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by

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Demographics and Sectoral Reallocations: A Search Theory with Immobile Workers*

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February 15, 2018

Abstract

We propose a mechanism via which a decline in the share of young workers slows employment growth in expanding sectors, and exacerbates sectoral reallocation costs. To quantify this mechanism, we develop a search model with perpetual youth, three sectors and endogenous separations of worker-firm matches. Our model incorporates three important features: *sector-specific human capital*; *sectoral preferences*, which imply that only some workers are mobile across sectors; and a *wage bargaining distortion*, whereby unskilled workers' wages in shrinking sectors are determined by mobile workers. In our parameterized model, output losses after a sectoral reallocation shock are significant and rise substantially at low population growth rates. While this result is driven by the interactions between the three model features, the wage distortion for unskilled wages is responsible for amplifying the sectoral reallocation costs. The model generates a rise in unemployment and a fall in vacancies after a sectoral shock, counter to conventional wisdom.

Keywords: demographics, sectoral reallocation, wage bargaining distortion.

JEL Classifications: E24, J11, J6

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1 Introduction

The decline in fertility in many countries since the 1950s has led to significant demographic changes (Feyrer, Sacerdote, and Stern (2008), Lee (2016)). Growth rates of the working-age population in G-7 countries have declined from highs of 2.7 percent in Canada over 1966 to 1975 to lows of -1 percent in Japan over 2007 to 2017. Moreover, recent projections from the United Nations imply that the young share of the population will remain near historical lows in the coming decades.

In this paper, we examine how demographics impact the sectoral reallocation of workers.¹ We propose a mechanism via which a decline in the share of young workers slows employment growth in expanding sectors, and exacerbates the costs of sectoral reallocations. Using cross-country data from IPUMS International and the U.S. CPS, we build on Murphy and Topel (1987) and document that industries that grow faster than average have a higher than average share of young workers.

To investigate this mechanism, we develop a perpetual youth three-sector model with sector-specific human capital and search and matching frictions. Workers are born young and stochastically become middle-aged, when they face a constant probability of dying. Workers are born unskilled and stochastically accumulate sector-specific skills via working. Our labor search framework builds on Mortensen and Pissarides (1994). Firms post vacancies in markets which specify the sector, skill and age of the worker.² Unemployed workers choose which market to search in. Existing worker-firm matches produce a sectoral good, and may separate endogenously next period, due to idiosyncratic fluctuations in match-specific productivity. Workers are risk-neutral, and consume a final good produced by a competitive firm using the sectoral goods.

Our benchmark model incorporates two interconnected frictions which affect mobility and wages of unskilled workers. First, fraction π_j of workers are born with a sectoral preference to work in sector j . These *immobile* unskilled workers only search for a job in their preferred sector. The remaining unskilled workers are *mobile* and maximize the expected value of searching for a job across all sectors. Second, the Nash bargaining over unskilled wages cannot condition on workers' sectoral preferences. Hence, mobile and immobile workers with the same match-specific productivity receive the same wage.

¹See Gallipoli and Pelloni (2013) for a survey of the literature on sectoral reallocations.

²Menzio, Telyukova, and Visschers (2016) note that while age discrimination is often illegal, in practice firms can target vacancies to workers of different ages by specifying minimum years of work experience, or avoid hiring older workers by saying they were less qualified than a younger applicant.

The workers' outside option in the Nash bargaining is determined by mobile workers who search in the sector with the highest return. This leads to a *wage bargaining distortion* during reallocations, since a better outside option for mobile workers in the growing sector puts upward pressure on immobile workers' wages in the shrinking sector.

We introduce sectoral preferences to capture impediments to geographical and industry mobility of workers. Some workers may prefer attributes of specific jobs, or have a mix of skills that are more valuable in some industries. Sectoral preferences may also arise due to the geographical concentration of industries and workers' reluctance to move.³ For example, recent Canadian evidence shows that two-thirds of unemployed workers would not move to another province to accept a job offer (Morissette (2017)).⁴ Our model's wage bargaining distortion—which drives up immobile workers' wages—is consistent with evidence from the Canadian reallocation from manufacturing to resources in the early 2000s. Specifically, the growth in resource jobs in Western provinces enabled workers in Eastern non-resource provinces to bargain for higher wages, despite high rates of unemployment (Green, Morissette, and Sand (2017)).⁵

To explore how demographics interact with sectoral reallocation, we parameterize the benchmark model to the U.S. economy and conduct numerical experiments for population growth rates ranging from -1 to 3 percent. We model a *sectoral reallocation shock* as a permanent and unanticipated switch in sector 1 and 3's expenditure share in the production of the final good. Our solution assumes perfect foresight, and we solve for the transition from an initial to a new symmetric steady state. The steady states differ only in their sectoral allocations of workers, which are proportional to the sectoral outputs' shares in the final good's production. Prices and wages are equalized across sectors, so unskilled mobile workers are indifferent over which sector to search in. As a result, neither the wage distortion nor sectoral preferences impact steady states.⁶

We find that sectoral shocks result in a deeper fall in output on impact, and more

³Lewis and Frank (2002) report differences across Americans in their desire to work in a government or private sector job. Delfgaauw (2007) finds that workers who are dissatisfied with attributes of their jobs are more likely to search in a different industry. A large literature on workers' cognitive, interpersonal and physical skills points to these factors' impact on educational attainment, and thus the industry of employment (Autor and Dorn (2013), Heckman, Lochner, and Taber (1998)).

⁴Cadena and Kovak (2016) show that low-skilled Mexican immigrants have higher geographic mobility in response to local labor market conditions than the native-born.

⁵Beaudry, Green, and Sand (2012) also provide evidence that the workers' outside option in the Nash bargaining impacts wages across U.S. cities, following changes in the industrial composition of jobs.

⁶Demographics have an intuitive impact on the symmetric steady state. Lower population growth decreases the fraction of young workers, which increases per capita output, since young workers have lower average human capital and higher unemployment rates than middle-aged workers.

protracted transitions at lower population growth rates. Cumulative output losses—measured as deviations from steady state, summed over the transition—roughly double as population growth falls from 3 to 1 percent, and more than triple as growth falls from 1 to -1 percent. In our benchmark economy with a 3 percent reallocation shock, these losses are significant, ranging from 1.3 to nearly 10 percent of annualized steady state output, as we lower the population growth rate from 3 to -1 percent.⁷

In our experiments, employment along the transition mirrors output. For our benchmark 3 percent reallocation shock, employment falls between 3 and 4 percent on impact. This large fall in employment is due to a spike in job destruction after the shock. The recovery in employment tracks job creation, and becomes more protracted at low population growth rates. While employment returns to its steady state level in 1 year at 3 percent population growth, the recovery takes 2 years at 1 percent population growth and 7 years at -1 percent. These changes are concentrated in the unskilled market. Since most young are unskilled, the fall in the young employment rate is much larger than that of the middle-aged. Moreover, as young workers become relatively scarcer at lower population growth rates, sectoral shocks results in larger falls in their employment rate.

What is the intuition for the large impact of demographics after a shock reallocates workers from sector 3 to 1? A lower population growth rate increases the middle-aged population share and implies that there are fewer unskilled and more skilled workers. In our experiments, only unskilled mobile workers switch sectors.⁸ After the shock, sector 1 has too few of the more productive skilled workers relative to the new steady state, and thus needs an inflow of unskilled to grow. A smaller share of unskilled at low population growth sees a larger fraction of unskilled matches in sector 3 terminated so as to induce mobile unskilled workers to shift sectors.

The large output and employment declines in our benchmark are driven by the interactions between sector-specific human capital, sectoral preferences and the wage distortion. Removing sectoral preferences—which effectively eliminate the wage distortion—isolates the direct contribution of sector-specific human capital, a modest 6 percent of output losses. Adding sectoral preferences, while allowing wages to differ between mobile and immobile unskilled workers—which we refer to as *preference-specific wages*—increases output losses, but only to about 25 percent of the benchmark. Consequently, the wage distortion is the main friction which amplifies output and employment losses.

⁷In our benchmark, 40 percent of unskilled workers are mobile. In Section 5.4, we show that varying the fraction of mobile unskilled workers has large effects on the level of output losses.

⁸The skill premium is sufficiently large that skilled workers choose not to switch industries.

Why does incorporating the wage distortion have such a large impact? Since wage bargaining does not condition on workers' sectoral preferences, mobile and immobile workers receive the same wage. After the reallocation shock, sector 1's price and wages rise, pushing up unskilled wages in sector 3. Combined with a fall in sector 3's price, this reduces the surplus of sector 3's unskilled matches and increases job destruction for both mobile and immobile workers. While unemployed mobile workers search in sector 1, unskilled workers with a sectoral preference drive up *rest unemployment*, as they continue to search in sector 3, despite the low probability of matching. Although sector 2 is not directly impacted by the reallocation shock, a similar mechanism pushes up sector 2's rest unemployment. The rise in rest unemployment of unskilled workers has persistent effects on the economy, through its impact on the future stocks of skilled workers. Skills are acquired via work experience, so lower unskilled employment today translates into fewer skilled workers tomorrow, lengthening the transition to the new steady state.

Our model yields novel insights into the implications of sectoral shocks for unemployment, vacancies and worker reallocation. Contrary to [Abraham and Katz \(1986\)](#), we show that a permanent sectoral reallocation shock can result in a rise in unemployment and a fall in vacancies. The duration of high unemployment and low vacancies increases as population growth declines. Moreover, we show that the rise in unemployment following a sectoral reallocation shock is mainly driven by workers with a sectoral preference, rather than by workers switching sectors. This is consistent with the finding in [Murphy and Topel \(1987\)](#) and [Loungani and Rogerson \(1989\)](#), that most of the rise in U.S. unemployment over 1968 – 1985 is accounted for by workers who do not switch industries. While [Murphy and Topel \(1987\)](#) argue that this points to a small role of sectoral shocks in accounting for the rise in unemployment, our model implies the opposite.

After a reallocation shock, gross flows of workers across sectors—defined by adding up workers who switched sectors and matched with firms—spike.⁹ Within a year after the shock, gross flows decline and remain below their steady state level for an extended period. The path of gross flows is driven by the sectoral allocation of unskilled employment and vacancies. Since there are too many (too few) skilled workers in the shrinking (growing) sector, the demand for unskilled workers pushes down (up) vacancies during the transition. Consequently, no unskilled unemployed mobile workers search in sector 3 for several periods. Gross flows fall, as no workers move to sector 3. Ultimately, gross flows recover along with sector 3's vacancies and unskilled employment.

⁹Net flows of workers—inflows into sector 1 and outflows from sectors 2 and 3—also spike immediately after the shock, but fall quickly to their steady state value of zero.

Our model predicts that the young account for much of job separations and sector switches. [Menzio, Telyukova, and Visschers \(2016\)](#) find that the rate at which workers move from employment to unemployment and switch employers declines with age, and that a life-cycle model of directed search can match these facts.¹⁰ In our model, the rate at which workers switch jobs also declines with age, and is driven by the stochastic accumulation of sector-specific human capital and the longer job tenure of skilled compared to unskilled workers. The important role of sector-specific human capital is broadly consistent with empirical work ([Sullivan \(2010\)](#)).¹¹

Closest in spirit to our paper is [Rogerson \(2005\)](#), who builds a two-period overlapping generations model with sector-specific human capital to show that the entry of young unskilled workers facilitates sectoral reallocation.¹² Our work complements Rogerson’s insights, as we find that the inflow of unskilled workers into expanding sectors has a significant quantitative impact on sectoral reallocation costs, for a range of population growth rates. Similar to [Phelan and Trejos \(2000\)](#) and [Tapp \(2011\)](#), we find large output losses from sectoral reallocation. Losses in [Phelan and Trejos \(2000\)](#) are driven by a convex cost of hiring, which slows employment growth in expanding sectors, while [Tapp \(2011\)](#) relies on sector-specific human capital and permanent exogenous shifts in sectoral prices. In contrast, we emphasize the role of demographics in slowing down transitions following a permanent sectoral reallocation. Our labor search framework incorporates sectoral preferences which restrict mobility of unskilled workers, and a Nash bargaining distortion which raises immobile workers’ wages in the shrinking sector.

The quantitative relevance of sectoral reallocation shocks has been debated since [Lilien \(1982\)](#). A long-standing literature indicates sectoral shocks have important aggregate effects ([Mills, Pelloni, and Zervoyianni \(1995\)](#), [Shin \(1997\)](#), [Foerster, Sarte, and Watson \(2011\)](#), [Garin, Pries, and Sims \(2018\)](#)). A critique of this literature is that since gross flows of workers across sectors exceed net flows, shocks which result in net reallocations of workers—with net outflows from some sectors and net inflows in others—should have small aggregate impacts ([Pilossoph \(2014\)](#)). In our model, demographics and the wage bargaining distortion amplify the costs of sectoral reallocations, even

¹⁰[Kambourov and Manovskii \(2008\)](#) document large age effects for occupational mobility.

¹¹There is an ongoing debate about the relative importance of occupation-specific versus industry-specific human capital, but [Sullivan \(2010\)](#) finds both are important.

¹²Surprisingly little work has examined the interaction between demographics and sectoral reallocation. [Börsch-Supan \(2003\)](#) and [Fougère, Mercenier, and Mérette \(2007\)](#) examine the impact of population aging on the demand for different goods, but do not consider the costs of reallocating labor. [Yoon \(2017\)](#) examines the fall in the share of goods producing industries in the U.S. Rust Belt, in a multi-region, multi-sector migration model, where regions differ in their supply of young workers.

though gross flows of workers exceeds net flows.

[Chodorow-Reich and Wieland \(2016\)](#) analyze sectoral reallocation shocks in a two-region, multi-sector search model with gross flows of workers in excess of net flows. They show that the impact of reallocation shocks on regional unemployment rates depends on whether the economy is in a recession or an expansion. Our work differs in that we highlight the interaction between demographics and sectoral reallocation. While we share the finding that wage frictions are important for sectoral reallocations, our modelling approaches differ. Our wage distortion arises as the outcome of Nash bargaining, while [Chodorow-Reich and Wieland \(2016\)](#) impose a downward wage rigidity. [Hobijn, Schoellman, and Vindas Q. \(2017\)](#) examine the interaction between demographics and long-run structural transformation in a life-cycle model with sector-specific skills and retraining costs. We differ in our focus on sectoral reallocation shocks and the role of worker mobility in a search framework with bargaining distortions.

Several papers examine the role for industry (occupation) shocks and worker mobility on unemployment. Motivated by the idea that some workers have specific skills or fixed geographical location, [Shimer \(2007\)](#) examines the implications of a mismatch between the number of workers and jobs in a labor market. [Alvarez and Shimer \(2011\)](#) incorporate a cost for workers switching industries into a [Lucas and Prescott \(1974\)](#) style island economy, and show that this can generate significant rest unemployment. [Carrillo-Tudela and Visschers \(2014\)](#) extend [Alvarez and Shimer \(2011\)](#) to allow for aggregate shocks and find that rest unemployment is a main driver of unemployment over the business cycle. [Wiczer \(2015\)](#) builds on [Carrillo-Tudela and Visschers \(2014\)](#) framework to examine the impact of worker heterogeneity in occupational switching costs on unemployment dynamics over the business cycle. Although our work is focused on demographics, we show that heterogeneity in worker mobility and wage bargaining can amplify the rise in rest unemployment during a sectoral reallocation.

The remainder of this paper is organized as follows. Section 2 documents the fall in the young share of the working-age population and the relationship between industry employment growth and young workers. Section 3 outlines the model and briefly characterizes the symmetric steady state. Section 4 presents our parameters and discusses the impact of population growth on the steady state. Section 5 presents our numerical experiments and discusses the economic impact of sectoral shocks. Section 6 concludes.

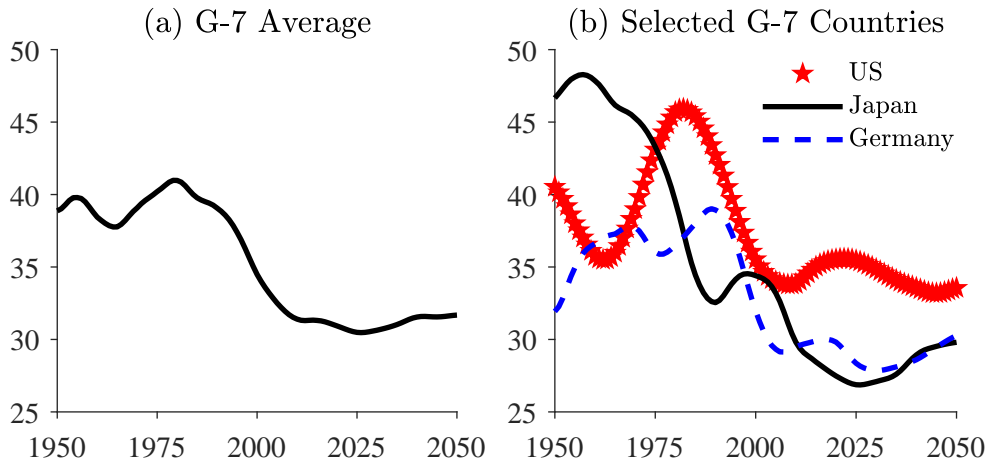
2 Data

We document the substantial decline in the young share of the working-age population, and the role young workers play in accounting for industry employment growth.

2.1 Young Share of the Working-Age Population

The decline in fertility in developed countries since the 1950s (e.g., see [Feyrer, Sacerdote, and Stern \(2008\)](#)) resulted in a significant fall in the young (20–34) share of the working-age population (20–64).¹³ Figure 1.a plots historical data to 2015 on the average of young shares for the G-7, as well as U.N. projections to 2050. These projections indicate that developed countries face an extended period of historically low shares of young.

Figure 1: YOUNG (20 – 34) SHARE OF THE WORKING-AGE (20 – 64) POPULATION



NOTES: Population data to 2015 and projections are from the United Nations, “World Population Prospects: The 2017 Revision” and are available at: <https://esa.un.org/unpd/wpp/>. The G-7 series is an average of young shares across these countries.

The timing and magnitude of the changes in fertility vary across countries. Figure 1.b plots the young shares of the working-age population for U.S., Germany, and Japan. The more pronounced baby boom in the U.S. is evidenced by the larger rise in the 20–34 share of the working-age population. Since the beginning of the 1960s, the U.S. young share rose by over 10 percentage points, before falling back to about 34 percent by early 2000s. The smaller baby boom in Germany implied a more modest rise in the young

¹³We define the young to cover ages 20 – 34, to account for education delaying labor market entry.

share, and a lower level by early 2000s. Japan’s experience is more dramatic, with the young share falling from 48 percent in the mid-1950s to 28 percent by 2015.

As noted by Lee (2016), Figure 1 shows that the largest demographics changes are still to come, as low fertility contributes to sustained declines in the working-age population. For example, Japan’s working-age population has been shrinking by about 1 percent annually since 2007, and its young share is projected to decline until 2030.

2.2 Young Workers’ Contribution to Industry Employment Growth

We use cross-country data from IPUMS International and the U.S. CPS to examine how the age composition of an industry relative to the aggregate varies with changes in that industry’s share of total employment. We find that growing sectors have larger shares of young workers compared to aggregate employment. In contrast, shrinking sectors have lower shares of young workers.

We divide workers into three age groups: the young (20 – 34), the middle-aged (35 – 49) and the old (50 – 64). For each industry j , we compute the share of total employment at time t , $\frac{E_{j,t}}{E_t}$. For each age group a of industry j , we compute its share of total employment in industry j at t , $\frac{E_{j,t}^a}{E_{j,t}}$. Lastly, for each age group a , we compute its share of total employment at t , $\frac{E_t^a}{E_t}$. We regress the difference in industry j ’s share of workers of age a relative to the aggregate, $\frac{E_{j,t}^a}{E_{j,t}} - \frac{E_t^a}{E_t}$, on the change in industry j ’s share of total employment between period t and $t - s$ and industry fixed effects, γ_j .

$$\frac{E_{j,t}^a}{E_{j,t}} - \frac{E_t^a}{E_t} = \alpha + \beta \left(\frac{E_{j,t}}{E_t} - \frac{E_{j,t-s}}{E_{t-s}} \right) + \gamma_j + \epsilon_{j,t} \quad (1)$$

Table 1 shows that the coefficient, β , on the change in the industry share of total employment is positive for young workers and negative for the middle-aged and the old. Looking at the young share regression using IPUMS (CPS) data, a 1 percentage point rise in an industry’s share of total employment is associated with a 0.4 (1.6) percentage point rise in that industry’s young share relative to total employment.

Our findings are consistent with work looking at the relationship between young workers and industry employment growth. Murphy and Topel (1987) use the U.S. CPS to look at industry mobility of workers over 1968 to 1985. They document that younger workers are more likely to leave the shrinking U.S. manufacturing sectors, and that the

fraction of new entrants to the labor force beginning their careers in manufacturing fell.¹⁴ Han and Suen (2011) find that growing (shrinking) industries in Hong Kong have larger shares of younger (older) workers, with a 1 percentage point rise in an industry's share of total employment lowering the average workforce age by 0.6 years. Similarly, Kim and Topel (1995) find that most intersectoral mobility in South Korea over 1970 – 1990 was accounted for by younger workers, which shifted the age distribution of the shrinking agricultural workforce relative to the growing manufacturing sector.

Table 1: INDUSTRY AGE SHARE AND INDUSTRY EMPLOYMENT GROWTH

	IPUMS International [†]			U.S. CPS [‡]		
	Young	Middle-aged	Old	Young	Middle-aged	Old
α	-0.071*** (0.009)	-0.029*** (0.004)	0.099*** (0.009)	-0.017* (0.009)	-0.025*** (0.005)	0.042*** (0.012)
β	0.423*** (0.098)	-0.118*** (0.045)	-0.304*** (0.089)	1.569*** (0.482)	-0.999*** (0.377)	-0.569 (0.463)
R^2	0.453	0.358	0.501	0.723	0.494	0.634
N	681	681	681	104	104	104

NOTES: We run equation 1 for 3 age groups. The industry fixed effects coefficients are not reported. Three stars (***) denotes significance at 1 percent, and one star (*) at the 10 percent. Standard errors are in parentheses. [†]The IPUMS regressions include: Argentina (1970, 1980, 1991, 2001); Brazil (1960, 1970, 1980, 1991, 2000, 2010); Canada (1971, 1981, 1991, 2001); Chile (1960, 1970, 1982, 1992, 2002); France (1962, 1968, 1975, 1982, 1990, 1999, 2006, 2011); Greece (1971, 1981, 1991, 2001); Ireland (1971, 1981, 1986, 1991, 1996, 2002, 2006, 2011); Mexico (1970, 1990, 1995, 2000, 2010); Spain (1981, 1991, 2001); Switzerland (1970, 1980, 1990, 2000) and the U.S. (1960, 1970, 1980, 1990, 2000, 2005, 2010). The IPUMS regressions use 17 industries: agriculture, fishing, and forestry; mining; manufacturing; electricity, gas and water; construction; wholesale and retail trade; hotels and restaurants; transportation and communications; financial services and insurance; public administration and defense; services, not specified; real estate and business services; education; health and social work; other services; private household services; other industry, not elsewhere classified. [‡]The CPS regressions use 5-year averages of annual data from 1970 – 2015 and 13 industries: agriculture, fishing and forestry; mining; construction; manufacturing; transportation; communication and utilities; wholesale and retail trade; finance, insurance and real estate; business and repair services; personal services; entertainment and recreation; professional related; and public administration.

¹⁴Autor and Dorn (2009) find a similar pattern for occupations, with the average age rising with declines in an occupation's share of total employment. Fallick (1996) finds that U.S. industries whose share of employment rises, disproportionately hire new entrants. Lee and Wolpin (2006) and Hoffmann, Laptiev, and Shi (2016) examine the rise in U.S. service sector employment and find that net flows into services were the largest for younger workers in the 1970s and the 1980s.

3 Model Environment

The economy is populated by a continuum of workers who are heterogenous in age, skills and sectoral preference. Each period, young workers are born unskilled, and face a constant probability of transitioning to middle-aged, while middle-aged workers face a constant probability of dying.¹⁵ Workers are risk neutral and consume the final good.

The final good is a composite of three sectoral goods produced using labor. Firms post vacancies which specify the sector, $j \in \{1, 2, 3\}$, the skill level, $k = l$ for unskilled or $k = h$ for skilled, and the age of eligible workers, $a = y$ for young or $a = m$ for middle-aged. Workers direct their search efforts to a specific market, defined as a trio (j, k, a) . Since skills are sector-specific, workers who choose to switch sectors must search for an unskilled job. A fraction of the unskilled have a preference for working in sector j (denoted by $\zeta = j$) and only search in that sector. Unskilled with no sectoral preference for working (denoted by $\zeta = 0$) are mobile across sectors. Unskilled employed workers face a constant probability of acquiring sector-specific skills via learning-by-doing. The productivity of worker-firm matches is stochastic, and matches below an endogenous productivity threshold are terminated. Separated workers search for new jobs.

Time is discrete and indexed by t . The timing of model events is presented in Table 2.

3.1 Demographics

The total population at the beginning of period t is composed of measures of young and middle-aged workers, i.e., $N_t = N_t^y + N_t^m$. New workers, ηN_t , are born at the end of each period and enter the labor market the following period. Stochastic aging shocks are realized at the end of each period. Young workers age with probability $\delta^y \in (0, 1)$. Middle-aged workers die with probability $\delta^m \in (0, 1)$. These probabilities are independent across periods and workers.

The laws of motion for the young, middle-aged and total population are:

$$\begin{aligned} N_{t+1}^y &= (1 - \delta^y) N_t^y + \eta N_t \\ N_{t+1}^m &= (1 - \delta^m) N_t^m + \delta^y N_t^y \\ N_{t+1} &= (1 + \eta) N_t - \delta^m N_t^m \end{aligned}$$

¹⁵Stochastic aging allows us to capture demographic changes in a tractable manner.

The growth rate of the total population is $\gamma_N = 1 + \eta - \delta^m \cdot \frac{\eta + \delta^m + \delta^y - \sqrt{(\eta + \delta^m + \delta^y)^2 - 4\delta^y \delta^m}}{2\delta^m}$.

Table 2: TIMING OF MODEL EVENTS

BEGINNING OF PERIOD t

Initial stocks of employed and unemployed are $E_{j,t}^{h,a}$, $E_{j,t}^{l,a,\zeta=0}$, $E_{j,t}^{l,a,\zeta=j}$, $U_{j,t}^{h,a}$, $U_t^{l,a,\zeta=0}$, $U_{j,t}^{l,a,\zeta=j}$

Match productivities $z_{j,t}^{k,a}$ are drawn from $G(z)$

Matches below $\underline{z_{j,t}^{k,a}}$ break endogenously

Separated workers $E_{j,t}^{h,a}G(\underline{z_{j,t}^{h,a}})$, $E_{j,t}^{l,a,\zeta=0}G(\underline{z_{j,t}^{l,a}})$, and $E_{j,t}^{l,a,\zeta=j}G(\underline{z_{j,t}^{l,a}})$ search for new jobs

Vacancies are posted $V_{j,t}^{h,a}$, $V_{j,t}^{l,a}$

Unemployed (including the newly separated) $\tilde{U}_{j,t}^{h,a}$, $\tilde{U}_{j,t}^{l,a,\zeta=0}$, and $\tilde{U}_{j,t}^{l,a,\zeta=j}$ search for jobs

New matches are formed $\tilde{U}_{j,t}^{h,a}\theta_{j,t}^{h,a}q(\theta_{j,t}^{h,a})$, $\tilde{U}_{j,t}^{l,a,\zeta=0}\theta_{j,t}^{l,a}q(\theta_{j,t}^{l,a})$, and $\tilde{U}_{j,t}^{l,a,\zeta=j}\theta_{j,t}^{l,a}q(\theta_{j,t}^{l,a})$

Production takes place

END OF PERIOD t

Births: ηN_t young are born as unskilled

Sectoral preference: newborns draw ζ

Aging: $\delta^y N_t^y$ young become middle-aged

Deaths: $\delta^m N_t^m$ middle-aged die

Skills: ϱ^a unskilled employed become skilled

Laws of motion give stocks for next period $E_{j,t+1}^{h,a}$, $E_{j,t+1}^{l,a,\zeta=0}$, $E_{j,t+1}^{l,a,\zeta=j}$, $U_{j,t+1}^{h,a}$, $U_{t+1}^{l,a,\zeta=0}$, $U_{j,t+1}^{l,a,\zeta=j}$

3.2 Worker Preferences and Heterogeneity

Workers' preferences are defined over the final good, which is produced using sectoral intermediate goods. Given our interest in reallocation, we make a common assumption in the search literature that workers are risk neutral. Workers draw at birth a sectoral preference shock. Fraction π_j of newborns draw $\zeta = j \in \{1, 2, 3\}$ and are immobile, since they prefer a job in sector j . Fraction $1 - \pi_1 - \pi_2 - \pi_3$ of newborns draw $\zeta = 0$ and are mobile across sectors, since they are indifferent about the sector in which they work.

Workers are indexed by their age $a \in \{y, m\}$, skill $k \in \{h, l\}$, sector (if any) in which they skilled, and sectoral preference $\zeta \in \{0, 1, 2, 3\}$. Let s^l denote the human capital of an unskilled worker, and s^h the human capital of a skilled worker. For simplicity,

the human capital levels do not depend on sector j . At birth, agents are young and unskilled. Unskilled employed workers become skilled with probability ϱ^a . Skills are sector-specific, so that if a skilled worker chooses to search for a job in a different sector, they must do so as an unskilled worker.

3.3 Production Technology

Each of the sectors has a continuum of risk neutral firms who produce a nonstorable intermediate good. Intermediate goods are sold in a competitive market and transformed into a final nonstorable good.

The technology of production for the final good is given in equation 2:

$$\begin{aligned} Y_t &= \left[\alpha_1 \cdot \tilde{Y}_{1,t}^\rho + \alpha_2 \cdot \tilde{Y}_{2,t}^\rho + \alpha_3 \cdot \tilde{Y}_{3,t}^\rho \right]^{1/\rho} \\ \tilde{Y}_{j,t} &= Y_{j,t} - \sum_{a \in \{y,m\}} \sum_{k \in \{h,l\}} c^k V_{j,t}^{k,a} \text{ for } j \in \{1, 2, 3\} \end{aligned} \quad (2)$$

where $Y_{j,t}$ is the gross output of intermediate input j at time t , $c^k V_{j,t}^{k,a}$ is the cost of posting vacancies in sector j for skill k and age a , and $\tilde{Y}_{j,t}$ is gross output net of vacancy costs. The sectoral good weights in equation 2 sum to one, i.e., $\sum_j \alpha_j = 1$, and the elasticity of substitution between sectoral goods is $\frac{1}{1-\rho}$. The final good price is normalized to 1. The competitive prices of the intermediate goods are given by: $p_{j,t} = \alpha_j \cdot \left(\frac{Y_t}{\tilde{Y}_{j,t}} \right)^{1-\rho}$.

The technology of production for sector j 's intermediate goods firms is given in 3:

$$y_{j,t}^{k,a} = z_{j,t}^{k,a} \cdot s^k \quad (3)$$

where $y_{j,t}^{k,a}$ is the output produced by a job filled with a worker of skill k and age a , $z_{j,t}^{k,a}$ is the idiosyncratic productivity of the job (i.e., the worker-firm match), which is drawn from the distribution $G(z)$, and s^k is the human capital of the worker with skill k .

3.4 Search and Matching

In our benchmark model, markets do not condition on a worker's sectoral preference. Workers apply to one market each period (e.g., $j = 2, k = h, a = m$). Let $M(\tilde{U}_{j,t}^{k,a}, V_{j,t}^{k,a})$ denote worker-firm matches in market (j, k, a) at time t , where $\tilde{U}_{j,t}^{k,a}$ is the number of

unemployed workers applying for jobs and $V_{j,t}^{k,a}$ is the number of vacancies available. Labor market tightness is denoted by $\theta_{j,t}^{k,a} \equiv \frac{V_{j,t}^{k,a}}{U_{j,t}^{k,a}}$. Vacancies are filled with probability $q(\theta_{j,t}^{k,a}) \equiv \frac{M}{V_{j,t}^{k,a}}$. Unemployed workers find a job with probability $\frac{M}{U_{j,t}^{k,a}} = \theta_{j,t}^{k,a} q(\theta_{j,t}^{k,a})$.

Worker-firm matches can be terminated endogenously by firms or exogenously if middle-aged workers die. Firms break matches whose productivity is below an endogenous threshold denoted by $\underline{z}_{j,t+1}^{k,a}$. With probability $(1 - \delta^m) G(\underline{z}_{j,t+1}^{k,a})$ the match breaks and with probability δ^m the worker dies. In either case, the firm has a vacant job at $t + 1$.

3.5 Hiring, Firing and Vacancies

There is free entry in the posting of vacancies. The expected value of a vacant job in sector j at time t , to be filled by a worker with skill $k \in \{h, l\}$ and age $a \in \{y, m\}$ is $\Phi_{j,t}^{V,k,a}$.

$$\Phi_{j,t}^{V,k,a} = -c^k \cdot p_{j,t} + q(\theta_{j,t}^{k,a}) \cdot \Phi_{j,t}^{J,k,a}(\xi^k) + [1 - q(\theta_{j,t}^{k,a})] \cdot \beta \Phi_{j,t+1}^{V,k,a} \quad (4)$$

Posting a vacancy for a job of skill k in sector j at time t costs a firm $c^k \cdot p_{j,t}$. With probability $q(\theta_{j,t}^{k,a})$, the vacancy is filled and the expected value to the firm of having a filled job is denoted by $\Phi_{j,t}^{J,k,a}(z)$ where z is the idiosyncratic productivity of the match. New jobs start at the productivity level $\xi^k \in (0, 1)$. With probability $1 - q(\theta_{j,t}^{k,a})$, the vacancy is reposted at $t + 1$.

With free entry, the profit from vacancy posting is zero in equilibrium, $\Phi_{j,t}^{V,k,a} = 0$, so:

$$\frac{c^k \cdot p_{j,t}}{q(\theta_{j,t}^{k,a})} = \Phi_{j,t}^{J,k,a}(\xi^k) \quad (5)$$

The labor market tightness is such that the expected value of a job equals the expected cost of hiring a worker, whenever vacancies are strictly greater than zero (equation 5).

The productivity of worker-firm matches is stochastic. Each period, firms break matches whose productivity is lower than an endogenous threshold denoted by $\underline{z}_{j,t}^{k,a}$. This means that productivity thresholds solve $\Phi_{j,t}^{J,k,a}(\underline{z}_{j,t}^{k,a}) = 0$.

Equation 6 summarizes the expected value of a skilled job filled by young ($a = y$) or

middle-aged workers ($a = m$), with productivity above the threshold, i.e., $z \geq \underline{z}_{j,t}^{h,a}$.

$$\begin{aligned} \Phi_{j,t}^{J,h,a}(z) = & p_{j,t} z s^l - w_{j,t}^{h,a}(z) + \beta(1 - \delta^a) \left[\int_{\underline{z}_{j,t+1}^{h,a}}^1 \Phi_{j,t+1}^{J,h,a}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,a}) \Phi_{j,t+1}^{V,h,a} \right] \\ & + I^y \cdot \beta \delta^y \left[\int_{\underline{z}_{j,t+1}^{h,m}}^1 \Phi_{j,t+1}^{J,h,m}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,m}) \Phi_{j,t+1}^{V,h,m} \right] + I^m \cdot \beta \delta^m \Phi_{j,t+1}^{V,h,m} \quad (6) \end{aligned}$$

The indicator function I^y equals 1 if age $a = y$ or 0 otherwise. I^m is defined analogously. These functions help us express $\Phi_{j,t}^{J,h,y}(z)$ and $\Phi_{j,t}^{J,h,m}(z)$ concisely, in one equation.

Letting $a = y$ in equation 6 shows that the value of a job filled by a skilled, young worker, i.e., $\Phi_{j,t}^{J,h,y}(z)$, equals the value of output produced minus the wage cost, plus the discounted value of having either a filled job or a vacancy at $t + 1$. With probability $(1 - \delta^y)$ the worker remains young at $t + 1$. The job is filled by the same skilled, young worker if the productivity of the match remains above the threshold $\underline{z}_{j,t+1}^{h,y}$, or the match breaks with probability $G(\underline{z}_{j,t+1}^{h,y})$ and the firm has a vacancy.¹⁶ With probability δ^y , the worker is middle-aged at $t + 1$. The job is filled by a skilled, middle-aged worker if the productivity is above the threshold $\underline{z}_{j,t+1}^{h,m}$, or the match breaks with probability $G(\underline{z}_{j,t+1}^{h,m})$ and the firm has a vacancy. The value of a job filled by a skilled, middle-aged worker, i.e., $\Phi_{j,t}^{J,h,m}(z)$, is defined similarly, by letting $a = m$ in equation 6, except that in this case with probability δ^m the worker dies and the firm has a vacancy at $t + 1$.

The expected value of an unskilled job filled by a worker of age $a \in \{y, m\}$ with $z \geq \underline{z}_{j,t}^{l,a}$ is given in equation 7. For unskilled jobs, we account for the probability ϱ^a that

¹⁶The firm could post a vacancy for a different skill k or age a once a match breaks. However, since the equilibrium value of posting a vacancy is 0 due to free entry, the $\Phi_{j,t+1}^{V,k,a}$ terms drop out for any k or a .

the worker becomes skilled. The skill shock occurs at the end of the period (Table 2).

$$\begin{aligned}
\Phi_{j,t}^{J,l,a}(z) &= p_{j,t} z s^l - w_{j,t}^{l,a}(z) + \beta(1 - \delta^a)(1 - \varrho^a) \left[\int_{\underline{z}_{j,t+1}^{l,a}}^1 \Phi_{j,t+1}^{J,l,a}(x) dG(x) + G(\underline{z}_{j,t+1}^{l,a}) \Phi_{j,t+1}^{V,l,a} \right] \\
&+ \beta(1 - \delta^a) \varrho^a \left[\int_{\underline{z}_{j,t+1}^{h,a}}^1 \Phi_{j,t+1}^{J,h,a}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,a}) \Phi_{j,t+1}^{V,h,a} \right] \\
&+ I^y \cdot \beta \delta^y (1 - \varrho^y) \left[\int_{\underline{z}_{j,t+1}^{l,m}}^1 \Phi_{j,t+1}^{J,l,m}(x) dG(x) + G(\underline{z}_{j,t+1}^{l,m}) \Phi_{j,t+1}^{V,l,m} \right] \\
&+ I^y \cdot \beta \delta^y \varrho^y \left[\int_{\underline{z}_{j,t+1}^{h,m}}^1 \Phi_{j,t+1}^{J,h,m}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,m}) \Phi_{j,t+1}^{V,h,m} \right] + I^m \cdot \beta \delta^m \Phi_{j,t+1}^{V,l,m} \quad (7)
\end{aligned}$$

3.6 Wage Bargaining

Wages $w_{j,t}^{k,a}(z)$ are determined through Nash bargaining. Unskilled jobs can be filled by immobile workers with a sectoral preference ($\zeta = j$) or mobile workers with no sectoral preference ($\zeta = 0$). We assume that wage bargaining cannot condition on the value of ζ . The surplus generated by unskilled jobs is that of mobile workers, who maximize the expected value of searching for a job across all sectors. In the steady state, this assumption is innocuous. However, along the transition following a sectoral shock, this assumption generates a wage distortion that affects unskilled immobile workers.

The Nash bargaining problem for unskilled jobs filled by workers of age a splits the surplus of the match—denoted by $S_{j,t}^{l,a}$ —between workers and firms according to bargaining power γ and $1 - \gamma$, respectively.

$$\begin{aligned}
&\max \left[\Phi_{j,t}^{E,l,a,\zeta=0}(z) - \max_j \left\{ \Phi_{j,t}^{U,l,a,\zeta=0} \right\} \right]^\gamma \cdot \left[\Phi_{j,t}^{J,l,a}(z) \right]^{1-\gamma} \quad (8) \\
&\text{subject to: } S_{j,t}^{l,a}(z) = \Phi_{j,t}^{J,l,a}(z) + \Phi_{j,t}^{E,l,a,\zeta=0}(z) - \max_j \left\{ \Phi_{j,t}^{U,l,a,\zeta=0} \right\}
\end{aligned}$$

The expected value of employment for an unskilled worker with no sectoral preference (equation 9), accounts for the probability that the worker becomes skilled and/or middle-aged, and also embeds the different search patterns of skilled and unskilled. Skilled search for a job only in their sector. However, unskilled mobile search in any sector j that maximizes their value of being unemployed (equation 10). The outside option

in the Nash bargaining problem also reflects this search pattern.

$$\begin{aligned}
\Phi_{j,t}^{E,l,a,\zeta=0}(z) &= w_{j,t}^{l,a}(z) + \beta(1-\delta^a)\varrho^a \left[\int_{\underline{z}_{j,t+1}^{h,a}}^1 \Phi_{j,t+1}^{E,h,a}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,a})\Phi_{j,t+1}^{U,h,a} \right] \\
&+ \beta(1-\delta^a)(1-\varrho^a) \left[\int_{\underline{z}_{j,t+1}^{l,a}}^1 \Phi_{j,t+1}^{E,l,a,\zeta=0}(x) dG(x) + G(\underline{z}_{j,t+1}^{l,a}) \max_j \{ \Phi_{j,t+1}^{U,l,a,\zeta=0} \} \right] \\
&+ I^y \cdot \beta\delta^y(1-\varrho^y) \left[\int_{\underline{z}_{j,t+1}^{l,m}}^1 \Phi_{j,t+1}^{E,l,m,\zeta=0}(x) dG(x) + G(\underline{z}_{j,t+1}^{l,m}) \max_j \{ \Phi_{j,t+1}^{U,l,m,\zeta=0} \} \right] \\
&+ I^y \cdot \beta\delta^y\varrho^y \left[\int_{\underline{z}_{j,t+1}^{h,m}}^1 \Phi_{j,t+1}^{E,h,m}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,m})\Phi_{j,t+1}^{U,h,m} \right] \tag{9}
\end{aligned}$$

$$\begin{aligned}
\Phi_{j,t}^{U,l,a,\zeta=0} &= \theta_{j,t}^{l,a} q(\theta_{j,t}^{l,a}) \cdot \Phi_{j,t}^{E,l,a,\zeta=0}(\xi^k) + [1 - \theta_{j,t}^{l,a} q(\theta_{j,t}^{l,a})] \cdot \left[b^l + \beta(1-\delta^a) \max_j \{ \Phi_{j,t+1}^{U,l,a,\zeta=0} \} \right] \\
&+ I^y \cdot [1 - \theta_{j,t}^{l,y} q(\theta_{j,t}^{l,y})] \cdot \beta\delta^y \max_j \{ \Phi_{j,t+1}^{U,l,m,\zeta=0} \} \tag{10}
\end{aligned}$$

The Nash bargaining problem for skilled workers of age a splits the surplus $S_{j,t}^{h,a}(z) = \Phi_{j,t}^{J,h,a}(z) + \Phi_{j,t}^{E,h,a}(z) - \Phi_{j,t}^{U,h,a}$ according to the same bargaining powers. The expected values of employment and unemployment for skilled are given in equations 11 and 12.

$$\begin{aligned}
\Phi_{j,t}^{E,h,a}(z) &= w_{j,t}^{h,a}(z) + \beta(1-\delta^a) \left[\int_{\underline{z}_{j,t+1}^{h,a}}^1 \Phi_{j,t+1}^{E,h,a}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,a})\Phi_{j,t+1}^{U,h,a} \right] \\
&+ I^y \cdot \beta\delta^y \left[\int_{\underline{z}_{j,t+1}^{h,m}}^1 \Phi_{j,t+1}^{E,h,m}(x) dG(x) + G(\underline{z}_{j,t+1}^{h,m})\Phi_{j,t+1}^{U,h,m} \right] \tag{11}
\end{aligned}$$

$$\begin{aligned}
\Phi_{j,t}^{U,h,a} &= \theta_{j,t}^{h,a} q(\theta_{j,t}^{h,a}) \cdot \Phi_{j,t}^{E,h,a}(\xi^k) \\
&+ [1 - \theta_{j,t}^{h,a} q(\theta_{j,t}^{h,a})] \cdot \left[b^h + \beta(1-\delta^a) \Phi_{j,t+1}^{U,h,a} + I^y \cdot \beta\delta^y \Phi_{j,t+1}^{U,h,m} \right] \tag{12}
\end{aligned}$$

3.7 Market Clearing Conditions

Population at t equals the stocks of employed and unemployed workers who are young or middle-aged, skilled or unskilled. We describe how these stocks evolve over time.

Skilled workers choose to search in the sector where they acquired skills. The stock of skilled unemployed at the beginning of period t is denoted by $U_{j,t}^{h,a}$, with $a \in \{y, m\}$. After productivity shocks are drawn, $E_{j,t}^{h,a} G(\underline{z}_{j,t}^{h,a})$ matches separate, where $E_{j,t}^{h,a}$ is the

beginning of period stock of skilled employed and $G(\underline{z}_{j,t}^{h,a})$ is the fraction of matches below the endogenous productivity threshold. These newly separated workers enter the pool of unemployed and search for a job. Thus, the number of skilled workers of age a searching in sector j during period t is denoted by $\tilde{U}_{j,t}^{h,a} = U_{j,t}^{h,a} + E_{j,t}^{h,a} G(\underline{z}_{j,t}^{h,a})$.

Fraction $\theta_{j,t}^{h,a} q(\theta_{j,t}^{h,a})$ of the unemployed, skilled workers find a job, while the remainder still search next period. The law of motion for the beginning of period stock of unemployed skilled workers of age a is given in equation 13. Unemployed skilled middle-aged ($a = m$) workers at $t + 1$ are the skilled middle-aged workers who haven't matched or died (first term in 13) plus the unskilled young workers who haven't matched, but became middle-aged at the end of period t (second term in 13). Unemployed skilled, young ($a = y$) at $t + 1$ are those who haven't matched or died by the end of t . The second term in equation 13 drops out, since $I^m = 1$ only for $a = m$.

$$U_{j,t+1}^{h,a} = (1 - \delta^a) \tilde{U}_{j,t}^{h,a} \left[1 - \theta_{j,t}^{h,a} q(\theta_{j,t}^{h,a}) \right] + I^m \cdot \delta^y \tilde{U}_{j,t}^{h,y} \left[1 - \theta_{j,t}^{h,y} q(\theta_{j,t}^{h,y}) \right] \quad (13)$$

The stocks of unskilled unemployed searching at t are $\tilde{U}_{j,t}^{l,a,\zeta=0}$ and $\tilde{U}_{j,t}^{l,a,\zeta=j}$, $a \in \{y, m\}$. The beginning of period stock of unskilled unemployed with sectoral preference ($\zeta = 0$) plus the newly separated can search in any sector for unskilled vacancies (equation 14). Unskilled with sectoral preference ($\zeta = j$) search only in sector j (equation 15).

$$U_t^{l,a,\zeta=0} + \sum_{j \in \{1,2,3\}} E_{j,t}^{l,a,\zeta=0} G(\underline{z}_{j,t}^{l,a}) = \tilde{U}_{1,t}^{l,a,\zeta=0} + \tilde{U}_{2,t}^{l,a,\zeta=0} + \tilde{U}_{3,t}^{l,a,\zeta=0} \quad (14)$$

$$U_{j,t}^{l,a,\zeta=j} + E_{j,t}^{l,a,\zeta=j} G(\underline{z}_{j,t}^{l,a}) = \tilde{U}_{j,t}^{l,a,\zeta=j} \quad (15)$$

The laws of motion for the beginning of period unskilled unemployed workers account for aging, births, and job finding probabilities (equations 16–19).

$$U_{t+1}^{l,m,\zeta=0} = \sum_j \left\{ (1 - \delta^m) \tilde{U}_{j,t}^{l,m,\zeta=0} \cdot [1 - \theta_{j,t}^{l,m} q(\theta_{j,t}^{l,m})] + \delta^y \tilde{U}_{j,t}^{l,y,\zeta=0} \cdot [1 - \theta_{j,t}^{l,y} q(\theta_{j,t}^{l,y})] \right\} \quad (16)$$

$$U_{j,t+1}^{l,m,\zeta=j} = (1 - \delta^m) \tilde{U}_{j,t}^{l,m,\zeta=j} \cdot [1 - \theta_{j,t}^{l,m} q(\theta_{j,t}^{l,m})] + \delta^y \tilde{U}_{j,t}^{l,y,\zeta=j} \cdot [1 - \theta_{j,t}^{l,y} q(\theta_{j,t}^{l,y})] \quad (17)$$

$$U_{t+1}^{l,y,\zeta=0} = (1 - \delta^y) \sum_j \tilde{U}_{j,t}^{l,y,\zeta=0} \cdot [1 - \theta_{j,t}^{l,y} q(\theta_{j,t}^{l,y})] + (1 - \pi_1 - \pi_2 - \pi_3) \eta N_t \quad (18)$$

$$U_{j,t+1}^{l,y,\zeta=j} = (1 - \delta^y) \tilde{U}_{j,t}^{l,y,\zeta=j} \cdot [1 - \theta_{j,t}^{l,y} q(\theta_{j,t}^{l,y})] + \pi_j \eta N_t \quad (19)$$

Stocks of employed workers at the beginning of $t + 1$ equal the employed from t

whose match wasn't terminated (i.e., whose productivity stayed above the threshold) plus the unemployed who newly matched. Stocks of skilled workers given in equation 20 also account for aging and skill acquisition. Unskilled workers with $\zeta = 0$ are given in equation 21, while stocks of unskilled with $\zeta = j$ are defined similarly.

$$\begin{aligned}
E_{j,t+1}^{h,a} &= (1 - \delta^a) \left\{ E_{j,t}^{h,a} \cdot [1 - G(\underline{z}_{j,t}^{h,a})] + \varrho^a (E_{j,t}^{l,a,\zeta=j} + E_{j,t}^{l,a,\zeta=0}) \cdot [1 - G(\underline{z}_{j,t}^{l,a})] \right\} \\
&\quad + (1 - \delta^a) \left\{ \tilde{U}_{j,t}^{h,a} \cdot \theta_{j,t}^{h,a} q(\theta_{j,t}^{h,a}) + \varrho^a (\tilde{U}_{j,t}^{l,a,\zeta=j} + \tilde{U}_{j,t}^{l,a,\zeta=0}) \cdot \theta_{j,t}^{l,a} q(\theta_{j,t}^{l,a}) \right\} \\
&\quad + I^m \cdot \delta^y \left\{ E_{j,t}^{h,y} \cdot [1 - G(\underline{z}_{j,t}^{h,y})] + \varrho^y (E_{j,t}^{l,y,\zeta=j} + E_{j,t}^{l,y,\zeta=0}) \cdot [1 - G(\underline{z}_{j,t}^{l,y})] \right\} \\
&\quad + I^m \cdot \delta^y \left\{ \tilde{U}_{j,t}^{h,y} \cdot \theta_{j,t}^{h,y} q(\theta_{j,t}^{h,y}) + \varrho^y (\tilde{U}_{j,t}^{l,y,\zeta=j} + \tilde{U}_{j,t}^{l,y,\zeta=0}) \cdot \theta_{j,t}^{l,y} q(\theta_{j,t}^{l,y}) \right\} \quad (20)
\end{aligned}$$

$$\begin{aligned}
E_{j,t+1}^{l,a,\zeta=0} &= (1 - \delta^a) (1 - \varrho^a) \left\{ E_{j,t}^{l,a,\zeta=0} \cdot [1 - G(\underline{z}_{j,t}^{l,a})] + \tilde{U}_{j,t}^{l,a,\zeta=0} \cdot \theta_{j,t}^{l,a} q(\theta_{j,t}^{l,a}) \right\} \\
&\quad + I^m \cdot \delta^y (1 - \varrho^y) \left\{ E_{j,t}^{l,y,\zeta=0} \cdot [1 - G(\underline{z}_{j,t}^{l,y})] + \tilde{U}_{j,t}^{l,y,\zeta=0} \cdot \theta_{j,t}^{l,y} q(\theta_{j,t}^{l,y}) \right\} \quad (21)
\end{aligned}$$

In the goods market, unemployed workers of skill k who are newly matched with firms produce ξ^k units of the good. The other employed workers produce z units of the good, where z is drawn from $G(z)$. We define $\tilde{U}_{j,t}^{l,a} \equiv \tilde{U}_{j,t}^{l,a,\zeta=0} + \tilde{U}_{j,t}^{l,a,\zeta=j}$ and $E_{j,t}^{l,a} \equiv E_{j,t}^{l,a,\zeta=0} + E_{j,t}^{l,a,\zeta=j}$. Then, sectoral output is given in equation 22.

$$Y_{j,t} = \sum_{a \in \{y,m\}} \sum_{k \in \{h,l\}} \left[\tilde{U}_{j,t}^{k,a} \theta_{j,t}^{k,a} q(\theta_{j,t}^{k,a}) \xi^k s^k + E_{j,t}^{k,a} \int_{\underline{z}_{j,t}^{k,a}}^1 z s^k dG(z) \right] \quad (22)$$

3.8 Symmetric Steady State

We detrend our model's variables by the total population. Given identical production technologies across sectors and no aggregate uncertainty, we characterize a symmetric steady state in our detrended economy. In Section 5, we examine transitions between symmetric steady states that result from unanticipated one-time reallocation shocks.

In the symmetric steady state, a positive mass of mobile ($\zeta = 0$) unskilled workers apply to jobs in each sector. Skilled separated workers and unskilled workers with sectoral preference ($\zeta = j$) apply in their own sector. Given the identical production and matching technologies, it follows that sectoral goods prices are equated, $p_1 = p_2 = p_3$. Wage rates are the same, $w_1^{k,a}(z) = w_2^{k,a}(z) = w_3^{k,a}(z)$ for any productivity z , labor market tightness are equated across sectors, $\theta_1^{k,a} = \theta_2^{k,a} = \theta_3^{k,a}$, as are the productivity thresholds

$\underline{z}_1^{k,a} = \underline{z}_2^{k,a} = \underline{z}_3^{k,a}$, for all k and a .¹⁷

The remainder of the steady state allocation is pinned down by the sectoral weights in the final good production function, α_j . In particular, for any $k \in \{h, l\}$ and $a \in \{y, m\}$, the ratios of allocations from any two sectors, $i, j \in \{1, 2, 3\}$ satisfy equation 23:

$$\frac{E_j^{k,a}}{E_i^{k,a}} = \frac{E_j^{k,a,\zeta=j}}{E_i^{k,a,\zeta=i}} = \frac{E_j^{k,a,\zeta=0}}{E_i^{k,a,\zeta=0}} = \frac{\tilde{U}_j^{k,a}}{\tilde{U}_i^{k,a}} = \frac{V_j^{k,a}}{V_i^{k,a}} = \frac{\tilde{Y}_j}{\tilde{Y}_i} = \frac{Y_j}{Y_i} = \left(\frac{\alpha_j}{\alpha_i} \right)^{\frac{1}{1-\rho}} \quad (23)$$

Sectors with larger weights α_j have relatively larger steady state stocks of workers (e.g., total employed, employed with sectoral preference, employed without sectoral preference, unemployed), post more vacancies and produce more output.

4 Parameterizing the Symmetric Steady State

Our experiments in Section 5 focus on the implications of demographics for the transition between symmetric steady states in response to one-time sectoral reallocation shocks. In Section 4.1, we outline the model parameterization, while in Section 4.2 we discuss the impact of population growth on the model's symmetric steady state.

4.1 Benchmark Model Parameters

We choose commonly used functional forms and parameterize the model to match targets in U.S. data. When parameterizing the model, we set the population growth rate to 1 percent annually ($\gamma_N = 1$), which is the U.S. average in the 2000s. We summarize the model's parameters in Table 3.

The model period is one quarter. The household discount factor, β , is set to 0.99 to match an annual risk-free interest rate of 4 percent. We set the probability of a young worker aging and that of a middle-aged worker dying so that youth spans the ages of 20 to 34, and middle age runs from 35 to 64. This leads us to set $\delta_m = \frac{1}{60}$ and $\delta_d = \frac{1}{120}$.

¹⁷See the supplementary online appendix for our constructive proof. We first assume that in a steady state the value of a new match is equated across sectors, and mobile unskilled workers apply for jobs in all of the sectors. We then verify that these assumptions hold.

The production function parameters α_1 and α_3 are chosen so that the GDP shares of sectors 1 and 3 are 30.5% and 33.5%, respectively. This roughly corresponds to a grouping of U.S. industries where sector 3 is agriculture, mining, manufacturing, construction, sector 1 is wholesale, retail, fire insurance and real estate, and personal services and sector 2 is the remaining service sectors, over 1990 – 2005. We set the elasticity of substitution, $\frac{1}{1-\rho}$, to 0.2. This lies between the value of 0.11 used by [Phelan and Trejos \(2000\)](#) and the [Stockman and Tesar \(1995\)](#) estimate of 0.44 between goods and services. In [section 5.4](#), we conduct sensitivity analysis with respect to this elasticity.¹⁸ For analytical tractability, the productivity distribution $G(z)$ is uniform over $[0, 1]$.

Estimates of the costs of hiring skilled workers typically exceed those for unskilled ([Manning \(2011\)](#)). [Blatter, Muehlemann, and Schenker \(2012\)](#) and [Muehlemann and Pfeifer \(2016\)](#) report that hiring a skilled worker costs from 8 to 13 weeks of salary. Roughly a third of these costs are due to recruitment, with the remainder due to firm specific hiring and training costs.¹⁹ We parameterize the vacancy posting c^k so that the expected recruitment cost per worker hired equals 3 weeks of wages, which—given 13 weeks in a quarter—represents 23 percent of the average wage. Since a large share of recruitment costs are due to training which take place after a vacancy has been filled, we scale down the initial match productivity for a new match from the average productivity of matches by one third for skilled matches (equivalent to roughly 4.5 weeks of a quarter) and one-ninth for unskilled matches. This implies $z_{new}^h = 0.36$ and $z_{new}^l = 0.56$.

[Kambourov and Manovskii \(2009\)](#) and [Auray et al. \(2015\)](#) parameterize the probability of becoming skilled to match the observation that earnings profiles flatten after 11 to 13 years of tenure. This leads us to set $\varrho^y = \frac{1}{11*4} = 0.0227$, and $\varrho^m = \frac{1}{10*4} = 0.025$. Given these probabilities, we set the ratio of skilled to unskilled productivity equal to 2.2 to generate a growth of average cohort wages after twenty years of roughly 75 percent, close to the value implied by [Lagakos et al. \(2017\)](#) for Canada and the U.S.²⁰

The matching functions are Cobb-Douglas, $M(\tilde{U}_{j,t}^{k,a}, V_{j,t}^{k,a}) = A^k (\tilde{U}_{j,t}^{k,a})^{1-\psi} (V_{j,t}^{k,a})^\psi$ for $k \in \{h, l\}$, $a \in \{y, m\}$ and $j \in \{1, 2, 3\}$. We set $\psi = 0.4$, which is the mid-point of the range (0.3 – 0.5) in [Petrongolo and Pissarides \(2001\)](#). We set the Nash bargaining weight

¹⁸[Ngai and Pissarides \(2008\)](#) calibrate an elasticity of substitution in a three-sector model of 0.1, while [Amaral and MacGee \(2017\)](#) parameterize an elasticity of substitution of 0.45 in a two-sector model.

¹⁹[Dube, Freeman, and Reich \(2010\)](#) report somewhat lower estimates using data on Californian firms, with the mean cost of hiring workers between 35 and 40 percent of quarterly wages.

²⁰[Williams \(2009\)](#) finds that 30 years of work experience increases wages by roughly 60 percent in the UK. [Rupert and Zanelaa \(2015\)](#) find a similar experience profile for men in the PSID, with workers in their early forties earning wages roughly 60 percent more than workers in their early twenties.

on the worker to $\gamma = 0.6$, which is the same as the weight on unemployment in the matching function. In Section 5.4, we show our results are robust to the choice of γ .

We assume the matching productivity A^k is the same across sectors and ages, but differs by worker skill. Based on Menzio, Telyukova, and Visschers (2016), we target a quarterly probability of transitioning from unemployed to employed of 0.61 for young and 0.56 for middle-aged workers.²¹ We target a separation rate of 10.4 percent, consistent with Shimer (2005). Exogenous separations arise via the death of employed middle-aged workers and account for about 4.8 percent of separations. The remaining separations arise endogenously due to productivity realizations less than the thresholds $\underline{z}_{j,t+1}^{k,a}$. We treat workers who separate and match within a period as job-to-job transitions. By construction, net job creation equals the growth rate of the working-age population.

The benefit received while unemployed varies by skill, but not age. We set the benefit for skilled workers to the average net replacement ratio of 53 percent for U.S. workers with earnings of 150 percent of median wage OECD (2007).²² The parameterized benefit for unskilled workers is 85 percent of the average unskilled wage.²³ This implies an average replacement rate of unemployed workers of 55 percent of mean wages.

The fraction of immobile workers with sectoral preference does not impact the symmetric steady state. Our parametrization targets the fraction of mobile workers to gross flows between industries, and then allocates the fraction of workers with a preference for sector j proportionately to each sector's share of steady state employment. To compute gross flows in the model, we assume that when unemployed mobile workers are indifferent over which sector to search in (as they are in the steady state), they randomize between sectors, with probabilities proportional to sector shares. We use data from the CPS Occupational Mobility Supplements to construct a target of gross industry switching rates for 1996, 2000, 2002, 2004 and 2006.²⁴ Specifically, we compute the fraction of workers aged 20 to 64 who work in a different 1-digit industry than one year earlier.

²¹Although the A^k do not vary by age, the fraction of workers who are skilled varies by age.

²²Specifically, we use the net replacement rates reported in Table 3.1 of OECD (2007), and average across the rates for single and married households with no children or two children.

²³Our parametrization of the benefit level for unskilled workers is somewhat lower than that of Hagedorn and Manovskii (2008), who show that a high outside option for workers improves the extent to which the Mortensen-Pissarides framework can account for cyclical fluctuations in employment.

²⁴We use the Occupational Mobility Supplements due to concerns over measurement error in industry and occupational switching in the CPS (e.g., Kambourov and Manovskii (2013)). However, these data are only available for 1996 and bi-annually since 2000. Our parametrization is conservative, as we exclude the post-2006 years of low gross flows, since otherwise our fraction of mobile workers would be lower and reallocation costs would be larger.

Since a higher number of mobile workers lowers the costs of sectoral adjustment, we conservatively target the average annual gross switching rate across all 1-digit industries (3.9 percent) rather than that of our three sector composite (2.9 percent). Consistent with the Occupational Mobility Supplements data which tracks workers who were employed at t and $t - 1$, we compute employment to employment switchers in our model. We set a benchmark value of 40 percent mobile workers, which implies a quarterly gross switching rate of 1.1 percent in the steady state.

Table 3: BENCHMARK PARAMETERIZATION

Parameter	Value	Target
β	0.99	Annual risk-free rate of 4%
δ^y	$\frac{1}{15 \times 4} = 0.0167$	Youth expectancy of 15 years (ages 20 – 34)
δ^m	$\frac{1}{30 \times 4} = 0.0083$	Middle age expectancy of 30 years (ages 35 – 64)
η	0.007543	Annual population growth rate $\gamma_N = 1\%$
α_1	0.2045	Sector 1 expenditure share of 0.305
α_2	0.4685	Sector 2 expenditure share of 0.36
α_3	0.3269	Sector 3 expenditure share of 0.335
ψ	0.4	Petrongolo and Pissarides (2001)
γ	0.6	Unemployment weight in matching function, $1 - \psi$
b^l	0.17	10.4% separation rate
b^h	0.2	53% of average skilled wages
ϱ^y	$\frac{1}{11 \times 4} = 0.0227$	11 years to become skilled
ϱ^m	$\frac{1}{10 \times 4} = 0.025$	10 years to become skilled
s^h	2.2	Average life-cycle wage growth
c^l	0.025	Recruitment cost $\simeq 23\%$ unskilled wage
c^h	0.165	Recruitment cost $\simeq 23\%$ skilled wage
z^l	0.56	Training new worker $\simeq 33\%$ unskilled wage
z^h	0.36	Training new worker $\simeq 11\%$ skilled wage
A^h	0.575	Quarterly UE probability for middle-aged of 0.56
A^l	0.407	Quarterly UE probability for young of 0.61
$\sum_j \pi_j$	0.6	Quarterly sectoral switching of 1.1%

4.2 Population Growth and the Symmetric Steady State

Before examining sectoral reallocation shocks, we outline how population growth impacts the symmetric steady state. In our experiments, we hold all parameters fixed at their benchmark values, except for the birth rate, η , which we choose to target population growth rates ranging from -1 percent to 3 percent.

Population growth has an intuitive impact on steady state variables (Table 4). A higher population growth rate increases the fraction of young workers, which lowers per capita output for two reasons. First, relatively fewer young workers are skilled than middle-aged workers. This lowers average human capital per worker, and thus measured labor productivity.²⁵ Second, young workers have higher unemployment rates than middle-aged workers. Thus, a larger fraction of young workers at high population growth results in lower employment rates, and thus lower output per capita.

This pattern is consistent with Feyrer (2007), who examines the impact of the age composition of the workforce on productivity for 97 countries over 1965 to 1995. Feyrer finds that countries with larger fractions of younger workers have lower average productivity. The quantitative impacts of demographics in our model are arguably conservative compared to Feyrer (2007) who finds that a 5 percentage point shift in the share of workers in their 30s to workers in their 40s implies a 16 percentage point rise in productivity.²⁶ In our parameterized economy, a fall in population growth from 2 to 1 percent yields a 5 percentage point drop in the young share and a 2.6 percent rise in labor productivity.

Table 4: STEADY STATE AS POPULATION GROWTH RATES VARY

	Population Growth Rate, γ_N				
	-1%	0%	1%	2%	3%
OUTPUT	107.5	103.5	100	96.9	94.1
LABOR PRODUCTIVITY	106.0	102.8	100	97.5	95.3
COMPOSITION OF POPULATION					
Young Share	25.9	33.3	39.4	44.4	48.6
Skilled Share	82.8	76.5	70.9	66.0	61.6
UNEMPLOYMENT RATES					
Young	8.6	9.2	9.7	10.2	10.6
Middle-aged	5.0	5.2	5.5	5.8	6.0

NOTES: All parameters except for γ_N are fixed at their benchmark values (e.g., sectoral shock is 3 percent). Output and labor productivity with 1 percent population growth are normalized to 100.

²⁵Our model abstracts from some channels via which demographics can impact labor productivity. For example, Prskawetz, Fent, and Guest (2008) point out that if workers of different ages are imperfect substitutes in production, then shifts in the ratio of age cohorts can raise or lower average labor productivity.

²⁶Our division of workers into two age groups differs from Feyrer (2007) who uses ten-year age groups.

5 Experiments

To quantify how demographics interact with sectoral reallocation shocks, we conduct numerical experiments for (annual) population growth rates ranging from -1 to 3 percent. This range spans the working-age population growth rates in G-7 countries over the past fifty years, with highs of 2.7 percent in Canada over 1966 to 1975 and lows of -1 percent in Japan over 2007 to 2017.

Our experiments begin with the economy in the symmetric steady state corresponding to the population growth rate. The *sectoral reallocation shock* is modelled as a permanent and unanticipated one-time switch in sector 1 and 3's expenditure share in the production of the consumption good (i.e., we switch α_1 and α_3 in equation 2).²⁷ We then solve for the transition to the new steady state. By design, the sectoral reallocation shock has no impact on aggregate steady state output, employment, prices, or wages. The only difference between the two steady states is in the relative allocations of sectors 1 and 3.

We find that the population growth rate significantly impacts the depth and duration of output declines after a sectoral reallocation shock. In our parameterized economy with a 3 percent reallocation shock, the cumulative output losses vary from 1.3 to nearly 10 percent of annualized steady state GDP, as we lower the population growth rate from 3 to -1 percent. Roughly 94 percent of output losses are due to the Nash bargaining distortion in unskilled worker's wages and sectoral preferences.

Our experiments offer new insights into the debate over whether sectoral shocks are consistent with the relationship between unemployment and vacancies. In Section 5.3 we show that in our benchmark economy, a sectoral reallocation shock can generate a rise in unemployment and a fall in vacancies. This relationship is impacted by demographics, as the number of periods with below average levels of vacancies rises when population growth rates decline.

5.1 Benchmark Experiment

To illustrate the key model mechanisms, we discuss a benchmark experiment where the annual population growth rate is 1 percent, which is close to the recent U.S. experience.

²⁷This approach to modelling sectoral shocks as a shift in the expenditure shares in the production function is also used by Phelan and Trejos (2000). Our approach is consistent with Figura and Wascher (2010), who find that shifts in U.S. sectoral demand drive differences in employment growth.

The reallocation shock is an increase (decrease) in the expenditure share of sector 1 (3) by 3 percentage points, which is of similar magnitude to that of [Phelan and Trejos \(2000\)](#).²⁸

In our experiments, we assume the shock occurs at the beginning of period 5, so for the first four periods (1 year) the economy is in steady state. Solving for the transition to the new steady state is complicated by the need to check for corner solutions in the allocation of unskilled mobile ($\zeta = 0$) workers. We guess and verify the number of periods in which zero unemployed unskilled mobile workers search for a job in sectors 2 and 3. As a measure of total reallocation costs, we compute the cumulative output loss by summing deviations from the steady state for 25 years following the shock.

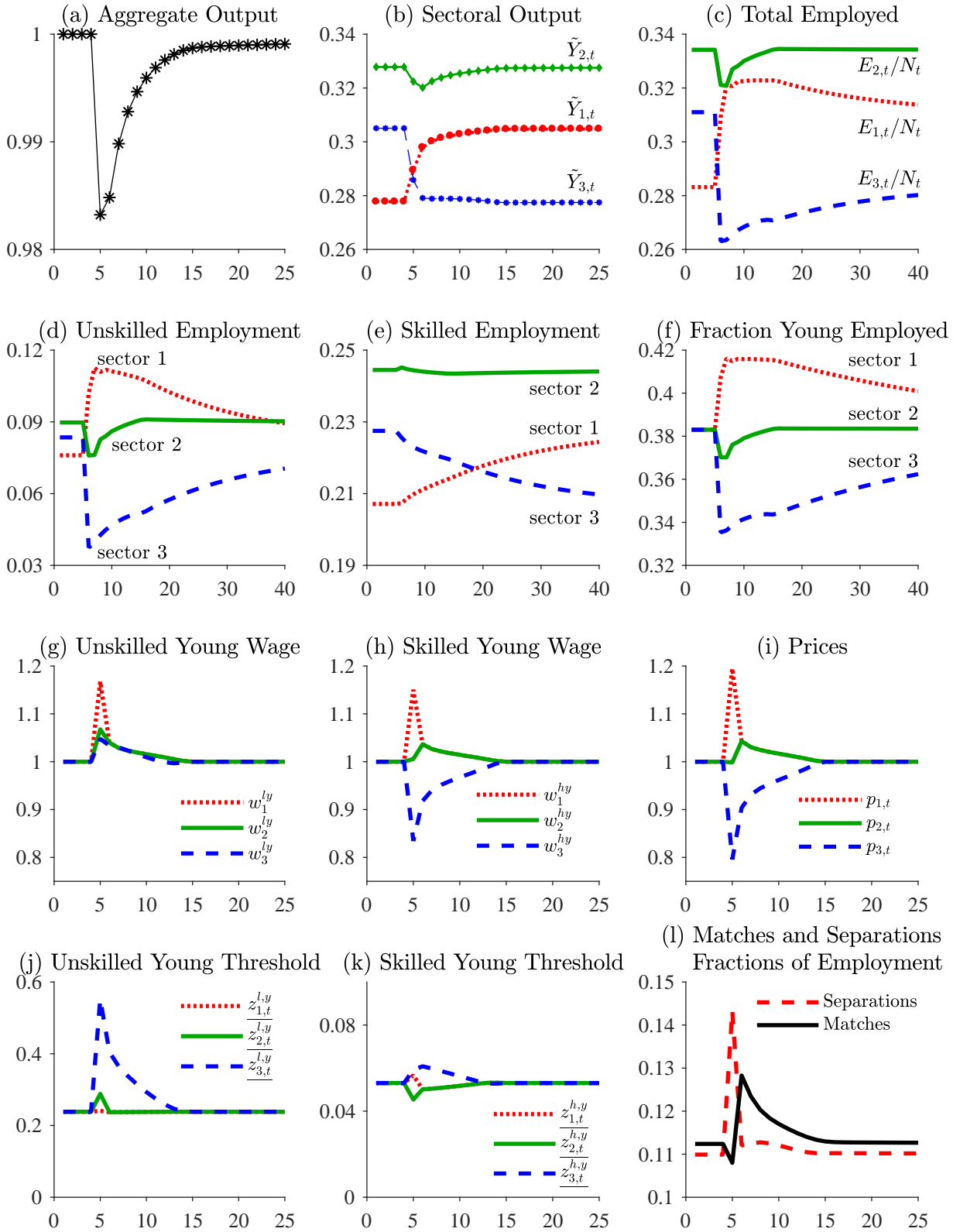
On impact, final output falls by 1.7 percent (Figure 2.a). After 6 periods (1.5 years in the model), final output is still roughly half a percent below its steady state. Although sector 3's output declines quickly, the growing sector 1 takes 8 periods to close within half a percentage point of its new steady state (Figure 2.b). The gradual adjustment of sectoral outputs following the shift in sectoral shares α_1 and α_3 , drives up (down) sector 1 (3)'s price along the transition to the new steady state (Figure 2.i). Somewhat surprisingly, sector 2—which was not directly impacted by the reallocation shock—sees a fall in output followed by a prolonged recovery and an increase in its price.

The sectoral output dynamics reflect the interaction between the reallocation of unskilled workers and the sector-specific human capital of skilled workers. In the shrinking sector, the initial fall in output is largely due to a decline in unskilled employment, which falls below the new steady state level before slowly recovering (Figure 2.d). Despite fewer unskilled and lower total employment (Figure 2.c), sector 3's output then remains near the new steady state level as employment of the more productive skilled workers declines gradually (Figure 2.e). At first glance, the slow decline of skilled employment seems puzzling, as the death rate ($\delta^m \simeq 0.8$ percent) of the middle-aged implies a quicker shrinkage rate. However, the deaths of the skilled middle-aged are partially offset by the aging of skilled young and the inflow of newly skilled workers.

In the growing sector, total and unskilled employment rise above the new steady state level due to the inflow of mobile workers who only search in sector 1 in the periods immediately following the reallocation shock. Since the unskilled are less productive and skilled employment grows slowly as it requires work experience, sector 1's output

²⁸[Phelan and Trejos \(2000\)](#) examine a sectoral shift of 3.5 percentage points due to a decrease in U.S. military spending from roughly 6.5 percent of GDP in 1987 to 3 percent in 2000. [Tapp \(2011\)](#) targets a shift of 3 percentage points (in employment) from manufacturing to resources in Canada over 2001 – 2005.

Figure 2: BENCHMARK MODEL RESULTS



NOTES: Results are quarterly, for benchmark parameters (e.g., 1% population growth and 3% sectoral shock). Wages and prices are normalized to 1 in the steady state.

rises slower than total employment. As most unskilled are young, the young share of sector 1's total employment jumps and remains high along the transition to the new steady state (Figure 2.f). Similar to sector 3, sector 2's output fall is initially accounted for by lower unskilled employment, which leads to fewer skilled workers in subsequent periods and a slow recovery in output.

Unskilled wages rise not only in the growing sector, but also in sectors 2 and 3. Figure 2.g plots the average wage of matches that produce with young workers. Average wages shift due to changes in average productivity of matches that produce as $\underline{z}_{3,t}^{l,y}$ varies, and changes in the bargained match-specific wages. Holding the productivity threshold for unskilled young workers in sector 3 fixed at the steady state level, would lower average wages by about 9 percent after the reallocation shock. However, the bargained match-specific real product wage— $w_{3,t}^{l,y}$ divided by $p_{3,t}$ —rises.²⁹

Higher wages in sector 3 are due to a Nash bargaining distortion arising from the inability to condition the wage on a worker's sectoral preference (equation 8). The surplus in the Nash bargaining for unskilled workers of age a , $S_{3,t}^{l,a}(z)$, is the value of a match with productivity z less the outside option of a mobile unemployed worker who chooses in which of the three sectors to search for a job: $\max_{j \in \{1,2,3\}} \left\{ \Phi_{j,t}^{U,l,a,\zeta=0} \right\}$. The shift in relative prices after the reallocation shock raises the value of searching for an unskilled job in sector 1, $\Phi_{1,t}^{U,l,a,\zeta=0}$. The higher price of sector 1's good raises the value of a new match today (second term in equation 24 where age is $a = y$) and the value of searching for an unskilled job next period—either as a young or as a middle-aged worker.

$$\Phi_{1,t}^{U,l,y,\zeta=0} = b^l + \frac{\gamma}{1-\gamma} \frac{p_{1,t} c^l \theta_{1,t}^{l,y}}{1 - \theta_{1,t}^{l,y} q(\theta_{1,t}^{l,y})} + \beta (1 - \delta^y) \Phi_{1,t+1}^{U,l,y,\zeta=0} + \beta \delta^y \Phi_{1,t+1}^{U,l,m,\zeta=0} \quad (24)$$

A young worker's share of the surplus, $\gamma S_{3,t}^{l,y}(z) = \gamma \left[\Phi_{3,t}^{J,l,y}(z) + \Phi_{3,t}^{E,l,y,\zeta=0}(z) - \Phi_{1,t}^{U,l,y,\zeta=0} \right]$,

²⁹The patterns for middle-aged workers are similar.

is combined with equation 24 to obtain the unskilled young worker's wage in sector 3.

$$w_{3,t}^{l,y}(z) = (1 - \gamma) b^l + \underbrace{\gamma p_{3,t} z s^l}_{\text{sector 1 value}} + \frac{\gamma p_{1,t} c^l \theta_{1,t}^{l,y}}{1 - \theta_{1,t}^{l,y} q(\theta_{1,t}^{l,y})} \quad (25)$$

$$- \beta (1 - \gamma) \varrho^y \left\{ \delta^y \left[\Phi_{3,t+1}^{U,h,m} - \underbrace{\Phi_{1,t+1}^{U,l,m,\zeta=0}}_{\text{sector 1 value}} \right] + (1 - \delta^y) \left[\Phi_{3,t+1}^{U,h,y} - \underbrace{\Phi_{1,t+1}^{U,l,y,\zeta=0}}_{\text{sector 1 value}} \right] \right\}$$

The unskilled wage, $w_{3,t}^{l,y}$, in equation 25 varies with the sectoral price through $\gamma p_{3,t} z s^l$. Sector 1's price raises the unskilled wage both directly through its impact on the outside option of a new match (third term in 25) and indirectly, through its impact on the expected future value of the match (which transitions to a skilled match with probability ϱ^y , while workers age with probability δ^y). The rise in $p_{1,t}$ increases the value of searching for an unskilled job next period—either as a young, $\Phi_{1,t+1}^{U,l,y,\zeta=0}$, or as a middle-aged worker, $\Phi_{1,t+1}^{U,l,m,\zeta=0}$,—relative to the value of working as skilled in sector 3, $\Phi_{3,t+1}^{U,h,y}$ or $\Phi_{3,t+1}^{U,h,m}$. A similar mechanism pulls up unskilled wages in sector 2 as sector 1's price rises.

The decline in unskilled employment in sector 3 is due to the rise in unskilled wages (Figure 2.g) relative to $p_{3,t}$, which lowers the value of a sector 3 unskilled job, $\Phi_{3,t}^{J,l,a}(z)$ (equation 7). Since $\Phi_{3,t}^{J,l,a}(z_{3,t}^{l,a}) = 0$ this pushes up the unskilled productivity threshold, $z_{3,t}^{l,a}$ (see Figure 2.j). The higher threshold makes posting vacancies less attractive, so that new unskilled matches decline. Unskilled workers with preference for sector 3 (i.e., with $\zeta = 3$) continue to search for a job in sector 3. This gives rise to what Alvarez and Shimer (2011) term *rest unemployment*, as unskilled workers continue to search for a job in the shrinking sector despite a low probability of finding one. Similar effects occur in sector 2, but are smaller since the wage rises less relative to the sectoral price.

The wages for skilled workers track sectoral prices (see Figure 2.h, i). An unemployed skilled worker chooses between searching for a skilled job in their own sector or an unskilled job in different sector. Despite the shift in relative prices, even in sector 3 the value of searching for a skilled job, $\Phi_{3,t}^{U,h,a}$, exceeds that of an unskilled job in the growing sector. Hence, the Nash bargaining problem for skilled workers of age a splits the surplus $S_{j,t}^{h,a}(z) = \Phi_{j,t}^{J,h,a}(z) + \Phi_{j,t}^{E,h,a}(z) - \Phi_{j,t}^{U,h,a}$, which implies the wage for a skilled worker in a match with idiosyncratic productivity z is: $w_{j,t}^{h,a}(z) = (1 - \gamma) b^h + \gamma p_{j,t} z s^h + \frac{\gamma p_{j,t} c^h \theta_{j,t}^{h,a}}{1 - \theta_{j,t}^{h,a} q(\theta_{j,t}^{h,a})}$. The resulting close co-movement of sectoral prices and skilled wages imply

relatively small changes in productivity thresholds for skilled workers (Figure 2.k).

The reallocation of labor has large impacts on job creation and destruction (Figure 2.l). The initial jump in separations is mainly driven by the increase in sector 3's unskilled productivity threshold.³⁰ New unskilled matches in sector 3 fall with the jump in the productivity threshold, and then gradually recover. Aggregate new matches also fall on impact as the rise in unskilled matches in sector 1 is dominated by the fall in sector 3. Subsequently, aggregate new matches rise as sector 1's hiring remains high, while sector 3's hiring partially recovers.

The benchmark experiment illustrates how sectoral reallocation varies the age composition of growing and shrinking sectors. Consistent with the empirical findings in Section 2.2, the fraction of young workers in the growing sector rises relative to the young share of total employment. After the reallocation shock, sector 1 sees an overshooting of their steady state fraction of young workers (Figure 2.f), which allows the stock of skilled workers and output to gradually accumulate to the new steady state. As the unskilled leave the shrinking sector and since most unskilled are young, sector 3 sees an undershooting of their steady state fraction of young workers.

5.2 Demographics and Reallocation Costs

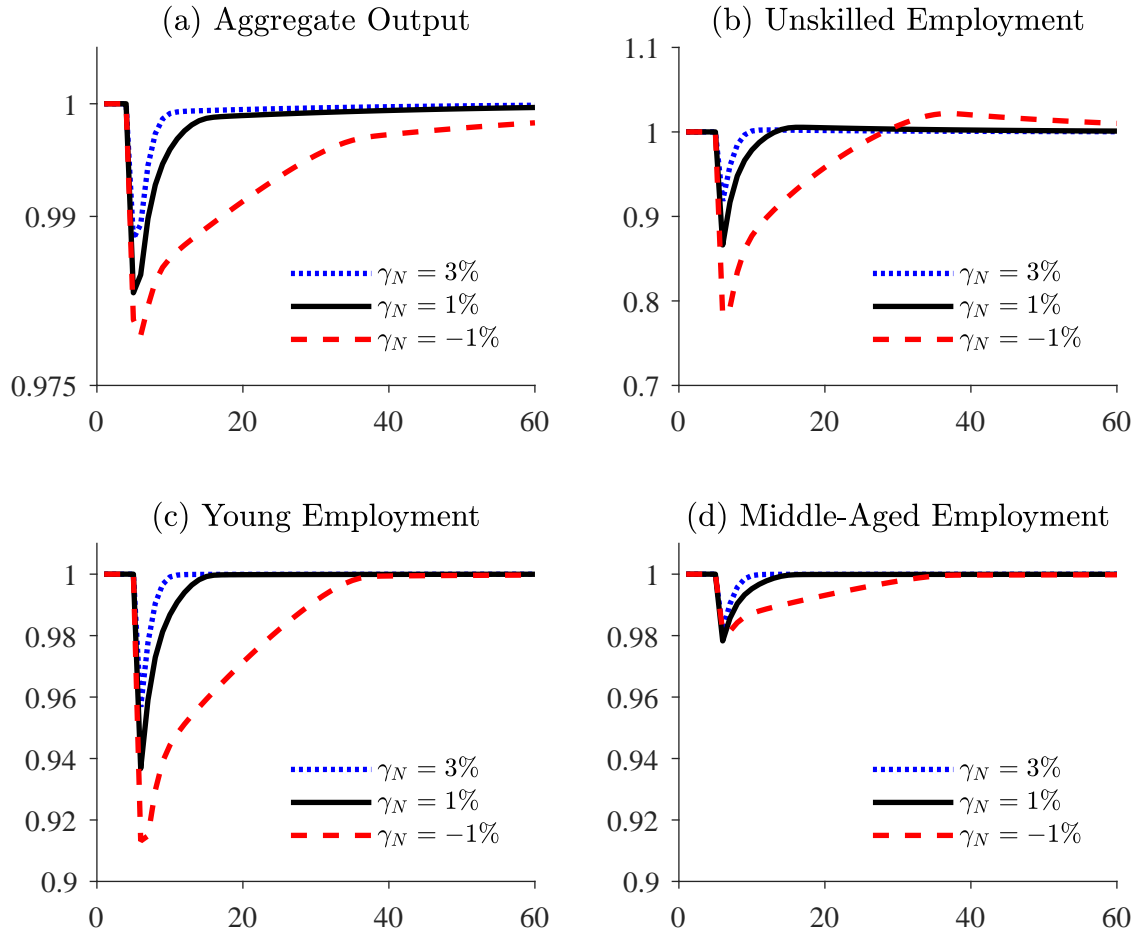
To show that demographics have a quantitatively significant impact on sectoral reallocation costs, we repeat our benchmark experiment for population growth rates ranging from -1 to 3 percent and sectoral reallocation shocks of 2 , 3 , and 4 percent.

Aggregate output shows deeper falls on impact, and more protracted transitions at lower population growth rates (Figure 3.a). Cumulative output losses roughly double as population growth falls from 3 to 1 percent, and more than triple as growth falls from 1 to -1 percent, for any reallocation shock (Table 5, benchmark experiments).

What is the intuition for the large impact of demographics on reallocation costs? As the population growth rate declines, the fraction of unskilled workers in the workforce falls (Figure 3.b), which amplifies the cost of reallocation in two ways. First, the smaller share of unskilled workers sees a larger fraction of unskilled matches in sector 3 terminated so as to induce mobile unskilled workers to shift sectors. This results in a larger

³⁰Separations are the sum of endogenous and exogenous separations due to middle-aged workers dying. Exogenous separations are roughly constant and account for about 4.8 percent of total separations.

Figure 3: BENCHMARK MODEL RESULTS AS DEMOGRAPHICS VARY



NOTES: Results are quarterly. Except for γ_N , we use benchmark parameters (e.g., sectoral shock is 3%).

initial decline in output. Second, fewer unskilled means that a larger fraction of steady state output is accounted for by skilled workers. Since stocks of skilled workers adjust slowly via death (exit) and skill accumulation (entry), lower population growth results in more prolonged deviations from the steady state ratio of sector 1 to sector 3's output.

Counterintuitively, as young workers become relatively scarcer at lower population growth rates, their employment rate falls by more after a sectoral shock (Figure 3.c). With lower population growth, the fraction of unskilled mobile ($\zeta = 0$) workers declines and the productivity thresholds for unskilled matches in sectors 2 and 3 increase. Of the endogenously separated workers, the unskilled mobile search in sector 1, while

the unskilled with sectoral preference search in their own sector. This drives up *rest unemployment*, which results in a lower fraction of young workers being employed. Although unskilled middle-aged workers' unemployment also rises, most middle-aged workers are skilled. Thus, the average employment of middle-aged workers falls by less than that of the young, and also varies less with population growth (Figure 3.d).

Table 5 shows that, not surprisingly, larger sectoral shocks imply larger output declines. However, the rise—by roughly a factor of 10—in the cumulative output losses as sectoral shocks increase from 2 to 4 percent in our benchmark experiments is surprising. This result is due to the interaction between sector-specific human capital and the fraction of unskilled workers. As population growth rates decline, the smaller share of unskilled mobile workers and the higher fraction of skilled workers result in larger and more prolonged deviations of relative sectoral outputs from the steady state.

Table 5: CUMULATIVE OUTPUT LOSSES (IN PERCENT OF ANNUAL GDP)
THE CONTRIBUTION OF WAGE DISTORTION AND SECTORAL PREFERENCE

	Population growth rate, γ_N				
	-1%	0%	1%	2%	3%
Benchmark					
2% reallocation shock	-2.39	-1.08	-0.69	-0.50	-0.38
3% reallocation shock	-9.84	-4.71	-2.68	-1.77	-1.33
4% reallocation shock	-23.07	-12.66	-7.33	-4.67	-3.38
Preference-Specific Wages					
2% reallocation shock	-0.43	-0.24	-0.17	-0.13	-0.10
3% reallocation shock	-2.01	-0.89	-0.55	-0.41	-0.34
4% reallocation shock	-6.34	-2.87	-1.51	-0.97	-0.74
Zero Sectoral Preferences					
2% reallocation shock	-0.12	-0.08	-0.06	-0.04	-0.03
3% reallocation shock	-0.38	-0.24	-0.18	-0.13	-0.12
4% reallocation shock	-0.81	-0.52	-0.38	-0.31	-0.25

NOTES: Output losses are expressed as a fraction of annualized steady state GDP (i.e., quarterly steady state output times 4), and are cumulated over a simulation of 100 quarters. In all experiments, sector 2's expenditure share is fixed at 0.36. Sector 1 and 3's shares sum to 0.64 and differ by the size of the sectoral shock (i.e., 2, 3 or 4 percentage points). Except for γ_N and the shock, we use benchmark parameters.

5.2.1 Role of Wage Distortion and Sectoral Preference

The unskilled labor market wage distortion and sectoral preferences have large impacts on labor reallocation. To decompose their contribution to our results, we repeat our experiments without these features.³¹

To remove the wage distortion, we extend labor markets to include a worker's sectoral preference. Unskilled workers thus apply for jobs in markets indexed by industry, skill, age and sectoral preference (i.e., an unskilled market is a quartet (j, l, a, ζ)). Unlike in our benchmark, the outside option in the Nash bargaining for workers with sectoral preference $\zeta = j \in \{1, 2, 3\}$ is the value of searching for a job in sector j , $\Phi_{j,t}^{U,l,a,\zeta=j}$.

The wage distortion significantly amplifies the costs of a sectoral reallocation shock. As Table 5 reports, the reallocation costs in the *preference-specific wage* experiments, where wages condition on workers' sectoral preferences, are roughly a quarter as large as in the benchmark. The qualitative pattern of rising reallocation costs—with lower population growth rates or larger sectoral shocks—is robust. The magnitudes of losses at low population growth rates remain large (e.g., 6.3 percent cumulative output losses in the experiment with 4 percent reallocation shock and -1 percent population growth).

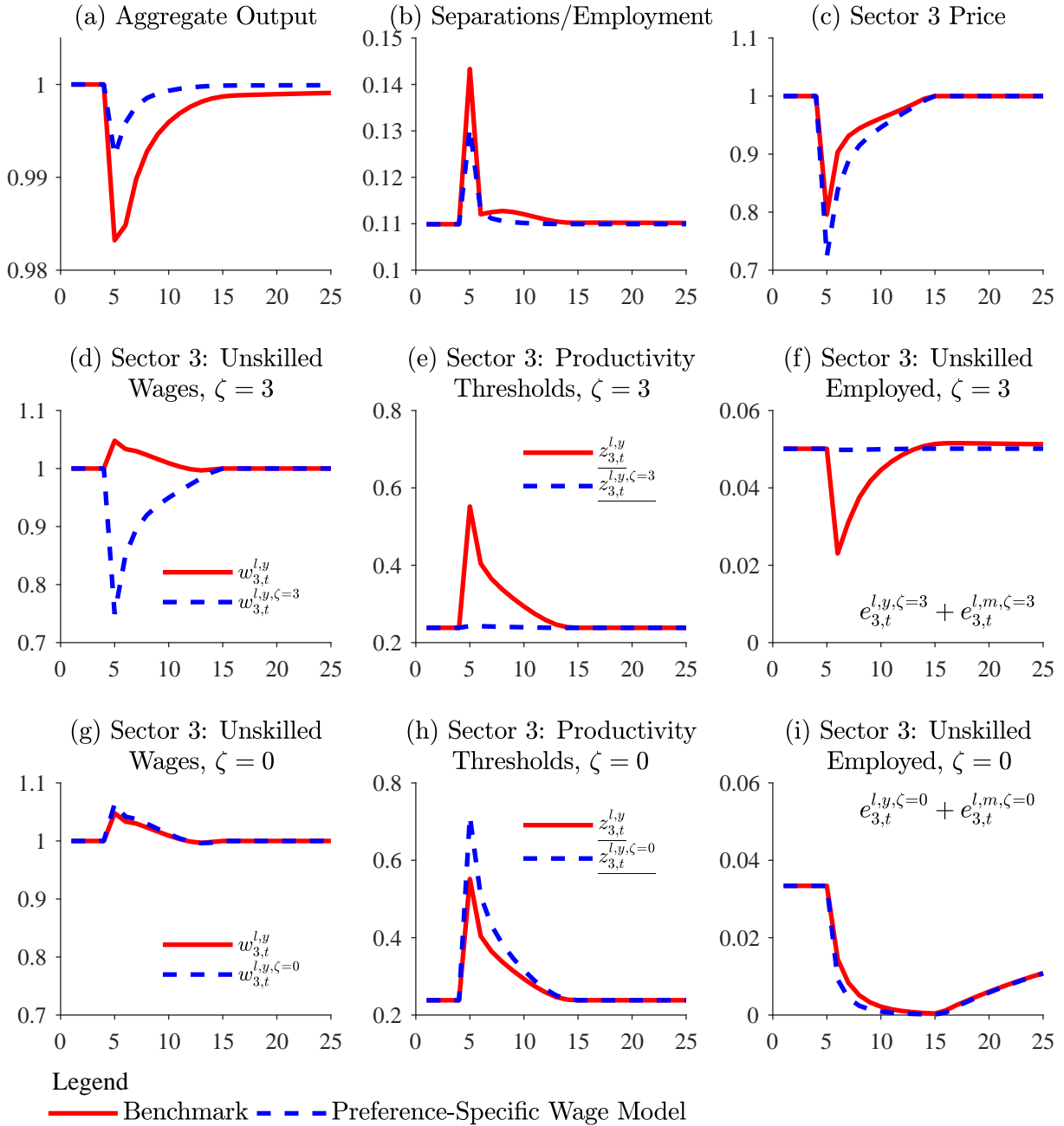
In the preference-specific wage experiments, the fraction of workers born with a sectoral preference is held fixed at the benchmark value of 0.6. To isolate the effect of some unskilled workers being unwilling to switch sectors, we examine zero sectoral preference experiments, in which all the unskilled are mobile. Not surprisingly, output losses are lower, on average 6 percent of benchmark (Table 5). Since these experiments directly remove the wage distortion, one can interpret the difference between the preference-specific wages and zero sectoral preferences as the direct contribution of immobile workers. On average, about one sixth of benchmark losses are thus directly attributable to workers' sectoral preferences. The interaction between the wage bargaining distortion and sectoral preferences accounts for about three quarters of benchmark losses.

To elucidate why the wage bargaining distortion has such a large impact, we compare the benchmark and the preference-specific wage experiments in more detail.³² With 1 percent population growth and 3 percent reallocation shock, the initial fall in output with preference-specific wages is less than half of the benchmark, and the recovery is largely complete within a year (Figure 4.a).

³¹Reparameterizing the model is not required since neither the wage distortion nor sectoral preference impact the symmetric steady state.

³²In these experiments, we keep the sectoral preference fixed at the benchmark value of 0.6.

Figure 4: BENCHMARK AND PREFERENCE-SPECIFIC WAGE MODEL



NOTES: Results are quarterly, for benchmark parameters (e.g., 1% population growth and 3% sectoral shock). Wages and sector 3's price are normalized to 1 in the steady state.

With preference-specific wages, the Nash bargaining for the unskilled with $\zeta = 3$ internalizes that their outside option is to search for a new match in sector 3 next period, $\Phi_{3,t+1}^{U,l,y,\zeta=3}$. Unlike our benchmark expression for sector 3 wages (equation 25), the wage for the unskilled with $\zeta = 3$ depends only on sector 3 prices and continuation values.

$$w_{3,t}^{l,y,\zeta=3}(z) = (1 - \gamma) b^l + \gamma p_{3,t} z s^l + \frac{\gamma p_{3,t} c^l \theta_{3,t}^{l,y,\zeta=3}}{1 - \theta_{3,t}^{l,y,\zeta=3} q(\theta_{3,t}^{l,y,\zeta=3})} - \beta (1 - \gamma) \varrho^y \left\{ \delta^y \left[\Phi_{3,t+1}^{U,h,m} - \Phi_{3,t+1}^{U,l,m,\zeta=3} \right] + (1 - \delta^y) \left[\Phi_{3,t+1}^{U,h,y} - \Phi_{3,t+1}^{U,l,y,\zeta=3} \right] \right\} \quad (26)$$

The direct impact of a lower $p_{3,t}$ in equation 26 is to push down the wage—through lower values of output and of a new match (second and third term in 26). This drives the fall in the wages for unskilled with $\zeta = 3$ relative to the increase in the benchmark (Figure 4.d). The fall in the wage offsets the lower price of output, $p_{3,t}$, so that the value of a sector 3 unskilled job filled by a worker with $\zeta = 3$, $\Phi_{3,t}^{J,l,a,\zeta=3}(z)$, varies little.³³ Since $\Phi_{3,t}^{J,l,a,\zeta=3}(\underline{z}_{3,t}^{l,a,\zeta=3}) = 0$, there is little change in the productivity threshold (Figure 4.e), and thus little change in employment of unskilled with sectoral preference (Figure 4.f). The higher unskilled employment in sector 3 results in a smaller decline in sectoral output, and a larger decline in the price compared to our benchmark economy (Figure 4.c).

The bottom row of plots in Figure 4 compare sector 3's unskilled market for mobile ($\zeta = 0$) workers with or without preference-specific wages. Since unemployed mobile workers search where the return is highest, their wage—given by the same benchmark equation 25—is similar across the two experiments (Figure 4.g). However, the larger decline in $p_{3,t}$ with preference-specific wages pushes up the productivity threshold, $\underline{z}_{3,t}^{l,a,\zeta=0}$, that solves $\Phi_{3,t}^{J,l,a,\zeta=0}(\underline{z}_{3,t}^{l,a,\zeta=0}) = 0$ (Figure 4.h). Thus, sector 3's employment of unskilled mobile workers falls faster than in the benchmark (Figure 4.i), which speeds up the transition by increasing unskilled workers and the growth of skilled workers in sector 1.

Although allowing matches to condition on workers' sectoral preferences reduces the output loss from sectoral reallocation, it increases volatility along other dimensions. Both wages and prices vary more in the preference-specific wage model than in our benchmark, which has implications for productivity thresholds and endogenous separations. Total separations are larger on impact in the benchmark than with preference-specific wages (Figure 4.b). To facilitate the reallocation of workers towards the growing sector, some matches of unskilled mobile workers are terminated endogenously. The

³³The equation for $\Phi_{3,t}^{J,l,a,\zeta=3}(z)$ is similar to that of $\Phi_{3,t}^{J,l,a}(z)$ in equation 7 except that the current and future values of a job, wages and thresholds are indexed by $\zeta = 3$.

larger fall in $p_{3,t}$ with preference-specific wages initially increases the number of $\zeta = 0$ matches destroyed, as the productivity threshold, $\frac{z_{3,t}^{l,a,\zeta=0}}{z_{3,t}^{l,a}}$, jumps higher than $\frac{z_{3,t}^{l,a}}{z_{3,t}^{l,a}}$ in the benchmark, but remains below 1. This effect is offset by a decline in separations of unskilled workers with $\zeta = 3$, as their wage falls relative to that in the benchmark.

We use the experiments in Figure 4 to decompose the contributions of skilled and unskilled workers to output losses in response to a 3 percent reallocation shock, when population grows at 1 percent per year.³⁴ Roughly three quarters of the smaller initial decline in output with preference-specific wages is due to a smaller fall in sector 3's employment of unskilled workers with a sectoral preference ($\zeta = 3$). The higher separation rate of mobile workers in sector 3 pushes up unskilled employment in sector 1, which accounts for roughly a fifth of the smaller fall in output, on impact.

Indirect effects from unskilled workers' reallocation on the stocks of skilled workers account for most of the differences in cumulative output losses between the two experiments. With preference-specific wages, unskilled employment is higher in sectors 1 and 3. Since skill acquisition is stochastic, this translates into a higher stock of skilled workers in the following periods. The higher productivity of skilled workers compared to unskilled implies that the stocks of skilled workers have a large impact on output. About 55 percent of the differences in cumulative output losses are driven by a higher stock of skilled in sector 3 under preference-specific wages, while 20 percent are due to more skilled in sector 1. Interestingly, although sector 2 was not directly impacted by the reallocation shock, mobile workers flow from sector 2 to the growing sector, which disrupts the skill distribution. More skilled in sector 2 under preference-specific wages account for 20 percent of the smaller losses relative to the benchmark.

5.3 Unemployment, Vacancies and Reallocation

We highlight several key predictions of our model for the impact of sectoral shocks on unemployment and worker reallocation. First, we show that, contrary to [Abraham and Katz \(1986\)](#), sectoral shocks can result in a rise in unemployment and a fall in vacancies. Moreover, the duration of high unemployment and low vacancies depends on

³⁴Aggregate output in the benchmark (denoted by B) and the preference-specific wage experiment (denoted by PSW) can be written as: $Y_t^e = \sum_j p_{j,t} (\tilde{Y}_{j,t}^{h,e} + \tilde{Y}_{j,t}^{l,e})$, where $e \in \{B, PSW\}$ and where h or l denote the skilled or unskilled components of output net of reallocation costs in sector j . We decompose the differences in output, $Y_t^B - Y_t^{PSW}$, in the contributions of unskilled, skilled and prices.

demographics. Second, we show that the rise in unemployment following a sectoral reallocation shock is mainly driven by workers with a sectoral preference, rather than by workers switching sectors. Third, we show that net reallocation and gross flows of workers across sectors spike as mobile ($\zeta = 0$) workers reallocate to sector 1.

In an influential paper, [Lilien \(1982\)](#) argues that the positive correlation between the cross-industry dispersion of employment growth and the unemployment rate implies that sectoral shocks contributed to high U.S. unemployment in the 1970s. [Abraham and Katz \(1986\)](#) argue that both sectoral and aggregate shocks generate the positive correlation reported by Lilien, but have opposite predictions for the correlation between unemployment and vacancies. In their view, sectoral reallocation implies a *positive* relationship between unemployment and vacancies. Since U.S. vacancies fell, while unemployment rose, [Abraham and Katz \(1986\)](#) rejected [Lilien \(1982\)](#)'s sectoral shock story.

In our benchmark economy, sectoral shocks can generate a rise in unemployment and a fall in vacancies.³⁵ The top panels of [Figure 5](#) plot the unemployment rate and the vacancy rate (defined as total vacancies divided by total employment) in our benchmark, for annual population growth rates of -1 , 1 and 3 percent. Both variables are normalized by their steady state values. In all scenarios, the initial response to the sectoral shock is a rise in the unemployment rate and a fall in the vacancy rate. Lower population growth results in longer periods of high unemployment and low vacancies.³⁶

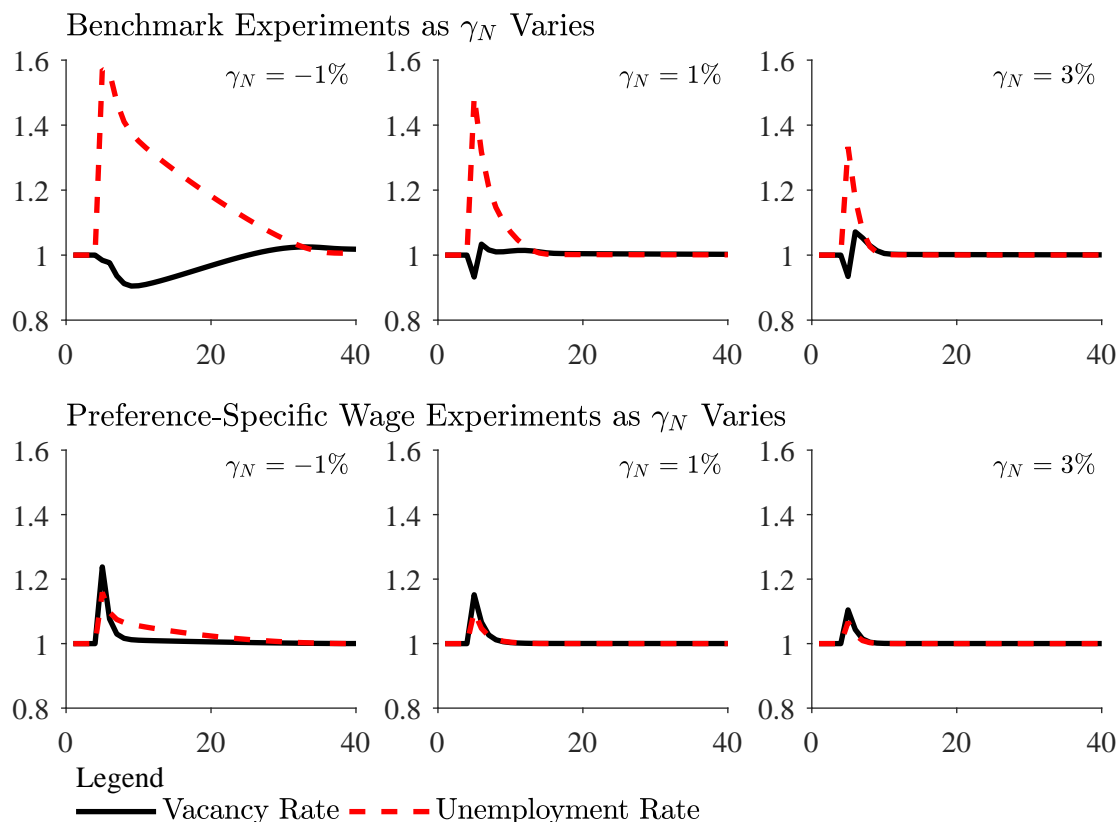
The wage bargaining distortion is crucial for our benchmark economy to deliver a fall in vacancies in response to a sectoral shock. In the shrinking sector 3, vacancies for unskilled workers plunge, since the rise in wages (discussed in [Section 5.1](#), [Figure 2.g](#)) reduces the expected surplus from posting a vacancy. In the growing sector 1, the rise in unskilled vacancies is dampened, since unemployed unskilled workers with a sectoral preference (i.e., $\zeta = 2$ or 3) continue searching for a job in their sectors. The bottom panels of [Figure 5](#) plot the unemployment and vacancy rates when wages (and vacancies) condition on unskilled workers' sectoral preference. In this case, consistent with [Abraham and Katz \(1986\)](#), sectoral reallocation shocks in our model deliver a rise in vacancies and unemployment, irrespective of demographics.³⁷

³⁵[Hosios \(1994\)](#) also shows that sectoral shocks can generate a rise in unemployment while vacancies fall. His result relies on transitory relative price shocks and temporary layoffs, whereas our permanent shift in sectoral shares is closer in spirit to [Abraham and Katz \(1986\)](#).

³⁶We define the vacancy rate using end of period employment. Our conclusions are similar if we use beginning of period employment, or simply the level of vacancies.

³⁷[Wiczer \(2015\)](#) follows [Abraham and Katz \(1986\)](#)'s view that industries (and occupations) differ in their

Figure 5: VACANCIES AND UNEMPLOYMENT IN THE BENCHMARK AND PREFERENCE-SPECIFIC WAGE MODEL AS POPULATION GROWTH RATE VARIES



NOTES: Results are quarterly. Except for γ_N , we use benchmark parameters (e.g., sectoral shock is 3%).

Murphy and Topel (1987) pioneered a different argument against the importance of sectoral shocks based on the small impact of workers who switch industries on unemployment. Using CPS data from 1968 – 1985, they find that most U.S. unemployment is accounted for by workers who transitioned between jobs in the same industry rather than by those who switched industries. Similarly, Loungani and Rogerson (1989), who use PSID data, conclude that most of the rise in U.S. unemployment over this period is accounted for by workers who do not switch industries.³⁸

sensitivity to cyclical shocks, to account for the observation that the dispersion of unemployment duration is countercyclical. Wiczer (2015)'s model shares similarities with ours, as some workers have a preference for specific occupations, and occupation-specific human capital. Hence, some workers continue to search in occupations where the job finding probability is lower than average, which leads to longer than average spells of unemployment. Switchers start with low productivity, so the surplus of their matches is smaller than for skilled workers. Thus, during recessions, the fall in vacancies for unskilled switchers is large.

³⁸Murphy and Topel (1987) define an industry switcher as one whose current 2-digit industry differs from that of the previous year. In comparison, Loungani and Rogerson (1989) exclude temporary switch-

We provide a different interpretation of these findings. In our benchmark economy, the rise in unemployment following the sectoral shock is mostly driven by workers who do not change sectors. In Table 6, we attribute changes in unemployment 1 quarter and 1 year after the shock to workers who switch the sector in which they search for a job and workers who do not. Similar to the empirical literature, we find that switchers' contribution to the rise in unemployment is modest. Instead, workers who do not search in the growing sector due to their sectoral preference (i.e., the unskilled with $\zeta = j$) account for 87 percent of the initial rise in unemployment. One year after the sectoral shock, these workers' contribution rises to 95 percent. Once again, these results are due to the wage bargaining distortion. If wages condition on workers' sectoral preferences, about 84 percent of the initial (smaller) rise in unemployment is due to mobile workers.

Table 6: BENCHMARK MODEL: UNEMPLOYMENT AFTER SECTORAL SHOCK, CONTRIBUTIONS BY SKILL AND SECTORAL PREFERENCE

	1 quarter after shock	1 year after shock
CHANGE IN UNEMPLOYMENT	3.5	1.1
CONTRIBUTIONS (IN PERCENT)		
Unskilled mobile ($\zeta = 0$)	12.3	2.8
Unskilled immobile ($\zeta = j$)	87.2	94.8
Skilled	0.6	2.5

NOTES: Results are for benchmark parameters (e.g., 1% population growth and 3% sectoral shock).

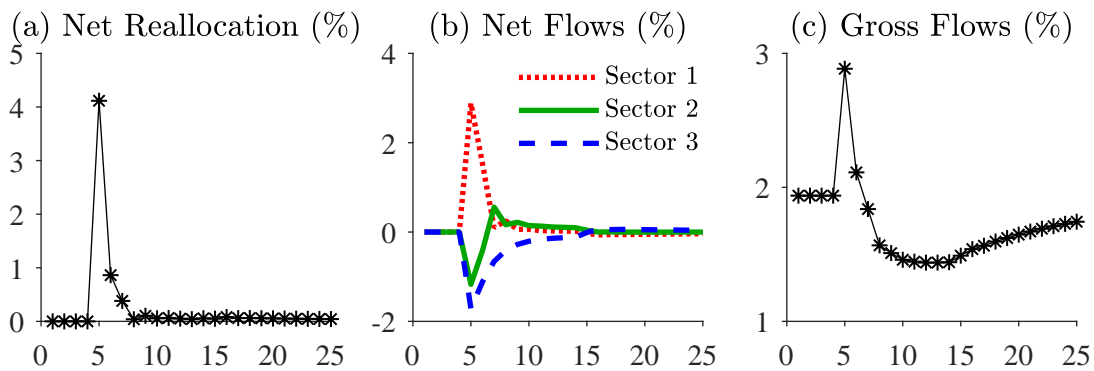
Net reallocation of workers across sectors occurs surprisingly fast in our benchmark model, returning to the steady state level of zero one year after the shock (Figure 6.a). We use a common measure of net reallocation, the sum of the absolute changes in sectoral employment shares, $\frac{1}{2} \sum_{j=1}^3 \left| \frac{E_{j,t}}{E_t} - \frac{E_{j,t-1}}{E_{t-1}} \right|$. Following the sectoral reallocation shock, the employment share in sector 1 (3) jumps (falls) by about 4 percent, while that of sector 2 remains unchanged. While measured net reallocation occurs quickly, sectoral employment levels take much longer to converge to the new steady state (Figure 2). This indicates that measures of reallocation motivated by Lilien (1982) could underestimate the persistent impacts of sectoral shocks.

The slow recovery from a sectoral shock in our model is apparent in the net flows of

 ers (i.e. individuals who return to their original industry within 2 years) and include individuals who switch industries after a long spell of unemployment.

workers switching sectors, which take considerably longer to return to zero, especially in sector 3 (Figure 6.b). When unemployed unskilled mobile workers are indifferent between searching for a job in multiple sectors, we assume they randomize so that the share of workers going to each sector equals that sector's share of employment.³⁹ In the symmetric steady state, this results in the net flow of workers switching into a sector being equal to the outflow. However, along the transition, mobile workers initially only search in sector 1, and then in sectors 1 and 2. During these periods, one sees a large net inflow into sector 1 accompanied by outflows from sectors 2 and 3. Since sector 2 is only indirectly impacted by the sectoral shock, it returns to net inflows very quickly, while sector 3 sees net outflows for prolonged periods.

Figure 6: BENCHMARK: NET SECTORAL REALLOCATION AND WORKER FLOWS



NOTES: Results are quarterly, for benchmark parameters (1% population growth and 3% sectoral shock).

The initial jump in gross reallocation across sectors is followed by a prolonged period of low gross flows. In steady state, roughly 1.9 percent of employees switch industries each quarter (with a net flow of zero). Immediately after the reallocation shock, gross flows across sectors jump as unemployed mobile workers all search for jobs in sector 1 (Figure 6.c). However, gross flows fall *below* the steady state level within 3 quarters of the shock, and remain low for over 8 years. The stock of mobile workers when the shock hits is significant—roughly 12 percent. However, the first two periods after the shock sees many mobile workers switch to working in sector 1, resulting in roughly two-thirds of mobile workers accumulating in sector 1. Since few unemployed unskilled mobile workers search for a job in sector 2 (or 3) along the transition, the flow of workers out of sector 1 remains low. Since the stock of mobile workers in sectors 2 and 3 is low along

³⁹We assume the past industry of work for newly born to be the steady state employment distribution.

the transition, there is also a small outflow from these sectors to sector 1. Somewhat paradoxically, a large sectoral shock is followed by a longer period of low gross flows.

The implications of our model for unemployment, vacancies and the reallocation of workers provide an alternative interpretation of empirical evidence. Unlike the original interpretation of sectoral shocks offered by [Lilien \(1982\)](#), that workers searching for new jobs in growing sectors drives higher unemployment, our model implies that most of the impacts are borne by workers who are not willing to search in the growing sectors. Our model illustrates the challenges of using measures of workers flows or the correlation between unemployment and vacancies to dismiss the importance of sectoral shocks.

5.4 Sensitivity Analysis

In this section, we explore the sensitivity of our results as we vary the elasticity of substitution between sectoral outputs, ρ , the share of workers with sectoral preference, $\sum_j \pi_j$, and the Nash bargaining power of workers, γ . Our analysis focuses on the benchmark model with the 3 percent reallocation shock.

Table 7 reports output losses as we vary the elasticity of substitution around our benchmark value of 0.2, while holding other parameters fixed. An elasticity of 0.1 (close to the value of 0.11 used in [Phelan and Trejos \(2000\)](#)) delivers cumulative output losses that are about a fifth larger than the benchmark. Higher elasticities of substitution deliver lower losses, as the sectoral price adjustments in response to the reallocation shock are smaller, while the sectoral output adjustments are larger: sector 3 shrinks by more and sector 1 grows by more under a Cobb-Douglas specification than the benchmark.

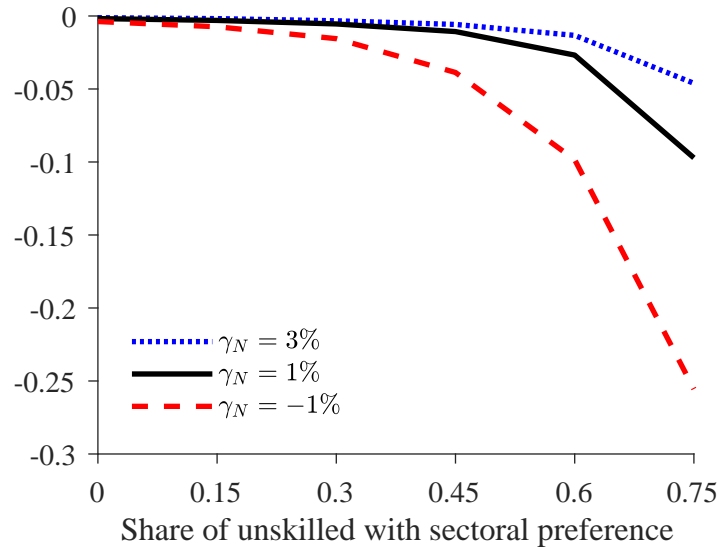
Table 7: OUTPUT LOSSES (IN PERCENT OF ANNUAL GDP) FOR DIFFERENT ELASTICITIES OF SUBSTITUTION BETWEEN SECTORAL OUTPUTS

	Elasticity of Substitution, $\frac{1}{1-\rho}$					
	0.1	0.2	0.4	0.6	0.8	1
Cumulative Output Losses	3.25	2.68	1.81	1.38	1.11	0.93

NOTES: Output losses are expressed as a fraction of annualized steady state GDP (i.e., quarterly steady state output times 4), and are cumulated over a simulation of 100 quarters. Except for ρ , we use benchmark parameters (e.g., 1% population growth and 3% sectoral shock).

The fraction of unskilled workers with a sectoral preference has a large impact on output losses. Figure 7 reports cumulative output losses as the share of workers with sectoral preference varies from 0 to 75 percent, for population growth rates of 3, 1 and -1 percent. Regardless of demographics, a more mobile workforce reduces the impact of sectoral shocks. Conversely, when few workers are mobile, output losses are large, especially at low population growth rates. For a 3 percent reallocation shock, when the sectoral preference is 75 percent, cumulative output losses are 4.6 percent, 9.7 percent or 25.6 percent of annualized steady state GDP, for the different population growth rates.

Figure 7: OUTPUT LOSSES (IN PERCENT OF ANNUAL GDP)
BY DEMOGRAPHICS AND SECTORAL PREFERENCE



NOTES: Output losses are expressed as a fraction of annualized steady state GDP (i.e., quarterly steady state output times 4), and are cumulated over a simulation of 100 quarters. Except for γ_N , we use benchmark parameters (e.g., sectoral shock is 3%).

Our results are robust to varying the Nash bargaining power of workers, γ . Since varying the value of γ impacts the productivity cut-offs, we reparameterize unemployment benefits to hold the cut-offs fixed.⁴⁰ Lowering γ from 0.6 to 0.5, or increasing it to 0.7, results in modest changes to output losses. Cumulative output losses are 2.7 percent for $\gamma = 0.5$, and 2.55 percent for $\gamma = 0.7$. These results are similar to the benchmark which delivers cumulative output losses of 2.68 percent for $\gamma = 0.6$.

⁴⁰We vary the unemployment benefit since this does not directly impact matching and hiring of new workers in the growing sector after the shock.

6 Conclusion

Population growth rates have fallen sharply in many developed countries. We show that the resulting decline in the fraction of young threatens to increase the costs of future sectoral reallocations. As population growth declines, the recovery of output and employment after a sectoral shock becomes more prolonged. Counterintuitively, these adverse effects are disproportionately concentrated on young workers, despite their relative shortage at lower population growth rates.

We interpret the markets in our model as industrial sectors. While this allows us to link our work to the debate on sectoral reallocation, one could also interpret our sectors as corresponding to occupations. To the extent that sectors differ in their mix of occupational employment, our analysis on sectoral shocks could be extended to occupational reallocations.

Our work points to promising directions of future research. We find that barriers to sectoral mobility of workers amplify the cost of sectoral reallocations. In an era of low population growth, policies that encourage mobility of workers across regions and sectors are even more important. Our results indicate that increasing worker mobility can alleviate wage bargaining distortions and lower rest unemployment. An alternative direction for future research is to investigate whether immigration policies that target skilled workers in rapidly growing industries can alleviate the costs of sectoral reallocation in a low population growth environment.

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