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# International endogenous growth, macro anomalies, and asset prices

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# International Endogenous Growth, Macro Anomalies, and Asset Prices

SAFE Working Paper No. 83

SAFE | Sustainable Architecture for Finance in Europe
A cooperation of the Center for Financial Studies and Goethe University Frankfurt

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## Non-Technical Summary

Technology spillover and international patent diffusion are important sources of economic growth. Moreover, international and domestic financial markets are highly developed and close-to-perfect in developed economies nowadays. Especially over the last decade, financial markets have become increasingly integrated. Asset returns are also typically quite highly correlated across countries. Another stylized fact in international macroeconomics is the robustly observed spread between the cross-country correlations of consumption growth to the one of output growth. Consumption growth is typically less correlated across countries than output growth.

So far, little is known about the equilibrium effects of technology spillover on asset prices and, in particular, on the cross-section of equity returns. This study develops a two-country endogenous growth economy with complete and frictionless financial markets that matches major stylized facts in international macroeconomics and provides a comprehensive analysis of cross-sectional and aggregate asset prices.

In the model, each country is populated by a single representative household, a single perfectly competitive final good firm, and a continuum of monopolistically competitive intermediate goods producers. The final good firm uses capital, labor, and a composite of intermediate goods to produce the respective country's final good. The production output is subject to both common and idiosyncratic productivity shocks affecting both countries simultaneously and only one country, respectively. The monopolistically competitive intermediate goods producers ("incumbents") each produce a single intermediate good. These incumbents can innovate on the quality of their products themselves or they can be displaced by new firms ("entrants") if these potential entrants make a successful innovation. Households consume both countries' consumption goods and display home bias for their domestic goods. The innovation technology of incumbents and the one of potential entrants use goods from both countries to capture endogenous technology spillover effects.

There are two important mechanisms at the core of this model. First, there is realistic competition between incumbents and potential entrants in innovations. Second, households and innovators share their risks internationally by using both final goods for consumption and research and development (R&D). Due to these risk-sharing opportunities, innovations in one country thus induce a positive effect also on the productivity of the other country.

Three different versions of the model are analyzed. The initial model features no technology spillover. Next, homogenous technology spillovers are added by allowing incumbents and entrants to use both goods for their R&D technologies. Finally, the home bias parameters of incumbents' and entrants' R&D technologies are assumed to be heterogeneous.

The first model only allows a small amount of final good trade and thus the households mainly resort to financial markets to share their risks. This induces a high volatility of the exchange rate and negative long-run spillover across countries in consumption dynamics driven by increased exports from the country experiencing a positive productivity shock to the other. This positive correlation between net exports and output is counterfactual and is

resolved by adding technology spillover to the model. The increased quantities in good trade lead to a much smoother exchange rate and net exports now deteriorate in response to positive productivity news, consistent with the data. Furthermore, this model generates a considerable amount of positive spillover in R&D expenditures providing a realistic description of international technology diffusion. Heterogeneity in the home bias of firms' R&D technologies enable the model to replicate the empirically rather mild correlation of incumbents' and entrants' innovation probabilities within a country.

Regarding international asset pricing dynamics, the three model specifications yield different implications. The first model without technology spillover does not match the high cross-country correlation of excess market returns. This is resolved by all models with technology spillover. Excess returns of incumbents are positively correlated without technology spillover and highly negatively correlated with technology spillover.

To summarize, the novel technology spillover channel in the model provides a micro-foundation why returns, growth rates, and macroeconomic quantities co-move across developed economies. A higher degree of international co-movement would be the likely consequence of policies that reduce barriers to international cooperation in R&D.

# International Endogenous Growth, Macro Anomalies, and Asset Prices

Patrick Grüning*

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**Abstract**

This paper studies a two-country production economy with complete and frictionless financial markets and international trade in which competition in R&D leads to endogenous new firm creation and economic growth. Current monopolists ("incumbents") and potential new firms ("entrants") compete in developing patents domestically. These innovative firms use both consumption goods in their R&D technologies to capture international technological spillovers. In the model specifications with technology spillover one obtains that (i) the cross-country correlation of consumption growth is lower than the one of output growth; (ii) net exports are negatively correlated with output; (iii) the model matches the high co-movement of stock returns across countries. Furthermore, heterogeneity in the R&D technology bundle home bias parameters for incumbents and entrants enables the model to replicate the empirically rather moderate correlation between the R&D innovation probabilities of incumbents and entrants within a country. Moreover, the model produces a positive value premium. Finally, the exchange rate volatility is decreasing in the amount of technology spillovers.

**Keywords:** Innovation, Technology Spillover, Endogenous Growth, Long-run Risk, International Finance

**JEL:** E22, F31, G12, O30, O41

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# 1 Introduction

Technology spillover and international patent diffusion have been identified as important sources of economic growth as documented by, for example, Coe, Helpman, and Hoffmaister (1997) and Santacreu (2015). However, little is known about the equilibrium effects of technology spillover on asset prices and, in particular, on the cross-section of equity returns. In this study, I develop a two-country endogenous growth economy with complete and frictionless financial markets that matches major stylized facts in international macroeconomics and that provides a comprehensive analysis of cross-sectional and aggregate asset prices. Major stylized facts in international macroeconomics include: (i) the cross-country correlations of macroeconomic quantities are moderate (Rabanal, Rubio-Ramírez, and Tuesta, 2011); (ii) asset markets highly co-move (Colacito and Croce, 2013); (iii) asset markets are highly integrated nowadays among developed countries (Fitzgerald, 2012); (iv) the exchange rate is very volatile (Tretvoll, 2013); and (v) the cross-country correlation of consumption growth is significantly lower than the one of output growth (Backus, Kehoe, and Kydland, 1994).

In the model, each country is populated by a representative household with recursive preferences, a final goods sector, and an intermediate goods sector. In the final goods sector, a perfectly competitive representative final goods firm uses capital, labor, and a composite of intermediate goods to produce the respective country's final good. The production output is subject to stochastic productivity shocks, which can be either restricted to one country ("idiosyncratic" shocks) or affect both countries simultaneously ("common" shocks). The intermediate goods sector is populated by a continuum of monopolistically competitive producers ("incumbents") each producing a single intermediate good. These incumbents can incrementally innovate on the quality of their products themselves or they can be displaced by new firms ("entrants") if these potential entrants make a successful radical innovation.[1] Hence, each period, a fraction of the technology capital improves quickly (caused by entrants' innovations), another fraction improves slowly (caused by incumbents' innovations), and the remaining part of the products' quality depreciates slightly capturing patent obsolescence. Households consume an aggregate of the two consumption goods and have access to a full set of state-contingent international Arrow-Debreu securities. Hence, financial markets are complete, both domestically and internationally. Technology spillovers are introduced in the full model by allowing both incumbents and

[1]The continuing process of creation and destruction of companies due to technical obsolescence is a key feature of economic growth in developed countries. This process of *creative destruction* has already been emphasized by Schumpeter (1934, 1942).

potential entrants to also use both consumption goods in their research and development (R&D) technologies.

There are two important mechanisms at the core of my model. First, there is competition between incumbents and entrants in the innovation of intermediate goods. The composition of the total innovative activity in a given country, i.e. the split of total R&D efforts between incumbents and entrants, therefore, has direct consequences on the patent value and, consequently, on aggregate quantities and asset prices. Second, households and innovators share their risks internationally by using both final goods for consumption and R&D. Thus, the composition between expenditure on domestic goods and on foreign goods is also an important risk factor. Therefore, innovations in either country induce endogenous technology spillover effects, which affect both domestic and foreign dynamics of macroeconomic growth rates and returns.

The model is calibrated to feature two symmetric equally-sized countries, which are highly developed to admit perfectly integrated financial markets.[2] I study three different versions of the model. The initial model does not feature a technology spillover channel and, thus, innovation technologies only use domestic goods. Next, technology spillovers are added by allowing incumbents and entrants to use both goods in their R&D technologies with identical home bias parameters. Finally, I will explore another channel of heterogeneity in innovation by studying the effects of heterogeneous home bias parameters in the R&D technologies of incumbents and entrants to capture empirically documented different levels of internationalization of R&D efforts across different types of firms.

Implications for both macroeconomic quantities and asset prices are analyzed. The main research question is understanding the equilibrium effects of the proposed technology spillover channel. In particular, can this channel explain the high co-movement of stock returns across countries while keeping the fundamentals moderately correlated, as the data suggest? Furthermore, can this channel explain other moments of interest such as the net export dynamics, domestic R&D investment dynamics, and the cross-country correlation of R&D investment? The answer to these questions is yes, the proposed channel helps in bringing the model-implied dynamics closer to their empirical counterparts. Specifically, the main results implied by the model can be summarized as follows: (i) the cross-country

[2]This is a reasonable assumption for at least developed countries, as the empirical evidence in Fitzgerald (2012) suggests that risk-sharing via financial markets among developed economies is close to being optimal. Moreover, Caporale, Donadelli, and Varani (2015) show that the financial liberalization in China and the increased financial integration of emerging economies has led to a number of shifts in macroeconomic dynamics in Chinese data, which can be explained by accounting for complete financial markets in a two-country production economy.

correlation of consumption growth is considerably lower than the one of output growth in the models with technology spillover; (ii) net exports are negatively correlated with output as in the data, also once the technology spillover channel is accounted for; (iii) the model matches the high co-movement of stock returns across countries when technology spillovers are present; (iv) heterogeneity in the R&D technology bundle home bias parameters for incumbents and entrants enables the model to replicate the empirically rather moderate correlation between incumbents' and entrants' R&D innovation probabilities within a country; (v) the model produces a positive value premium, i.e. a positive return spread between final good firm returns and incumbent returns; and (vi) the exchange rate volatility is decreasing in the amount of technology spillovers.

Specifically, the first model only allows limited risk-sharing via trading goods and thus the households mainly resort to financial markets to share their risks. This induces a very high volatility of the exchange rate and *negative* long-run spillover across countries in consumption dynamics, which are mainly driven by increased exports to the country experiencing a positive productivity shock. Nevertheless, through counter-acting movement in the terms of trade, there is positive correlation between net exports and output, which is counterfactual. This can be resolved by adding homogenous technology spillovers to the model. When including technology spillovers, the increased quantities in goods trade lead to a much smoother exchange rate and net exports now deteriorate in response to positive productivity news, consistent with the data. Furthermore, this channel generates a considerable amount of *positive* spillover in R&D expenditures, providing a realistic description of international technology diffusion.

These first two model specifications, however, produce a mechnically perfect correlation between the innovation probabilities of incumbents and entrants. Allowing for heterogeneity in the spillover intensity for the two types of innovative firms provides a better fit of the model along this dimension. Moreover, the assumption of heterogeneity in the internationalization of R&D activities is both intuitive and confirmed by empirical evidence.[3] For larger and more established firms, it is easier to coordinate innovation efforts on an international scale than for smaller and younger firms. Hence, heterogeneous spillovers are modeled in such a way that incumbents have a lower degree of home bias in their R&D technologies than entrants.

In the data, aggregate R&D expenditure growth rates are only moderately correlated across countries. The economy without technology spillover produces too much co-movement

[3]See the studies by Gertler, Wolfe, and Garkut (2000), Ramondo (2009), Fernández-Ribas (2010), as well as Guadalupe, Kuzmina, and Thomas (2012).

of these growth rates due to productivity shocks affecting innovation intensities being absorbed only domestically. Opening international trade for R&D investment induces productivity shocks to be shared more strongly internationally through the terms of trade and, therefore, leads to a more moderate correlation, as in the data.

All models produce a positive value premium, as the final goods firm is more strongly affected by the productivity shocks as compared to incumbents, and final goods firms having larger book-to-market ratios than incumbents. Furthermore, the final goods firm faces rigidities in the form of capital adjustment costs, whereas incumbents enjoy free entry into research. Regarding international asset pricing dynamics, these three model specifications yield significantly different implications. The first model without technology spillover does not succeed in explaining the high cross-country correlations of returns. Including technology spillover leads to more highly correlated endogenous long-run risks and thus yields significantly better results. Incumbent returns are positively correlated without technology spillover and negatively correlated with technology spillover, but positively correlated again when accounting for a realistic heterogeneity in the internationalization of innovation activities.

This study is structured as follows. Section 2 provides a literature review. Next, I present my model in Section 3. It is followed by a discussion of the model's calibration and its economic implications in Section 4. Section 5 concludes.

# 2 Literature Review

This study is related to several strands of the literature on macroeconomics and finance. First, it builds on the literature on economic growth, which can be subdivided into the literature on expanding product variety economies, started by the seminal work of Romer (1990), and Schumpeterian growth theory, as introduced by the seminal studies of Grossman and Helpman (1991), as well as Aghion and Howitt (1992). Within the context of an expanding product variety model, in which growth is created by an increasing number of intermediate goods, Kung and Schmid (2015) show that endogenous growth endogenously creates a persistent component in expected consumption growth. Thus, they provide a microeconomic foundation of consumption-based models with long-run risks.[4] Jinnai (2015)

[4]The long-run risks model introduced by Bansal and Yaron (2004) is one of the leading asset pricing models in finance. Long-run risks are small but persistent deviations in expected consumption or productivity growth that affect the growth prospects in the economy for a quite a long time, hence the name. Moreover, long-run productivity shocks have been introduced in production economies to jointly study

extends their model by accounting for the product cycle, i.e., the transition from monopoly to perfect competition in the intermediate goods sector. He demonstrates that this boosts risk premia by a factor of roughly two. By accounting for endogenous labor decisions and wage rigidities, Donadelli and Grüning (2016) show that risk premia can also roughly double and that labor and wage dynamics can be reasonably matched within the model.

Schumpeterian growth theory, where innovations in product quality and not in the number of products create economic growth, allows for an empirically plausible positive relation between competition and growth in contrast to the aforementioned models with expanding product variety as demonstrated by Aghion, Harris, Howitt, and Vickers (2001). The dynamics of firm exit and entry are intensively studied in this literature. A firm enters with a new product using a superior, more productive technology and replaces the current firm ("creative destruction"). My model with innovating incumbents and potential entrants in each country is similar to the innovation dynamics in Klette and Kortum (2004), Akcigit and Kerr (2012), Acemoglu and Cao (2015), as well as Bena, Garlappi, and Grüning (2016). The latter study shows that Schumpeterian growth also creates the aforementioned persistent component in expected consumption growth and hence endogenous long-run risks. The amount of heterogeneity in the innovation efforts by incumbents and entrants has sizeable effects on the asset pricing dynamics in their model.[5] In contrast to these two studies, I study a two-country economy where R&D expenditures are fueled by both consumption goods to capture R&D spillovers as evidenced by Coe, Helpman, and Hoffmaister (1997).

The second strand of the literature my model relates to is the literature on international macroeconomics. It is closely related to recent papers using exogenous long-run productivity risks and recursive preferences in open two-country endowment and production economies, where households are allowed to share consumption risk internationally. To capture two famous empirical anomalies in international macroeconomics, Colacito and Croce (2013) compare a regime under financial autarky to a regime with complete markets.[6] In Colacito, Croce, Ho, and Howard (2014), the differential reactions of countries with respect to short-run and long-run producitivity news is studied. The model is carefully calibrated to account for the empirically observed international responses of capital

asset prices and the business cycle: Croce (2014), Ai, Croce, and Li (2013), Ai and Kiku (2013).

[5] A summary of the literature on Schumpeterian growth is provided by Acemoglu (2010) and Aghion and Howitt (1998, 2009).

[6] Specifically, the following anomalies are explained: the correlation between consumption differentials and exchange rates has decreased over time (Backus and Smith, 1993), and the tendency of currencies in high real interest rate countries to appreciate has become more severe over time (Fama, 1984).

and investment flows. However, the model does not account for the fact that output growth is internationally more correlated than consumption growth, which is called the quantity anomaly by Backus, Kehoe, and Kydland (1994). Colacito and Croce (2011) study exchange rates in a suitable exchange economy and explain the high variability of exchange rate growth relative to consumption growth ("exchange rate volatility puzzle"). Tretvoll (2013) explains the exchange rate volatility puzzle in a related production economy and provides a good fit to cross-country correlations of growth rates. A survey on asset pricing in multi-country economies is provided by Lewis (2011).

Third, the literature on trade and spillovers combines endogenous growth theory and multi-country production economies as I do in this study as well. The model in Gavazzoni and Santacreu (2015) is probably the closest to my model. They study a two-country endogenous growth economy with recursive preferences and adoption of technologies across countries to jointly explain the co-movement in asset prices and macroeconomic dynamics. In contrast to my model, the households in their model only consume their respective country's final good and innovations are only driven by new firms as in Kung and Schmid (2015). Therefore, the assumption in my model that households consume both countries' goods seems to be slightly more realistic, given that households in developed economies usually consume goods from multiple countries. Moreover, their model ignores the importance of established firms for innovations, as documented by Bena, Garlappi, and Grüning (2016). Additionally, Garcia-Macia, Hsieh, and Klenow (2015) demonstrate using plant-level data and a growth model based on the framework of Klette and Kortum (2004) that most growth seems to originate from incumbents and through incremental innovations, and that own-production innovations by incumbents dominate innovations causing creative destruction.

To the best of my knowledge, my model is the first international endogenous growth model with heterogeneity in the innovation process. Liao and Santacreu (2015) show empirically and within the context of a multi-country model that the intensiveness of trade affects the cross-country correlation of business cycles. The adoption of foreign technologies and thus the diffusion of technology across countries is studied by Santacreu (2015). In particular, fast-growing developing countries seem to benefit from importing foreign technologies (see Coe, Helpman, and Hoffmaister (1997)). This literature is typically quite silent about the asset pricing implications of technology spillover with the notable exception of Gavazzoni and Santacreu (2015). Ghironi and Melitz (2005) and Alessandria and Choi (2007) employ only international bond trading in their models.

The value-added of my model to the literature is, therefore, the explicit introduction of het-

erogeneous innovation technologies into a two-country stochastic endogenous growth general equilibrium model by simultaneously accounting for international technology spillover effects, which can differ in intensity for the involved heterogeneous innovative firms.

# 3 Model

My model features two equally-sized countries, the home and the foreign country. Each country is populated by a representative household and two producing sectors. The first sector admits a perfectly competitive firm producing the respective country's final good using labor, capital, and a composite of intermediate goods. The monopolistically competitive intermediate goods sector provides a continuum of intermediate goods for production in the final goods sector. The monopolists ("incumbents") in this sector are threatened to be displaced by a more efficient firm entering the market ("entrant"). The households supply labor inelastically. All quantities in my model are real since there is no inflation in the model.

Financial markets are assumed to be complete and frictionless, both domestically and internationally. This is why the countries represent developed economies with highly functional and integrated financial markets, which admit perfect risk-sharing possibilities. Moreover, households and intermediate goods producers can share their risks internationally by trading in goods markets, i.e. the consumption of households, as well as incumbents' and potential entrants' effective R&D expenditures, are given as Cobb-Douglas aggregates of both final goods.

Domestic entrants are only allowed to enter the domestic intermediate goods sector. Furthermore, incumbents only produce intermediate goods for the final goods sector in their respective country. Therefore, competition in the intermediade goods sector is restricted to one country. This is not unrealistic, as only a relatively small fraction of firms actually exports as suggested by the evidence in Eaton, Kortum, and Kramarz (2004). The assumption that R&D expenditures are Cobb-Douglas aggregates over both final goods introduces a novel way to model technology spillover, which introduces a higher correlation of innovation technologies across countries than in an economy where R&D expenditures are only comprised of the domestic final good. Hence, product innovations discovered in one country significantly and positively affect growth in the other country.

The basic model structure is depicted in Figure 1 to provide an overview of the model ingredients.

## 3.1 Households

The representative households in the home and foreign country have recursive preferences over a Cobb-Douglas aggregate of the final home and foreign good (Epstein and Zin, 1989; Weil, 1990). Variables and parameters with index $h$ denote home country's quantities and those with index $f$ denote the foreign country's ones, respectively. Households' preferences in country $k = h, f$ are given by

$$U_t^{(k)} = \left\{ (1-\beta_k) \left( C_t^{(k)} \right)^{\frac{1-\gamma_k}{\theta_k}} + \beta_k \left( \mathbb{E}_t \left[ \left( U_{t+1}^{(k)} \right)^{1-\gamma_k} \right] \right)^{\frac{1}{\theta_k}} \right\}^{\frac{\theta_k}{1-\gamma_k}}, \tag{1}$$

where $\gamma_k$ is the coefficient of relative risk aversion, $\beta_k$ the subjective time preference parameter, and $\psi_k$ the elasticity of intertemporal substitution. Finally, $\theta_k = \frac{1-\gamma_k}{1-\frac{1}{\psi_k}}$. The households choose the amount of the home final good, $\mathcal{Y}_{h,t}^{(k)}$, and of the foreign final good, $\mathcal{Y}_{f,t}^{(k)}$, for consumption to maximize lifetime utility. The consumption bundle is given by

$$C_t^{(k)} = \left( \mathcal{Y}_{k,t}^{(k)} \right)^{\phi_{\mathrm{C,k}}} \left( \mathcal{Y}_{-k,t}^{(k)} \right)^{1-\phi_{\mathrm{C,k}}},$$

where $-k = f$ if $k = h$ and $-k = h$ if $k = f$. Home bias in consumption is captured by assuming $\phi_{\mathrm{C,k}} > 0.5$. Market clearing conditions dictate that the net output of country $k$'s final goods available for consumption, $\mathcal{Y}_t^{(k)}$,[7] is allocated among both households, i.e.

$$\mathcal{Y}_t^{(k)} = \mathcal{Y}_{k,t}^{(k)} + \mathcal{Y}_{k,t}^{(-k)}. \tag{2}$$

My assumption of complete and frictionless financial markets for trading final goods across countries implies that there is a complete set of Arrow-Debreu securities available to households in both countries. These claims are denoted by $Q_{t+1}(\chi_{t+1})$ where $\chi_{t+1}$ is the state of the economy at time $t+1$. If a household holds one unit of $Q_{t+1}(\chi_{t+1})$ between time $t$ and $t+1$, it receives one unit of the home country's final good if the economy is in state $\chi_{t+1}$ at time $t+1$ and zero otherwise. Country $k$'s household's holdings of these assets are given by $A_{t+1}^{(k)}(\chi_{t+1})$. The budget constraints of both households are therefore

[7] Specifically, net output is given by final good output minus capital investment, final good input to production in the intermediate goods sector and total R&D expenditures (see Equation (17)).

given by

$$\mathcal{Y}_{h,t}^{(h)} + P_t^{(h)}\mathcal{Y}_{f,t}^{(h)} + \int_{\chi_{t+1}} A_{t+1}^{(h)}(\chi_{t+1})Q_{t+1}(\chi_{t+1}) = A_t^{(h)} + \mathcal{Y}_t^{(h)},$$
$$P_t^{(h)}\mathcal{Y}_{f,t}^{(f)} + \mathcal{Y}_{h,t}^{(f)} + \int_{\chi_{t+1}} A_{t+1}^{(f)}(\chi_{t+1})Q_{t+1}(\chi_{t+1}) = A_t^{(f)} + P_t^{(h)}\mathcal{Y}_t^{(f)}.$$

$P_t^{(h)}$ ($P_t^{(f)}$) denotes the terms of trade or, equivalently, the price of the foreign (home) final good in home (foreign) final good units. These prices are determined by

$$P_t^{(h)} = \frac{1-\phi_{C,h}}{\phi_{C,h}}\frac{\mathcal{Y}_{h,t}^{(h)}}{\mathcal{Y}_{h,t}^{(f)}}, \qquad P_t^{(f)} = \frac{1}{P_t^{(h)}}.$$

In Appendix A.1, I solve the international consumption allocation problem of these households. An important quantity in this context is the "pseudo" Pareto share $S_t$ measuring the relative performance of the home country to the foreign country. It is determined by the following recursion (see Equation (A9) in the appendix)

$$S_t = S_{t-1}\frac{\mathbb{M}_{t-1,t}^{(h)}}{\mathbb{M}_{t-1,t}^{(f)}}\frac{C_t^h}{C_{t-1}^h}\frac{C_{t-1}^{(f)}}{C_t^{(f)}}.$$

$S_t > 1$ implies that the home country's household currently consumes more than the foreign country's one and is thus relatively richer. Moreover, since financial markets are complete, exchange rate growth $\Delta e_t$ is given by

$$\Delta e_t = \log\left(\mathbb{M}_{t-1,t}^{(f)}\right) - \log\left(\mathbb{M}_{t-1,t}^{(h)}\right).$$

Finally, the stochastic discount factor expressed in units of the consumption aggregate, $C_t^{(k)}$, implied by above preferences using standard derivations can be expressed as

$$\mathbb{M}_{t,t+1}^{(k)} = \frac{\beta_k\left(U_{t+1}^{(k)}\right)^{\frac{1}{\psi_k}-\gamma_k}}{\left(\mathbb{E}_t\left[\left(U_{t+1}^{(k)}\right)^{1-\gamma_k}\right]\right)^{\frac{\frac{1}{\psi_k}-\gamma_k}{1-\gamma_k}}}\left(\frac{C_{t+1}^{(k)}}{C_t^{(k)}}\right)^{-\frac{1}{\psi_k}} = \beta_k^{\theta_k}\left(\frac{C_{t+1}^{(k)}}{C_t^{(k)}}\right)^{-\frac{\theta_k}{\psi_k}}\left(R_{c,t+1}^{(k)}\right)^{\theta_k-1}, \quad (3)$$

where $R_{c,t+1}^{(k)} = \frac{W_{c,t+1}^{(k)}}{W_{c,t}^{(k)}-C_t^{(k)}}$ is the return on wealth. Household's wealth $W_{c,t}^{(k)}$ is defined as the present value of future consumption, $W_{c,t}^{(k)} = \mathbb{E}_t\left[\sum_{s=0}^{\infty} M_{t,t+s}^{(k)}C_{t+s}^{(k)}\right]$. The stochastic

discount factor expressed in units of country $k$'s final good is given by

$$\mathbb{M}_{t,t+1}^{(k),loc} = \mathbb{M}_{t,t+1}^{(k)} \frac{\partial C_{t+1}^{(k)}/\partial \mathcal{Y}_{k,t+1}^{(k)}}{\partial C_t^{(k)}/\partial \mathcal{Y}_{k,t}^{(k)}} = \beta_k^{\theta_k} \left(\frac{C_{t+1}^{(k)}}{C_t^{(k)}}\right)^{1-\frac{\theta_k}{\psi_k}} \left(R_{c,t+1}^{(k)}\right)^{\theta_k - 1} \left(\frac{\mathcal{Y}_{k,t+1}^{(k)}}{\mathcal{Y}_{k,t}^{(k)}}\right)^{-1}.$$

## 3.2 Final goods sectors

I closely follow Kung and Schmid (2015), as well as Bena, Garlappi, and Grüning (2016) in modeling the final goods sector, the intermediate goods sector, and the R&D technologies. There is a representative perfectly competitive firm in the final goods sector of each country $k = h, f$ producing the respective final good using capital $K_t^{(k)}$, labor $L_t^{(k)}$, and a composite of local intermediate goods $G_t^{(k)}$. Output is produced according to

$$Y_t^{(k)} = \left[\left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k}\right]^{1-\xi_k} \left[G_t^{(k)}\right]^{\xi_k}, \tag{4}$$

where $\alpha_k \in (0,1)$ is the capital share, $\xi_k \in (0,1)$ is the share of intermediate goods, and $\Omega_t^{(k)} = e^{z_t + a_t^{(k)}}$ is a productivity shock with two components. First, the common or world shock $z_t$ affects the productivity in both countries. It follows a strictly stationary AR(1) process

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t},\ \varepsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2). \tag{5}$$

Second, the two idiosyncratic shocks, $a_t^{(k)}$, $k = h, f$, are determined by similar processes

$$a_t^{(k)} = \rho_{a,k} a_{t-1}^{(k)} + \varepsilon_{a,t}^{(k)},\ \varepsilon_{a,t}^{(k)} \sim \mathcal{N}(0, \sigma_{a,k}^2). \tag{6}$$

These three productivity shocks are mutually independent.[8] The existence of the common component induces a positive correlation in productivity levels in the two countries that is crucial for reconciling the empirical evidence of rather highly correlated output growth across countries with my model. Liao and Santacreu (2015), for example, provide this empirical evidence on the correlation of TFP and output.

I assume that in each country the intermediate goods sector is composed of a continuum of firms with measure one indexed by $i_k \in [0,1]$. Each intermediate good firm in each country

[8] Equivalently, one could remove the common shock from the model and instead use only country-specific productivity shocks that are correlated.

produces a single intermediate good. The intermediate goods are aggregated according to

$$G_t^{(k)} = \left[ \int_0^1 \left( q_{i_k,\mathrm{t}}^{(\mathrm{k})} \right)^{1-\frac{1}{\nu_k}} \left( x_{i_k,\mathrm{t}}^{(\mathrm{k})} \right)^{\frac{1}{\nu_k}} \, di_k \right]^{\nu_k}, \tag{7}$$

where $q_{i_k,\mathrm{t}}^{(\mathrm{k})}$ denotes the highest available quality of the respective intermediate good, $x_{i_k,\mathrm{t}}^{(\mathrm{k})}$ is the quantity of intermediate good $i_k$ produced, and $\frac{\nu_k}{\nu_k - 1}$ is the elasticity of substitution between any two intermediate goods. I assume $\nu_k > 1$ to imply that an increase in the quality of intermediate goods leads to a more productive final goods firm and thus fosters economic growth in the economy of the respective country. The final goods sector only uses local goods and intermediate goods cannot be exported to or imported from the other country. This allows me to clearly differentiate between the effects of productivity shocks on the final goods sector and product innovations in the intermediate goods sector.[9]

The final goods firm in each country takes the pricing kernel of the country's household in local units $\mathbb{M}_t^{(k),loc}$ as given and chooses investment $I_t^{(k)}$, labor $L_t^{(k)}$, next period's capital $K_{t+1}^{(k)}$, and the quantity of each intermediate good $i_k$, $x_{i_k,\mathrm{t}}^{(\mathrm{k})}$, to maximize its value

$$\max_{\left\{ I_t^{(k)}, L_t^{(k)}, K_{t+1}^{(k)}, x_{i_k,\mathrm{t}}^{(\mathrm{k})} \right\}_{i_k \in [0,1]; t \geq 0}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \mathbb{M}_t^{(k),loc} D_t^{(k)} \right], \tag{8}$$

where dividends (in country-specific final good units) are given by

$$D_t^{(k)} = Y_t^{(k)} - I_t^{(k)} - \omega_t^{(k)} L_t^{(k)} - \int_0^1 p_{i_k,\mathrm{t}}^{(\mathrm{k})} x_{i_k,\mathrm{t}}^{(\mathrm{k})} \, di_k,$$

and where the price of intermediate good $i_k$ with quality $q_{i_k,\mathrm{t}}^{(\mathrm{k})}$ at time $t$ is denoted by $p_{i_k,\mathrm{t}}^{(\mathrm{k})}$. Capital accumulates according to

$$K_{t+1}^{(k)} = (1 - \delta_k) K_t^{(k)} + \Lambda^{(k)} \left( \frac{I_t^{(k)}}{K_t^{(k)}} \right) K_t^{(k)}, \tag{9}$$

[9]This assumption is also broadly consistent with the fact that many manufacturing firms do not export at all and only serve domestic markets. Eaton, Kortum, and Kramarz (2004) report that the fraction of exporting manufacturing firms is only around 17.4% in France and 14.6% in the US, respectively. Moreover, Bernard, Eaton, Jensen, and Kortum (2003) show that exports in the manufacturing sector only represent a small share of total revenue for a large sample of countries in 1992. This measure is highly heterogeneous across countries and typically quite low (around 10% or below for most countries). Furthermore, own calculations using US data between 1985 and 2008 reported in Table 4 reveal that the mean ratio of total exports to GDP is quite low at 8.81% and the mean ratio of total imports to GDP, respectively, is also only 11.45%.

where the capital depreciation rate is given by $\delta_k$, and the convex capital adjustment cost function is specified as in Jermann (1998).[10] The optimization problem of the final goods sector is standard and solved in Appendix A.2.

## 3.3 Intermediate goods sectors and R&D technologies

There is a monopolistically competitive intermediate goods sector in each country, in which a continuum of incumbent firms produces intermediate goods for the respective final goods sector. At the same time entrants deploy R&D giving them a chance to replace the respective incumbent and take over its monopoly.

### 3.3.1 Incumbents

At time 0, intermediate good $i_k$ in country $k$ starts with quality $q^{(k)}_{i_k,0} > 0$ and is produced by an incumbent firm in country $k$ which holds a fully enforced patent on the initial quality. Incumbents need $\mu_k$ units of the final good to produce one unit of its respective intermediate good. Incumbent $i_k$ sets the price $p^{(\mathrm{k})}_{i_k,\mathrm{t}}$ to maximize its profits

$$\pi^{(\mathrm{k})}_{i_k,\mathrm{t}} = \max_{\left\{p^{(\mathrm{k})}_{i_k,\mathrm{t}}\right\}} \left\{p^{(\mathrm{k})}_{i_k,\mathrm{t}} x^{(\mathrm{k})}_{i_k,\mathrm{t}} - \mu_k x^{(\mathrm{k})}_{i_k,\mathrm{t}}\right\}, \tag{10}$$

taking the demand schedule $x^{(\mathrm{k})}_{i_k,\mathrm{t}}$ for intermediate good $i_k$ of quality $q^{(\mathrm{k})}_{i_k,\mathrm{t}}$ as determined by the final good firm as given (see Equation (A15)). The optimal price is derived in Appendix A.2.

At each date $t$, the incumbent firm can improve its product quality by investing in R&D. To capture technology spillover, the incumbent firm uses both the home and foreign country's final good in its innovation technology.[11] If the incumbent spends $s^{(\mathrm{k})}_{\mathrm{I,k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$ units of country $k$'s and $s^{(\mathrm{k})}_{\mathrm{I,-k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$ of the other country's final good on R&D to improve its intermediate good with quality $q^{(\mathrm{k})}_{i_k,\mathrm{t}}$, the probability of a successful incremental product

[10] Specifically, the functional form is $\Lambda^{(k)}\left(\frac{I^{(k)}_t}{K^{(k)}_t}\right) = \frac{\alpha_{1,k}}{1-\frac{1}{\zeta_k}}\left(\frac{I^{(k)}_t}{K^{(k)}_t}\right)^{1-\frac{1}{\zeta_k}} + \alpha_{2,k}$, where the constants are given by $\alpha_{1,k} = \left(Q^{(k)}_{ss} + \delta_k - 1\right)^{\frac{1}{\zeta_k}}, \alpha_{2,k} = \frac{1}{1-\zeta_k}\left(Q^{(k)}_{ss} + \delta_k - 1\right)$. The constant $Q^{(k)}_{ss}$ is chosen such that there are no adjustment costs in the deterministic steady state. The limiting cases $\zeta_k \to 0$ and $\zeta_k \to \infty$ represent infinitely costly adjustment and frictionless adjustment, respectively.

[11] R&D and technology spillover have been identified as key sources of economic growth in, e.g., Coe, Helpman, and Hoffmaister (1997) and Santacreu (2015).

innovation by this incumbent is equal to $\Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I},i_k,\mathrm{t}}^{(\mathrm{k})})$, where[12]

$$\Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I},i_k,\mathrm{t}}^{(\mathrm{k})}) = \eta_{\mathrm{I,k}} \left(s_{\mathrm{I},i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{\omega_{\mathrm{I,k}}}, \quad s_{\mathrm{I},i_k,\mathrm{t}}^{(\mathrm{k})} = \left(s_{\mathrm{I,k},i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{\phi_{\mathrm{I,k}}} \left(s_{\mathrm{I,-k},i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{1-\phi_{\mathrm{I,k}}}.$$

A successful innovation creates a patent to intermediate good $i_k$ with quality $\kappa_{\mathrm{I,k}} q_{i_k,\mathrm{t}}^{(\mathrm{k})}$, where $\kappa_{\mathrm{I,k}} > 1$. The total amount of R&D expenditure of country $k$'s ($-k$'s) final good by incumbent firms in country $k$ is

$$S_{\mathrm{I,k,t}}^{(\mathrm{k})} = \int_0^1 s_{\mathrm{I,k},i_k,\mathrm{t}}^{(\mathrm{k})} q_{i_k,\mathrm{t}}^{(\mathrm{k})} \, di_k, \qquad S_{\mathrm{I,-k,t}}^{(\mathrm{k})} = \int_0^1 s_{\mathrm{I,-k},i_k,\mathrm{t}}^{(\mathrm{k})} q_{i_k,\mathrm{t}}^{(\mathrm{k})} \, di_k. \tag{11}$$

The level of technology capital in country $k$ is defined by the aggregate product quality of intermediate goods

$$Q_t^{(k)} = \int_0^1 q_{i_k,\mathrm{t}}^{(\mathrm{k})} \, di_k. \tag{12}$$

These aggregate levels of technology capital in the home and foreign country are the key state variables in the model capturing endogenous economic growth.

### 3.3.2 Entrants

For each intermediate good $i_k$ in country $k$ at each date $t$, there is furthermore an infinite supply of atomistic entrants who deploy R&D in order to radically improve the intermediate good's quality and to take over the monopoly of the current incumbent due to the high productivity of this new product. If all intermediate good $i_k$ entrants together spend $s_{\mathrm{E,k},i_k,\mathrm{t}}^{(\mathrm{k})} q_{i_k,\mathrm{t}}^{(\mathrm{k})}$ units of country $k$'s and $s_{\mathrm{E,-k},i_k,\mathrm{t}}^{(\mathrm{k})} q_{i_k,\mathrm{t}}^{(\mathrm{k})}$ units of the other country's final good on R&D, the probability with which an entrant makes a discovery is $\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})} \equiv s_{\mathrm{E},i_k,\mathrm{t}}^{(\mathrm{k})} \Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E},i_k,\mathrm{t}}^{(\mathrm{k})})$, where

$$\Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E},i_k,\mathrm{t}}^{(\mathrm{k})}) = \eta_{\mathrm{E,k}} \left(s_{\mathrm{E},i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{\omega_{\mathrm{E,k,t}}-1}, \quad s_{\mathrm{E},i_k,\mathrm{t}}^{(\mathrm{k})} = \left(s_{\mathrm{E,k},i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{\phi_{\mathrm{E,k}}} \left(s_{\mathrm{E,-k},i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{1-\phi_{\mathrm{E,k}}},$$

and where the function $\Phi_{\mathrm{E}}^{(\mathrm{k})}(\cdot)$ is chosen such that the innovation probability of entrants has the same functional form as the one of incumbents.[13]

Entrants, by using both final goods, also enjoy the benefits of technology spillover, exactly

[12] This functional form of the R&D technology $\Phi_{\mathrm{I}}^{(\mathrm{k})}$ has also been used by Comin and Gertler (2006), Comin, Gertler, and Santacreu (2009), Acemoglu and Cao (2015), as well as Kung and Schmid (2015), among others. Specifically, it takes into account an empirically plausible decreasing marginal gain from R&D investment and positive spillover from the aggregate stock of innovations as also in Romer (1990).

[13] $\Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E},i_k,\mathrm{t}}^{(\mathrm{k})})$ is taken as given by entrants in the optimization problem (16). See also the discussion in Appendix A.3.2.

as incumbents do. The total amount of R&D expenditure of country $k$'s ($-k$'s) final good by entrants in country $k$ is

$$S^{(\mathrm{k})}_{\mathrm{E,k,t}} = \int_0^1 s^{(\mathrm{k})}_{\mathrm{E,k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}\, di_k, \qquad S^{(\mathrm{k})}_{\mathrm{E,-k,t}} = \int_0^1 s^{(\mathrm{k})}_{\mathrm{E,-k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}\, di_k. \tag{13}$$

## 3.4 Valuation of patents and R&D expenditures

Incumbent firms invest the total amount $s^{(\mathrm{k})}_{\mathrm{I,k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} + P^{(k)}_t s^{(\mathrm{k})}_{\mathrm{I,-k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$ in R&D and thus make a net profit of $\pi^{(\mathrm{k})}_{i_k,\mathrm{t}} - s^{(\mathrm{k})}_{\mathrm{I,k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} - P^{(k)}_t s^{(\mathrm{k})}_{\mathrm{I,-k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$ at time $t$. Due to the competition structure in the intermediate goods sector, incumbent $i_k$'s value in period $t+1$ is a random variable as of time $t$ and can take on three values. First, the incumbent might be displaced by an entrant with probability $\hat{\Phi}^{(\mathrm{k})}_{\mathrm{E}}(s^{(\mathrm{k})}_{\mathrm{E},i_k,\mathrm{t}}) := s^{(\mathrm{k})}_{\mathrm{E},i_k,\mathrm{t}} \Phi^{(\mathrm{k})}_{\mathrm{E}}(s^{(\mathrm{k})}_{\mathrm{E},i_k,\mathrm{t}})$, where the incumbent takes the potential entrants' R&D expenditure $s^{(\mathrm{k})}_{\mathrm{E},i_k,\mathrm{t}}$ as given. In case of displacement, the incumbent's value drops to zero. Second, the incumbent might not be displaced but innovates itself. With probability $\Phi^{(\mathrm{k})}_{\mathrm{I}}(s^{(\mathrm{k})}_{\mathrm{I},i_k,\mathrm{t}})$, the incumbent improves on its product and the quality increases to $q^{(k)}_{i_k,t+1} = \kappa_{\mathrm{I,k}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$. Third, the incumbent might neither be displaced nor innovate itself. In this case, which happens with probability $1 - \Phi^{(\mathrm{k})}_{\mathrm{I}}(s^{(\mathrm{k})}_{\mathrm{I},i_k,\mathrm{t}}) - \hat{\Phi}^{(\mathrm{k})}_{\mathrm{E}}(s^{(\mathrm{k})}_{\mathrm{E},i_k,\mathrm{t}})$, the quality depreciates to $q^{(k)}_{i_k,t+1} = \kappa_{\mathrm{D,k}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$, where $\kappa_{\mathrm{D,k}} < 1$. This depreciation factor captures patent expiration and general obsolescence of products over time. Hence, the incumbent $i_k$'s value function $v^{(\mathrm{k})}_{i_k,\mathrm{t}}$ solves the following Bellman equation

$$\begin{aligned} v^{(\mathrm{k})}_{i_k,\mathrm{t}}(q^{(\mathrm{k})}_{i_k,\mathrm{t}}) = \max_{\left\{s^{(\mathrm{k})}_{\mathrm{I,k},i_k,\mathrm{t}}, s^{(\mathrm{k})}_{\mathrm{I,-k},i_k,\mathrm{t}}\right\}} \Big\{ & \pi^{(\mathrm{k})}_{i_k,\mathrm{t}} - s^{(\mathrm{k})}_{\mathrm{I,k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} - P^{(k)}_t s^{(\mathrm{k})}_{\mathrm{I,-k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} \\ & + \mathbb{E}_t \left[ \mathbb{M}^{(k),loc}_{t,t+1} \left( \Phi^{(\mathrm{k})}_{\mathrm{I}} v^{(\mathrm{k})}_{i_k,\mathrm{t}+1}(\kappa_{\mathrm{I,k}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}) + \left(1 - \Phi^{(\mathrm{k})}_{\mathrm{I}} - \hat{\Phi}^{(\mathrm{k})}_{\mathrm{E}}\right) v^{(\mathrm{k})}_{i_k,\mathrm{t}+1}(\kappa_{\mathrm{D,k}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}) \right) \right] \Big\}. \end{aligned} \tag{14}$$

From Equations ($A16$) and ($A18$), net profit is linear in quality. This implies $\pi^{(\mathrm{k})}_{i_k,\mathrm{t}} = \pi^{(k)}_t q^{(\mathrm{k})}_{i_k,\mathrm{t}}$. By focusing on a linear balanced growth path equilibrium,[14] the following homogeneity properties emerge: $s^{(\mathrm{k})}_{\mathrm{I,k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} = s^{(\mathrm{k})}_{\mathrm{I,k,t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$, $s^{(\mathrm{k})}_{\mathrm{I,-k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} = s^{(\mathrm{k})}_{\mathrm{I,-k,t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$, $s^{(\mathrm{k})}_{\mathrm{E,k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} = s^{(\mathrm{k})}_{\mathrm{I,k,t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$, and $s^{(\mathrm{k})}_{\mathrm{E,-k},i_k,\mathrm{t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}} = s^{(\mathrm{k})}_{\mathrm{E,-k,t}} q^{(\mathrm{k})}_{i_k,\mathrm{t}}$. This implies $s^{(\mathrm{k})}_{\mathrm{I},i_k,\mathrm{t}} = s^{(\mathrm{k})}_{\mathrm{I,t}}$, $s^{(\mathrm{k})}_{\mathrm{E},i_k,\mathrm{t}} = s^{(\mathrm{k})}_{\mathrm{E,t}}$ as well. Furthermore, the incumbent's value is also linear in quality and thus $v^{(\mathrm{k})}_{i_k,\mathrm{t}}(q^{(\mathrm{k})}_{i_k,\mathrm{t}}) = v^{(k)}_t q^{(\mathrm{k})}_{i_k,\mathrm{t}}$ for all $t$ and $i_k \in [0,1]$. By plugging this into (14) and dividing by $q^{(\mathrm{k})}_{i_k,\mathrm{t}}$, the Bellman equation

[14] This conjecture follows Acemoglu and Cao (2015). The authors discuss that the linear balanced growth path equilibrium is the outcome of any equilibrium in their economy, although they cannot prove it if the R&D technology of incumbents is non-linear. Nevertheless, I rely on this equilibrium concept although I am not able to prove these homogeneity properties in my model either.

defining the incumbent's value, $v_t^{(k)}$, becomes

$$v_t^{(k)} = \max_{\left\{s_{\mathrm{I,k,t}}^{(\mathrm{k})}, s_{\mathrm{I,-k,t}}^{(\mathrm{k})}\right\}} \left\{ \pi_t^{(k)} - s_{\mathrm{I,k,t}}^{(\mathrm{k})} - P_t^{(k)} s_{\mathrm{I,-k,t}}^{(\mathrm{k})} \right. \tag{15}$$
$$\left. + \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \left( \Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) \kappa_{\mathrm{I,k}} + (1 - \Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) - \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})})) \kappa_{\mathrm{D,k}} \right) \right] \right\}.$$

The first order conditions of this Bellman equation are supplied in Appendix A.3.1. Potential entrants enjoy free entry to the R&D technology and thus they maximize the net present value of future profits achieved if they become incumbents

$$\max_{\left\{s_{\mathrm{E,k,t}}^{(\mathrm{k})}, s_{\mathrm{E,-k,t}}^{(\mathrm{k})}\right\}} \left\{ \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}) \kappa_{\mathrm{E,k}} \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \right] - s_{\mathrm{E,k,t}}^{(\mathrm{k})} - P_t^{(k)} s_{\mathrm{E,-k,t}}^{(\mathrm{k})} \right\}. \tag{16}$$

The first order conditions of this optimization problem are supplied in Appendix A.3.2.

## 3.5 Resource constraint

To close the model, resource constraints in both countries need to be specified. Net output $\mathcal{Y}_t^{(k)}$, which is available for both households' consumption, is final good output minus capital investment, final good input to production in the intermediate goods sector, and total R&D expenditures. Hence, it is given by

$$\mathcal{Y}_t^{(k)} = Y_t^{(k)} - I_t^{(k)} - \mu_k X_t^{(k)} - S_{\mathrm{I,k,t}}^{(\mathrm{k})} - S_{\mathrm{I,-k,t}}^{(\mathrm{k})} - S_{\mathrm{E,k,t}}^{(\mathrm{k})} - S_{\mathrm{E,-k,t}}^{(\mathrm{k})}. \tag{17}$$

## 3.6 Technology capital and equilibrium

Since the incumbent's value $v_t^{(k)}$ and the R&D expenditure bundles of incumbents and entrants are independent of the distribution of qualities $q_{i_k,\mathrm{t}}^{(\mathrm{k})}$ across incumbent firms, the dynamics of aggregate technology capital growth are given by

$$\frac{Q_{t+1}^{(k)}}{Q_t^{(k)}} = \kappa_{\mathrm{D,k}} + (\kappa_{\mathrm{I,k}} - \kappa_{\mathrm{D,k}}) \Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) + (\kappa_{\mathrm{E,k}} - \kappa_{\mathrm{D,k}}) \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}). \tag{18}$$

This equation determines the growth rate of technology capital and of the economy in the respective country. It depends on the level of R&D expenditures by incumbents and entrants. Over a period of time, $\Phi_{\mathrm{I}}^{(\mathrm{k})}$ intermediate good sectors experience an innovation by incumbents who increase quality by $\kappa_{\mathrm{I,k}}$, another $\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}$ sectors experience an innovation

by entrants who increase quality by $\kappa_{\mathrm{E,k}}$ and displace the respective incumbents, and the remaining sectors' qualities depreciate by the factor $\kappa_{\mathrm{D,k}}$. The growth rate is thus endogenously determined by heterogeneous innovations of incumbents and entrants.

The definition of the decentralized equilibrium in this economy is stated in Appendix A.4. The pricing kernels are used to price a number of assets. I exactly specify which assets are priced and how the resulting returns are computed in Appendix A.5. As each economy is growing, solving for the equilibrium requires normalizing of the growing real quantities by technology capital in order to make them stationary.

# 4 Results

I discuss how I calibrate the model and inspect the economic implications and intuition of a number of different model specifications in this section. For each calibration, the model is solved using third order perturbation methods.[15]

## 4.1 Calibration

The model is calibrated to feature symmetric countries. For within-country dynamics I refer to US data for the calibration and the evaluation of the model's fit. For international quantities the moments for calibration and evaluation are based on the average of the moments for a large sample of developed economies (see Appendix B for details on the data used). The home country is thus best represented by the United States in the real world and the foreign country probably by a major developed economy in Europe.

Table 1 reports the parameters of four different calibrations. The model calibration is annual, as the data used for the calibration of the model is only available at annual frequency. Moreover, creating an innovation takes quite some time and thus the annual frequency is better compatible with the long horizons of investments in R&D. In the simple economy without R&D spillover, incumbents and entrants only use local goods for their R&D expenditures.[16] For the second model specification labeled "Technology Spillover" the R&D

[15]These perturbation methods are implemented using Dynare++, version 4.4.3. All results are based on 3,000 simulations of the model with each sample being 112 years long from which the first 50 years are used as a "burn-in" period leaving 62 years of simulated data for computing the moments. For each simulation, I draw random sequences of the involved stochastic shocks and compute the state and policy variables using Dynare++'s Dynare_simul.m function.

[16]This economy represents a limiting case of the model presented in Section 3, i.e. one has to set the home bias parameter in the R&D expenditure bundles to $\phi_{\mathrm{I,k}} = \phi_{\mathrm{E,k}} = 1$.

expenditures of both incumbents and entrants are Cobb-Douglas aggregates of both final goods. For this model specification I differentiate between homogenous and heterogeneous spillovers. In the former case, the home bias parameters in R&D technologies are restricted to be identical, i.e. $\phi_{\mathrm{I,k}} = \phi_{\mathrm{E,k}}$. Two different sets of values are used: for the economy with low technology spillover I assume $\phi_{\mathrm{I,k}} = \phi_{\mathrm{E,k}} = 0.90$, and for the one with high spillover a value of 0.80 is used.

The fourth calibration features heterogeneous spillovers. In this model, I assume $\phi_{\mathrm{I,k}} = 0.85$ and $\phi_{\mathrm{E,k}} = 0.95$. Hence, incumbents have a higher degree of internationalization in their R&D technologies than entrants. This is done to capture the intuitive notion that it should be easier for incumbents to coordinate research efforts across countries by having employees in different countries working on the same research project than for entrants. Incumbents have more capital available to build such a global infrastructure. This intuitive notion is supported by empirical evidence given in Ramondo (2009) as well as Guadalupe, Kuzmina, and Thomas (2012) that multinational firms are successful in spreading R&D benefits across countries. Furthermore, another existing line of empirical research shows that indeed research activities by bigger and more established firms, proxied by incumbents in the model, are undertaken to a larger extent internationally than research by smaller and younger firms, which are proxied by entrants. For example one can refer to the studies by Fernández-Ribas (2010) as well as Gertler, Wolfe, and Garkut (2000).

The preference parameters are in line with the literature on long-run risks (Bansal and Yaron, 2004), i.e., the elasticity of intertemporal substitution (EIS), $\psi_k = 1.1$, is above 1, the relative risk aversion parameter is set to $\gamma_k = 10$, and the subjective discount factor is $\beta_k = \sqrt[4]{0.984}$.

The capital share and depreciation rate of physical capital are set to standard values used in the literature and have been taken from Kung and Schmid (2015), together with the marginal cost parameter of intermediate goods production. Specifically: $\alpha_k = 0.35$, $\delta_k = 0.02$, and $\mu_k = 1$. The adjustment cost elasticity parameter is equal to $\zeta_k = 1.15$, a value in line with existing empirical evidence.[17] The monopoly markup is given by $\nu_k = 1.25$ and thus identical to the one used by Bena, Garlappi, and Grüning (2016). This implies $\xi_k = 0.72$ since the following parameter restriction needs to be satisfied to ensure balanced growth:

$$\frac{(\nu_k - 1)\xi_k}{1 - \xi_k} = 1 - \alpha_k, \quad k = h, f. \tag{19}$$

[17] Eberly (1997), for instance, reports estimates ranging between 1.08 and 1.36. In an earlier empirical work, Abel (1980) reports value for $\zeta_k$ ranging between 0.5 and 1.14.

My model thus features a realistic moderate monopoly markup for intermediate good firms and a empirically plausible amount of physical capital adjustment costs.

The autocorrelations of productivity shocks also follow Kung and Schmid (2015), i.e. $\rho_z = \rho_{a,k} = \sqrt[4]{0.95}$. The volatilities of the country-specific shocks $\sigma_{a,k}$ and of the common productivity shock $\sigma_z$ are chosen to obtain a very moderate value for the output growth volatility as observed in US data for the recent period 1985–2008 and the observed average cross-country output growth correlation of 0.46 across all calibrations. The relative size of the common component $z_t$ in the exogenous productivity processes mainly determines the output growth correlation and thus the calibrated volatilities imply that the ratio $\frac{\sigma_z^2}{\sigma_z^2+\sigma_{a,k}^2}$ is roughly equal to 0.46.

The R&D related parameters are calibrated identically to Bena, Garlappi, and Grüning (2016). The values $\kappa_{\mathrm{D,k}} = 0.91, \kappa_{\mathrm{E,k}} = 2.50$ and $\kappa_{\mathrm{I,k}} = 1.25$ are in proximity to their empirically estimated ones. Furthermore, $\eta_{\mathrm{I,k}} = 17.50$ and $\eta_{\mathrm{E,k}} = 2.00$ are set to obtain a similar fit to average innovation probabilities in the United States as in the aforementioned study. Furthermore, $\omega_{\mathrm{I,k}}$ and $\omega_{\mathrm{E,k}}$ are determined to match the empirically observed average output growth rate of 1.84 percentage points and the average ratio between entrants' innovation probability to the total innovation probability in US data. Finally, the home bias parameter for consumption is $\phi_{\mathrm{C,k}} = 0.95$. As documented by Erceg, Guerrieri, and Gust (2008) only about 3–5% of the US consumption bundle consists of foreign goods. I confirm this assessment by calculating the share of foreign goods in US households' consumption bundles. This share is depicted in Figure 2 for the period 1995–2008. The average of the share across theses years is 5.07% and thus the home bias is chosen to be 95%, consistent with these data. This parameter is also in line with the parameterization of Colacito and Croce (2013).

## 4.2 The simple economy without technology spillover

This economy features only low trading turnover in goods markets due to the high home bias in consumption and the absence of trading for R&D expenditures. Financial markets, however, are perfectly integrated and allow households to efficiently share their risks via trading Arrow-Debreu securities.

As documented first in the literature by Kung and Schmid (2015), endogenous growth coupled with recursive preferences creates a small persistent component in expected consumption growth endogenizing the long-run risk model of Bansal and Yaron (2004) in a

production economy. A fraction of short-run productivity shocks is transformed into long-run shocks affecting the economy for quite a long period of time. The channel operates through positive innovations to total factor productivity (TFP) leading to higher R&D expenditures and innovation probabilities which, in turn, lead to persistent increases in expected growth rates due to the one-period time lag in innovation. Hence, the expected consumption growth volatility is substantial and amounts to 0.24 percentage points as reported in the second panel of Table 2.

Furthermore, the model closely replicates the empirically observed average output growth rate and innovation probabilities due to calibration choices. Moreover, consumption growth volatility is as low as in the recent data. The volatilities of the innovation probabilities are only about one third of the data counterparts, and the correlation of incumbents' and entrants' innovation probabilities is mechanically equal to 1. How to resolve these issues by using an exogenous shock to the barriers of entry is discussed in Bena, Garlappi, and Grüning (2016) in detail. I provide an alternative endogenous resolution to the correlation issue in Section 4.3.2, where the degree of internationalization of the R&D technologies of entrants and incumbents is assumed to be heterogeneous.

Since there is no international trade of goods for R&D expenditures, the innovation probabilities of both incumbents and entrants react strongly positively and persistently to domestic productivity shocks but only marginally positively to foreign shocks, as depicted in the first rows of Figures 3 and 4, respectively. The small positive spillover from foreign productivity shocks is due to additional co-movement in the pricing kernels as will become clear below.

The model setup here is similar to the endowment economy in Colacito and Croce (2013), i.e. there is limited trading in consumption goods and correlated short- and long-run productivity shocks. In contrast to their model with exogenous long-run risks, the long-run risks in my model arise endogenously through heterogeneous R&D decisions by incumbents and entrants. However, the intuition carries over to my setup. These long-run risks are endogenously correlated due to co-movement in the pricing kernels (fueled, in turn, by the presence of common productivity shocks). A positive home idiosyncratic productivity shock induces positive long-run news and thus home household's continuation utility rises in anticipation of higher future consumption. This implies a lower marginal utility and by assumption of complete markets the exchange rate $\Delta e_t$ increases (see Figure 5). As a result of this, there are considerably higher consumption good exports from the foreign to the home country. However, the increase of domestic consumption is relatively weak. Thus, the normalized consumption bundle of the home country only slightly increases

initially and later on decreases as the growth rate of the home economy is larger than the growth rate in consumption.

In sum, this constitutes a small *negative* spillover in consumption. It is depicted in the impulse reponse functions of Figure 6. On the one hand, the normalized home consumption bundle $\widehat{C}_t^{(h)}$ decreases in the long-run (and increases in the short-run) in response to home TFP shocks despite the strong increase in imports from the foreign country, $\widehat{\mathcal{Y}}_{f,t}^{(h)}$, due to the relatively low increase in the contribution of the home final good to home consumption, $\widehat{\mathcal{Y}}_{h,t}^{(h)}$. On the other hand, the normalized home consumption bundle increases in response to foreign TFP shocks due a significant increase in the domestic good contribution dominating a strong decline in imports.

Moreover, there is a small *positive* spillover in expected consumption growth (Figure 5), i.e. home expected consumption growth also slightly increases in response to foreign TFP shocks. This small positive spillover effect originates from the co-movement of the pricing kernels, also visible in Figure 5. The Pareto share $S_t$ drives this co-movement by introducing a common endogenous component in the international consumption good allocation. Consequently, the valuation of patents, the R&D expenditures, and the expected consumption growth rate co-move across countries. Taken together, the higher supply of home consumption goods are mainly used for domestic R&D expenditures and for foreign consumption good imports, but to some extent also for higher consumption of the domestic good.

Since consumption reacts less than proportionally to output in response to idiosyncratic TFP shocks, the within-country correlation of consumption with output growth is only 0.55. Due to the endogenous growth mechanism, output and R&D expenditures are close to being perfectly correlated, which implies that consumption and R&D expenditures exhibit essentially the same correlation as consumption and output growth.

Table 3 reports domestic asset pricing moments. The excess returns are lower and the risk-free rate is higher than usual in endogenous growth models with recursive preferences due to the possibility of smoothing consumption internationally via both complete international financial markets and goods markets, and since the model is calibrated to match the very smooth output growth dynamics in the recent data. This lowers the market price of long-run risk substantially in relation to closed economy models. Hence, the risk-free rate is quite high at around 3 percentage points and the market excess return is only 0.4 percentage points. Appendix C reveals that a much better fit of those basic asset pricing moments can be achieved by calibrating the model to the high output growth volatility

as observed in the sample starting from the Great Depression period, a calibration choice quite common in the asset pricing literature (see, for example, Croce (2014)).

There is a notable return spread between expected excess returns of the final good firm and of the incumbent firm. The incumbent excess return is only about one third of the final good firm's excess return. The high adjustment costs for capital investment are responsible for the high expected excess return of the final good firm. Free entry in R&D and the negative impact of entrants lead to rather low excess incumbent returns. Interpreting this spread as a value premium, the model is in line with empirical evidence. This seems reasonable, as the final good firm has a higher book-to-market ratio than the incumbent firm.[18] Hence, the model replicates the empirical anomaly that firms with higher book-to-market ratios earn higher expected returns than firms with lower book-to-market ratios.

The substantial pressure for risk-sharing via financial markets leads to an exchange rate volatility of about 0.079, reported in Table 4, which is of roughly the same magnitude as in the data. However, the model fails to explain the quantity anomaly of Backus, Kehoe, and Kydland (1994) by matching output growth correlation of 0.46, but obtaining a consumption growth correlation of 0.67 (vs. 0.34 empirically). The EIS above 1 induces substitution motives for households, i.e. it is utility-enhancing for them to substitute consumption for R&D expenditures to profit from higher future consumption. However, as analyzed before, the resulting negative consumption spillover only materializes only about 4 years after the shock. In the first 3 years, one has excess correlation in consumption, causing a consumption growth correlation in excess of the one of output growth. Accounting for technology spillovers in the next section induces consumption to be less correlated internationally than output, as will be discussed later.

The small positive spillover in R&D technologies implies cross-country correlations of R&D innovation probabilities in excess of 0.46 (i.e. 0.47). Similarly, this holds for the cross-country correlation of the aggregate R&D expenditure growth rate. Furthermore, expected consumption growth is considerably more correlated internationally than output growth (i.e. 0.53) due to the additional pricing kernel co-movement induced by the Pareto

[18]The book value of the final good firm is equal to capital $K_t$. The ratio of $K_t$ to the ex-dividend price of the final good firm, $V_{d,t} - D_t$, is about 0.98 in the stochastic steady state of the model. The book value of an incumbent firm is not straightforwardly defined. I define it as the profits generated by the patents without accounting for the growth options of incumbents. Applying appropriate risk-adjusted discounting by accounting for the obsolescence of patents and the displacement risk induced by entrants, I compute the book value of incumbents in the stochastic steady state as $\frac{\Pi_{ss}^{(k)}}{1-M_{ss}^{(k),loc}\left(\kappa_D-\hat{\Phi}_{\mathrm{E,ss}}^{(\mathrm{k})}\right)}$. The ratio of this quantity to the ex-dividend incumbent value, $v_{ss}^{(k)} - s_{\mathrm{I,h,ss}}^{(\mathrm{h})} - P_{ss}^{(h)} s_{\mathrm{I,h,ss}}^{(\mathrm{f})}$ is about 0.87. This spread is also present and of roughly the same magnitude in the other model specifications discussed further below.

share $S_t$.

The correlation of the pricing kernels leads to a moderate co-movement of risk-free rates and returns. The correlation of incumbent returns is 0.46 as R&D expenditure growth is highly correlated. For the final good firm return and aggregate market returns, the correlations are of the same magnitude (0.46 and 0.45, respectively). The correlation of the risk-free rates is equal to 0.43. Although a positive domestic productivity shock induces a small decrease in the net exports of consumption goods, $NX_t^{(h)} = \mathcal{Y}_{h,t}^{(f)} - P_t^{(h)}\mathcal{Y}_{f,t}^{(h)}$, occuring together with an increase in output, the other two productivity shocks in the model, i.e. common and foreign productivity shocks induce a positive correlation and thereby dominating the effect of home productivity shocks. Hence, the correlation of output and the net exports to GDP ratio is slightly positive and equal to 0.27, which is counterfactual, given the data. Introducing technology spillover in the next section resolves this issue, and it also allows me to obtain higher cross-country correlations of returns.

## 4.3 Introducing technology spillover

In the following, I will investigate the properties of the full model with the proposed endogenous technology spillover channel, i.e. incumbents and potential entrants use both final goods in their R&D technologies. First, Section 4.3.1 studies homogenous technology spillover by imposing the restriction $\phi_{\mathrm{I,k}} = \phi_{\mathrm{E,k}}$. Second, Section 4.3.2 relaxes this assumption and studies heterogeneous spillovers.

### 4.3.1 Homogenous technology spillover

In this section, I study how adding technology R&D spillovers and, in turn, a higher degree of international connectedness, affect the economic implications of the model analyzed in the last section by setting $\phi_{\mathrm{I,k}} = \phi_{\mathrm{E,k}} = 0.80$ or $\phi_{\mathrm{I,k}} = \phi_{\mathrm{E,k}} = 0.90$ instead of 1 (i.e. the low and high homogenous technology spillover calibrations). The models are recalibrated to match the average output growth rate, the average innovation probabilities and the cross-country correlation of output growth by adjusting $\sigma_{a,k}, \sigma_z, \omega_{\mathrm{I,k}}$ and $\overline{\omega_{\mathrm{E,k}}}$ as before. All other parameters are kept fixed.

By inspecting the impulse response functions of incumbents' and entrants' innovation probabilities and R&D expenditures in Figures 3 and 4, one notices the large increase in the contribution of the home good in R&D expenditures in response to a home TFP shock, i.e. increases in $s_{\mathrm{I,h,t}}^{(\mathrm{h})}$ and $s_{\mathrm{E,h,t}}^{(\mathrm{h})}$. However, there are lower imports from the foreign country,

i.e. $s^{(\mathrm{h})}_{\mathrm{I,f,t}}$ and $s^{(\mathrm{h})}_{\mathrm{E,f,t}}$ decrease. In response to foreign TFP shocks, the home country imports more of the foreign good, whereas the contribution of the home good basically does not change. This means that net exports of final goods for R&D deteriorate in response to domestic productivity news. This is due to the long-run component of these shocks.[19] The short-run component of these shocks, on the other hand, implies that a significant fraction of the immediate increase in output is still kept within the more productive country. In sum, the R&D innovation probabilities of incumbents and entrants increase in response to both home and foreign productivity shocks, and therefore, there is significant *positive* spillover in R&D.

In Section 4.2, the technology spillover was also positive albeit small. Adding technology spillover leads to additional endogenous co-movement between the countries. Hence, the cross-country correlations of the pricing kernels and of expected consumption growth rates increase significantly to 0.99 and 0.81, respectively, in the low technology spillover calibration (see Table 4). Similar numbers are obtained when the amount of technology spillovers is assumed to be high. The correlation of the net exports to GDP ratio with output is now significantly negative and thus broadly consistent with the data as domestic shocks imply an outflow of goods, mainly for R&D expenditures (see the lowest panel of Table 4).[20] The main reason for this is that although imports are reduced, exports increase more.[21]

Furthermore, the amount of exports and imports as a fraction of output is a bit closer to the data counterpart due to the fact that now also goods for R&D investment are imported and exported. However, it still falls several factors short of the amount of exports in the US economy since R&D investment accounts for only a relatively small fraction of total output. In order to account for more exports and imports, one could assume investment in both countries as also being a bundle of both final goods in the spirit of Colacito, Croce, Ho, and Howard (2014). Since the focus of this study is the equilibrium effects of technology spillovers, this additional export and import channel is not added

[19] As decribed in Colacito, Croce, Ho, and Howard (2014), the households face a tradeoff after any innovation to productivity. On the one hand, resources should flow to the country with higher productivity (*productivity* channel). On the other hand, the resources should flow to the country with higher marginal utility (*risk-sharing* channel). They show that for long-run productivity news the risk-sharing channel dominates and countries see their net exports deteriorate, and that for short-run productivity news the productivity channel is quantitatively more important implying an inflow of capital goods.

[20] Total net exports of the home country are now: $NX^{(h)}_t = \mathcal{Y}^{(f)}_{h,t} + S^{(\mathrm{h})}_{\mathrm{I,f,t}} + S^{(\mathrm{f})}_{\mathrm{E,h,t}} - P^{(h)}_t (\mathcal{Y}^{(h)}_{f,t} + S^{(\mathrm{h})}_{\mathrm{I,f,t}} + S^{(\mathrm{h})}_{\mathrm{E,f,t}})$.

[21] This can be seen by inspecting the impulse response functions of $s^{(\mathrm{h})}_{\mathrm{I,f,t}}$ or $s^{(\mathrm{h})}_{\mathrm{E,f,t}}$ in response to foreign productivity shocks, which are identical to the impulse response functions of $s^{(\mathrm{f})}_{\mathrm{I,h,t}}$ and $s^{(\mathrm{f})}_{\mathrm{E,h,t}}$, respectively, in response to domestic productivity shocks due to the assumed symmetry of the countries.

to the model. Moreover, expected consumption growth increases significantly in response to foreign shocks (by about one half of the response to domestic shocks) as displayed in Figure 5. This represents a remarkable international diffusion of innovation activities and is consistent with the empirical fact that many firms simultaneously file for patents in major developed countries, and that a significant fraction of growth is induced by foreign inventions (see the evidence in Eaton and Kortum (1999) and the references therein).

The increased importance of risk-sharing in good markets lowers the importance of risk-sharing via financial markets. Consequently, the exchange rate volatility drops by a factor of roughly 3 to 0.029 for the low technology spillover calibration, far below the empirical counterpart (Table 4). An even lower exchange rate growth volatility is observed in the economy with high technology spillovers. A higher degree of home bias in R&D expenditures is therefore needed to match exchange rate data. This is in line with the empirical evidence in Gavazzoni and Santacreu (2015) that countries trading more with each other have a lower exchange rate volatility in the data.

Since there are now opportunities to invest in R&D by investing into foreign goods, the *risk-sharing* channel becomes stronger on international goods markets. Thus, as already discussed above, the goods flow to the country with the higher marginal utility since investment in R&D is more profitable. Since the households have an EIS above 1, the substitution effect implies that resources are diverted away from consumption. Hence, imports for consumption slightly decrease in response to a domestic TFP shock. Moreover, the substitution effect implies that the fraction of the home good's contribution to the home consumption bundle $\mathcal{Y}_{h,t}^{(h)}$ in response to a domestic TFP shock increases significantly more than in the simple economy of Section 4.2 (see Figure 6). Since domestic R&D investment is not as attractive as before in the simple economy, the household consumes a larger fraction of the increased supply of final goods.

Due to the larger fraction of the domestic productivity shock absorbed by home quantities, the within-country correlation of consumption growth with output growth becomes close to unity (0.98 and 1.00 in the two calibrations as reported in Table 2). Since the marginal utility decreases a bit more for the foreign country than for the home country due to the technology spillover channel, exchange rate growth now slightly decreases in response to domestic productivity shocks, as can be seen from the lower panel of Figure 5. The response is relatively weak due to the aforementioned decreased incentives to share risks via financial markets.

The higher degree of technology spillover, relative to the simple economy, induces higher

correlations of innovation probabilities (an increase from 0.49 to 0.79 or 0.73, depending on the amount of spillovers) as reported in the third panel of Table 4. Subsequently, the correlation of aggregate R&D expenditure growth increases to 0.83 for a low amount of technology spillover and remains at 0.47 for a high amount of technology spillover. The counter-acting forces of decreasing imports in response to domestic shocks kick in when the amount of technology spillover is high, therefore reducing this correlation relative to the economy with low amount of technology spillover.

Asset pricing dynamics only marginally change due to slightly better smoothing possibilities for households as reported in Table 3. The risk-free rate is slightly higher and less volatile. The risk premia are a little bit lower as well. Since incumbents are now also exposed to foreign productivity shocks directly, the volatility of incumbents' R&D expenditure growth is higher. Due to incumbents' R&D expenditure being an outflow to aggregate dividends, excess market returns' volatility decreases slightly. As incumbent's profit becomes more volatile, the volatility of incumbent returns increases relative to the economy without technology spillover.

The technology-spillover economy is now successful in matching the quantity anomaly of Backus, Kehoe, and Kydland (1994) as reported in the second Panel of Table 4. The cross-country correlations of consumption growth are now 0.14 and 0.31, respectively. Hence, the economy with high technology spillovers matches the anomaly almost exactly, due to the now significantly different reaction of the consumption bundle with respect to home and foreign productivity shocks, respectively.

Due to the opposite directions of R&D expenditure imports to domestic and foreign productivity shocks, the cross-country correlation of excess incumbent returns becomes slightly negative. Since R&D expenditures are an outflow to aggregate dividends, this effect and the larger co-movement of the pricing kernels induce a large cross-country correlation of excess market returns. Furthermore, the excess final good firm return correlation basically does not change. Finally, the cross-country correlation of the risk-free rates is now also much higher, i.e. 0.77 or 0.76. This number does not fit well the average of the correlations between the US risk-free rate and the other economies as this empirical moment is only 0.27. However, for some country pairs, such as the US and Canada or the US and the UK the correlations of the risk-free rates are 0.61 and 0.58, respectively. All individual correlations that have been computed are reported in Table 5. Hence, the model with technology spillover matches this data moment well for countries that are traditionally close to each other. The average of these correlations is matched quite well in the simple economy as the model predicts only a correlation of 0.43 (data: 0.27) when

technology spillovers are not accounted for.

At this stage it is worthwile to relate my findings to the implications of the model developed by Gavazzoni and Santacreu (2015) and their empirical evidence on the impact of international trade on asset prices and macroeconomic quantities in greater detail. As in their model and empirical evidence, exchange rate growth volatility decreases and asset return correlations increase with the international amount of trade. Moreover, economies that are closer in their R&D intensity also show more co-movement in asset returns and lower exchange rate growth volatility. In my model these empirical facts are reconciled by a higher amount of trade directed towards R&D investment, whereas in their model this is explained by the possibility to adopt technologies from abroad rather than inventing them domestically. Hence, the sources of the results are very different, but the results are comparable.

In contrast to their results, incumbent returns co-move much less (in fact, negatively) than final good firm returns when technology spillovers are accounted for. They show that the returns on intangible capital are much more correlated than final good firm returns in their model. This result is obtained due to the presence of entrants and heterogeneous innovations as the technology spillover for entrants reduce the cross-country correlation of incumbent returns due to the Schumpeterian nature of their innovations, which imply a negative effect for the valuation of incumbents. If the technology spillover for entrants is weaker than for incumbents, the correlation increases, as will be discussed in Section 4.3.2.

### 4.3.2 Heterogeneous technology spillover

This model specification explores the economic implications of heterogeneous technology spillovers. The R&D technologies of incumbents are calibrated to be subject to a lower home bias of only $\phi_{\mathrm{I,k}} = 0.85$, whereas entrants are subject to higher home bias in their R&D technologies, i.e. $\phi_{\mathrm{E,k}} = 0.95$. This captures the intuition and empirical evidence that more established firms (proxied by incumbents in the model) are able to coordinate R&D efforts more easily internationally than smaller or young firms (proxied by entrants).

Since entrants absorb more of the domestic productivity shocks and incumbents can better smooth that shock internationally, entrants' innovation probability becomes slightly more volatile and the incumbents' one slightly less volatile compared to the homogenous technology spillover economies as reported in Table 2. Due to the now heterogeneous exposure to productivity shocks of incumbents and entrants, this also breaks the perfect

correlation of the innovation probabilities of those innovative firms and thus with a value of 0.49 becomes close to the empirical counterpart in US data (0.47). This heterogeneity in the internationalization of innovation activities thus offers an alternative explanation to the inclusion of exogenous "stochastic barriers to entry" in the study by Bena, Garlappi, and Grüning (2016).

The asset pricing results reported in Table 3 do not materially change compared to the homogenous technology economies since the amount of risk is not changed significantly and the home bias parameters are not out of the range of the previously used values.

In terms of international quantities, the moments of which are reported in Table 4, one first notices that the exchange rate growth volatility is 1.12 percentage points and thus slightly lower than in the low technology spillover economy due to the higher degree of internationalization of innovation efforts for incumbents. Since incumbents account for a higher fraction of good imports than entrants, incumbents drive the residual desired amount of risk sharing via financial markets more than entrants and thus the exchange rate volatility is mainly driven by incumbents. Since their home bias is between the low and high technology spillover calibration, so too is the exchange rate growth volatility.

The cross-country correlations of growth rates do not change materially. The innovation probabilities of incumbents are now more positively correlated than the ones of entrants across countries due to the high level of technology spillover for incumbents, but the rather low level for entrants. Due to these observations, incumbent returns are moderately positively correlated in contrast to all homogenous spillover calibrations (i.e. 0.14). The reason being that then the value-enhancing incremental innovations tend to co-move more but the value-reducing effect of entrants' innovations tends to co-move less internationally, both leading to a higher correlation of incumbents' returns.

As the focus of this study is to provide potential channels for the resolution of macroeconomic and asset pricing anomalies, Table 6 summarizes the insights obtained in this section with respect to a number of famous anomalies in international economics, and with respect to the observed heterogeneous innovation dynamics.

An additional sensitivity analysis is provided in Appendix C, which analyzes the model when consumption growth volatility is calibrated to a much higher value to match the data for the period 1929–2008, which includes the period of the Great Depression.

# 5 Conclusion

This paper studies asset prices, macroeconomic quantities, and risk-sharing dynamics in a two-country endogenous growth economy with technology spillover, where innovations are driven by heterogeneous innovations of competing firms, and the amount of technology spillover is allowed to be heterogeneous for these two types of firms.

Endogenous growth, coupled with recursive preferences and technology spillover, generates a small persistent component in expected consumption growth, inducing realistic asset pricing dynamics by means of endogenous long-run risks. On the one hand, there is positive spillover in R&D expenditures implying that shocks to the innovation process are partly transmitted from the home to the foreign country. This creates a realistic technology diffusion process that furthermore allows the model to match the empirically observed high co-movement of returns across countries. On the other hand, productivity shocks spill over to the other country with the opposite sign for consumption.

Modeling heterogeneous technology spillovers allows the model to better match the empirical evidence on the cross-sectional properties of the innovation process. Removing this technology spillover channel induces a much higher exchange rate volatility and too low cross-country correlations of returns. Finally, the model is consistent with the quantity anomaly of Backus, Kehoe, and Kydland (1994): consumption growth is internationally less correlated than output growth in all model specifications with technology spillovers.

Taken together, this study shows that international technology spillovers are an important channel for explaining the co-movement of innovation dynamics and asset prices. Higher international co-movement of economic growth and asset prices would be the likely consequence of policies that reduce international barriers in R&D investments.

Table 1: Calibrated parameters

This table reports the parameters used in the calibrations of my model, as outlined in Section 4.1. The calibration is annual and countries are symmetric in all calibrations.

| Parameter | Description | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|---|
| | | | Homogenous | | Heterogeneous |
| | | | Low | High | |
| $\beta_k$ | Subjective discount factor | 0.984 | 0.984 | 0.984 | 0.984 |
| $\gamma_k$ | Risk aversion | 10 | 10 | 10 | 10 |
| $\psi_k$ | Elasticity of intertemporal substitution | 1.1 | 1.1 | 1.1 | 1.1 |
| $\alpha_k$ | Capital share | 0.35 | 0.35 | 0.35 | 0.35 |
| $\xi_k$ | Intermediate goods share | 0.72 | 0.72 | 0.72 | 0.72 |
| $\nu_k$ | Elasticity of substitution between intermediate goods | 1.25 | 1.25 | 1.25 | 1.25 |
| $\mu_k$ | Marginal cost of producing an intermediate good | 1 | 1 | 1 | 1 |
| $\delta_k$ | Depreciation rate of physical capital | 0.08 | 0.08 | 0.08 | 0.08 |
| $\zeta_k$ | Investment adjustment costs parameter | 1.15 | 1.15 | 1.15 | 1.15 |
| $\rho_{a,k}$ | Persistence of country-specific technology shock $a_t^{(k)}$ | 0.95 | 0.95 | 0.95 | 0.95 |
| $\sigma_{a,k}$ | Volatility of country-specific technology shock $\varepsilon_{a,t+1}^{(k)}$ | 0.01530 | 0.01521 | 0.01521 | 0.01521 |
| $\rho_z$ | Persistence of common technology shock $z_t$ | 0.95 | 0.95 | 0.95 | 0.95 |
| $\sigma_z$ | Volatility of common technology shock $\varepsilon_{z,t+1}$ | 0.01410 | 0.01425 | 0.01425 | 0.01425 |
| $\kappa_{\mathrm{I,k}}$ | Size of incumbent firms' innovation upon success | 1.25 | 1.25 | 1.25 | 1.25 |
| $\kappa_{\mathrm{E,k}}$ | Size of potential entrants' innovation upon success | 2.50 | 2.50 | 2.50 | 2.50 |
| $\kappa_{\mathrm{D,k}}$ | Depreciation rate of technology capital | 0.91 | 0.91 | 0.91 | 0.91 |
| $\eta_{\mathrm{I,k}}$ | Incumbents' R&D productivity shift parameter | 17.50 | 17.50 | 17.50 | 17.50 |
| $\eta_{\mathrm{E,k}}$ | Entrants' R&D productivity shift parameter | 2.00 | 2.00 | 2.00 | 2.00 |
| $\omega_{\mathrm{I,k}}$ | Incumbents' R&D elasticity | **0.8782** | **0.8228** | **0.7955** | **0.8077** |
| $\omega_{\mathrm{E,k}}$ | Entrants' R&D elasticity | **0.8470** | **0.7950** | **0.7690** | **0.8167** |
| $\phi_{\mathrm{C,k}}$ | Home bias in consumption | 0.95 | 0.95 | 0.95 | 0.95 |
| $\phi_{\mathrm{I,k}}$ | Home bias in incumbents' R&D | **1** | **0.90** | **0.80** | **0.85** |
| $\phi_{\mathrm{E,k}}$ | Home bias in entrants' R&D | **1** | **0.90** | **0.80** | **0.95** |

Table 2: Country-specific macro quantities

This table reports the simulated moments for consumption dynamics, innovation dynamics, and within-country correlations of macroeconomic quantities of the model developed in Section 3 and the calibrations reported in Table 1. The model is simulated 3,000 times with each sample being 112 years long after where the first 50 years are used as a "burn-in" period leaving 62 years of simulated data for computing the moments. The moments of the model and the data are annual. Details on the sources for the values in the data column are provided in Appendix B.

| | Data | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|---|
| | | | Homogenous | | Heterogeneous |
| | | | Low | High | |
| | | | *Output* | | |
| $\mathbb{E}[\Delta y^{(k)}]$ | 1.84 | 1.84 | 1.84 | 1.84 | 1.84 |
| $\sigma_{\Delta y^{(k)}}$ | 1.35 | 1.35 | 1.36 | 1.35 | 1.36 |
| | | | *Consumption* | | |
| $\sigma_{\Delta c^{(k)}}$ | 1.10 | 1.04 | 1.26 | 1.18 | 1.19 |
| $\sigma(\mathbb{E}_t[\Delta c_{t+1}^{(k)}])$ | — | 0.24 | 0.21 | 0.20 | 0.21 |
| | | | *Innovation process* | | |
| $\mathbb{E}[\Phi_{\mathrm{I}}^{(\mathrm{k})}]$ | 13.86 | 13.98 | 13.97 | 13.96 | 13.95 |
| $\mathbb{E}[\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}]$ | 3.82 | 3.84 | 3.84 | 3.84 | 3.84 |
| $\sigma_{\Phi_{\mathrm{I}}^{(\mathrm{k})}}$ | 2.20 | 0.60 | 0.49 | 0.47 | 0.49 |
| $\sigma_{\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}}$ | 0.56 | 0.13 | 0.11 | 0.11 | 0.15 |
| $\mathrm{corr}(\Phi_{\mathrm{I}}^{(\mathrm{k})}, \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})})$ | 0.47 | 1.00 | 1.00 | 1.00 | 0.49 |
| | | | *Correlation of growth rates* | | |
| $\mathrm{corr}(\Delta c^{(k)}, \Delta y^{(k)})$ | 0.90 | 0.55 | 0.98 | 1.00 | 0.99 |
| $\mathrm{corr}(\Delta c^{(k)}, \Delta s^{(k)})$ | 0.19 | 0.55 | 0.97 | 0.99 | 0.98 |
| $\mathrm{corr}(\Delta y^{(k)}, \Delta s^{(k)})$ | 0.34 | 1.00 | 1.00 | 1.00 | 1.00 |

Table 3: Asset prices

This table reports the simulated moments for the risk-free rate, the excess return on consumption, the excess market return, the excess final good firm return, and the excess return of incumbents of the model developed in Section 3 and the calibrations reported in Table 1. The model is simulated 3,000 times with each sample being 112 years long after where the first 50 years are used as a "burn-in" period leaving 62 years of simulated data for computing the moments. The moments of the model and the data are annual. Returns in the model are levered following Boldrin, Christiano, and Fisher (2001). Details on the sources for the values in the data column are provided in Appendix B.

| | Data | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|---|
| | | | Homogenous | | Heterogeneous |
| | | | Low | High | |
| | | | *Risk-free rate* | | |
| $\mathbb{E}[r_f^{(k)}]$ | 4.58 | 3.07 | 3.09 | 3.08 | 3.09 |
| $\sigma_{r_f^{(k)}}$ | 1.98 | 0.23 | 0.20 | 0.19 | 0.19 |
| | | | *Return on consumption* | | |
| $\mathbb{E}[r_c^{(k)} - r_f^{(k)}]$ | — | 0.38 | 0.35 | 0.36 | 0.36 |
| $\sigma_{r_c^{(k)} - r_f^{(k)}}$ | — | 1.74 | 2.03 | 1.93 | 1.94 |
| | | | *Market return* | | |
| $\mathbb{E}[r_a^{(k)} - r_f^{(k)}]$ | 6.91 | 0.40 | 0.36 | 0.37 | 0.37 |
| $\sigma_{r_a^{(k)} - r_f^{(k)}}$ | 18.27 | 2.02 | 1.86 | 1.97 | 2.01 |
| | | | *Final goods firm return* | | |
| $\mathbb{E}[r_d^{(k)} - r_f^{(k)}]$ | — | 0.43 | 0.37 | 0.38 | 0.38 |
| $\sigma_{r_d^{(k)} - r_f^{(k)}}$ | — | 2.14 | 2.21 | 2.24 | 2.21 |
| | | | *Incumbent firm return* | | |
| $\mathbb{E}[r_I^{(k)} - r_f^{(k)}]$ | — | 0.13 | 0.12 | 0.15 | 0.13 |
| $\sigma_{r_I^{(k)} - r_f^{(k)}}$ | — | 0.64 | 1.09 | 1.07 | 0.83 |

Table 4: International quantities

This table reports the simulated moments for exchange rate growth, cross-country correlations of various log macroeconomic growth rates, of innovation probabilities, of the expected consumption growth rates, of the pricing kernels, and of returns, the mean total exports to GDP ratio, the mean total imports to GDP ratio, as well as the correlation of the net exports to GDP ratio with output of the model developed in Section 3 and the calibrations reported in Table 1. Total R&D expenditures in country $k$ are defined by $S_t^{(k)} = S_{\mathrm{I,k,t}}^{(\mathrm{k})} + S_{\mathrm{E,k,t}}^{(\mathrm{k})} + P_t^{(k)}(S_{\mathrm{I,-k,t}}^{(\mathrm{k})} + S_{\mathrm{E,-k,t}}^{(\mathrm{k})})$ and its log growth rate is denoted by $\Delta s^{(k)}$. The model is simulated 3,000 times with each sample being 112 years long after where the first 50 years are used as a "burn-in" period leaving 62 years of simulated data for computing the moments. The moments of the model and the data are annual. Returns in the model are levered following Boldrin, Christiano, and Fisher (2001). Details on the sources for the values in the data column are provided in Appendix B.

| | Data | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|---|
| | | | Homogenous | | Heterogeneous |
| | | | Low | High | |
| | | *Exchange rate growth* | | | |
| $\sigma_{\Delta e}$ | 12.70 | 7.94 | 2.86 | 0.49 | 1.12 |
| | | *Growth rates* | | | |
| $\mathrm{corr}(\Delta c^{(h)}, \Delta c^{(f)})$ | 0.34 | 0.67 | 0.14 | 0.31 | 0.26 |
| $\mathrm{corr}(\Delta y^{(h)}, \Delta y^{(f)})$ | 0.46 | 0.46 | 0.46 | 0.46 | 0.46 |
| $\mathrm{corr}(\Delta s^{(h)}, \Delta s^{(f)})$ | 0.28 | 0.47 | 0.83 | 0.47 | 0.34 |
| | | *Innovation probabilities* | | | |
| $\mathrm{corr}(\Phi_{\mathrm{I}}^{(\mathrm{h})}, \Phi_{\mathrm{I}}^{(\mathrm{f})})$ | — | 0.47 | 0.79 | 0.73 | 0.37 |
| $\mathrm{corr}(\hat{\Phi}_{\mathrm{E}}^{(\mathrm{h})}, \hat{\Phi}_{\mathrm{E}}^{(\mathrm{f})})$ | — | 0.47 | 0.79 | 0.73 | 0.29 |
| | | *Expected consumption growth* | | | |
| $\mathrm{corr}(\mathbb{E}_t[\Delta c_{t+1}^{(h)}], \mathbb{E}_t[\Delta c_{t+1}^{(f)}])$ | — | 0.53 | 0.81 | 0.80 | 0.80 |
| | | *Returns and pricing kernel* | | | |
| $\mathrm{corr}(r_I^{(h)} - r_f^{(h)}, r_I^{(f)} - r_f^{(f)})$ | — | 0.46 | -0.28 | -0.14 | 0.14 |
| $\mathrm{corr}(r_d^{(h)} - r_f^{(h)}, r_d^{(f)} - r_f^{(f)})$ | 0.66 | 0.46 | 0.45 | 0.44 | 0.46 |
| $\mathrm{corr}(r_a^{(h)} - r_f^{(h)}, r_a^{(f)} - r_f^{(f)})$ | 0.66 | 0.45 | 0.89 | 0.68 | 0.64 |
| $\mathrm{corr}(r_f^{(h)}, r_f^{(f)})$ | 0.27 | 0.43 | 0.77 | 0.76 | 0.76 |
| $\mathrm{corr}(M^{(h)}, M^{(f)})$ | — | 0.93 | 0.99 | 1.00 | 1.00 |
| | | *Exports and imports* | | | |
| $E[\mathrm{Exports}^{(k)}/Y^{(k)}]$ | 8.81 | 1.96 | 2.59 | 3.70 | 2.15 |
| $E[\mathrm{Imports}^{(k)}/Y^{(k)}]$ | 11.45 | 1.96 | 2.59 | 3.70 | 2.15 |
| $\mathrm{corr}(NX^{(k)}/Y^{(k)}, Y^{(k)})$ | -0.82 | 0.27 | -0.42 | -0.45 | -0.45 |

Table 5: Cross-country return correlations (data)

This table reports cross-country correlations of returns, risk-free rates, and excess returns for a number of countries for the period 1990–2008. Specifically, the moments for all country pairs from the following set of countries are reported: Australia, Belgium, Canada, Japan, the Netherlands, Norway, Singapore, Switzerland, the United Kingdom, and the United States. Details on the data sources are provided in Appendix B.

| **Risk-free rates** | AUS | BEL | CAN | JPN | NLD | NOR | SGP | CHE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|
| AUS | 1.00 | | | | | | | | | |
| BEL | 0.77 | 1.00 | | | | | | | | |
| CAN | 0.79 | 0.75 | 1.00 | | | | | | | |
| JPN | 0.64 | 0.73 | 0.49 | 1.00 | | | | | | |
| NLD | 0.83 | 0.88 | 0.75 | 0.70 | 1.00 | | | | | |
| NOR | 0.54 | 0.66 | 0.70 | 0.48 | 0.67 | 1.00 | | | | |
| SGP | -0.11 | -0.13 | 0.01 | -0.24 | -0.30 | 0.13 | 1.00 | | | |
| CHE | 0.51 | 0.24 | 0.39 | 0.16 | 0.33 | 0.51 | 0.26 | 1.00 | | |
| GBR | 0.71 | 0.69 | 0.87 | 0.37 | 0.69 | 0.65 | 0.62 | 0.42 | 1.00 | |
| USA | 0.39 | 0.26 | 0.61 | -0.14 | 0.18 | 0.13 | 0.10 | 0.29 | 0.58 | 1.00 |
| **Market returns** | AUS | BEL | CAN | JPN | NLD | NOR | SGP | CHE | GBR | USA |
| AUS | 1.00 | | | | | | | | | |
| BEL | 0.67 | 1.00 | | | | | | | | |
| CAN | 0.88 | 0.55 | 1.00 | | | | | | | |
| JPN | 0.66 | 0.41 | 0.66 | 1.00 | | | | | | |
| NLD | 0.79 | 0.83 | 0.79 | 0.48 | 1.00 | | | | | |
| NOR | 0.82 | 0.54 | 0.89 | 0.64 | 0.74 | 1.00 | | | | |
| SGP | 0.83 | 0.47 | 0.70 | 0.72 | 0.65 | 0.65 | 1.00 | | | |
| CHE | 0.54 | 0.75 | 0.60 | 0.28 | 0.83 | 0.50 | 0.41 | 1.00 | | |
| GBR | 0.77 | 0.83 | 0.78 | 0.48 | 0.94 | 0.75 | 0.62 | 0.75 | 1.00 | |
| USA | 0.62 | 0.75 | 0.66 | 0.41 | 0.86 | 0.51 | 0.46 | 0.76 | 0.89 | 1.00 |
| **Equity premium** | AUS | BEL | CAN | JPN | NLD | NOR | SGP | CHE | GBR | USA |
| AUS | 1.00 | | | | | | | | | |
| BEL | 0.67 | 1.00 | | | | | | | | |
| CAN | 0.88 | 0.54 | 1.00 | | | | | | | |
| JPN | 0.66 | 0.42 | 0.66 | 1.00 | | | | | | |
| NLD | 0.80 | 0.84 | 0.81 | 0.52 | 1.00 | | | | | |
| NOR | 0.78 | 0.51 | 0.88 | 0.62 | 0.74 | 1.00 | | | | |
| SGP | 0.81 | 0.46 | 0.68 | 0.68 | 0.64 | 0.61 | 1.00 | | | |
| CHE | 0.53 | 0.75 | 0.59 | 0.29 | 0.82 | 0.49 | 0.44 | 1.00 | | |
| GBR | 0.76 | 0.82 | 0.77 | 0.50 | 0.94 | 0.73 | 0.62 | 0.75 | 1.00 | |
| USA | 0.61 | 0.76 | 0.65 | 0.44 | 0.85 | 0.48 | 0.48 | 0.76 | 0.88 | 1.00 |

Table 6: Model performance with respect to macroeconomic and asset pricing anomalies

This table summarizes the performance of the model in explaining famous international macroeconomics and asset pricing anomalies, and in matching the heterogeneous innovation dynamics.

| | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|
| | | Homogenous | | Heterogeneous |
| | | Low | High | |
| **Anomaly** | | | | |
| $E[\Phi_{\mathrm{I}}^{(\mathrm{k})}]/E[\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}]$ | ✓ | ✓ | ✓ | ✓ |
| $\mathrm{corr}(\Phi_{\mathrm{I}}^{(\mathrm{k})}, \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})})$ | | | | ✓ |
| $\mathrm{corr}(\Delta y^{(h)}, \Delta y^{(f)}) - \mathrm{corr}(\Delta c^{(h)}, \Delta c^{(f)})$ | | | ✓ | ✓ |
| $\mathrm{corr}(\Delta s^{(h)}, \Delta s^{(f)})$ | | | | ✓ |
| $\mathrm{corr}(NX^{(k)}/Y^{(k)}, Y^{(k)})$ | | ✓ | ✓ | ✓ |
| $E[\mathrm{Exports}^{(k)}/Y^{(k)}]$ | | | (✓) | |
| $\sigma(\Delta e)$ | ✓ | | | |
| $\mathrm{corr}(r_f^{(h)}, r_f^{(f)})$ | (✓) | (✓) | (✓) | (✓) |
| $\mathrm{corr}(r_a^{(h)}, r_a^{(f)})$ | | | ✓ | ✓ |

Figure 1: Model structure

This figure depicts the structure of the model presented in Section 3. The arrows within the countries depict the flows of goods and labor within a country. The arrows across countries depict the physical goods markets and the international financial market in which the countries can trade with each other, i.e. the trade for households' consumption, incumbents' R&D investments, entrants' R&D investments, and international bonds. Moreover, the common shock that affects the exogenous productivity processes of both countries' final goods firms simultaneously is visualized.

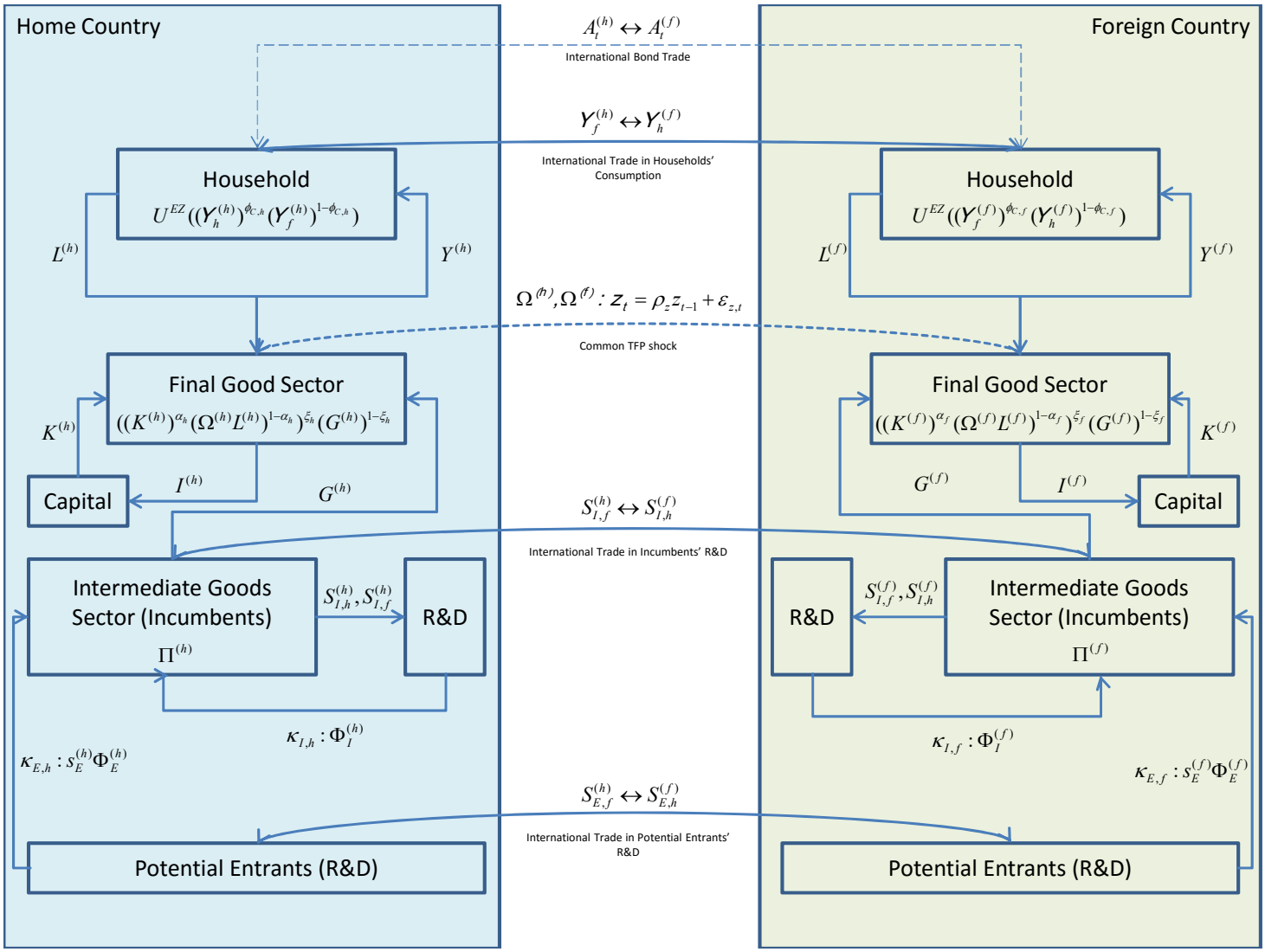


Figure 2: Home bias in U.S. households' consumption bundles

Using the World Input-Output Database, available from `http://www.wiod.org/new_site/home.htm`, the share of foreign goods in the consumption bundle of US households is computed for the period 1995–2011. Details on the World Input-Output Database are provided by Timmer, Dietzenbacher, Los, Stehrer, and de Vries (2015).

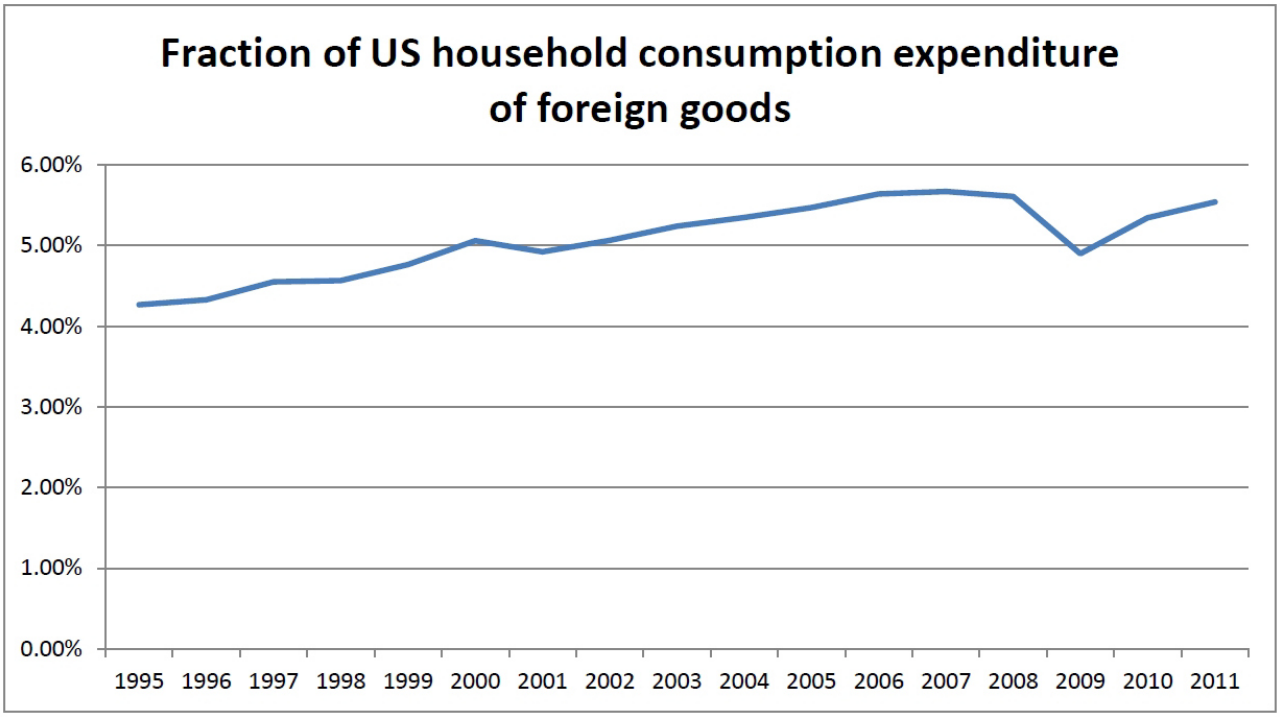


Figure 3: Incumbents' R&D expenditure – impulse response functions

This figure depicts the impulse response functions of the home country's incumbents' innovation probability $\Phi_{\mathrm{I}}^{(\mathrm{h})}$, of the home country's incumbents' normalized R&D expenditure of the home good $s_{\mathrm{I,h,t}}^{(\mathrm{h})}$, and of the home country's normalized incumbents' R&D expenditure of the foreign good $s_{\mathrm{I,f,t}}^{(\mathrm{h})}$, to a positive one-standard-deviation shock to the exogenous component of home country's productivity $a_t^{(h)}$ $(\varepsilon_{a,t}^{(h)})$, and to the exogenous component of foreign country's productivity $a_t^{(f)}$ $(\varepsilon_{a,t}^{(f)})$, respectively. The parameter values reported in Table 1 are used and the impulse response functions for three calibrations are depicted (simple economy, medium-strength homogenous technology spillover economy, and heterogeneous spillover economy). The impulse response functions are expressed in log deviations from the steady state in percentage points.

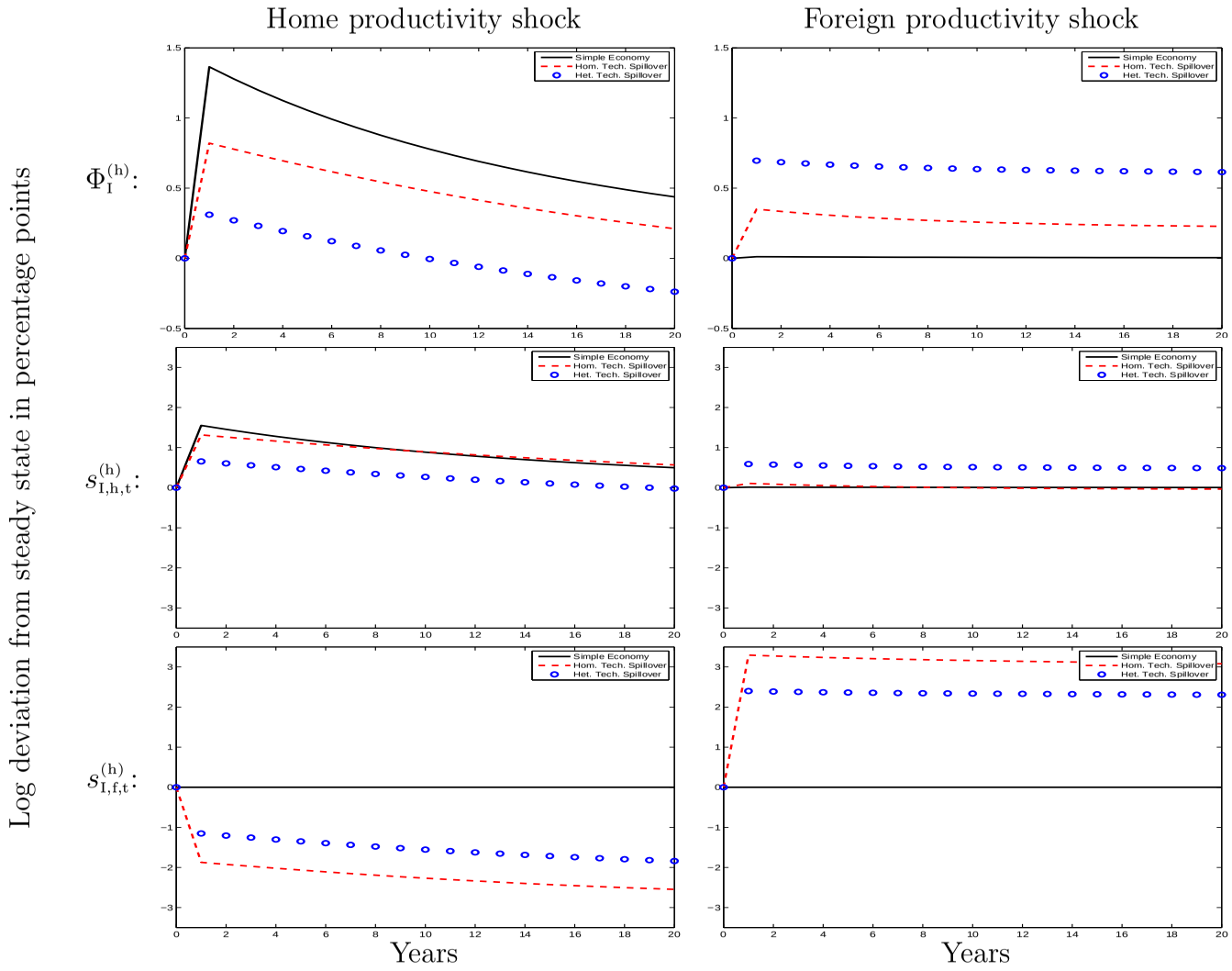


Figure 4: Entrants' R&D expenditure – impulse response functions

This figure depicts the impulse response functions of the home country's entrants' R&D innovation probability $\hat{\Phi}_{\mathrm{E}}^{(\mathrm{h})}$, of the home country's entrants' normalized R&D expenditure of the home good $s_{\mathrm{E,h,t}}^{(\mathrm{h})}$, and of the home country's normalized entrants' R&D expenditure of the foreign good $s_{\mathrm{E,f,t}}^{(\mathrm{h})}$, to a positive one-standard-deviation shock to the exogenous component of home country's productivity $a_t^{(h)}$ $(\varepsilon_{a,t}^{(h)})$, and to the exogenous component of foreign country's productivity $a_t^{(f)}$ $(\varepsilon_{a,t}^{(f)})$, respectively. The parameter values reported in Table 1 are used and the impulse response functions for three calibrations are depicted (simple economy, medium-strength homogenous technology spillover economy, and heterogeneous spillover economy). The impulse response functions are expressed in log deviations from the steady state in percentage points.

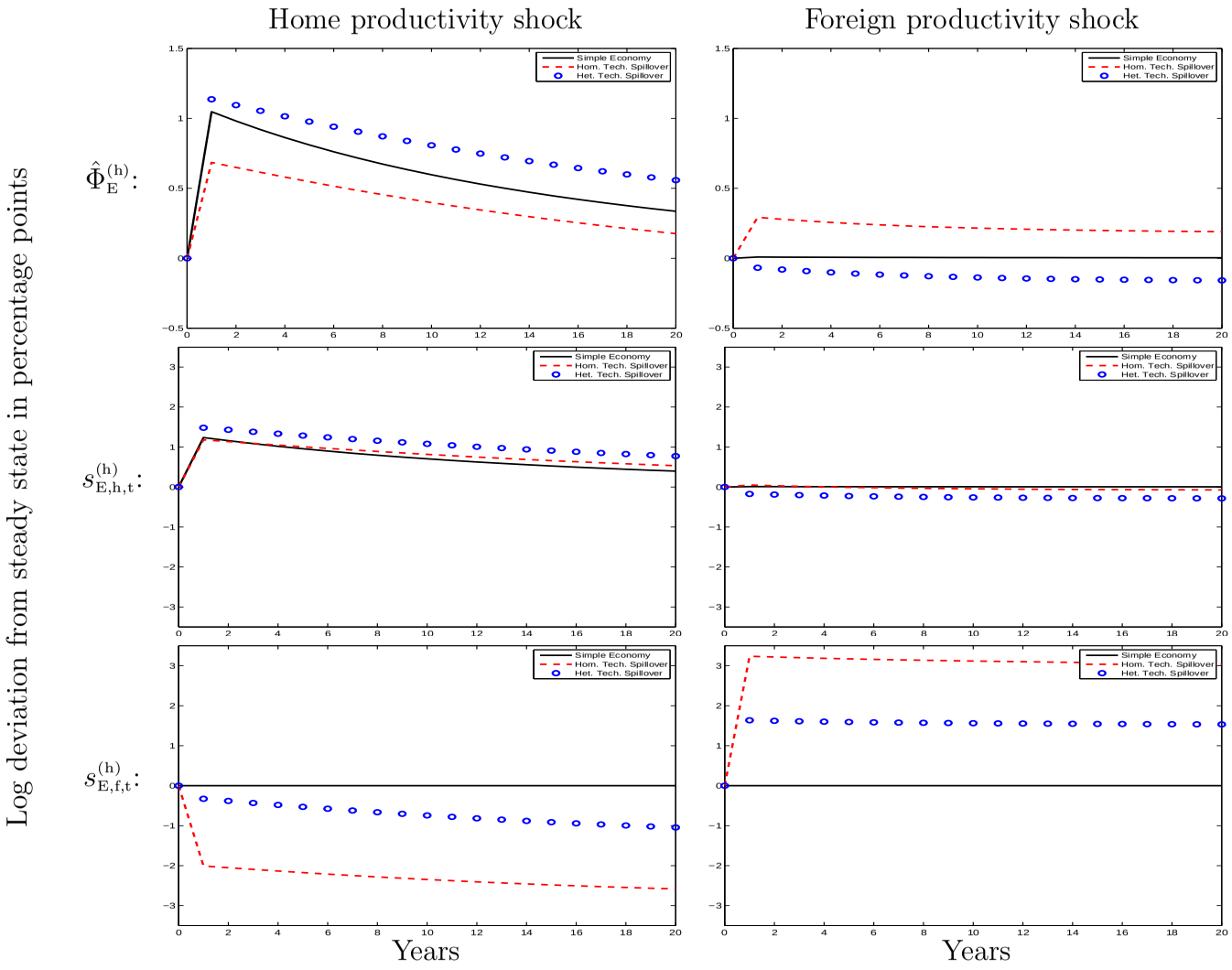


Figure 5: Expected consumption growth, pricing kernel, exchange rate – impulse response functions

This figure depicts the impulse response functions of the expected home country's log consumption growth $\mathbb{E}_t[\Delta c_{t+1}^{(h)}]$, of the home country's pricing kernel $\mathbb{M}_{t,t+1}^{(h)}$, and of exchange rate growth $\Delta e_t$, to a positive one-standard-deviation shock to the exogenous component of home country's productivity $a_t^{(h)}$ $(\varepsilon_{a,t}^{(h)})$, and to the exogenous component of foreign country's productivity $a_t^{(f)}$ $(\varepsilon_{a,t}^{(f)})$, respectively. The parameter values reported in Table 1 are used and the impulse response functions for these calibrations are depicted (simple economy, medium-strength homogenous technology spillover economy, and heterogeneous spillover economy). The impulse response functions are expressed in log deviations from the steady state in percentage points for the pricing kernel and in absolute deviations in percentage points for expected consumption and exchange rate growth.

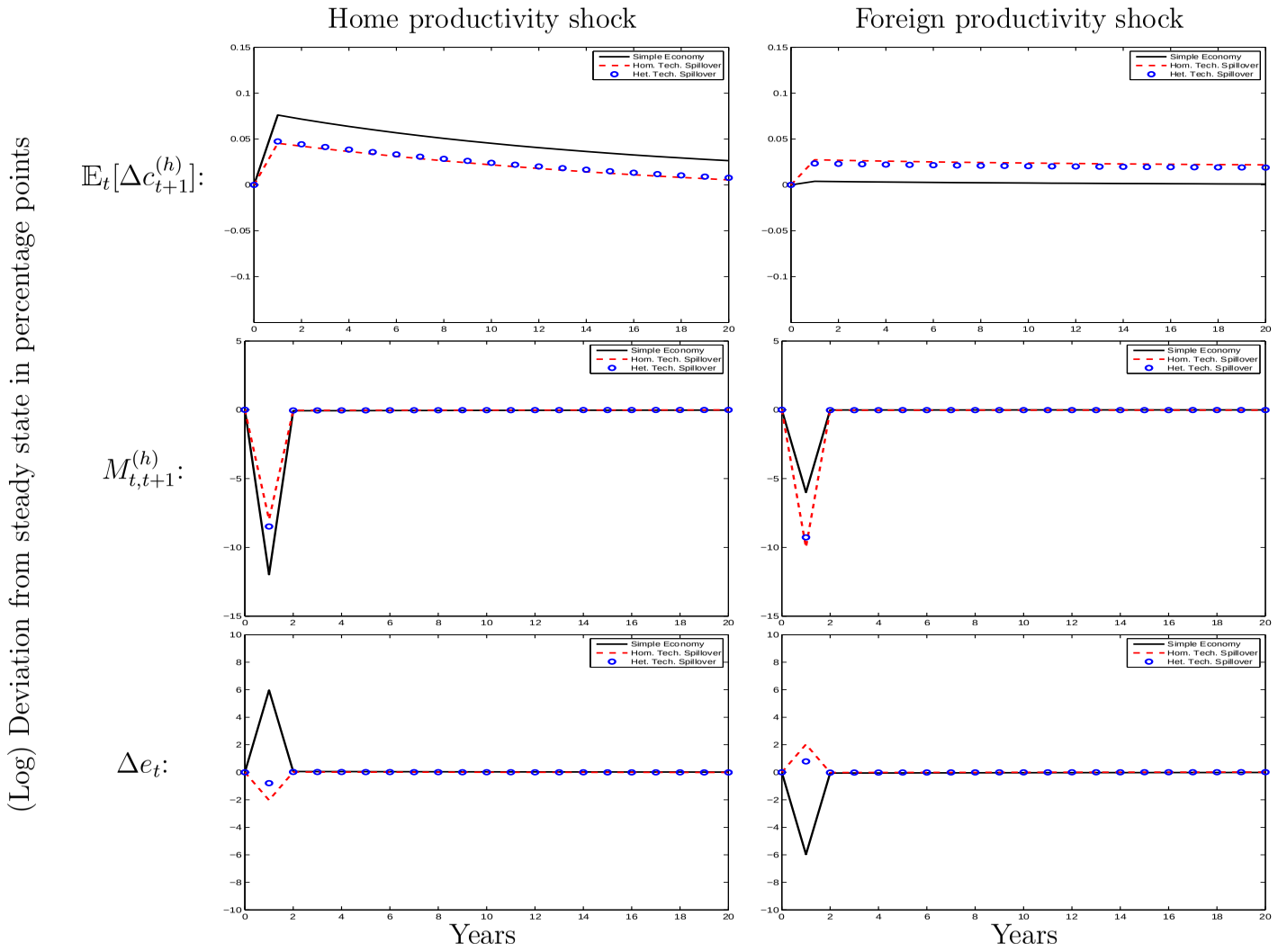
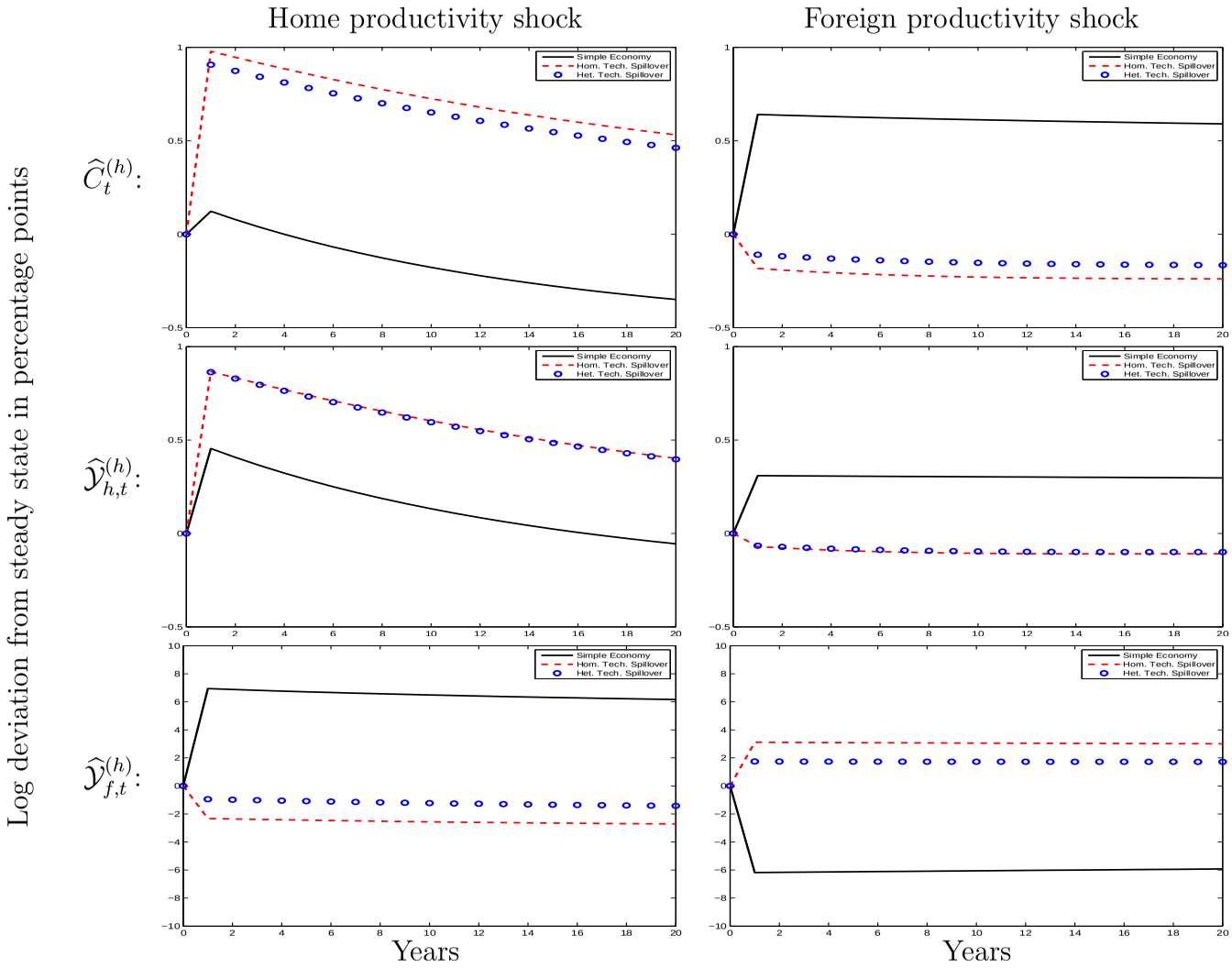


Figure 6: Consumption – impulse response functions

This figure depicts the impulse response functions of the home country's consumption bundle $\widehat{C}_t^{(h)}$, of the home country's normalized consumption of the home good $\widehat{\mathcal{Y}}_{h,t}^{(h)}$, and of the home country's normalized consumption of the foreign good $\widehat{\mathcal{Y}}_{f,t}^{(h)}$, to a positive one-standard-deviation shock to the exogenous component of home country's productivity $a_t^{(h)}$ $(\varepsilon_{a,t}^{(h)})$, and to the exogenous component of foreign country's productivity $a_t^{(f)}$ $(\varepsilon_{a,t}^{(f)})$, respectively. The parameter values reported in Table 1 are used and the impulse response functions for three calibrations are depicted (simple economy, medium-strength homogenous technology spillover economy, and heterogeneous spillover economy). The impulse response functions are expressed in log deviations from the steady state in percentage points.



# A Derivations and Equilibrium

## A.1 Solving households' consumption allocation problem

There is a complete set of Arrow-Debreu securities (i.e. one-period-ahead claims to state-contingent final goods) available to households in both countries. These claims are denoted by $Q_{t+1}(\chi_{t+1})$ where $\chi_{t+1}$ is the state of the economy at time $t+1$. If a household holds one unit of $Q_{t+1}(\chi_{t+1})$ between time $t$ and $t+1$, it receives one unit of the home country's final good if the economy is in state $\chi_{t+1}$ at time $t+1$ and zero otherwise. Country $k$'s household's holdings of these assets from time $t$ to time $t+1$ are given by $A^{(k)}_{t+1}(\chi_{t+1})$. $P^{(h)}_t$ denotes the terms of trade or, equivalently, the price of the foreign final good in home final good units. Moreover, final good trading and financial markets are assumed to be frictionless.

Therefore, the home country's consumption household's allocation problem is

$$\max_{\left\{\mathcal{Y}^{(h)}_{h,t},\mathcal{Y}^{(h)}_{f,t},A^{(h)}_{t+1}(\chi_{t+1})\right\}} \left\{ \left\{ (1-\beta_h)\left(C^{(h)}_t\right)^{\frac{1-\gamma_h}{\theta_h}} + \beta_h \left( \mathbb{E}_t\left[\left(U^{(h)}_{t+1}\right)^{1-\gamma_h}\right]\right)^{\frac{1}{\theta_h}} \right\}^{\frac{\theta_h}{1-\gamma_h}} \right\}, \tag{A1}$$

subject to the budget constraint

$$\mathcal{Y}^{(h)}_{h,t} + P^{(h)}_t \mathcal{Y}^{(h)}_{f,t} + \int_{\chi_{t+1}} A^{(h)}_{t+1}(\chi_{t+1}) Q_{t+1}(\chi_{t+1}) = A^{(h)}_t + \mathcal{Y}^{(h)}_t,$$

where

$$C^{(h)}_t = \left(\mathcal{Y}^{(h)}_{h,t}\right)^{\phi_{\mathrm{C,h}}} \left(\mathcal{Y}^{(h)}_{f,t}\right)^{1-\phi_{\mathrm{C,h}}}.$$

Attaching the Lagrange-multiplier $\mu^{(h)}_t$ to the budget constraint, the first order conditions of this optimization problem are given by

$$\begin{aligned}
\mu^{(h)}_t &= \left(U^{(h)}_t\right)^{1-\frac{1-\gamma_h}{\theta_h}} (1-\beta_h)\left(C^{(h)}_t\right)^{-\frac{1}{\psi_h}} \phi_{\mathrm{C,h}} \frac{C^{(h)}_t}{\mathcal{Y}^{(h)}_{h,t}} \\
P^{(h)}_t \mu^{(h)}_t &= \left(U^{(h)}_t\right)^{1-\frac{1-\gamma_h}{\theta_h}} (1-\beta_h)\left(C^{(h)}_t\right)^{-\frac{1}{\psi_h}} (1-\phi_{\mathrm{C,h}}) \frac{C^{(h)}_t}{\mathcal{Y}^{(h)}_{f,t}} \\
Q_{t+1}(\chi_{t+1})\mu^{(h)}_t &= \beta_h \left(U^{(h)}_t\right)^{1-\frac{1-\gamma_h}{\theta_h}} \left( \mathbb{E}_t\left[\left(U^{(h)}_{t+1}\right)^{1-\gamma_h}\right]\right)^{\frac{1}{\theta_h}-1} \mathbb{E}_t\left[\left(U^{(h)}_{t+1}\right)^{-\gamma_h} \frac{\partial U^{(h)}_{t+1}}{\partial A^{(h)}_{t+1}(\chi_{t+1})}\right].
\end{aligned}$$

Using the envelope theorem, I obtain $\frac{\partial U^{(h)}_{t+1}}{\partial A^{(h)}_{t+1}(\chi_{t+1})} = \mu^{(h)}_{t+1}$. Moreover, by noting that the last first order condition determining the Arrow-Debreu price $Q_{t+1}$ holds for each state $\chi_{t+1}$, one obtains by combining all first conditions and, for the ease of exposition, by not explicitly writing the dependence on the state $\chi_{t+1}$

$$P^{(h)}_t \phi_{\mathrm{C,h}} \mathcal{Y}^{(h)}_{f,t} = (1-\phi_{\mathrm{C,h}}) \mathcal{Y}^{(h)}_{h,t} \tag{A2}$$

$$Q_{t+1} = \mathbb{M}^{(h)}_{t,t+1} \frac{C^{(h)}_{t+1}}{C^{(h)}_t} \frac{\mathcal{Y}^{(h)}_{h,t}}{\mathcal{Y}^{(h)}_{h,t+1}}, \tag{A3}$$

where the standard functional form of the pricing kernel given in Equation (3) has been used.

Similarly, the foreign country's household's consumption allocation problem is

$$\max_{\left\{\mathcal{Y}_{f,t}^{(f)},\mathcal{Y}_{h,t}^{(f)},A_{t+1}^{(f)}(\chi_{t+1})\right\}}\left\{\left\{(1-\beta_f)\left(C_t^{(f)}\right)^{\frac{1-\gamma_f}{\theta_f}}+\beta_f\left(\mathbb{E}_t\left[\left(U_{t+1}^{(f)}\right)^{1-\gamma_f}\right]\right)^{\frac{1}{\theta_f}}\right\}^{\frac{\theta_f}{1-\gamma_f}}\right\}, \tag{A4}$$

subject to the budget constraint (written in units of the home country's final good)

$$P_t^{(h)}\mathcal{Y}_{f,t}^{(f)}+\mathcal{Y}_{h,t}^{(f)}+\int_{\chi_{t+1}}A_{t+1}^{(f)}(\chi_{t+1})Q_{t+1}(\chi_{t+1})=A_t^{(f)}+P_t^{(h)}\mathcal{Y}_t^{(f)},$$

where

$$C_t^{(f)}=\left(\mathcal{Y}_{f,t}^{(f)}\right)^{\phi_{\mathrm{C,f}}}\left(\mathcal{Y}_{h,t}^{(f)}\right)^{1-\phi_{\mathrm{C,f}}}.$$

Attaching the Lagrange-multiplier $\mu_t^{(f)}$ to the budget constraint, the first order conditions of this optimization problem are given by

$$\begin{aligned}
P_t^{(h)}\mu_t^{(f)}&=\left(U_t^{(f)}\right)^{1-\frac{1-\gamma_f}{\theta_f}}(1-\beta_f)\left(C_t^{(f)}\right)^{-\frac{1}{\psi_f}}\phi_{\mathrm{C,f}}\frac{C_t^{(f)}}{\mathcal{Y}_{f,t}^{(f)}}\\
\mu_t^{(f)}&=\left(U_t^{(f)}\right)^{1-\frac{1-\gamma_f}{\theta_f}}(1-\beta_f)\left(C_t^{(f)}\right)^{-\frac{1}{\psi_f}}(1-\phi_{\mathrm{C,f}})\frac{C_t^{(f)}}{\mathcal{Y}_{h,t}^{(f)}}\\
Q_{t+1}(\chi_{t+1})\mu_t^{(f)}&=\beta_f\left(U_t^{(f)}\right)^{1-\frac{1-\gamma_f}{\theta_f}}\left(\mathbb{E}_t\left[\left(U_{t+1}^{(f)}\right)^{1-\gamma_f}\right]\right)^{\frac{1}{\theta_f}-1}\mathbb{E}_t\left[\left(U_{t+1}^{(f)}\right)^{-\gamma_f}\frac{\partial U_{t+1}^{(f)}}{\partial A_{t+1}^{(f)}(\chi_{t+1})}\right].
\end{aligned}$$

Using again the envelope theorem implies $\frac{\partial U_{t+1}^{(f)}}{\partial A_{t+1}^{(f)}(\chi_{t+1})}=\mu_{t+1}^{(f)}$. With this and the same notational conventions as used for the home country's problem, I obtain

$$\phi_{\mathrm{C,f}}\mathcal{Y}_{h,t}^{(f)}=P_t^{(h)}(1-\phi_{\mathrm{C,f}})\mathcal{Y}_{f,t}^{(f)} \tag{A5}$$

$$Q_{t+1}=\mathbb{M}_{t,t+1}^{(f)}\frac{C_{t+1}^{(f)}}{C_t^{(f)}}\frac{\mathcal{Y}_{f,t}^{(f)}}{\mathcal{Y}_{f,t+1}^{(f)}}\frac{P_t^{(h)}}{P_{t+1}^{(h)}}. \tag{A6}$$

In equilibrium, the market for Arrow-Debreu securities clears, i.e. $A_t^{(h)}+A_t^{(f)}=0$ at each time $t\geq 0$. Moreover, the final good markets in both countries have to clear, i.e. for each time $t$ the following restrictions apply

$$\mathcal{Y}_t^{(h)}=\mathcal{Y}_{h,t}^{(h)}+\mathcal{Y}_{h,t}^{(f)} \tag{A7}$$

$$\mathcal{Y}_t^{(f)}=\mathcal{Y}_{f,t}^{(f)}+\mathcal{Y}_{f,t}^{(h)}. \tag{A8}$$

Finally, equating conditions (A3) and (A6) and defining the "pseudo" Pareto share $S_t=\frac{1}{P_t^{(h)}}\frac{\mathcal{Y}_{h,t}^{(h)}}{\mathcal{Y}_{f,t}^{(f)}}$ implies the recursion

$$S_t=S_{t-1}\frac{\mathbb{M}_{t-1,t}^{(h)}}{\mathbb{M}_{t-1,t}^{(f)}}\frac{C_t^{(h)}/C_{t-1}^{(h)}}{C_t^{(f)}/C_{t-1}^{(f)}}. \tag{A9}$$

Rewriting equations (A2) and (A5) using this "pseudo" Pareto share gives

$$S_t \phi_{\mathrm{C,f}} \mathcal{Y}_{h,t}^{(f)} = (1-\phi_{\mathrm{C,f}}) \mathcal{Y}_{h,t}^{(h)} \tag{A10}$$

$$S_t (1-\phi_{\mathrm{C,h}}) \mathcal{Y}_{f,t}^{(f)} = \phi_{\mathrm{C,h}} \mathcal{Y}_{f,t}^{(h)}. \tag{A11}$$

These two conditions jointly with the market clearing conditions (A7) and (A8) determine the allocation of the two final goods for consumption in each country given the recursion for the "pseudo" Pareto share (A9). The "pseudo" Pareto share includes the stochastic discount factors of the countries and thus is responsible for risk adjustments in the consumption good allocation.

## A.2 Equilibrium of the final and intermediate Goods Sectors

The first order conditions of the final good firm's problem,

$$\max_{\left\{I_t^{(k)}, K_{t+1}^{(k)}, L_t^{(k)}, x_{i_k,\mathrm{t}}^{(\mathrm{k})}\right\}_{i_k \in [0,1]; t \geq 0}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \mathbb{M}_t^{(k),loc} \left( Y_t^{(k)} - I_t^{(k)} - \omega_t^{(k)} L_t^{(k)} - \int_0^1 p_{i_k,\mathrm{t}}^{(\mathrm{k})} (q_{i_k,\mathrm{t}}^{(\mathrm{k})}) x_{i_k,\mathrm{t}}^{(\mathrm{k})} \, di_k \right) \right],$$

subject to

$$K_{t+1}^{(k)} = (1-\delta_k) K_t^{(k)} + \Lambda^{(k)} \left( \frac{I_t^{(k)}}{K_t^{(k)}} \right) K_t^{(k)},$$

are given by (denoting by $q_t$ the Lagrange multiplier attached to above capital accumulation equation)

$$q_t = \frac{1}{\left(\Lambda_t^{(k)}\right)'}$$

$$w_t^{(k)} = (1-\alpha_k)(1-\xi_k) Y_t^{(k)} \tag{A12}$$

$$1 = \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} \left(\Lambda_t^{(k)}\right)' \left( \frac{\alpha_k (1-\xi_k) Y_{t+1}^{(k)} - I_{t+1}^{(k)}}{K_{t+1}^{(k)}} + \frac{\Lambda_{t+1}^{(k)} + 1 - \delta_k}{\left(\Lambda_{t+1}^{(k)}\right)'} \right) \right] \tag{A13}$$

$$p_{i_k,\mathrm{t}}^{(\mathrm{k})} = \left[ \left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k} \right]^{1-\xi_k} \xi_k \left(G_t^{(k)}\right)^{\xi_k - \frac{1}{\nu_k}} \left(q_{i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{1-\frac{1}{\nu_k}} \left(x_{i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{\frac{1}{\nu_k}-1}, \tag{A14}$$

where $\Lambda_t^{(k)} \equiv \Lambda^{(k)} \left( \frac{I_t^{(k)}}{K_t^{(k)}} \right)$, $\left(\Lambda_t^{(k)}\right)' \equiv \left(\Lambda^{(k)}\right)' \left( \frac{I_t^{(k)}}{K_t^{(k)}} \right)$. From the final good firm's first order condition $(A14)$, the final good firm's demand for intermediate good $i_k$, $x_{i_k,\mathrm{t}}^{(\mathrm{k})}$, is given by

$$x_{i_k,\mathrm{t}}^{(\mathrm{k})} = (\xi_k)^{\frac{\nu_k}{\nu_k - 1}} \left( \left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k} \right)^{\frac{(1-\xi_k)\nu_k}{\nu_k - 1}} \left(G_t^{(k)}\right)^{\frac{\xi_k \nu_k - 1}{\nu_k - 1}} \left(p_{i_k,\mathrm{t}}^{(\mathrm{k})}\right)^{\frac{\nu_k}{1-\nu_k}} q_{i_k,\mathrm{t}}^{(\mathrm{k})}. \tag{A15}$$

Using $(A15)$ in the incumbent's problem

$$\pi_{i_k,\mathrm{t}}^{(\mathrm{k})} = \max_{\left\{p_{i_k,\mathrm{t}}^{(\mathrm{k})}\right\}} \left\{ p_{i_k,\mathrm{t}}^{(\mathrm{k})} x_{i_k,\mathrm{t}}^{(\mathrm{k})} - \mu_k x_{i_k,\mathrm{t}}^{(\mathrm{k})} \right\},$$

which is also given in Equation (10), leads to the following price and profit

$$p_{i_k,\mathrm{t}}^{(\mathrm{k})} = \nu_k \mu_k, \quad \pi_{i_k,\mathrm{t}}^{(\mathrm{k})} = (\nu_k - 1) \mu_k x_{i_k,\mathrm{t}}^{(\mathrm{k})}. \tag{A16}$$

Hence, the monopoly price is $\nu_k$ multiplied by marginal costs $\mu_k$. Thus, the parameter $\nu_k$ has two properties. First, it measures the monopoly markup or market power of incumbents. Second, it determines the elasticity of substitution between any two intermediate goods given by $\frac{\nu_k}{\nu_k-1}$. Substituting $(A15)$ and $(A16)$ into (7) yields the following expression for the composite of intermediate goods

$$G_t^{(k)} = \left(\frac{\xi_k}{\nu_k \mu_k}\right)^{\frac{1}{1-\xi_k}} \left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k} \left(Q_t^{(k)}\right)^{\frac{\nu_k-1}{1-\xi_k}}. \tag{A17}$$

Using again $(A16)$ and $(A17)$, I can rewrite the demand for intermediate good $i_k$ in country $k$, $x_{i_k,\mathrm{t}}^{(\mathrm{k})}$, as a linear function of quality which substantially facilitates aggregation

$$x_{i_k,\mathrm{t}}^{(\mathrm{k})} = \left(\frac{\xi_k}{\mu_k \nu_k}\right)^{\frac{1}{1-\xi_k}} \left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k} \left(Q_t^{(k)}\right)^{\frac{\xi_k \nu_k - 1}{1-\xi_k}} q_{i_k,\mathrm{t}}^{(\mathrm{k})}. \tag{A18}$$

Linearity in intermediate good demand translates into the following aggregate quantities by using the definition of aggregate output (4), the definition of aggregate quality (12), as well as the just derived individual firm quantities $(A16)$, $(A17)$ and $(A18)$

$$\begin{aligned}
X_t^{(k)} &= \int_0^1 x_{i_k,\mathrm{t}}^{(\mathrm{k})}\, di_k = \left(\frac{\xi_k}{\mu_k \nu_k}\right)^{\frac{1}{1-\xi_k}} \left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k} \left(Q_t^{(k)}\right)^{\frac{\xi_k(\nu_k-1)}{1-\xi_k}} \\
\Pi_t^{(k)} &= \int_0^1 \pi_{i_k,\mathrm{t}}^{(\mathrm{k})}\, di_k = (\nu_k - 1)\mu_k X_t^{(k)} \\
Y_t^{(k)} &= \left(\frac{\xi_k}{\mu_k \nu_k}\right)^{\frac{\xi_k}{1-\xi_k}} \left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k} \left(Q_t^{(k)}\right)^{\frac{\xi_k(\nu_k-1)}{1-\xi_k}}.
\end{aligned}$$

To ensure balanced growth, I need to impose a restriction on the exponent of technology capital $Q_t^{(k)}$ in each country and thus I assume

$$\frac{(\nu_k - 1)\xi_k}{1-\xi_k} = 1 - \alpha_k, \quad k = h, f. \tag{A19}$$

This condition implies the following functional forms for final good output and the demand for intermediate goods

$$\begin{aligned}
Y_t^{(k)} &= \left(\frac{\xi_k}{\mu_k \nu_k}\right)^{\frac{\xi_k}{1-\xi_k}} \left(K_t^{(k)}\right)^{\alpha_k} \left(\Omega_t^{(k)} Q_t^{(k)} L_t^{(k)}\right)^{1-\alpha_k} \\
X_t^{(k)} &= \frac{\xi_k}{\nu_k \mu_k} Y_t^{(k)}.
\end{aligned}$$

Technology capital $Q_t^{(k)}$ thus acts as an endogenous "labor augmenting" productivity factor. It determines total factor productivity (TFP) alongside exogenous shocks. Furthermore, it measures the size of the economy in country $k$ as it determines the endogenously growing part of TFP. The exogenous shocks have zero mean and thus do not induce growth in the economy. Nevertheless, the productivity shocks $\varepsilon_{z,t}, \varepsilon_{a,t}^{(h)}$ and $\varepsilon_{a,t}^{(f)}$ are essential to make productivity growth stochastic.

## A.3 R&D decisions by incumbents and entrants

### A.3.1 Incumbents

The value of each incumbent is determined by the Bellman equation

$$v_t^{(k)} = \max_{\left\{s_{\mathrm{I,k,t}}^{(\mathrm{k})}, s_{\mathrm{I,-k,t}}^{(\mathrm{k})}\right\}} \left\{ \pi_t^{(k)} - s_{\mathrm{I,k,t}}^{(\mathrm{k})} - P_t^{(k)} s_{\mathrm{I,-k,t}}^{(\mathrm{k})} \right. \tag{A20}$$
$$\left. + \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \left( \Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) \kappa_{\mathrm{I,k}} + (1 - \Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) - \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})})) \kappa_{\mathrm{D,k}} \right) \right] \right\}.$$

The net profit $\pi_t^{(k)} - s_{\mathrm{I,k,t}}^{(\mathrm{k})} - P_t^{(k)} s_{\mathrm{I,-k,t}}^{(\mathrm{k})}$ can thus be thought of as the dividends distributed by the incumbent. This is equivalent to incumbent's value $v_{t+1}^{(k)}$ evolving as a random variable with the following distribution

$$v_{t+1}^{(k)} = \begin{cases} 0 & \text{with probability} \quad \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}) \\ \kappa_{\mathrm{I,k}} v_t^{(k)} & \text{with probability} \quad \Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) \\ \kappa_{\mathrm{D,k}} v_t^{(k)} & \text{otherwise.} \end{cases}$$

The first order conditions of the Bellman equation $(A20)$ with respect to incumbents' R&D expenditure of the home and foreign consumption good imply

$$1 = \left( \Phi_{\mathrm{I}}^{(\mathrm{k})} \right)' (s_{\mathrm{I,t}}^{(\mathrm{k})}) \phi_{\mathrm{I,k}} \frac{s_{\mathrm{I,t}}^{(\mathrm{k})}}{s_{\mathrm{I,k,t}}^{(\mathrm{k})}} (\kappa_{\mathrm{I,k}} - \kappa_{\mathrm{D,k}}) \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \right] \tag{A21}$$

$$P_t^{(k)} = \left( \Phi_{\mathrm{I}}^{(\mathrm{k})} \right)' (s_{\mathrm{I,t}}^{(\mathrm{k})}) (1 - \phi_{\mathrm{I,k}}) \frac{s_{\mathrm{I,t}}^{(\mathrm{k})}}{s_{\mathrm{I,-k,t}}^{(\mathrm{k})}} (\kappa_{\mathrm{I,k}} - \kappa_{\mathrm{D,k}}) \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \right], \tag{A22}$$

where $\left( \Phi_{\mathrm{I}}^{(\mathrm{k})} \right)' (s_{\mathrm{I,t}}^{(\mathrm{k})}) = \eta_{\mathrm{I,k}} \omega_{\mathrm{I,k}} (s_{\mathrm{I,t}}^{(\mathrm{k})})^{\omega_{\mathrm{I,k}} - 1}$.

### A.3.2 Entrants

Entrants maximize the net present value of future profits achieved if they become incumbents

$$\max_{\left\{s_{\mathrm{E,k,t}}^{(\mathrm{k})}, s_{\mathrm{E,-k,t}}^{(\mathrm{k})}\right\}} \left\{ \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}) \kappa_{\mathrm{E,k}} \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \right] - s_{\mathrm{E,k,t}}^{(\mathrm{k})} - P_t^{(k)} s_{\mathrm{E,-k,t}}^{(\mathrm{k})} \right\}.$$

Since potential entrants are atomistic, each potential entrant takes the component $\Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})})$ in $\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}) \equiv s_{\mathrm{E,t}}^{(\mathrm{k})} \Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})})$ as given. This assumption represents a congestion externality and captures the fact that all potential entrants in the intermediate good $i_k$ product line are likely to try similar ideas. Under this assumption, the first order conditions lead to the following two free entry conditions for potential entrants determining their optimal R&D expenditure of the home and foreign consumption good

$$1 = \Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}) \phi_{\mathrm{E,k}} \frac{s_{\mathrm{E,t}}^{(\mathrm{k})}}{s_{\mathrm{E,k,t}}^{(\mathrm{k})}} \kappa_{\mathrm{E,k}} \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \right] \tag{A23}$$

$$P_t^{(k)} = \Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}) (1 - \phi_{\mathrm{E,k}}) \frac{s_{\mathrm{E,t}}^{(\mathrm{k})}}{s_{\mathrm{E,-k,t}}^{(\mathrm{k})}} \kappa_{\mathrm{E,k}} \mathbb{E}_t \left[ \mathbb{M}_{t,t+1}^{(k),loc} v_{t+1}^{(k)} \right]. \tag{A24}$$

R&D expenditures of incumbents and potential entrants are thus chosen such that the marginal benefits of R&D are equal to the marginal costs.

## A.4 Definition of equilibrium

The decentralized equilibrium allocation in this economy consists of (i) time paths of the net output available for consumption and the allocation of final goods $\left\{\mathcal{Y}_t^{(k)}, \mathcal{Y}_{f,t}^{(k)}, \mathcal{Y}_{h,t}^{(k)}\right\}_{k\in\{h,f\},t=0}^{t=\infty}$; (ii) time paths of consumption levels, physical capital and investment $\left\{C_t^{(k)}, K_t^{(k)}, I_t^{(k)}\right\}_{k\in\{h,f\},t=0}^{t=\infty}$; (iii) time paths of quality growth $\left\{Q_{t+1}^{(k)}/Q_t^{(k)}\right\}_{k\in\{h,f\},t=0}^{t=\infty}$; (iv) time paths of incumbents' R&D expenditures $\left\{s_{\mathrm{I},i_k,\mathrm{t}}^{(\mathrm{k})}, s_{\mathrm{I,k},i_k,\mathrm{t}}^{(\mathrm{k})}, s_{\mathrm{I,-k},i_k,\mathrm{t}}^{(\mathrm{k})}\right\}_{k\in\{h,f\},i_k\in[0,1],t=0}^{t=\infty}$; (v) time paths of potential entrants' R&D expenditures $\left\{s_{\mathrm{E},i_k,\mathrm{t}}^{(\mathrm{k})}, s_{\mathrm{E,k},i_k,\mathrm{t}}^{(\mathrm{k})}, s_{\mathrm{E,-k},i_k,\mathrm{t}}^{(\mathrm{k})}\right\}_{k\in\{h,f\},i_k\in[0,1],t=0}^{t=\infty}$; (vi) time paths of prices and quantities for each intermediate good and incumbent value functions $\left\{p_{i_k,\mathrm{t}}^{(\mathrm{k})}, x_{i_k,\mathrm{t}}^{(\mathrm{k})}, v_{i_k,\mathrm{t}}^{(\mathrm{k})}\right\}_{k\in\{h,f\},i_k\in[0,1],t=0}^{t=\infty}$; (vii) time paths of wages and pricing kernels $\left\{\omega_t^{(k)}, M_{t,t+1}^{(k)}, M_{t,t+1}^{(k),loc}\right\}_{k\in\{h,f\},t=0}^{t=\infty}$; and (viii) time paths of the Pareto share $\{S_t\}_{t=0}^{t=\infty}$, such that (a) the representative household maximizes lifetime utility by optimally choosing the allocation of final goods via trading in Arrow-Debreu securities (Equations (1), (A9), (A10) and (A11)); (b) the final good firm maximizes the present value of future dividends by choosing labor, capital investment and the demand for intermediate goods (Equations (8), (A12), (A13) and (A14)) subject to the capital accumulation equation (9); (c) incumbents and potential entrants maximize present values of their future net profits by choosing the monopoly price and R&D expenditures (Equations (10), (15), (16), (A16), (A21), (A22), (A23) and (A24)); (d) the labor market clears (i.e., $L_t^{(k)} \equiv 1$ for $k = h, f$); and (e) the final good market clears (Equations (2) and (17)), given the vector of exogenous state variables $\left\{z_t, a_t^{(k)}\right\}_{k=h,f}$, whose processes are stated in Equations (5) and (6).

## A.5 Asset prices

Using the stochastic discount factors, every payoff stream can be priced. Additionally to the incumbent's value $v_t^{(k)}$, I will price the final good firm's dividend stream using the following Bellman equation

$$V_{d,t}^{(k)} = D_t^{(k)} + \mathbb{E}_t\left[\mathbb{M}_{t,t+1}^{(k),loc} V_{d,t+1}^{(k)}\right].$$

Aggregate dividends in each country are defined by the sum of the profits of the final goods and intermediate goods sector. Therefore, the aggregate stock market value in country $k$ is given by

$$D_{a,t}^{(k)} = D_t^{(k)} + \Pi_t^{(k)} - S_{I,k,t}^{(k)} - P_t^{(k)} S_{I,-k,t}^{(k)}, \qquad V_{a,t}^{(k)} = D_{a,t}^{(k)} + \mathbb{E}_t\left[\mathbb{M}_{t,t+1}^{(k),loc} V_{a,t+1}^{(k)}\right]. \tag{A25}$$

Log returns of the final good firm and the aggregate stock market, and the risk-free rate are defined by

$$r_{d,t+1}^{(k)} = \log\left(\frac{V_{d,t+1}^{(k)}}{V_{d,t}^{(k)} - D_t^{(k)}}\right), \qquad r_{a,t+1}^{(k)} = \log\left(\frac{V_{a,t+1}^{(k)}}{V_{a,t}^{(k)} - D_{a,t}^{(k)}}\right), \qquad r_{f,t}^{(k)} = -\log\left(\mathbb{E}_t\left[\mathbb{M}_{t,t+1}^{(k),loc}\right]\right).$$

The incumbent's log return is defined as

$$r_{I,t+1}^{(k)} = \log\left(\frac{\left(\kappa_{\mathrm{I,k}}\Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) + \kappa_{\mathrm{D,k}}(1 - \Phi_{\mathrm{I}}^{(\mathrm{k})}(s_{\mathrm{I,t}}^{(\mathrm{k})}) - s_{\mathrm{E,t}}^{(\mathrm{k})}\Phi_{\mathrm{E}}^{(\mathrm{k})}(s_{\mathrm{E,t}}^{(\mathrm{k})}))\right) v_{t+1}^{(k)}}{v_t^{(k)} - \left(\pi_t^{(k)} - s_{\mathrm{I,k,t}}^{(\mathrm{k})} - P_t^{(k)} s_{\mathrm{I,-k,t}}^{(\mathrm{k})}\right)}\right).$$

# B Data

This appendix describes the sources for the values in the "Data" columns of the tables. Within-country moments are computed for US annual data which implies that the US economy is the benchmark home economy in my model. Cross-country moments are then computed by using averages for the correlations between the US economy on the one hand and other developed economies on the other side. Data for these countries are used: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Japan, the Netherlands, Norway, New Zealand, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States. The exact data used and comments on missing data follow below. The data sources are

- the World Development Indicators (WDI) supplied by the World Bank (`http://data.worldbank.org/data-catalog/world-development-indicators`),
- Kenneth R. French's webpage (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html`),
- Global Financial Data (GFD) (`https://www.globalfinancialdata.com/index.html`).

**Within-country macroeconomic dynamics (Table 2)**

- Log consumption growth is computed using annual US final consumption expenditure data in constant 2005 US dollars (NE.CON.TETC.KD) for the period 1985–2008 from WDI. Final consumption expenditure is divided by total population (SP.POP.TOTL). The resulting consumption per capita series is used to calculate log growth rates.
- The within-country correlations of annual growth rates are computed using consumption data as explained above. Similarly, log output growth and log investment growth is calculated using data on GDP in purchaser's prices (NY.GDP.MKTP.KD) and gross capital formation as % of GDP (NE.GDI.TOTL.ZS). The latter series is multiplied by GDP before dividing by total population size.
- The annual moments for the innovation probabilities of incumbents and entrants are taken from Bena, Garlappi, and Grüning (2016) for US data between 1985 and 2009.

**Asset pricing moments (Table 3)**

- The moments for the US market excess return are computed using annual data for the first factor in the Fama-French 3-Factor Model available from Kenneth French's webpage for the period 1985–2008.
- The risk-free rate is constructed using annual data from WDI using the lending interest rate and subtracting the risk premium on lending (i.e. $r_f = \text{FR.INR.LEND} - \text{FR.INR.RISK}$). The risk-free rate is thus proxied by the US treasury bill rate. The period used is again 1985–2008.

**International moments (Tables 4 and 5)**

- To calculate the volatility of exchange rate growth, I use quarterly exchange rate data from GFD for the US dollar vs. German Deutschmark exchange rate (which from the first quarter of 1999 is the same as the US dollar vs. euro exchange rate) with ticker USDDEM. The first differences of the log data are taken to obtain the quarterly exchange rate growth. This series is then annualized (and thus identical to using annual data) to compute exchange rate growth volatility. Data between 1985 and 2008 are used. Just using the US dollar vs. euro exchange rate between only 1999 and 2008 or computing the quarterly volatility of exchange rate growth and then multiplying the resulting number by $\sqrt{4}$ gives rise to very similar results.
- The cross-country correlations between aggregate per capita log consumption, output and investment growth are computed using annual data from WDI for all countries mentioned above and for the period 1985–2008. The series are constructed similarly to the US data as explained above for within-country moments in US data. The correlation is computed using US data as the home economy and the other countries as the foreign economy. The average correlation across these pairs is reported in the table.
- The cross-country correlations for aggregate per capita log R&D expenditure growth are computed using annual data from WDI for the period 1997–2008 due to data unavailability prior to 1997. The series for research and development expenditure as % of GDP (GB.XPD.RSDV.GD.ZS) are used and then appropriately converted to R&D expenditure per capita growth as previously outlined. The data for Australia, Denmark, Norway, New Zealand, and Switzerland are incomplete and thus have been dropped from the sample of countries mentioned above. The correlation is computed using US data as the home economy and the other countries as the foreign economy. The average correlation across these pairs is reported in the table.
- The net exports series for the US is computed as exports minus imports of goods and services in constant 2005 US dollars (NE.EXP.GNFS.KD and NE.IMP.GNFS.KD, respectively). Annual WDI data for the period 1985–2008 are used and the correlation between the ratio of US net exports to US GDP and US GDP is computed and reported in the table. Moreover, the mean of the ratio of total exports to GDP and the mean of the ratio of total imports to GDP in the period 1985–2008 for US data are reported in the table.
- The correlation between risk-free rates is computed using annual data from WDI for the period 1990–2008. Data on the real interest rate (FR.INR.RINR) are used for Australia, Belgium, Canada, Japan, the Netherlands, Norway, Singapore, Switzerland, the United Kingdom, and the United States. The correlation is computed using US data as the home economy and the other countries as the foreign economy. The average correlation across these pairs is reported in the table.
- Market returns are computed using annual data from WDI for the period 1990–2008. Data on the annual % change of the S&P Global Equity Indices (CM.MKT.INDX.ZG) are used for all the countries mentioned above. Data prior to 1990 were not available. The correlation of excess market returns by subtracting the risk-free rate above from the market returns is computed using US data as the home economy and the other countries as the foreign economy. The average correlation across these pairs is reported in the table.

# C Sensitivity Analysis: High-Volatility Calibrations

The value of output growth volatility in US data varies quite a lot with respect to the specific data sample used. In the very recent sample 1985–2008 output growth volatility has been extremely low (1.35 percentage points) due to the phenomenon termed the Great Moderation. The results discussed in Section 4 were obtained from models calibrated to this low output growth volatility, and equity premia were not successfully matched by these models. If one includes the time of the Great Depression, output growth volatility is a lot higher (3.56 percentage points as reported by Croce (2014) for the period 1929–2008). However, there is only a small difference in average output growth (1.84 vs. 1.80 percentage points). In this appendix I calibrate the models to match the high volatility of output growth in the long sample and re-do the simulations for the four model specifications to assess the model's performance in a regime with a high output and consumption growth volatility, especially regarding asset pricing dynamics. Note that it is a quite common approach in the macro-finance literature to account for the turbulant times at the beginning of the century. The parameters used for this exercise are reported in Table 7 and the results are reported in Tables 8–10 having the same structure as Tables 2–4.

For single-country macro quantities (Table 8), no surprising conclusions can be drawn. Due to the higher exogenous volatility the innovation probabilities are also more volatile. The mechanism that reduces the correlation of the innovation probabilities of incumbents and entrants in the heterogeneous spillover economy is operating more strongly and thus the correlation is reduced from 0.49 in the calibration with low output growth volatility to 0.34 in the high volatility calibration. Finally, the correlations of growth rates, if not equal to 1 in Table 2, are also slightly reduced now.

As regards the asset pricing implications, the results are also in line with expectations. As shown in Table 9, the risk-free rate is reduced, and the excess returns and their volatilities are now at reasonable levels. Naturally, this is due to the non-linearity coming from the curvature in the utility functions. It is important to note here, however, that the levels of the risk premia in my model are now broadly in line with the majority of the studies on production-economy asset pricing. Just a higher level of exogenous volatility is needed to compensate for the additional consumption smoothing introduced by employing an open economy instead of a closed economy.

Finally, the results for international quantities are also extremely robust to assuming a high exogenous volatility as can be seen from inspecting the numbers in Table 10. Exchange rate growth becomes much more volatile, and the cross-country correlations of macroeconomic growth rates and returns are comparable to the low volatility calibrations.

Taken together, this exercise reveals that the fit of the model can be improved upon by assuming a higher exogenous volatility, which is consistent with the output growth volatility observed in the period including the Great Depression, but that the main results are robust with respect to the level of this volatility.

Table 7: Parameters for high-volatility cases

This table reports the parameters used for the high-volatility calibrations of my model, as discussed in Appendix C. All other parameters not reported in this table are calibrated as in Table 1. The calibration is annual and countries are symmetric in all calibrations.

| Parameter | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|
| | | Homogenous | | Heterogeneous |
| | | Low | High | |
| $\sigma_{a,k}$ | 0.0404 | 0.0394 | 0.0394 | 0.0394 |
| $\sigma_z$ | 0.0375 | 0.0378 | 0.0378 | 0.0378 |
| $\omega_{\mathrm{I,k}}$ | 0.8874 | 0.8309 | 0.8019 | 0.8149 |
| $\omega_{\mathrm{E,k}}$ | 0.8558 | 0.8029 | 0.7753 | 0.8242 |

Table 8: Country-specific macro quantities – high-volatility cases

This table reports the simulated moments for consumption dynamics, innovation dynamics, and within-country correlations of macroeconomic quantities of the model developed in Section 3 and the high-volatility calibrations reported in Table 7. The model is simulated 3,000 times with each sample being 112 years long after where the first 50 years are used as a "burn-in" period leaving 62 years of simulated data for computing the moments. The moments of the model and the data are annual. Details on the sources for the values in the data column are provided in Appendix B with the exception of average output growth, output growth volatility, and consumption growth volatility that have been taken from Croce (2014) for the sample period 1929–2008 using US data.

| | Data | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|---|
| | | | Homogenous | | Heterogeneous |
| | | | Low | High | |
| | | | *Output* | | |
| $\mathbb{E}[\Delta y^{(k)}]$ | 1.80 | 1.80 | 1.80 | 1.80 | 1.80 |
| $\sigma_{\Delta y^{(k)}}$ | 3.56 | 3.56 | 3.56 | 3.55 | 3.55 |
| | | | *Consumption* | | |
| $\sigma_{\Delta c^{(k)}}$ | 2.53 | 3.55 | 3.59 | 3.08 | 3.13 |
| $\sigma(\mathbb{E}_t[\Delta c_{t+1}^{(k)}])$ | — | 0.63 | 0.54 | 0.52 | 0.53 |
| | | | *Innovation process* | | |
| $\mathbb{E}[\Phi_{\mathrm{I}}^{(\mathrm{k})}]$ | 13.86 | 14.10 | 14.04 | 14.01 | 14.11 |
| $\mathbb{E}[\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}]$ | 3.82 | 3.83 | 3.83 | 3.84 | 3.84 |
| $\sigma_{\Phi_{\mathrm{I}}^{(\mathrm{k})}}$ | 2.20 | 1.58 | 1.25 | 1.23 | 1.36 |
| $\sigma_{\hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})}}$ | 0.56 | 0.32 | 0.28 | 0.29 | 0.39 |
| $\mathrm{corr}(\Phi_{\mathrm{I}}^{(\mathrm{k})}, \hat{\Phi}_{\mathrm{E}}^{(\mathrm{k})})$ | 0.47 | 1.00 | 1.00 | 1.00 | 0.34 |
| | | | *Correlation of growth rates* | | |
| $\mathrm{corr}(\Delta c^{(k)}, \Delta y^{(k)})$ | 0.90 | 0.24 | 0.95 | 0.99 | 0.99 |
| $\mathrm{corr}(\Delta c^{(k)}, \Delta s^{(k)})$ | 0.19 | 0.27 | 0.92 | 0.99 | 0.98 |
| $\mathrm{corr}(\Delta y^{(k)}, \Delta s^{(k)})$ | 0.34 | 1.00 | 0.99 | 1.00 | 1.00 |

Table 9: Asset Prices – high-volatility cases

This table reports the simulated moments for the risk-free rate, the excess return on consumption, the excess market return, the excess final good firm return, and the excess return of incumbents of the model developed in Section 3 and the high-volatility calibrations reported in Table 7. The model is simulated 3,000 times with each sample being 112 years long after where the first 50 years are used as a "burn-in" period leaving 62 years of simulated data for computing the moments. The moments of the model and the data are annual. Returns in the model are levered following Boldrin, Christiano, and Fisher (2001). The data values have been taken from Croce (2014) for the sample period 1929–2008 using US data.

| | Data | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|---|
| | | | Homogenous | | Heterogeneous |
| | | | Low | High | |
| | | | *Risk-free rate* | | |
| $\mathbb{E}[r_f^{(k)}]$ | 0.65 | 1.82 | 1.94 | 1.94 | 1.97 |
| $\sigma_{r_f^{(k)}}$ | 1.86 | 0.60 | 0.48 | 0.46 | 0.47 |
| | | | *Return on consumption* | | |
| $\mathbb{E}[r_c^{(k)} - r_f^{(k)}]$ | — | 2.66 | 2.46 | 2.44 | 2.37 |
| $\sigma_{r_c^{(k)} - r_f^{(k)}}$ | — | 0.57 | 5.68 | 5.01 | 5.05 |
| | | | *Market return* | | |
| $\mathbb{E}[r_a^{(k)} - r_f^{(k)}]$ | 4.71 | 2.73 | 2.56 | 2.50 | 2.47 |
| $\sigma_{r_a^{(k)} - r_f^{(k)}}$ | 20.89 | 5.41 | 5.10 | 4.94 | 4.98 |
| | | | *Final good firm return* | | |
| $\mathbb{E}[r_d^{(k)} - r_f^{(k)}]$ | — | 2.90 | 2.51 | 2.58 | 2.51 |
| $\sigma_{r_d^{(k)} - r_f^{(k)}}$ | — | 5.66 | 5.80 | 5.84 | 5.74 |
| | | | *Incumbent firm return* | | |
| $\mathbb{E}[r_I^{(k)} - r_f^{(k)}]$ | — | 0.85 | 0.72 | 0.95 | 0.81 |
| $\sigma_{r_I^{(k)} - r_f^{(k)}}$ | — | 1.57 | 3.44 | 2.90 | 2.13 |

Table 10: International quantities – high-volatility cases

This table reports the simulated moments for exchange rate growth, cross-country correlations of various log macroeconomic growth rates, of innovation probabilities, of the expected consumption growth rates, of the pricing kernels, and of returns, the mean total exports to GDP ratio, the mean total imports to GDP ratio, as well as the correlation of the net exports to GDP ratio with output of the model developed in Section 3 and the high-volatility calibrations reported in Table 7. Total R&D expenditures in country $k$ are defined by $S_t^{(k)} = S_{\mathrm{I,k,t}}^{(\mathrm{k})} + S_{\mathrm{E,k,t}}^{(\mathrm{k})} + P_t^{(k)}(S_{\mathrm{I,-k,t}}^{(\mathrm{k})} + S_{\mathrm{E,-k,t}}^{(\mathrm{k})})$ and its log growth rate is denoted by $\Delta s^{(k)}$. The model is simulated 3,000 times with each sample being 112 years long after where the first 50 years are used as a "burn-in" period leaving 62 years of simulated data for computing the moments. The moments of the model and the data are annual. Returns in the model are levered following Boldrin, Christiano, and Fisher (2001). Details on the sources for the values in the data column are provided in Appendix B.

| | Data | Simple Economy | Technology Spillover | | |
|---|---|---|---|---|---|
| | | | Homogenous | | Heterogeneous |
| | | | Low | High | |
| | | | *Exchange rate growth* | | |
| $\sigma_{\Delta e}$ | 12.70 | 28.53 | 14.18 | 2.23 | 3.75 |
| | | | *Growth rates* | | |
| $\mathrm{corr}(\Delta c^{(h)}, \Delta c^{(f)})$ | 0.34 | 0.11 | 0.00 | 0.31 | 0.24 |
| $\mathrm{corr}(\Delta y^{(h)}, \Delta y^{(f)})$ | 0.46 | 0.46 | 0.46 | 0.47 | 0.47 |
| $\mathrm{corr}(\Delta s^{(h)}, \Delta s^{(f)})$ | 0.28 | 0.53 | -0.07 | 0.40 | 0.28 |
| | | | *Innovation probabilities* | | |
| $\mathrm{corr}(\Phi_{\mathrm{I}}^{(\mathrm{h})}, \Phi_{\mathrm{I}}^{(\mathrm{f})})$ | — | 0.52 | 0.94 | 0.80 | 0.20 |
| $\mathrm{corr}(\hat{\Phi}_{\mathrm{E}}^{(\mathrm{h})}, \hat{\Phi}_{\mathrm{E}}^{(\mathrm{f})})$ | — | 0.52 | 0.94 | 0.80 | 0.25 |
| | | | *Expected consumption growth* | | |
| $\mathrm{corr}(\mathbb{E}_t[\Delta c_{t+1}^{(h)}], \mathbb{E}_t[\Delta c_{t+1}^{(f)}])$ | — | 0.52 | 0.78 | 0.84 | 0.82 |
| | | | *Returns and pricing kernel* | | |
| $\mathrm{corr}(r_I^{(h)} - r_f^{(h)}, r_I^{(f)} - r_f^{(f)})$ | — | 0.49 | -0.54 | -0.24 | 0.09 |
| $\mathrm{corr}(r_d^{(h)} - r_f^{(h)}, r_d^{(f)} - r_f^{(f)})$ | — | 0.44 | 0.44 | 0.45 | 0.46 |
| $\mathrm{corr}(r_a^{(h)} - r_f^{(h)}, r_a^{(f)} - r_f^{(f)})$ | 0.65 | 0.35 | 0.72 | 0.79 | 0.85 |
| $\mathrm{corr}(r_f^{(h)}, r_f^{(f)})$ | 0.27 | 0.28 | 0.79 | 0.79 | 0.76 |
| $\mathrm{corr}(M^{(h)}, M^{(f)})$ | — | 0.84 | 0.96 | 1.00 | 1.00 |
| | | | *Net exports* | | |
| $E[\mathrm{Exports}^{(k)}/Y^{(k)}]$ (%) | 8.81 | 1.01 | 2.38 | 3.74 | 2.08 |
| $E[\mathrm{Imports}^{(k)}/Y^{(k)}]$ (%) | 11.45 | 1.01 | 2.38 | 3.74 | 2.08 |
| $\mathrm{corr}(NX^{(k)}/Y^{(k)}, Y^{(k)})$ | -0.82 | 0.37 | -0.44 | -0.42 | -0.37 |

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