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$$\frac{n!}{(n-1)!} p^{m-1} (1-p)^{n-m} = p \sum_{\ell=0}^{n-1} \frac{\ell+1}{n} \frac{(n-1)!}{(n-1-\ell)! \ell!} p^{\ell} (1-p)^{n-1-\ell}$$
$$= p \frac{n-1}{n} \sum_{\ell=0}^{n-1} \left[ \frac{\ell}{n-1} + \frac{1}{n-1} \right] \frac{(n-1)!}{(n-1-\ell)! \ell!} p^{\ell} (1-p)^{n-1-\ell} = p^2 \frac{n-1}{n} +$$

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# Sectoral Impacts of International Labour Migration and Population Ageing in the Czech Republic

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## **Abstract:**

This study assesses macroeconomic and sectoral impacts of demographic changes in the Czech Republic, as a result of population ageing and international migration. To do so, it develops a unique dynamic Overlapping Generations Computable General Equilibrium (OLG-CGE) model with detailed representation of individuals of different ages, educational attainment and occupations, as well as interrelations among industrial sectors in producing intermediate and final outputs. The numeric simulations show that the Czech economy will face a substantial reduction in its effective labour supply and changes in sectoral demand patterns, leading to an increase in unit labour costs and consequent shift towards more capitalbased production, price increase for the consumers, and a long-term decrease in demand particularly for agricultural products. While international migration may alleviate the pressure, the annual net immigration would need to increase by at least 8 thousand individuals on average in the 2020-2035 period and by 17 thousand individuals in the 2036-2050 period to offset the negative effects in the long term.

**JEL:** C68, E27, J11

**Keywords:** OLG, CGE, migration, labour force, economic impact

# 1 Introduction

The population in the Czech Republic, similarly to many other countries in Europe, has been going through unprecedented changes in the recent years, characterised principally by ageing, a result of long-term below-replacement fertility rates, and international migration, especially following the 2004 enlargement of the European Union. The median age in the Czech Republic increased from 35.3 in 1990 to 41.4 in 2015 (United Nations, 2017), while the average life expectancy increased from 71.4 in 1990 to 78.6 in 2015, resulting in a shift in the old-age dependency ratio — the number of retired to working-age individuals — from 19.3% to 26.9% in that time period (World Bank, 2019). In addition, the number of foreign nationals with various forms of residence statuses increased from just over 80,000 in 1993 to 425,000 in 2008 and 515,000 in 2018, while the number of Czech nationals living in another EU country reached approx. 125,000 in 2018 (MICZ, 2019; European Commission, 2019).

The exact effects of demographic changes and international migration on the economy and society in both the receiving country and the country of origin are difficult to estimate, not only because a large proportion of the relevant data, such as the share of spending in the receiving country, are often not collected, and may range from changes in wages (Nagarajan et al., 2016; Ratha et al., 2011) and tax revenues (Martinsen and Pons Rotger, 2017; Vargas-Silva, 2015) to human capital accumulation (Beine et al., 2008; Dinkelman and Mariotti, 2016) and innovation (Bosetti et al., 2015; Capello and Lenzi, 2019). In the context of the Czech Republic, population ageing has affected particularly the pension system, which will require a substantial reform in order to maintain a consistent pension-wage ratios in the future, and the labour force, which has been shrinking both in absolute and relative terms within the population as a whole. Net immigration, on the other hand, may help to counterbalance these effects by bringing additional working-age population in the economy.

Which effects will dominate and what will be the impact on the economy and the society as a whole? Understanding the potential implications of demographic changes is essential for academics, businesses and policy-makers alike, yet an empirical impact assessment is difficult due to data limitations, leading to a gap in analytical evidence essential for long-term planning. This study develops a dynamic Overlapping Generations Computable General Equilibrium (OLG-CGE) model used to evaluate the micro- and macroeconomic impacts of population ageing and international migration in the context of the Czech Republic, analysing the potential effects on population welfare and production, as well as public finances. This is done through computer simulations of various scenarios of population ageing and migration patterns, analysing specifically the possibility of a return of a large number of Czechs working abroad, a sudden outflow of foreign workers, and a status quo scenario summarising the consumption and labour force shifts due to population ageing. The model is calibrated in detail using the latest population and productivity estimates for 350 population groups differing in their education, occupation and sectoral affiliation, providing an accurate representation of the Czech economy and its internal economic mechanisms.

The analysis clearly outlines the extent of the underlying population changes, characterised by a projected 5.4% decrease in the overall population size and over 6% decrease in the effective labour supply in the next 30 years (keeping the default retirement age unchanged), and 4-year increase in the average age. This results in higher unit labour costs and a shift towards production relying more heavily on capital across all industries, price increase, and industrial sectors being affected by various levels of initial increase and subsequent decrease in domestic demand for their products and services.

The next section provides a brief overview of the relevant literature and sources for this study, followed by a description of the simulation model, its calibration, the three modelled scenarios with varying levels of migration, and an overview of the simulation results across a range of variables with a concluding summary.

## 2 Existing research

The impacts of population ageing have been thoroughly discussed in the academic literature from multiple angles, particularly regarding the negative supply side effects (e.g. [Bloom et al., 2010](#); [Culling and Skilling, 2018](#); [Fougère and Mérette, 1999](#)), pension system (e.g. [Foster, 2018](#); [Hess et al., 2017](#); [Miles and Černý, 2006](#)) and healthcare (e.g. [Ashby and Beech, 2016](#); [Geue et al., 2014](#); [Lim et al., 2017](#)). The general conclusion is that the projected demographic changes will result in higher costs to the public as higher proportion of the population will be in retirement and thus not contributing to the public finances. The literature also suggests that there is often an insufficient reaction from the policy makers in introducing reforms that may shift the burden more equally across the current and future generations.

Similarly, the effects of international migration on labour supply (e.g. [Barrell et al., 2010](#); [Czaika, 2018](#); [Issac, 2013](#)) or public finances (e.g. [Aldén and Hammarstedt, 2016](#); [Harper, 2016](#); [Preston, 2014](#)) have been studied in detail, indeed often in relation to population ageing (e.g. [Hoff, 2016](#); [Karl and Torres, 2015](#)), noting that immigration may help to reduce the burden of population ageing, while emigration of young people may do the opposite. A specific strand of the literature then analyses the brain drain phenomena (e.g. [Horvat, 2004](#); [Veugelers, 2017](#); [Zhang and Lucey, 2017](#)), a human capital flight of skilled individuals providing significant economic benefits for both the migrants themselves and the receiving country - often at the expense of the country of origin. At the same time, emigration remittances and return migration can also have various positive effects on the country of origin ([Barsbai et al., 2017](#) or [Ratha et al., 2011](#)).

The methodology used in this study combines the explicit modelling of age effects in OLG models with the simultaneous assessment of all economic activities - production, consumption, employment, taxes, savings and trade - in CGE models. The baseline OLG model used in this study was developed by [Samuelson \(1958\)](#) and [Diamond \(1965\)](#) and recently used to evaluate the effects of population ageing e.g. by [Catalano and Pezzolla \(2016\)](#) or [Kim et al. \(2016\)](#). The main sources of inspiration for the model were then the seminal work of [Auerbach and Kotlikoff \(1987\)](#), as well as newer publications of [Heer and Mauner \(2009\)](#), [Ihori \(1996\)](#), [Lisenkova et al. \(2013\)](#), [Zodrow et al. \(2013\)](#) and others. CGE models have an equally long history, originating in input-output and linear programming models, with [Dervis et al. \(1982\)](#), [Lofgren et al. \(2002\)](#) and [Nordhaus and Yang \(1996\)](#) representing some of the main pieces of literature. The the CGE model variation used in this study is broadly based on [Burfisher \(2017\)](#), [Corong et al. \(2017\)](#), [Dixon and Jorgenson \(2012\)](#), [Garau et al. \(2013\)](#) and [Lecca et al. \(2013\)](#). Both OLG and CGE models have been previously used in the context of the Czech Republic to address e.g. the issues of pension system, R&D investment or trade and environmental policies by [Kiuila \(2015\)](#), [Křístková and Habrychova \(2011\)](#), [Křístková et al. \(2012\)](#), [Martin and Skinner \(1998\)](#), [Marek \(2008\)](#) and [Snudden and Klyuev \(2011\)](#).

## 3 The model

This section describes the OLG-CGE model used in the simulation analysis. The computer script is based on [Stepanek \(2019\)](#); additional model details and approach to its estimation are described in the Appendix. The simulations were done in MATLAB R2018b.

The model is set up as follows. There are two regions actively trading with one another: the Czech Republic (CZ) and Rest of the World (ROW), each with  $p = \{1..13\}$  economic sectors. Each period (year), the remainder of the oldest cohort dies and a new generation is born. Agents live for maximum of 60 periods and spend the first  $T = 43$  years working and the last  $T^r = 17$  years retired, broadly corresponding to the default minimum retirement age of 62-64 years and the average life expectancy of 78 years in the Czech Republic ([World Bank, 2019](#)). In line with the existing literature, the first 18 years of actual life are not modelled. Agents face positive probability of death each year, given by an exogenous unconditional survival function calibrated using the real-world mortality rate projections. Size of the first newborn generation ( $t = 1$ ) is normalised to one, while size of the subsequent generations ( $t = 2, \dots$ ) evolves according to the real-world demographic projections.

Besides age, agents differ in their education, production sector and occupation, all of which define their productivity and income both within and across cohorts.<sup>1</sup> The full list of categories is presented in Table 4; given the 13 production sectors, 5 education groups and 5 occupation groups, each cohort is broken down into 325 separate units. In addition, agent's income evolves over time, representing human capital accumulation, and has the characteristic hump-shaped profile with average wages first increasing with age and subsequently decreasing again at higher ages.

### 3.1 Household optimisation

New agents are born without wealth, but may start accumulating capital through savings. Analogously, in the last period, agents sell all of their remaining capital stock for consumption. In absence of inflation in the model, the total annual income of an agent aged  $s$ , with education  $e$ , occupation  $o$ , in production sector  $p$ , in any given period  $t$ , is a combination of the baseline average income  $e_{s,e,o,p}$  and period-specific equilibrium wage  $w_{e,o,p,t}$  (see Equation 11):

$$I_{s,z,t} = (1 - \tau) e_{s,z} w_{z,t}, \quad (1)$$

where  $\tau$  is the effective tax rate for the given level of income, calculated using flat social security and progressive marginal tax rates (see Section 3.3 for details) and  $z$  is a composite subscript indicating one of the 325 income/productivity groups of agents determined by production sector, education and occupation.

Agents are assumed to be rational and to optimise their utility over life cycle using a standard utility function common to all households:

$$U(s, z) = \mathbb{E}_s \left[ \sum_{j=s}^{T+T^r} \pi_{j,t} \frac{C(j, z_j)^{1-1/\sigma_u}}{(1 - 1/\sigma_u)(1 + \rho)^{j-s}} \right], \quad (2)$$

where  $C(j, z_j)$  is the aggregate consumption across all production sectors at age  $j$ , conditional on being in income group  $z_j$  at that age,  $\sigma_u$  is the intertemporal elasticity of substitution, and  $\pi_{j,t}$  represents probability of surviving additional year at age  $j$ , which effectively acts as an additional discounting factor in addition to the pure time preference discounting  $\rho$ . The survival probability rate depends both on the probability of death at age  $s$ ,  $q_{s,t}$ , and the probability of surviving up to that age:

$$\pi_{s,j,t} = \prod_{k=s}^{j+1} (1 - q_{k,t}) \quad (3)$$

Agents maximise their lifetime utility subject to a dynamic lifetime budget constraint consisting of labour income  $I_{s,z,t}$  (if working), pension transfers  $p_{s,z,t}$  (if retired) and interest payments from asset holdings:

$$A_{s+1,z,t+1} = A_{s,z,t} (1 + r_{t+1}) + I_{s,z,t} - P_t^c C(s, z) \quad (4)$$

for workers and

$$A_{s+1,z,t+1} = A_{s,z,t} (1 + r_{t+1}) + p_{s,z,t} - P_t^c C(s, z) \quad (5)$$

for pensioners. Additionally,  $A_{1,z,t} = A_{T+T^r+1,z,t+1} = 0$ , i.e. agents start with no wealth and consume all of their remaining wealth in the last period of their lives. In the equations,  $P_t^c$  represents price of the aggregate

<sup>1</sup>The classification of educational attainment follows the International Standard Classification of Education (ISCE; UNESCO Institute for Statistics, 2012), industrial sectors are categorised following the Statistical Classification of Economic Activities in the European Community (NACE; European Commission, 2007), and occupations using the International Standard Classification of Occupations (ISCO; International Labour Organization, 2016).

consumption good. The maximisation procedure yields the standard time path of consumption:

$$\frac{C_t}{C_{t+1}} = \left[ \frac{1 + \rho}{1 + r_{t+1}} \frac{P_t^c}{P_{t+1}^c} \right]^{\frac{1}{\sigma_u}} \quad (6)$$

Each sector in the economy produces a representative good  $Q_{p,t}$  with price  $P_{p,t}^q$ . All of the goods can be purchased domestically ( $Q_{p,t}^d$  priced  $P_{p,t}^d$ ) and abroad ( $Q_{p,t}^f$  priced  $P_{p,t}^f$ ), i.e. no theoretical distinction is made between traded and non-traded sectors, although services are practically not traded following the real-world parametrisation (see Section 3.4). Following empirical findings from [CSO \(2018\)](#), agents' consumption behaviour is assumed to change with age – with younger individuals spending proportionally more on leisure, clothing or transport than older individuals, who spend more on housing or healthcare – but is otherwise constant across income groups  $z$ . Domestic and imported goods are considered imperfect substitutes and agents combine domestic and imported goods under the Armington trade assumption through a CES function. The composition of overall consumption then depends both on agents' preferences and the relative price of goods and services. Following [Lecca et al. \(2013\)](#):

$$Q_{s,p,t} = \delta_{s,p}^q \sigma_d \left( \frac{P_t^c}{P_{p,t}^q} \right)^{\sigma_d} C_t \quad (7)$$

$$Q_{s,p,t} = \gamma_{s,p} \left[ \delta_{s,p}^d P_{p,t}^d Q_{s,p,t}^\lambda + \delta_{s,p}^f P_{p,t}^f Q_{s,p,t}^\lambda \right]^{\frac{1}{\lambda}} \quad (8)$$

where  $\sigma_d$  is the income elasticity of demand;  $\lambda = (\sigma_a - 1)/\sigma_a$ , with  $\sigma_a$  being the Armington elasticity of substitution between domestic goods and imports from ROW;  $\gamma_{s,p}$  is a scaling parameter; and  $\delta_{s,p}^q$ ,  $\delta_{s,p}^d$  and  $\delta_{s,p}^f$  are share parameters. In words, the demand for any particular sectoral good  $Q_{s,p,t}$  depends on its relative price compared to other goods, while the amount purchased domestically and imported from abroad depends on the domestic and foreign prices.

### 3.2 Firm optimisation

The production sector consists of a representative firm in each sector  $p$ , producing goods and services  $X_{p,t}$  using intermediate inputs  $V_{i,p,t}$  in combination with its own value added  $Y_{p,t}$ . Value added is obtained through effective labour  $N_t$  and capital  $K_t$ , combined in a CES production function allowing their substitution as necessary:

$$Y_{p,t} = \left[ \delta_p^k K_{p,t}^\zeta + \delta_p^l L_{p,t}^\zeta \right]^{1/\zeta} \quad (9)$$

where  $\zeta = (\sigma_y - 1)/\sigma_y$ , with  $\sigma_y$  being the value-added elasticity of substitution; and  $\delta_p^k$  and  $\delta_p^l$  are share parameters. Intermediate goods and value added are then combined using a Leontief production function:

$$X_{p,t} = \min \left( \frac{Y_{p,t}}{\alpha_p^y}; \frac{V_{i,p,t}}{\alpha_p^v} \right), \quad (10)$$

where  $V_{i,p,t}$  is intermediate input produced in sector  $i$  and used in sector  $p$ , and  $\alpha_p^y$  and  $\alpha_p^v$  are scaling constants. That is, unlike value added, the final goods can be produced only using a constant ratio of intermediate inputs and value added.

Firms hire region-specific labour at the total cost of  $(1 + \tau^r) e_{s,z} w_{z,t}$ , where  $\tau^r$  is the social security contribution paid by firm and  $w_{p,t}$  is the sector-specific equilibrium wage, and rent physical capital paying

economy-wide rental rate  $r_t$ . The equilibrium prices can be derived using the firm's maximisation problem:

$$\frac{w_{p,t}}{P_{p,t}} = \frac{\delta_p^l}{1 + \tau^r} \left( \frac{Y_{p,t}}{L_{p,t}} \right)^{1-\zeta} \quad (11)$$

$$\frac{r_t}{P_t} = \delta^k \left( \frac{Y_t}{K_t} \right)^{1-\zeta} - \delta, \quad (12)$$

where  $K$  and  $L$  are the aggregate capital and effective labour stock per production sector and  $\delta$  is the depreciation rate of capital. The equilibrium wage is set to one in each sector initially, whereas  $e_{s,e,o,p}$  is set to the average gross wage in the Czech Republic in 2017 for each group  $\{s, e, o, p\}$ . In the subsequent periods,  $e_{s,e,o,p}$  remains constant to maintain the baseline differences among individuals with different characteristics, while the sectoral equilibrium wage, determined endogenously within the model, also starts to vary by age and productivity group based on relative changes in their representation in the overall sectoral labour force:

$$w_{s,z,t} = w_{t,p} \frac{L_{s,z,t}}{L_{s,z,1}} \frac{L_{p,1}}{L_{p,t}} \quad (13)$$

Prices of domestic goods and services,  $P_{p,t}^c$ , evolve over time, balancing changes in domestic demand and supply in each sector. Prices of foreign goods are considered constant over time.

### 3.3 Government

Government's role in the model consists of collecting contributions to the social security system, paying out pensions and purchasing goods and services both domestically and from abroad. For simplicity, the model assumes that the government's total consumption is always proportional to that of the households' in the same ratio as in  $t = 1$ . The initial pension system specification follows the actual implementation in the Czech Republic. Pension transfers are determined by income history of new pensioners and exogenously given replacement rates  $h_{z,t}$ , which are in form of marginal rates and decrease with income. An effective replacement rate,  $h_t^e$  can be calculated for each level of income using the income thresholds translated into the model as percentage of the average wage. Pension transfers in period  $t$  can be determined as:

$$p_{s,z,t} = h_t^e \sum_{j=1}^T \frac{I_{j,z,t-T+j}}{T} \quad (14)$$

The pension system budget is modelled to be in a balance initially. In the subsequent periods, pensions are indexed at the level of nominal wages, so that the pension replacement rates do not decrease, and the pension system budget may be in a deficit or surplus following the balance equation:

$$\sum_{s=1}^T \sum_{z \in Z} (\tau^r + \tau^e) I_{s,z,t} \mu_{s,z,t} \leq \sum_{s=T}^{T+T^r} \sum_{z \in Z} \kappa p_{s,z,t} \mu_{s,z,t}, \quad (15)$$

where  $\mu_{s,z,t}$  is the measure of generation  $s$  in productivity class  $z$  and year  $t$ ,  $\tau_t^e$  is the social security contribution paid by employers, and  $\kappa$  is a scaling parameter reflecting the discrepancy between the old-age dependency ratio in the model and in reality, caused by difference in life expectancy vs retirement age and the implicit assumption that every household of working age is employed in the model, as opposed to positive unemployment rates in reality. The scaling parameter is calculated endogenously within the model.

### 3.4 Calibration

The model is calibrated to correspond, in its initial steady state, to the Social Accounting Matrix (SAM) for the Czech Republic and the rest of the World in 2011 as obtained from the Global Trade Analysis Project (GTAP, [Aguilar et al., 2016](#)). Specifically, the parameters obtained are primarily the total expenditure on domestic and foreign goods and services by private households and the government, as well as the amount of



intermediate goods, capital and labour (differentiated by the five occupation groups) required for production. Data on average earnings, number of employees by productivity group  $z$ , and consumption patterns by age were obtained from the Czech Statistical Office for year 2017 (CSO, 2018). Other economic parameters, such as income and social security tax rates or pension replacement rates are set to their actual values in the Czech Republic as of March 2019.

The behavioural parameters were obtained from the relevant academic literature and databases. Following Zodrow et al. (2013), the rate of time preference  $\rho$  is set equal to 0.011, the intertemporal elasticity of substitution  $\sigma_u$  to 0.5 as estimated by Havranek et al. (2015); depreciation rate  $\delta$  to 4.46% following the Penn World Tables (Feenstra et al., 2015); marginal income tax rates vary across income classes between 1.9% and 35%, with social security taxes set to 6.5% paid by the employee and 25% paid by the employer as in reality; and the sector-specific value-added elasticity of substitution  $\sigma_y$ , the Armington elasticity of substitution  $\sigma_a$ , and the income elasticity of demand  $\sigma_d$  were obtained from Dimaranan et al. (2006) and are presented in Table 5. Changes in agents' consumption with age follow data on average annual expenses by expenditure group obtained by CSO (2018). A summary overview of the number of employees and median wage by sector are presented in Figure 5 in the Appendix.

Population predictions are based on the total population, mortality, and fertility indicators from United Nations (2017) for years 1996-2050. Following the model specification, fertility rates from year  $t - 18$  are used to reflect that  $s = 1$  corresponds to the real life age of 19. The initial mortality rates are set according to the UN data for 2019. From the second period onwards, size of the newborn generation evolves according to the ratio of fertility rates compared to the initial period, mortality rates evolve according to the UN data, and the population structure is defined endogenously within the model. The maximum age does not change. The population predictions highlight three separate trends: decrease in fertility, increase in mortality, and a shift in population structure with large cohorts reaching retirement age in the next decades. Data on migration patterns were obtained from the Eurostat database (European Commission, 2019) and are further described in the next section.

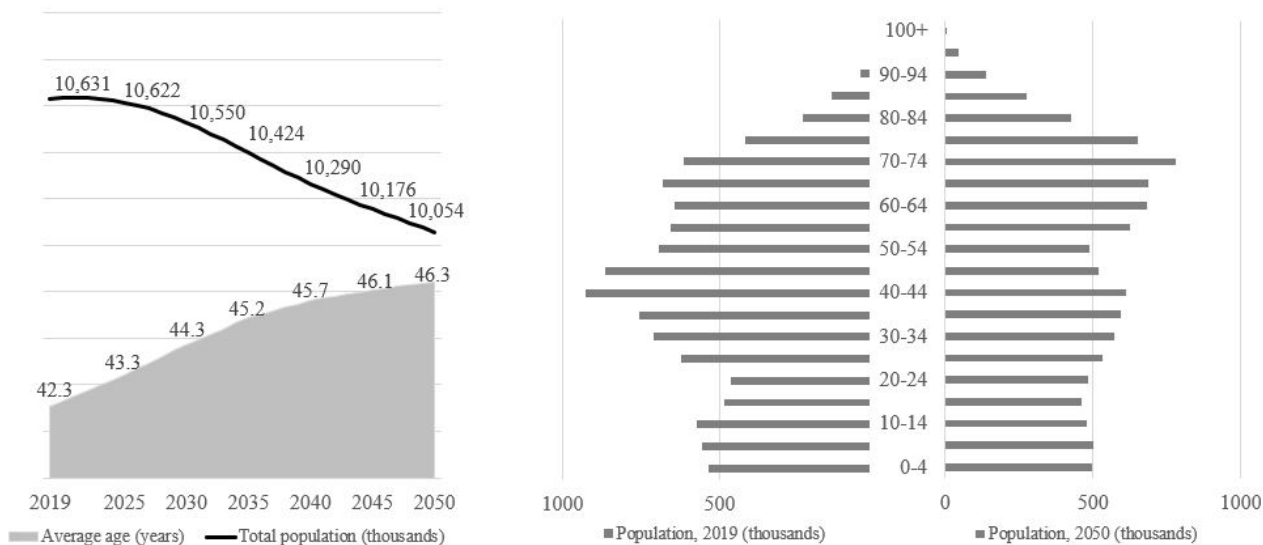
### 3.5 Scenarios

Three scenarios are considered in the study. In the first, baseline scenario, population development, including migration patterns, follows the medium variant projected by United Nations (2017); the scenario thus outlines patterns in the Czech economy if nothing unexpected happens. Since the population projections assume that the net number of migrants (i.e. the number of immigrants minus the number of emigrants) in the Czech Republic will remain more or less constant at the current approx. 60 thousand over the considered time period (i.e. the model assumes effectively no net immigration or emigration), changes in the total population effectively reduce to the difference in the birth and death rates, while the average age of the population as a whole is assumed to increase over time. The overall change in population size and the age structure in the initial (2019) and last period (2050) are depicted in Figure 1. The total population is expected to decrease by approx. 5.4% over the next 30 years, with the average age increasing by more than 4 years. Importantly, the large post-war cohorts currently aged 40-50 will substantially shift the old-age dependency ratio as they retire in the next decades.

The second scenario considers a large inflow of migrants from abroad. Given that the Czech Republic is relatively unattractive for foreigners outside of the Eastern Europe and former communist countries – more than 54% of the total 468 thousand foreign-born individuals in the Czech Republic in 2018 come from Slovakia, Ukraine and Vietnam (European Commission, 2019) – it is unlikely that the recent surge in immigration to other European countries such as Germany or France would be replicated in the Czech Republic. Instead, the scenario assumes that 90% of the 45 thousand Czechs aged 19+ living in the UK (42% of all such Czechs living in another EU/EFTA country) returns due to Brexit. This represents approx. 1% of all employees.

The third scenario assumes an opposite trend, specifically that 40% of all foreign-born individuals aged 19+ living in the Czech Republic (approx. 177 thousand, or nearly 5% of all employees) suddenly leave. In

Figure 1: Total population size and average age (left graph) and population age distribution (right graph) in the Czech Republic in 2019-2050 as used in the baseline scenario.



Source: [United Nations \(2017\)](#)

both cases, the model assumes the same proportion of individuals to migrate across all age categories (e.g. 90% of individuals aged 21-30, 90% of those aged 31-40 and so on). For simplicity, it also assumes that the distribution across industries, occupations and education groups corresponds to that of the Czech Republic as a whole. The migration is assumed to happen at the beginning of the second period, i.e.  $t = 2$ . For comparison purposes, none of the scenarios considers a shift in the default retirement age.

## 4 Simulation results

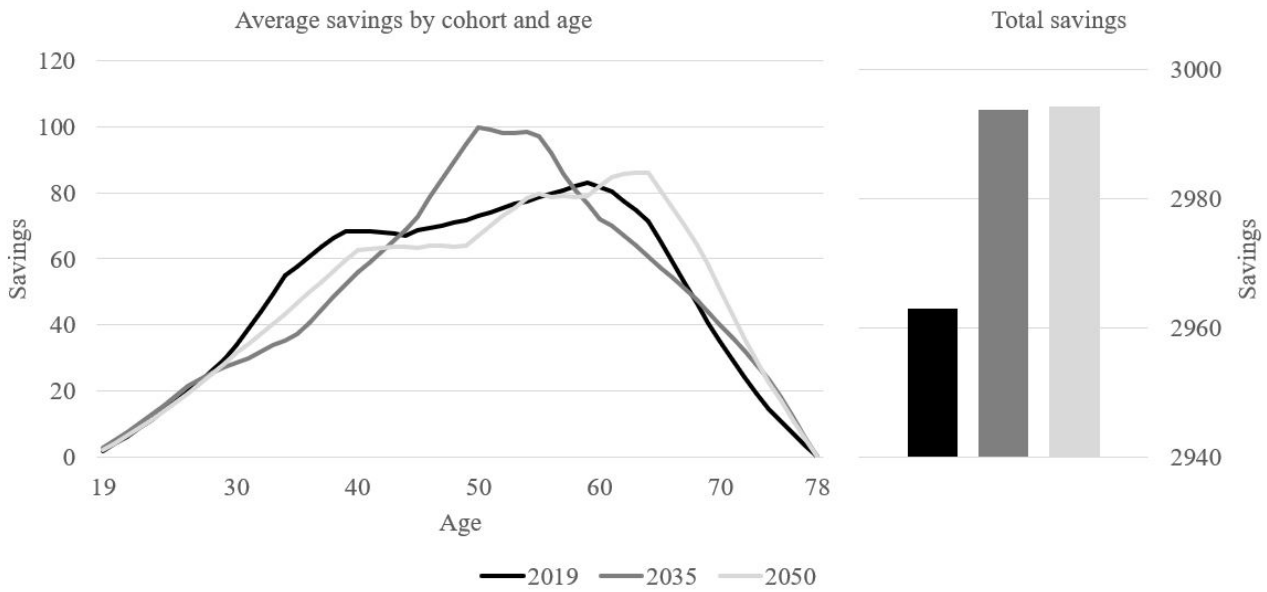
This section reports on the economic impacts of the assumed demographic changes on individuals in the Czech society, industrial sectors, and the economy as a whole. First, we analyse results of a baseline scenario following the UN population projections, followed by the two alternative scenarios characterised by higher-than-expected immigration and emigration.

### 4.1 Baseline scenario

The baseline scenario represents the expected population and migration development projected by [United Nations \(2017\)](#), characterised by a continuous population decrease (5.4% by 2050) and ageing (4 years increase in average age, see Figure 1).

In the model, population ageing results in comparatively higher costs of labour due to the shape of agents' income curves (see Figure 6 in the Appendix), which, on the agents' side, results in increasing income per capita and, on the firms' side, increases costs of production and prompts substitution effects in the production technology. In addition, agents' savings increase with age (up to retirement) as they put a share of their income aside for later use each month, resulting in higher average savings per capita as well in the slowly ageing society. Both factors contribute to higher overall accumulation of capital in the economy. We can see this in Figure 2; the total savings increase by 0.8% between 2019 and 2035 and then plateau, principally as a result of increasing number of individuals being in retirement and slowly consuming their savings. Indeed, we can see that the overall distribution of savings by age cohort shifts to the right, with the total amount of capital held by retirees increasing from 21% in 2019 to 27% in 2050. As a consequence, the equilibrium interest rate decreases from 3.5% to 3.36% over the assumed time period.

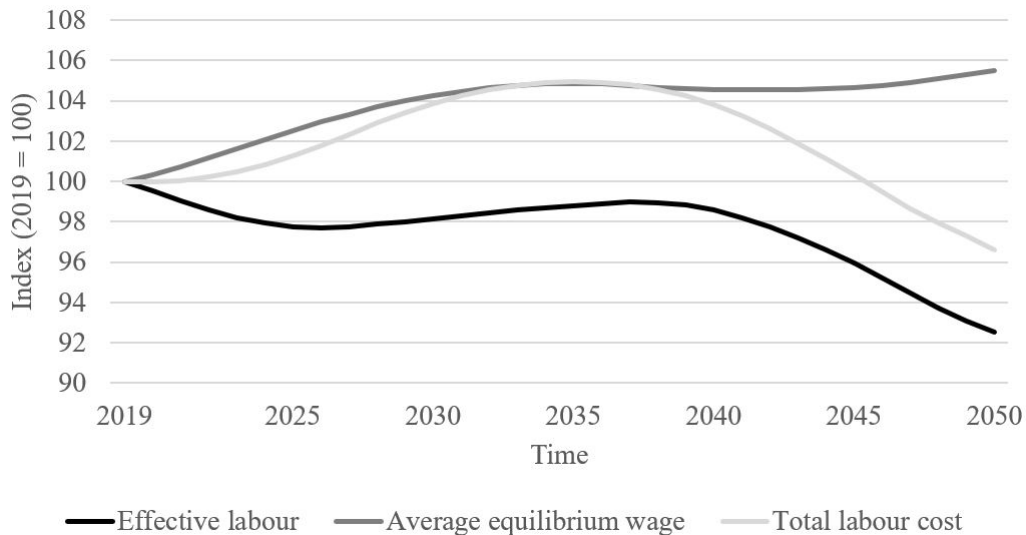
Figure 2: Savings, baseline scenario.



Values standardised using agents' maximum achieved average savings (maximum = 100).

At the same time, the economy faces a gradual decrease in labour force and amount of effective labour available to firms, only partially offset by higher proportion of older, more productive workers. This is shown in Figure 3. The average equilibrium wage (across all sectors in the economy) increases by more than 5% over time as a result, shifting the cost of production factors from capital towards labour. Analysing the results in detail, there are subtle differences in the elements affecting the average equilibrium wage and total labour cost; the initial dip in effective labour supply is immediately reflected in rising wages (years 2020-2025), yet the subsequent increase is not (years 2030-2035), due to a higher proportion of individuals reaching a peak in their consumption profile (around retirement age), resulting in higher domestic demand for goods and services (see also Table 1). Finally, the further decrease in labour supply leads to even higher equilibrium wage while the overall labour costs decrease due to the lower number of agents paid and, to a lesser extent, sector differentiation (see below).

Figure 3: Effective labour supply, average equilibrium wage and total labour costs, baseline scenario.



The underlying changes in the population size and age structure with the associated shifts in income and consumption preferences (see Section 3.1) have a profound effect on the aggregate domestic demand and, in turn, on performance of the 13 production sectors, as outlined in Table 1. Indeed, while the profile of aggregate domestic demand broadly follows that of aggregate effective labour supply, the matrix of sector-wide effects is more complex as sectoral demand figures cover both final and intermediate goods, i.e. the indirect effects of higher demand (and therefore production) in one sector on the other associated sectors. Here, the model results show an overall decrease of domestic demand for agricultural products and higher-than-average domestic demand for wholesale and retail trade goods, with other production sectors being affected equally, with an initial increase in domestic demand followed by a decrease below the 2019 values.

Table 1: Sectoral (table) and aggregate (graph) domestic demand changes (% change compared to 2019), baseline scenario.

	<b>Final household demand in 2019</b> (% of total)	<b>% change in demand</b>					
		2025	2030	2035	2040	2045	2050
Agriculture, forestry and fishing	1.0	-1.04	-0.11	0.69	-0.20	-3.16	-6.11
Mining and quarrying	0.1	0.22	0.84	1.38	1.16	-0.33	-2.14
Manufacturing	37.6	0.17	0.80	1.36	1.11	-0.45	-2.32
Electricity, gas, air	6.6	0.25	0.82	1.41	1.20	-0.28	-2.12
Water supply, sewerage	0.9	0.17	0.79	1.34	1.08	-0.48	-2.35
Construction and real estate	15.3	0.18	0.82	1.36	1.12	-0.40	-2.26
Wholesale and retail trade	11.2	0.54	1.08	1.54	1.50	0.39	-1.16
Transportation and storage	6.7	0.33	0.95	1.43	1.28	-0.08	-1.82
Recreation	3.1	0.24	0.88	1.36	1.15	-0.30	-2.09
Information and communication	6.0	0.22	0.77	1.40	1.15	-0.37	-2.26
Financial and insurance activities	1.6	0.16	0.72	1.38	1.11	-0.50	-2.43
Professional activities	5.5	0.19	0.80	1.39	1.16	-0.38	-2.28
Public	4.5	0.02	0.69	1.26	0.94	-0.77	-2.77

This is a result of the agricultural production, which includes primarily growing crops and animal production, being used virtually only in manufacturing (production of food and drinks) and direct sales to the end customer as opposed to wholesale and retail trade, which is intertwined in production processes of other industries (particularly manufacturing, transportation services and professional activities). Growing domestic demand in other sectors thus leads to a secondary, indirect increase in the total demand for the trade sector. Secondly, the income elasticity of demand of agricultural products is estimated at only 0.53 (see Table 5) as opposed to other industries with elasticities greater than one. The increase in equilibrium wages and average income is thus disproportionately reflected in rising domestic demand for agricultural products. Note, that since the economy is assumed to be in equilibrium in every period, the aggregate supply matches demand and the results also represent the total output generated.

Changes in the equilibrium wage by sector are depicted in Table 2, resembling the overall pattern from Figure 3 as well as the domestic demand changes by sector from Table 1. In particular, wages are projected to gradually increase over time across all sectors as a result of changes in demand and decline in labour force. Changes in the agricultural sector are again out of the ordinary as the projected wage increase is lower than in the other sectors, yet the proportional difference is smaller than in case of domestic demand changes. The trade sector also no longer stands out as other sectors are suggested to see higher increase in wage too, partially due to the capital/labour ratio differences across sectors, which is also visible in the variation between the final domestic demand and share of effective labour per sector. The overall changes in the aggregate demand, average capital/labour ratio and consumer price index across all sectors are presented in Figure 7 in the Appendix.

To sum up, the Czech economy is projected to be substantially affected by changes in the demographic

Table 2: Changes in the equilibrium wage by sector (% change compared to 2019), baseline scenario.

	Share of effective labour in 2019 (% of total)	% change in equilibrium wage					
		2025	2030	2035	2040	2045	2050
Agriculture, forestry and fishing	2.0	0.00	2.12	3.49	3.63	3.16	2.90
Mining and quarrying	0.7	0.00	2.66	3.95	4.06	3.89	4.30
Manufacturing	28.4	0.00	2.61	3.91	4.02	3.82	4.18
Electricity, gas, air	1.2	0.00	2.69	3.94	4.09	3.94	4.37
Water supply, sewerage	1.2	0.00	2.60	3.89	4.00	3.79	4.16
Construction and real estate	5.6	0.00	2.50	3.82	3.93	3.68	3.93
Wholesale and retail trade	12.6	0.00	2.56	3.86	3.98	3.75	4.06
Transportation and storage	6.2	0.00	2.41	3.75	3.86	3.57	3.72
Recreation	3.0	0.00	2.67	3.99	4.04	3.89	4.33
Information and communication	4.9	0.00	2.67	3.90	4.08	3.90	4.30
Financial and insurance activities	3.3	0.00	2.61	3.85	4.04	3.83	4.14
Professional activities	5.3	0.00	2.62	3.91	4.06	3.87	4.24
Public	25.5	0.00	2.48	3.79	3.91	3.65	3.87

structure – population shrinking and ageing – in the next decades, resulting in higher capital accumulation, lower labour supply associated with higher costs of each unit of labour, varying domestic demand for products and services, and higher prices. The agriculture, forestry and fishing industry is projected to be affected the most, with the total domestic demand for its products and services dropping by more than 6% by 2050. Other industries will see a modest increase and subsequent decrease in domestic demand and increase in unit cost of labour, pushing their production more towards utilising capital instead. The next section analyses the extent to which further changes in immigration/emigration patterns contribute to the overall picture.

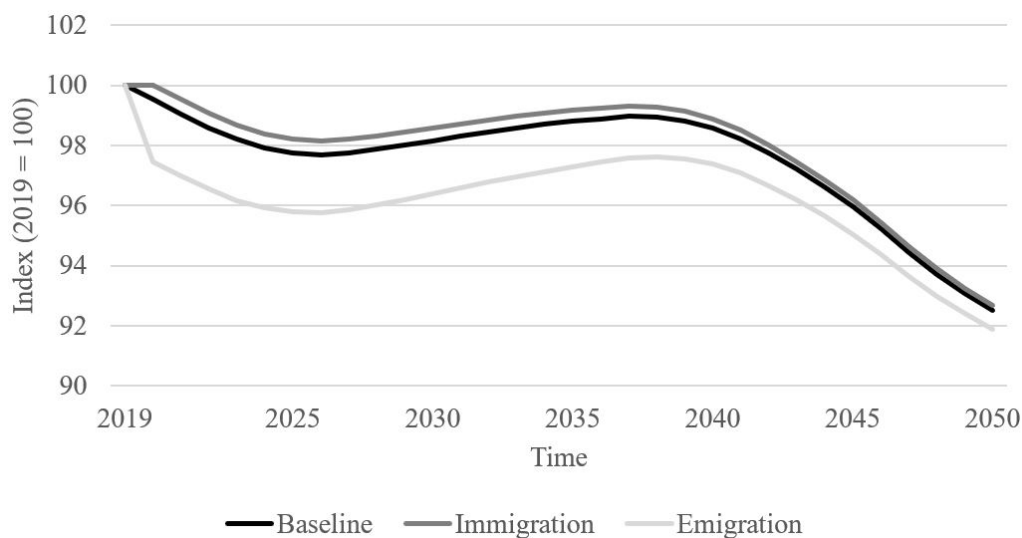
## 4.2 Scenarios with migration changes

This section considers two alternative scenarios with higher-than-expected inflow/outflow of workers to/from the Czech Republic in period  $t = 2$ , as described in Section 3.5 and depicted in Figure 4 using the amount of effective labour available in the economy. As we can see, the differences, amounting to approx. 0.5% higher and 2.5% lower labour supply in the immigration and emigration scenarios at first, respectively, gradually diminish over time as the older generations retire. The higher labour supply in the immigration scenario is insufficient to fully offset the natural decrease due to demographic changes, whereas the assumed emigration scenario leads to labour supply levels at the 2040 level in the baseline scenario.

The simulation results for the main variables of interest across the three scenarios are presented in Table 3. As in the baseline scenario results, all of the variables closely follow changes in the population size and structure, although the extent of changes varies by variable. Starting with the aggregate capital stock, the immigration scenario shows a slightly higher accumulation of capital than in the baseline scenario, as opposed to the emigration scenario, where the capital stock is below the baseline. Similarly, the aggregate domestic demand reflects the relatively higher/lower number of potential customers on the market, resulting in higher/lower aggregate domestic demand and, in turn through the market clearing conditions, higher/lower aggregate output in the economy in the immigration/emigration scenarios, respectively.

Importantly, for none of the variables the higher immigration offsets the natural demographic changes. On the contrary, we can use the projected loss in effective labour force to calculate an increase in net inflow of migrants in productive age that would maintain the current level of effective labour supply and thus help to prevent increasing unit labour costs and prices of goods and services. Such an increase in the number of workers would also lead to aggregate domestic demand above the 2019 levels and improve the old-age dependency ratio, lowering the pressure on the public pension system. Given the average of 1.3% decrease in the effective labour

Figure 4: Effective labour supply as a result of migration.



in the 2020-2035 period and 2.8% in the 2036-2050 period, net immigration would need to increase by at least 8 thousand individuals per year on average (15.4% of the current gross immigration, assuming the same distribution of skills as in the overall Czech population; [European Commission, 2019](#)) in the 2020-2035 period and by 17 thousand (33.5%) in the 2036-2050 period in order to offset the negative effects of population ageing and shrinking in the long term.

## 5 Conclusions

This study analyses the consequences of the expected demographic changes – population ageing and shrinking – as well as international migration on the example of the Czech Republic, with the aim to highlight the impacts on the consumers, production sectors and the economy as a whole and thus contribute to the debate on the need for a long-term solution regarding demographic changes. The Czech population is projected to see approx. 5.4% decrease in its overall population size over the next 30 years, with nearly null net immigration and the average age increasing by more than 4 years over the same period ([United Nations, 2017](#)). In absence changes in the default retirement age and with a large Generation X cohort slowly approaching retirement age, the demographic changes will result in disproportionately higher impact on the labour force, with the effective labour supply projected to decrease by more than 7% in the model.

The study utilises numeric simulations of a bespoke OLG-CGE model explicitly taking into account both the changes in the population size and age structure and the various interlinked activities in the economy: production, consumption, employment, taxes, savings and trade. This allows assessment of counterfactual scenarios in a perfectly controlled environment with only one changing exogenous variable – the Czech population. Three distinct scenarios are analysed: a baseline scenario corresponding to the medium variant projected by the United Nations; a scenario with higher-than-expected immigration, assuming arrival of 90% of the 45 thousand Czechs aged 19+ living in the UK; and a scenario with an unexpected outflow of 40% of the approx. 443 thousand foreign-born individuals aged 19+ living in the Czech Republic.

The model results show that, on the positive side, the ageing population will hold comparatively higher savings, promoting capital accumulation in the economy, and demand more goods and services, leading to higher output of the economy as a whole. This effect will, however, be eventually outweighed by the decreasing population size, leading to an ultimate drop in the aggregate domestic demand of more than 2% by 2050. At the same time, the decreasing labour supply will lead to higher equilibrium wages and cost of production,

Table 3: Main simulation results, comparison of the three scenarios.

Scenario	Variable	Year						
		2019	2025	2030	2035	2040	2045	2050
<b>Baseline</b>	Effective labour	100.0	97.8	98.2	98.8	98.6	96.0	92.5
	Capital stock	100.0	102.2	102.7	103.0	103.2	103.1	102.4
	Equilibrium wage	100.0	102.5	104.3	104.9	104.6	104.6	105.5
	Aggregate demand	100.0	100.2	100.6	101.1	101.1	99.9	97.8
	Consumer price index	100.0	99.9	99.8	99.6	99.6	100.1	101.0
<b>Immigration</b>	Effective labour	100.0	98.2	98.6	99.2	98.9	96.2	92.7
	Capital stock	100.0	102.7	103.2	103.5	103.7	103.6	102.5
	Equilibrium wage	100.0	102.5	104.2	104.9	104.6	104.7	105.6
	Aggregate demand	100.0	100.7	101.1	101.6	101.5	100.2	98.1
	Consumer price index	100.0	99.8	99.6	99.4	99.4	100.0	100.9
<b>Emigration</b>	Effective labour	100.0	95.8	96.4	97.3	97.4	95.0	91.9
	Capital stock	100.0	100.3	100.7	101.0	101.3	101.4	100.9
	Equilibrium wage	100.0	102.7	104.3	104.7	104.3	104.3	104.9
	Aggregate demand	100.0	98.3	98.7	99.4	99.5	98.6	96.8
	Consumer price index	100.0	100.8	100.6	100.3	100.2	100.7	101.5

All results shown as index of the 2019 baseline scenario value for the given variable.

which will force firms to substitute some of the labour input in their production for capital, increasing the average capital/labour ratio by 10% by 2050.

The impacts are estimated to differ by sector as a result of shifting consumer preferences with age and complex interrelations between production sectors, where production in one sector often requires substantial intermediate inputs from another. The agricultural sector is projected to be affected the worst, with domestic demand of its products decreasing by more than 6% by 2050, particularly due to low income elasticity of demand. The trade sector, on the other hand, is estimated to see an increase in domestic demand up until 2046, with the other sectors seeing approx. 1.5% increase in domestic demand at first, followed by more than 2% decrease by 2050. The effects are then amplified in the scenario with unexpected emigration, while the assumed higher immigration in the other alternative scenario is insufficient to outweigh the long-term negative effects. Overall, net immigration would need to increase by at least 8 thousand individuals per year on average in the 2020-2035 period and by 17 thousand in the 2036-2050 period to negate the long-term demographic changes.

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## Appendix

The following text expands on the model description from Section 3 and provides additional details on the computation methodology.

### Model equilibrium

The economy is assumed to be in an equilibrium in each period. The concept of equilibrium uses a recursive representation of the consumer's problem following Heer and Mauner (2009) and is characterised by the following properties:

1. Individual and aggregate behaviour are consistent:

$$N_t = \sum_{s=1}^T \sum_{z \in Z} e_{s,z} w_{z,t} \quad (16)$$

$$K_t = \sum_{s=1}^{T+T^R} \sum_{z \in Z} A_{s,z,t} \quad (17)$$

$$C_t = \sum_{s=1}^{T+T^R} \sum_{z \in Z} C(s, z, t) \quad (18)$$

2. Agents' dynamic programs and firms' optimisation problems are solved by satisfying Equations (2)–(13) using the relative prices  $w_t, r_t$ , pensions, and the individual policy rules  $C_s(\cdot)$  and  $A_{s+1}(\cdot)$ .
3. The goods market clears:

$$X_t + M_t = C_t + E + K_{t+1} - (1 - \delta)K_t, \quad (19)$$

where  $M_t = \sum_{p=1}^P Q_{p,t}^f$  are the total imports, sum of sectoral imports across all sectors  $p = \{1..P\}$ , and  $E$  are the total exports, set constant at their  $t = 1$  level.

4. Intermediate goods  $V_{i,p,t}$  and value added  $Y_{p,t}$  are sufficient to generate production in each sector:

$$X_t \leq \alpha_p^y \cdot Y_{p,t} \quad (20)$$

$$X_t \leq \alpha_p^v \cdot V_{i,p,t} \quad (21)$$

5. Prices of domestic goods and services  $P_{p,t}^c$  are set so that the sectoral domestic goods market clears given Equations 7-8:

$$X_{p,t} = Q_{p,t}^d + E_p \quad (22)$$

### Solution method

The simulation algorithm used in this study is based on Stepanek (2019), which follows earlier works of Heer and Mauner (2009) and Nishiyama and Smetters (2007), and further developed to work in a multi-sectoral environment following principally the works of Garau et al. (2013) and Lecca et al. (2013). It utilises value function iteration to compute agents' policy functions governing their optimal consumption and savings patterns conditional on the economic situation in each period. With a set of policy functions for each group of agents  $z$ , firms set prices  $P_{p,t}^c$  to put the sectoral demand and supply in equilibrium.

Specifically, the agent's decision functions are calculated using backward induction, i.e. by analysing the optimal behaviour in the last period of agent's live and, conditional on that, in all preceding periods. Let  $V_s(A_{s,z}, z_s)$  be the value of the objective function of an  $s$ -year old agent from group  $z$  with wealth  $A_{s,z}$ .

$V_s(A_{s,z}, z)$  is defined as the solution to the dynamic program:

$$V_s(A_{s,z}, z) = \max_{A_{s+1}, c_t} \{U(s, z) + \pi_s \mathbb{E}[V_{s+1}(A_{s+1}, z)(1 + \rho)^{-1} \mid A_{s,z}, z]\} \quad (23)$$

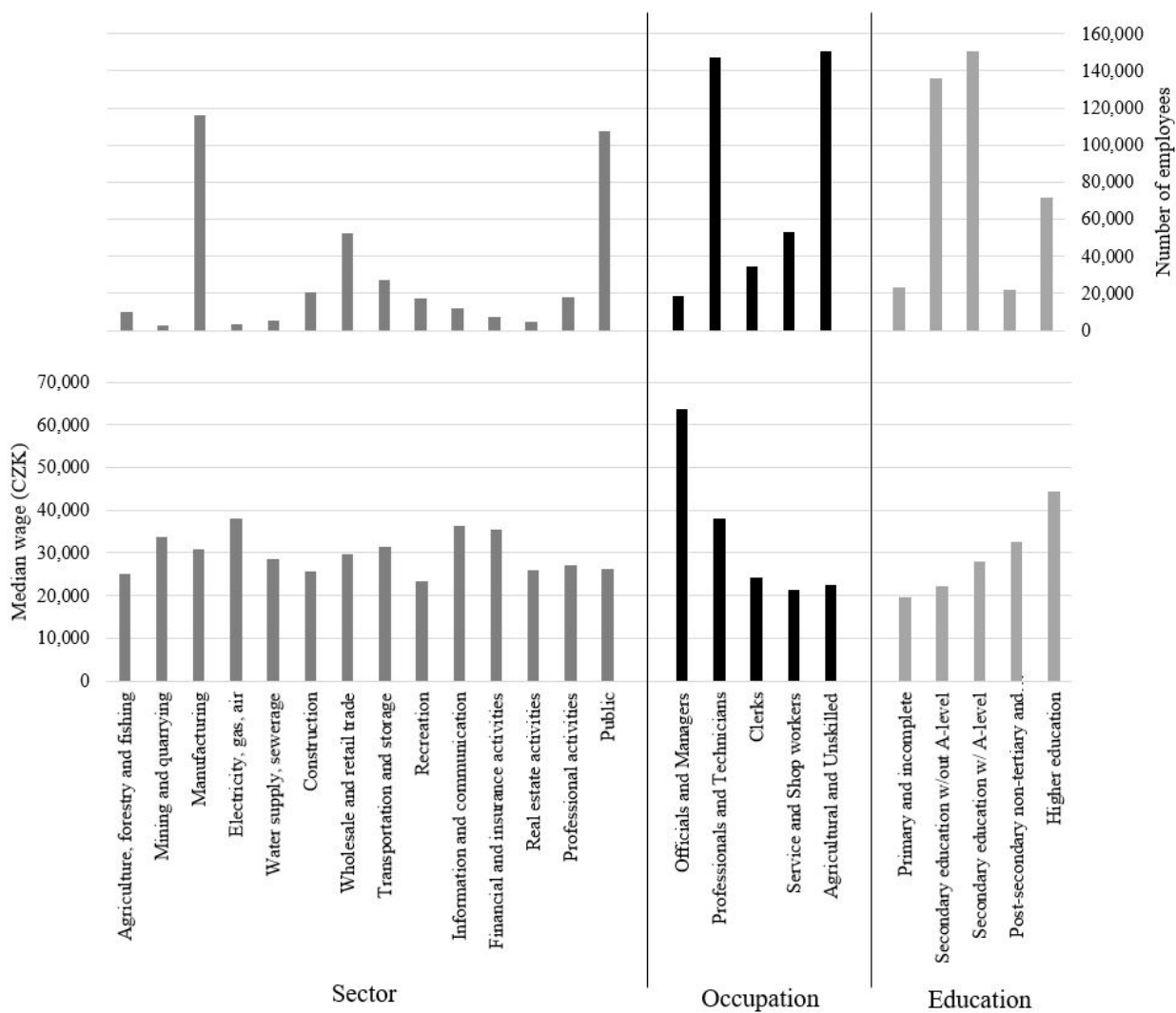
That is, subject to the budget constraints, optimal decision rules for consumption and next-period capital stock are functions of wealth and the idiosyncratic productivity shock, and associated with every optimal next period capital stock  $A_{s+1}(A_{s,z}, z)$  is an optimal consumption policy  $C(s, z)$ . Consequently, in each period, all agents can calculate the optimal aggregate consumption and saving behaviour in that period given their age, income group, probability of death, and other variables in the model.

Given the total consumption policy  $C(s, z)$ , agents then determine their sectoral consumption given the set of prices  $P_{p,t}^c$ , as well its distribution over domestic and foreign supply (see Equations 7-8). At the same time, firms use the available capital and effective labour to produce goods and services for domestic consumption and export. If there is a mismatch between demand and supply at the sectoral and/or aggregate level, prices  $P_{p,t}^c$ ,  $w_t$  and  $r_t$  adjust accordingly.

The main simulation process can thus be characterised as follows:

1. Parametrise the model so that the outputs correspond to the empirical data in period  $t = 1$ .
2. In each period, use  $t = 1$  parametrisation as a starting point and demographic changes as a source of variation to compute  $w_t$  and  $r_t$  and, consequently, the set of endogenous parameters, such as pensions.
3. Compute the household's decision functions by backward induction, resulting in the optimal aggregate consumption and saving behaviour for each cohort alive in period  $t$ .
4. Calculate the optimal sectoral demand and supply of goods and services.
5. Update prices  $P_{p,t}^c$ ,  $w_t$  and  $r_t$  to increase/decrease the amount of consumption, savings and production so to minimise the demand/supply differences.
6. Repeat steps 2-5 until the sectoral and aggregate demand and supply are in balance.

Figure 5: Total number of employees and median wage by sector, occupation group and educational attainment.



Source: CSO (2018)

Table 4: Industry, education and occupation classifications

Original classification		Adjusted classification		
Category name	ID	Category name	ID	ID
<b>Production sectors</b>	Agriculture, forestry and fishing	1	Agriculture, forestry and fishing	1
	Mining and quarrying	2	Mining and quarrying	2
	Manufacturing	3	Manufacturing	3
	Electricity, gas, air	4	Electricity, gas, air	4
	Water supply, sewerage	5	Water supply, sewerage	5
	Construction	6	Construction and real estate (groups 6, 12)	6
	Wholesale and retail trade	7	Wholesale and retail trade	7
	Transportation and storage	8	Transportation and storage	8
	Accommodation and food services	9	Recreation (groups 9, 18)	9
	Information and communication	10	Information and communication	10
	Financial and insurance activities	11	Financial and insurance activities	11
	Real estate activities	12	Professional activities	12
	Professional activities	13	Public (groups 14, 15, 16, 17, 19)	13
	Administrative and support services	14		
	Public administration and defence	15		
	Education	16		
	Human health and social work	17		
	Arts, entertainment and recreation	18		
	Other service activities	19		
<b>Educational attainment levels</b>	Primary and incomplete education	1	Primary and incomplete education	1
	Secondary education without A-levels	2	Secondary education without A-levels	2
	Secondary education with A-levels	3	Secondary education with A-levels	3
	Post-secondary and bachelor's education	4	Post-secondary and bachelor's education	4
	Higher education	5	Higher education	5
<b>Occupation groups</b>	Armed forces occupations	1	Officials and Managers	1
	Managers	2	Professional, Technicians (groups 3, 4)	2
	Professionals	3	Clerks	3
	Technicians and associate professionals	4	Service and Shop workers	4
	Clerical support workers	5	Agricultural and Unskilled (groups 7, 8, 9, 10)	5
	Service and sales workers	6		
	Skilled agricultural workers	7		
	Craft and related trades workers	8		
	Plant and machine operators	9		
	Elementary occupations	10		

Note: The sectoral names were shortened for presentation purposes. The full classifications are available in [UNESCO Institute for Statistics \(2012\)](#), [European Commission \(2007\)](#) and [International Labour Organization \(2016\)](#).

Table 5: Elasticity parameters used in the model.

Sector	$\sigma_a$ <sup>a</sup>	$\sigma_y$ <sup>b</sup>	$\sigma_d$ <sup>c</sup>
Agriculture, forestry and fishing	0.66	-3.55	0.53
Mining and quarrying	0.85	0.21	1.09
Manufacturing	0.69	0.21	1.09
Electricity, gas, air	0.64	0.21	1.09
Water supply, sewerage	0.64	0.21	1.09
Construction	0.47	0.29	1.09
Wholesale and retail trade	0.47	0.40	1.16
Transportation and storage	0.47	0.40	1.04
Recreation	0.47	0.21	1.14
Information and communication	0.47	0.21	1.04
Financial and insurance activities	0.47	0.21	1.16
Professional activities	0.47	0.21	1.14
Public	0.47	0.21	1.14

<sup>a</sup> Armington elasticity of substitution.

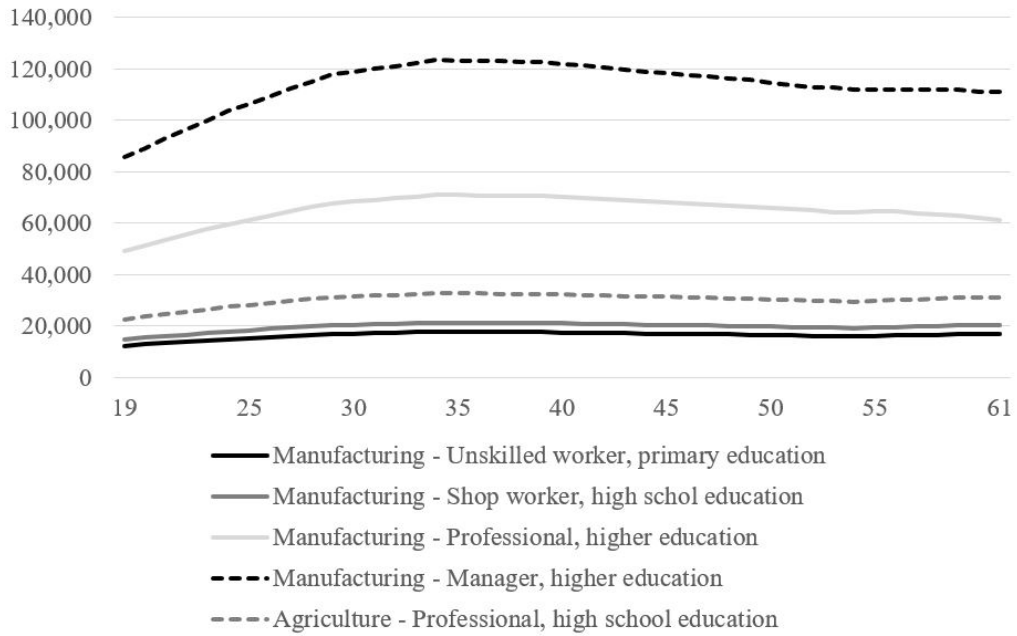
<sup>b</sup> Value-added elasticity of substitution.

<sup>c</sup> Income elasticity of demand.

Source: [Dimaranan et al. \(2006\)](#)

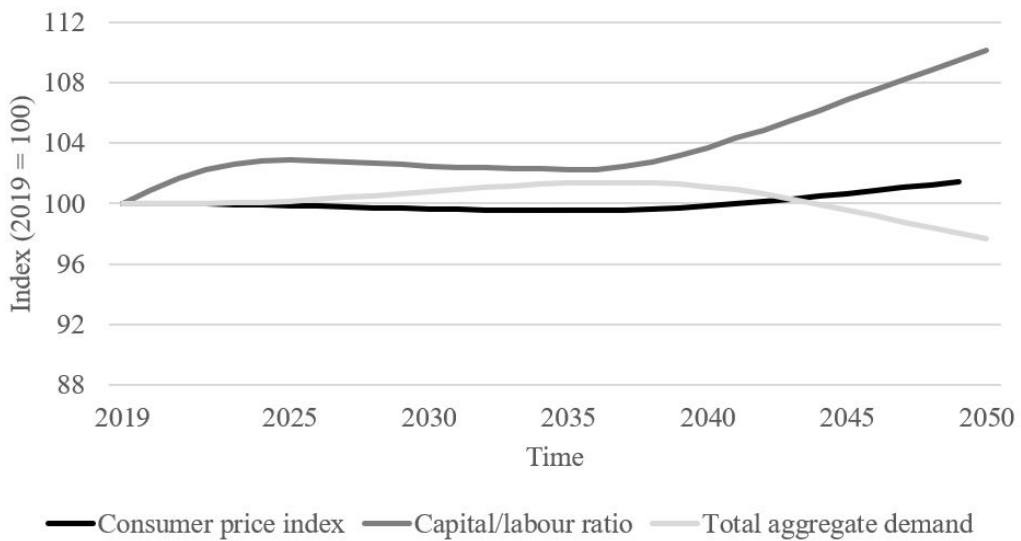


Figure 6: Selected income profiles used in the model.



Source: CSO (2018)

Figure 7: Aggregate demand, capital/labour ratio and price changes, baseline scenario.



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