A_{jt}K_{jt}^{\alpha_K}L_{jt}^{\alpha_L}$  at the establishment level. Denoting the price of the output good as  $p_{jt}$ , firm revenue is  $R_{jt} = p_{jt}Y_{jt} = p_{jt}A_{jt}K_{jt}^{\alpha_K}L_{jt}^{\alpha_L}$ . Diving by  $L_{jt}$  and taking the logarithm yields

$$\ln r_{jt} = \alpha_K \ln k_{jt} + \delta \ln L_{jt} + Z'_{jt}\gamma + \varphi_j + \varepsilon_{jt}, \quad (10)$$

where  $\delta := \alpha_K + \alpha_L - 1$  and  $\ln(p_{jt}A_{jt}) = Z'_{jt}\gamma + \varphi_j + \varepsilon_{jt}$ . Estimating (10) in first differences generates an endogeneity problem because the change in employment,  $\Delta \ln L_{jt}$ , is correlated with  $\Delta \varepsilon_{jt}$ . In the literature, this is commonly resolved by using appropriate lags of the variable

	(a) static FE model		(b) dynamic FE model	
	coefficient	std. err.	coefficient	std. err.
$\ln r_{j,t-1}$	—	—	0.2503***	0.0378
$\ln k_{jt}$	0.3205***	0.0289	0.3021***	0.0233
$\ln L_{jt}$	0.0234	0.0380	−0.0206	0.0318
	statistic	<i>p</i> -value	statistic	<i>p</i> -value
AR(2) test	−2.77	0.006	1.81	0.070
AR(3) test	−1.55	0.120	−0.69	0.493
AR(4) test	0.72	0.471	1.10	0.270
Hansen <i>J</i> test	44.70	0.211	80.66	0.252

two-step difference GMM, corrected standard errors clustered at the establishment level, significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table B.2. GMM regressions of firm revenue including employment

in levels ( $\ln L_{jt}$ ) as instruments for the difference  $\Delta \ln L_{jt}$ . This yields a set of moment conditions  $\mathbb{E}[L_{j,t-s}\Delta \varepsilon_{jt}] = 0$  that (together with the ones belonging to the exogenous regressors  $\Delta Z_{jt}$ ) are used to apply GMM estimation. However, difference-in-Hansen tests reveal that any lag of  $\ln L_{jt}$  is itself correlated with  $\Delta \varepsilon_{jt}$  and therefore not a valid instrument.

Another possibility is to adopt ideas of [Blundell and Bond \(1998\)](#) and complement the first differenced equation by a level equation,

$$\Delta \ln r_{jt} = \alpha_K \Delta \ln k_{jt} + \Delta Z'_{jt} \gamma + \Delta \varepsilon_{jt}, \quad (11)$$

$$\ln r_{jt} = \delta \ln L_{jt} + Z'_{jt} \gamma + \varphi_j + \varepsilon_{jt}, \quad (12)$$

The difference equation (11) excludes the endogenous employment change, while the level equation (12) contains the employment variable and the strictly exogenous regressors  $Z_{jt}$ . The capital-labor ratio has to be excluded from the level equation since it is likely to be correlated with  $\varphi_j$ . Identification of  $\alpha$  therefore only uses moments from the difference equation, where  $\Delta \ln k_{jt}$  can be regarded as exogenous, as indicated by a series of difference-in-Hansen tests with different instrument choices. Identification of  $\delta$  is still an open issue, since  $\ln L_{jt}$  is likely to be correlated with the joint error term  $\varphi_j + \varepsilon_{jt}$ . [Blundell and Bond \(1998\)](#) suggest to instrument the variable in levels with its lagged first differences and to use moment conditions of the form  $\mathbb{E}[(\varphi_j + \varepsilon_{jt})\Delta \ln L_{j,t-s}] = 0$ . By (2), the time-varying error term  $\varepsilon_{jt}$  accumulates permanent shocks,  $\varepsilon_{jt} = \zeta_{j1} + \sum_{s=1}^t \tilde{u}_{js} + \tilde{v}_{jt}$ . Since period  $t$  employment is likely to depend on the permanent innovation  $\tilde{u}_{jt}$ , lagged employment changes are unlikely to be orthogonal to  $\varepsilon_{jt}$ . However, future employment changes might be. Provided that  $\Delta \ln L_{jt}$  is uncorrelated with  $\varphi_j$  and future errors  $\varepsilon_{js}$  ( $s > t + 1$ ), the moment conditions  $\mathbb{E}[(\varphi_j + \varepsilon_{jt})\Delta \ln L_{j,t+k}] = 0$  with  $k > 1$  can be used to identify  $\delta$ . In the estimation I use the 2nd, 3rd and 4th *leads* of  $\Delta \ln L_{jt}$  as instruments for  $\ln L_{jt}$ .

The results are reported in Table B.2(a). The coefficient estimate on the capital-labor ratio is close to the baseline of Table 1, while the coefficient on employment is insignificant and close to zero. The Hansen test does not reject the validity of the overidentifying moment

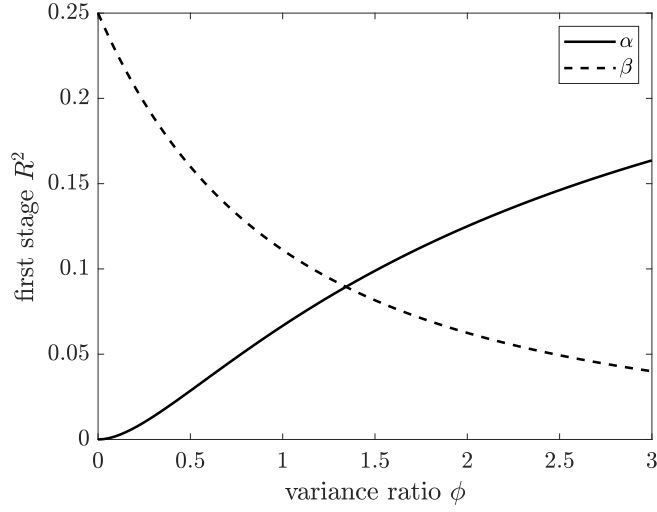


Figure C.1. First stage  $R^2$  for  $\alpha$  and  $\beta$  as a function of the variance ratio  $\phi = \sigma_u^2/\sigma_v^2$

restrictions. These observations remain valid if the lagged dependent variable is included among the explanatory variables, see Table B.2(b). Constant returns to scale at the establishment level,  $\delta = 0$ , cannot be rejected by a  $t$ -test. Since the point estimates of  $\delta$  are also not significant in economic terms, I impose constant returns to scale from the outset, which boils down to the regression equation (1).

## C When the indirect method fails

A common measure to detect weakness of an instrument is the first stage  $F$  statistic, which is a function of the  $R^2$  statistic. In the linear regression model  $x = \pi z + \xi$  the population coefficient of determination,  $R^2$ , equals the square of  $\rho = \mathbb{E}[xz]/\sqrt{\mathbb{E}x^2\mathbb{E}z^2}$ . Provided that  $\mathbb{E}x = \mathbb{E}z = 0$ ,  $\rho$  is simply the population-equivalent of the correlation coefficient between  $x$  and  $z$ .

The first stage regression that is fit to identify  $\beta$  is  $\Delta\varepsilon_{jt} = \pi\Delta\varepsilon_{j,t+1} + \xi_{jt}$ . Note that  $\mathbb{E}\Delta\varepsilon_{jt}^2 = \mathbb{E}\Delta\varepsilon_{j,t+1}^2 = \sigma_u^2 + 2\sigma_v^2$  and  $\mathbb{E}[\Delta\varepsilon_{jt}\Delta\varepsilon_{j,t+1}] = -\sigma_v^2$ . Therefore,

$$R_\beta^2 = \frac{(\sigma_v^2)^2}{(\sigma_u^2 + 2\sigma_v^2)^2} = \frac{1}{(2 + \phi)^2}$$

where  $\phi := \sigma_u^2/\sigma_v^2$ . The first stage regression that is fit to identify  $\alpha$  is  $\Delta\varepsilon_{jt} = \pi \sum_{k=-1}^1 \Delta\varepsilon_{j,t+k} + \xi_{jt}$ . Note that  $\mathbb{E}[(\sum_{k=-1}^1 \Delta\varepsilon_{j,t+k})^2] = 3\sigma_u^2 + 2\sigma_v^2$  and  $\mathbb{E}[\Delta\varepsilon_{jt} \sum_{k=-1}^1 \Delta\varepsilon_{j,t+k}] = \sigma_u^2$ . Therefore,

$$R_\alpha^2 = \frac{(\sigma_u^2)^2}{(\sigma_u^2 + 2\sigma_v^2)(3\sigma_u^2 + 2\sigma_v^2)} = \frac{\phi^2}{(2 + \phi)(2 + 3\phi)}.$$

Figure C.1 shows that  $R_\alpha^2$  is increasing in  $\phi$  while  $R_\beta^2$  is decreasing in  $\phi$ . This suggests a trade-off between the accuracy of the estimated  $\alpha$  and the accuracy of the estimated  $\beta$ .

## D Supplementary tables

### Distribution of shocks

percentile	total shock, $\Delta\hat{\varepsilon}_{jt}$	permanent shock, $\Delta\hat{P}_{jt}$	transitory shock, $\Delta\hat{T}_{jt}$
5%	−0.3970	−0.0902	−0.3000
10%	−0.2568	−0.0621	−0.1925
25%	−0.1028	−0.0260	−0.0763
50%	0.0053	0.0030	0.0024
75%	0.1086	0.0316	0.0792
90%	0.2504	0.0653	0.1862
95%	0.3797	0.0911	0.2836

Table D.3. Percentiles of the shock distributions

### Robustness of wage elasticity estimates

	(a) homoscedastic		(b) heteroscedastic: establishment size + industry	
	coefficient	std. err.	coefficient	std. err.
$\alpha$	0.0701***	0.0162	0.0626***	0.0170
$\beta$	0.0201**	0.0091	0.0192*	0.0101

bootstrapped standard errors clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table D.4. Wage elasticity estimates for different variance structures

### Heterogeneity by firm characteristics

industry	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
	coefficient	std. err.	coefficient	std. err.
manufacturing	0.0615***	0.0162	0.0204*	0.0121
construction	0.0950***	0.0313	0.0113	0.0136
sales	0.0599**	0.0236	0.0015	0.0116
services	0.0228	0.0344	0.0259	0.0246
total	0.0625***	0.0143	0.0189*	0.0102

separate wage regressions by industry; bootstrapped standard clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table D.5. Wage elasticity by industry

size category	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
	coefficient	std. err.	coefficient	std. err.
1–9 employees	0.0545	0.0407	0.0069	0.0097
10–99 employees	0.0681***	0.0177	0.0148***	0.0057
100–199 employees	0.0593***	0.0155	0.0110	0.0087
200+ employees	0.0588***	0.0178	0.0231	0.0146
total	0.0625***	0.0143	0.0189*	0.0102

separate wage regressions by size category; bootstrapped standard clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table D.6. Wage elasticity by establishment size category

	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
	coefficient	std. err.	coefficient	std. err.
(baseline)	0.0708*	0.0382	0.0179*	0.0105
CBA industry	−0.0142	0.0354	0.0035	0.0188
CBA firm	−0.0936	0.0801	0.0073	0.0291
WC	0.0104	0.0386	−0.0004	0.0209

baseline = no CBA and no WC; establishments in the manufacturing sector only; bootstrapped standard clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table D.7. Wage elasticity by industrial relations (direct method)

	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
	coefficient	std. err.	coefficient	std. err.
(baseline)	0.0741*	0.0397	0.0101	0.0188
CBA industry	−0.0257	0.0392	0.0133	0.0457
CBA firm	−0.0923	0.0879	0.0201	0.0489
WC	−0.0008	0.0458	−0.0121	0.0397
K-P Wald $F$ stat.	1.688		4.620	
Hansen $J$ stat. ( $p$ val.)	15.41 (0.212)		13.78 (0.315)	

baseline = no CBA and no WC; establishments in the manufacturing sector only; bootstrapped standard clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table D.8. Wage elasticity by industrial relations (indirect method)



## Heterogeneity by gender

	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
	coefficient	std. err.	coefficient	std. err.
male	0.0628***	0.0143	0.0188*	0.0101
female	0.0443***	0.0170	0.0025	0.0072

interactions with gender; bootstrapped standard clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table D.9. Wage elasticity by gender

	permanent shock, $\Delta P_{jt}$		transitory shock, $\Delta T_{jt}$	
	coefficient	std. err.	coefficient	std. err.
male	0.0722*	0.0376	0.0169	0.0106
female	0.0741**	0.0376	-0.0014	0.0146
CBA industry $\times$ male	-0.0132	0.0348	0.0041	0.0188
CBA industry $\times$ female	-0.0712*	0.0413	0.0244	0.0189
WC $\times$ male	0.0079	0.0381	0.0000	0.0206
WC $\times$ female	0.0371	0.0453	-0.0171	0.0199

interactions with gender and industrial relations; establishments in the manufacturing sector only; bootstrapped standard clustered at the establishment level, coefficient significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table D.10. Wage elasticity by gender and industrial relations

# Web appendix to “Size and persistence matter: Wage and employment insurance at the micro level”

Martin Kerndler\*

*(Intended for web publication only)*

## 1 Maximum likelihood estimation

Assume the following linear state space system is valid for every firm  $j$ ,

$$\begin{aligned} y_{jt} &= Bz_{jt} + u_{jt}, & u_{jt} &\sim N(0, \sigma_{ju}^2 R) \\ z_{jt} &= Az_{j,t-1} + e_{jt}, & e_{jt} &\sim N(0, \sigma_{je}^2 Q), \end{aligned}$$

where  $\ln \sigma_{ju}^2 = D'_{ju} \lambda_u$  and  $\ln \sigma_{je}^2 = D'_{je} \lambda_e$ . The unknown parameter vectors  $\lambda_u \in \mathbb{R}^{K_u \times 1}$  and  $\lambda_e \in \mathbb{R}^{K_e \times 1}$  are estimated from the data by maximum likelihood.

### 1.1 Evaluation of the likelihood

Since shocks are independent across firms, the joint log-likelihood  $\ell\ell$  is the sum of the firm-specific log-likelihoods,

$$\ell\ell(\lambda_u, \lambda_e) = \sum_{j=1}^J \ell\ell_j(\lambda_u, \lambda_e) \omega_j,$$

where  $\omega_j$  may be a firm-specific weight. At the firm level, the log-likelihood can be evaluated using the Kalman filter. For  $t = 1, \dots, t_j$ , the algorithm proceeds in three steps:

1. prediction:

$$\begin{aligned} \hat{z}_{jt} &= Az_{j,t-1} \\ \hat{P}_{jt} &= AP_{j,t-1}A' + \sigma_{je}^2 Q \end{aligned}$$

2. update states:

$$\begin{aligned} \tilde{u}_{jt} &= y_{jt} - B\hat{z}_{jt} \\ S_{jt} &= B\hat{P}_{jt}B' + \sigma_{ju}^2 R \end{aligned}$$

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$$\begin{aligned}
K_{jt} &= \hat{P}_{jt} B' S_{jt}^{-1} \\
z_{jt} &= \hat{z}_{jt} + K_{jt} \tilde{u}_{jt} \\
P_{jt} &= (I - K_{jt} B) \hat{P}_{jt}
\end{aligned}$$

3. update log-likelihood:

$$\ell\ell_{jt} = \ell\ell_{j,t-1} - \frac{1}{2} \left[ \tilde{u}_{jt}' S_{jt}^{-1} \tilde{u}_{jt} + \ln |S_{jt}| + \ln(2\pi) \right]$$

The recursive system is initialized with  $z_{j0} = 0$ ,  $P_{j0} = \sigma_{je}^2 I$ , and  $\ell\ell_{j0} = 0$  for all  $j = 1, \dots, J$ . Clearly,  $\ell\ell_j(\lambda_u, \lambda_e) = \ell\ell_{jt_j}$ .

## 1.2 First order derivatives

Let  $h \in \{e, u\}$ . Since any parameter  $\lambda_{hk}$ ,  $k \in \{1, \dots, K_h\}$  only affects the likelihood via  $\sigma_{jh}^2$ , the gradient vector satisfies

$$\frac{\partial \ell\ell(\lambda_u, \lambda_e)}{\partial \lambda_h} = \sum_{j=1}^J \frac{\partial \ell\ell_j(\lambda_u, \lambda_e)}{\partial \lambda_h} \omega_j = \sum_{j=1}^J \frac{\partial \ell\ell_j(\lambda_u, \lambda_e)}{\partial \sigma_{jh}^2} \frac{\partial \sigma_{jh}^2}{\partial \lambda_h} \omega_j.$$

The functional form of  $\sigma_{jh}^2$  implies

$$\frac{\partial \sigma_{jh}^2}{\partial \lambda_h} = \sigma_{jh}^2 D_{jh} \in \mathbb{R}^{K_h \times 1},$$

and therefore the gradient vector is just a linear combination of the regressors,

$$\frac{\partial \ell\ell(\lambda_u, \lambda_e)}{\partial \lambda_h} = \sum_{j=1}^J \underbrace{\frac{\partial \ell\ell_j}{\partial \sigma_{jh}^2} \sigma_{jh}^2}_{\in \mathbb{R}} D_{jh} \omega_j \in \mathbb{R}^{K_h \times 1}.$$

For each firm, the derivative  $\partial \ell\ell_j / \partial \sigma_{jh}^2$  can be computed recursively by differentiating the Kalman filter equations,<sup>1</sup>

1. prediction:

$$\begin{aligned}
\frac{\partial \hat{z}_{jt}}{\partial \sigma_{jh}^2} &= A \frac{\partial z_{j,t-1}}{\partial \sigma_{jh}^2} \\
\frac{\partial \hat{P}_{jt}}{\partial \sigma_{jh}^2} &= A \frac{\partial P_{j,t-1}}{\partial \sigma_{jh}^2} A' + \delta_{he} Q
\end{aligned}$$

---

<sup>1</sup>  $\delta_{hg}$  refers to Kronecker's delta for  $h, g \in \{e, u\}$ .

2. update states:

$$\begin{aligned}
\frac{\partial \tilde{u}_{jt}}{\partial \sigma_{jh}^2} &= -B \frac{\partial \hat{z}_{jt}}{\partial \sigma_{jh}^2} \\
\frac{\partial S_{jt}}{\partial \sigma_{jh}^2} &= B \frac{\partial \hat{P}_{jt}}{\partial \sigma_{jh}^2} B' + \delta_{hu} R \\
\frac{\partial K_{jt}}{\partial \sigma_{jh}^2} &= \frac{\partial \hat{P}_{jt}}{\partial \sigma_{jh}^2} B' S_{jt}^{-1} - \hat{P}_{jt} B' S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \\
\frac{\partial z_{jt}}{\partial \sigma_{jh}^2} &= \frac{\partial \hat{z}_{jt}}{\partial \sigma_{jh}^2} + \frac{\partial K_{jt}}{\partial \sigma_{jh}^2} \tilde{u}_{jt} + K_{jt} \frac{\partial \tilde{u}_{jt}}{\partial \sigma_{jh}^2} \\
\frac{\partial P_{jt}}{\partial \sigma_{jh}^2} &= -\frac{\partial K_{jt}}{\partial \sigma_{jh}^2} B \hat{P}_{jt} + (I - K_{jt} B) \frac{\partial \hat{P}_{jt}}{\partial \sigma_{jh}^2}
\end{aligned}$$

3. update first order derivatives of the log-likelihood:

$$\frac{\partial \ell_{jt}}{\partial \sigma_{jh}^2} = \frac{\partial \ell_{j,t-1}}{\partial \sigma_{jh}^2} - \frac{1}{2} \left[ 2 \frac{\partial \tilde{u}'_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \tilde{u}_{jt} - \tilde{u}'_{jt} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \tilde{u}_{jt} + \text{tr} \left( S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} \right) \right]$$

The recursion is initialized with

$$\frac{\partial z_{j0}}{\partial \sigma_{jh}^2} = 0, \quad \frac{\partial P_{j0}}{\partial \sigma_{jh}^2} = \delta_{he} I, \quad \frac{\partial \ell_{j0}}{\partial \sigma_{jh}^2} = 0.$$

Again,  $\partial \ell_j / \partial \sigma_{jh}^2 = \partial \ell_{jt_j} / \partial \sigma_{jh}^2$ .

### 1.3 Second order derivatives

Let  $g, h \in \{e, u\}$ . The second derivative of the likelihood function is

$$\frac{\partial^2 \ell(\lambda_u, \lambda_e)}{\partial \lambda'_g \partial \lambda_h} = \sum_{j=1}^J \frac{\partial^2 \ell_j(\lambda_u, \lambda_e)}{\partial \lambda'_g \partial \lambda_h} \omega_j$$

where

$$\frac{\partial^2 \ell_j(\lambda_u, \lambda_e)}{\partial \lambda'_g \partial \lambda_h} = \frac{\partial^2 \ell_j(\lambda_u, \lambda_e)}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} \frac{\partial \sigma_{jg}^2}{\partial \lambda'_g} \frac{\partial \sigma_{jh}^2}{\partial \lambda_h} + \frac{\partial \ell_j(\lambda_u, \lambda_e)}{\partial \sigma_{jh}^2} \frac{\partial^2 \sigma_{jh}^2}{\partial \lambda'_g \partial \lambda_h}.$$

Since

$$\frac{\partial \sigma_{jh}^2}{\partial \lambda_h} = \sigma_{jh}^2 D'_{jh} \quad \text{and} \quad \frac{\partial^2 \sigma_{jh}^2}{\partial \lambda'_g \partial \lambda_h} = \begin{cases} \sigma_{jh}^2 D_{jh} D'_{jh} & \text{if } g = h, \\ 0 & \text{if } g \neq h \end{cases}$$

the Hessian is a linear combination of outer products of the regressor matrices,

$$\frac{\partial^2 \ell(\lambda_u, \lambda_e)}{\partial \lambda'_g \partial \lambda_h} = \sum_{j=1}^J \underbrace{\left[ \frac{\partial^2 \ell_j}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} \sigma_{jg}^2 \sigma_{jh}^2 + \frac{\partial \ell_j}{\partial \sigma_{jh}^2} \sigma_{jh}^2 \delta_{gh} \right]}_{\in \mathbb{R}} D_{jg} D'_{jh} \omega_j \in \mathbb{R}^{K_g \times K_h}$$

The second order derivatives  $\partial^2 \ell_j / (\partial \sigma_{jg}^2 \partial \sigma_{jh}^2)$  can again be computed recursively,

1. prediction:

$$\begin{aligned}\frac{\partial^2 \hat{z}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= A \frac{\partial^2 z_{j,t-1}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} \\ \frac{\partial^2 \hat{P}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= A \frac{\partial^2 P_{j,t-1}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} A'\end{aligned}$$

2. update states:

$$\begin{aligned}\frac{\partial^2 \tilde{u}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= -B \frac{\partial^2 \hat{z}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} \\ \frac{\partial^2 S_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= B \frac{\partial^2 \hat{P}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} B' \\ \frac{\partial^2 K_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= \frac{\partial^2 \hat{P}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} B' S_{jt}^{-1} - \frac{\partial \hat{P}_{jt}}{\partial \sigma_{jh}^2} B' S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \\ &\quad - \frac{\partial \hat{P}_{jt}}{\partial \sigma_{jg}^2} B' S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} + \hat{P}_{jt} B' S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \\ &\quad - \hat{P}_{jt} B' S_{jt}^{-1} \frac{\partial^2 S_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} S_{jt}^{-1} + \hat{P}_{jt} B' S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \\ \frac{\partial^2 z_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= \frac{\partial^2 \hat{z}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} + \frac{\partial^2 K_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} \tilde{u}_{jt} + \frac{\partial K_{jt}}{\partial \sigma_{jh}^2} \frac{\partial \tilde{u}_{jt}}{\partial \sigma_{jg}^2} + \frac{\partial K_{jt}}{\partial \sigma_{jg}^2} \frac{\partial \tilde{u}_{jt}}{\partial \sigma_{jh}^2} + K_{jt} \frac{\partial^2 \tilde{u}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} \\ \frac{\partial^2 P_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= -\frac{\partial^2 K_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} B \hat{P}_{jt} - \frac{\partial K_{jt}}{\partial \sigma_{jh}^2} B \frac{\partial \hat{P}_{jt}}{\partial \sigma_{jg}^2} - \frac{\partial K_{jt}}{\partial \sigma_{jg}^2} B \frac{\partial \hat{P}_{jt}}{\partial \sigma_{jh}^2} + (I - K_{jt} B) \frac{\partial^2 \hat{P}_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2}\end{aligned}$$

3. update second order derivatives of the log-likelihood:

$$\begin{aligned}\frac{\partial \ell_{jt}^2}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} &= \frac{\partial \ell_{j,t-1}^2}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} - \frac{1}{2} \left[ 2 \frac{\partial^2 \tilde{u}'_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} S_{jt}^{-1} \tilde{u}_{jt} - 2 \frac{\partial \tilde{u}'_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \tilde{u}_{jt} + 2 \frac{\partial \tilde{u}'_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \frac{\partial \tilde{u}_{jt}}{\partial \sigma_{jh}^2} \right. \\ &\quad - 2 \frac{\partial \tilde{u}'_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \tilde{u}_{jt} + 2 \tilde{u}'_{jt} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} S_{jt}^{-1} \tilde{u}_{jt} \\ &\quad \left. - \tilde{u}'_{jt} S_{jt}^{-1} \frac{\partial^2 S_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} S_{jt}^{-1} \tilde{u}_{jt} + \text{tr} \left( -S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jg}^2} S_{jt}^{-1} \frac{\partial S_{jt}}{\partial \sigma_{jh}^2} \right) + \text{tr} \left( S_{jt}^{-1} \frac{\partial^2 S_{jt}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} \right) \right]\end{aligned}$$

The recursion is initialized with

$$\frac{\partial^2 z_{j0}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} = 0, \quad \frac{\partial^2 P_{j0}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} = 0, \quad \frac{\partial^2 \ell_{j0}}{\partial \sigma_{jg}^2 \partial \sigma_{jh}^2} = 0.$$

Again,  $\partial \ell_j^2 / (\partial \sigma_{jg}^2 \partial \sigma_{jh}^2) = \partial^2 \ell_{jt_j} / (\partial \sigma_{jg}^2 \partial \sigma_{jh}^2)$ .

## 2 Decomposing firm productivity

Suppose that idiosyncratic firm productivity  $Y_{jt}$  follows the dynamic model for  $t \geq 1$

$$\begin{aligned} Y_{jt} &= \rho Y_{j,t-1} + \varepsilon_{jt}, \\ \varepsilon_{jt} &= \xi_{jt} + \tilde{v}_{jt}, & \tilde{v}_{jt} &\sim N(0, \sigma_{j\tilde{v}}^2), \\ \xi_{jt} &= \xi_{j,t-1} + \tilde{u}_{jt}, & \tilde{u}_{jt} &\sim N(0, \sigma_{j\tilde{u}}^2), \end{aligned}$$

where  $|\rho| < 1$  and  $Y_{j0}$  is the first available observation. The productivity process is integrated of order 1 and can be decomposed into a transitory and a permanent stochastic component by the Granger representation theorem,

$$Y_{jt} = T_{jt} + P_{jt},$$

where

$$\begin{aligned} P_{jt} &= (1 - \rho)^{-1} \xi_{jt}, \\ T_{jt} &= \rho T_{j,t-1} + \tilde{v}_{jt} - \rho(1 - \rho)^{-1} \tilde{u}_{jt}. \end{aligned}$$

Taking the AR parameter  $\rho$  as given, the goal is to decompose the observed time series of  $Y_{jt}$  into its unobserved components  $T_{jt}$  and  $P_{jt}$ . This can be achieved with the above version of the Kalman filter, which estimates the firm-specific variances  $(\sigma_{j\tilde{v}}^2, \sigma_{j\tilde{u}}^2)$  *en lieu* for given matrices of explanatories  $D_{j\tilde{v}}$  and  $D_{j\tilde{u}}$ . For the filter to be applicable, the system is first differenced to render it stationary. The error process is then

$$\Delta \varepsilon_{jt} = \tilde{u}_{jt} + \Delta \tilde{v}_{jt} = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} \tilde{v}_{jt} \\ \tilde{v}_{j,t-1} \end{pmatrix} + \tilde{u}_{jt}, \quad t \geq 2,$$

Defining  $z_{jt} := (\tilde{v}_{jt}, \tilde{v}_{j,t-1})'$  as the unobserved state of the Kalman filter gives rise to the state equation

$$z_{jt} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} z_{j,t-1} + \begin{pmatrix} \tilde{v}_{jt} \\ 0 \end{pmatrix}, \quad t \geq 2,$$

Since  $\Delta \varepsilon_{jt} = \Delta Y_{jt} - \rho \Delta Y_{j,t-1}$  is known, the Kalman filter described above can be applied to the problem and used to predict the state sequence  $\hat{z}_{jt}$  as well as the residuals  $\hat{\tilde{u}}_{jt}$  for  $t \geq 2$ . The sequence  $\hat{\tilde{v}}_{jt}$  for  $t \geq 1$  can be reconstructed from the states. Assuming a starting value  $\hat{T}_{j1} = \hat{\tilde{v}}_{j1}$ , this allows to construct  $\hat{T}_{jt}$  and  $\hat{P}_{jt} = Y_{jt} - \hat{T}_{jt}$  for  $t \geq 1$ .