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DISCUSSION PAPER SERIES

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# Efficiency Wages in Cournot-Oligopoly

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## ABSTRACT

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### **Efficiency Wages in Cournot-Oligopoly\***

In a Cournot-oligopoly with free but costly entry and business stealing, output per firm is too low and the number of competitors excessive, assuming labor productivity to depend on the number of employees only or to be constant. However, a firm can raise the productivity of its workforce by paying higher wages. We show that such efficiency wages accentuate the distortions occurring in oligopoly. Specifically, excessive entry is aggravated and the welfare loss due to market power rises.

**JEL Classification:** D43, J31, L13

**Keywords:** oligopoly, efficiency wages, excessive entry, welfare

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# 1 Introduction

Digitization is becoming one of the most important facets of economic life. It, for example, comprises the rise of networks and artificial intelligence as well as the evaluation of big data. These developments also affect the nature of labor as input. There are mixed findings about whether digitization will reduce or raise aggregate employment, but there is consensus that a reallocation of tasks within and across occupations has already started and will continue (see, for instance, Akerman et al., 2015, Dauth et al., 2017, Michaels et al., 2014). One implication is that non-routine tasks are rapidly expanding, as pointed out by Dustmann et al. (2009) or Autor and Dorn (2013).

Such non-routine tasks, in particular abstract ones which are complementary to computer technology, make the effort of workers an increasingly important factor of production. In addition, they alter the observability of an individual worker's performance. On the one hand, information and communications technologies (ICT) reduce the cost of controlling effort. On the other hand, the change in a job's content implies that the importance of activities rises, for which the input of effort is basically unobservable. As pointed out by McKinsey&Company (2017), ICT and digitization imply that workers become more critical for a firm's success and that managers have to find ways to incentivize their workforce accordingly. One way to do so is to offer a performance pay scheme, for example efficiency wages, i.e. to use wages in order to enhance a worker's productivity.<sup>1</sup>

At the same time during which the digitization process has accelerated, markets have become less competitive (Autor et al., 2017). Accordingly, many, if not most, product markets feature an oligopolistic structure (Head and Spencer, 2017). Therefore, this type of market and the associated welfare losses deserve special attention. Take a free entry Cournot-oligopoly as an example. In such a market, two types of inefficiencies occur: Output per firm is too low and the number of firms too high if there is business stealing. Such a business stealing effect exists if an exogenous increase in the number of competitors lowers output per firm. The inefficiency result has been derived for a variety of settings

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<sup>1</sup>There is strong evidence that performance related wage schedules lead to higher effort. Lazear (2000) and Shearer (2004) find empirical support for manufacturing workers, while Lavy (2009) and Gielen et al. (2010) show a positive relationship for high-skilled employees and for a representative sample of workers with different skills, respectively.

(see, inter alia, Perry, 1984, Varian, 1985, Mankiw and Whinston, 1986, Mukherjee, 2012b, Amir et al., 2014), mostly without considering specific characteristics of inputs.

This neglect may, however, affect the nature of inefficiencies and is particularly relevant with regard to the most important factor of production, i.e. labor. In a digitalized world, in which the importance of (abstract) non-routine tasks rises, firms may not only view wages as costs, but can use them more and more to improve employee productivity, as pointed out above. Higher productivity raises output per firm, possibly mitigating or eradicating the output inefficiency. Moreover, profits rise with greater productivity such that the incentive to enter the market is enhanced, suggesting that the second inefficiency, excessive entry, may be aggravated.

Although productivity enhancing wage strategies could thus potentially influence the distortions in an oligopolistic market, there is, to the best of our knowledge, no study that systematically investigates this relationship. First evidence that labor market concentration is associated with higher demand for cognitive and social skills (Hershbein and Macaluso, 2018) and more prevalent for workers performing non-routine cognitive tasks (Bachmann et al., 2019) indicates the empirical relevance of combining both aspects. Our contribution fills part of the gap in knowledge identified above by analyzing theoretically how positive productivity effects of wages alter the distortions resulting in a homogeneous Cournot-oligopoly with free but costly entry. We do not only take the effect on the market equilibrium into account but also look at the impact on socially optimal outcomes and their relation to the equilibrium counterparts.

In our analysis, we show that efficiency wages do not eradicate oligopoly distortions, but make them more pronounced in all dimensions. In particular, the difference between optimal output per firm and the market outcome increases, excessive entry is aggravated and, consequently, the welfare loss due to market power rises. These effects occur because efficiency wages raise productivity and make entry more attractive. This impact is stronger in market equilibrium than in social optimum because part of the productivity gain of each firm displaces output by competitors, which each individual firm does not fully take into account. Our findings indicate that gains from policy interventions in oligopolistic markets, for example by restricting the number of competitors or allowing

mergers, depend on the productivity effects of wages. More specifically, our investigation clarifies that the welfare gains from preventing excessive entry are larger in a world in which firms pay efficiency wages to incentivize employees, relative to a setting in which labor markets are competitive. Another implication is that policies that aim to boost the use of productivity-related pay schemes, as e.g. tax incentives for profit-sharing, could have drawbacks in oligopolistic markets. Furthermore, the findings suggest that the process of digitalization may aggravate the welfare losses arising due to oligopolistic market structures.

Our analysis is primarily related to contributions which consider the excess entry prediction in the presence of input market imperfections and focus mainly on non-labor inputs (Okuno-Fujiwara and Suzumura, 1993, Ghosh and Morita, 2007a,b, Basak and Mukherjee, 2016). Labor as input has rarely been looked at and if so, the focus has been on trade unions (de Pinto and Goerke, 2016, Marjit and Mukherjee, 2013), profit-sharing (Suzumura, 1995, Chap. 8) and labor-managed firms (Hamada et al., 2018). There also exist studies that consider costly R&D investments which reduce marginal production costs (Okuno-Fujiwara and Suzumura, 1993, Haruna and Goel, 2011, Mukherjee, 2012a, Chao et al., 2017). In these contributions, the inefficiencies resulting in oligopoly often depend on the extent of cost asymmetries and of knowledge spillovers. Both aspects play no role in our analysis. Furthermore, Corchon and Fradera (2002) clarify that lower variable costs tend to raise the number of firms, output per firm and aggregate output in market equilibrium. They also show that these predictions do not necessarily extend to a reduction in the costs of market entry. This is important because our analysis effectively combines a change of variable costs and (fixed) market entry costs, as we clarify below (see footnote 7). Furthermore, we consider changes both in the market outcome and the socially optimal situation. The previous contributions have not undertaken such comparison.

Regarding the literature on efficiency wages, the vast majority of contributions assume competitive output markets. This may be the case because there are no repercussions from the output market on wage formation in standard models of efficiency wages (see Nickell, 1999). Notable exceptions, such as Amable and Gatti (2002, 2004) and Chen and

Zhao (2014), consider changes in the intensity of competition. In contrast to our setting, they take the number of firms as exogenously given and, thereby, neglect an important determinant of welfare, namely market entry costs.

The remainder of our paper is structured as follows. Section 2 describes the basic assumptions of our model. In Section 3, we determine the equilibrium and socially optimal outcomes. Section 4 analyzes the effects of efficiency wages on the oligopoly distortions. Section 5 concludes.

## 2 Model

We consider a market in which  $j = 1, \dots, n$ ,  $n > 1$ , firms produce a homogenous consumption good. Output of firm  $j$  is denoted by  $x_j$  and aggregate output equals  $X = \sum_{j=1}^n x_j$ . The inverse demand curve is given by  $p(X) = q - X$ , with  $p$  denoting the market price and  $q$  the choke price. There is Cournot-competition. Profits of firm  $j$  are

$$\pi_j = p(X)x_j - w_j l_j - k, \tag{1}$$

where  $w_j$  and  $l_j$  denote wages and employment, respectively, and  $k$  ( $> 0$ ) market entry costs.

We incorporate the notion of efficiency wages by assuming that output  $x_j$  is an increasing function of employment  $l_j$  and of effort  $e$  per employee. Effort, in turn, rises with the wage  $w_j$  paid by firm  $j$ . There are a variety of approaches which rationalize the nature of such efficiency wage mechanism (cf. Schlicht, 2016, for a short survey). One of the most prominent ones is the shirking model (Shapiro and Stiglitz, 1984). According to this approach, each firm in the labor market has an incentive to raise wages above the full employment level because there is imperfect information about a worker's effort. Since all firms face the same incentives, unemployment will result. In the model of dichotomous effort proposed by Shapiro and Stiglitz (1984), unemployment disciplines workers and ensures a positive effort level as well as an equilibrium in the labor market. Effort, however, does not vary incrementally with the wage. Thus, the approach has been extended to allow for a continuous choice of effort, implying that effort increases in wages (Altenburg



and Straub, 1998).

While we are agnostic about the source of the effort relationship, we take up the above idea and subsequently assume that effort is costly to the worker and that these costs of effort decrease with the wage and unemployment, relative to the gain say from shirking. Consequently, the effort function  $e$  is increasing in the wage  $w_j$  and the unemployment rate  $u$ , implying that  $e = e(w_j, u)$  and  $\partial e/\partial w_j, \partial e/\partial u > 0$  hold. Moreover, we assume that effort is strictly convex in the wage, i.e.  $\partial^2 e/\partial w_j^2 < 0$ . This approach is compatible with a shirking model of efficiency wages and also with other underlying mechanisms, such as the exchange of gifts or a reduction in turnover. Hence, our approach commands substantial empirical relevance.

Using this specification of effort, the production function can be expressed as

$$x_j = F(e(w_j, u)^\gamma l_j). \quad (2)$$

As usual in efficiency wage models, output increases in the product of employment  $l_j$  and the measure of effort  $e(w_j, u)^\gamma$ . We therefore assume  $F'(E_j) > 0$ , with  $E_j$  being effective labor input,  $E_j = e(w_j, u)^\gamma l_j$ . Output can either be concave in  $E_j$ , i.e.  $F'(E_j) > 0 > F''(E_j)$ , or linear in  $E_j$ , implying for simplicity that  $F'(E_j) = 1$  holds. We also consider the case of  $F''(E_j) > 0$ , but then assume that output is not too convex in  $E_j$ ; otherwise, second-order conditions could be violated.

The parameter  $\gamma$ ,  $0 < \gamma \leq 1$ , indicates how sensitive output reacts to changes in effort. For  $\gamma \rightarrow 0$ , labor productivity,  $F(E_j)/l_j$ , only depends on the number of employees, or is constant if  $F'(E_j) = 1$ . This scenario can be considered as describing a world in which there are only routine tasks, or in which effort is perfectly observable such that there is no need to incentivize workers by increasing wages, i.e. there are no efficiency wages. The higher the parameter  $\gamma$  is, the more important effort becomes for the level of effective labor input  $E_j$  and the more relevant is the efficiency wage mechanism.

We follow the traditional approach in the industrial organization literature and assume that the market under consideration is small, relative to the rest of the economy. This allows us to determine welfare with reference solely to this market, since repercussions

on other markets are negligible. Accordingly, welfare  $W$  can be defined as the sum of consumer surplus and aggregate profits.<sup>2</sup> As a further consequence, also changes in wages and employment in the oligopoly we look at have (virtually) no impact on the economy-wide unemployment rate  $u$ , which is, therefore, fixed from the perspective of all oligopolists.<sup>3</sup> Hence, we can simplify the effort function to  $e(w_j)$ , with  $e' > 0 > e''$ . Output can then be rewritten as

$$x_j = F(e(w_j)^\gamma l_j), \quad (3)$$

with  $\partial x_j / \partial w_j \equiv x_w = F'(E_j) \gamma e(w_j)^{\gamma-1} e'(w_j) l_j > 0$  and  $\partial x_j / \partial l_j \equiv x_l = F'(E_j) e(w_j)^\gamma > 0$ . Further, we assume  $e(w_j) > 1$  such that  $\partial x_j / \partial \gamma > 0$ , i.e. efficiency wages have positive output effects.

For the solution of our model, we distinguish between a free entry equilibrium and the social optimum. In the former case, firms initially enter the market as long as this is profitable.<sup>4</sup> Subsequently, they maximize profits with respect to employment and wages, while taking the choices of other firms as given. In the latter case, a social planner selects the number of entrants. Given this choice, all firms allowed to compete in the market set employment and wages, i.e. we consider a second-best scenario (see, inter alia, Perry, 1984, Varian, 1985, Mankiw and Whinston, 1986, Amir et al., 2014).

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<sup>2</sup>If effort is associated with disutility, firms pay higher wages to compensate for the loss resulting from the exertion of effort. Moreover, firms set wages such that the utility of wages minus disutility of effort is at least as high as the worker's outside option, i.e. the worker's utility of being employed elsewhere. Because this participation constraint is binding, wages and effort have no direct effect on welfare. Note that we could explicitly derive this result by introducing an outside sector with perfectly competitive markets and worker mobility.

<sup>3</sup>Suppose instead that the unemployment  $u$  were increasing in the wage  $w_j$  such that  $e = e(w_j, u(w_j))$ . In this case, the basic features of our simplified effort function would survive since  $de/dw_j = \partial e / \partial w_j + (\partial e / \partial u)(\partial u / \partial w_j) > 0$ , as long as the second derivative of effort with respect to wages,  $d^2 e / d(w_j)^2$ , is negative.

<sup>4</sup>We ignore the integer constraint and consider  $n$  as a continuous variable (Seade, 1980).

### 3 Solution

#### 3.1 Free Entry Equilibrium

Because firms are identical, we can suppress the index  $j$ . Maximizing (1) with respect to  $w$  and  $l$  yields

$$\frac{\partial \pi}{\partial w} \equiv \pi_w = (p(nx) - x) \underbrace{F'(\cdot) \gamma e(w)^{-(1-\gamma)} e'(w)}_{=x_w/l} - 1 = 0, \quad (4)$$

$$\frac{\partial \pi}{\partial l} \equiv \pi_l = (p(nx) - x) \underbrace{F'(\cdot) e(w)^\gamma}_{=x_l} - w = 0. \quad (5)$$

This implies that in an interior solution  $p - x > 0$  holds, which requires the choke price  $q$  to be sufficiently high.<sup>5</sup> Combining (4) and (5) leads to

$$\gamma \frac{e'(w^*) w^*}{e(w^*)} = 1, \quad (6)$$

where the superscript  $*$  indicates equilibrium outcomes. The equilibrium wage  $w^*$  is thus determined by the generalized Solow condition (6) (cf. Solow, 1979 or Layard et al., 1991) and depends on  $\gamma$  but not on the number of firms. Free entry implies that

$$p(nx)x - wl - k = 0. \quad (7)$$

Differentiating (5) at  $w = w^*$ , we obtain

$$\frac{dx}{dn} = \frac{x F'(\cdot) e(w^*)^\gamma}{\pi_{lx}} < 0, \quad (8)$$

with  $\pi_{lx} = \pi_{ll}/x_l < 0$  due to the second-order conditions. Hence, our framework exhibits business stealing, i.e. an exogenous increase in the number of firms reduces output per

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<sup>5</sup>Second-order conditions are given by  $\pi_{ww}, \pi_{ll} < 0$  and  $|H| > 0$ , where  $|H|$  is the determinant of the Hesse-matrix. It is straightforward to show that  $\pi_{ww} = -2x_w^2 + x_{ww}l/x_w < 0$ ,  $\pi_{ll} = -2x_l^2 + x_{ll}w/x_l < 0$  (where we implicitly assume that in case of  $F'' > 0$ , the degree of convexity is sufficiently weak). The determinant of the Hesse-matrix reads

$$|H| = \frac{x_{ww}x_{ll}}{x_w x_l} lw - 2 \left( x_w^2 \frac{x_{ll}}{x_l} w + x_l^2 \frac{x_{ww}}{x_w} l \right) > 0.$$

Second-order conditions are thus fulfilled.

firm. Accordingly, the pre-condition for excessive entry in a world without efficiency wages is fulfilled (Amir et al., 2014).

Given  $w^*$ , equilibrium employment  $l^*$ , output  $x^*$  and the number of firms  $n^*$  are jointly determined by the Eqs. (3), (5) and (7). A closed-form solution is, however, only possible in case of  $F' = 1$ , i.e. if output is linear in employment. Combining (3), (5) and (7) then yields (cf. Etro, 2014)

$$\underline{x}^* = k^{0.5}, \quad (9)$$

$$\underline{n}^*(\gamma) = \frac{1}{\underline{x}^*} (q - w^*(\gamma)e(w^*(\gamma))^{-\gamma}) - 1, \quad (10)$$

where we use the underline notation to indicate that  $F' = 1$  is assumed. From (10), we can further derive that the number of firms  $\underline{n}^*$  increases in  $\gamma$  (for the proof, see below). Equilibrium employment follows from (3) and reads  $\underline{l}^*(\gamma) = e(w^*)^{-\gamma}\underline{x}^*$ .

To provide intuition for the outcome, note that the worker's effort rises if  $\gamma$  goes up. This has, ceteris paribus, an output-enhancing effect. Profits increase, which incentivizes more firms to enter the market. These firms steal business of competitors, which has, ceteris paribus, an output-reducing effect. In the case of a linear demand schedule and a linear production function, the two effects exactly offset each other, as Eq. (9) clarifies. If output per firm does not vary with  $\gamma$ , higher productivity will raise profits because employment declines. In consequence, more firms enter the market.

Irrespective of the curvature of the production function, profits are zero in equilibrium. Accordingly, welfare equals consumer surplus

$$W^*(\gamma) = \int_0^{n^*x^*} p(\tilde{X})d\tilde{X} - p(n^*x^*)n^*x^*. \quad (11)$$

### 3.2 Social Optimum

From (4) and (5) it can be observed that the firm's trade-off between  $w$  and  $l$  is independent of  $n$ . Accordingly, socially optimal wages and equilibrium wages coincide, i.e.  $w^{opt} = w^*$ , where the superscript *opt* denotes socially optimal outcomes. Output and employment, in contrast, depend on  $n$  and, therefore, on whether the number of firms is

determined in market equilibrium or in a socially optimal manner.

The social planner's objective function is given by

$$\begin{aligned} W(n) &= \int_0^{nx(n)} p(\tilde{X})d\tilde{X} - p(x(n)n) + n\pi(n) \\ &= \int_0^{nx(n)} p(\tilde{X})d\tilde{X} - w^*nl(n) - nk. \end{aligned} \quad (12)$$

The first-order condition reads

$$\begin{aligned} \frac{dW}{dn} &= \frac{d}{dn} \int_0^{nx(n)} p(\tilde{X})d\tilde{X} - w^* \left( l(n) + n \frac{dl}{dx} \frac{dx}{dn} \right) - k \\ &= p(nx(n)) \left( n \frac{dx}{dn} + x(n) \right) - w^* \left( l(n) + n \frac{1}{x_l(n)} \frac{dx}{dn} \right) - k \\ &= \left( p(nx(n)) - \frac{w^*}{x_l(n)} \right) n \frac{dx}{dn} + \pi(n) \\ &= nx(n) \underbrace{\frac{dx}{dn}}_{<0} + \pi(n) = 0, \end{aligned} \quad (13)$$

where we used (1) and (5). We assume that the second-order condition is fulfilled.<sup>6</sup>

There is no closed-form solution for the optimal number of firms in social optimum, unless output is linear in employment. In this case ( $F' = 1$ ), we can use the demand function to express output per firm as

$$\underline{x}(n) = \frac{1}{1+n} (q - w^*(\gamma)e(w^*(\gamma))^{-\gamma}). \quad (14)$$

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<sup>6</sup>Differentiating (13) with respect to  $n$  yields

$$\begin{aligned} \frac{d^2W}{dn^2} &= x(n) \frac{dx}{dn} + n \left( \frac{dx}{dn} \right)^2 + nx(n) \frac{d^2x}{dn^2} + \frac{\partial \pi(n, l, w)}{\partial n} + \frac{\partial \pi(n, l, w)}{\partial l} \frac{\partial l}{\partial n} + \frac{\partial \pi(n, l, w)}{\partial w} \frac{\partial w}{\partial n} \\ &= \frac{dx}{dn} \underbrace{\left( x(n) + n \frac{dx}{dn} \right)}_{dX/dn} + nx(n) \frac{d^2x}{dn^2} - x(n)^2, \end{aligned}$$

where we used (4) and (5). Therefore, the second-order condition will surely be fulfilled if there is business stealing, i.e.  $dx/dn < 0$ , and  $dX/dn > 0$  as well as  $d^2x/dn^2 \leq 0$  hold. If output is linear in employment ( $F' = 1$ ), we have  $d\underline{x}/dn = -\underline{x}(n)/(1+n)$  (see (16) below) and thus  $d^2\underline{x}/dn^2 = 2\underline{x}(n)(1+n)^{-2}$ . Replacing these terms in the second-order condition and simplifying the resulting expression yields

$$\frac{d^2W}{dn^2} = - \left( \frac{\underline{x}(n)}{1+n} \right)^2 (2+n^2) < 0,$$

which shows that the second-order condition is always fulfilled in this case. The first-order condition then implicitly defines the number of firms in the social optimum,  $n^{opt}$ . Intuitively, a marginal increase in  $n$  raises welfare by the amount of profits generated, but reduces welfare because of the business stealing effect,  $dx/dn < 0$ .

Observing (10), we obtain

$$\underline{x}(\underline{n}) = \frac{1}{1 + \underline{n}} (\underline{n}^*(\gamma) + 1) \underline{x}^*, \quad (15)$$

$$\frac{d\underline{x}}{d\underline{n}} = -\frac{1}{(1 + \underline{n})^2} (\underline{n}^*(\gamma) + 1) \underline{x}^* = -\frac{\underline{x}(\underline{n})}{1 + \underline{n}}. \quad (16)$$

Combining (15) and (16), making use of the definition of profits and subsequently substituting in accordance with (14) and then (9), we obtain

$$\begin{aligned} & -\frac{\underline{n}}{1 + \underline{n}} \underline{x}^2 + \underline{\pi} = 0 \\ \Leftrightarrow & -\frac{\underline{n}}{1 + \underline{n}} \underline{x}^2 + q\underline{x} - \underline{n}\underline{x}^2 - e(w^*)^{-\gamma} w^* \underline{x} - k = 0 \\ \Leftrightarrow & -\frac{\underline{n}}{1 + \underline{n}} \underline{x}^2 - \underline{n}\underline{x}^2 + \underline{x}^2(1 + \underline{n}) - k = 0 \\ \Leftrightarrow & \frac{\underline{x}^2}{1 + \underline{n}} - k = 0. \end{aligned} \quad (17)$$

Using (9) and (15), we can therefore express the number of firms in social optimum for the case of a linear production function as a function of the number of firms in market equilibrium (cf. Etro, 2014)

$$\underline{n}^{opt}(\gamma) = (\underline{n}^*(\gamma) + 1)^{2/3} - 1. \quad (18)$$

As shown in Appendix A.1, output per firm and welfare can then be calculated as

$$\underline{x}^{opt}(\gamma) = (\underline{n}^{opt}(\gamma) + 1)^{0.5} \underline{x}^*, \quad (19)$$

$$\underline{W}^{opt}(\gamma) = (\underline{x}^{opt}(\gamma)^2 - \underline{x}^{*2}) \underline{n}^{opt}(\gamma) + 0.5[\underline{n}^{opt}(\gamma) \underline{x}^{opt}(\gamma)]^2. \quad (20)$$

In contrast to the market equilibrium, output per firm rises in  $\gamma$  in the social optimum if output is linear in employment. This is because the planner internalizes the business stealing effect and allows a lower number of new competitors into the market (relative to the equilibrium case) as  $\gamma$  rises. Therefore, the output-enhancing effect due to higher labor productivity dominates the output-reducing effect of fiercer competition.

## 4 Effects of Efficiency Wages

### 4.1 A General Result

How do efficiency wages affect oligopoly distortions? To answer this question, we focus on a rise of the parameter  $\gamma$ . If  $\gamma$  is sufficiently small, firms have no incentive to pay efficiency wages because output is (virtually) unaffected by effort. As  $\gamma$  increases, effort as an input factor becomes increasingly important and firms employ wages to increase productivity.

One element of the response to the above question is provided by

#### Proposition 1

*An increase in  $\gamma$  does not eradicate oligopoly distortions.*

#### Proof 1

*Evaluating (13) at  $n = n^*$  and noting that  $\pi(n^*) = 0$  implies*

$$\frac{dW}{dn}_{n=n^*} = n^* x(n^*) \frac{dx}{dn} < 0. \quad (21)$$

*Therefore,  $n^*(\gamma) > n^{opt}(\gamma) \forall \gamma$  holds, i.e. market entry is excessive. Because the difference between the equilibrium and socially optimal output per firm is determined solely by the number of firms [cf. (8)], we obtain  $x^*(\gamma) < x^{opt}(\gamma) \forall \gamma$ , i.e. output per firm is insufficient. Note that in the special case where output is linear in employment, (18) and (19) immediately imply  $\underline{n}^*(\gamma) > \underline{n}^{opt}(\gamma)$  and  $\underline{x}^*(\gamma) < \underline{x}^{opt}(\gamma)$ .*

### 4.2 Further Analytical Results

Next, we consider how efficiency wages affect the magnitudes of the various distortions. To obtain analytical results, we rely on the scenario with a linear production function and, thus, assume  $F' = 1$ . In Section 4.3, we verify the robustness of the analytical findings by solving our model numerically and considering non-linear relationships between output and employment.

## Proposition 2

Assume a linear production function ( $F' = 1$ ). An increase in  $\gamma$

- (i) raises  $\underline{n}^*$  by more than  $\underline{n}^{opt}$ , implying that excessive entry becomes more pronounced,
- (ii) has no effect on  $\underline{x}^*$  but raises  $\underline{x}^{opt}$ , implying that the insufficiency of output per firm becomes more pronounced,
- (iii) raises the welfare loss due to Cournot-competition.

**Proof 2** See Appendix A.2.

The intuition for the results stated in Proposition 2 is as follows. If  $\gamma$  increases, wages and effort rise. Hence, output and profits increase. In the free entry equilibrium, more firms enter the market and steal business of incumbents. This effect is not internalized by an entrant. The output-reducing effect due to more entry exactly offsets the output-enhancing effect of higher effort. This is due to the unit elasticity of output with respect to labor and the linearity of the inverse demand curve. Consequently, the market equilibrium is characterized by an increase in  $\underline{n}^*$ , while  $\underline{x}^*$  remains constant.

From the social planner's perspective, the increase in effort raises the marginal gain of entry, while marginal costs  $k$  remain constant. The planner increases  $\underline{n}^{opt}$ , taking into account that this, ceteris paribus, reduces output per firm  $\underline{x}^{opt}$ . This implies that a) the increase in  $\underline{n}^{opt}$  is weaker than the increase in  $\underline{n}^*$  and b) the output-enhancing effect of higher effort dominates the output-reducing impact of higher competition, i.e.  $\underline{x}^{opt}$  increases in  $\gamma$ . As the distortions due to excessive entry and insufficient output both become more pronounced, the welfare loss resulting from Cournot-competition increases.<sup>7</sup>

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<sup>7</sup>In our setting, a rise in  $\gamma$  reduces the wage per efficiency unit of labor  $w/(e(w)^\gamma)$ , as inspection of (10) and (A.7) clarifies, assuming that the free entry condition (7) holds. Given the linearity of output in employment, an increase in  $\gamma$  is, thus, tantamount to a simultaneous reduction in unit costs  $w/(e(w)^\gamma)$  and the fixed costs of entry. It is straightforward to show that our findings are determined by the joint impact on both costs components as, for example, the changes in market outcome and socially optimal situation owing to a variation in entry costs can differ from those with respect to efficiency wages derived above. In Davidson and Mukherjee (2007), it is shown that privately beneficial mergers always raise welfare because they entail savings in production costs. Our predictions indicate that this result may be sensitive to the exact way in which cost savings come about.



## 4.3 Numerical Results

### 4.3.1 Quantitative Effects

To evaluate the results quantitatively for the linear production function ( $F' = 1$ ), as illustrated in Proposition 2, we solve our model numerically. To that end, we set  $q = 5$  (which ensures an interior solution),  $k = 2$  (as done by Bernard et al., 2007) and  $e(w) = \ln(w)/0.01 + 1$ . In order to measure the effects of efficiency wages, we compare the case of  $\gamma = 0.01$  (virtually no efficiency wage) with a setting in which  $\gamma = 1$  (where the production elasticities of  $e$  and  $l$  are identical).

As shown in the first column of Table 1, the equilibrium number of firms rises by 37.6%, while the increase in the socially optimal number of firms equals 31.2%. This results in an increase of excessive entry by 45.3%. Moreover, the difference between socially optimal output per firm and the market outcome rises by 25.7%, which is driven by the fact that the latter remains constant while the former goes up. The welfare loss in the presence of efficiency wages is then 66.8% higher than in their absence, although efficiency wages on their own raise welfare because of their productivity effect.<sup>8</sup>

Table 1: Alternative Production Functions

	linear	concave	convex
$\Delta n^*$	37.6	36.8	36.6
$\Delta n^{opt}$	31.2	19.9	34.1
$\Delta(n^* - n^{opt})$	45.3	61.6	39.5
$\Delta x^*$	0	9.8	-5.1
$\Delta x^{opt}$	7.5	26.4	-0.5
$\Delta(x^{opt} - x^*)$	25.7	78.3	9.6
$\Delta \pi^{opt}$	31.2	90.7	10.6
$\Delta W^*$	89.4	125.4	68.2
$\Delta W^{opt}$	85.7	129.2	62.8
$\Delta(W^{opt} - W^*)$	66.8	154	37.3

Note:  $\Delta$  indicates percentage changes of the respective variable, when comparing  $\gamma = 1$  to  $\gamma = 0.01$ . Demand is linear in all specifications. See Section 4.3.2 for information on the specification of the production function in columns 2 and 3.

<sup>8</sup>Note that higher values of market entry costs (higher than  $k = 2$ ) and lower values of the choke price (higher than  $q = 5$ ) aggravate the distortions. The numerical results are available upon request.

### 4.3.2 Non-linearities

Our analytical results summarized in Proposition 2 are based on the assumptions that (a) profits are linear in input, for a given price, and (b) the price decreases linearly with aggregate output. These assumptions suffice to ensure the existence of a Cournot-equilibrium (Novshek, 1985). The first simplification is the limiting case of many seminal contributions which usually assume a convex but not necessarily a strictly convex cost function (Mankiw and Whinston, 1986, Suzumura and Kiyono, 1987) and has often been employed in subsequent analyses (inter alia by Amir et al., 2014, Amir and Burr, 2015, Marjit and Mukherjee, 2013, Mukherjee, 2012b). It is generally without impact because profits will surely decrease in output or input at some production level due to the impact of each firm's output on the market price.<sup>9</sup> However, when comparing the extent of market inefficiency in two different situations, the degree of the convexity of the cost function or, alternatively, of the concavity of the production function, plays a role. This is the case because the deviation of the market outcome from the socially optimal situation depends on how strongly output varies with an input adjustment. Therefore, in one of the numerical exercises below we analyze a specific production function and assume  $x = e(w)^\gamma l^\beta$ . This implies that the marginal product of labor decreases (increases) in  $l$  if  $\beta < 1$  ( $\beta > 1$ ).<sup>10</sup>

The second simplification, linear demand, also represents a limiting case which is usually allowed for (Mankiw and Whinston, 1986, Okuno-Fujiwara and Suzumura, 1993, Ghosh and Morita, 2007a,b) and often analyzed (Amir and Burr, 2015, Basak and Mukherjee, 2016, Haruna and Goel, 2011, Marjit and Mukherjee, 2013, Mukherjee, 2012b). Frequently, the inverse demand function is supposed to be not too convex in output and to be log-concave, such that  $p'(X) + Xp''(X) \leq 0$  applies (Ara et al., 2017, Ghosh and Saha, 2007, Matsumura and Okumura, 2014, Mizuno, 2003). For all these specifications, the excessive entry prediction can be derived. Since the extent of the distortions resulting from oligopoly may also depend on the curvature of the demand schedule, we

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<sup>9</sup>As an exception, Basak and Mukherjee (2016) show that if a supplier of an oligopolist has market power, the excessive entry prediction can depend on whether the production function exhibits constant or decreasing returns to scale.

<sup>10</sup>Note that we can rewrite the production function as  $x = F(e(w)^\gamma l) = [e(w)^\gamma l]^\beta = e(w)^\gamma l^\beta$ . Thus, the above specification is compatible with (3) and the assumption that  $F' > 0 > F''$ .

consider an according generalization as well. More precisely, we specify (inverse) demand as  $p(X) = q - X^{1+\alpha}$ .<sup>11</sup> Thus, the demand curve is strictly convex (concave) if  $\alpha < 0$  ( $\alpha > 0$ ). Moreover, it is log-concave, as  $\log(X) = \log(q - p)^{1/(1+\alpha)}$  is decreasing in the price at an increasing rate.

We subsequently consider the two generalizations in turn. As before, the impact of efficiency wages is measured by comparing outcomes in the case of  $\gamma = 0.01$  with the respective values in the case of  $\gamma = 1$ . First, we focus on a non-linear production function and maintain the assumption of linear demand, i.e. we assume  $\alpha = 0$ . Table 1, column two, illustrates the case in which output is concave in employment ( $\beta = 0.8$ ). Excessive entry rises by 61,6% and the difference between  $x^{opt}$  and  $x^*$  increases by 78,3%. The welfare loss due to oligopoly is 154% higher in the presence of efficiency wages than in their absence. Setting  $\beta = 1.2$ , i.e. assuming a convex relationship between employment and output (see column 3 of Table 1), we find that efficiency wages raise  $n^* - n^{opt}$  by 39.5% and  $x^{opt} - x^*$  by 9.6%. The increase in the welfare loss owing to oligopoly at about 37.3% is substantially lower than for a concave production function. Therefore, our analytical results are qualitatively robust with respect to variations in the marginal product of labor. Quantitatively, however, we observe sizable differences. If labor productivity increases at an increasing (decreasing) rate, the distortions and the resulting welfare loss due to oligopoly is lower (much higher) in presence of efficiency wages than in our benchmark setting with a constant marginal product of labor.

The intuition for the quantitative differences is as follows. Assuming  $\beta = 0.8$  and moving from a world without efficiency wages to a framework in which they have strong productivity effects (i.e. to a setting with  $\gamma = 1$ ) is tantamount to substituting a decreasing returns to scale technology ( $\gamma + \beta < 1$ ) by an increasing returns to scale production function ( $\gamma + \beta > 1$ ). In case of increasing returns, the business stealing effect is more

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<sup>11</sup>With this generalization, we can also analyze the impact of the oligopoly distortion in a world without efficiency wages. In our baseline setting with a linear demand curve, the equilibrium number of firms exceeds in this case their socially optimal number by about 83%. In addition, welfare in the social optimum is about 20% higher than welfare resulting in market equilibrium. Using the parameter values which we will introduce below in detail, excessive entry is about 10% higher (lower) if the demand curve is concave (convex) in comparison to the linear case. Similarly, the welfare loss, i.e. the difference between  $W^{opt}$  and  $W^*$ , is about 5% greater (smaller) (the detailed computation is available upon request). As such, we find that the curvature of the demand curve changes the consequences of the oligopoly distortion in a quantitative manner, while the qualitative results remain unaffected.

pronounced than if there are decreasing returns to scale. This is because the use of a given amount of inputs by an entrant reduces the output of incumbents more strongly. Consequently, the difference between optimal and equilibrium output per firm is greater in the case of increasing returns. Therefore, the incentives of the social planner to limit entry are more pronounced. We can conclude that the ensuing welfare loss, because such entry restriction does not occur, is particularly pronounced if efficiency wages change the nature of the production technology.

Table 2: Alternative Demand Functions

	linear	concave	convex
$\Delta n^*$	37.6	35.8	40.2
$\Delta n^{opt}$	31.2	31.7	31.2
$\Delta(n^* - n^{opt})$	45.3	40.1	52.3
$\Delta x^*$	0	-2.7	3.8
$\Delta x^{opt}$	7.5	3.2	13.4
$\Delta(x^{opt} - x^*)$	25.7	15.7	38.5
$\Delta \pi^{opt}$	31.2	25.7	37.5
$\Delta W^*$	89.4	84.4	96.6
$\Delta W^{opt}$	85.7	79	94.2
$\Delta(W^{opt} - W^*)$	66.8	57.7	77.8

Note:  $\Delta$  indicates percentage changes of the respective variable, when comparing  $\gamma = 1$  to  $\gamma = 0.01$ . Technology is linear in labor in all specifications.

Second, we focus on a non-linear demand function and maintain the assumption of a linear relationship between output and employment (see Table 2). Column one repeats the findings for the linear case (cf. Table 1, column one). Setting  $\alpha = 0.2$ , i.e. considering a concave inverse demand curve, we observe from column two in Table 2 that efficiency wages increase excessive entry by 40.1% and raise the output difference  $x^{opt} - x^*$  by 15.7%. The welfare loss resulting from a Cournot-oligopoly in the presence of efficiency wages is then 57.7% higher than in their absence. If, instead, we consider a convex inverse demand curve and set  $\alpha = -0.2$  (column three), excessive entry increases by 52.3% and  $x^{opt} - x^*$  by 38.5% if  $\gamma$  rises from 0.01 to 1. The welfare loss is about 77.8% higher if firms pay efficiency wages. Comparing these findings with the baseline specification (column one), we see that a convex (concave) demand curve slightly increases (decreases) the distortions and the ensuing welfare loss due to oligopoly when comparing a world with efficiency

wages ( $\gamma = 1$ ) with a setting in which no such productivity-enhancing mechanism exists ( $\gamma = 0.01$ ). Therefore, our results illustrated in Proposition 2 are robust with respect to the curvature of the demand curve as well.

## 5 Conclusion

Do efficiency wages affect the distortions in a free entry Cournot-oligopoly? To answer this question, we set up a model in which firms can raise labor productivity by increasing wages. Comparing a world in which labor productivity depends on the number of employees only or is constant on the one hand, with an efficiency wage framework, on the other, we obtain the following results.

First, efficiency wages enhance the incentives to enter the market more strongly in market equilibrium than is socially optimal. This implies that excessive entry arising in a free entry Cournot-oligopoly is aggravated. Second, efficiency wages, *ceteris paribus*, raise the incentives in market equilibrium to increase output per firm, while the increase in the number of competitors reduces these incentives. The net effect depends on the curvature of the demand schedule and the production function. Since it is socially optimal to restrict entry to below a level occurring in market equilibrium, the output-reducing effect of entry is smaller in the social optimum. In consequence, the difference between the socially optimal output per firm and the market outcome rises with efficiency wages. Therefore, third, the welfare loss arising due to market power is greater in the presence of efficiency wages than in their absence.

In order to relate our findings to other studies of free entry Cournot-oligopoly outcomes, we may interpret efficiency wages as a form of market imperfection. This is instructive because efficiency wages imply that firms do not take the price of inputs as given. Previous contributions focusing on imperfect input markets tend to derive conditions under which the excess entry outcome occurring in a world with competitive input market continues to hold. To illustrate the different mechanisms at work, we subsequently consider two important contributions by Ghosh and Morita (2007b) and Okuno-Fujiwara and Suzumura (1993). Ghosh and Morita (2007b) assume that each downstream firm

purchases inputs from an upstream firm and bargains with the upstream counterpart over the price and the quantity supplied by the upstream firm. The important feature of their model is that downstream firms create business for upstream firms. In a world of imperfect competition, downstream firms do not fully internalize the resulting business creation effect via the price they pay for inputs. Therefore, the business creation effect must not be too strong for the excess entry prediction to hold. Okuno-Fujiwara and Suzumura (1993), in contrast, do not explicitly model an input market imperfection but assume instead that firms can reduce marginal production costs by a costly R&D investment. Hence, in their model the input price is not exogenously given, as it is in our setting. Again, under additional assumptions, the excess entry result can be shown to hold.

Our contribution differs from these – and other – previous studies in a number of conceptually important aspects. First, labor income is welfare neutral for a given output level because it lowers profits by the same amount by which consumer surplus is raised. This is in contrast to the above-mentioned studies as they are based on the assumption that lower production costs, *ceteris paribus*, raise welfare. One may conjecture that due to this effect the conditions for excessive entry to occur are more stringent than in a world without input market imperfections. Second, efficiency wages can be interpreted as a (relative) decline both in marginal production costs and in fixed costs. In other contributions on input market imperfections, usually only marginal costs are affected. In studies incorporating R&D investments, generally a decline in marginal costs is achieved at the expense of higher (fixed) costs, which are unrelated to the production level. Therefore, the channels by which efficiency wages affect the profit-maximizing and socially optimal decisions relating to output and entry differ from those looked at in earlier contributions on input market imperfections and R&D investments. Third, our analysis goes beyond previous investigations with regard to the comprehensiveness of predictions. We can not only show how efficiency wages affect the market outcome and the excess entry result, but additionally demonstrate that the welfare loss due to market power on the output market increases with input market imperfections.

# A Appendix

Throughout the Appendix, we consider the case of a linear production function, i.e.  $F' = 1$ . For simplicity, we suppress the respective underline notation (as employed in the main text).

## A.1 Derivation of Eqs. (19) and (20)

Using (18), we can rewrite (15) as

$$\begin{aligned}
 x^{opt} &= \frac{n^* + 1}{n^{opt} + 1} x^* \\
 &= \frac{(n^{opt} + 1)^{1.5}}{n^{opt} + 1} x^* \\
 &= (n^{opt} + 1)^{0.5} x^*,
 \end{aligned} \tag{A.1}$$

which is identical to Eq. (19). To compute (20), note that in case of a linear production function, the social planner's objective can be rewritten as

$$\begin{aligned}
 W &= 0.5(q - p)nx + n\pi \\
 &= qnx - nwl - 0.5(nx)^2 - nk \\
 &= (n^*(\gamma) + 1)x^*nx - 0.5(nx)^2 - nk,
 \end{aligned} \tag{A.2}$$

where we have used (1), (3) and (10). Combining (A.2) with (A.1) and (9) yields

$$\begin{aligned}
 W^{opt} &= (n^* + 1)x^*n^{opt}x^{opt} - 0.5(n^{opt}x^{opt})^2 - n^{opt}k \\
 &= (n^{opt} + 1)x^{opt}n^{opt}x^{opt} - 0.5(n^{opt}x^{opt})^2 - n^{opt}k \\
 &= (n^{opt}x^{opt})^2 + x^{opt}n^{opt}x^{opt} - 0.5(n^{opt}x^{opt})^2 - n^{opt}k \\
 &= x^{opt}n^{opt}x^{opt} - n^{opt}k + 0.5(n^{opt}x^{opt})^2 \\
 &= ((x^{opt})^2 - k)n^{opt} + 0.5(n^{opt}x^{opt})^2 \\
 &= ((x^{opt})^2 - (x^*)^2)n^{opt} + 0.5(n^{opt}x^{opt})^2.
 \end{aligned} \tag{A.3}$$

## A.2 Derivation of Proposition 2 part (i)

Differentiating (10) with respect to  $\gamma$  yields

$$\frac{dn^*}{d\gamma} = -\frac{1}{x^*} \left[ \frac{dw^*}{d\gamma} e(w^*)^{-\gamma} + w^* \frac{d}{d\gamma} e(w^*(\gamma))^{-\gamma} \right]. \quad (\text{A.4})$$

The second derivative in square brackets can be expressed as

$$\frac{d}{d\gamma} e(w^*(\gamma))^{-\gamma} = e(w^*)^{-\gamma} \left[ -\ln(e(w^*)) - \frac{\gamma}{e(w^*)} \frac{de(w^*(\gamma))}{d\gamma} \right]. \quad (\text{A.5})$$

Inserting  $de(w^*(\gamma))/d\gamma = de/dw \times dw^*/d\gamma$  into (A.5) and substituting the result into (A.4) yields

$$\begin{aligned} \frac{dn^*}{d\gamma} &= -\frac{1}{x^*} \left[ \frac{dw^*}{d\gamma} e(w^*)^{-\gamma} - w^* e(w^*)^{-\gamma} \left( \ln(e(w^*)) + \frac{\gamma}{e(w^*)} \frac{de}{dw} \frac{dw^*}{d\gamma} \right) \right] \\ &= -\frac{1}{x^*} e(w^*)^{-\gamma} \left[ \frac{dw^*}{d\gamma} - \gamma \frac{de}{dw} \frac{dw^*}{d\gamma} \frac{w^*}{e(w^*)} - w^* \ln(e(w^*)) \right] \\ &= -\frac{1}{x^*} e(w^*)^{-\gamma} \left[ \frac{dw^*}{d\gamma} \left( 1 - \gamma \frac{de}{dw} \frac{w^*}{e(w^*)} \right) - w^* \ln(e(w^*)) \right]. \end{aligned} \quad (\text{A.6})$$

Using the Solow-condition implies

$$\frac{dn^*}{d\gamma} = \frac{1}{x^*} e(w^*)^{-\gamma} w^* \ln(e(w^*)) > 0. \quad (\text{A.7})$$

Differentiating (18) with respect to  $\gamma$  yields

$$\frac{dn^{opt}}{d\gamma} = \underbrace{\frac{2}{3}(n^* + 1)^{-1/3}}_{<1} \underbrace{\frac{dn^*}{d\gamma}}_{>0} > 0. \quad (\text{A.8})$$

This shows that

$$\frac{d(n^* - n^{opt})}{d\gamma} = \frac{dn^*}{d\gamma} - \frac{dn^{opt}}{d\gamma} > 0, \quad (\text{A.9})$$

which proves the first part of Proposition 2.



### A.3 Derivation of Proposition 2 part (ii)

From (9) and (19), we obtain  $dx^*/d\gamma = 0$  and  $dx^{opt}/d\gamma > 0$ . This immediately proves the second part of Proposition 2.

### A.4 Derivation of Proposition 2 part (iii)

Using (11) and (20), we can write the welfare loss as

$$\begin{aligned} W^* - W^{opt} &= - \left( (x^{opt} x^{opt} - x^* x^*) n^{opt} + 0.5 \left( (n^{opt} x^{opt})^2 - (n^* x^*)^2 \right) \right) \\ &= - \left( (x^{opt} + x^*) (x^{opt} - x^*) n^{opt} + 0.5 (n^{opt} x^{opt} + n^* x^*) (n^{opt} x^{opt} - n^* x^*) \right). \end{aligned} \quad (\text{A.10})$$

From (18) and (19), we find:

$$\frac{x^{opt}}{x^*} = (n^{opt} + 1)^{0.5}, \quad (\text{A.11})$$

$$n^* = (n^{opt} + 1)^{1.5} - 1. \quad (\text{A.12})$$

This leads to:

$$\begin{aligned} n^{opt} x^{opt} - n^* x^* &= x^* \left( n^{opt} \frac{x^{opt}}{x^*} - n^* \right) \\ &= x^* \left( n^{opt} (n^{opt} + 1)^{0.5} - (n^{opt} + 1)^{1.5} + 1 \right) \\ &= x^* + x^* \left( n^{opt} (n^{opt} + 1)^{0.5} - (n^{opt} + 1)^{1.5} \right) \\ &= x^* - x^* (n^{opt} + 1)^{0.5} \\ &= -(x^{opt} - x^*). \end{aligned} \quad (\text{A.13})$$

Given (A.11), (A.12) and (A.13), we can rewrite (A.10) as

$$\begin{aligned}
W^* - W^{opt} &= -(x^{opt} - x^*) \left( (x^{opt} + x^*)n^{opt} - 0.5(n^{opt}x^{opt} + n^*x^*) \right) \\
&= -(x^{opt} - x^*) \left( 0.5x^{opt}n^{opt} + x^*n^{opt} - 0.5n^*x^* \right) \\
&= -(x^{opt} - x^*)x^* \left( 0.5(n^{opt} + 1)^{0.5}n^{opt} + n^{opt} - 0.5(n^{opt} + 1)^{1.5} + 0.5 \right) \\
&= -(x^{opt} - x^*)x^* \left( 0.5(n^{opt} + 1)^{0.5}n^{opt} + n^{opt} + 1 - 0.5(n^{opt} + 1)^{1.5} - 0.5 \right) \\
&= -(x^{opt} - x^*)x^* \left( (n^{opt} + 1) \left( 1 + 0.5(n^{opt} + 1)^{-0.5}n^{opt} - 0.5(n^{opt} + 1)^{0.5} \right) - 0.5 \right) \\
&= -(x^{opt} - x^*)x^* \left( (n^{opt} + 1) \left( 1 - 0.5(n^{opt} + 1)^{-0.5} \right) - 0.5 \right)
\end{aligned} \tag{A.14}$$

Defining  $\Theta \equiv (n^{opt}(\gamma) + 1)[1 - 0.5(n^{opt}(\gamma) + 1)^{-0.5}] - 0.5$ , we can calculate

$$W^* - W^{opt} = -(x^{opt}(\gamma) - x^*)x^*\Theta(\gamma), \tag{A.15}$$

$$\frac{d(W^* - W^{opt})}{d\gamma} = -x^* \left( \underbrace{\frac{dx^{opt}}{d\gamma}}_{>0} \Theta + (x^{opt} - x^*) \underbrace{\frac{d\Theta}{dn^{opt}}}_{>0} \underbrace{\frac{dn^{opt}}{d\gamma}}_{>0} \right) < 0, \tag{A.16}$$

which proves the last part of Proposition 2.

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