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Interbank network and bank bailouts: Insurance mechanism for non-insured creditors? $\stackrel{\diamond}{\sim}$

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Abstract

This paper presents a new theory that explains why it is beneficial for banks to be highly interconnected and to engage in herding behavior. It shows that these two important causes of systemic risk are interdependent and thus cannot be considered in isolation. The reason is that banks have an incentive to exploit their implicit government guarantees by artificially channeling funds through the interbank market, which leads to high interconnectedness. Moreover, given that banks are highly interconnected, they are incentivized to invest in correlated portfolios to minimize contagion risks and thereby maximize the government subsidy per invested unit of capital.

Keywords: bailout, systemic risk, interconnectedness, herding, interbank network *JEL:* G01, G21, G28

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1. Introduction

The 2008-2009 financial crisis vividly exposed the various ways in which shocks can be propagated through the financial system, which gave systemic risk a high priority on the agenda of policy makers. While systemic risk can be difficult to define and measure, two main channels of systemic risk can be identified. First, financial institutions are highly interconnected, which creates a contagion risk because idiosyncratic shocks might be transmitted through the interbank market. Second, financial institutions tend to undertake similar activities, thereby creating the risk that they fail jointly, which, in turn, may amplify the impact of common shocks.

Considering that concerns such as "too interconnected to fail" and "too correlated to fail" have been widespread, the question arises of why market solutions did not emerge to an extent that would have avoided these concerns? This paper provides a theoretical underpinning for why it is beneficial for banks to be highly interconnected on the interbank market and to engage in herding behavior. First, the papers shows that banks can significantly increase the expected repayment of uninsured creditors by artificially channeling funds through the interbank market before investing in loan portfolios. Thereby, banks exploit their implicit government guarantees to increase the insurance coverage of the creditor funds and are thus better able to attract funds from uninsured creditors. Second, given this incentive to become highly interconnected exists, the paper shows that banks can maximize the government subsidy per invested unit of capital by having correlated portfolios. Taken together, this bank behavior results in a high level of systemic risk caused, on the one hand, by the risk of a joint bank failure due to high correlation on the asset side and, on the other hand, by the risk of contagion due to high interconnectedness on the liability side. So far the literature on systemic risk considers these two types of systemic risk individually, but as shown by this paper, this approach is too narrow, since the two channels are closely interrelated and cannot be considered in isolation.

The mechanism presented in this paper differs from the effects of government bailouts on bank behavior considered in the literature so far. It has been argued that banks might try to increase the probability of a bailout by becoming very large and/or highly interconnected (e.g., Freixas, 1999) and by engaging in herding behavior (e.g., Farhi and Tirole, 2012). We show that, even if we abstract from these well-known moral hazard channels, there is still an incentive for banks to be highly interconnected and to invest in correlated portfolios since this still increases the value of government bailouts for individual banks.

Even if the bailout probability is not increasing with either balance sheet size, inter-

connectedness, or the number of failing banks, we show that the wealth transfer from the government to the private sector increases with the degree of interconnectedness. In a nutshell, instead of directly investing in an asset, non-insured market participants can increase their expected returns by lending funds to banks that, in turn, invest these funds in the asset. Thereby, non-insured market participants can benefit from the bank's government guarantees. If the bank is only partially insured by the government (e.g., through an implicit bailout guarantee) it can increase the insurance coverage of the creditor funds even further by lending the funds to another at least partially insured bank before the funds are finally invested in the asset. Therefore, by artificially channeling funds through the interbank market before investing in an asset, banks can significantly increase the expected repayment to their uninsured creditors. This mechanism works even if we allow the interbank market to exist for a different reason (e.g., liquidity coinsurance). Due to the resulting high interconnectedness, banks lend large amounts among themselves, leading to increased leverage for each bank and high systemic risk. The resulting superfluous interbank liabilities can be observed both, bilaterally between banks as well as in the form of large structural cycles throughout the financial system. For example, after the removal of explicit public guarantees for German Landesbanken had been announced in 2001, these banks started to issue longterm debt and invest the proceeds in bonds of other Landesbanken (Fitch, 2006). In a broader perspective, Heijmans, Pröpper, and van Lelyveld (2008) show the existence of large circular interbank net flows (up to EUR 90 billion) domestically and across the entire TARGET system.

Given that a high degree of interconnectedness creates an additional transfer from the government to the private sector, in a second step, we show that banks can maximize the government subsidy per invested unit of capital by investing in risky, correlated assets. The intuition for this result is as follows. If banks are connected and invest in uncorrelated assets, they are not always successful in the same states. This yields a contagion risk, that is, a bank might fail even though it has a successful investment due to contagion on the interbank market. This in turn lowers the expected residual bank profits, compared to the case where the banks invest in correlated assets because in the latter case the banks always default jointly and contagion cannot occur. In a nutshell, if bank health is already intertwined via a high degree of interconnectedness, banks can maximize residual profits by choosing a correlated asset structure. It is again important to note that this investment behavior does not rely on the conjecture of the existing literature that the individual bailout probability is potentially increasing with the number of failing banks (e.g., Acharya and Yorulmazer, 2007 and Farhi and Tirole, 2012).

Therefore, the mechanism presented in this paper shows that the two types of systemic risk cannot be considered individually, since it provides an incentive for banks to increase both types of systemic risk as the benefits from high interconnectedness are maximized by investing in correlated loan portfolios. Our model thus helps explain why banks were highly interconnected and invested in risky correlated investments (e.g., US subprime loans) in the run-up to the financial crisis.

The rest of the paper is organized as follows. Section 2 provides an overview of the related literature. Using a simple example, Section 3 shows how artificially channeling funds through the interbank market creates an additional wealth transfer from the government to the private sector in case there is a positive bailout probability. Section 4 develops our main model and shows that banks can maximize the value of government bailouts by investing in correlated assets. Section 5 provides two extensions of our main model. First, we introduce asymmetric bank bailout probabilities and analyze the impact on the optimal level of interbank exposure. Second, we show that, given that banks are interconnected, they have an incentive to engage in risk shifting. Section 6 discusses policy implications, while Section 7 concludes.

2. Related literature

Our paper is related to several strands of the theoretical literature. First, it adds to the literature on interbank network formation. Pioneering work in this area has been accomplished by Allen and Gale (2000), who show that banks can coinsure each other through an interbank market against liquidity shocks as long as these shocks are not perfectly correlated. This theme has been taken on by many other papers. Dasgupta (2004) and Babus (2013) determine the optimal level of interconnectedness if interbank deposits can be used by banks to hedge against shocks but at the same time expose them to the risk of contagion. Freixas and Holthausen (2005) analyze the scope for international interbank market integration when cross-border information about banks is less precise than home country information. Here, banks can cope with these shocks by investing in a storage technology or can use the interbank market to channel liquidity. Finally, Zawadowski (2013) analyzes how banks use OTC contracts to hedge their portfolio risks.

Furthermore, our paper relates to the literature on bank bailouts. Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) focus on whether governments have an incentive to bail out banks ex post if they engaged in herding behavior ex ante. Diamond and Rajan (2002) show that bailouts alter available liquidity in the economy and distinguish between well targeted bailouts (which can be beneficial) and poorly targeted ones that can lead to a systemic crisis. Gorton and Huang (2004) argue that there is a potential role for governments to provide liquidity through, for example, bank bailouts to reduce the problem of agents hoarding liquidity inefficiently. In contrast to these studies, we use a constant exogenously given bailout probability to avoid mingling the mechanism presented in this paper with the incentive to become interconnected that results from an increase in the individual bailout probability. Leitner (2005) and David and Lehar (2011) show that interbank linkages can be optimal ex ante because they act as a commitment device to facilitate mutual private sector bailouts. In contrast, we investigate the effect of government bailouts on the incentives of banks to create such liabilities. Finally, Niepmann and Schmidt-Eisenlohr (2013) study the incentives of governments to bail out banks if there are international spillover effects.

Our paper also provides a theoretical underpinning for several empirical findings. Cai, Saunders, and Steffen (2014) show that, in fact, a larger overlap of banks' loan portfolio makes them greater contributors to systemic risk and Iyer and Peydro (2011) find robust evidence for financial contagion due to interbank linkages.

Regarding bank interconnectedness, there is ample evidence that the global banking network has a very high density and a high degree of concentration. Using locational statistics from the Bank for International Settlements (BIS), Minoiu and Reyes (2011) analyze the global banking network and find that, besides a high network density, there exists a positive correlation between network density and the circularity of liabilities (measured by the network's clustering coefficient). For the overnight market in the United Kingdom, Soramäki, Wetherlit, and Zimmermann (2010) find that the net lending/borrowing amounts are much lower than the gross trades, implying many superfluous liabilities in this market. Similar evidence can be found for national interbank markets (Wells, 2004; Mueller, 2006; Arnold, Bech, Beyeler, Glass, and Soramäki, 2006). Furthermore, there is also a very high interconnectedness in other interbank markets besides the traditional interbank lending market (see for example Markose, Giansante, and Shaghaghi, 2012 for the CDS market).

Empirical studies analyzing the extent to which banks engage in herding behavior find, consistent with the predictions of our model, that banks tend to herd more when economic conditions are less favorable, the health of the banking industry is rather weak, and when they are systemically important (Stever and Wilcox, 2007; Liu, 2011; Bonfim and Moshe, 2012).

3. Main idea

We use a very simple framework to illustrate how interbank connections create an additional wealth transfer from the government to the private sector, that is, to banks and creditors. The main model then extends this setup and analyzes how this mechanism affects the banks' investment behavior. For now, we assume that the interbank market consists of a few banks and an uninsured creditor (e.g., mutual fund, bondholder, smaller bank) who is endowed with one unit of capital. There is an investment project available that costs one unit in the first period and generates a return R > 1 in the second period with probability λ and a return of zero otherwise. All parties are risk-neutral.

We develop the intuition of our mechanism in three steps. If the uninsured creditor (C) decides to directly invest in the project (P), the expected return for the private sector is λR . Next, consider the case in which the creditor lends the one unit of capital to a bank (B_A) at t = 0 in exchange for a repayment R_D at t = 1, which the bank then invests in the project. In the second period, the cash flow from the project is realized. If the project is successful, the bank receives an amount R and is able to fully repay its uninsured creditor. If the project fails and the bank is not bailed out, the uninsured creditor receives no repayment. Conversely, if the government bails out the bank (i.e., takes over the bank and settles all its liabilities), the creditor again receives his full repayment (see Fig. 1). Therefore, by lending to the bank, the private sector return increases to $\lambda R + (1 - \lambda)\alpha R_D > \lambda R$, where α is the probability that the bank is bailed out.

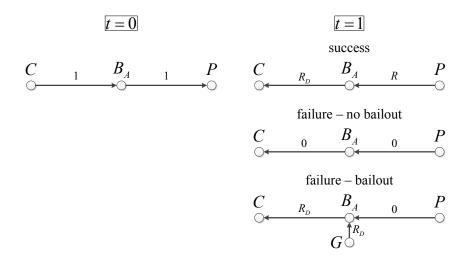


Figure 1: Capital flows without interbank market

In a third step, we allow banks to establish an interbank network at t = 0 by lending

funds in a circular way before investing into the project.¹ To be precise, bank B_A lends the funds it receives from its creditor to bank B_B , which in turn lends it to bank B_C , from which the capital flows back to B_A and is then invested into the project. For entering into an interbank exposure of K, this circular lending procedure has to be repeated K times. For now, we assume that banks B_B and B_C do not have any other investments. We relax this assumption in the main model. If the project is successful, B_A receives the project return R and uses it to settle its liabilities with B_C ². After receiving the payment from B_B , B_A repays its uninsured creditor. If the project fails, bank B_A defaults since it cannot repay its creditors. If the government steps in and bails out bank B_A , both the uninsured creditor of B_A and bank B_C receive their full repayment, implying that all claims are settled in this case. If the government refuses to bail out B_A , B_C defaults as well. Now it depends on whether the government (not necessarily the same one as in the case of B_A , since B_C could be established in another country) bails out B_C . If it does, it takes over B_C and settles its liabilities. Therefore, B_B receives K from B_C and hence B_B can pay back its debt to B_A . However, B_A has total liabilities of $R_D + K$ and is therefore still unable to meet all its obligations. Consequently, the funds B_A received from B_B must be divided among the creditors of B_A , that is, the uninsured creditor of B_A , on the one hand, and B_C , on the other hand.

The common procedure in bankruptcy proceedings is for debt to be paid back on a pro rata basis once a default occurs. Therefore, the uninsured creditor of B_A receives $K \cdot R_D/(R_D + K)$ and bank B_C receives $K \cdot K/(R_D + K)$. Hence, even though the uninsured creditor's own bank fails and is not bailed out, he receives a positive repayment due to the existence of the interbank network. Furthermore, this repayment increases with the level of interbank exposure. Since the government takes over B_C , it receives $K \cdot K/(R_D + K)$ from B_C . However, it has to pay $R_D + K$ to bail out the bank and hence records a loss. The case in which B_C is not bailed out but B_B is can be described analogously. The corresponding

¹For ease of illustration, we assume throughout the paper that interbank exposure stems from interbank lending. Of course, our mechanism also works with every other kind of interbank exposure.

²Throughout the paper we assume that, as soon as there exists a clearing payment vector, the banks use this vector to settle all liabilities in the network. If the sequence of payments is chosen in a less sophisticated manner, banks can still default, even though there is enough liquidity in the system to settle all claims. However, an unsophisticated settlement process would only reinforce our mechanism, since it would increase the value of the government's implicit guarantee.

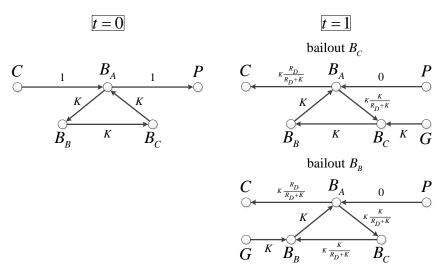


Figure 2: Capital flows with interbank market

cash flows are presented in Fig. 2. The expected private sector return thus becomes

$$\lambda R + (1-\lambda) \left[\alpha R_D + (1-\alpha)\alpha K \frac{R_D}{R_D + K} + (1-\alpha)^2 \alpha K \frac{R_D}{R_D + K} \right] > \lambda R + (1-\lambda)\alpha R_D$$
(1)

Hence, in case there is a positive probability for a government bailout if a bank defaults, the expected private sector return can be increased considerably by first channeling funds through the interbank market and only investing them into the project afterwards. The reason is that, by channeling the funds through several partially insured banks, the insurance coverage can be increased. Furthermore, the interbank exposure increases the banks' liabilities and thus increases the amount of cash governments have to inject to bail out a bank. As a result of the banks' interconnectedness, this extra cash trickles down to other banks in the network, benefiting them and their creditors and thereby increasing the wealth transfer from the government to the private sector even further. Finally, it is important to note that this mechanism works with any other sharing rule during bankruptcy proceedings and becomes even stronger if interbank funding has a lower seniority than the liabilities of uninsured creditors. The reason is that C's share $R_D/(R_D + sK)$ of the bailout funds received from B_B is higher the lower the interbank funding seniority s.

The allocation of the additional private sector wealth, which can be gained by artificially channeling funds through the interbank market, depends on the distribution of the bargaining power between the banking sector and bank creditors. If banks have the bargaining power, creditors will demand a lower interest rate (risk premium) given the existence of an interbank network (the participation constraint of uninsured creditors is already binding for lower values of R_D), which considerably reduces the bank's borrowing cost. This reduction in turn leads to higher bank profits, which can help explain the comparatively high return-on-equity ratios of banks. If, on the other hand, uninsured creditors have the bargaining power, they will increase their expected repayment by increasing R_D until the participation constraint of the bank owners is just binding. Furthermore, creditors will only deposit money in banks that are part of a highly connected interbank network, since the expected repayment in this case is higher than when the bank is not connected to others via an interbank market.

4. The main model

Having described how artificially channeling funds through the interbank market creates an additional wealth transfer from the government to the private sector, we now investigate how this mechanism is interrelated with the banks' investment behavior. We consider an economy that consists of two dates t = 0 and t = 1 and two different regions, A and B. Each region is comprised of a continuum of identical banks. We assume that, due to competition, all banks adopt the same behavior and can thus be described by a representative bank (protected by limited liability). The representative bank in region A (B) is denoted by B_A (B_B) .

| t = 0 | t = 1 | | | | |
|--|------------------------------|--|--|--|--|
| | | | | | |
| Investors provide equity \underline{e} | Cash flows are realized | | | | |
| Banks raise debt capital c | Possibly government bailouts | | | | |
| Banks exchange K | | | | | |
| Banks invest in loan portfolio | | | | | |

Figure 3: Timing of the model

Furthermore, we assume that there exists a risk neutral uninsured creditor and one investor who provides equity financing to the bank in each region. Creditors are denoted C_A and C_B in regions A and B, respectively, and their outside option is to invest in a risk-free asset which yields $R_F = 1$. The contract between the uninsured creditor and the bank takes the form of a standard debt contract; that is, it specifies the interest payment R_D and it cannot be made contingent on either the realization of the investment or the realization of the state of nature. However, the parties can contractually specify the bank's interbank exposure and the structure of its loan portfolio. We abstract from deposit financing, since such funds are explicitly protected by a deposit insurance scheme and thus depositors are not affected by the banks' repayment abilities. Therefore, including deposits in the model does not affect our results qualitatively as long as banks also borrow from non-insured creditors. The timing of our model is depicted in Fig. 3.

Each bank has access to two scalable investment possibilities in two different industries (denoted 1 and 2) at t = 0. One can think of these investment opportunities as portfolios of loans to firms in one of the two industries. More precisely, bank B_A (B_B) can lend to firms in industry A_1 or A_2 (B_1 and B_2). If in equilibrium banks decide to lend to firms in the same industry, that is, they either lend to A_1 and B_1 or to A_2 and B_2 , then the returns of their loan portfolios are assumed to be perfectly correlated ($\rho = 1$). However, if they decide to invest in different industries, we assume that the returns are uncorrelated ($\rho = 0$). Similar to Rochet and Tirole (1996), we assume that both investments are stochastic decreasingreturns-to-scale technologies, which return $I \cdot R$ with probability λ (where $\lambda R > 1$) and yield a return of zero with probability ($1 - \lambda$) at t = 1. The costs for an initial investment of size I are $\psi(I)$ at t = 0, where $\psi(0) = 0$, $\psi'(0) = 1$, and $\psi'' > 0$. Consequently, the decision, in which industry to invest, only affects the correlation of returns, but not their magnitude. This structure allows us to determine how interbank connections influence the banks' incentive to invest, that is, the size and the correlation of their loan portfolios.

In line with Allen and Gale (2000), the banks can establish an interbank market (network) by contracting the exchange of an arbitrary amount of interbank deposits K at t = 0, which have to be repaid at $t = 1.^3$ When increasing interbank deposits, the banks incur transaction costs $\tau(K)$, where $\tau(0) = \tau'(0) = 0$ and $\tau'' > 0$. These costs include a variety of expenses associated with trading funds, such as brokerage and CHIPS or Fedwire transaction fees or the costs of searching for banks with matching liquidity needs. The convex form of $\tau(K)$ represents the increasing marginal costs of searching for trade partners and those resulting from the need to split large interbank transactions into many small ones to work around credit lines (e.g., Never and Wiemers, 2004).

Lastly, we assume that, due to regulatory requirements, banks need at least an equity contribution of \underline{e} . To model equity investors we follow Allen and Gale (2005) and Brusco and Castiglionesi (2007) in that we assume that the equity investor E_A (E_B) in region A(B) is endowed with $e \geq \underline{e}$ units of capital at t = 0 and has no endowment at date t = 1. The investors can use their endowment for either consumption or to buy bank shares. In

³Since the setup is symmetric and banks have equal bargaining power the banks are indifferent with respect to the interest rate on interbank loans, which is in line with the literature (see for example Allen and Gale (2000) and Babus (2013) for interbank deposits and Zawadowski (2013) for OTC contracts). For simplicity, we thus assume that the interbank rate is the risk free rate.

the latter case the investors are entitled to receive dividends at t = 1 (denoted by d_1). Their utility is then given by

$$u(d_0, d_1) = d_0 \lambda R + d_1 \tag{2}$$

Since an investor can obtain a utility of $e\lambda R$ by immediately consuming his initial endowment (consumption at t = 0 is denoted by d_0), he has to earn an expected return of at least λR on the invested capital to give up consumption at t = 0. By investing an amount \underline{e} at t = 0, the equity investor obtains a lifetime utility of $(e - \underline{e})\lambda R + d_1$. Hence, the investors will only buy bank shares if the expected utility from doing so is higher than the utility they would get from immediately consuming their endowment, that is, if

$$(e - \underline{e})\lambda R + E[d_1] \ge e\lambda R \tag{3}$$

holds. This setup leads to the following participation constraint for investors:

$$E[d_1] \ge \underline{e}\lambda R \tag{4}$$

Under the assumption of perfect competition in the banking market (i.e., creditors have all the bargaining power), this constraint will be binding.⁴ Therefore, if a bank wants to invest I and have an interbank exposure of K, it has to raise $c = \psi(I) + \tau(K) - \underline{e} > 0$ from the uninsured creditor. Increasing the equity level above the required minimum cannot be optimal, since equity raises the marginal investment costs and thus decreases the expected creditor payment. In the following, we assume that $\psi'(\underline{e}) < \lambda R - R_F$, which ensures that it is always optimal to raise debt from the uninsured creditor and that it is always optimal for the creditors to lend a positive amount to the banks.

If both investments are successful, the banks are able to settle their interbank claims, repay the uninsured creditors, and pay the investors a positive dividend. If, however, the investment of one or both banks fails, either one or both banks may not be able to meet their liabilities and will consequently default. In case a bank defaults, the regulator has to decide whether to bail out the bank by settling its liabilities, or whether to let the bank fail. We follow the literature on the too-big-to-fail problem in that we assume that, when making this decision, the regulator has to trade off the costs of transferring funds from the public to the private sector, which are given by a fraction $\chi \in [0, 1]$ of the transferred funds

⁴Shifting the bargaining power to the equity investors does not affect bank behavior qualitatively. It only changes the distribution of the benefits from implicit government guarantees.

(e.g., costs that originate from taxation), and the costs of a bank failure (e.g., Freixas, 1999). Since in reality interbank exposures are highly complex, intransparent, and of course involve more than two banks, netting these flows before bailing out a bank is impossible. Therefore, we assume that the regulator can only bail out a bank by settling all its liabilities without prior netting. For the costs of a bank failure, we again follow the literature on the too-bigto-fail problem and assume that the negative externalities of a bank failure increase with the bank's balance sheet size, which in the bad state equals its liabilities, that is, the social costs of a bank failure are given by the fraction β of the banks' liabilities ($R_D + K$). Since these costs are driven by bank specific factors (e.g., availability of outside investors, asset liquidity, lending relationships with the non-financial sector) that are only revealed in times of distress, we assume that at t = 0 only the distribution of β is known, which is a uniform distribution between zero and some upper limit $\overline{\beta} \geq 1$ (i.e., $\beta = \mathcal{U}(0, \overline{\beta})$). Therefore, at t = 1, the regulator decides to bail out a distressed bank if

$$\chi(R_D + K) \le \beta(R_D + K) \tag{5}$$

Hence, the ex-ante probability of a bank in distress being bailed out at t = 1 is equal to $\alpha = 1 - \chi/\overline{\beta}$ and thus increases with the government's ability to raise bailout funds and with the expected negative externalities of a bank failure. It would be reasonable to assume that the negative externalities of a bank failure (and thus $\overline{\beta}$) are also affected by the interconnectedness of the bank (too interconnected to fail) and the number of failing banks (too many to fail). However, to isolate the direct effect that artificially channeling funds through the interbank market has on the uninsured creditors' expected repayment from the indirect effect that comes from the change in the bailout probability, we refrain from the conjecture that the bailout probability increases with these factors. Including the conjecture would only reinforce our results.

Consequently, the payments to the uninsured creditors and investors depend on the performance of the loan portfolio and on whether a bank is bailed out if a default occurs. Due to perfect competition in the banking sector, banks thus seek to maximize the repayment to uninsured creditors by choosing the parameters R_D , ρ , I, and K. Having described the setup, we now return to our main questions in this section: What level of interbank exposure do banks choose, which investment size (and, in turn, which amount of creditor funds) is optimal, and do banks prefer to invest in correlated or uncorrelated assets to optimally exploit implicit bailout guarantees?

All aspects are important to consider, since they all increase systemic risk. On the one

hand, interconnectedness and high leverage lead to systemic risk resulting from spillover effects that are transmitted through the interbank market (even without correlation on the asset side of the banks' balance sheet). On the other hand, even without being interconnected, investment correlation increases systemic risk due to possible joined bank failures. The following analysis investigates the interaction between these sources of systemic risk and determines how interconnectedness influences the banks' investment decision, that is, whether they invest in correlated or uncorrelated loan portfolios. To analyze this issue, we derive the highest expected repayment banks can achieve with an investment correlation of zero and one, respectively. Then we compare the resulting repayments to determine which of the two yields a higher expected return for uninsured creditors.

4.1. Positively correlated investments

Consider first the situation in which bank investments are perfectly positively correlated, that is, $\rho = 1$. In this case there are five different outcomes (depending on the success of the investments and whether the banks are bailed out or not), depicted in Table 1.

| $\rho = 1$ | Probability | L_A | L_B | B_A | B_B | C_A | C_B | E_A | E_B |
|------------|-------------------------------|-------|-------|-------|-------|-------------------------|-------------------------|------------|------------|
| S_1 | λ | | | | N | R_D | R_D | $IR - R_D$ | $IR - R_D$ |
| S_2 | $(1-\lambda)\alpha^2$ | F | F | B | B | R_D | R_D | 0 | 0 |
| S_3 | $(1-\lambda)(1-\alpha)\alpha$ | F | F | B | N | R_D | $K \frac{R_D}{R_D + K}$ | 0 | 0 |
| S_4 | $(1-\lambda)(1-\alpha)\alpha$ | F | F | N | B | $K \frac{R_D}{R_D + K}$ | R_D | 0 | 0 |
| S_5 | $(1-\lambda)(1-\alpha)^2$ | F | F | N | N | 0 | 0 | 0 | 0 |

Table 1: Capital flows for investment correlation of $\rho = 1$

Column 1 presents the five different states, while Column 2 presents the probability of each given state occurring. Columns L_A and L_B show whether the investments of banks B_A and B_B are successful (S) or not (F). Columns B_A and B_B indicate whether banks B_A and B_B are bailed out by the government (B) or not (N). The Columns C_A and C_B show the repayment to uninsured creditors, while Columns E_A and E_B show the dividends the equity holders receive. To understand the cash flows presented in Table 1, first note that if either both investments are successful (S_1) or both banks are bailed out (S_2) , the uninsured creditors of both banks will receive their full repayment. These states only differ with respect to the dividend paid to the investor, since in the case of a bailout the government takes over the bank and thus has the residual claim. Assuming that equity is only partially wiped out after a default would only reinforce our results, since this would relax the participation constraint of the equity investor. If only one bank is bailed out $(S_3$ and $S_4)$, then the creditor of this bank will receive the full repayment whereas the creditor of the other bank will still receive a fraction $K/(R_D + K)$ of his claim R_D , despite the fact that his own bank is not bailed out. Therefore, as shown in the last section, artificially channeling funds through the interbank market increases the insurance coverage of the creditor funds. Since the model is symmetric, it is sufficient to focus on the optimization problem of one of the banks (we relax this assumption in Section 5). Hence, we only analyze the behavior of bank B_A . Due to perfect competition, bank B_A wants to maximize the expected utility of its uninsured creditor C_A . Thus, the optimization problem at t = 0 becomes:

$$\max_{R_D,I,K} U_1 = \lambda R_D + (1-\lambda) \left[\alpha R_D + (1-\alpha) \alpha K \frac{R_D}{R_D + K} \right] - c(I,K,\underline{e})$$
(6)

subject to

$$E[d_1] \ge \underline{e}\lambda R \tag{7}$$

The objective function consists of the following parts: With probability λ the investment of the bank is successful and creditors receive their contractually specified repayment R_D . With probability $(1-\lambda)$ the investment fails. In this case the return of the creditors depends on whether the banks are bailed out or not. Specifically, if bank B_A is bailed out (which happens with probability α), the government repays all liabilities and hence its creditors again receive the full repayment. If, however, the government does not bail out bank B_A , the repayment depends on whether bank B_B is bailed out. If bank B_B is not bailed out either, the repayment is clearly zero. However, if bank B_B is bailed out, the government injects $R_D + K$. This bailout then allows bank B_B to settle all its claims. Therefore, B_A receives K and has to split these proceeds between its uninsured creditor C_A and bank B_B . As described before, in bankruptcy proceedings this splitting is usually done on a pro rata basis, that is, the uninsured creditor of bank B_A receives a share $R_D/(R_D + K)$ of the funds bank B_A received from B_B .

Furthermore, the binding participation constraint of the equity holder implies

$$E[d_1] = \underline{e}\lambda R \Rightarrow \lambda \left[IR - R_D \right] = \underline{e}\lambda R \Rightarrow R_D = (I - \underline{e})R \tag{8}$$

Inserting $R_D = (I - \underline{e})R$ and c into Eq. (6) yields the following maximization problem:

$$\max_{I,K} U_1 = \lambda(I - \underline{e})R + (1 - \lambda) \left[\alpha(I - \underline{e})R + (1 - \alpha)\alpha K \frac{(I - \underline{e})R}{(I - \underline{e})R + K} \right] - \psi(I) - \tau(K) + \underline{e} \quad (9)$$

The first-order conditions lead to:

Proposition 4.1.

- a) If the banks choose correlated investments, there exist unique, interior optimal levels of the investment size, I_1^* , the creditor liabilities, $(I_1^* \underline{e})R$, and the interbank exposure, K_1^* .
- b) Larger interbank exposure K incentivizes banks to have more creditor liabilities and to invest more and vice versa.
- c) Higher equity requirements \underline{e} incentivize banks to lower their interbank exposure and to invest more.

Proof See the Appendix. QED

For high interbank exposures, the governments have to inject more funds in the banking sector in case of a bailout. Hence, if the banks default and only B_B is bailed out, the amount B_A receives from B_B increases with the interbank exposure. If this amount is large, bank B_A is also incentivized to increase its creditor liabilities and invest more funds since then a larger share $(I - \underline{e})R/((I - \underline{e})R + K)$ of the funds bank B_A receives from B_B is paid to the creditor of B_A . Hence, creditor liabilities and interbank exposure are mutually reinforcing each other, implying that banks with a high interbank exposure have an incentive to increase creditor liabilities and vice versa. This mechanism thus results in high bank leverage and interconnectedness. Furthermore, higher equity requirements decrease the face value of creditor liabilities and, in turn, the share of the other bank's bailout funds that is paid to the creditor of B_A . This incentivizes banks to lower their interbank exposure and, due to its higher marginal effect on the creditor's bailout share, they are incentivized to increase the investment size. We derive comparative statics for changes in the bailout probability α in Section 5.1, where the implications of a change in the bailout probability can be analyzed in more detail due to asymmetric bailout probabilities.

Therefore, the highest expected utility for the creditor that can be achieved when choosing correlated investments is:

$$\overline{U_1} = \lambda (I_1^* - \underline{e})R + (1 - \lambda) \left[\alpha (I_1^* - \underline{e})R + \alpha (1 - \alpha) K_1^* \frac{(I_1^* - \underline{e})R}{(I_1^* - \underline{e})R + K_1^*} \right] - \psi (I_1^*) - \tau (K_1^*) - \underline{e}$$
(10)

4.2. Uncorrelated investments

We next turn to the case in which banks decide to invest in different industries, that is, $\rho = 0$. Here, two scenarios must be considered. On the one hand, the interbank exposure can be chosen such that even if one bank's investment is successful but the other bank's investment fails, the first bank will be unable to repay its obligations and hence financial contagion will occur. On the other hand, if the exposure is low enough, a successful bank will stay solvent no matter what happens to the other bank. Let K^t denote the "switching point", that is, the level of interbank exposure where a successful bank will just stay solvent, even if the other bank fails (see the Appendix for the derivation of K^t). The different possibilities for the cash flows are presented in Tables 2 and 3, where the notation is as described before. It is crucial to note that the interest rate R_D differs between the two possibilities, since the participation constraints of the equity investors differ. Table 2 presents the cash flows for $K < K^t$.

| $\rho = 0$ | Probability | L_A | L_B | B_A | B_B | C_A | C_B | E_A | E_B |
|------------|---------------------------------|-------|-------|-------|-------|-----------------------------------|--|-----------------|-----------------|
| S_1 | λ^2 | S | S | N | N | R_D^{nc} | R_D^{nc} | $IR - R_D^{nc}$ | $IR - R_D^{nc}$ |
| S_2 | $(1-\lambda)^2 \alpha^2$ | F | F | B | B | $R_D^{\overline{n}c}$ | $R_{D_{\alpha}}^{\overline{n}c}$ | 0 | 0 |
| S_3 | $(1-\lambda)^2(1-\alpha)\alpha$ | F | F | B | N | R_D^{nc} | $K \frac{R_D^{\overline{nc}}}{R_D^{nc}} K \frac{R_D^{nc}}{R_D^{nc} + K}$ | 0 | 0 |
| S_4 | $(1-\lambda)^2(1-\alpha)\alpha$ | F | F | N | B | $K \frac{R_D^{nc}}{R_D^{nc} + K}$ | R_D^{nc} | 0 | 0 |
| S_5 | $(1-\lambda)^2(1-\alpha)^2$ | F | F | N | N | $\tilde{0}$ | 0 | 0 | 0 |
| S_6 | $\lambda(1-\lambda)lpha$ | S | F | N | B | R_D^{nc} | R_D^{nc} | $IR - R_D^{nc}$ | 0 |
| S_7 | $\lambda(1-\lambda)lpha$ | F | S | B | N | $R_D^{\overline{n}c}$ | $R_{Dnc}^{\overline{n}c}$ | 0 | $IR-R_D^{nc}$ |
| S_8 | $\lambda(1-\lambda)(1-\alpha)$ | S | F | N | N | R_D^{nc} | $K \frac{R_D^{nc}}{R_D^{nc}} K \frac{R_D^{nc}}{R_D^{nc} + K}$ | X_0 | 0 |
| S_9 | $\lambda(1-\lambda)(1-\alpha)$ | F | S | N | N | $K \frac{R_D^{nc}}{R_D^{nc} + K}$ | R_D^{nc} | 0 | X_0 |

Table 2: Outcomes for $K < K^t$, where $X_0 = IR - R_D^{nc} - K \frac{R_D^{nc}}{R_D^{nc} + K}$ - No contagion

States $S_1 - S_5$ parallel the respective outcomes in Table 1. Things differ from the results of Table 1 if only one investment fails, depending on whether the successful bank stays solvent (no contagion; see Table 2) or also becomes insolvent (see Table 3). If the interbank exposure is low enough ($K < K^t$) such that there is no contagion, then the successful bank can always fully repay its uninsured creditor, whereas the creditor of the unsuccessful bank will only receive the full amount if this bank is bailed out (S_6 and S_7 in Table 2). If the unsuccessful bank is not bailed out, its creditor will get just a fraction of his repayment (S_8 and S_9 in Table 2). If, on the other hand, the interbank exposure is higher than the threshold K^t , the successful bank will not be able to settle its interbank liabilities and, on top of that, will be unable to fully repay its creditor. Depending on which bank (if any) is bailed out, the creditors of both the successful and the failed bank receive either their full repayment or just a fraction ($S_6 - S_{11}$ in Table 3). In a next step, we compare the expected repayments of the uninsured creditor in these two scenarios, that is, $K < K^t$ and $K \ge K^t$.

The interest rate R_D^{nc} (no contagion) follows from the binding participation constraint of

| $\rho = 0$ | Probability | L_A | L_B | B_A | B_B | C_A | C_B | E_A | E_B |
|------------|--------------------------------------|-------|-------|-------|-------|---|---|--------------|--------------|
| S_1 | λ^2 | S | S | N | N | R_D^c | R_D^c | $IR - R_D^c$ | $IR - R_D^c$ |
| S_2 | $(1-\lambda)^2 \alpha^2$ | F | F | B | B | $R_D^{\widetilde{c}}$ | $R_{D_a}^{c}$ | 0 | 0 |
| S_3 | $(1-\lambda)^2(1-\alpha)\alpha$ | F | F | B | N | R_D^c | $K \frac{R_D^c}{\frac{R_D^c}{R_D^c + K}}$ | 0 | 0 |
| S_4 | $(1-\lambda)^2(1-\alpha)\alpha$ | F | F | N | B | $K \frac{R_D^c}{R_D^c + K}$ | R_D^c | 0 | 0 |
| S_5 | $(1-\lambda)^2(1-\alpha)^2$ | F | F | N | N | 0 | 0 | 0 | 0 |
| S_6 | $\lambda(1-\lambda)\alpha$ | S | F | N | B | R_D^c | $R^c_{\underline{D}_c}$ | $IR - R_D^c$ | 0 |
| S_7 | $\lambda(1-\lambda)(1-\alpha)\alpha$ | S | F | B | N | R_D^c | $K \frac{R_D^c}{\frac{R_D^c}{R_D^c + K}}$ | 0 | 0 |
| S_8 | $\lambda(1-\lambda)(1-\alpha)^2$ | S | F | N | N | $IR \frac{R_D^c + K}{R_D^c + 2K}$ | $IR \frac{K}{R_D^c + 2K}$ | 0 | 0 |
| S_9 | $\lambda(1-\lambda)lpha$ | F | S | B | N | | $\tilde{R_D^c}$ | 0 | $IR - R_D^c$ |
| S_{10} | $\lambda(1-\lambda)(1-\alpha)\alpha$ | F | S | N | B | $K \frac{\bar{R_D^c}}{R_D^c} K \frac{\bar{R_D^c}}{R_D^c + K}$ | R_D^c | 0 | 0 |
| S_{11} | $\lambda(1-\lambda)(1-\alpha)^2$ | F | S | N | N | $IR \frac{K}{R_D^c + 2K}$ | $IR \frac{R_D^c + K}{R_D^c + 2K}$ | 0 | 0 |

Table 3: Outcomes for $K \geq K^t$ - Contagion

the equity holder. If $K < K^t$, Constraint (7) implies that

$$\lambda^2 \left(IR - R_D^{nc} \right) + \lambda (1 - \lambda) \left[\alpha \left(IR - R_D^{nc} \right) + (1 - \alpha) \left(IR - R_D^{nc} - K \frac{R_D^{nc}}{R_D^{nc} + K} \right) \right] = \underline{e} \lambda R \quad (11)$$

Therefore, if the investment correlation is zero and $K < K^t$, the overall utility of the uninsured creditors is

$$U_0(K < K^t) = [\lambda + (1 - \lambda)\alpha]R_D^{nc} + (1 - \lambda)(1 - \alpha)[\lambda + (1 - \lambda)\alpha]K\frac{R_D^{nc}}{R_D^{nc} + K} - \psi(I) - \tau(K) + \underline{e}$$
(12)

Rearranging Eq. (11) to

$$(1-\lambda)(1-\alpha)[\lambda+(1-\lambda)\alpha]K\frac{R_D^{nc}}{R_D^{nc}+K} = [\lambda+(1-\lambda)\alpha][(I-\underline{e})R-R_D^{nc}]$$
(13)

and plugging this expression into Eq. (12) yields

$$U_0(K < K^t) = [\lambda + (1 - \lambda)\alpha](I - \underline{e})R - \psi(I) - \tau(K) + \underline{e}$$
(14)

Eq. (14) implies that, for $K < K^t$, the expected repayment of the creditor C_A does not depend on the interbank exposure. The reason is that, due to the participation constraint of the equity investor E_A , his loss in dividends caused by the payment to the failed bank B_B (S_8 in Table 2) has to be offset by a reduction in the creditor's interest rate, R_D^{nc} . The resulting decrease in the repayment to C_A in the success states S_1 , S_6 , and S_8 is exactly offset by the additional payment in S_9 that arises due to the interbank exposure:

$$\left(\lambda^2 + \lambda(1-\lambda)\alpha + \lambda(1-\lambda)(1-\alpha)\right)\left(R_D - R_D^{nc}\right) = \lambda(1-\lambda)(1-\alpha)K\frac{R_D^{nc}}{R_D^{nc} + K}$$
(15)

Furthermore, the reduction in the interest rate has the disadvantage that it lowers the value of the implicit government guarantees, since the face value of the creditor's liabilities is decreased. This reduces the expected repayment to C_A in states S_2 , S_3 , and S_7 , when his own bank is bailed out. However, this loss is exactly offset by the additional payment in state S_4 that arises from the interbank exposure and the bailout of B_B . Due to the symmetry of our model, the same holds for investor E_B and creditor C_B . Taken together, the expected repayment to the creditors becomes independent from the interbank exposure. Hence, due to transaction costs, it is always optimal to choose $K_0^{nc} = 0$ when $K < K^t$.

Furthermore, the Appendix shows that there exists a unique and interior maximum I_0^{nc} for the investment size. Hence, for $K < K^t$, the highest expected utility for the non-insured creditor is

$$\overline{U_0}(K < K^t) = [\lambda + (1 - \lambda)\alpha](I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e}$$
(16)

Next, we derive the interest rate R_D^c for the contagion case. When $K \ge K^t$, the equity investors do not receive a dividend payment as soon as one of the banks fails and is not bailed out and we thus obtain for the interest rate

$$\lambda^{2} (IR - R_{D}^{c}) + \lambda (1 - \lambda) \alpha (IR - R_{D}^{c}) \geq \underline{e} \lambda R$$
$$\Rightarrow R_{D}^{c} = \left[I - \frac{\underline{e}}{\lambda + (1 - \lambda)\alpha} \right] R < R$$
(17)

Therefore, as soon as $K \ge K^t$, a change in K does not alter the dividend payment to E_A and hence no longer changes the interest rate R_D^c . Compared to the no contagion case, where investor E_A receives at least a partial repayment if B_A is successful and B_B defaults and is not bailed out (S_8 in Table 2), E_A receives nothing in this situation in the contagion case. Hence, the interest rate R_D^c is even lower than R_D^{nc} . In the contagion case, the overall utility of the uninsured creditors will be:

$$U_{0}(K \geq K^{t}) = \left[(1+\lambda)\alpha + \lambda^{2}(1-2\alpha) - (1-\lambda)\lambda\alpha^{2} \right] R_{D}^{c} + \lambda(1-\lambda)(1-\alpha)^{2}IR + (1-\lambda)(1-\alpha)\alpha K \frac{R_{D}^{c}}{R_{D}^{c} + K} - \psi(I) - \tau(K) + \underline{e}$$
(18)
$$= \left[\lambda + (1-\lambda)\alpha \right] (I-\underline{e})R + (1-\lambda)(1-\alpha)\alpha K \frac{R_{D}^{c}}{R_{D}^{c} + K} - \alpha \underline{e}R \frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} - \psi(I) - \tau(K) + \underline{e}$$
(19)

Since $R_D^c < R_D^{nc}$, the face value of the creditors' liabilities is even further reduced and thus the value of their own banks' bailout is smaller than in the no contagion case (third term in Eq. (19)). However, a bailout of the other bank becomes more valuable since higher interbank exposure implies that a higher fraction of the other bank's bailout funds is transferred to the creditor (second term in Eq. (19)).

Again, the Appendix shows that also for $K \ge K^t$ there exists a unique and interior maximum I_0^c for the investment size and a unique optimal level of interbank exposure K_0^c . Hence, for $K \ge K^t$, the highest expected utility for the non-insured creditor that can be achieved is

$$\overline{U_0}(K \ge K^t) = [\lambda + (1-\lambda)\alpha] (I_0^c - \underline{e})R + (1-\lambda)(1-\alpha)\alpha K_0^c \frac{\left(I_0^c - \frac{\underline{e}}{\lambda + (1-\lambda)\alpha}\right)R}{\left(I_0^c - \frac{\underline{e}}{\lambda + (1-\lambda)\alpha}\right)R + K_0^c} - \alpha \underline{e}R \frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} - \psi(I_0^c) - \tau(K_0^c) + \underline{e}$$

$$(20)$$

To determine whether or not banks choose a level of interbank exposure that leads to contagion, we now compare the utility of creditors for the different levels of interbank deposits from Eq. (16) and Eq. (20). In the Appendix, we show that choosing (I_0^c, K_0^c) dominates the alternative of having no interbank exposure and choosing I_0^{nc} if the expected additional gain from the interbank exposure, due to the higher value of a bailout of the other bank, outweighs the loss in value of the own bank's bailout due to the contagion risk and the resulting lower interest rate $R_D^c < R_D^{nc} < (I - \underline{e})R$. Otherwise, the banks do not enter into interbank connections and choose an investment size of I_0^{nc} . These findings can be summarized in the following proposition.

Proposition 4.2. If banks invest in uncorrelated portfolios (given a positive bailout probability), they choose an interbank exposure of K_0^c and an investment size of I_0^c if $\overline{U_0}(K \ge K^t) > \overline{U_0}(K < K^t)$ and $K_0^c \ge K^t$. Otherwise, the banks choose to have no interbank exposure and an investment size of I_0^{nc} .

Proof See the Appendix. QED

4.3. Comparison of correlated and uncorrelated investments

What remains is to show under which correlation structure uninsured creditors receive a higher expected repayment. In the Appendix, we formally prove that $\overline{U_1} > \overline{U_0}$ always holds, implying that banks will always choose perfectly correlated investments. This main finding can be summarized in the following proposition.

Proposition 4.3. If there is a nonzero bailout probability, banks have an incentive to increase their interbank exposure to $K_1^* > 0$ and choose the investment size I_1^* and the creditor liabilities $(I_1^* - \underline{e})R$. Moreover, it is optimal for them to invest in correlated assets.

Proof See the Appendix. QED

To understand why this result holds, recall that the investment correlation only alters the expected bailout funds and not the investment returns. Hence, banks choose the investment correlation that maximizes the value of implicit government guarantees and, in turn, the total expected inflows into the private sector. If banks are connected through interbank liabilities and decide to invest in uncorrelated assets, they are not always successful in the same states. This yields a contagion risk, that is, a bank might fail even though it has a successful investment due to contagion on the interbank market. This in turn lowers the expected dividend payments to equity investors, compared to the case where the banks invest in correlated assets because in the latter case the banks always default jointly and contagion cannot occur. Hence, in case banks invest in uncorrelated portfolios, the creditors' interest rates have to be lowered, which, in turn, decreases the face value of debt and thus the value of the implicit government guarantees. This relation incentivizes banks to invest in correlated portfolios. Therefore, in our model, the incentive to herd results from bank interconnectedness and not from the time-inconsistency in bank bailout policies as in Acharya and Yorulmazer (2007) and Farhi and Tirole (2012). If banks were not interconnected, there would be no incentive for banks to invest in correlated portfolios in our model.

In this section, we demonstrate that banks always have an incentive to artificially channel funds through the interbank market to increase the value of government guarantees. The benefit of being connected to other banks can be further enhanced by choosing correlated assets, which gives banks an incentive to herd. We can thus provide an additional explanation for the herding behavior of banks beyond the rationale that herding might increase the banks' bailout probability. Furthermore, interbank exposure incentivizes banks to invest more and, in turn, to take on more creditor liabilities and vice versa. This positive link between interbank exposure and creditor liabilities might help explain the increase in the density of the interbank network in the last decades. According to our model, this might be a result of the increase in bank leverage and size in this period.

Hence, the mechanism described in this paper leads to an overall increase in systemic risk that results from interconnectedness, higher leverage, and herding behavior. However, the incentive of being highly interconnected can be mitigated by raising the minimum equity requirements. Since banks always choose correlated investment, given interbank connections and a positive bailout probability, we restrict our analysis to this case in the next section.

5. Extensions

This section provides two extensions to our main model. In the first part, we introduce asymmetric bailout probabilities and in the second part, we analyze the effect of interbank connections on risk shifting incentives.

5.1. Asymmetric bailout probabilities

In this section, we analyze the implications of banks having different bailout probabilities on their incentive to be interconnected. Without loss of generality, we now assume that bank $B_A(B_B)$ has the probability $\alpha_A(\alpha_B)$ of being bailed out in case of a default with $\alpha_A = \alpha + \delta$ and $\alpha_B = \alpha - \delta$. The parameter $\delta > 0$ thus captures the difference in bailout probabilities. This difference arises if the banks are established in different countries and the respective governments differ with respect to their ability to raise bailout funds (i.e., different χ 's) or if the banks differ with respect to the expected negative externalities they cause in case of a default (i.e., different $\overline{\beta}$'s).

For simplicity, we assume from now on that the investment is not scalable and instead needs an initial amount of one unit of capital (with $\psi(1) = 1$), such that $c = 1 + \tau(K) - \underline{e}$. Thus, the optimization problems at t = 0 now become:

$$\max_{R_D,K} U_A = \lambda R_D + (1-\lambda) \left[\alpha_A R_D + (1-\alpha_A) \alpha_B K \frac{R_D}{R_D + K} \right] - c(K,\underline{e})$$
(21)

$$\max_{R_D,K} U_B = \lambda R_D + (1-\lambda) \left[\alpha_B R_D + (1-\alpha_B) \alpha_A K \frac{R_D}{R_D + K} \right] - c(K,\underline{e})$$
(22)

subject to the participation constraints of the equity investors

$$E[d_1] \ge \underline{e}\lambda R \Rightarrow \lambda(R - R_D) = \underline{e}\lambda R \tag{23}$$

and the interest rate thus becomes $R_D = (1 - \underline{e})R$. Therefore, the desired interbank exposure of bank B_B , K_B^{α} , implied by the first-order condition:

$$\frac{\partial U_B}{\partial K} = (1-\lambda)(1-(\alpha-\delta))(\alpha+\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R+K_B^{\alpha})^2} - \tau'(K_B^{\alpha}) = 0$$
(24)

is higher than the desired exposure of B_A , K_A^{α} :

$$\frac{\partial U_A}{\partial K} = (1-\lambda)(1-(\alpha+\delta))(\alpha-\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R+K_A^{\alpha})^2} - \tau'(K_A^{\alpha}) = 0$$
(25)

The reason is that interbank exposure increases the value of the other bank's bailout, in case the creditor's own bank is not bailed out. Given that the creditor's own bank's bailout probability is very low and the other bank's bailout probability is very high, the likelihood of this case occurring is very high and, in turn, so is the additional value of having interbank exposure. Therefore, the desired interbank exposure increases with the other bank's bailout probability and decreases with the own bank's bailout probability:

$$\frac{\partial K_A^{\alpha}}{\partial \delta} = -\frac{(1-\lambda)(1-2\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R+K_A^{\alpha})^2}}{(1-\lambda)(1-(\alpha+\delta))(\alpha-\delta)R^2 \frac{2(1-\underline{e})^2}{((1-\underline{e})R+K_A^{\alpha})^3} + \tau''(K_A^{\alpha})} < 0$$
(26)

$$\frac{\partial K_B^{\alpha}}{\partial \delta} = \frac{(1-\lambda)(1+2\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R+K_B^{\alpha})^2}}{(1-\lambda)(1-(\alpha+\delta))(\alpha-\delta)R^2 \frac{2(1-\underline{e})^2}{((1-\underline{e})R+K_B^{\alpha})^3} + \tau''(K_B^{\alpha})} > 0$$
(27)

To incentivize B_A to enter into higher interbank liabilities, B_B can compensate B_A for the additional costs by paying the amount η :

$$\eta = \tau(K) - \tau(K_A^{\alpha}) - (1-\lambda)(1-(\alpha+\delta))(\alpha-\delta) \left[K \frac{(1-\underline{e})R}{(1-\underline{e})R+K} - K_A^{\alpha} \frac{(1-\underline{e})R}{(1-\underline{e})R+K_A^{\alpha}} \right]$$
(28)

The right hand side of Eq. (28) represents the additional costs for B_A of having an interbank exposure of $K > K_A^{\alpha}$ instead of K_A^{α} , which are given by the additional transaction costs minus the additional benefit of a higher interbank exposure.

Hence, incorporating the additional payment, the optimization problem of bank B_B be-

comes:

$$\max_{K} U_{B} = \lambda (1 - \underline{e})R + (1 - \lambda)(\alpha - \delta)(1 - \underline{e})R + (1 - \lambda)(1 - (\alpha - \delta))(\alpha + \delta)K \frac{(1 - \underline{e})R}{(1 - \underline{e})R + K} - \eta - \tau(K) - (1 - \underline{e})$$
(29)

This optimization problem yields the following proposition.

Proposition 5.1. If bank B_A has a higher probability of being bailed out than B_B then

- a) B_B desires more interbank exposure than B_A , where the banks' desired level increases with the other bank's bailout probability and decreases with the bank's own bailout probability.
- b) bank B_B incentivizes B_A to increase the interbank exposure to $K^{\alpha} = K^{\alpha}_A + \Delta^{\alpha}$ by paying the amount η^{α} , where Δ^{α} is the interbank exposure that B_B wants to have in addition to K^{α}_A .
- c) the desired additional interbank exposure Δ^{α} and the respective compensation payment η^{α} both increase with the difference between the bailout probabilities δ .

Proof See the Appendix. QED

Hence, by paying an additional fee and thereby incentivizing other banks to enter into higher interbank exposure, banks with lower bailout probabilities can utilize higher bailout probabilities of other banks. The simplest way to implement such a compensation fee is an interest payment on interbank deposits/loans, such that banks with lower bailout probabilities (i.e., smaller, non-systemic banks in poorer countries) pay higher interbank interest rates than banks with higher bailout probabilities (i.e., larger, systemic banks in richer countries). This interest rate gap then increases with the difference of the banks' bailout probabilities, implying that implicit bailout guarantees are priced into a bank's interbank liabilities.

5.2. The interbank network and risk shifting

In the following, we show that the incentive to engage in risk-shifting increases with K. To model the riskiness of the investment decision, we consider two assets: a risk-free storage technology that transfers one unit of wealth today into one unit of wealth tomorrow, and a risky negative NPV investment that generates a return $R_R > 1$ with probability $\lambda_R < 1$ where $\lambda_R R_R < 1$.

For ease of illustration, we neglect transaction costs and thus $c = 1 - \underline{e}$. Given that there is no bailout possibility, the bank can offer creditors either a repayment of c (if it invests

in the safe asset) or R_D^R with probability λ_R if it invests in the risky negative NPV asset. The promised repayment R_D^R results from the binding participation constraint of the equity holder. We assume that the outside option of the equity holder is now given by the risk-free storage technology. Therefore, the participation constraint for an investment in the risky asset becomes

$$E[d_1] = \underline{e} \Rightarrow \lambda_R \left[R_R - R_D^R \right] = \underline{e} \Rightarrow R_D^R = R_R - \frac{\underline{e}}{\lambda_R}$$
(30)

In the following, we assume that R_R is at least high enough such that the uninsured creditor receives a return larger than one in the success state (i.e., $R_D^R > c$). We first consider a scenario without a bailout possibility and no interbank network. Here, it can be easily seen that the expected repayment of the creditors is higher if the bank invests in the safe asset since

$$c > \lambda_R R_D^R = \lambda_R R_R - \underline{e} \tag{31}$$

Hence, without the possibility of a bailout, banks will always choose the safe investment.

Next, we consider the case in which the bank has a positive probability of being bailed out by the government but still no connections to other banks. Now it can become profitable to switch to the negative NPV investment if the bailout probability is high enough. More precisely, a bank will switch to the negative NPV investment if the expected repayment of creditors for this investment is higher than for the safe repayment c, that is,

$$\lambda_R R_D^R + (1 - \lambda_R) \alpha R_D^R > c \tag{32}$$

Besides the state of nature in which the investment is successful, creditors now also receive the higher return R_D^R when the bank is bailed out by the government. The critical α , that is, the bailout probability where the bank is indifferent between the two investments is given by

$$\alpha^* = \frac{c - \lambda_R R_D^R}{(1 - \lambda_R) R_D^R} < 1 \tag{33}$$

which is true since $R_D^R > c$. Hence, for $\alpha > \alpha^*$ it is always profitable to switch to the negative NPV investment.

Now, we again allow the bank to exchange funds with the bank in the other region. Whether banks will switch to the negative NPV investment again depends on α . Whenever the expected repayment of the uninsured creditor from investing in the negative NPV investment opportunity is higher, banks will shift away from the risk-free investment. Formally, the following condition must be satisfied:

$$\lambda_R R_D^R + (1 - \lambda_R) \left[\alpha R_D^R + \alpha (1 - \alpha) K \frac{R_D^R}{R_D^R + K} \right] > c \tag{34}$$

Rearranging this equation yields

$$\alpha^{**} = \frac{c - \lambda_R R_D^R}{(1 - \lambda_R) R_D^R \left(1 + (1 - \alpha^{**}) \frac{K}{R_D^R + K} \right)} < \alpha^*$$
(35)

Hence, the critical α^{**} is strictly smaller if a bank is connected (i.e., K > 0) to another bank on the interbank market, that is, $\alpha^* > \alpha^{**}$. Hence, the critical threshold α is lower once a bank enters into connections with other banks. Put differently, a lower bailout probability is sufficient to make the bank switch to the negative NPV investment. The positive bailout probability can turn a negative NPV investment into a positive NPV investment from the perspective of the uninsured creditors since they will receive the high repayment with a higher probability. This effect is reinforced once the bank is connected to another bank if this other bank has a positive bailout probability as well. Our results are summarized in the following proposition.

Proposition 5.2. The more interconnected a bank becomes, the lower the critical bailout probability that makes it profitable for the bank to engage in risk shifting, that is, to switch to risky negative NPV investments.

Risk shifting thus becomes more attractive for banks since the downside risk is limited by two factors. First, the downside risk is limited by the positive bailout probability because creditors receive their full repayment after the bank is bailed out. Second, the interbank connection further reduces the downside risk by increasing the insurance coverage, since it adds an additional state in which the creditor receives a positive repayment. These two effects turn a negative NPV investment into a positive NPV investment (from the perspective of the uninsured creditors).

6. Discussion and policy implications

This paper shows that banks have an incentive to create a high degree of interconnectedness by artificially channeling funds through the interbank market. This holds true even if we allow the interbank market to exist for other reasons than simply exploiting implicit government guarantees (e.g., liquidity co-insurance, see the online appendix for further details). Given the high degree of interconnectedness, banks are incentivized to engage in herding behavior. Several policy implications can be derived from our results. Generally, most of these policy implications aim at reducing the banks' incentive to create excessive interbank exposures, because this, in turn, also decreases the incentive to engage in herding behavior, which helps to reduce systemic risk.

First of all, as shown in Section 4.1, raising the the minimum equity requirement reduces the banks' incentive to by highly interconnected. Similarly, one can think about increasing the risk weights for interbank liabilities under the Basel accord and thereby increase the amount of equity necessary to satisfy minimum capital requirements. If interbank liabilities get a higher risk weight, banks are incentivized to reduce their excessive lending activities and hence reduce systemic risk in the interbank market. However, banks could potentially counter this regulatory measure by engaging in cross equity holdings in addition to interbank debt liabilities. By investing equity in a cyclical way, banks can reach any desired equity ratio without being dependent on outside investors.

Second, as long as the interbank liabilities only exist bilaterally between two banks, regulators could potentially net these exposures before deciding upon bank bailouts. In reality, however, the interbank flows of course involve more than two banks, implying that regulators would need to know the entire network topology to be able to cancel out superfluous flows. Since interbank exposures are highly complex, intransparent, and often involve banks in different countries, canceling out these flows before bailing out a bank is impossible. However, the creation of a centralized clearing house for interbank activities can potentially eliminate the perverse incentives described in the paper. If all interbank activities are channeled through a clearing house, the regulator knows the complete interbank network topology and is thus able to cancel matching interbank deposits of the various banks. However, this approach would require a global clearing house and thus a collaboration of all involved bank regulators.

Furthermore, one of the key topics in the current discussion in the European Union is the introduction of a financial transaction tax to limit speculative trading activities. Since interconnectedness can not only be created via interbank loans, but also by using derivatives like for example CDS, such a tax could be a potential mechanism to reduce the high interconnectedness by adding additional transaction costs and therefore mitigate the systemic risk problems that result from investing in highly correlated low-quality assets.

A fourth possibility to mitigate the incentives to artificially channel funds through the interbank market would be the introduction of the widely discussed bank levy. Charging banks with large balance sheets (that can very well result from high interconnectedness) higher taxes for their systemic risk can potentially mitigate the incentive to create these large interbank flows in the first place.

From the results derived in Section 5.1., it follows that if a government lowers bailout expectations (e.g., by credibly committing to a no-bailout policy), this actually leads to higher interconnectedness when other governments do not act in a similar way. The reason is that the incentive to be interconnected increases if a bank's own bailout probability is lowered. Hence, if the bailout probability of banks is reduced in only one country, these banks then want to have more interbank exposure to banks in other countries to benefit from their bailouts. Hence, reducing the interconnectedness on the interbank market by lowering bailout exceptions can only be realized when governments use a coordinated approach and expectations are lowered in all countries simultaneously.

Finally, to jointly aim at the banks' incentive to be highly interconnected as well as engage in herding behavior, regulators could introduce a systemic-risk-based capital requirement based on a financial firm's marginal contribution to systemic risk. For a comprehensive overview about possibilities to introduce such a capital surcharge see IMF (2010).

7. Conclusion

This paper sheds light on the puzzle why banks have an incentive to be highly interconnected on the interbank market and to invest in correlated portfolios. First, we show that banks have an incentive to artificially channel funds through the interbank market, because this behavior increases the value of implicit government bailout guarantees. Such guarantees shift the probability distribution of the returns of risky investments and thereby increase the expected repayment of uninsured creditors. This behavior considerably increases systemic risk and leverage without altering the aggregate relation with the real economy. Second, given that bank health is already intertwined via a high degree of interconnectedness, the paper shows that banks can maximize residual profits by choosing risky assets and a correlated portfolio structure since high asset correlation minimizes contagion risks. Hence, the presented mechanism leads to an overall increase in systemic risk that results from interconnectedness, risky assets, as well as herding behavior. Therefore, our model helps explain why banks invested in risky correlated investments (e.g., US subprime loans) in the run-up to the financial crisis.

Appendix

A.1. Proof of Proposition 4.1

The first-order condition with respect to I for an optimum is:

$$\frac{\partial U_1}{\partial I} = \lambda R + (1 - \lambda) \left[\alpha R + (1 - \alpha) \alpha R \frac{K^2}{((I - \underline{e})R + K)^2} \right] - \psi'(I) = 0$$
(36)

The corresponding second-order condition is:

$$SOC_{I} \equiv \frac{\partial^{2} U_{1}}{\partial^{2} I} = -(1-\lambda)(1-\alpha)\alpha R^{2} \frac{2K^{2}}{((I_{1}^{*}-\underline{e})R+K)^{3}} - \psi''(I) < 0$$
(37)

which is satisfied since $\psi'' > 0$. Treating K as exogenous and using the implicit function theorem yields for the partial derivative of I with respect to interbank exposure K:

$$\frac{\partial I}{\partial K} = \frac{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I-\underline{e})K}{((I-\underline{e})R+K)^3}}{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2K^2}{((I-\underline{e})R+K)^3} + \psi''(I)} > 0$$
(38)

Hence, the size of the investment depends positively on the interbank exposure.

The first-order condition with respect to K for an optimum is:

$$\frac{\partial U_1}{\partial K} = (1-\lambda)(1-\alpha)\alpha R^2 \frac{(I-\underline{e})^2}{((I-\underline{e})R+K)^2} - \tau'(K) = 0$$
(39)

The corresponding second-order condition is:

$$SOC_{K} \equiv \frac{\partial^{2} U_{1}}{\partial^{2} K} = -(1-\lambda)(1-\alpha)\alpha R^{2} \frac{2(I-\underline{e})^{2}}{((I_{1}^{*}-\underline{e})R+K)^{3}} - \tau''(K) < 0$$
(40)

which is satisfied since $\tau'' > 0$. Treating *I* as exogenous and using the implicit function theorem yields for the derivative of *K* with respect to the size of the investment *I*:

$$\frac{\partial K}{\partial I} = \frac{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I-\underline{e})K}{((I-\underline{e})R+K)^3}}{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I-\underline{e})^2}{((I_1^*-\underline{e})R+K)^3} + \tau''(K)} > 0$$
(41)

Hence, the interbank exposure depends positively on the size of the investment.

Hence, the optimal choice (I_1^*, K_1^*) must satisfy the equations:

$$\lambda R + (1 - \lambda) \left[\alpha R + (1 - \alpha) \alpha R \frac{(K_1^*)^2}{((I_1^* - \underline{e})R + K_1^*)^2} \right] - \psi'(I_1^*) = 0$$
(42)

$$(1-\lambda)(1-\alpha)\alpha R^2 \frac{(I_1^* - \underline{e})^2}{((I_1^* - \underline{e})R + K_1^*)^2} - \tau'(K_1^*) = 0$$
(43)

which is a unique and interior maximum since

$$\Delta \equiv \frac{\partial^2 U_1}{\partial^2 I} \frac{\partial^2 U_1}{\partial^2 K} - \left(\frac{\partial^2 U_1}{\partial I \partial K}\right)^2 > 0 \tag{44}$$

and $\partial^2 U_1 / \partial^2 I < 0$ with

$$\frac{\partial^2 U_1}{\partial I \partial K} = (1 - \lambda)(1 - \alpha)\alpha R^2 \frac{2(I_1^* - \underline{e})K_1^*}{((I_1^* - \underline{e})R + K_1^*)^3}$$
(45)

Lastly, we determine the total derivative of I_1^* and K_1^* with respect to \underline{e} . The total derivative of Eq. (36) with respect to \underline{e} is given by

$$\frac{d \frac{\partial U_1}{\partial I}}{d \underline{e}} = \frac{\partial \frac{\partial U_1}{\partial I}}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial I}}{\partial K} \frac{\partial K}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial I}}{\partial I} \frac{\partial I}{\partial \underline{e}} = 0$$

$$\Rightarrow \frac{\partial K}{\partial \underline{e}} \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K(I-\underline{e})}{((I-\underline{e})R+K)^3} \right)$$

$$- \frac{\partial I}{\partial \underline{e}} \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} + \psi''(I) \right) = -\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} \quad (46)$$

Furthermore, the total derivative of Eq. (39) with respect to \underline{e} is given by

$$\frac{d}{\partial K} \frac{\partial U_1}{\partial K} = \frac{\partial \frac{\partial U_1}{\partial K}}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial K}}{\partial K} \frac{\partial K}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial K}}{\partial I} \frac{\partial I}{\partial \underline{e}} = 0$$

$$\Rightarrow -\frac{\partial K}{\partial \underline{e}} \left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^2}{((I-\underline{e})R+K)^3} + \tau''(K) \right)$$

$$+ \frac{\partial I}{\partial \underline{e}} \left(\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3} \right) = \frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3} \quad (47)$$

Hence, the derivative of I_1^* with respect to \underline{e} becomes

$$\frac{\partial I_{1}^{*}}{\partial \underline{e}} = \frac{\begin{vmatrix} -\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K^{2}}{((I-\underline{e})R+K)^{3}} & \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K(I-\underline{e})}{((I-\underline{e})R+K)^{3}}\right) \\ \frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^{3}} & -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^{2}}{((I-\underline{e})R+K)^{3}} + \tau''(K)\right) \end{vmatrix}}{\left| -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K^{2}}{((I-\underline{e})R+K)^{3}} + \psi''(I)\right) & \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K(I-\underline{e})}{((I-\underline{e})R+K)^{3}}\right) \\ \left| \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K(I-\underline{e})}{((I-\underline{e})R+K)^{3}} - \left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^{2}}{((I-\underline{e})R+K)^{3}} + \tau''(K)\right) \right) \right| \\ = \frac{\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K^{2}}{((I-\underline{e})R+K)^{3}} \tau''(K)}{\frac{\partial^{2}U_{1}}{\partial^{2}I} \frac{\partial^{2}U_{1}}{\partial^{2}K} - \left(\frac{\partial^{2}U_{1}}{\partial I\partial K}\right)^{2}} > 0 \tag{49}$$

and the derivative of K_1^* with respect to \underline{e} becomes

$$\frac{\partial K_{1}^{*}}{\partial \underline{e}} = \frac{\begin{vmatrix} -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K^{2}}{((I-\underline{e})R+K)^{3}} + \psi''(I)\right) & -\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K^{2}}{((I-\underline{e})R+K)^{3}} \\ \left(\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^{3}}\right) & \frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^{3}} \end{vmatrix}}{\left| -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K^{2}}{((I-\underline{e})R+K)^{3}} + \psi''(I)\right) & \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K(I-\underline{e})}{((I-\underline{e})R+K)^{3}}\right) \\ \left| \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^{2}K(I-\underline{e})}{((I-\underline{e})R+K)^{3}}\right) & -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^{2}}{((I-\underline{e})R+K)^{3}} + \tau''(K)\right) \right| \end{aligned}$$
(50)
$$= -\frac{\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)}\psi''(I)}{\frac{\partial^{2}U_{1}}{\partial^{2}I}\frac{\partial^{2}U_{1}}{\partial^{2}K} - \left(\frac{\partial^{2}U_{1}}{\partial I\partial K}\right)^{2}} < 0$$
(51)

A.2. Switching point K^t in Section 4.2

Here, we will formally derive the critical threshold of interbank deposits K^t that just allows a successful bank to stay solvent if the bank it is connected to defaults and is not bailed out. The critical cases to derive this threshold are those in which only one investment fails and neither of the banks is bailed out, i.e., S_8 and S_{11} . Here, the bank with the successful investment will pay the following amount to the bank with the failed investment:

$$\min\left\{K, IR\frac{K(R_D+K)}{R_D(R_D+2K)}\right\}$$
(52)

The first term represents the amount the successful bank owes to the failed bank and the second term results from:

$$\sum_{i=0}^{\infty} IR\left(\frac{K}{R_D + K}\right)^{(1+2i)} = IR\frac{K}{R_D + K}\frac{1}{1 - \frac{K^2}{(R_D + K)^2}} = IR\frac{K(R_D + K)}{R_D(R_D + 2K)}$$
(53)

Hence, the failing bank receives either its full repayment (if there are enough funds available to settle all claims), i.e., $K \leq IR \ K(R_D + K)/(R_D(R_D + 2K))$ or receives a payment of

 $IR K(R_D + K)/(R_D(R_D + 2K))$. The critical threshold up to which the bank receives its full repayment can be written as:

$$K_1^t = IR \frac{K_1^t (R_D + K_1^t)}{R_D (R_D + 2K_1^t)} \Rightarrow K_1^t = \frac{R_D [IR - R_D]}{2R_D - IR}$$
(54)

From Eq. (54) we can see that the successful bank can always pay back its liabilities to the unsuccessful bank as long as $IR > 2R_D$. Thus, it will never default in this case. In what follows we will focus on the more interesting case in which a default is possible depending on the level of K. Hence, from now on we will assume that $IR < 2R_D$. We next consider the repayment the uninsured creditor gets from the successful bank, which is given by:

$$\min\left\{R_D, IR\frac{R_D + K}{R_D + 2K}\right\}$$
(55)

The first term is the total amount owed to the uninsured creditor and the second term comes from:

$$\sum_{i=0}^{\infty} IR \frac{R_D}{R_D + K} \left(\frac{K}{R_D + K}\right)^{2i} = IR \frac{R_D}{R_D + K} \frac{1}{1 - \frac{K^2}{(R_D + K)^2}} = IR \frac{(R_D + K)}{R_D + 2K}$$
(56)

Hence, as long as K is small enough such that $R_D \leq IR (R_D + K)/(R_D + 2K)$ the successful bank can fully repay its uninsured creditor. However if K exceeds a critical threshold, the bank is unable to settle all its claims and can only repay $IR (R_D + K)/(R_D + 2K)$ to its creditor. The critical switching point is given by:

$$R_D = IR \frac{(R_D + K_2^t)}{R_D + 2K_2^t} \Rightarrow K_2^t = \frac{R_D [IR - R_D]}{2R_D - IR}$$
(57)

As can be seen from Eq. (54) and Eq. (57), the thresholds K_1^t and K_2^t are equal. We now turn to the repayment of the uninsured creditor of the failed bank, which is given by:

$$\min\left\{R_D, K\frac{R_D}{R_D + K}, IR\frac{K}{R_D + 2K}\right\} = \min\left\{K\frac{R_D}{R_D + K}, IR\frac{K}{R_D + 2K}\right\}$$
(58)

where the first term is again the total amount owed to the uninsured creditor, the second term is the maximal payment from the bank with the successful investment to the bank with the failed investment times the fraction the insured creditor gets from this payment, and the last term comes from:

$$\sum_{i=0}^{\infty} IR \frac{R_D}{R_D + K} \left(\frac{K}{R_D + K}\right)^{(1+2i)} = IR \frac{R_D K}{(R_D + K)^2} \frac{1}{1 - \frac{K^2}{(R_D + K)^2}} = IR \frac{K}{R_D + 2K}$$
(59)

One can immediately see that the unsuccessful bank can never fully repay its uninsured creditors. Furthermore, as long as K is small enough such that

$$K\frac{R_D}{R_D + K} \le IR\frac{K}{R_D + 2K} , \qquad (60)$$

the payment of the unsuccessful bank to its uninsured creditors is $KR_D / (R_D + K)$. If K is too high, the payment is $IR K / (R_D + 2K)$. The critical switching threshold is given by

$$R_D \frac{K_3^t}{R_D + K_3^t} = IR \frac{K_3^t}{R_D + 2K_3^t} \Rightarrow K_3^t = \frac{R_D \left[IR - R_D\right]}{2R_D - IR}$$
(61)

Hence, all three thresholds are the same, which is why we will denote them in the following K^t . Plugging the value of R_D^{nc} (since we approach K_t from below) into the formula for the contagion threshold K_t in Eq. (61) yields for this threshold

$$K^{t} = \frac{R_{D}^{nc}\left[IR - R_{D}^{nc}\right]}{2R_{D}^{nc} - IR} = \frac{\underline{e}R}{\lambda + (1 - \lambda)\alpha} \frac{I\left[\lambda + (1 - \lambda)\alpha\right] - \underline{e}}{I\left[\lambda + (1 - \lambda)\alpha\right] - 2\underline{e}}$$
(62)

Hence, there exists a positive interbank exposure K^t for which the successful bank stays solvent (in case one bank is successful and the other is not) if

$$I\left[\lambda + (1-\lambda)\alpha\right] - 2\underline{e} > 0 \tag{63}$$

Conversely, if Condition (63) does not hold, we can restrict our analysis to the contagion case $K \ge K^t$. This completes the derivation of K^t .

A.3. Proof of Proposition 4.2

For $K < K^t$, the first-order condition implies for the optimal investment size I_0^{nc} :

$$\frac{\partial U_0}{\partial I}(K < K^t) = [\lambda + (1 - \lambda)\alpha]R - \psi'(I_0^{nc}) = 0$$
(64)

where the second order derivative is negative and the determinant positive. Thus, I_0^{nc} is a unique and interior maximum.

For $K \geq K^t$, the first-order condition with respect to the level of creditor funds I yields:

$$\frac{\partial U_0}{\partial I}(K \ge K^t) = [\lambda + (1-\lambda)\alpha]R + (1-\lambda)(1-\alpha)\alpha R \frac{K^2}{\left(\left(I - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right) + K\right)^2} - \psi'(I) = 0$$
(65)

The respective second-order condition is:

$$\frac{\partial^2 U_0}{\partial^2 I} (K \ge K^t) = -(1-\lambda)(1-\alpha)\alpha R^2 \frac{2K^2}{\left(\left(I - \frac{e}{\lambda(1-\lambda)\alpha}\right) + K\right)^3} - \psi''(I) < 0$$
(66)

which is satisfied since $\psi'' > 0$.

Furthermore, the first-order condition with respect to the interbank exposure K implies:

$$\frac{\partial U_0}{\partial K}(K \ge K^t) = (1 - \lambda)(1 - \alpha)\alpha R^2 \frac{\left(I - \frac{e}{\lambda(1 - \lambda)\alpha}\right)^2}{\left(\left(I - \frac{e}{\lambda(1 - \lambda)\alpha}\right)R + K\right)^2} - \tau'(K) = 0$$
(67)

The second-order condition is:

$$\frac{\partial^2 U_0}{\partial^2 K} (K \ge K^t) = -(1-\lambda)(1-\alpha)\alpha R^2 \frac{2\left(I - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)^2}{\left(\left(I - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R + K\right)^3} - \tau''(K) < 0$$
(68)

which is satisfied since $\tau'' > 0$.

Hence, the optimal choice (I_0^c, K_0^c) must satisfy the equations:

$$[\lambda + (1 - \lambda)\alpha]R + (1 - \lambda)(1 - \alpha)\alpha R \frac{(K_0^c)^2}{\left(\left(I_0^c - \frac{e}{\lambda(1 - \lambda)\alpha}\right) + K_0^c\right)^2} - \psi'(I_0^c) = 0$$
(69)

$$(1-\lambda)(1-\alpha)\alpha R^2 \frac{\left(I_0^c - \frac{e}{\lambda(1-\lambda)\alpha}\right)^2}{\left(\left(I_0^c - \frac{e}{\lambda(1-\lambda)\alpha}\right)R + K_0^c\right)^2} - \tau'(K_0^c) = 0$$
(70)

which is a unique and interior maximum if $K_0^c \geq K^t$ since

$$\Delta \equiv \frac{\partial^2 U_0}{\partial^2 I} \frac{\partial^2 U_0}{\partial^2 K} - \left(\frac{\partial^2 U_0}{\partial I \partial K}\right)^2 > 0 \tag{71}$$

and $\partial^2 U_0 / \partial^2 I < 0$ with

$$\frac{\partial^2 U_0}{\partial I \partial K} = (1 - \lambda)(1 - \alpha)\alpha R^2 \frac{2\left(I_0^c - \frac{e}{\lambda(1 - \lambda)\alpha}\right)K_0^c}{\left(\left(I_0^c - \frac{e}{\lambda(1 - \lambda)\alpha}\right)R + K_0^c\right)^3}$$
(72)

If $K_0^c \ge K^t$ and

$$\overline{U_0}(K \ge K^t) > \overline{U_0}(K < K^t)$$

$$\begin{bmatrix}
[\lambda + (1 - \lambda)\alpha](I_0^c - \underline{e})R \\
+ (1 - \lambda)(1 - \alpha)\alpha K_0^c \frac{(I_0^c - \frac{\underline{e}}{\lambda(1 - \lambda)\alpha})R}{(I_0^c - \frac{\underline{e}}{\lambda(1 - \lambda)\alpha})R + K_0^c} \\
- \alpha \underline{e}R \frac{(1 - \lambda)(1 - \alpha)}{\lambda + (1 - \lambda)\alpha} - \psi(I_0^c) - \tau(K_0^c) + \underline{e}
\end{bmatrix} > [\lambda + (1 - \lambda)\alpha](I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e} \\
(1 - \lambda)(1 - \alpha)\alpha K_0^c \frac{(I_0^c - \frac{\underline{e}}{\lambda(1 - \lambda)\alpha})R}{(I_0^c - \frac{\underline{e}}{\lambda(1 - \lambda)\alpha})R + K_0^c} - \tau(K_0^c) > \begin{bmatrix}
\int_{I_0^{nc}}^{I_0^c} \psi(x)dx \\
- [\lambda + (1 - \lambda)\alpha](I_0^c - I_0^{nc})R \\
+ \alpha \underline{e}R \frac{(1 - \lambda)(1 - \alpha)}{\lambda + (1 - \lambda)\alpha}
\end{bmatrix}$$
(73)

hold, choosing the amount K_0^c of interbank deposits dominates the alternative of having no interbank exposure. As shown by Condition (74), choosing K_0^c dominates if the expected additional gain from the interbank exposure, due to the higher value of a bailout of the other bank, outweighs the loss in value of the own bank's bailout due to the contagion risk and the resulting lower interest rate $R_D^c < R_D^{nc} < (I - \underline{e})R$.

Hence, if $K_0^c \ge K^t$ and Condition (73) holds, the banks will choose to have the interbank exposure K_0^c . If, on the other hand, on of these conditions does not hold, they will chose to have no interbank exposure.

A.4. Proof of Proposition 4.3

First, we prove that $\overline{U_1} > \overline{U_0}(K < K^t)$ and then that $\overline{U_1} > \overline{U_0}(K \ge K^t)$. We prove that $\overline{U_1} > \overline{U_0}(K < K^t)$ in two steps. First, we show that correlated investments yield the same expected utility as uncorrelated investments when the banks choose $(I_0^{nc}, 0)$ instead of (I_1^*, K_1^*) when investing in correlated portfolios (and $K < K^t$). Second, due to the fact that (I_1^*, K_1^*) is a unique optimum, it has to hold that the expected utility when choosing (I_1^*, K_1^*) has to be higher than for the case that the bank chooses $(I_0^{nc}, 0)$ when investing in correlated investments. Comparing $U_1(I_0^{nc}, 0)$ and $\overline{U_0}(K < K^t)$ from Eq. (16) it follows that

$$U_1(I_0^{nc}, 0) = \overline{U_0}(K < K^t)$$

$$\lambda(I_0^{nc} - \underline{e})R + (1 - \lambda)\alpha(I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e} = [\lambda + (1 - \lambda)\alpha](I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e}$$

$$(75)$$

Since $\overline{U_1} > U_1(I_0^{nc}, 0)$, it follows that $\overline{U_1} > \overline{U_0}(K < K^t)$.

Similarly, we show that $\overline{U_1} > \overline{U_0}(K \ge K^t)$ in two steps. First, we show that correlated investments yield a higher expected utility than uncorrelated investments even when the banks choose (I_0^c, K_0^c) instead of (I_1^*, K_1^*) when investing in correlated investments (and $K \ge K^t$). Second, due to the fact that (I_1^*, K_1^*) is a unique optimum, it has to hold that the expected utility when choosing (I_1^*, K_1^*) has to be higher than for the case that the bank chooses (I_0^c, K_0^c) when investing in correlated investments.

Comparing $U_1(I_0^c, K_0^c)$ and $\overline{U_0}(K \ge K^t)$ from Eq. (20) it follows that

$$U_{1}(I_{0}^{c}, K_{0}^{c}) > \overline{U_{0}}(K \ge K^{t})$$

$$\begin{bmatrix} \lambda(I_{0}^{c} - \underline{e})R + (1 - \lambda)\alpha(I_{0}^{c} - \underline{e})R \\ + (1 - \lambda)(1 - \alpha)\alpha K_{0}^{c}\frac{(I_{0}^{c} - \underline{e})R}{(I_{0}^{c} - \underline{e})R + K_{0}^{c}} \end{bmatrix} > \begin{bmatrix} [\lambda + (1 - \lambda)\alpha] (I_{0}^{c} - \underline{e})R \\ + (1 - \lambda)(1 - \alpha)\alpha c_{0}^{c}K_{0}^{c}\frac{(I_{0}^{c} - \frac{\underline{e}}{\lambda + (1 - \lambda)\alpha})R}{(I_{0}^{c} - \frac{\underline{e}}{\lambda + (1 - \lambda)\alpha})R + K_{0}^{c}} \\ - \psi(I_{0}^{c}) - \tau(K_{0}^{c}) + \underline{e} \end{bmatrix} > \begin{bmatrix} (1 - \lambda)(1 - \alpha)\alpha K_{0}^{c}\frac{(I_{0}^{c} - \underline{e})R}{(I_{0}^{c} - \underline{e})R + K_{0}^{c}} \\ - \alpha \underline{e}R\frac{(1 - \lambda)(1 - \alpha)}{\lambda + (1 - \lambda)\alpha} - \psi(I_{0}^{c}) - \tau(K_{0}^{c}) + \underline{e} \end{bmatrix} \end{bmatrix} > (1 - \lambda)(1 - \alpha)\alpha K_{0}^{c}\frac{\left(I_{0}^{c} - \frac{\underline{e}}{\lambda + (1 - \lambda)\alpha}\right)R}{\left(I_{0}^{c} - \frac{\underline{e}}{\lambda + (1 - \lambda)\alpha}\right)R + K_{0}^{c}}$$
(76)

which is true since $\lambda + (1-\lambda)\alpha < 1$. Since $\overline{U_1} > U_1(I_0^c, K_0^c)$, it follows that $\overline{U_1} > \overline{U_0}(K \ge K^t)$.

A.5. Proof of Proposition 5.1

Inserting the expression from Eq. (28) and $K = K_A^{\alpha} + \Delta$ into Eq. (29) yields

$$\max_{\Delta} U_B = \lambda (1 - \underline{e})R + (1 - \lambda)(\alpha - \delta)(1 - \underline{e})R + (1 - \lambda)(1 - (\alpha - \delta))(\alpha + \delta) \frac{(K_A^{\alpha} + \Delta)(1 - \underline{e})R}{(1 - \underline{e})R + (K_A^{\alpha} + \Delta)} + (1 - \lambda)(1 - (\alpha + \delta))(\alpha - \delta) \left[\frac{(K_A^{\alpha} + \Delta)(1 - \underline{e})R}{(1 - \underline{e})R + (K_A^{\alpha} + \Delta)} - \frac{K_A^{\alpha}(1 - \underline{e})R}{(1 - \underline{e})R + K_A^{\alpha}} \right] - 2\tau (K_A^{\alpha} + \Delta) + \tau (K_A^{\alpha})$$
(77)

where Δ is the interbank exposure that B_B wants to have in addition to K_A^{α} . The first order condition of Eq. (77) with respect to Δ yields the optimal additional interbank exposure Δ^{α} :

$$\frac{\partial U_B}{\partial \Delta} = 2(1-\lambda) \frac{(1-\underline{e})^2 R^2}{((1-\underline{e})R + (K_A^{\alpha} + \Delta^{\alpha}))^2} \left[\delta^2 + (1-\alpha)\alpha \right] - 2\tau' (K_A^* + \Delta^{\alpha}) = 0$$
(78)

Solving Eq. (78) for Δ^{α} and plugging it into Eq. (28) gives the optimal compensation fee η^{α} . From Eq. (78) follows for the derivative of Δ^{α} with respect to δ :

$$\frac{\partial \Delta^{\alpha}}{\partial \delta} = \frac{-\frac{4(1-\lambda)(1-\underline{e})^{2}R^{2}}{((1-\underline{e})R+(K_{A}^{\alpha}+\Delta^{\alpha}))^{3}}\frac{\partial K_{A}^{\alpha}}{\partial \delta}\left[\delta^{2}+(1-\alpha)\alpha\right]+\frac{4(1-\lambda)(1-\underline{e})^{2}R^{2}}{((1-\underline{e})R+(K_{A}^{\alpha}+\Delta^{\alpha}))^{2}}\delta}{\frac{4(1-\lambda)(1-\underline{e})^{2}R^{2}}{((1-\underline{e})R+(K_{A}^{\alpha}+\Delta^{\alpha}))^{3}}\left[\delta^{2}+(1-\alpha)\alpha\right]+2\tau''(K_{A}^{\alpha}+\Delta^{\alpha})}>0$$
(79)

where the derivative is positive since $\partial K_A^*/\partial \delta < 0$. Therefore, the interbank exposure that B_B wants to have in addition to K_A^{α} increases with the difference between the bailout probabilities of the two banks, δ . Furthermore, from Eq. (28) it follows directly that η^{α} increases with δ .

Online appendix: Risk averse creditors

In the following, we allow uninsured creditors to be risk averse (in line with the literature on interbank networks and financial contagion, e.g., Allen and Gale, 2000; Brusco and Castiglionesi, 2007) to demonstrate the robustness of our results. Here, the interbank market not only is present for the reasons discussed previously, but also allows banks to coinsure against regional liquidity shocks as in Allen and Gale (2000). We show that even if the interbank market has a different reason to exist, our main mechanism is still present. Specifically, we show that banks have an incentive to increase their interbank exposure beyond the level that would be sufficient to perfectly coinsure against liquidity shocks. Our economy in this section now consists of three dates t = 0, 1, 2 and, again, two regions A and B, each with a continuum of identical banks that all adopt the same behavior and can thus be described by a representative bank (protected by limited liability). Furthermore, there are now n ex ante identical uninsured creditors and again one risk-neutral investor in each region. Creditors have Diamond-Dybvig (1983) preferences, that is,

$$U(c_1, c_2) = \begin{cases} u(c_1) \text{ with probability } \omega^i \text{ (early creditors)} \\ u(c_2) \text{ with probability } 1 - \omega^i \text{ (late creditors)} \end{cases}$$
(80)

where $i \in \{A, B\}$ and the utility function $u(\cdot)$ is defined for nonnegative numbers, strictly increasing, strictly concave, and twice continuously differentiable and satisfies Inada conditions. Consumption at t = 1 (t = 2) is denoted by c_1 (c_2). Each creditor is endowed with one unit of capital at t = 0. Of the *n* creditors in each region there are n_e^i early creditors and n_l^i late creditors. Thus, $\omega^i \equiv n_e^i/n$ represents the fraction of early creditors, where ω^i can be either high or low ($\omega_H > \omega_L$). There are two equally likely states S_1 and S_2 . At t = 1 state-dependent liquidity preferences are revealed (see Table 4).

| | ω^A | ω^B |
|-------|------------|------------|
| S_1 | ω_H | ω_L |
| S_2 | ω_L | ω_H |

Table 4: Liquidity shocks

Each region has the same ex ante probability of facing a high liquidity shock. A creditor's type is private information and the proportion of early creditors in the whole economy is given by $\gamma = (\omega_H + \omega_L)/2$. Thus, there is no aggregate uncertainty. At t = 1 all liquidity-related uncertainty is resolved and creditors learn their type.

There are two types of investment opportunities: a risk-free, liquid type and a risky, illiquid one (generating only a return of r < 1 if liquidated at t = 1). The risk-free asset is a storage technology that transfers one unit of capital at a certain period into one unit of capital in the following period. The illiquid asset is only available at t = 0 and generates a return of either R > 1 with probability λ or zero with probability $(1 - \lambda)$ at t = 2 for each unit of capital invested. We assume that the illiquid asset has a positive NPV, that is, $\lambda R > 1$, and that investment outcomes are perfectly positively correlated across regions.

Since our model now has three dates, the equity investors are entitled to receive dividends at t = 1 and t = 2. Hence, the investor's utility is now

$$u(d_0, d_1, d_2) = \lambda R d_0 + d_1 + d_2 \tag{81}$$

As before, since investors can obtain a utility of $\lambda e R$ by immediately consuming the initial endowment, they must earn an expected return of at least λR on their invested money to give up consumption at t = 0. Hence, the participation constraint for investors becomes

$$E[d_1 + d_2] \ge \lambda \underline{e}R \tag{82}$$

As shown by Brusco and Castiglionesi (2007), we can restrict attention to policies paying no dividends at t = 1. Therefore, the bank has to invest the whole equity contribution into the illiquid asset and the full proceeds from this investment have to be paid to the equity investors to satisfy their participation constraint. Hence, we only have to analyze the bank's decision regarding the allocation of the debt contribution.

Central planner economy

In this economy the Pareto-efficient allocation can be characterized as the solution to the problem of a planner maximizing the creditors' expected utility. By pooling resources the planner can overcome the problem of the regions' asymmetric liquidity needs. Let y and x denote the per capita amounts invested in the risk-free and risky assets, respectively. Furthermore, let c and cr_d denote the amounts creditors can withdraw to satisfy their liquidity needs at t = 1 and t = 2, respectively. In this context, r_d can be understood as the interest rate creditors earn by not withdrawing their funds for an additional period. The planner's problem can then be written as

$$\max_{x,y,c,r_d} U = \gamma u(c) + (1 - \gamma) \lambda u(cr_d)$$
(83)

subject to

$$2x + 2y \le 2n, \ \gamma \, 2nc \le 2y, \ (1 - \gamma) \, 2ncr_d \le 2xR, \tag{84}$$

$$x \ge 0, \ y \ge 0, \ c \ge 0, \ r_d \ge 0.$$
 (85)

The first set of constraints represents budget constraints for periods 0, 1 and 2. Since optimality requires that the constraints be binding, the optimization problem can be rewritten as

$$\max_{y} \gamma u\left(\frac{y}{\gamma n}\right) + (1-\gamma) \ \lambda u\left(\frac{R(n-y)}{(1-\gamma)n}\right) \tag{86}$$

Given the utility function's properties this optimization problem has a unique interior solution. The optimal value $y^* \in (0, 1)$ can be obtained from the first-order condition

$$u'\left(\frac{y^*}{\gamma n}\right) = \lambda R u'\left(\frac{R(n-y^*)}{(1-\gamma)n}\right)$$
(87)

Once y^* has been determined, we can use the remaining constraints to determine the optimal values of the other variables. Hence, we obtain

$$c^* = \frac{y^*}{\gamma n}, \ r_d^* = \frac{R(n-y^*)}{(1-\gamma)nc^*}, \ \text{and} \ x^* = n-y^*$$
(88)

Since $\lambda R > 1$, we can conclude that $u'(c) > u'(cr_d)$ and hence $r_d > 1$, implying that consumption is higher at t = 2 than at t = 1. Consequently, late creditors have no incentive to mimic early creditors. We denote the first-best allocation as $\delta^* = (y^*, x^*, c^*, r_d^*)$.

Decentralized economy with an interbank market and no bailout possibility

Allen and Gale (2000) show that this first-best allocation can be achieved by allowing banks in a decentralized economy to coinsure against liquidity shocks. Coinsurance is possible since the liquidity needs of the two regions are negatively correlated. In contrast to Allen and Gale (2000), we again allow banks to exchange an arbitrary amount of deposits K at t = 0, and not only the amount necessary to achieve first-best. However, we show that exchanging funds above the level of the first best solution does not increase the utility of uninsured creditors if there is no bailout possibility. Let k denote the amount of interbank deposits that is withdrawn by the bank that faces a high liquidity shock at t = 1.

Figure 4: Capital flows in the two region economy

The capital flows are depicted in Fig. 4. At t = 0 the two banks exchange deposits K. At t = 1 the bank with the high liquidity shock $(B_A \text{ in Fig. 4})$ withdraws an amount k from the other bank to satisfy the liquidity needs of its creditors. In the final period bank B_A receives its remaining deposits (K - k) from bank B_B and pays back the deposits that bank B_B deposited in bank B_A . From now on, we follow Allen and Gale (2000) in that we assume that these remaining deposits also yield the interest r_d and that interbank deposits incur no transaction costs. Hence, each bank can offer a contract $\delta = (y, x, c, r_d, K)$ to its creditors and the bank in the other region. With perfect competition in the banking sector, the banks will offer their creditors a contract that replicates the first-best outcome. The optimization problem of a bank can then be written as

$$\max_{x,y,c,r_d,K,k} U = \frac{1}{2} [\omega_H u(c) + (1 - \omega_H)\lambda u(cr_d)] + \frac{1}{2} [\omega_L u(c) + (1 - \omega_L)\lambda u(cr_d)]$$
(89)

subject to

$$\omega_H nc \le y + k \tag{90}$$

$$\omega_L nc + k \le y \tag{91}$$

$$(1 - \omega_H)ncr_d + Kr_d \le Rx + (K - k)r_d \tag{92}$$

$$(1 - \omega_L)ncr_d + (K - k)r_d \le Rx + Kr_d \tag{93}$$

$$x \ge 0, \ y \ge 0, \ c \ge 0, \ r_d \ge 0, \ x + y \le n, \ k \le K$$
 (94)

Constraints (90) and (91) represent budget constraints at t = 1 and Constraints (92) and (93) represent budget constraints at t = 2. As shown by Allen and Gale (2000), optimality requires that $k^* = (\omega_H - \gamma)nc^*$. As long as there is no positive bailout probability, the actual amount of funds exchanged, K, does not alter the utility of the creditors as long as $K \ge k^*$. These findings lead to the following proposition.

Proposition 7.1. If there is no possibility for banks to be bailed out and the two representative banks exchange an amount K of deposits, then the first-best allocation δ^* can be implemented by a decentralized banking system offering standard deposit contracts. Moreover, banks have no incentive to exchange more funds than required to achieve first-best, that is, they will only exchange $K = k^* = (\omega_H - \gamma)nc^*$.

Proof For the proof of the first part of the proposition, we refer to the proof of Proposition 3 of Brusco and Castiglionesi (2007). To see why the second part is true, that is, why banks do not exchange more than necessary to achieve first-best, note that optimality again requires the constraints to be binding. Then the amount of funds actually exchanged, K, drops out of the optimization problem. Hence, the amount that is actually exchanged does not influence the utility of the creditors. Therefore, banks have no incentive to exchange more funds than necessary to achieve first-best, which implies that $K = k^* = (\omega_H - \gamma)nc^*$. QED

Decentralized economy with an interbank market and positive bailout probability

So far we have assumed that after a bank failure occurs, creditors receive no repayment at t = 2. Now we investigate how the results change if there is the possibility that a bank will be bailed out by the government after a default. As before, the bailout probability is then α . Therefore, the optimization problem becomes

$$\max_{x,y,c,r_d,K,k} U = \frac{1}{2} \left[\omega_H u(c) + (1 - \omega_H) \left[\begin{array}{c} \lambda u(cr_d) + (1 - \lambda)[(1 - \alpha)^2 u(0) \\ + \alpha (1 - \alpha) u(cr_d) \\ + (1 - \alpha) \alpha u (\theta_1 cr_d) + \alpha^2 u(cr_d)] \end{array} \right] \right] + \frac{1}{2} \left[\omega_L u(c) + (1 - \omega_L) \left[\begin{array}{c} \lambda u(cr_d) + (1 - \lambda)[(1 - \alpha)^2 u(0) \\ + \alpha (1 - \alpha) u(cr_d) \\ + (1 - \alpha) \alpha u (\theta_2 cr_d) + \alpha^2 u(cr_d)] \end{array} \right] \right]$$
(95)

with

$$\theta_1 = \frac{K - k}{(1 - \omega_H)ncr_d + K} \text{ and } \theta_2 = \frac{K}{(1 - \omega_L)ncr_d + (K - k)}$$

subject to

$$\omega_H nc \le y + k \tag{96}$$

$$\omega_L nc + k \le y \tag{97}$$

$$(1 - \omega_H)ncr_d + Kr_d \le Rx + (K - k)r_d \tag{98}$$

$$(1 - \omega_L)ncr_d + (K - k)r_d \le Rx + Kr_d \tag{99}$$

$$x \ge 0, \ y \ge 0, \ c \ge 0, \ r_d \ge 0, \ x + y \le n, \ k \le K$$
 (100)

Eq. (95) is the objective function of the optimization problem of the representative bank in region i. The bank in region i is equally likely to face a high or a low liquidity shock. If a high liquidity shock occurs in, for example, region A, a fraction ω_H of the creditors will withdraw their funds at t = 1 and the remaining creditors will demand repayment in t = 2. At t = 2 several cases must be considered. The risky asset yields a positive return R with probability λ and creditors receive their promised repayment cr_d . If the risky asset yields a zero payoff, the return of the creditor depends on whether the banks are bailed out or not. If neither of the two banks is bailed out, creditors receive no payment. If the bank in region A is bailed out, the government steps in and creditors receive their full repayment cr_d . If only the bank in region B is bailed out, bank B_A receives the funds still owed to it by B_B (see Fig. 4). Since B_A has already withdrawn an amount k at t = 1, it receives the

remaining funds $(K - k)r_d$. Since B_A has two creditors, namely, its uninsured creditor and bank B_B , funds are again split on a pro rata basis. Hence, creditors receive a fraction θ_1 of their promised repayment. Finally, if both banks are bailed out, then creditors again receive the full amount. The second case (where B_A faces a low liquidity shock) can be described analogously.

All constraints are as in the previous case without a bailout possibility. By examining the optimization problem, it becomes obvious that the amount of funds exchanged, K, now has an influence on the utility of the creditors. Although K again drops out of the constraints (optimality again requires the constraints to be binding), it now also enters the objective function directly because it determines the amount that creditors receive in the case of a default if only one bank is bailed out. Before the repayment in this state of nature was zero.

Again, optimality requires that banks choose $k^* = (\omega_H - \gamma)nc^*$. Hence, the optimization problem (95) can be simplified to

$$\max_{x,y,c,r_d,K} U = \gamma u(c) + (1-\gamma) \left[\lambda + (1-\lambda)\alpha\right] u(cr_d) + \frac{1}{2} (1-\lambda)(1-\alpha)\alpha \left[(1-\omega_H)u(\theta_1 cr_d) + (1-\omega_L)u(\theta_2 cr_d)\right]$$
(101)

subject to

$$x + y \le n, \ \gamma nc \le y, \ (1 - \gamma)ncr_d \le xR,$$

$$(102)$$

$$x \ge 0, \ y \ge 0, \ c \ge 0, \ r_d \ge 0.$$
 (103)

Since the constraints in the respective periods again have to be binding, we can solve them for c and r_d , respectively and can plug these values into the objective function, which yields:

$$\max_{y,K} U = \gamma u \left(\frac{y}{\gamma n}\right) + (1-\gamma) \left[\lambda + (1-\lambda)\alpha\right] u \left(\frac{R(n-y)}{(1-\gamma)n}\right) + \frac{1}{2}(1-\omega_H)(1-\lambda)(1-\alpha)\alpha u \left(\theta_1 \frac{R(n-y)}{(1-\gamma)n}\right) + \frac{1}{2}(1-\omega_L)(1-\lambda)(1-\alpha)\alpha u \left(\theta_2 \frac{R(n-y)}{(1-\gamma)n}\right)$$
(104)

The first order condition with respect to y then yields:

$$u'\left(\frac{y}{\gamma n}\right) = \left[\lambda + (1-\lambda)\alpha\right]u'\left(\frac{R(n-y)}{(1-\gamma)n}\right)R$$

+
$$\frac{1}{2}\frac{(1-\omega_H)}{(1-\gamma)}(1-\lambda)(1-\alpha)\alpha u'\left(\theta_1\frac{R(n-y)}{(1-\gamma)n}\right)\theta_1R$$

+
$$\frac{1}{2}\frac{(1-\omega_L)}{(1-\gamma)}(1-\lambda)(1-\alpha)\alpha u'\left(\theta_2\frac{R(n-y)}{(1-\gamma)n}\right)\theta_2R$$
 (105)

where the second order conditions are satisfied. Looking at this first order condition one can see that the marginal utility of consumption at t = 1 is higher now, implying that consumption is lower. Hence, if it is more likely to get the higher repayment at t = 2creditors want to shift more consumption to this later period. Hence, the optimal amount of funds withdrawn at t = 1 is now smaller than in the situation without bailout. Furthermore, we obtain the following first-order condition for K:

$$\frac{\partial U}{\partial K} = \frac{1}{2} \frac{(1-\omega_H)}{(1-\gamma)} (1-\lambda)(1-\alpha)\alpha u' \left(\theta_1 \frac{R(n-y)}{(1-\gamma)n}\right) \frac{\partial \theta_1}{\partial K} \frac{R(n-y)}{(1-\gamma)n} + \frac{1}{2} \frac{(1-\omega_L)}{(1-\gamma)} (1-\lambda)(1-\alpha)\alpha u' \left(\theta_2 \frac{R(n-y)}{(1-\gamma)n}\right) \frac{\partial \theta_2}{\partial K} \frac{R(n-y)}{(1-\gamma)n} > 0$$
(106)

which is true since $\partial \theta_1 / \partial K > 0$ and $\partial \theta_2 / \partial K > 0$. As we can see from the first-order condition, the utility of the creditor is now increasing in K (i.e., the funds exchanged at t = 0), since K increases the amount that the creditor receives in case of default of the risky asset (although the amount needed to satisfy the consumption needs of creditors is now actually smaller, banks have an incentive to increase their interbank exposure). Therefore, banks have an incentive to increase the amount of interbank deposits and hence their connectivity to a level that exceeds the first-best solution derived before. These findings yield the following proposition.

Proposition 7.2. Given a positive bailout probability, banks have an incentive to increase their interbank exposure beyond the first-best level.

Hence, even if the interbank market does not exist only as an insurance for non-insured creditors but also to coinsure against regional liquidity shocks, as in Allen and Gale (2000), the main mechanism is still present.

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