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## Time-frequency response analysis of monetary policy transmission

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IES Working Paper: 30/2018



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# Time-Frequency Response Analysis of Monetary Policy Transmission

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## **Abstract:**

In our study, we consider a new approach to quantify the effects of economic shocks on monetary transmission. We analyse the widely known phenomenon of price puzzle in a time-varying environment using the frequency decomposition. We use the frequency response function to measure the power of shocks transferred to different economic cycles. Considering both time and frequency domains, we quantify the dynamics of shocks implied by monetary policy within an economic system. While studying the monetary policy transmission of the U.S., the empirical evidence shows that low-frequency cycles of output are prevalent and have positive transfers. Examination of the inflation reveals that the frequency responses vary significantly in time and alter the direction of transmission for all cyclical lengths.

**Keywords:** cyclical, frequency, economic systems, monetary policy

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# 1 Introduction

A large amount of research has been done on the monetary policy transmission aiming to explain the interaction between economic variables. Regarding the monetary policy, a particular increase in interest links to the study of [Sims \(1980\)](#). The vector autoregression (VAR) models are the cornerstone of the modern macroeconomic studies disentangling effects of monetary policy ([Sims, 1992](#); [Castelnuovo and Surico, 2010](#)).

The finding of the so-called "price puzzle" phenomenon that an increase in prices follows a monetary tightening is present in many studies that employ a VAR framework and analyse impulse response functions (IRFs) to monetary policy. Attempts to quantify the monetary policy responses have been many and vary with a large scale of modifications of estimation, identification schemes, different data used, leading naturally to multiple results and implications. Since the final explanation is still in question, there is a space to labour on this story from a different perspective.

In this paper, we study the evolving macroeconomic dynamics through the lens of frequency domain analysis. Our study relies on a linear representation of an economic model, which we use to obtain impulse response functions. Traditionally, the impulse response functions are a prominent tool in the analysis of monetary policy.

First, the literature of time-series filtering, where the impulse responses also belong goes in hand with frequency domain. This text aims to work with IRFs counterparts in the frequency domain – frequency response functions. To start, we point out conceptual works of [Baxter and King \(1999\)](#); [Christiano and Fitzgerald \(2003\)](#); [Murray \(2003\)](#) where a reader can find a spectral domain<sup>1</sup> representations of economic variables together with filters used to define cyclical components, for example. The tool we want to highlight is the frequency response function, which provides a way how to filter data or quantify how the data has been filtered ([Dew-Becker and Giglio, 2016](#)). We find potential in this interpretation and employ the frequency responses in an attempt to quantify information in the macroeconomic dynamics. The dynamics can be studied via exogenous shocks to the economy and depicted in both time and frequency domains. Moreover, we use additional spectral measures as the dynamic correlation to look at cyclical dependencies between monetary variables.

Second, to enable the quantification of the frequency dependency of economic dynamics, we need to estimate structures of a model. In the literature, the finding of the evolving structure of economies has been recognised ([Koop et al., 2009](#); [Cogley and Sargent, 2005](#)). Hence, we implement a time-varying parameter vector autoregressive model (TVP-VAR) to capture most of the evolution in the macroeconomic dynamics. Further, we aim to complement this

---

<sup>1</sup>The labels spectral domain and frequency domain are interchangeable.

1 modern tool using a structural vector autoregressive model identified using sign restrictions.  
2 To allow for time-variation, the authors estimate structural models using rolling windows.  
3 However, rolling windows might suffer from outliers observations when small subsample is  
4 selected, and on the other hand, if the window is wide, it does not capture much of the vari-  
5 ability (Geraci and Gnabo, 2016). Nevertheless, we will compare our time-varying results to  
6 the literature that shows a state of the US economy at different periods of time, mainly esti-  
7 mated at different subsamples. For example, Castelnovo and Surico (2010) shows different  
8 responses for subsamples before and after Paul Volcker’s era.

9 The literature on the vector autoregressive model and monetary policy is broad, and its  
10 multivariate modelling started several decades ago (Sims, 1980, 1992). Yet it is not ending  
11 journey on proper identification of shocks in models using zero restrictions Christiano et al.  
12 (1999) or sign-restrictions identification schemes of Uhlig (2005); Rubio-Ramirez et al. (2010)  
13 or a combination. We focus our analysis of monetary policy, particularly on the responses  
14 of endogenous variables as the output, prices, and the interest rate. Since Sims (1992), the  
15 price puzzle represented by the increase in prices after monetary tightening has been a long-  
16 term subject to research because the unintuitive direction of inflation when doing monetary  
17 tightening. Nonetheless, Rusnák et al. (2013) compared more than 70 studies and point out  
18 an interesting result that the price puzzle arises rather from misspecification of the model  
19 than from price behaviour in the economy. There are many ways how to approach this  
20 problem. For example, in a recent study, Koop et al. (2009) compare time-variant and time-  
21 invariant settings of the Primiceri (2005), and they found a strong preference for a model  
22 that allows for bot time-varying coefficient and covariance matrix. Alternatively, Baumeister  
23 and Hamilton (2015) provide a recommendation that the researcher should acknowledge  
24 their prior believes about structural conclusions. The results suggest a Bayesian vector  
25 autoregression. Hence, we are going to accommodate both modelling features and take a  
26 traditional setup of TVP-VAR with stochastic volatility and sign restrictions.

27 A similar connection of frequency domain and vector autoregression modelling has been  
28 made by Gehrke and Yao (2017) who examines the role of supply shocks in real exchange  
29 rates. They use an SVAR model with sign restriction to identify structural shocks. They  
30 obtain spectral variance decomposition (SVD) showing the importance of productivity shock  
31 in the persistence of the real exchange rate. Further, Ellington (2018) uses TVP-VAR and  
32 time-frequency coherence to study dynamics between monetary variables with Divisia money.  
33 New in the financial literature, Geraci and Gnabo (2016) employ Bayesian TVP-VAR and  
34 defines time-varying spillovers. The time-varying work of Geraci and Gnabo (2016) also  
35 relates to rolling window frequency connectedness measure of Barunik and Krehlik (2015).  
36 Lovcha and Perez-Laborda (2018) uses a spectral matrix of the reduced-form model in the

1 frequency domain Whittle log-likelihood function to estimate monetary policy Fractionally  
2 integrated VAR.

3 The main contribution of this research is that we show how to decompose responses of  
4 structural monetary policy and measure the power of shocks transferred to economic cycles.  
5 We use the frequency response functions to describe the evolution of monetary policy impacts  
6 on inflation and output. The findings show that the monetary policy transmission effects  
7 on macroeconomic variables vary in time, frequency and intensity. We show that even with  
8 controlling for the sign of the inflation impulse response to monetary policy the frequency  
9 response is both negative and positive. Thus, the impacts are frequency-dependent and  
10 the direction changes. Additionally, we quantify cyclical behaviour using time-frequency  
11 correlation that captures dependence between output, inflation, and interest rates, which is  
12 rather weak at short cycles and much stronger at business cycles.

13 The text structure is the following. Section 2 starts with relevant methodological ap-  
14 proaches. We introduce a traditional vector autoregression and show the definition of the  
15 frequency responses. Further, we propose TVP-VAR model and frequency dependent mea-  
16 sures. In Section 3, we present the data and estimation procedure. Section 5 provides results,  
17 and the last section concludes.

## 18 **2 Methodology**

19 Studying a dynamics of an economic system and cyclical responses to monetary policy require  
20 an estimation framework. Since the literature has shown that economic systems evolve  
21 over time, e.g. (Canova and Gambetti, 2009), we build on the time-varying parameters  
22 vector autoregressive model of Primiceri (2005); Cogley and Sargent (2005), which allows  
23 for capturing gradual evolution parameters. Further, we use the specification of Benati and  
24 Mumtaz (2007), who, among others, featured the model using the identification scheme of  
25 sign restrictions of Uhlig (2005); Rubio-Ramirez et al. (2005).

26 First, we present the traditional vector autoregressive model, in which we introduce the  
27 central concept of the frequency response functions. The traditional measure of impulse  
28 responses is decomposed into frequencies corresponding to economic cycles, which appear in  
29 the monetary policy. Second, we define the time-varying parameters model with stochastic  
30 volatility and the reduced-form spectral measures capturing the co-movement between vari-  
31 ables. The time-frequency representation of the correlation measure is insightful due to its  
32 detailed information about different cyclical length. Lastly, we describe the data and the  
33 estimation strategy.

## 2.1 VAR frameworks

The framework of the vector autoregressive (VAR) process is convenient for demonstration and definition of the frequency response functions and their interpretations. Let  $y_t = (y_{1,t}, y_{2,t}, \dots, y_{n,t})^T$  be a stationary  $n$ -variate process, which is defined as a  $p$ -th order VAR model

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + u_t, \text{ for } t = 1, \dots, T, \quad (1)$$

where  $A_1, A_2, \dots, A_p$  are  $n \times n$  coefficient matrices, and  $u_t$  is a white noise with covariance matrix  $\Sigma$ . The coefficient matrices contain information about the relationship of each variable to its own lags of order  $p$  as well as about the link to lags of other variables. The matrices of autoregression coefficients can be represented in the form of lag-polynomials  $A(L) = I_n - A_1 L - \dots - A_p L^p$ , which rewrites the model into  $A(L)y_t = u_t$ . If the system of variables is stationary, the VAR process can be represented as an infinite vector moving average process (MA( $\infty$ )) such that

$$y_t = \Psi(L)u_t = \sum_{i=1}^{\infty} \Psi_i u_{t-i} + u_t, \quad (2)$$

where  $\Psi(L)$  is  $n \times n$  infinite matrix lag polynomial,  $\Psi(L) = \sum_{h=0}^{\infty} \Psi_h L^h$ , which we can obtain from  $\Psi(L) = [A(L)]^{-1}$ .

## 2.2 Frequency response function

We look at an economic system as a filter formed by economic variables, which carries on the transmission of new information from one variable to the other. To quantify the effects transmitted through such a system, we propose to use the apparatus known in the filtering domain as the impulse transfer function (ITF) or frequency response function (FRF).<sup>2</sup>

First, let us define the impulse transfer for the system from Eq. (2). The moving average representation is a key since the coefficient matrices,  $\Psi(L)$ , capture the dynamics of the system. The frequency response function is a spectral representation of the  $\Psi_{h \rightarrow \infty}$  coefficients,

$$\Psi(e^{-i\omega}) = \sum_h \Psi_h e^{-i\omega h}. \quad (3)$$

Since  $h$  corresponds to infinite lags, we approximate the coefficient matrices for sufficient number of lags  $H$  allowing the calculation of the impulse transfer functions.

Traditionally, the VAR provides impulse response functions to study the propagation of

---

<sup>2</sup>In this section, we define the impulse transfer function (ITF) because of the filtering literature. And further we equivalently name it as the frequency response function (FRF) because *frequency* corresponds to cycles.



1 shock for given time horizon. The impulse transfer function<sup>3</sup> is a spectral counterpart of an  
 2 impulse response function. Hence, in the structural analysis, we have an impulse transfer  
 3 function of  $i^{th}$  variable to a shock to the  $j^{th}$  variable as

$$4 \quad ITF_{ij}(\omega) = \sum_h \Psi_{ij,h} e^{-i\omega h},$$

5 Since, we work with real economic variables, we have  $\Psi_{ij} = 0$  for the horizon  $h < 0$ , then  
 6 the  $ITF_{ij}(\omega)$  might be also represented as the real part of the Fourier transform of the MA  
 7 coefficients (Dew-Becker and Giglio, 2016),

$$8 \quad ITF_{ij}(\omega) = Re \left( \sum_h \Psi_{ij,h} e^{-i\omega h} \right) = \sum_{h=0}^{\infty} \cos(\omega h) \Psi_{ij,h} \quad (4)$$

9 The transfer function measures how the filter (coefficients) process the information through  
 10 itself at each frequency. In the time-series analysis, the frequencies are mostly viewed as  
 11 economic cycles of different length. Spectral representation of a time-series process known  
 12 as the power spectrum is a common measure to understand distribution of variance across  
 13 frequencies. The transfer function  $\Psi(e^{-i\omega h})$  forms the power spectrum of the given VAR  
 14 model (Stiassny, 1996), such that

$$15 \quad f(\omega) = \Psi(e^{-i\omega h}) \Sigma \Psi'(e^{+i\omega h}), \quad (5)$$

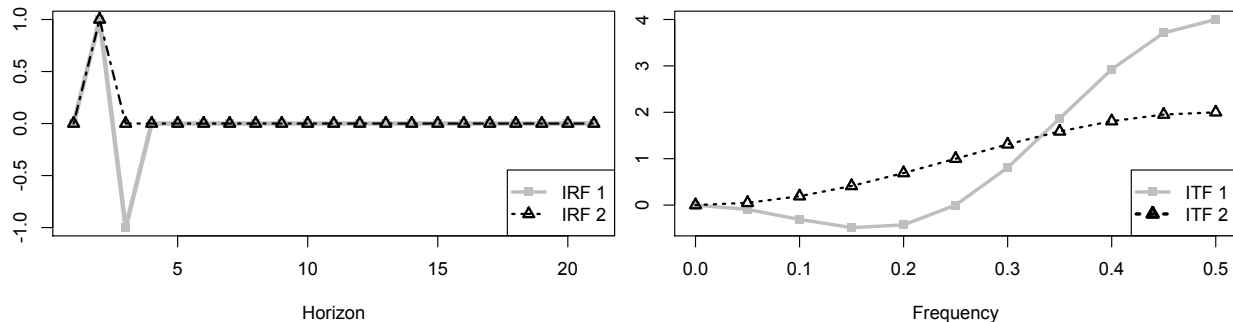
16 where  $\Sigma$  is the covariance matrix of VAR in Eq. (1). We present similar representation of  
 17 the power spectrum in relation to TVP-VAR later in Section 2.5.

### 18 **2.3 Examples of transfer functions**

19 We demonstrate the relationship between the impulse response function and the impulse  
 20 transfer function in Fig 1. We use two shocks with transitory effects. In Figure 1 (left), the  
 21 impulse response 2 (IRF2) reacts to a shock of size 1 at time  $h = 0$  and size of -1 at time  
 22  $h = 1$ , which translates the shock into IRF2 values of 1 and 0 a times  $h = 1$  and  $h = 2$ .  
 23 A reader might recognise the representation of the first difference filter of the time domain.  
 24 Alternatively, in the frequency domain, it filters out the information at the lowest frequency  
 25 components (or the longest cycles). The impulse response 1 reacts to a different shock, which  
 26 lasts longer – three periods. Moreover, in the frequency domain, this shock quadruples the

---

<sup>3</sup>Filtering techniques use the power transfer function, or equivalently the gain function,  $H(\omega) = |\Psi(e^{-i\omega})|^2$ .



**Figure 1:** Left: Impulse response functions with  $\Psi_h$  coefficients of IRF 1:  $\{1, -2, 1, 0, 0, \dots\}$ ,  $\Psi_h$  coefficients of IRF 2:  $\{1, -1, 0, 0, 0, \dots\}$ . Right: Corresponding impulse transfer functions. Examples of [Dew-Becker and Giglio \(2016\)](#).

1 effect of high frequencies (at frequency 0.5), diminishes the low frequencies, and puts some  
 2 negative weight on frequencies approximately between 0.1 and 0.2.

3 The benefit of frequency responses, in comparison to IRFs, is that using IRFs we only  
 4 observe reactions to shocks and their evolution in time for a given horizon  $1, \dots, H$ . However,  
 5 we do not distinguish the real transfer for given economic cycles. The frequency responses  
 6 provide insights upon frequency-specific decomposed information of the effects of shocks on  
 7 the system transmission.

## 8 2.4 TVP-VAR framework

Since our aim is to capture time-frequency dynamics, we consider a time-varying parameters  
 VAR with stochastic volatility. We work with TVP-VAR of  $n$  variables and  $k$  lags:

$$y_t = B_{0,t} + B_{1,t}y_{t-1} + \dots + B_{k,t}y_{t-k} + u_t, \text{ for } t = 1, \dots, T, \quad (6)$$

$$= X_t' \theta_t + u_t, \quad (7)$$

9 where  $y_t$  is an  $n \times 1$  vector of endogenous variables,  $B_{0,t}$  are  $n \times 1$  time varying inter-  
 10 cepts,  $B_{i,t}$  are  $n \times n$  matrices of time-varying coefficients for  $i = 1, \dots, k$  lags, and  $u_t \sim$   
 11  $N(0, \Omega_t)$  are unobservable shocks with time-varying covariance matrix  $\Omega_t$ . The specification  
 12 of the model follows [Primiceri \(2005\)](#); [Cogley and Sargent \(2005\)](#) and [Benati and Mumtaz](#)  
 13 [\(2007\)](#). It is assumed that parameters  $\theta_t$  evolve as a drift-less random walk constrained by  
 14  $p(\theta_t | \theta_{t-1}, Q) = I(\theta_t) f(\theta_t | \theta_{t-1}, Q)$ , where  $I(\theta_t)$  is an indicator function that rejects unstable  
 15 draws and  $f(\theta_t | \theta_{t-1}, Q) \sim N(\theta_t, Q)$  ([Cogley and Sargent, 2005](#)). Hence, parameters  $\theta_t$  follow

$$16 \theta_t = \theta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q). \quad (8)$$

1 The covariance matrix

$$2 \quad \text{Var}(u_t) = \Omega_t = A_t^{-1} H_t (A_t^{-1})' \quad (9)$$

3 can be factorized such that  $u_t = A_t^{-1} H_t^{\frac{1}{2}} \zeta_t$ , with  $\zeta_t \sim N(0, I)$ . Further, the matrix  $A_t$  is  
 4 a lower triangular matrix with elements  $\alpha_t = [\alpha_{2,1}, \alpha_{3,1}, \alpha_{3,2}]$  below diagonal that depict  
 5 contemporaneous relations and evolve as a random walk

$$6 \quad \alpha_t = \alpha_{t-1} + \xi_t, \quad \xi_t \sim N(0, S). \quad (10)$$

7 The matrix  $H_t$  is diagonal such that

$$8 \quad H_t = \begin{pmatrix} h_{1t} & 0 & 0 \\ 0 & h_{2t} & 0 \\ 0 & 0 & h_{3t} \end{pmatrix}, \quad (11)$$

9

$$10 \quad \log(h_{i,t}) = \log(h_{i,t-1}) + \eta_{i,t}, \quad \eta_t \sim N(0, W). \quad (12)$$

11 and its elements on the diagonal evolve as a geometric random walk. All innovations of in  
 12 the model form a matrix  $S$ , which is a block diagonal. And all the innovations that are  
 13 serially uncorrelated follow a joint normal distribution with the matrix  $S$  with the elements  
 14  $\text{Var}([u_t, \varepsilon_t, \xi_t, \eta_t]^T)$ . Since, we follow the specification of [Benati and Mumtaz \(2007\)](#) we  
 15 refer a reader to consult [Benati and Mumtaz \(2007\)](#); [Del Negro and Primiceri \(2015\)](#) for  
 16 further specification of priors and calibration of the models.

## 17 2.5 Spectral dependent measures

18 For further analysis of frequency dependent dynamics, we describe the time-varying (tempo-  
 19 rary) spectrum following [Cogley and Sargent \(2005\)](#); [Primiceri \(2005\)](#) of the reduced form  
 20 model such as

$$21 \quad f_{t|T}^{(ii)}(\omega) = s_i (I - B_{t|T} e^{-i\omega})^{-1} \frac{\Omega_{t|T}}{2\pi} ((I - B_{t|T} e^{+i\omega})^{-1})' s_i' \quad (13)$$

22 where  $s_i$  is selecting vector of variable  $i$ ,  $B_{t|T} e^{-i\omega} = B_{t|T,p=1} e^{-i\omega} + B_{t|T,p=2} e^{-i\omega 2} + \dots + B_{t|T,p=k} e^{-i\omega k}$ ,  
 23  $\Omega_{t|T}$  is a time-varying reduced-form VAR covariance matrices. Studies in the literature use  
 24 the temporal, spectral density to access the persistence of variables, which is studied around  
 25 frequency zero, for example in [Cogley and Sargent \(2005\)](#).

26 We study connections between variables at different frequencies that are known in the  
 27 economic literature. Traditionally, the link is assessed via correlations coefficient on filtered

1 series to see the dependence at different horizons, for example. [Croux et al. \(2001\)](#) provide  
2 frequency dependent measures such as a dynamic correlation and coherence. For instance,  
3 to study the co-movement between output and inflation using the coherence measure, which  
4 depicts strength of the relationship. The time-varying coherence based on the temporal  
5 spectra is defined ([Mumtaz and Sunder-Plassmann, 2013](#); [Ellington, 2018](#)) as

$$6 \quad \hat{h}_{t|T}^{(ij)}(\omega) = \sqrt{\frac{\text{Re}\{\hat{f}_{t|T}^{(ij)}(\omega)\}^2 + \text{Im}\{\hat{f}_{t|T}^{(ij)}(\omega)\}^2}{\hat{f}_{t|T}^{(ii)}(\omega)\hat{f}_{t|T}^{(jj)}(\omega)}}, \quad (14)$$

7 where  $\text{Re}\{\}$  is the real part of the spectrum (co-spectrum),  $\text{Im}\{\}$  is the imaginary part  
8 of the spectrum (quadrature spectrum), and hence the dynamic coherence  $\hat{h}_{t|T}^{(ij)}(\omega) \in [0, 1]$ .  
9 Similar type of coherence measure is nowadays often in time-frequency (wavelet) analysis to  
10 study co-movement between variables.<sup>4</sup> However, the coherence measure depicts only the  
11 strength not the direction of the relationship, thus we present the time-frequency correlation.  
12 Accordingly to the time-invariant dynamic correlation of [Croux et al. \(2001\)](#), we write

$$13 \quad \rho_{t|T}^{(ij)}(\omega) = \frac{\text{Re}\{\hat{f}_{t|T}^{(ij)}(\omega)\}}{\sqrt{\hat{f}_{t|T}^{(ii)}(\omega)\hat{f}_{t|T}^{(jj)}(\omega)}}, \quad (15)$$

14 which range is as for the traditional correlation measure, such that  $-1 \leq \rho_{ij,t|T}^{(ij)}(\omega) \leq 1$  for  
15 all  $\omega$ . The local correlation provides unconditional prospects to see relationship between  
16 the output, inflation, and interest rate at given time,  $t$ , and frequency horizon,  $\omega$ . Further,  
17 we use the time-frequency correlation averaged at given frequency band. Such a correlation  
18 averages values of  $\rho_{t|T}^{(ij)}(\omega_{band})$  for given  $\omega_{band} \in [\omega_1, \omega_2]$ , where  $\omega_1, \omega_2$  correspond to desired  
19 frequencies.

## 20 **3 Data and estimation**

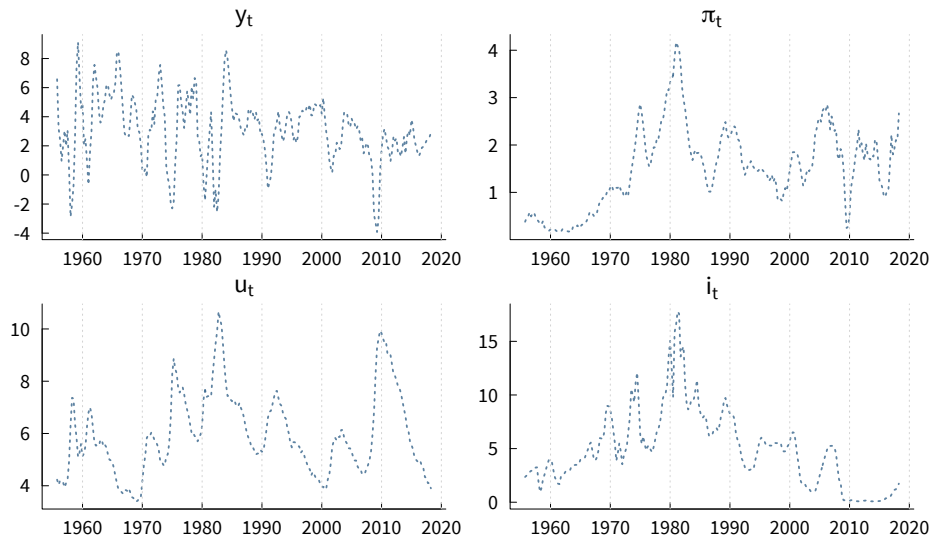
### 21 **3.1 Data**

22 We use the data standard for the literature ([Primiceri, 2005](#); [Koop et al., 2009](#); [Cogley](#)  
23 [and Sargent, 2005](#); [Canova and Gambetti, 2009](#)). For the analysis, we employ the output  
24 growth of real GDP, GDP deflator inflation, unemployment rate, and the effective Federal  
25 Funds rate, Figure 2. The data are at the quarterly frequency and spans from 1955:Q4 to

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<sup>4</sup>For example, [Aguilar-Conraria et al. \(2008\)](#); [Ramsey and Lampart \(1998\)](#)

1 2018:Q2.<sup>5</sup> This data set is small nevertheless it provides sufficient information for monetary  
 2 policy analysis and the frequency measures we emphasise.



**Figure 2:** US macroeconomic data spanning 1955:Q4–2018:Q2. In the first row, there is output growth (left) and the GDP deflator inflation. The second row depicts the unemployment rate (left) and the effective Federal Funds rate (right).

### 3 3.2 Estimation

4 We estimate the TVP-VAR model in line with Benati and Mumtaz (2007)<sup>6</sup>, variables include  
 5 the output growth, inflation, and the interest rate, in this order, which matters (Del Negro  
 6 and Primiceri, 2015). The number of lags is set to 2, consistently with the literature due  
 7 to the parsimonious and computational reasons. Moreover, the two lags allow capturing the  
 8 essential economic dynamics of the multivariate system, of which we estimate frequency-  
 9 specific features.

10 The model is estimated using Bayesian techniques that have shown to be useful in vector  
 11 autoregressions especially when the number of parameters is large. We take the first ten  
 12 years of the data as a training sample to obtain OLS estimates for calibration the TVP-  
 13 VAR model. Subtracting those ten years, we analyse the rest of the sample spanning from  
 14 1965:Q2 to 2018:Q2. We use the MCMC algorithms of Cogley and Sargent (2005) to draw  
 15 parameters sequentially from different distributions conditional on remaining parameters of

<sup>5</sup>The data used are downloaded from the FRED St. Louis website.

<sup>6</sup>We use the Matlab and Julia codes of Harood Mumtaz, available at <https://sites.google.com/site/hmumtaz77/code>.

1 the model (Koop et al., 2009). We simulate posterior distributions of parameters using 20000  
 2 iterations.

3 To identify the structural monetary policy shocks, we stay in line with the literature  
 4 and theory and impose sign restrictions on a period-by-period basis (Uhlig, 2005; Benati  
 5 and Mumtaz, 2007; Rubio-Ramirez et al., 2010). We impose contemporaneous restrictions  
 6 in a way that positive monetary policy shocks have non-negative effects on interest rates  
 7 and non-positive effects on inflation and output. The restrictions of structural shocks are  
 outlined in Table 1.

**Table 1:** Sign restrictions

	GDP, $y_t$	Inflation, $\pi_t$	Interest rate, $i_t$
<b>Monetary policy shock</b>	$\leq$	$\leq$	$\geq$

*Note:* The restrictions are imposed contemporaneously on the endogenous variables.

8

## 9 4 Results

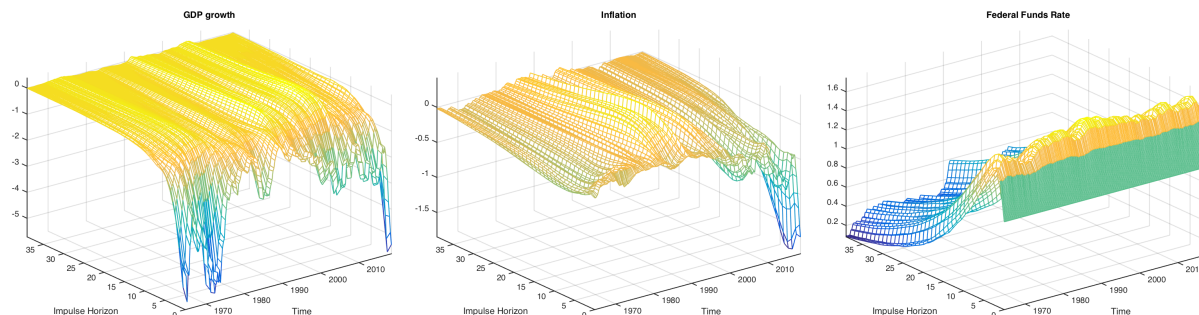
10 In this section, we present the monetary policy analysis in time and frequency domains.  
 11 All results are based on medians of posterior estimates computed via the time-varying VAR  
 12 model. Our primary focus is on the decomposition of dynamic responses to different fre-  
 13 quency horizons. Traditionally impulse responses often do not look farther than a given  
 14 time horizon, at which the researchers observe limited information. In our case, we ide-  
 15 ally decompose an infinite horizon coefficient to capture the longest relationships between  
 16 variables. Similar intuition lies behind picturing the extent of local correlations between  
 17 macroeconomic variables in time and frequency<sup>7</sup> meaning that the longest possible cycle  
 18 that can be measures equals the half of our sample length.

### 19 4.1 Monetary policy in time and frequency

20 In this section, we present propagation of identified unitary monetary policy shocks in the  
 21 form of impulse response functions in Figure 3. The IRFs are computed as medians at  
 22 each point of time based on posterior distributions. Shocks might be considered as a policy  
 23 action which is controlled by the Fed. Concerning the assumptions on the model, the impulse

<sup>7</sup>We work with frequencies  $\omega$  on the interval of  $[0, \pi]$ . We define particular economic cycles on this scale and having quarterly data such that business cycles (2-8 years):  $\omega \in [\frac{\pi}{16}, \frac{\pi}{4}]$ , long cycles ( $\geq 8$  years):  $\omega \leq \frac{\pi}{16}$ , and short cycles ( $< 2$  years):  $\omega > \frac{\pi}{4}$ .

1 response functions have the expected signs with imposed contemporaneous restrictions. In  
 2 Figure 4, we depict the frequency response functions of the monetary policy transmission  
 related to the impulse response functions in Figure 3.



**Figure 3:** Time-varying median impulse responses to unitary monetary policy shocks.

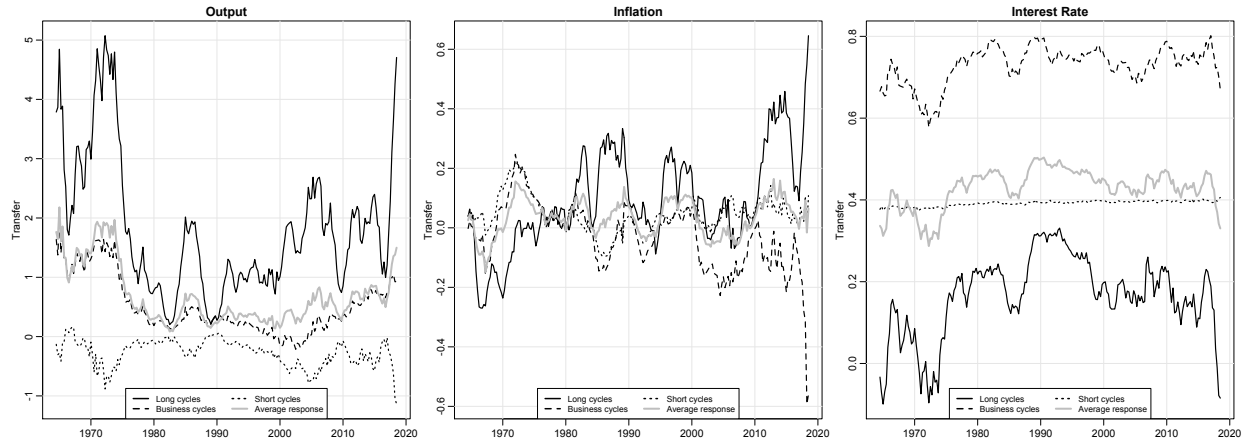
3

4 The impulse responses of interest rates are positive and stable over time with a slow  
 5 decay. The propagation of the monetary policy shock is also positive at all frequency band,  
 6 except for long cycles in the first half of the 1970s.

7 Impulse responses of the output vary during the sample and react extensively to the policy  
 8 shocks, especially during the 1960s and 1970s. After this period, the relationship stabilises  
 9 until the 2000s. Similarly, [Castelnuovo and Surico \(2010\)](#) find larger falls of output gap  
 10 before 1979 than in the sub-sample afterwards. During the Great Moderation, we observe  
 11 the lowest effects of shocks on output, which appear to recover quickly and increase the  
 12 output after several quarters. These findings are in line with [Belongia and Ireland \(2016\)](#)  
 13 who show practically no differences in output responses between the years 2000 and 2007.  
 14 While the transmission depicted in the impulse response functions returns the economy to the  
 15 origin,<sup>8</sup> observing the frequency response functions we decompose the frequency-horizons,  
 16 which determine the adjustment. The monetary policy has the strongest impact on the  
 17 output in the period before 1980. The transfer of the policy was the lowest around 1983 and  
 18 before 1990.

19 Examining frequency responses of the inflation to monetary policy shocks, there are  
 20 evident patterns of switching the direction of transfers. During the first years, the inflations  
 21 responses negatively at almost all cycles. Before 1980 the situation changes and the responses  
 22 are all nearly positive, which corresponds to the debated empirical evidence of price puzzle  
 23 prior 1979. Additional to this result, we can see that short and business cycles responses  
 24 are negative. This frequency observation may explain the post-Volcker era without the price  
 25 puzzle since researchers draw such conclusion from IRFs based on the first several quarters,

<sup>8</sup>Assuming estimation of a stable model.



**Figure 4:** Time-varying frequency response functions of output (left), inflation (centre), and interest rate (right) in response to monetary policy. Frequency responses are shown at different cyclical intervals, long-, business-, short-terms corresponding to averages over intervals of longer than 8 years, between 2 to 8 years, and shorter than 2 years.

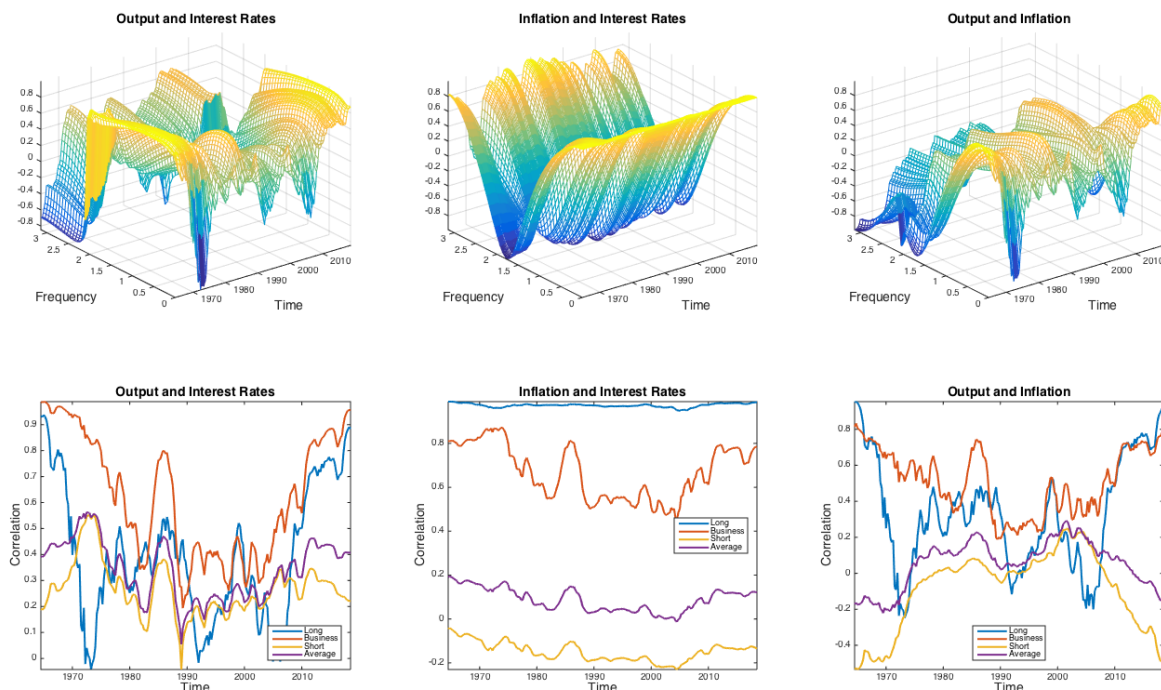
1 which are mostly not induced by long cycles. Combining the results from IRFs and frequency  
 2 responses during late 1980, we see inflation positive response at long cycles together with  
 3 its gradual increase after 15 quarters in IRFs, Figure 3. Similar explanation follows for first  
 4 quarters and short cycles. However, this comparison cannot be used in general. For example  
 5 in years after 2010, the inflation responses positively at short cycles, even there is a large fall  
 6 for the first horizons and no puzzling result in the time domain, but all other cycles, it is. The  
 7 alternating behaviour of inflation frequency responses creates evidence that at different point  
 8 of time even the impulse response functions fulfil the sign restriction of no price puzzle and  
 9 yet in the frequency domain, we observe with both positive and negative transfers. Hence,  
 10 given the frequency-specific transfers, these findings provide a better understanding of the  
 11 impact of structural shocks for policy-making.

## 12 4.2 Time-varying frequency dependence

13 We complement the previous structural analysis with the evidence about local correlations of  
 14 output and inflation with interest rates. We draw the correlations from the spectral matrix  
 15 (Eq. 13) obtained from posterior time-varying coefficients and covariance matrix. Figure 5  
 16 depicts complete time-frequency plane of output, inflation, and interest rates correlations  
 17 (top row) and time-varying correlations at specific frequency horizons – economic cycles  
 18 (bottom row). The correlations depict the link and direction between the variables. We  
 19 observe changes from pro-cyclical to counter-cyclical and vice-versa behaviour with changing  
 20 magnitudes in both time and frequency.



1 Correlations between pairs of variables show similar patterns of strong relationship at the  
 2 beginning of the sample, in the mid-1980s, and after 2010. During the Great Moderation,  
 3 particularly in the 1990s, we see that correlations are more stable, it is also when the transfer  
 4 functions of interest rates (Figure 4, right) are strongest and stable. One might link the  
 5 change in correlations during 1980 the Volcker disinflation because in all case, previous to  
 6 1984, the correlations had been decreasing, however, after that the correlations increase for  
 7 several years. Moreover, the three correlations share the same pattern at business cycle  
 frequencies, which is strongest in the 1970s, lowest in 1990s, and from 2000 rises again.



**Figure 5:** Posterior median (unconditional) time-varying correlations between pairs of GDP growth, inflation, interest rates. Top row shows complete TV time-frequency correlations according to Eq. 15, where z-axis is the value of correlation. Bottom row depicts the same but averaged information over particular frequency bands – different cyclical lengths: long cycles – greater than 8Y, business cycles – from 2Y to 8Y, and short cycles – lower than 2Y.

8

9 Empirical literature often presents time-varying correlations between monetary variables  
 10 calculated directly from the posterior distribution of the time-varying covariance matrix.  
 11 For example, the correlations between inflation and interest rates are similar in shape to  
 12 [Cogley and Sargent \(2005\)](#), but in our case, the inflation is on average less correlated with  
 13 the interest rate. Unfortunately, their sample ended in 2000. Thus, we do not compare the  
 14 phase of strengthening.

15 Interestingly, related to the price puzzle phenomenon, we see the relatively clear pattern  
 16 in time-frequency dependence between the inflation and the interest rates, Figure 5 (top-

1 centre). The correlation is strong and positive at long, business and the shortest cycles  
2 ( $\omega = \pi \approx 2$  quarters); and it is also strong but negative for all cycles around one-year length  
3 over the whole sample. This horizon specific outlook might help in policy making that in  
4 long cycles inflation co-moves with interest rates. As [Benati and Mumtaz \(2007\)](#), we observe  
5 a sign change in the correlation between output and inflation in the mid-1970s. The average  
6 correlation is very similar to their findings in shape and magnitudes.

7 We observe the time-varying frequency responses of inflation and interest rates that  
8 change vividly (Figure 4), and when looking at time-varying correlations, we do not observe  
9 such changing behaviour. The reason is that the two measures use different underlying  
10 information. Correlations are measured by the covariance structure, and the frequency  
11 response functions try to capture the filtering process in coefficients of the model.

## 12 **5 Conclusion**

13 In this paper, we have used the frequency-specific methodology to provide an additional way  
14 of looking at the monetary policy transmission dynamics. We employ the frequency response  
15 functions to decompose the transfers of monetary policy in the framework that allows for  
16 time-varying coefficients and covariance structure. The time-varying model empowers us to  
17 quantify dependence measures in both time and frequency domains. To assess the frequency  
18 domain monetary transmission, we estimate a traditional TVP-VAR identified with sign  
19 restrictions. The propagations are studied on the U.S. data.

20 Firstly, we find a substantial variation in output and inflation in response to shocks in  
21 time, which is in line with the literature. The frequency transmission of monetary policy  
22 pronounces the largest positive impacts in output at economic cycles longer than eight years.  
23 The frequency response of output is overall positive, only for cycles shorter than two years  
24 is negative over time. The shocks affect the interest rates most at business cycles of length  
25 from 2 to 8 years, with positive impact at all frequencies.

26 Moreover, using the time-varying approach, we observe a negative propagation of shocks  
27 to the inflation meaning that the price puzzle phenomenon is not observed for business  
28 cycles after 1985. However, the average frequency transmission varies, and in every decade  
29 we see that prices rise in response to monetary policy, except the years between 2000 and  
30 2010. This leads us to a new result that the price puzzle phenomenon may have frequency  
31 dependent effect and be propagated at different cycles, which is not observed using only  
32 impulse response functions, even with sign restrictions imposed.

33 Lastly, we estimated reduced-form time-frequency correlation at each point of time that  
34 quantifies local dependence structure between variables. We find a characteristic pattern of

1 strong dependence at the beginning and the end of the studied period and smaller dependence  
2 from 1980 to 2000. The relationship between inflation and interest rate is strong, they move  
3 pro-cyclically at long and short cycles, and counter-cyclically at cycles around one year.

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