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Quality and its impact on efficiency

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Quality and its Impact on Efficiency

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Abstract

The issue of quality and its relationship with efficiency and performance is a crucial operational issue in many fields of study including production economics, operations research, engineering and business management. In this paper we provide a methodology for identifying latent quality factors, estimate their statistical significance and analyze their impact on the performance of the production process. This methodology is based on up-to-date computational methods and statistical tests for directional distances. We illustrate the approach using real data to evaluate the performance of European Universities.

Key Words: nonparametric efficiency, performance assessment, quality, benchmarking, directional distances, conditional efficiency, observed and unobserved heterogeneity, separability condition, European universities.

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1 Introduction

Efficiency, performance evaluation and benchmarking exercises abound in the empirical literature. There are many performance evaluation systems both for business (Neely, Gregory and Platts, 1995; Neely et al., 2000; Neely, 2002) and for the public sphere with the advent of the so called *New Public Management* (Lane, 2000; for an overview and recent trends, see Van Dooren, Bouckaert and Halligan, 2015). A common approach in both practices is to define one or more Key Performance Indicators (KPIs) and compare them for the different units. While this approach is useful in very simple cases, it has some drawbacks: it presumes constant returns to scale, it does not facilitate a comprehensive view of the unit under analysis that accounts for all inputs and outputs, and different KPIs may point to different ideal units. It is difficult to evaluate an organization’s performance when there are multiple performance metrics related to a system or operation. The difficulties are further enhanced when the relationships among the performance metrics are complex and involve unknown trade-offs.

In all the cases, and in particular for the analysis of the performance of services, it is important to describe a general model of production on the base of which to run the empirical analyses. Performance is a broad concept which includes productivity and efficiency. The productivity of a unit is defined as the ratio of its outputs to its inputs. Efficiency instead is the distance between the outputs/inputs ratio of a unit and the outputs/inputs ratio of the best possible or *efficient frontier* for the unit. As discussed in Daraio and Simar (2007, p. 14), productivity and efficiency are two cooperating concepts for analysing the performance of producing units. Efficiency measures are more accurate then those of productivity. This is because efficiency measures involve a comparison with the most efficient frontier and for that they can complete those of productivity based on the ratios of outputs/inputs.

Frontier efficiency analysis, introduced and developed in the economics of production, operational research and management science, and based on nonparametric quantitative methods (e.g. see Bogetoft, 2012; Zhu, 2014), are widely used in the context of performance evaluation and benchmarking for many reasons. First of all, because it offers a rigorous analytical framework for representing a general model of production. Second, because of their empirical orientation and nonparametric nature, typical of nonparametric efficiency estimators such as Data Envelopment Analysis (DEA, Farrell, 1957; Charnes et al. 1978) and Free Disposal Hull (FDH, Deprins et al., 1984), that is the absence of a priori assumptions about the functional relationships between inputs and outputs. Third, because it allows identification of best practices as a means to improve performance and increase productivity. Finally, frontier efficiency analysis is particularly valuable for service operations, where identifying
benchmarks or standards is more difficult than in a manufacturing context.

Nonparametric efficiency analysis is more and more used in studies involving best-practice identification in the nonprofit sector including education, higher education, the healthcare sector, in the regulated sector, and in the private sector. Robust nonparametric techniques, based on the so-called partial frontiers, have also been introduced (see e.g. Daraio and Simar, 2007 for an introduction) to overcome some of the limits of the traditional nonparametric approach, namely the influence of extreme values and outliers. When directional distance functions introduced by Chambers et al. (1996) are used, the target is then defined as the virtual unit obtained by the projection of the evaluated unit to the efficient frontier along the chosen direction. The directional distance function approach provides a general and flexible way to use a benchmarking model as a learning lab (Bogetoft, 2012). By changing the direction of improvement the user can learn about the possibilities available and choose a production target based on this interaction. Recent surveys (e.g., Emrouznejad and Yang, 2017) show an increasing trend in applications of nonparametric efficiency analysis in all kind of services.

A major challenge in benchmarking and performance assessment of services is accounting for quality. One of the main critics that is made to benchmarking analyses of all kinds is that they are not able to adequately take into account the peculiarities of the assessed units and/or the various aspects of quality. The quantitative evaluations and comparison should take into account the main features of the assessed units, or in other words, should account for their heterogeneity and the efficiency analysis should include also quality dimensions. Quality is important but not easy to measure. Quality is linked to efficiency and performance but there may be trade-offs between quality and efficiency. The role played by quality is far from being unambiguously determined. We propose an approach to identify unobserved quality factors that may have an impact on the efficiency and performance, although its impact is not a priori known and must be empirically estimated. In the next section, we will discuss about the ambiguity of the definition of quality and will show how quality can be introduced in a general model of production.

While the earliest analyses of efficiency in the service sector (e.g. Ruggiero, 1996) have been mostly concerned with comparing input to output quantities, subsequent studies have tried to integrate output quality using various methods (e.g. Färe et al., 2006). Lee and Kim (2014) propose a DEA-based approach to aggregate and benchmark different measures of service quality. Efforts have been made recently to estimate the quality of managerial practices in the frontier analysis framework (Delis and Tsonias, 2018), considering that it is difficult to measure them, when data on firms and their managers are not available. There are indeed different ways to include quality in efficiency analysis. The most used within the
nonparametric efficiency literature are i) one-stage approach, in which the quality variables are included in the efficiency estimation as outputs; ii) two-stage approach, in which the (unconditional) efficiency scores are estimated including only the inputs and outputs and afterwards are regressed, in a second stage, against quality variables; iii) conditional approach that includes quality variables by conditioning to their values the production process.

By considering quality as an output, according to the one-stage approach, we are not able to investigate if there are trade-offs between efficiency and quality. It is well known that the two-stage approach suffers from different limitations (see Simar and Wilson, 2007 and 2011) and is based on the so called separability assumption which, as we will see in the following of the paper, assumes that quality does not affect the efficient frontier of the best practice, but may affect only the distribution of the distances of the units from this efficient frontier. Varabyova and Schreyögg (2017) show that the conditional approach of Daraio and Simar (2007) extended in Badin, Daraio and Simar (2012 and 2014) may be helpful in disentangling the impact of efficiency and quality in the health care sector.

Although there have been recent developments in the frontier-based quantitative analysis of performance, the issue of quality and its relationship to efficiency and performance remains crucial and open in many fields of study. As a matter of fact, the investigation of quality and the development of methods for including quality in empirical analysis is a crucial operational issue at the intersection of the Economics of Production, Management Science and Operational Research with other disciplines, such as Operations Management, Engineering and Business Management. Table 1 summarises the main fields of study interested in quality and provides a few key references for each of them. We believe that the approach proposed in this paper could be interesting for all these streams of literature.

In the next section we describe how we model quality, allowing for the estimation of latent quality factors. This means that we recognize that it may be difficult to directly observe all the quality features of a production process, in particular those that are related to human capital. We propose then a general and flexible approach to estimate these unobserved quality factors which can be related to inputs and/or outputs of the production process (this is described in Section 4, while Section 3 introduces the basic notions of the flexible nonparametric directional distance estimators adopted). Both observed and unobserved quality factors may have an impact on the efficiency and performance, although their impact is not a priori known and must be empirically estimated. At this purpose we propose in Section 4 a statistical test of the impact of quality on the efficiency. Section 5 illustrates the proposed approach on data about the performance of European Universities, while the last section summarizes and concludes the paper.
Table 1: A selected overview on the main fields of study interested in quality.

<table>
<thead>
<tr>
<th>Literature Stream</th>
<th>References</th>
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</tr>
<tr>
<td><strong>Engineering</strong></td>
<td>Chandrupatla (2009)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Econometrics</strong></td>
<td>Mundlak (1961)</td>
</tr>
<tr>
<td>(Panel data econometrics consider fixed effects of managerial “quality”)</td>
<td>Powell (1995)&lt;sup&gt;b&lt;/sup&gt;; Bloom, and Van Reenen (2007)&lt;sup&gt;c&lt;/sup&gt;; Neely, Gregory and Platts (1995); Neely et al. (2000); Neely (2002)</td>
</tr>
<tr>
<td><strong>Business Management</strong></td>
<td>Lane (2000); Van Dooren, Bouckaert and Halligan (2015)</td>
</tr>
<tr>
<td><strong>New Public Management</strong></td>
<td>Gale and Wood (1994); Mukherjee and Nath (2005)&lt;sup&gt;d&lt;/sup&gt;; Seth, Deshmukh and Vrat (2005)</td>
</tr>
<tr>
<td><strong>Quality in Efficiency Analysis</strong></td>
<td>Färe et al. (2006); Lee and Kim (2014); Varabyova and Schreyögg (2017)</td>
</tr>
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<td><strong>Knowledge Management</strong></td>
<td>Delis and Tsionas (2018)</td>
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<tr>
<td><strong>Intangibles and intellectual capital measurement</strong></td>
<td>Tiwana (2000); Alavi and Leidner (2001) and Liebowitz (2012)</td>
</tr>
<tr>
<td></td>
<td>Bontis (2001); Bryl (2018); Boufoun and Edvinsson (2012); Guthrie and Dumay (2015); Dumay, Guthrie and Puntillo (2015); Secundo, Lombardi and Dumay (2018)</td>
</tr>
</tbody>
</table>

<sup>a</sup>“Quality improvement is an ongoing process and the implementation of “quality” principles is not limited to industry - these principles are for all businesses, offices, services, education, healthcare, and others”.

<sup>b</sup>In the *Strategic Management Journal* finds that “certain tacit, behavioural, imperfectly imitable features - such as open culture, employee empowerment and executive commitment, can produce advantage; these tacit resources drive the success of TQM (Total Quality Management).”

<sup>c</sup>Note that while *business and management* have highlighted the importance of good management, empirical economists have had relatively little to say about management practices, due to the absence of “quality” data.

<sup>d</sup>In the marketing area observe that a single overall measure of observed service “quality” based on one indicator is over-simplistic. It would be more useful to explore a richer profile of customer service “quality” provided by different measures. But then of course, the question that remains is which one of the available indicators to consider.
2 Ambiguity of quality and our approach to handle it

Before describing the main elements of our approach, we introduce the concept of quality and its ambiguity. The Oxford Dictionary (https://en.oxforddictionaries.com/definition/quality last accessed 25 Jan 2019) defines quality as “1) The standard of something as measured against other things of a similar kind; the degree of excellence of something, and 2) A distinctive attribute or characteristic possessed by someone or something.” From this definition emerges the first ambiguity of quality. Quality is on the one hand (definition 1) related to how good or bad a unit is operating with respect to other similar units. It is synonym of standard, grade, classification, rank, level and it is close to the concept of efficiency (how good or bad the outputs are produced given the available inputs with respect to a benchmark frontier). On the other hand (definition 2), it is a distinctive characteristic or feature of someone or something, synonym of attribute, property, peculiarity, quite different from the previous one.

We propose a broader definition that encompasses both definitions of quality. Figure 1 shows the main building blocks of our notion of quality. We take the view that quality does not coincide with efficiency, but it may be linked to it. How this could be the case is a matter of empirical investigation. For example, in automobile manufacturing quality may be reliability. In steel production quality involves the metallurgical properties of the steel being produced, such as hardness, malleability, and so on.

![Figure 1: An illustration of the concept of quality.](image)

Another ambiguity of quality is related to the fact that it must be defined in terms of the context being examined. Moving from standard production activities towards services the component and the role of human capital increase their importance and increase the ambiguity of the definition of what quality is and how it can be quantitatively assessed. In the most simple production cases, as those recalled above, quality features, that may be connected to people or other factors of production, may be directly observed and quantified.
Indicators may be calculated and included in the performance model. Nevertheless, when people are involved, it is more difficult to collect all the information related for instance to their efforts, motivation, skills and ability. Human capital and managerial tasks (coordination and activities related to people) in general are very difficult to measure quantitatively. See also the notes of Table 1 in this respect.

Current evaluation models impose precise definitions and standardization of the dimensions in which the activities are organized. This is very difficult for activities related to human capital such as services. Vidaillet (2013, p.120) observes that “Working implies cultivating some secrets.” Therefore, in evaluating performance, factors and characteristics not directly observed, related to the human capital involved, must also be considered.

Intangibles and intellectual capital have always been considered as relevant factors to the productivity and competitiveness of the private sector as well as of the public sector (Guthrie and Dumay, 2015; Dumay, Guthrie and Puntillo, 2015; Secundo, Lombardi and Dumay, 2018). The measurement of intellectual capital (Bryl, 2018) is an emerging research area in knowledge management (Tiwana, 2000; Alavi and Leidner, 2001 and Liebowitz, 2012). However, being at its infant stage, it still lacks a rigorous methodology for being assessed, as also managerial quality, that remains difficult to be directly measured and included in a more general performance measurement system.

The approach we propose in this paper, outlined below and described at length in the next sections, tries to extend the latest available non-parametric efficiency analysis techniques to model quality features (both observed and unobserved) within the performance assessment (efficiency) of units. This approach may be useful for different streams of literature (see Table 1) for which quality is a crucial operational issue.

Figure 2 illustrates our approach at a glance. The top of Figure 2 represents the model of production process based on the activity analysis framework that will be formally introduced in the next section. It is based on the transformation of ‘p’ inputs (or resources, in what follows denoted by X) that are used for the realization of ‘q’ outputs (in what follows denoted by Y), that may be products or services. The measurement of the efficiency consists in the estimation of an efficient frontier over the observed units, that is the frontier of the best practices, those that produce the maximum outputs given the available inputs or those that produce their level of outputs with the minimum amount of resources. In this model we may include also heterogeneity or conditioning factors (in what follows denoted by Z) that are neither inputs nor outputs of the production but may affect the production process. As we will rigorously show in Section 4, we can introduce in this model of production unobserved

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1 According to Drucker and Macariello (2008, p. xxvi), management (whether we talk of a business, a government agency, or a nonprofit organization) is “to make human resources productive”.

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quality factors related to some inputs and/or outputs (in what follows denoted by \( V \)).

In this frontier context (outlined in the top of Figure 2), adopting an output orientation means that we look at the maximum expansion of the outputs that is achievable given the available resources (inputs). In our approach, we may identify a latent quality factor \((V')\) that is linked to some inputs. The rationale for this is that when we want to expand our outputs, we have to consider what is the current quality of our available inputs. On the contrary, when we are in an input orientation, that is we look at the minimum level of inputs (or resources) that is achievable, given the outputs realized, we may identify a latent quality factor \((V'')\) linked to some outputs. The rationale for this is that when we want to reduce our inputs, we have to consider what is the current quality of our outputs.

![Model of the production process](image)

**Main steps of our approach**

**If output orientation, then:**

1. Define X and Y
2. Identify a latent quality factor \((V')\) linked to some input \((X)\)

**If input orientation, then:**

1. Define X and Y
2. Identify a latent quality factor \((V'')\) linked to some output \((Y)\)

**After that:**

3. Include, eventually, other observed quality factors \((Q)\) and Heterogeneity factors \((Z)\)
4. Test the statistical significance of the impact of the quality factors (separability test)
5. Choose the direction (strategy comparison) and estimate the efficiency scores
6. Calculate the gaps

Figure 2: A simple illustration of our approach.

Once these latent quality factors \((V)\) have been identified, we can estimate what is their impact on the production process and if there are trade-offs with the efficiency (performance) of the production process. After that we can calculate and compare the efficiency measures related to different paths towards the efficient frontier, selecting different directions towards
the benchmarking frontier. Finally we can analyse the obtained gaps of the assessed units.
The bottom of Figure 2 summarizes the main steps of our approach that will be detailed in
the next sections and will be illustrated on real data in Section 5.

3 Basic notions on frontier and conditional frontier models

This section introduces and summarizes the basic setup and notation for frontier, conditional
frontier models and their robust version. Here we present a comprehensive summary of
concepts developed in Cazals et al. (2002), Daraio and Simar (2005), Simar and Wilson
(2012). Below, section 4 introduces the methodology to include quality in this setup.

3.1 Introducing heterogeneity in frontier models

Production may be characterized by a process generating a vector of inputs and outputs
defined over an appropriate probability space. Let $X \in \mathbb{R}^p$ denotes inputs and $Y \in \mathbb{R}^q$ the
outputs and we can define the attainable set

$$\Psi = \{(x, y) \in \mathbb{R}^{p+q} | x \text{ can produce } y\}, \quad (3.1)$$

as the set of values $(x, y)$ which are technically possible.

The attainable set $\Psi$ is the support of the joint distribution of $(X,Y)$ which can be
described, e.g. by the joint probability $H_{XY}(x, y) = \text{Prob}(X \leq x, Y \geq y)$, which is the
probability of finding a unit $(X,Y)$ dominating the point $(x,y)$. As shown in Cazals et al.
(2002),

$$\Psi = \{(x, y) \in \mathbb{R}^{p+q} | H_{XY}(x, y) > 0\}, \quad (3.2)$$

under the free disposability assumption.\(^2\)

In the presence of external or environmental factors $Z \in \mathcal{Z} \subset \mathbb{R}^r$ that may introduce
heterogeneity by influencing the production process, the probability space to consider has to
be augmented. The random variables $X,Y,Z$ are defined on the probability space $(\Omega, \mathcal{F}, P)$
and we denote by $\mathcal{P}$ the support of the joint distribution of $(X,Y,Z)$. Let $\Psi^z$ denote the
support of $(X,Y)$ given that $Z = z$. Thus the attainable set for units facing external

\(^2\)The free disposability of inputs and outputs assumes that if $(x, y) \in \Psi$, then $(\tilde{x}, \tilde{y}) \in \Psi$ for all $(\tilde{x}, \tilde{y})$ such that $\tilde{x} \geq x$ and $\tilde{y} \leq y$. In a sense, it assumes the possibility of wasting resources.
conditions $Z = z$ is

$$\Psi^z = \{ (x, y) \in \mathbb{R}^{p+q} | x \text{ can produce } y \text{ if } Z = z \},$$

$$= \{ (x, y) \in \mathbb{R}^{p+q} | H_{XY|Z}(x, y|z) > 0 \},$$

(3.3)

where $H_{XY|Z}(x, y|z) = \text{Prob}(X \leq x, Y \geq y|Z = z)$. The variables in $Z$ can affect the production process either (i) only through $\Psi^z$ the support of $(X,Y)$, or (ii) only through the conditional distribution $(X,Y)$ given $Z$, affecting e.g. only the probability of a unit to reach its optimal boundary, or (iii) through both. It is easy to see that $\Psi = \bigcup_{z \in Z} \Psi^z$, so that $\Psi^z \subseteq \Psi$, for all $z \in Z$. In the very particular case where the joint support of $(X,Y,Z)$ can be written as a cartesian product $P = \Psi \times Z$, then $Z$ will have no impact on the boundaries of $\Psi$ and $\Psi^z = \Psi$ for all $z \in Z$ (this is called the “separability condition” in this literature; see for example, Simar and Wilson, 2007, 2011). In the latter case, $Z$ may eventually influence the production process only through the probability of reaching its optimal boundary.

The performance of a unit operating at level $(x, y)$ can be measured by its distance to its optimal boundary defining a measure of efficiency. Several measures have been proposed in the literature (see e.g. Fried, Lovell and Schmidt, 2008). We will focus our presentation to flexible directional distances (see e.g. Chambers et al. 1998 and Färe, Grosskopf and Margaritis, 2008). The choice of the directions $d_x \in \mathbb{R}^p_+$ and $d_y \in \mathbb{R}^q_+$ for measuring the distance from the efficiency boundary of unit operating at level $(x, y)$ allows us to analyze different strategies of the units to reach the efficient frontier. The directional distance is defined by

$$\beta(x, y; d_x, d_y) = \sup\{ \beta > 0 | (x - \beta d_x, y + \beta d_y) \in \Psi \},$$

$$= \sup\{ \beta > 0 | H_{XY}(x - \beta d_x, y + \beta d_y) > 0 \},$$

(3.4)

where the second equality assumes free disposability of inputs and outputs (see Simar and Vanhems, 2012). Note that $\beta(x, y; d_x, d_y) \geq 0$ for $(x, y) \in \Psi$ and that a value of zero indicates a unit $(x, y)$ on the efficient boundary. It measures the distance of the unit $(x, y)$ toward the boundary of $\Psi$ along the path determined by $(d_x, d_y)$. Similarly, for conditional measures we add the conditioning on $Z = z$ to obtain

$$\beta(x, y; d_x, d_y|z) = \sup\{ \beta > 0 | H_{XY|Z}(x - \beta d_x, y + \beta d_y|z) > 0 \}. $$

(3.5)

It is well known that the particular case $d_x = 0$ and $d_y = y$ allows us to recover the popular output-oriented radial measures of Farrell-Debreu and of Shephard (the input-oriented case is given by $d_x = x$ and $d_y = 0$). Note that the additive nature of directional distances permits negative input and output quantities, which is not the case for radial distances.
Nonparametric estimators are obtained by substituting the nonparametric estimators \( \hat{H}_{XY} \) and \( \hat{H}_{XY|Z} \) in the expressions above (we give more details in the next section). As shown in Cazals et al. (2002), Daraio and Simar (2005) and Simar and Vanhems (2012) this allows us to recover the Free Disposal Hull (FDH, Deprins et al. 1984) estimators of the efficiency measures and even the Data Envelopment Analysis (DEA, Farrell, 1957, Charnes et al. 1978) estimators if we convexify the FDH estimator of the attainable set (see Simar, Vanhems and Wilson (2012) for their statistical properties). All of these nonparametric estimators have well-known asymptotic properties: to summarize, they suffer from the curse of dimensionality, and practical inference requires bootstrap techniques (see Simar and Wilson, 2015, and the references therein for a recent survey).

The analysis of the effect of \( Z \) on efficiency is based on the investigation of the ratios of the conditional on the unconditional efficiency scores (Daraio and Simar, 2005, 2007). Bádín et al. (2012, 2014) show that in the output orientation an increasing shape of the ratios (unconditional divided by conditional efficiency scores) as a function of \( Z \) corresponds to an unfavorable (negative) effect of \( Z \), while the opposite is true for a decreasing trend (positive effect of \( Z \)). Daraio and Simar (2014) extend this approach to directional distances, considering the differences between unconditional and conditional efficiency scores, and show that an increasing trend of these differences implies a negative impact of \( Z \) on the frontier, while a decreasing trend of these differences points to a positive impact of \( Z \).

### 3.2 Robust approach: partial frontiers

The nonparametric estimators (FDH or DEA type) are envelopment estimators in the sense that the corresponding estimate of \( \Psi \) (or of \( \Psi^z \)) envelopes the cloud of observed data points and so they are quite sensitive to extreme values and outliers. This is the major interest of the robust version of these estimators developed for radial measures (for an overview see Daraio and Simar, 2007). Simar and Vanhems (2012) extend these concepts to directional distances. The idea is to define a less extreme boundary as benchmark, here we define a partial-frontier by contrast to the full-frontier used above. It allows us to measure the distance of a unit to a partial-frontier allowing, by construction, some data points to be outside this partial-frontier. Two ways have been suggested in the literature: the order-\( \alpha \) quantile frontier and the order-\( m \) partial frontier. An introduction and an overview on these methods may be found in Daraio and Simar (2007). In this summary we give only some intuitive definitions for the case of one output and with the output orientation (e.g. \( d_x = 0 \))

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3For instance, for the FDH case we will follow below, the rate of convergence of the efficiency estimates is of the order \( n^{1/(p+q)} \) which becomes much less than the usual parametric rate of convergence \( (n^{1/2}) \) when the dimension of the problem is \( p + q > 2 \).
and $d_y = 1$) and for the unconditional to $Z$ case. In the next section we will describe the most general cases.

For any $\alpha \in (0, 1]$ the order-$\alpha$ measure of efficiency is given by

$$\beta_\alpha(x, y; 0, 1) = \sup \{ \beta | S_{Y|X}(y + \beta | x) > 1 - \alpha \},$$

(3.6)

where $S_{Y|X}(y|x) = \text{Prob}(Y \geq y|X \leq x) = H_{XY}(x, y)/F_X(x)$ is the conditional survival function of $Y$ given $X \leq x$. We remark that if $\alpha \to 1$, we are back to the usual full frontier measure (for $d = (0, 1)$). So for $\alpha < 1$, the benchmark frontier for the unit $(x, y)$ (i.e. where $\beta_\alpha(x, y; 0, 1) = 0$) corresponds to the $\alpha$-quantile of the conditional distribution of the output among the population of units using less inputs than $x$. So $\beta_\alpha(x, y; 0, 1)$ can take negative values if $y$ is large and the unit lies above this conditional quantile.

The order-$m$ frontier in the same situation (output orientation) can be defined, for any integer $m$, as

$$\varphi_m(x) = \mathbb{E}[\max(Y_1, \ldots, Y_m)|X \leq x],$$

(3.7)

where $Y_j$ are iid (independent and identically distributed) realizations of the output $Y$, conditionally on $X \leq x$. So that $\beta_m(x, y; 0, 1) = \varphi_m(x) - y$ which can take negative values for large values of $y$. Here, as $m \to \infty$, we are back to the usual full-frontier measure. So the benchmark frontier is the expected value of the maximum output among $m$ peers drawn from the population of units using less inputs than $x$. It can be shown that when $Y$ takes only positive values

$$\varphi_m(x, y) = \int_0^\infty [1 - (1 - S_{Y|X}(y|x))^m]dy.$$

(3.8)

Nonparametric estimators are obtained by plugging-in the empirical version of the conditional survival function ($\hat{S}_{Y|X}(y|x)$ in the previous equation. They share interesting properties, in particular they achieve the parametric $\sqrt{n}$ rate of convergence independently of the dimension of the problem. Their robustness properties rely on the fact that for large $\alpha$ or $m$ we estimate a partial frontier not far from the full one, but for $\alpha < 1$ and finite $m$, the estimators will not envelop all the data points and so are robust to extreme data points and outliers. Comparisons of the two concepts from a robustness point of view can be found in Daouia and Ruiz-Gazen (2006) and Daouia and Gijbels (2011).

Nonparametric frontier estimation, conditional and unconditional, and their robust versions, are widely applied. Examples of applications include Verschelde and Rogge(2012), Varabyova et al. (2017), Matousek and Tzeremes (2016), Minviel and De Witte (2017). Their estimation is obtained by replacing the unknown probabilities $H_{XY}$ and $H_{XY|Z}$ by their empirical versions, as proposed in the statistical approach to nonparametric frontier estimation (see the references cited at the beginning of Section 3). For fast computations and
exact formulas for the order\text{−}m and conditional order\text{−}m (including their Matlab codes) see Daraio et al. (2019).

4 Inclusion of quality

4.1 Identification of a latent quality factor

As pointed in Simar and Wilson (2007, 2011) and in Daraio et al. (2018), neglecting heterogeneity factors that are not separable may introduce problems. This happens if the boundary of the attainable set may vary with $Z$ ($\Psi^z \neq \Psi$ for some $z \in Z$). In fact, the problem is that the boundary of $\Psi$ considered by ignoring these factors may be not achievable for units facing particular external conditions described by $Z$ and hence, benchmarking units against such boundary has little economic meaning. We have to consider the boundary of $\Psi^z$ for units facing condition $Z = z$.

The problem is the same if we suspect that some unobserved (latent) factor of heterogeneity may affect the boundary of the attainable set. As we have seen in Section 2 quality may be such a factor. Since in our illustration below we will use an output orientation, we propose to use the approach suggested by Simar et al. (2016), which allows identification of a latent factor linked to some input (the converse would follow similar developments, i.e. input orientation and quality linked to some output).

Suppose without loss of generality that this latent quality factor, $V$, is linked to the input $X^1$ and that we can write the link through the following nonparametric model

$$X^1 = \phi(W,V),$$

(4.1)

where $W$ is an auxiliary variable, correlated to $X^1$ but independent of $V$. The model is nonseparable in $V$ and has been studied in econometrics (see e.g. Matzkin, 2003). The classical assumptions of the model are as follows: monotonicity (increasing) of $\phi$ with respect to $V$ and without loss of generality $V$ is uniformly distributed on $[0,1]$ (it is just a matter of scaling $V$ such that it can be interpreted as a quantile). It is known that under these assumptions $V$ is identified by the conditional distribution of $X^1$ given $W$

$$V = F_{X^1|W}.$$ 

(4.2)

So, we can see the latent quality variable $V$ as the part of $X^1$ which is independent of $W$. The choice of the input $X^1$ and of the auxiliary variable $W$ are crucial to identify the latent quality variable we are interested in. We may identify latent quality factors using a different auxiliary variable for each input (Simar et al. 2016) or we could even use the same auxiliary...
variable for identifying latent quality factors linked to different inputs. As pointed in Simar et al. (2016), it has to be noticed that the function \( \phi \) is unknown and in nonseparable models like (4.1) \( V \) plays the role of residual. Under the monotonicity assumption, \( V \) is identified by (4.2) and since \( V \) is uniform on \([0, 1]\), \( \phi \) can be interpreted as a quantile function. This is a nice duality property of these nonseparable models. The choice of the uniform distribution for \( V \) is not a limitation since it is just a matter of rescaling \( V \), but if we rescale it in another way, then we lose the natural interpretation in terms of quantile function and cdf (cumulative distribution function). We will see below how to estimate these unknown quantities.

As illustrated in Figure 2 the approach above may work *mutatis mutandis* in many setups. In the application to the activity of European Universities, we will choose \( X \) as the total number of academic staff and \( W \) as total enrolled students that represents a proxy for the size of the university. This \( W \) variable is correlated to \( X \) but independent from \( V \) and for this reason allows us to interpret the identified (estimated) \( V \) as a latent quality factor related to the quality of the human capital of the universities and their management, that is independent from their size. The identified \( V \) is what remains from the academic personnel once we have accounted for its volume component. In practice, we can check that our identified latent factor behaves as expected by model (4.1). We can also check empirically if the identified (unobserved quality factor) \( V \) may be related to some known partial indicators of quality. See Section 5). This approach to estimate latent quality factors identifying what remains from the volume of the human capital once we have accounted for its size component could be extended and tested also in other contexts and different services. This is left to further research.

### 4.2 Statistical issues and separability test

Nonparametric estimators of the unknown functions in (4.1) and (4.2) are obtained from a sample of observations \((X_i^1, W_i)\) by the following estimator

\[
\hat{V}_i = \hat{F}_{X_i^1|W_i}^W(X_i^1|W_i) = \frac{\sum_{k=1}^{n} I(X_k^1 \leq X_i^1)K_{h_w}(W_i - W_k)}{\sum_{k=1}^{n} K_{h_w}(W_i - W_k)},
\]

of \( V_i \), where \( I(\cdot) \) is the indicator function, \( K_{h_w}(W_i - W_k) = (1/h_w)K((W_i - W_k)/h_w) \) and \( K(\cdot) \) is an usual kernel function (we use an Epanechnikov kernel). Statistical properties of such estimators are derived in Li et al. (2013), in particular it is shown that the bandwidth determined by leave-one-out least-squares cross-validation has the optimal order \( n^{-1/5} \). Note
that an estimate of the function $\phi$ defined in (4.1) is obtained by the corresponding quantiles of the cdf $\hat{F}_{X|W}$.

Theorem 2.1 in Li et al. (2013) indicates that the error of estimation ($\hat{V}_i - V_i$) has an asymptotic normal distribution, with a bias term and a variance that have rather complicated expressions, but we could use the bootstrap to evaluate for each $i = 1, \ldots, n$ a probability interval of level $\gamma$ (e.g. $\gamma = 0.95$) for $V_i$. We should use here the bias-corrected percentile method (see Efron and Tibshirani, 1993) to account for the bias term and to achieve intervals included between the natural bounds $[0, 1]$.

Once the latent quality factor has been estimated, we can use the values $\hat{V}_i$ as an additional variable (like the observed external factor $Z_i$), and as shown in Simar et al. (2016), the fact that we use $\hat{V}_i$ in place of $V_i$ does not affect the asymptotic statistical properties of the nonparametric frontier estimators, nor of the resulting estimators of the conditional efficiency measures such as $\hat{\beta}(x, y; d_x, d_y|z, v)$, computed from the sample $\{(X_i, Y_i, Z_i, \hat{V}_i)\}_{i=1}^n$, where $d_x = 0$ since we have chosen the output orientation, the latent factor being identified through an input.

The effect of $(Z_i, V_i)$ on the efficiency measures is an empirical question. First we can test the separability assumption for $(Z_i, V_i)$ (does the boundary of the attainable set depends on $(z, v)$) and in a second stage we can analyze the links between the conditional efficiency scores with $(Z_i, V_i)$, by using appropriate nonparametric regressions (see e.g. Daraio and Simar, 2014).

In general setups, for testing separability by using directional distances we suggest taking a fixed direction $d$ (that may contain some zeros for inactive variables). This allows to give an interesting interpretation of the test statistics derived in Daraio et al. (2018). By doing so, the directional distances may be interpreted at a constant (the inverse of the norm of the direction vector, $||d||$ which does not depend on $(x, y)$) as the Euclidean distance between the point under evaluation and its projection in the direction $d$ on the efficient frontier. We have $\beta(x, y; d_x, d_y) = ||d||^{-1}|\Psi^{\theta}(x, y) - (x, y)||$ and similarly $\beta(x, y; d_x, d_y|z, v) = ||d||^{-1}|\Psi^{\theta,z,v}(x, y) - (x, y)||$. So the test statistics we use for the test (see Daraio et al. 2018) is an estimator of $E_{X|Z,V}(\beta(X, Y; d_x, d_y)) - E_{X|Z,V}(\beta(X, Y; d_x, d_y|Z, V))$ (where for the first term, the expectation in $Z, V$ is just an abuse of notation since $\beta(X, Y; d_x, d_y)$ does not depend on $Z, V$). This quantity can be interpreted as a constant multiplied by the expected value of the Euclidean distances between the projections of random $(X, Y, Z, V)$ on the unconditional and on the conditional frontiers. We reject the null hypothesis (separability: $(Z, V)$ has no influence on the frontier) if an estimator of this expected distance is too large.

For practical application, first split the sample $S_n = \{(X_i, Y_i, Z_i, \hat{V}_i)\}_{i=1}^n$ randomly into
two independent sub-samples, $S_{1,n_1}$, $S_{2,n_2}$ such that $n_1 = \lfloor n/2 \rfloor$, $n_2 = n - n_1$, $S_{1,n_1} \cup S_{2,n_2} = S_n$, and $S_{1,n_1} \cap S_{2,n_2} = \emptyset$. The $n_1$ observations in $S_{1,n_1}$ are used for the unconditional estimates, while the $n_2$ observations in $S_{2,n_2}$ are used for the conditional estimates.

After splitting the sample, compute for the chosen direction $d = (d_x, d_y)$, the estimators

$$\hat{\mu}_{n_1} = n_1^{-1} \sum_{(X_i, Y_i) \in S_{1,n_1}} \hat{\beta}(X_i, Y_i; d \mid S_{1,n_1})$$

and

$$\hat{\mu}_{c,n_2,h} = n_2^{-1} \sum_{(X_i, Y_i, Z_i, \hat{V}_i) \in S_{2,n_2,h}} \hat{\beta}(X_i, Y_i; d \mid Z_i, \hat{V}_i, S_{2,n_2}),$$

where $S_{2,n_2,h}$ in (4.5), is a random subsample from $S_{2,n_2}$ of size $n_2,h = \min(n_2, n_2 h^{r+1})$.

Here to simplify the notation, $h^{r+1}$ denotes the product of the bandwidths for the $r + 1$ conditioning variables $(Z_i, \hat{V}_i)$ obtained by least squares cross validation when computing the estimator of $H_{X,Y \mid Z,V}$. Consistent estimators of the variances in the two independent samples are given by

$$\hat{\sigma}^2_{n_1} = n_1^{-1} \sum_{(X_i, Y_i) \in S_{1,n_1}} \left( \hat{\beta}(X_i, Y_i; d \mid S_{1,n_1}) - \hat{\mu}_{n_1} \right)^2$$

and

$$\hat{\sigma}^2_{c,n_2,h} = n_2^{-1} \sum_{(X_i, Y_i, Z_i, \hat{V}_i) \in S_{2,n_2}} \left( \hat{\beta}(X_i, Y_i; d \mid Z_i, \hat{V}_i, S_{2,n_2}) - \hat{\mu}_{c,n_2} \right)^2$$

(respectively), where the full (sub) samples are used to estimate the variances.

Now the final form of test statistics depends on the value of $p + q$. As explained below, in our application we will use the FDH estimators so the rate of convergence is $n^\kappa$, where $\kappa = 1/(p + q)$.\(^4\) Then, if $\kappa \geq 1/3$,

$$T_{1,n} = \frac{(\hat{\mu}_{n_1} - \hat{\mu}_{c,n_2,h}) - (\hat{B}_{\kappa,n_1} - \hat{B}_{\kappa,n_2,h}^c)}{\sqrt{\frac{\hat{\sigma}^2_{n_1}}{n_1} + \frac{\hat{\sigma}^2_{c,n_2,h}}{n_2,h}}} \xrightarrow{L} N(0,1)$$

under the null. Alternatively, for larger values of $p + q$, when $\kappa < 1/2$,

$$T_{2,n} = \frac{(\hat{\mu}_{n_1} - \hat{\mu}_{c,n_2,h,\kappa}) - (\hat{B}_{\kappa,n_1} - \hat{B}_{\kappa,n_2,h}^c)}{\sqrt{\frac{\hat{\sigma}^2_{n_1}}{n_1,\kappa} + \frac{\hat{\sigma}^2_{c,n_2,h}}{n_2,h,\kappa}}} \xrightarrow{L} N(0,1)$$

\(^4\)For computing the directional distance estimators we used the fast and exact algorithms described in Daraio et al. (2019).
under the null, where \( n_{1,\kappa} = [n_1^{2\kappa}] \) with \( \hat{\mu}_{n_{1,\kappa}} = n_{1,\kappa}^{-1} \sum_{(X_i, Y_i) \in S_{n_{1,\kappa}}} \hat{\beta}(X_i, Y_i; d \mid S_{n_1}) \), and \( S_{n_{1,\kappa}}^* \) is a random subsample of size \( n_{1,\kappa} \) taken from \( S_{n_1} \). For the conditional part, we have similarly and as described in the preceding section, \( n_{2,h,\kappa} = [n_2^{2\kappa}] \), with \( \hat{\mu}_{c,n_{2,h,\kappa}} = n_{2,h,\kappa}^{-1} \sum_{(X_i, Y_i, Z_i, \tilde{V}_i) \in S_{n_{2,h,\kappa}}^*} \hat{\beta}(X_i, Y_i; d \mid Z_i, \tilde{V}_i, S_{n_2}) \) where \( S_{n_{2,h,\kappa}}^* \) is a random subsample of size \( n_{2,h,\kappa} \) from \( S_{n_2} \). Here the terms \( \hat{B}_{\kappa,n_1} \) and \( \hat{B}_{c,\kappa,n_2} \) are estimators of the corresponding bias correction. They are obtained by a generalized jackknife method described in Daraio et al. (2018); without these bias corrections, the above results do not hold (the limiting normal distributions will have an unknown mean different from zero).

Given a random sample \( S_n \), one can compute values \( \hat{T}_{1,n} \) or \( \hat{T}_{2,n} \) depending on the value of \( (p + q) \).\(^5\) The null should be rejected whenever \( 1 - \Phi(\hat{T}_{1,n}) \) or \( 1 - \Phi(\hat{T}_{2,n}) \) is less than the desired test size, e.g., .1, .05, or .01, where \( \Phi(\cdot) \) denotes the standard normal distribution function.

5 Illustration on European universities

In this section we illustrate the proposed methodology by analysing the efficiency of European universities. We first introduce the issue of quality and performance in Higher Education (HE). After that, we introduce the data. Then we estimate the unobserved quality factor and finally estimate the efficiency and complete the benchmarking analysis.

5.1 Quality in HE

Universities carry out a complex production process. They realize different activities, such as teaching, research and knowledge transfer (the so called third mission), by combining different resources: human capital, financial stocks and infrastructures. Their activities, realized within an heterogeneous environment, produce heterogeneous outputs, such as undergraduate degrees, PhD degrees, scientific publications, citations, service contracts, patents, spin off and so on. In this process, size and subject mix play also an important role (e.g. Daraio et al. 2015 a,b and the references cited there).

The concept of quality of HE institutions is difficult and problematic. Its modeling in quantitative analysis is compelling and challenging as it is the case in general for services (see Section 2). The task of defining quality in higher education is rather tricky, due to the complexity of the matter (Sarrico, 2018a,b; Sarrico et al. 2010): “A consensus seems to have emerged in recent years that attempts to define quality can be regarded as an unrewarding

\(^{5}\)Note that when \( p + q = 3 \) we can use both statistics, but it is better to use the test statistics \( T_{2,n} \) involving errors of approximation in the underlying Central Limit Theorem of smaller order (see Remark 4.1 in Daraio et al., 2018).
venture\[sic\], since quality does not appear to exist as something unique and absolute in higher education” (Sarrico et al. 2010, p. 40). There are several different meanings, from quality as academic excellence to quality as value for money. Quality seems to be not only an elusive concept, but also a complex one that can be perceived in very different ways (Westerheijden et al. 2007). According to this perspective quality is seen as a multidimensional concept that should take into account all these different perspectives about HE and its quality, going from quality to qualities of higher education (Blackmur 2007). Daraio (2017) proposes an overarching concept of quality to develop models for the quantitative assessment of research and Higher Education, based on a conceptual framework made by three dimensions: theory, methodology and data. From this framework it clearly appears the challenging role of the econometric modeling of quality from a methodological perspective.

Human capital, as we have seen in Section 2, is relevant to increase productivity and output of organizations as it includes natural ability, innate skills, knowledge, experience, talent and inventiveness. In the context of university education, it has been observed by Kucharčíková et al. (2015, p. 52) that there are “several approaches how to measure the value of human capital, but a single methodology has not yet been adopted”. This is because on the one hand there is a problem of quantification of knowledge, ability, skills, motivation and talent. On the other hand, the main models proposed in the literature, based on accounting, “have not achieved wider application in practice, due to largely subjectivism, uncertainty and lack of replicability” (Kucharčíková et al. 2015, p. 52).

Paradeise and Thoenig (2015, p. 1-2) stated that “Academic quality still remains a black box not only with regard to assessing the outputs, but also in terms of the formal and informal social, cultural and organizational processes adopted by specific university governance regimes”. Paradeise and Thoenig (2015) identifies two components of quality: reputation (internal component, the elitist oligarchy) and excellence (external component, rankings and Top of the Pile model). Quality is linked to the academic staff, it is a combination of the “iron law of talent”, and of a “post-excellence” quality which rests in administrators and faculty. Table 2 proposes a summary of the literature on quality in HE without any claim of completeness.
Table 2: Selected references on “quality” in Higher Education.

<table>
<thead>
<tr>
<th>Description</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptualization of “quality”</td>
<td>Harvey and Green (1993); Sarrico et al. (2010)</td>
</tr>
<tr>
<td></td>
<td>Williams and de Rassenfosse (2018)</td>
</tr>
<tr>
<td>Total Quality in HE</td>
<td>Lewis and Smith (1994)</td>
</tr>
<tr>
<td>Quality Assurance and regulation in HE</td>
<td>Westerheijden et al. (2007)</td>
</tr>
<tr>
<td>Total Quality Management in Education</td>
<td>Sallis (2002)</td>
</tr>
<tr>
<td>Quality Management in HE</td>
<td>Manatos, Sarrico and Rosa (2016); Sarrico (2018)</td>
</tr>
<tr>
<td>Econometric modelling of Quality</td>
<td>Daraio (2017, 2018a,b)</td>
</tr>
<tr>
<td>Human capital management and efficiency in HE</td>
<td>Kucharčíková et al. (2015)</td>
</tr>
<tr>
<td>Academic Quality (reputation and excellence)</td>
<td>Paradeise and Thoenig (2015)</td>
</tr>
</tbody>
</table>

In this paper we estimate an unobserved quality factor of HE institutions which is linked to the resources (input) of the HE institutions, in particular to the academic staff. We will investigate if it plays a role on the efficiency of HE systems (and which kind of role, i.e. if it is complementary to or a substitute for efficiency), and afterwards we will assess its impact on the benchmarking frontier, including also an observed factor of heterogeneity (Z) that is subject mix or specialization of the HE institutions (see the next section).

5.2 The data and the variables

Our data have been collected within the European Project ETER (European Tertiary Education Register) and have been validated by national statistical authorities. The ETER data were extracted in early 2016 and refer to year 2011 (academic year 2011/2012). They include as inputs total number of academic staff (ACAD) and total number of non-academic staff (NONAC), total expenditures (TEXP) that is the sum of all expenditures (includes expenditure for personnel, non-personnel, capital and unclassified expenditure); as outputs total number of degrees (TDEG) in all the educational levels without the PhDs which are considered as an additional output (PHD), and additional variables such as the share of Third party funding (in PPP) over Total revenues (in PPP, indicated as %REVTHIRD), the foundation year (F. Year) i.e. the year when the institution was established and a proxy of SIZE we built (that will act as an auxiliary variable in the following analysis), given by the total enrolled students ISCED 5-7 plus the PhD students.

6 For additional information and to download the data, see the project website: http://eter.foanneum.at/imdas-eter/ where one can find also additional information on the variables and the Data Quality Report.
These data were integrated with other data on the scientific activity of universities collected from the Scopus bibliometric database in the Scimago Global 2013 Rank (SCIMAGO in Table 3), whose data refer to outputs realized in the years 2007-2011. These scientific publication data include total number of publications (PUB) considered as an output which includes the total number of documents published in scholarly journals indexed in Scopus, the specialization index (SPEC) that indicates the extent of thematic concentration/dispersion of an institution’s scientific output (with values between 0 and 1, indicating generalist vs. specialized institutions respectively), that will be considered as a Z variable, and other variables considered as observed partial quality indicators, that are International Collaboration Institution’s output ratio (%IC), Normalized Impact of citations (NI), High quality Publications Ratio (publications in the first 25% of the distribution % Q1), Excellence Rate (percentage of publications among the most 10% of highly cited publications, %Exc.), Excellence with Leadership (%EwL) that indicates the amount of documents in the Excellence rate in which the institution is the main contributor, the placement of the institution in the Scimago ranking at world level (WR), the placement of the institution in the Scimago ranking at regional level (where region= Europe, RR). From these sources we have the data available for \( n = 337 \) European universities. See Table 3 for the list of variables we use in our illustration and their sources.

Due to the limited size of the available sample, and due to the high correlation between the three inputs and between the two research outputs (PUB and PHD), we use the dimension reduction based on factor analysis, suggested in Daraio and Simar (2007) and analyzed by Monte-Carlo analysis in Wilson (2018). For the input factor \( FX \), it is determined by the first eigenvector of the second moment matrix of the three inputs \( u_x = (0.5723 \ 0.6218 \ 0.5346)' \), which can roughly be interpreted as an average of the scaled inputs; it explains 96% of the total inertia and so little information is lost by using this single input factor. Its correlations with the three original inputs are 0.9777, 0.9474 and 0.9325 respectively. For the two research outputs we have similar results with \( u_y = (0.6986 \ 0.7155)' \) which explains 97% of the total inertia. This output research factor \( FY \) has correlations 0.9676 and 0.9691 with PUB and PHD, respectively. So we end up with 337 observations with one input \( X = FX \) and two outputs \( Y = (TDEG, FY) \) the first one being the teaching activity and the second summarizing the research activity.
### Table 3: Variables about European Universities used in the illustration.

<table>
<thead>
<tr>
<th>Role</th>
<th>Acron.</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inputs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACAD</td>
<td></td>
<td>Total number of academic staff</td>
<td>ETER</td>
</tr>
<tr>
<td>NONAC</td>
<td></td>
<td>Total number of non-academic staff</td>
<td>ETER</td>
</tr>
<tr>
<td>TEXP</td>
<td></td>
<td>Total expenditures in Euro PPP&lt;sup&gt;a&lt;/sup&gt;</td>
<td>ETER</td>
</tr>
<tr>
<td><strong>Outputs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TDEG</td>
<td></td>
<td>Total number of degrees ISCED5-7&lt;sup&gt;b&lt;/sup&gt;</td>
<td>ETER</td>
</tr>
<tr>
<td>PUB</td>
<td></td>
<td>Total number of publications</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td>PHD</td>
<td></td>
<td>Total number of PhD degrees</td>
<td>ETER</td>
</tr>
<tr>
<td><strong>“unobs. quality” factor</strong></td>
<td>V = UQUAL</td>
<td>estimated by $V_i \in [0,1]$ (see below)</td>
<td>our elab.</td>
</tr>
<tr>
<td><strong>Heterogeneity factor</strong></td>
<td>Z = SPEC</td>
<td>Degree of specialization $\in [0,1]$</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td><strong>Auxiliary variable</strong></td>
<td>SIZE</td>
<td>Total number of enrollments</td>
<td>ETER</td>
</tr>
<tr>
<td><strong>Observed “quality” partial indic.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%REVTHIRD</td>
<td></td>
<td>Share of third party funds</td>
<td>ETER</td>
</tr>
<tr>
<td>F. Year</td>
<td></td>
<td>Foundation year</td>
<td>ETER</td>
</tr>
<tr>
<td>%IC</td>
<td></td>
<td>International Collaboration rate</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td>NI</td>
<td></td>
<td>Normalized Citation Impact</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td>%Q1</td>
<td></td>
<td>High “quality” Publication ratio</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td>%Exc.</td>
<td></td>
<td>Excellence ratio</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td>%EwL.</td>
<td></td>
<td>Excellence with Leadership ratio</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td>WR</td>
<td></td>
<td>Scimago World Ranking</td>
<td>SCIMAGO</td>
</tr>
<tr>
<td>RR</td>
<td></td>
<td>Scimago European Ranking</td>
<td>SCIMAGO</td>
</tr>
</tbody>
</table>

<sup>a</sup>PPP stands for Purchasing Power Parity.

<sup>b</sup>ISCED is the International Standard Classification of Education maintained by the UNESCO. ISCED 5 is short cycle tertiary education, ISCED 6 corresponds to bachelor’s level and ISCED 7 to Master’s level.

The directional distance function approach provides a general and flexible way to use a benchmarking model as a learning lab (see Bogetoft, 2012), as introduced in Section 1. By changing the direction of improvement, the user can learn in an interactive manner about the possibilities available and choose a production target or budget based on this interaction. Addressing strategic issues through directional distances for outputs (because $d_y = 0$), we compare an egalitarian centralized path (median direction: $d_y = \text{med}(Y)$), as often used in analysis with directional distances, with the results obtained by using an autonomous paths (individual directions). This will allow us to analyse the difference of centralized directions towards a given output mix (egalitarian direction) versus autonomous directions of improvement selected by the units (individual directions) of the European Humboldtian university model of education production of teaching and research (Schimank and Winnes, 2000).
For identifying a latent quality factor $V$, we decide to select the input factor and try to identify the part of $FX$ which is independent of the SIZE of the university, which acts as an auxiliary variable according to the model described in Section 4. Due to the asymmetric nature of the size of universities, that is distributed as a lognormal, we work rather with $W = \log(\text{SIZE})$, which formally does not change anything, but simplifies the nonparametric estimation of $F_{X|W}$, avoiding huge universities isolated with large values of $W_i$.

5.3 Unobserved quality factor

We start our analysis by the estimation of the latent quality factor $V_i$. First, once the values of $\hat{V}_i$ are obtained we check if the assumption of independence between $V$ and the instrument $W$ is reasonable. As pointed in Simar et al. (2016), the theory for a test of independence has still to be provided, but we can at least compute the various correlations between $\hat{V}_i$ and $W_i$ and have a look on the $p$-values for the hypothesis that these correlations could be zero (as they would in case of independence). The results are shown in Table 5 (see the description of the variables in Table 3) and clearly indicate that the independence seems to be reasonable.

Table 4: Correlations of $\hat{V}_i$ with $W_i$ and $p$-values

<table>
<thead>
<tr>
<th></th>
<th>Pearson</th>
<th>Spearman</th>
<th>Kendall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlations</td>
<td>-0.0187</td>
<td>0.0311</td>
<td>0.0236</td>
</tr>
<tr>
<td>$p$-values</td>
<td>0.7329</td>
<td>0.5695</td>
<td>0.5186</td>
</tr>
</tbody>
</table>

Then we check if the identified quality factor can be interpreted, as we expect, as related to some observed partial quality factors. This is done by looking to the correlations (Pearson) between $\hat{V}_i$ and some proxies suggested in the literature to indicate some partial quality indicators of the university output production (see Moed, 2017 and the references in Table 2). The results are indicated in Table 5, where we also give the correlations with the two outputs ($Y_1$ is teaching (TDEG) and $Y_2$ is our research factor ($FY$)). We can see that all the correlations have the expected sign and are when needed clearly different from zero.

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Table 5: Correlations of $\hat{V}_i$ with outputs and some observed partial indicators of “quality”. Output $Y_1$ is the number of degrees ISCED5-7 and output $Y_2$ is the research factor.

<table>
<thead>
<tr>
<th></th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>%REVTHIRD</th>
<th>%IC</th>
<th>NI</th>
<th>%Q1</th>
<th>%EXC</th>
<th>%EWL</th>
<th>WR</th>
<th>RR</th>
<th>F. Year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0609</td>
<td>0.5817</td>
<td>0.5589</td>
<td>0.4405</td>
<td>0.4508</td>
<td>0.4785</td>
<td>0.4751</td>
<td>0.3549</td>
<td>-0.6139</td>
<td>-0.6050</td>
<td>-0.2626</td>
</tr>
</tbody>
</table>

We see that the estimated latent factor $V$ can be interpreted as the hidden component of the resources, after the elimination of the size component, that contributes to the quality of the university. Interestingly, the same results have been obtained if we estimate the latent factor not of the aggregated input factor (FY) but only of the total number of Academic Staff (ACAD). This could confirm that the estimated latent factor is mainly related to the unobserved or difficult to measure quality of the human capital and in particular of the academic staff of the universities. The quality of the academic staff is made by internal quality (elitism and reputation) and external quality (excellence and rankings) according to Paradeise and Thoenig (2015).

Now the role of our identified latent quality factor on the production process is still an open question. Does it act as a hidden input, or as a latent output? Does it influence the shape of the production possibilities (attainable set) of universities and/or the distribution of their efficiency scores? These questions are addressed in the next section.

5.4 Frontier estimation and benchmarking

Before starting our analysis, we performed a test of convexity due to Kneip et al. (2016) and the convexity assumption was highly rejected (with a $p$-value = 0.0000166). In all the frontier analysis then we use FDH-based estimators. These do not rely on the convexity assumption of the attainable set $\Psi$.

We test the separability condition where $(Z, V)$ have no influence on the boundary of the input × outputs attainable set. We perform the test of the separability, first for $V$ and $Z$ themselves and then jointly for $(Z, V)$. In all the cases we obtain $p$-values less or equal to $10^{-6}$ and so we reject the null hypothesis of separability. The test provides clear evidence that the variables $SPEC = Z$ and $UQUAL = V$, modify (have an impact on) the shape of the efficient benchmarking boundary.

We can investigate the effects of our variables $(Z, V)$ on potential shifts of the frontier by analyzing the nonparametric regression surface of estimates of $\beta(x, y; 0, d) - \beta(x, y; 0, d | z, v)$ on $(z, v)$ as explained in Bădăin et al. (2012) and Daraio and Simar (2014). Figure 3 displays the results. It illustrates the way in which the two variables affect the shift of the efficient
frontier by looking to the local linear regression of the differences $\hat{\beta}(x, y; 0, d) - \hat{\beta}(x, y; 0, d|z, v)$ on $(z, v)$ (see e.g. Bädin et al., 2012, and Daraio and Simar (2014)).

Of course the efficiency measures depend also of the input level $x$, so we should analyze these differences as a function of $(z, v)$ for fixed levels of $x$. We follow the strategy of Florens et al. (2014) and fix three levels of the input factor at its 3 quartiles ($Q_1, Q_2, Q_3$); we then take all the available measures for the observations $(X_i, Y_i, Z_i, V_i)$ such that $|X_i - Q_k| \leq h_x$, $k = 1, \ldots, 3$, where $h_x$ is the normal reference rule bandwidth for $X$. This yields three subsamples with 66, 85 and 48 observations respectively. From these we build the 3 local linear estimates of the regression of $\hat{\beta}(x, y; 0, d) - \hat{\beta}(x, y; 0, d|z, v)$ on $(z, v)$. The results are displayed in Figure 3.

Figure 3 shows that the effect on the efficient benchmarking frontier (shift) is present for all the values of $X$, but is much more important for the large units (with high level of staff). We see also that the latent quality factor $\hat{V}$ has a bigger effect than the specialization (SPEC). This effect (the shift) is more important for universities with high quality factor indicating a trade-off between quality and the efficiency of production.

To analyze the impact of $(Z, V)$ on the distribution of the efficiency scores, we will use robust estimators of the frontier to avoid that extreme data points or outliers hide some effects (see Daraio and Simar, 2007, for simple examples of these situations). We choose to perform the robust analysis by using the order-$m$ partial frontiers. We may also do similar analysis by using the order-$\alpha$ quantile frontier. Comparisons of the two concepts from a robustness point of view can be found in Daouia and Ruiz-Gazen (2006) and Daouia and Gijbels (2011). We prefer to focus the presentation with the order-$m$ case for two reasons. First for robustness properties: once the quantile based frontiers break down they become definitively less resistant to outliers than the order-$m$ frontiers. Second, the asymptotic theory linked with the identification of latent factors and its use in frontier estimation has been done in Simar et al. (2016) for order-$m$ only. We conjecture that the same theory is valid for order-$\alpha$, but it is only a conjecture, so we prefer to do the analysis with the order-$m$ robust frontiers.
Figure 3: Impact of $\hat{V} = UQUAL$ and $Z = SPEC$ on the shift of the full frontier $\beta(x, y; 0, d) - \beta(x, y; 0, d|z, v)$, where $d = \text{med}(y)$ for fixed values of the Input Factor (FX) at the 3 quartiles: from top to bottom, small, median and large level of labor (ACAD).
We select a value for the order \( m \) using the standard methods suggested in the literature (see e.g. Daraio and Simar, 2007; Daouia and Gijbels, 2011), i.e. by looking to the percentages of points lying above the estimated order-\( m \) frontier, as a function of \( m \). Of course this curve will converge to zero as \( m \to \infty \). This is shown in the left panel of Figure 4, when the curve indicate a shoulder effect (becomes more “flat”) it indicates that for letting the points outside the order-\( m \) frontier at this stage, we need to increase much more the value of \( m \), indicating that these points are really extreme data points and potential outliers. Here we select \( m = 310 \), letting around 24% of the data points outside the frontier.

Interestingly, when drawing the same picture for the conditional to \((Z,V)\) order-\( m \) frontier we see that with \( m = 310 \) almost all the points are under the frontier except eight of them (around 2%). This indicates that most of the heterogeneity which was present in the input \( \times \) outputs space has mostly disappeared when conditioning on \((Z,V)\). In the latter cases, the order-\( m \) estimates will be very similar to the full conditional frontier (for \( m \to \infty \), i.e. the conditional FDH frontier). This will be confirmed in the tables of results shown below.

![Figure 4: Percentage of points outside the order-\( m \) frontier. From the left panel (unconditional efficiencies), we select \( m = 310 \), around 24% points still outside the frontier. On the right panel, conditional to \((V,Z)\), with \( m = 310 \), only around 2% points outside the conditional frontier.](image)

We focus on the comparison of the averages of the efficiency scores by country, provided in Tables 6 and 7. Each table shows by column the country, the number of observations (\#obs, note that country with only one university such as CY, LU and MT are not displayed), averages of the full unconditional (\( \hat{\beta}(x,y) \)) and conditional (\( \hat{\beta}(x,y|z) \)) efficiency scores, their corresponding robust versions (\( \hat{\beta}_m(x,y) \) and \( \beta_m(x,y|z) \)) and their standard deviation (\( \text{std} \)).
The difference between the two tables rests in the direction chosen for reaching the efficient frontier. In Table 6 the directional vector is the same for all the universities (*egalitarian* centralized path) and is fixed at the European median level (med(Y)). While in Table 7 the direction is different for each university (*individual directions* given by the values of Y) showing *autonomous* paths.

Considering the values of robust conditional efficiency ($\hat{\beta}_m(x, y|z)$) and remembering that closer to zero is the value of $\hat{\beta}_m(x, y|z)$ the higher is the level of efficiency, we can compare the average values reported in Table 6 and Table 7. We note that in some countries (BE, CH, DE, DK, NL, NO and UK) passing from the egalitarian direction (Table 6) to the autonomous one (Table 7) we observe an increase in efficiency (reduction of the $\hat{\beta}_m(x, y|z)$ value), while for the other countries (IE, IT, LT, PT and SE) we have a reduction in efficiency (increase in the value of $\hat{\beta}_m(x, y|z)$) associated with the transition from the same direction for all (centralized path) to autonomous direction. HU remains almost unchanged. This is a striking result that may point to existing differences in the governance systems of the HE national systems: more *differentiated HE systems* including CH, NL and UK benefit from the autonomy in the choice of the path to follow in order to reach the best practice frontier, while *undifferentiated HE systems* such as IT and PT are not able to fully exploit their autonomy because of governance constraints. Of course this is just a conjecture that should be empirically validated with additional research and is outside the scope of the present paper. The inclusion in the analysis of variables on the governance of HE systems may represent an interesting line for further research. In aggregate, Europe improves its level of efficiency by moving from the same direction for all to the autonomous one (see the last row of Tables 6 and 7 corresponding to EU).
Table 6: Estimates of Efficiency, direction is egalitarian: averages by country and standard deviations of the conditional measures $\beta(x,y|z)$ and $\beta_m(x,y|z)$.

| Country | #obs | $\beta(x,y)$ | $\beta(x,y|z)$ | std | $\beta_m(x,y)$ | $\beta_m(x,y|z)$ | std |
|---------|------|--------------|----------------|-----|----------------|----------------|-----|
| BE      | 5    | 0.1687       | 0.1152         | 0.1293 | 0.1196       | 0.1152         | 0.1293 |
| CH      | 11   | 0.5883       | 0.2051         | 0.2207 | 0.5129       | 0.2051         | 0.2207 |
| DE      | 73   | 0.9908       | 0.6996         | 0.6140 | 0.8801       | 0.6887         | 0.6066 |
| DK      | 8    | 0.7121       | 0.4228         | 0.3848 | 0.6213       | 0.4179         | 0.3797 |
| HU      | 7    | 1.0406       | 0.5463         | 0.4533 | 0.9870       | 0.4237         | 0.2954 |
| IE      | 10   | 0.1293       | 0.0637         | 0.0990 | 0.1159       | 0.0637         | 0.0990 |
| IT      | 60   | 0.1976       | 0.1137         | 0.1788 | 0.1504       | 0.1060         | 0.1681 |
| LT      | 4    | 0.7334       | 0.2923         | 0.2242 | 0.7021       | 0.2923         | 0.2242 |
| NL      | 13   | 0.3250       | 0.0579         | 0.0959 | 0.2190       | 0.0576         | 0.0954 |
| NO      | 10   | 0.5508       | 0.4408         | 0.5428 | 0.4687       | 0.3733         | 0.5412 |
| PT      | 17   | 0.1219       | 0.0723         | 0.1059 | 0.1075       | 0.0721         | 0.1059 |
| SE      | 20   | 0.1445       | 0.2262         | 0.2866 | 0.3191       | 0.2260         | 0.2863 |
| UK      | 96   | 0.0972       | 0.0621         | 0.1305 | 0.0184       | 0.0522         | 0.1156 |
| EU      | 337  | 0.4072       | 0.2582         | 0.3374 | 0.2488       |                |      |

Table 7: Estimates of Efficiency, direction is autonomous: averages by country and standard deviations of the conditional measures $\beta(x,y|z)$ and $\beta_m(x,y|z)$.

| Country | #obs | $\beta(x,y)$ | $\beta(x,y|z)$ | std | $\beta_m(x,y)$ | $\beta_m(x,y|z)$ | std |
|---------|------|--------------|----------------|-----|----------------|----------------|-----|
| BE      | 5    | 0.1609       | 0.0648         | 0.0857 | 0.1443       | 0.0648         | 0.0857 |
| CH      | 11   | 0.3912       | 0.1699         | 0.2238 | 0.3411       | 0.1699         | 0.2238 |
| DE      | 73   | 0.6984       | 0.4914         | 0.4472 | 0.6416       | 0.4880         | 0.4480 |
| DK      | 8    | 0.5091       | 0.2981         | 0.3417 | 0.4633       | 0.2944         | 0.3408 |
| HU      | 7    | 1.1710       | 0.4907         | 0.3665 | 1.1074       | 0.3996         | 0.2903 |
| IE      | 10   | 0.1721       | 0.1264         | 0.2065 | 0.1618       | 0.1233         | 0.1982 |
| IT      | 60   | 0.2779       | 0.1638         | 0.2909 | 0.2574       | 0.1579         | 0.2879 |
| LT      | 4    | 1.6082       | 0.4993         | 0.4257 | 1.5668       | 0.4719         | 0.3993 |
| NL      | 13   | 0.2042       | 0.0375         | 0.0615 | 0.1562       | 0.0372         | 0.0608 |
| NO      | 10   | 0.7585       | 0.3205         | 0.3045 | 0.7342       | 0.3115         | 0.3059 |
| PT      | 17   | 0.3204       | 0.2158         | 0.3035 | 0.3122       | 0.2147         | 0.3038 |
| SE      | 20   | 0.4443       | 0.2684         | 0.2906 | 0.4223       | 0.2665         | 0.2882 |
| UK      | 96   | 0.0896       | 0.0496         | 0.0998 | 0.0668       | 0.0457         | 0.0976 |
| EU      | 337  | 0.3804       | 0.2245         | 0.3486 | 0.2184       |                |      |

In the next step, we analyze the impact of $(Z,V)$ on the efficiency measures $\beta_m(x,y,z,v)$ (see Bădin et al. 2012 and Daraio and Simar, 2014). As above for Figure 3 the efficiency measures depends on the input level $x$, so we analyze $\hat{\beta}_m(x,y,z,v)$ as a function of $(z,v)$ for fixed levels of $x$ at its three quartiles $(Q_1,Q_2,Q_3)$. From the three subsamples, as above we
build the three local linear estimates of the regression of $\hat{\beta}_m(x, y|z, v)$ on $(z, v)$. The results are displayed in Figure 5.

Globally, efficiency decreases ($\beta_m(x, y; 0, d|z, v)$ increases) when $X$ increases. We see an almost flat impact for $X = Q_1$ (first quartile of small universities with low academic staff). We observe a slight negative effect of quality on efficiency (as $V$ increases, $\beta_m(x, y; 0, d|z, v)$ increases) for $X = Q_2$ median-sized universities. There is also a modest effect of the specialization (SPEC). It seems that there is a trade-off between quality and efficiency: when quality ($V$) increases universities may decrease their efficiency levels (the value of $\beta_m(x, y; 0, d|z, v)$ increases), they may produce less of their output mix. In addition, for big universities (large staff number corresponding to the third quartile of the distribution ($X = Q_3$), there is an interaction between degree of specialization (SPEC) and quality: we observe a different effect for specialized university than for generalist ones, pointing globally to a trade-off of quality vs efficiency except for generalist (unspecialized) universities (with lower values of SPEC) which seem to combine efficiency and quality well.
Figure 5: Impact of $\hat{V} = UQUAL$ and $Z = SPEC$ on conditional order-$m$ efficiency measures $\beta_m(x, y; 0, d|z, v)$, where $d = \text{med}(y)$ for fixed values of the Input Factor at the 3 quartiles: from top to bottom, small, median and large levels of labor.
Figure 6: Estimated gaps in the outputs. Top panels report the boxplots of the European countries considered following an egalitarian centralized path (median direction). Bottom panels show the boxplots obtained by selecting autonomous path (individual directions).

Finally, Figure 6 gives, by country, the boxplots of the gaps for each university to reach the frontier according to the egalitarian and autonomous directions. They are given in the original units of the outputs, even for the research outputs that were transformed in the output factor ($FY$) in the analysis. The boxplots confirm the results reported in Tables 6 and in Table 7 but in addition give an idea of the efforts to be made (the gaps to fill) to reach the efficient frontier in terms of the original units of the outputs.

6 Conclusions

The investigation of the relationship between quality and efficiency is an intriguing and compelling issue at the core of many and different streams of literature, such as operational research and management science, production economics and business management (see
Table 1). There may be many ways, and different approaches because the investigation of observed and unobserved quality and its impact and relationship with efficiency is a critical operational issue difficult to handle. Since the most crucial and challenging part of the analysis relates to the inclusion of unobserved or latent quality factors, we propose a nonparametric procedure to estimate unobserved quality features, test their impact on the performance and analyse it, in a state-of-the-art nonparametric performance evaluation model based on up-to-date conditional and robust frontier estimation techniques.

In the application to the activity of European universities, we identified a latent quality variable related to the human capital of the universities and their management, that is independent from their size. We believe that this approach to estimate latent quality factors and this specific choice of identifying it as what remains from the volume of the human capital or labour once we have eliminated its size component could be particularly interesting in the area of quantitative assessment of intangibles, intellectual capital and knowledge management. It could be interesting to extend and test the proposed approach also in other contexts and different services\(^7\). This is left to further research.

The illustration on the European universities data showed globally some evidence of an existing trade-offs between quality and efficiency with the exception of generalist universities that seem to be better able to combine higher quality levels with high efficiency scores. Although these results are interesting, additional research and the extension of the investigations to consolidate it are required and left for further research.

**References**


\(^7\)We are well aware that this is a considerable challenge and for this reason the Matlab code for the implementation and extension of our approach in a broader set of application contexts is available upon requests to the authors.


