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BANK-SOURCED TRANSITION MATRICES: ARE BANKS' INTERNAL CREDIT RISK ESTIMATES MARKOVIAN?

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Bank-Sourced Transition Matrices: Are Banks' Internal Credit Risk Estimates Markovian?

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Abstract:
This study provides new insights into banks' credit risk models by exploring features of their credit risk estimates and assessing practicalities of transition matrix estimation and related assumptions. Using a unique dataset of internal credit risk estimates from twelve global A-IRB banks, covering monthly observations on 20,000 North American and EU large corporates over the 2015-2018 time period, the study empirically tests the widely used assumptions of the Markovian property and time homogeneity at a larger scale than previously documented in the literature. The results show that internal credit risk estimates do not satisfy these assumptions as they show evidence of both path-dependency and time heterogeneity. In addition, contradicting previous findings on credit rating agency data, banks tend to revert their rating actions.

JEL: C12, G12, G21, G32
Keywords: Risk management, credit risk, transition matrices

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1 Introduction

Credit risk, identified by Bank for International Settlements as the potential that a bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms, is one of the most researched topics in finance (Lando, 2009). The increased interest in credit risk research during the last two decades has been driven by development of portfolio risk measurement, growing credit derivatives trading, regulatory concerns and Basel II implementation.

Credit transition matrices are essential components of credit risk modelling and have been extensively covered in the prior literature (e.g. Bangia, Diebold, Kronimus, Schagen, & Schuermann, 2002; Jafry & Schuermann, 2004; Nickell, Perraudin, & Varotto, 2000; Trück & Rachev, 2009). They are used to characterise the expected changes in credit quality of obligors and have many practical applications including portfolio risk assessment, modelling of credit risk premia term structure, and pricing of credit derivatives (Bangia et al., 2002). Transition matrices are also used in bank stress-testing, which took on a prominent role within the regulatory toolkit after the financial crisis in 2008 (e.g. Varotto, 2012). Recently, transition matrices gained attention due to modelling of life-time credit losses required by the new IFRS9 and CECL regulations as outlined in several industry papers including Conze (2015), Cziraky and Zink (2017) and Chawla, Forest Jr, and Aguais (2015). Transition matrices are estimated using past credit risk data and the main sources of transition matrices are currently credit rating agencies. Banks also construct transition matrices using internal credit rating data, yet there is no public source for such matrices based on bank data, as bank-specific matrices are considered to be sensitive information. We propose estimation of transition matrices based on aggregated data sourced from multiple global advanced internal rating based (A-IRB) banks.

Given the limited information on banks' internal credit assessment systems and their potential heterogeneity, characteristics of banks' models and credit estimates need to be thoroughly investigated to ensure that bank-sourced transition matrices are unbiased. This study provides a unique insight into the essential features of banks' internal credit risk estimates and applicability of the existing transition matrix estimators on this type of data. Using a one-of-a-kind dataset of credit risk estimates from 12 global A-IRB banks, we empirically test the two main assumptions applied in the most common transition matrix estimators - the Markovian property and time homogeneity assumptions - and assess banks' credit rating behaviour patterns at a larger scale than covered by previous literature, which mostly covers local clusters of banks, e.g. Gómez-González and Hinojosa (2010) and Lu (2007).

The study focuses on banks' main corporate models and data on large North American and EU corporates. The final dataset includes monthly observations on 800-2,000 corporates from each bank over the 2015-2018 time period and covers more than 20,000 unique entities. The Markovian property is tested using conditional transition matrices (Bangia et al., 2002) and panel probit (Fuertes & Kalotychou, 2007) investigating momentum and duration effect hypotheses. We compare individual annual matrices to their long term average using the χ^2 test to assess the time homogeneity assumption (Trück & Rachev, 2009).

We show that the internal probability of default estimates from most of the 12 researched banks do not satisfy the tested assumptions as they reveal both path-dependency and time heterogeneity, effectively reducing the set of applicable transition matrix estimators that may be used on such data. This is in line with other studies using credit rating agencies data (see e.g. Bangia et al., 2002, or Nickell et al., 2000). Further, our analysis suggests that banks tend to revert their rating actions, which is in contradiction with findings on rating processes by credit rating agencies (see e.g. Lando & Skødeberg, 2002; Bangia et al., 2002; or Fuertes & Kalotychou, 2007). Our findings are essential for estimation of transition matrices based on banks' internal credit risk estimates as they show that one must employ more complex estimators (see e.g. Frydman & Schuermann, 2008, or Wei, 2003) that, unlike the more simplistic estimators, do not rely on the two assumptions.

2 Assumptions and Estimators

In this section we review the main assumptions and transition matrix estimators, introduce the concepts of Markovian property and time homogeneity, and present the estimation methods and approaches to matrix comparison. Each subsection then contains an overview of the relevant literature and technical details.

2.1 Notation and Main Assumptions

When defining a transition matrix, we consider a rating space $S = 1, 2, ..., K$ where 1 and $K-1$ represent the best and the worst credit quality, respectively, and K represents a default. $R(t)$ denotes rating of an entity at time t and takes values from the rating space S.

The $(K \times K)$ transition matrix $Q(t, t+\delta)$ describes all possible transitions and their probabilities over time horizon $(t, t+\delta)$:

$$Q(t, t+\delta) = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1K} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2K} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{K1} & p_{K2} & p_{K3} & \cdots & p_{KK} \end{bmatrix}, \tag{1}$$

where p_{ij} represents the transition probability from state i to state j within time period $(t, t+\delta)$ when $i \neq j$ and the probability of rating being preserved when $i = j$. The rows represent ratings of the entities at time t while the columns represent ratings at time $t+\delta$. It is often assumed for the sake of simplicity that the last row with defaults is an absorbing state, which means that defaulted entities cannot emerge from default. The transition rates satisfy $p_{ij} \geq 0$ for all i, j and $p_{ii} \equiv 1 - \sum_{j=1, j\neq i}^{K} p_{ij}$ for all i.

Starting with the works of Jarrow and Turnbull (1995) and Jarrow, Lando, and Turnbull (1997), the industry standard in description of credit rating dynamics has been based on time homogeneous Markov chain models. Consequently, one of the most discussed topics in the field of transition matrices is the Markovian chain assumption suggesting that the estimated migration probabilities are independent of the previous rating history. In addition, the assumption of time homogeneity suggests that the probabilities are constant over time. Even though the validity of the assumptions has been challenged by number of empirical studies (e.g. Kavvathas, 2001; Lando & Skødeberg, 2002; Nickell et al., 2000), the assumptions significantly simplify estimation of transition probabilities, and estimations based on the assumptions provide valuable insight into rating systems of banks.

Markovian Property Definition A stochastic process satisfies the first-order Markovian property if the probability of transition to a future state j depends only on the current state and is independent of the rating history:

$$P[R(t+\delta)=j|R(t),R(t-1),R(t-2),...]=P[R(t+\delta)=j|R(t)], \tag{2}$$

where $R(t)$ denotes rating of an entity at time t and takes the values from the rating space S.

Time Homogeneity Definition A Markovian chain is time homogeneous if transition probabilities depend only on the time horizon of interest, δ, and not on the initial date:

$$Q(\delta)\equiv Q(t,t+\delta)=Q(t-k,t-k+\delta). \tag{3}$$

Time homogeneous Markovian chain satisfies

$$P[R(t+\delta)=j|R(t)=i]=P[R(t-k+\delta)=j|R(t-k)=i]. \tag{4}$$

As explained in Fei, Fuertes, and Kalotychou (2012), time homogeneity implies that an n-year migration matrix is given by the n^{th} power of an annual one, defined as $Q(t,t+n)=Q(t,t+1)^n$ or the matrix product of n copies of $Q(t,t+1)$, and it allows the user to make statistical inference. Time homogeneous transition matrices are an important tool for measuring credit risk as they can be used for forecasting the future.

2.2 Estimation of Transition Matrices

Transition matrices can be estimated using either cohort or hazard rate (duration) approaches, which differ in their conception of time: the cohort approach is a discrete-time framework, whereas the hazard rate approach is a continuous-time framework. The basic versions of both of the estimators are based on the Markov and time homogeneity assumptions.

The cohort approach looks at the number of entities that migrated from rating i to rating j over a specific period of time $(t,t+\delta)$, where δ is a discrete number. $N_i(t)$ denotes the number of entities with rating i at time t, $R(t) = i$, and $N_{ij}(t, t+\delta)$ is a subset of such entities that migrated to rating j within the period $(t, t+\delta)$, $R(t) = i$ and $R(t+\delta) = j$.

Assuming a time homogeneous Markov rating process, the maximum-likelihood (ML) estimator of the credit migration probability is:

$$\hat{p}_{ij} \equiv \hat{p}_{ij}(\delta) = \sum_{t=1}^{T} w_i(t)\hat{p}_{ij}(t, t+\delta) = \frac{\sum_{t=1}^{T} N_{ij}(t, t+\delta)}{\sum_{t=1}^{T} N_i(t)} = \frac{N_{ij}}{N_i}, \tag{5}$$

where $w_i(t) = N_i(t)/\sum_{t=1}^{T} N_i(t)$ are yearly weights. Therefore, $\hat{p}_{ij}$ can be simply computed as the total number of migrations over a specific period from grade i to j, divided by the total number of obligors that were in grade i at the start of the sample period.

Anderson and Goodman (1957) show that the ML estimator is biased but consistent; large enough datasets thus allow estimation of consistent transition matrices. Bangia et al. (2002) conclude, based on the estimated coefficient of variation of transition matrix elements, that only the diagonal elements are estimated with high precision. Another weakness of the cohort method is that it neglects within-year transitions and rating duration information.

An alternative approach is a hazard rate (duration) estimator capturing transitions occurring at any time and taking into account duration and entities entering or ending the period with 'not rated' status. The approach also estimates positive probabilities of extreme transitions that are not observed in the data but can occur given a large dataset as illustrated by Lando and Skødeberg (2002). However, it requires higher frequency of observed data and the calculation is based on a more complex generator matrix. The cohort method is less efficient and Jafry and Schuermann (2004) find that the differences between the cohort and duration methods are larger than between different duration methods. The findings are confirmed by Fuertes and Kalotychou (2007).

Other estimation methods are used by Hanson and Schuermann (2006), who assess confidence intervals around probabilities of default using analytical approaches and parametric/ non-parametric bootstrap methods, finding that bootstrap intervals for the continuous estimates are tight compared to intervals around cohort estimators. In other studies, Stefanescu, Tunaru, and Turnbull (2009), Kadam and Lenk (2008), and McNeil and Wendin (2007) use Bayesian techniques for estimation of default and transition probabilities to mitigate the effect of data sparsity. There are multiple studies providing an alternative for data that are either time heterogeneous or non-Markovian. Bluhm and Overbeck (2007) calibrate a non-homogeneous time-continuous Markov chain. Frydman and Schuermann (2008) use Markov mixtures, and Giampieri, Davis, and Crowder (2005) model the occurrence of defaults within a bond portfolio as a simple hidden Markov process.

In our study, we use the cohort method for testing of the Markovian property and time homogeneity assumption despite its lower efficiency. Since we do not intend to produce a matrix best representing the credit risk transitions, the lower efficiency is not an essential issue for us, and the method allows us to focus on examination of characteristics of rating systems of individual banks and their comparison.

2.3 Comparing Transition Matrices

The simplest approach to transition matrices comparison uses the Euclidean distance (based on the average absolute difference) and the average root-mean-square difference between corresponding cells of the matrices. However, Jafry and Schuermann (2004) point out that these methods provide only a relative rather than absolute comparison, and thus only limited information on magnitude of the difference. As a result, they propose a singular value decomposition (SVD) metric based on a mobility matrix (defined as the original matrix minus an identity matrix) that approximates the average probability of migration and facilitates a meaningful comparison between transition matrices.

The metric is defined as the average of the singular values of the mobility matrix:

$$M_{SVD}(P) = \frac{\sum_{i=1}^{n} \sqrt{\lambda_i(\hat{P}'\hat{P})}}{n}, \tag{6}$$

where $\hat{P}$ is the $n \times n$ mobility matrix defined as the original transition matrix minus the identity matrix of the same dimension, i.e. $\hat{P} = P - I$, and $\lambda_i(...)$ denotes the i-th largest eigenvalue.

The SVD method captures the probability and size of migration but not the direction of the migration. Hence, we also save the percentage of entities upgrading and downgrading.

3 Analytical Approach

This section describes the applied methodology of testing time homogeneous Markov chain assumption. The text is thematically structured; we introduce the tests used for detection of the non-Markovian behaviour and time heterogeneity and review the relevant literature and technical details.

3.1 Testing the Markovian Property

The Markovian property of rating processes is challenged by multiple studies investigating the presence of non-Markovian effects using a variety of methodologies detecting the momentum effect and duration effects defined below. Specifically, Lando and Skødeberg (2002) and Kavvathas (2001) employ a semi-parametric multiplicative hazard model, Fuertes and Kalotychou (2007) and Lu (2012) use logit models, Bangia et al. (2002) estimate transition matrices dependent on previous developments and compare them, and Krüger, Stötzel, and Trück (2005) test the Markov property based on Likelihood Ratio Test and conditional transition matrices. Most of the studies find a strong support for a downgrade momentum and evidence of duration effect, although Krüger et al. (2005), who, unlike most of other studies, do not use rating agencies data and analyse a rating system based on balance-sheet data of Deustche Bundesbank, conclude that upgrades are more likely to be followed by downgrades and vice versa and identify a second-order Markov behaviour. A significant duration effect is described in e.g. Fuertes and Kalotychou (2007), Lando and Skødeberg (2002), or Kavvathas (2001), but the evidence on direction of the effect is mixed.

3.1.1 Momentum Effect

Rating momentum presupposes that prior changes in credit ratings have a predictive power regarding the direction of future rating changes. Specifically, downgrade momentum suggests that an entity downgraded to a given credit category is more likely to be downgraded than upgraded in the future. There are several approaches to testing the momentum effect; we present a comparison of conditional transition matrices and panel probit model.

Conditional Transition Matrices In our study, we first follow the approach shown by Bangia et al. (2002) and analyse up and down momentum transition matrices. The entities are separated into three groups based on their rating experience from the previous year - upgrading, downgrading, and stable. The groups are then followed for a year to capture subsequent rating changes and to construct group-specific transition matrices - up, down, and maintain momentum transition matrices.

We focus on a comparison of the conditional up and down momentum transition matrices to highlight the differences in rating behaviour following an upgrade and a downgrade. The comparison is based on the singular value decomposition (SVD) metric.

Annual credit migration probability is calculated based on transitions observed over the last 12 months covered by the datasets and the counts are conditioned by previous movements. The credit migration probability in an up momentum transition matrix is defined as

$$\hat{p}^u_{ijt}(t, t+12) \equiv \frac{N^u_{ij}(t, t+12)}{N^u_i(t)}, \tag{7}$$

where $N^u_i(t)$ denotes the number of entities with rating i at time t that were upgraded within the period $(t-13, t-1)$, $R(t) = i$, and $N^u_{ij}(t, t+12)$ is a subset of such entities that migrated to rating j within the period $(t, t+12)$, $R(t) = i$ and $R(t+\delta) = j$.

Panel Probit As a second step, we follow Fuertes and Kalotychou (2007) and Lu (2012) and test the Markovian chain assumption using a probit model to detect momentum effects in the internal rating data from global banks. Lu (2012) examines the rating momentum over 9 years and Fuertes and Kalotychou (2007) use 24 months of rating data. Due to the time limitations of our data and inconsistencies of sample history across banks, we analyse the momentum effect for rating changes over 36 and 12 month periods. To do so, we define the following four variables related to the current and historical rating changes:

- $U_{it} = 1$ if the borrower i was upgraded in month t and 0 otherwise;
- $D_{it} = 1$ if the borrower i was downgraded in month t and 0 otherwise;

- $M^u_{it} = 1$ if the borrower i was upgraded to the current rating over $(t - x, t - 1)$ and 0 otherwise, where x represents the number of months (36 or 12 as discussed above);[1]
- $M^d_{it} = 1$ if the borrower i was downgraded to the current rating in the period $(t - x, t - 1)$ and 0 otherwise, with x defined as for $M^u_{it} = 1$.

The M^u and M^d variables are upward and downward momentum indicators. The U and D variables represent the current upgrade and downgrade indicators.

When observing the full rating history (up to 36 months), we focus on entities with at least one rating change preceding the current upgrade or downgrade. As we show later in this study, time spent in a rating impacts the probability of an upgrade or downgrade, which might cause a bias in comparison of entities with a previous rating change and entities that have been stable for long time. The base group of the model with an upgrade dummy variable is 'downgraded to the current state'.[2] The respective probit regression for upgrade momentum estimation is defined as:

$$y_{it} = \alpha + \beta M^u_{it} + \epsilon_{it}, \tag{8}$$

where $\epsilon_{it} \equiv iid(0, \sigma^2)$ and y_{it} is a continuous latent variable such that $U_{it} = 1$ for $y_{it} \geq 0$ and $U_{it} = 0$ otherwise. The downgrade momentum model is analogous. We use a panel probit regression model.

When we focus on rating changes within the last year, we consider also entities that are stable (i.e. no rating change) and these form the base group of the model with downgrade and upgrade dummies. The probit regression for upgrade momentum estimation is then defined as:

$$y_{it} = \alpha + \beta M^u_{it} + \beta M^d_{it} + \epsilon_{it}, \tag{9}$$

where $\epsilon_{it} \equiv iid(0, \sigma^2)$ and y_{it} is a continuous latent variable such that $U_{it} = 1$ for $y_{it} \geq 0$ and $U_{it} = 0$ otherwise.

As the available data cover a 3 year time period, the number of observed upgrades and downgrades is limited and does not allow for assessing the momentum effect for each of the rating categories separately as presented in Lando and Skødeberg (2002). Hence, we use a panel probit model on all of the observed rating changes without rating differentiation.

3.1.2 Duration Effect

The duration effect refers to a link between time spent in a given rating category and the associated transition probability and, as discussed by Fuertes and Kalotychou (2007), it is another non-Markovian property. The duration measure d_{it} is defined as the number of months between the last transition and the current state. The effect of d_{it} is measured

[1] If previous 36 observations are not available in the analysed sample, the x is adjusted based on the data availability.

[2] We ran additional regressions using the entire sample with the base group 'downgraded to the current state or stable'. The results are consistent with the outputs reported in Section 5.

separately for upgraded and downgraded entities using a similar panel probit model as for detecting momentum effect:

$$y_{it} = \alpha + \beta d_{it} + \epsilon_{it}, \tag{10}$$

where $\epsilon_{it} \equiv iid(0, \sigma^2)$ and y_{it} is a continuous latent variable such that $U_{it} = 1$ for $y_{it} \geq 0$ and $U_{it} = 0$ otherwise. Analogous notation applies to downgrades.

Since the presented dataset of internal ratings starts in 2015, we are not able to determine the exact rating duration of some of the stable ratings that change only once during the observed period. In order to maintain consistency, we use estimate date as a proxy for the date of previous upgrade/downgrade/rating issuance even though estimate date may indicate a review without change.

3.2 Testing Time Homogeneity

The time homogeneity assumption has been extensively covered in the academic literature; it is mostly tested using eigenvalues or sensitivity of transition rates to the business cycle, yet some studies link transition matrices to specific macro- and micro-economic indicators. Specifically, Bangia et al. (2002), Kavvathas (2001), Fuertes and Kalotychou (2007), and Krüger et al. (2005) investigate time heterogeneity using eigenvalue and eigenvector tests or conditioning the hazard rates on time. Fuertes and Kalotychou (2007) find that eigenvalue and eigenvector tests supports the time homogeneous Markovian process, while Kavvathas (2001) and Krüger et al. (2005) identify time dependence.

Studies comparing transition matrices across the business cycle include Kavvathas (2001), Nickell et al. (2000), Bangia et al. (2002), Christensen, Hansen, and Lando (2004), Andersson and Vanini (2010), Gavalas and Syriopoulos (2014), Fei et al. (2012), Frydman and Schuermann (2008), and Amato and Furfine (2004). Most of the analyses conclude that there are significant differences between transition matrices estimated during recession and expansion periods. Amato and Furfine (2004) conclude that ratings vary according to the state of the business cycle, but this is driven by cyclical changes to business and financial risks rather than cycle-related changes to rating standards.

Finally, studies measuring the dependency of transition probabilities on various economic indicators include Gavalas and Syriopoulos (2014), Kavvathas (2001), Krüger et al. (2005), and Figlewski, Frydman, and Liang (2012); the studies show a correlation between transition probabilities and GDP growth or unemployment. Stefanescu et al. (2009) use a Bayesian model to describe the explanatory power of S&P500 returns; and Gómez-González and Hinojosa (2010), who include both macroeconomic and microeconomic variables into their model to obtain conditional time homogeneity.

Our dataset is too short to apply these methods of testing. Both the Centrum for Economic Policy Research (EU) and the National Bureau of Economic Research (US) mark the recent years covered in our dataset as a period of expansion, so, based on the time homogeneity assumption, the estimated transition matrices should not be significantly different. To test this hypothesis, we construct non-overlapping annual matrices, average them and compare the average to the individual matrices using the χ^2 test developed by Goodman (1958) and more recently applied by Trück and Rachev (2009) to transition matrices. Note that there are two to three non-overlapping annual transition matrices per bank, depending on the covered time period.

To check whether the individual transition matrices for time sub-samples significantly differ from the average transition matrix, we employ the following test statistics:

$$Q_t = \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j\epsilon V_i} n_i(t)\frac{(\hat{p}_{ij}(t)-\hat{p}_{ij})^2}{\hat{p}_{ij}}\ \chi^2\bigg(\sum_{i=1}^{N}(u_i-1)(v_i-1)\bigg), \tag{11}$$

where $\hat{p}_{ij}$ denotes the average probability of default representing the transition from rating i to j estimated based on the full sample, $\hat{p}_{ij}(t)$ is the corresponding transition rate estimated based on a sub-sample t, and $n_i(t)$ is the number of observations initially in the i-th rating class within the t-th sub-sample.

The test is based only on transition probabilities that are positive across the entire sample; hence, we define $V_i = \{j : p_{ij} > 0\}$. Q_t has an asymptotic χ^2 distribution with degrees of freedom equal to the number of summands in Q_t, corrected for the number of categories where $n_i(t) = 0$, number of estimated transition probabilities $\hat{p}_{ij}$ and the number of restrictions (i.e. $\sum_j \hat{p}_{ij}(t) = 1$ and $\sum_j \hat{p}_{ij} = 1$). Consequently, the degrees of freedom can be calculated as

$$\sum_{i=1}(u_i(v_i-1)-(v_i-1)) = (u_i-1)(v_i-1), \tag{12}$$

where v_i is the number of positive entries in the i-th row of the matrix for the entire sample ($v_i = |V_i|$ meaning v_i is the number of elements in V_i), and u_i is the number of sub-samples (t) in which observations for the i-th row are available ($u_i = |U_i|; U_i = t : n_i(t) > 0$).

4 Data

This section focuses on data used for the construction of transition matrices, it reviews the major existing data sources, describes the dataset used in this study and discusses issues related to bank-sourced credit risk data. Specifically we discuss transition to a 'not rated' category and the problem of banks recalibrating their rating scales.

4.1 Existing Data Sources

Existing studies on credit risk transition matrices use various types of data for the transition matrices analysis. The mainstream literature focuses on corporates, e.g. Bangia et al. (2002), Lando and Skødeberg (2002), Kavvathas (2001),Krüger et al. (2005), Jafry and Schuermann (2004), and Gavalas and Syriopoulos (2014). A minority of studies then analyses sovereigns, such studies include Fuertes and Kalotychou (2007), Nickell et al. (2000), and Wei (2003), who covers corporates, financials and sovereigns.

The most common source of rating and transition data are credit rating agencies. This data is employed by various papers including e.g. Nickell et al. (2000), Fuertes and Kalotychou (2007), Trück (2008), Kadam and Lenk (2008) (Moody's); Lando and Skødeberg (2002), Jafry and Schuermann (2004), Frydman and Schuermann (2008), Andersson and Vanini (2010), Stefanescu et al. (2009) (S&P). Several studies focus on internal bank estimates; Lu (2012) employs data of Taiwanese investment bank Chiao Tung Bank, Krüger et al. (2005) analyse rating system based on a balance-sheet data of Deustche Bundesbanks, Gómez-González and Hinojosa (2010) analyse a Colombian commercial loans sample, and Gavalas and Syriopoulos (2014) work with the internal rating data of four advanced European countries' central banks. Finally, Jones (2005) takes a different approach and estimates transition matrices using aggregate proportions data on US non-performing loans and corporate sector interest coverage.

4.2 Bank-Sourced Data

Our study is based on a unique dataset containing probability of default estimates (PDs) from the 2015-2018 period from 12 global banks provided by Credit Benchmark. The actual time frame for individual banks varies between two to three years. Credit Benchmark works with global advanced internal ratings based (A-IRB) banks,[3] pools together their internal estimates of hybrid through the cycle (H-TTC) one year PDs and aggregates them into an entity-level credit risk benchmark. Data used in this study are bank-specific. As regulators require an annual review of all credit risk estimates, we focus on annual transition matrices to ensure that all of the entities have been reviewed over the observed period.

Basel II introduced reduced risk weighting for small and medium-sized enterprises (SMEs) in line with their turnover.[4] Some of the banks analysed in this study use two different models for large corporates and SMEs. A similar distinction exists for developed and developing markets and the standard is to produce transition matrices separately for corporates, financials and governments. Hence, in our study we focus on banks' main corporate models and limit our dataset to large corporates from North America (NA) and the European Union (EU). The entity size and country are determined using information on annual sales, number of employees and family structure from Duns & Bradstreet and FactSet.[5] As we limit the sample to only large North American or EU corporates and the two economies are closely connected, the transition rates are still comparable across the banks and the differences in model behaviour should not be driven by sampling.

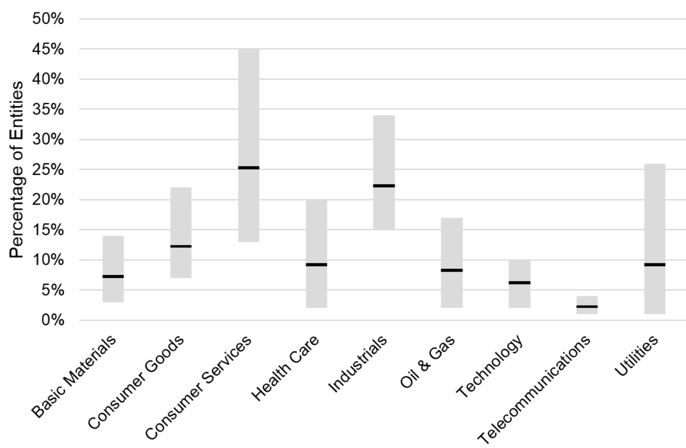
[3] A-IRB banks are allowed to use internal credit risk model to estimate credit risk parameters for calculation of regulatory capital. Banks need approval from the national regulator to use the A-IRB approach and their models are regularly assessed by regulators to ensure quality.

[4] Altman and Sabato (2007) argue that an SME-specific credit risk models are needed to minimise the expected and unexpected losses as they find that SMEs are riskier than large corporates.

[5] According to the European Commission (OJ L 124, 20.5.2003, pp. 3641), SMEs are companies with staff headcount lower than 250 and turnover below EUR 50 million or a balance sheet total below EUR 43 million. Companies that are a part of a larger family should be assessed based on the group data.

Each of the banks provides PD estimates for 800-2,000 large North American and European Union corporates covering in total more than 20,000 unique entities. The distribution between NA and EU entities is bank-specific and banks tend to have a much higher coverage of entities from the country of their domicile than from other countries. Around 90% of entities covered by EU banks come from within the EU; the NA banks show a similar portfolio structure in favour of NA entities. Figure 1 shows that the distribution across industries is more balanced, Industrials and Consumer Services are the most strongly represented industries with an average share of 34% and 45% respectively, while there are only a few entities in the Telecommunications (4%), Technology (10%) and Basic Materials (14%) industries.

Figure 1: Distribution of PD Estimates Across Industries - Ranges based on Individual Banks



The participating banks prefer to stay anonymous so we do not name them here. The banks are based in various countries. The order of presented results is random and changes for each set of results due to confidentiality.

4.3 Data Considerations

There are two issues to consider in relation to the presented data. Firstly, the set of entities covered by each of the banks changes over time as banks adjust their portfolios. This is essentially an equivalent of moving to a 'not rated' category in the assessment process of rating agencies. Literature suggests several approaches to the 'not rated' category (see Bangia et al. (2002) for more details). One possibility is to fix the sample of entities over the whole period but Bangia et al. (2002) argue that transition matrices should be based on a current sample of a rated universe because a fixed sample of entities suffers from several problems: the cohort quickly becomes outdated due to emerging issuers and mergers and acquisitions, the fundamental characteristics of entities evolve over time and it significantly reduces the sample of examined entities as issuers perish, default or retire their rating over time.

If the sample varies over time, entities transitioning into a 'not rated' category need a special handling - it has to be decided if the change is informative and if it should be viewed positively or negatively. Entities can be removed from a bank's portfolio for several different reasons including a change in the bank's strategy, an increase in the entity's credit risk, or its decision to change the lending bank. Unfortunately, details of a rating withdrawal are not known and it is not clear if the transition is favourable or not. Hence, we treat the exclusion of an entity from the portfolio as a non-informative action and distribute the probability of dropping from the sample among all states in proportion to their values, which is the industry standard. On average, 14% of entities covered by the examined banks drop from the sample annually, transitioning to the 'not rated' status. The percentage varies across banks; portfolios of some of the banks are very stable with only 2% churn, while other portfolios change more rapidly with up to 20% churn.

The second issue relates to the fact that transition matrices are based on a set of rating categories rather than continuous PDs. The banks in our sample produce only a limited number of PD values which are linked to specific credit categories based on banks' internal rating scales. The number of different rating categories varies across banks; the minimum number for the analysed banks is 13 and the maximum 26. Most of the banks use 16-21 categories.

Mapping of PDs to rating categories is not stable over time as banks recalibrate their models regularly. Recalibration usually causes less than 10% change in the PD estimate and the effect tends to be larger in low and high credit categories, which are more sensitive to changes in the number of defaults due to the limited number of observations. Changes in PD estimates driven by recalibrations are not linked to an upgrade or downgrade action; hence, we need to identify these PD changes to avoid creating a bias. We use PD transition matrices to match the pre- and post-recalibration PD estimates and replace the pre-recalibration PD values with post-recalibration PD values throughout our data sample, which removes potential bias from our analysis.

5 Results

The following section summarises the results of our analysis, structured according to Section 3. Specifically, first we investigate whether the PD estimates from the 12 banks satisfy the Markovian property, looking separately at the momentum effect (assessed through comparison of conditional transition matrices and panel probit regression) and the duration effect (panel probit regression). Subsequently, we assess the assumption of time homogeneity of the rating processes using the likelihood ratio test.

The order of the banks in the results tables is random and different in every subsection due to confidentiality.

5.1 Testing the Markovian Property

The Markovian property is one of the main assumptions used in the transition matrix estimation. Transition probabilities following a Markovian process depend only on the current state; they are independent of the rating history. We analyse the banks' rating processes using tests based on the momentum and duration effects.

5.1.1 Momentum Effect

The momentum effect states that the direction of future credit rating changes can be linked to prior changes in ratings. We employ the conditional transition matrices defined in Equation 7 and the panel probit regression models described in Equations 8 and 9 to test the effect in the bank-sourced data.

Conditional Transition Matrices In the first step, we investigate the differences between up and down momentum transition matrices. As described before, an up momentum transition matrix (UTM) is a conditional matrix based only on entities that were upgraded during a one year period preceding the period captured by the transition matrix, and similarly for a down momentum transition matrix (DTM).

We calculate the percentage of upgrades (UP) and downgrades (DP) for the up and down momentum matrices and compare them, which indicates whether it is more probable to see a downgrade/upgrade among entities that previously downgraded than for entities that previously upgraded. The significance level of these differences is determined using the test statistic for testing the difference in two population proportions.[6]

Further, we obtain the singular value decomposition (SVD) metric defined in Equation 6 for the up and down momentum transition matrices and compare them. The percentage of upgrades and downgrades does not reflect the size of the changes, which can span several rating categories. The SVD metric focuses on both frequency and size of migration but does not report on the direction of the changes. For reference, SVD metrics significantly differ across the analysed banks with a minimum of 0.15 and a maximum of 0.60 (not reported in Table 1), with lower values corresponding to fewer migrations. Jafry and Schuermann (2004) report values between 0.1 and 0.3 for transition matrices based on the S&P data.

Table 1 summarises the results. The analysis shows that banks more often revert their rating change than continue in the established trend. As shown in the column two comparing the upgrade probabilities in the up and down momentum transition matrices, an upgrade is more likely to come after a previous downgrade than after a previous upgrade; these results are significant for 6 out of 10 examined banks. The results for downgrades are less pronounced, it is significantly more likely to observe a downgrade after a previous upgrade than after a previous downgrade for 4 out of 10 banks, while 2 banks more often

[6] Defined as $Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}}$, where $\hat{p}_1$ and $\hat{p}_2$ stay for the two samples 'successes' proportions, n_1 and n_2 are the sample sizes, $\hat{p}$ is the proportion of 'successes' in the two samples combined and the null hypothesis assumes $p_1 = p_2$.

Table 1: Differences in Upgrades and Downgrades between the Conditional Transition Matrices

Bank	UP DTM - UP UTM		DW DTM - DW UTM		SVD DTM - SVD UTM	# obs
1	9%	***	-10%	***	-0.08	high
2	3%		-1%		0.02	med
3	14%	***	-8%	*	0.11	med
4	7%	**	-9%	**	0.03	high
5	-7%		4%		0.06	med
6	-6%		17%	*	0.04	low
7	16%	***	7%	***	0.03	med
8	19%	***	0%		0.20	med
9	14%	***	-8%	***	0.10	high
10	16%		-9%		0.00	low
11	insufficient data for analysis					
12	insufficient data for analysis					

. significant at p < 0.1; * signif. at p < 0.05;
** signif. at p < 0.005; *** signif. at p < 0.001
Notes:
The order of banks and labels differ from the other tables due to confidentiality.
UP DTM - upgrade percentage in down momentum matrix,
UP UTM - upgrade percentage in up momentum matrix,
DW - downgrade percentage,
SVD - singular value decomposition, see Subsection 2.3.
# obs - high = more than 1,000; med = 100 to 1,000;
low = less than 100

downgrade entities that were previously downgraded. Further, we find that down momentum matrices show more movements or more significant movements than up momentum matrices as the singular value decomposition metric is larger for the down momentum matrices resulting in positive differences shown in the column four.

Panel Probit To confirm the result, we employ a panel probit regression to analyse momentum effect using two cuts of data: tracking changes over the full sample of data ('Full Sample'), and over 12 months preceding the given upgrade or downgrade ('12 Months').

Firstly, we limit the sample to entities with at least two rating changes and check if the later change was preceded by an up or down movement using upward and downward momentum indicators. We regress the current upgrade indicator (U) on the upward momentum indicator (M^u) with downward momentum indicator (M^d) as the base group[7] using a panel probit model, similarly for the current downgrade indicator (D). This analysis is labelled 'Full Sample'.

[7] Base group is the group against which the comparisons are made.

Subsequently, we focus on the '12 Months' analysis and divide the entities with at least one change into three groups: upgraded, downgraded and stable during the 12 months preceding the last change. We again employ a panel logit model on the current upgrade and downgrade indicators (U and D) and upward and downward momentum indicators (M^u and M^d). The base group is defined as stable entities.

Table 2: Regression Analysis: Impact of Previous Upgrade and Downgrade on Probability of Rating Change

	Full Sample (See Equation 8)				12 Months (See Equation 9)							
	U_{it}		D_{it}		U_{it}				D_{it}			
Bank	M^u_{it}		M^u_{it}		M^u_{it}		M^d_{it}		M^u_{it}		M^d_{it}	
101	-0.283	***	0.121	***	0.166	***	0.445	***	0.337	***	0.228	***
	(0.031)		(0.032)		(0.035)		(0.032)		(0.033)		(0.035)	
102	-0.289	***	0.050		-0.095	.	0.190	***	0.149	**	0.074	
	(0.048)		(0.048)		(0.051)		(0.044)		(0.046)		(0.049)	
103	insufficient data for analysis				-0.056		0.101		0.091		-0.031	
					(0.084)		(0.073)		(0.08)		(0.078)	
104	-1.261	***	0.881	***	-0.622	.	0.607	***	0.981	***	-0.020	
	(0.316)		(0.166)		(0.321)		(0.104)		(0.109)		(0.192)	
105	-0.210	***	0.191	***	-0.059	*	0.186	***	0.190	***	-0.021	
	(0.025)		(0.026)		(0.027)		(0.024)		(0.025)		(0.028)	
106	insufficient data for analysis				0.162	.	0.581	***	-0.212	.	0.397	***
					(0.086)		(0.075)		(0.118)		(0.084)	
107	-0.639	***	0.599	***	-0.003		0.730	***	0.676	***	-0.036	
	(0.031)		(0.028)		(0.036)		(0.024)		(0.024)		(0.032)	
108	-0.151	*	0.022		0.098		0.134	.	0.082		0.028	
	(0.076)		(0.731)		(0.08)		(0.077)		(0.073)		(0.075)	
109	-0.053		-0.032		0.248		0.337		-0.005		0.154	
	(0.189)		(0.232)		(0.186)		(0.51)		(0.256)		(0.267)	
110	insufficient data for analysis				-0.387	**	0.332	**	0.269	**	0.175	
					(0.118)		(0.121)		(0.103)		(0.165)	
111	-0.631	**	-0.001		0.008		0.628	***	0.293		-0.284	
	(0.213)		(0.223)		(0.227)		(0.159)		(0.184)		(0.343)	
112	-0.396	***	0.012		-0.203	*	0.224	***	-0.177	*	-0.125	*
	(0.076)		(0.068)		(0.083)		(0.056)		(0.069)		(0.061)	

. significant at $p < 0.1$; * signif. at $p < 0.05$; ** signif. at $p < 0.005$; *** signif. at $p < 0.001$
Notes:
The order of banks and labels differ from the other tables due to confidentiality.
$U_{it} = 1$ if borrower i was upgraded in month t and 0 otherwise, similarly for $D_{it} = 1$;
$M^u_{it} = 1$ if borrower i was upgraded to the current rating over $[t-x, t-1]$ and 0 otherwise, similarly for M^d_{it}, x represents the number of months (36 for 'Full Sample' and 12 for '12 Months').
Full Sample - base group is 'previously downgraded entities'.
12 Months - base group is 'entities that have been stable over the last 12 months'.

The results are summarised in Table 2. The 'Full Sample' analysis directly compares previously upgrading and downgrading entities and the results are in line with the Conditional Transition Matrices analysis. We find that banks tend to reverse their rating change;

the probability of an upgrade is significantly lower for entities that were upgraded to the current state than for entities that were downgraded to the current state and similarly for downgrades. The reversion tendency is stronger for upgrades.

The '12 Months' analysis compares the previously upgrading and downgrading entities to the base group of stable entities and the model includes both upward and downward momentum indicators. 'Previously downgraded/upgraded' in this context means 'downgraded/upgraded over the last 12 months'. The upgrade model clearly shows that entities downgraded in the last 12 months are more likely to be upgraded than stable entities or entities experiencing an upgrade. The difference between stable and upgraded entities is not uniform across the banks. The direct comparison of entities previously upgrading and downgrading based on the 95% Wald confidence interval shows that an upgrade is more likely to occur after a previous downgrade than a previous upgrade for 8 out of the 12 banks.

The results in the downgrade model are less definite. The previous upgrade dummy variable has a positive effect on the downgrade probability for half of the banks, while a downgrade observed in the last 12 months has no clear impact on the downgrade probability across the banks. The 95% Wald confidence intervals imply that only 3 banks are significantly more likely to downgrade a previously upgraded entity than a previously downgraded entity, 1 bank shows a significantly higher probability of downgrade for previously downgrading entities, and the differences are not significant for the remaining banks.

The momentum effect assumes that the directions of previous and future rating changes are correlated. Analyses of this effect using conditional transition matrices and panel probit regression model lead to the same conclusion: it is more likely to observe an upgrade for previously downgraded entities compared to previously upgraded and stable entities. The impact of previous movements on downgrades is weaker and less definite but we can still conclude that previous upgrades have a positive impact on the downgrade probability compared to previous downgrades and stable periods.

5.1.2 Duration Effect

The duration effect links time spent in a given rating category or duration (d) with the associated transition probability and its existence indicates non-Markovian behaviour. Duration is defined as a number of months spent in a given rating before an upgrade or downgrade. We employ panel probit regression defined in Equation 10.

Table 3 shows that the effect of duration is not uniform across the banks but a negative impact of duration on the probability of any rating changes prevails, which means that recently upgraded or downgraded entities have a higher chance of another rating change than that of stable entities. These findings are in line with mixed evidence found by Fuertes and Kalotychou (2007), Lando and Skødeberg (2002) and Kavvathas (2001).

Table 3: Regression Analysis: Impact of Duration on Probability of Rating Change

	U_{it}		D_{it}	
Bank	d_{it}		d_{it}	
A	-0.005	.	0.001	
	(0.003)		(0.003)	
B	-0.027	***	-0.022	***
	(0.001)		(0.001)	
C	-0.013		-0.021	*
	(0.008)		(0.011)	
D	0.005		0.011	***
	(0.003)		(0.003)	
F	-0.004		-0.004	
	(0.004)		(0.005)	
G	0.006	**	0.000	
	(0.002)		(0.002)	
H	-0.042	***	-0.019	***
	(0.006)		(0.005)	
I	-0.003		0.003	
	(0.005)		(0.005)	
J	-0.014	***	-0.013	***
	(0.002)		(0.002)	
K	0.012	***	-0.014	***
	(0.003)		(0.004)	
L	-0.014	**	-0.004	
	(0.004)		(0.005)	
M	0.000		-0.004	**
	(0.001)		(0.001)	

. significant at $p < 0.1$; * signif. at $p < 0.05$;
** signif. at $p < 0.005$; *** signif. at $p < 0.001$
Notes:
The order of banks and labels differ from the other tables due to confidentiality.
$U_{it} = 1$ if borrower i was upgraded in month t and 0 otherwise, similarly for $D_{it} = 1$;
d_{it} is duration measure.

5.2 Testing Time Homogeneity

Time homogeneity is the second main assumption used for transition matrix estimation. A time homogeneous rating process depends only on the time horizon of interest and not on the initial date. The time homogeneity assumption is tested using the likelihood ratio test defined in Equations 11 and 12. We examine the difference between individual annual transition matrices and the average matrix, calculate the observed χ^2 test statistics and compare the values with the tabulated values for 99% confidence level.

The observed χ^2 values reported in Table 4 are larger than the tabulated ones for 7 out of 10 observed banks, which means that we can reject the null hypothesis of time

Table 4: Likelihood Ratio Test: Time Homogeneity of Transition Matrices

	Bank Z	Y	W	V	U	T	S	R	Q	P
Observed χ^2	116	1005	413	573	274	376	72	757	147	103
Tabulated $\chi^2_{99\%}$	105	739	383	300	274	362	93	557	121	147
DF	74	652	321	246	222	302	64	482	87	110
p-value	0.001	0.000	0.000	0.000	0.010	0.002	0.230	0.000	0.000	0.669
	**	***	***	***	*	**		***	***	

. significant at $p < 0.1$; * signif. at $p < 0.05$; ** signif. at $p < 0.005$; *** signif. at $p < 0.001$
Note:
The order of banks and labels differ from the other tables due to confidentiality.
Two banks are not included due to insufficient data.

homogeneous transition matrices at 99% level. The results indicate that bank-sourced transition matrices are not stable over time even across the recent period of economic expansion. The banks' PD estimates are hybrid through-the-cycle (H-TTC), which means that the sensitivity of the PD estimates to economic cycle is between the pure through-the-cycle (TTC) PDs (which express the same degree of creditworthiness at any time, regardless of the state of the economy) and point-in-time (PIT) PDs (which capture the variations in economic cycle) but the banks do not specify the level of impact of cyclical variables in their H-TTC PD estimates.

We show that banks' credit risk data have non-Markovian features and are time heterogeneous and that the credit risk estimates of the examined banks differ from credit rating agencies. We detect a momentum effect in the rating processes of 10 out of the 12 examined banks. Interestingly, we conclude that banks tend to reverse their rating changes and previously downgraded entities are more likely to upgrade than previously upgraded entities. This is similar for previously upgraded entities and downgrades, but the link is weaker. Studies focusing on rating agencies (e.g. Bangia et al., 2002; Carty & Fons, 1994; Lando & Skødeberg, 2002) mostly detect a downgrade momentum and conclude that a downgrade is more likely to be followed by another downgrade than by an upgrade. This difference in results for credit rating agencies and banks can be driven either by different approaches to credit risk estimation or by different timing of the studies as business cycle can significantly impact the transition rates as shown in multiple previous studies including Kavvathas (2001) and Christensen et al. (2004). Further, we describe a duration effect in the data as duration has a significant impact on the probability of rating change for 10 out of 12 banks but the direction of the effect is not uniform across the banks. This is in line with the mixed evidence found by Fuertes and Kalotychou (2007), Lando and Skødeberg (2002) and Kavvathas (2001). The time homogeneity test suggests that the banks' credit risk estimates are time heterogeneous.

6 Conclusion

Banks' internal credit risk estimates can be used to create an industry standard for transition matrices, overcoming the issue of data sparsity faced by rating agencies, which are currently the main source of transition matrices in the field. Indeed, data from banks provide greater detail than data from credit rating agencies and allow estimation of country- and industry-specific transition matrices, which may lead to improvements in the accuracy of forward-looking credit risk models.

This study provides an insight into some of the essential features of banks' internal credit models using a unique dataset of probability of default estimates from 12 global A-IRB banks. Specifically, it assesses the two main assumptions commonly used for estimation of transition matrices: the Markovian property and time homogeneity of the underlying rating processes. The existing literature, including e.g. Fuertes and Kalotychou (2007) and Bangia et al. (2002), documents extensive testing of these assumptions for credit rating agencies but the coverage of banks' internal rating processes is sparse and the relevant studies mostly analyse local clusters of banks such as in Gómez-González and Hinojosa (2010) or Lu (2007).

The dataset of credit risk estimates consists of large corporates in North America and the European Union, which are modelled by banks' main corporate models; SMEs and developing markets are often modelled separately. The final dataset covers 800-2,000 monthly observations from each of the 12 analysed banks for the 2015-2018 time period. The analysis explores the applicability of the existing transition matrix estimators on banks' internal credit risk data at an unprecedented scale, providing more robust results than in the previous literature.

We test the Markovian property assumption using the momentum and duration effects hypotheses; based on the comparison of conditional transition matrices and panel probit models, we conclude that banks' credit rating processes are not Markovian as previous ratings and time spent in a given rating (i.e. duration) have a significant impact on transition probabilities. The results are in line with previous studies on credit rating processes by major credit rating agencies (e.g. Bangia et al., 2002; Fuertes & Kalotychou, 2007; Lando & Skødeberg, 2002). At the same time, and in contradiction to the listed studies, our analysis suggests that the probability of an upgrade is higher for previously downgraded entities than for previously upgraded entities - and analogously for downgrades. That is, banks tend to revert their rating actions. Even though duration has a significant impact on the transition probability, results on direction of the impact are mixed.

The Likelihood ratio test indicates that the transition matrices are time heterogeneous, even though the analysis is limited to a three year period of economic expansion. This supports results of previous studies (e.g. Frydman & Schuermann, 2008; Gavalas & Syriopoulos, 2014; Nickell et al., 2000).

The findings are vital for estimation of transition matrices based on banks' internal credit risk estimates as they show that one must employ more complex estimators (e.g. Frydman & Schuermann, 2008; Wei, 2003) that, unlike the more simplistic estimators, do not rely on the two assumptions.

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