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by

**Simona E. Cociuba and Ananth Ramanarayanan**

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# International Risk Sharing with Endogenously Segmented Asset Markets\*

Simona E. Cociuba      Ananth Ramanarayanan

September, 2017

## Abstract

Asset price data imply a large degree of international risk sharing, while aggregate consumption data do not. We evaluate whether a model with trade in goods and endogenously segmented asset markets accounts for this puzzling discrepancy. *Active households* pay a fixed cost to transfer income into or out of assets. These households share risk within and across countries, and their marginal utility growth prices assets, so asset prices imply high international risk sharing. *Inactive households* consume current income and do not share risk, so aggregate consumption (which averages across all households) reflects lower risk sharing. Trade in goods is essential for generating these differences in the asset price-based and the consumption-based measures of risk sharing. Indeed, without trade, consumption is constrained by domestic resources and there is no international risk sharing. The calibrated model predicts risk sharing measures in line with data, and also partly resolves the Backus-Smith-Kollmann puzzle.

JEL codes: F36, F44, G15

Keywords: international risk sharing, real exchange rates, segmented asset markets, limited asset market participation, consumption-real exchange rate anomaly, Backus-Smith-Kollmann puzzle

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# 1 Introduction

How much do countries share risk through international financial markets, and how large are the gains from doing so? The answers to these questions depend on how risk sharing is measured. Brandt, Cochrane, and Santa-Clara (2006) show that measures of risk sharing based on asset price data imply significant international risk sharing, while measures based on aggregate consumption data display much less risk sharing.<sup>1</sup> As a consequence, as shown by Lewis (2000), welfare gains from risk sharing based on stock returns are higher than those based on aggregate consumption.

The discrepancy in these risk sharing measures is puzzling. In standard international macro models, consumption determines asset prices, rendering consumption-based and asset price-based calculations identical. Resolving this puzzle involves either changing the preferences used in standard models or modifying the asset market structure, as discussed by Brandt, Cochrane and Santa-Clara. In this paper, we take the latter route, and evaluate the extent to which frictions that endogenously limit participation in asset markets can account for the discrepancy between the asset price-based and consumption-based measures of international risk sharing.

We analyze a two-country model with international trade in financial assets along the lines of Alvarez, Atkeson, and Kehoe (2002) (henceforth AAK), in which households must pay a fixed cost to transfer current income into or out of interest-bearing assets. Households face idiosyncratic and aggregate income shocks, and asset markets are endogenously segmented because only a fraction of households at any point in time find it beneficial to pay the fixed cost associated with adjusting their asset holdings. We enrich the AAK model to allow for trade in goods, which is essential for generating risk sharing across countries.<sup>2</sup> Indeed, without trade, consumption is constrained by domestic resources and there is no risk sharing.

Limited asset market participation leads to differences in the asset price-based and consumption-based risk sharing measures. Households that actively adjust their asset holdings share risk among each other, both within and across countries. Since these households' marginal utility growth determines asset prices, asset prices imply a high degree of international risk sharing. On the other hand, these households account for only a (time-varying) fraction of aggregate consumption in each country, so measures of consumption risk sharing

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<sup>1</sup>In particular, Brandt, Cochrane, and Santa-Clara (2006) show that marginal utility growth (i.e. intertemporal marginal rates of substitution or discount factors) derived from stock prices is highly correlated across countries, indicating significant international risk sharing, while marginal utility growth derived from aggregate consumption is weakly correlated across countries, indicating much less risk sharing.

<sup>2</sup>AAK analyze a closed economy and a two-country model with only non-traded goods. A version of our model with only non-traded goods cannot account for the discrepancy in the risk sharing measures. For this reason, we incorporate both traded and non-traded goods.

imply a low degree of international risk sharing at the aggregate level.

We quantify this mechanism by calibrating our model to match the cross-sectional variance of household income and consumption in US data, along with the time series properties of aggregate traded and non-traded output in the US and an aggregate of 19 OECD trading partners. The model predicts a high cross-country correlation of the marginal utility growth of active households—about 0.95—and a cross-country correlation of aggregate consumption that is much lower—about 0.55. These values are in line with the empirical findings in Brandt, Cochrane, and Santa-Clara (2006). We contrast the results of our benchmark segmented markets model with two alternative environments. First, in a frictionless, complete markets model (with no asset market segmentation), the risk sharing measures are both about 0.88, reflecting relatively high risk sharing. Second, in a segmented markets model with no trade in goods, the risk sharing statistics are both about 0.17, equal to the calibrated cross-country correlation in aggregate income, reflecting no risk sharing. Therefore, both asset market segmentation and trade in goods are necessary to generate the discrepancy between consumption-based and asset price-based measures of risk sharing in our framework.

Given that our benchmark model can address the puzzling discrepancy between the two measures of risk sharing, we employ it to evaluate the welfare effects of access to international financial markets. We construct three alternate measures of welfare gains: (i) *actual gains* based on the model’s ex-ante expected utility of consumption for the representative household (since all households are ex-ante identical), (ii) *asset price-based gains* based on the utility of a hypothetical household who is active in each period and (iii) *consumption-based gains* based on the utility of a hypothetical household consuming the aggregate consumption each period. In each case, the welfare gain is measured relative to financial autarky.<sup>3</sup> The welfare gains in (ii) and (iii) are analogous to the gains calculated by Lewis (2000) from data on asset prices and aggregate consumption. We find that asset price-based welfare gains are significantly larger compared to consumption-based gains, consistent with the empirical finding in Lewis (2000). With a coefficient of relative risk aversion of two, the welfare gains measured from asset prices are three times larger, and this difference increases with higher levels of risk aversion. Moreover, we find that the asset price-based measure of welfare gains gives a more accurate picture of the actual welfare gains from access to international financial markets than the consumption-based welfare gains.

The fixed cost that limits asset market participation helps our model deliver a low correlation between the real exchange rate and the ratio of aggregate consumption across countries, partly resolving the Backus-Smith-Kollmann puzzle (Backus and Smith (1993), Kollmann

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<sup>3</sup>Section 3.4 provides details on the exact calculations of the welfare gains.

(1995)).<sup>4</sup> In the standard complete markets model, the ratio of two countries’ aggregate consumption is perfectly correlated with the real exchange rate, but in the data this correlation is close to zero (see calculations for G7 countries over the period 1960 to 2015 in Table 2). In our segmented assets markets model with traded and non-traded goods, the ratio of *active* households’ consumption is perfectly correlated with real exchange rate fluctuations, but active households only account for a fraction of aggregate consumption.<sup>5</sup> This mechanism lowers the correlation of relative aggregate consumption with the real exchange rate to 0.24. The correlation is low because traded shocks move the ratios of relative aggregate consumption and relative active consumption in opposite directions, but it remains positive because non-traded shocks move the ratios of relative consumption in the same direction.<sup>6</sup> In the other two environments we examine—the frictionless, complete markets model, or the segmented markets model with no trade in goods—the aggregate consumption-real exchange rate correlation is equal to one.<sup>7</sup>

We extend the model to include an exogenous fraction of households with no assets, and show that, when recalibrated to the same moments, varying this fraction barely changes the model’s implications for the discrepancy between the risk sharing measures and for the consumption-real exchange rate correlation. This is because a larger fraction of exogenous non-asset holders reduces the calibrated fixed cost for asset holders required to match the same overall dispersion in consumption. So while non-asset holders do not share risk, asset holders’ consumption risk sharing improves because they are more likely to be active. We use this extension to derive predictions on asset holders and non-asset holders that we compare to survey data from the US (the Consumer Expenditure Survey, or CEX) and UK (the Family Expenditure Survey, or FES). The model predicts average consumption of asset holders is more highly correlated across countries than non-asset holders’ consumption, a prediction that is borne out in the CEX and FES data.

Other papers that explain the discrepancy between consumption-based and asset price-

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<sup>4</sup>The Backus-Smith-Kollmann puzzle is also referred to in the literature as the consumption-real exchange rate anomaly.

<sup>5</sup>AAK conjecture that this mechanism can explain the Backus-Smith-Kollmann puzzle. In contrast to their paper, our model has trade in goods to allow for international risk sharing.

<sup>6</sup>The role of non-traded goods in reducing international consumption correlations has been well-studied, going back to, e.g. Stockman and Dellas (1989), Tesar (1993), and Stockman and Tesar (1995); however, while adding non-traded goods per se reduces consumption correlations, it does not account for a difference in consumption-based and asset price-based risk sharing measures or a low consumption-real exchange rate correlation in the absence of asset market frictions.

<sup>7</sup>An alternative asset market friction that has been used to explain the low consumption-real exchange rate correlation is incomplete markets (e.g., Corsetti, Dedola, and Leduc (2008)). However, Devereux, Smith, and Yetman (2012) (and references therein) show that a key prediction of an incomplete markets model—that conditional forecasts of the real exchange rate and the ratio of consumption are perfectly correlated—is not borne out in the data.

based measures of risk sharing include Colacito and Croce (2011) and Lewis and Liu (2015). In these papers, Epstein-Zin preferences and long-run consumption risk result in a separation between asset prices and contemporaneous aggregate consumption levels, which generates cross-country asset price correlations that are larger than consumption correlations. We complement these papers by evaluating how much we can explain with standard time-separable preferences and a single asset market friction—segmented markets due to fixed costs of transferring income—with otherwise complete asset markets.

Our use of an endogenously segmented asset markets model with heterogeneous households to study international risk sharing is novel.<sup>8</sup> Kollmann (2012) uses a model with an exogenous fraction of households participating in asset markets, and shows that exogenous shocks to this fraction along with shocks to investment and output are necessary to account for a low consumption-real exchange rate correlation. In contrast, in our model, the fraction of households actively participating endogenously varies over time, and we show that the endogenous movements in the fraction of inactive households and their average income actually reinforce risk sharing and raise the consumption-real exchange rate correlation. Other work that also incorporates household heterogeneity to address international risk sharing and the Backus-Smith-Kollmann puzzle includes Sungur (2004), and Kocherlakota and Pistaferri (2007).<sup>9</sup> Sungur tests the predictions of the Alvarez, Atkeson, and Kehoe (2002) model in Italian regional data and finds support for the model’s relationship between the real exchange rate across regions and active households’ consumption. Kocherlakota and Pistaferri test the implications of a model with private information using US and UK survey data and show that the ratio of higher moments of the consumption distribution across countries is linked to the real exchange rate.

The remainder of the paper is organized as follows. Section 2 presents our benchmark segmented asset markets model with traded and non-traded goods. Section 3 performs the quantitative analysis and discusses results for our benchmark model, as well as two alternative models: one with no asset market segmentation (frictionless, complete markets) and one with asset market segmentation, but no trade in goods. This section also computes welfare gains and considers the extension with non-asset holders. Section 4 concludes.

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<sup>8</sup>Endogenously segmented asset markets models have been used recently in studying the effects of monetary shocks on inflation and interest rates. In addition to Alvarez, Atkeson, and Kehoe (2002), recent examples are Alvarez, Atkeson, and Edmond (2009), Khan and Thomas (2015), and Dotsey and Guerron-Quintana (2016).

<sup>9</sup>A related set of papers characterize differences in the consumption behaviour of stockholders and non-stockholders (e.g., Vissing-Jørgensen (2002a)), or active and inactive stock market participants (e.g., Bonaparte and Cooper (2009)) in closed economy settings.

## 2 Model

We extend the two-country environment in Alvarez, Atkeson, and Kehoe (2002), which has trade in assets and non-traded goods only, to include traded goods. While non-traded goods generate real exchange rate movements that allow us to evaluate the relationship between consumption and real exchange rates, trade in goods is essential for generating risk sharing across countries. Indeed, without trade in goods, consumption is constrained by domestic resources, and there can be no risk sharing, even through trade in financial assets (Brandt, Cochrane, and Santa-Clara (2006)).

We examine an infinite horizon pure-exchange economy with three goods: one internationally traded good, and two non-traded goods.<sup>10</sup> We refer to the two countries as “home” and “foreign”, and label foreign variables with an asterisk (\*). In each country, there is a continuum of households who receive endowments of traded and non-traded goods. Each household’s endowments consist of an idiosyncratic component, which is i.i.d. across households and over time, and an aggregate component. Exogenous fluctuations in the aggregate components of endowments are the source of uncertainty in the economy.

Households value consumption of both traded goods and non-traded goods, and they can buy and sell internationally traded assets to insure against idiosyncratic and aggregate fluctuations. However, they must pay a fixed cost to transfer goods into or out of these assets. This segmentation of households into active participants and non-participants in the asset market disconnects asset prices from aggregate consumption. In Alvarez, Atkeson, and Kehoe (2002), a similar separation of goods and assets accounts is specified through a cash-in-advance restriction with a fixed cost motivated as in Baumol (1952) and Tobin (1956). We abstract from money, and simply require households to pay a fixed cost denominated in units of consumption whenever they consume more or less than their current period income. One motivation for such a cost is that there is a fixed cost to ensuring repayment of private debt, as described by Chatterjee and Corbae (1992). A fixed cost like this is also related to the stock market participation cost considered by Luttmer (1999).

### 2.1 Timing and Uncertainty

Time is discrete and infinite. At the beginning of each period  $t$ , the aggregate home and foreign endowments of traded goods,  $Y_{Tt}, Y_{Tt}^*$ , and non-traded goods,  $Y_{Nt}, Y_{Nt}^*$ , are realized, and each household receives a draw  $y_t$  of an idiosyncratic shock from a distribution with

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<sup>10</sup>We use a single traded good to focus on the difference between traded and non-traded goods, as in Tesar (1993) and Backus and Smith (1993). Adding multiple traded goods as in, for example, Stockman and Tesar (1995), would introduce an additional risk sharing channel through movements in the terms of trade (as in Cole and Obstfeld (1991)).



density function  $f$ . The idiosyncratic shock determines a household's endowments: a home household with current idiosyncratic shock  $y_t$  receives endowments  $y_t Y_{Tt}$ ,  $y_t Y_{Nt}$ , and a foreign household with shock  $y_t$  receives endowments  $y_t Y_{Tt}^*$ ,  $y_t Y_{Nt}^*$ . The mass of households in each country is normalized to 1, and the distribution of idiosyncratic endowments has mean 1, so that the aggregate endowments of the goods are in fact  $Y_{Tt}$ ,  $Y_{Nt}$ ,  $Y_{Tt}^*$ , and  $Y_{Nt}^*$ .

We refer to the aggregate *shock* in period  $t$  as the realization of the four aggregate endowments,  $s_t = (Y_{Tt}, Y_{Nt}, Y_{Tt}^*, Y_{Nt}^*)$ , and define the aggregate *state*  $s^t = (s_0, s_1, \dots, s_t)$  as the history up to date  $t$  of these shocks, with  $s_0$  given. We let  $g(s^t)$  denote the density of the aggregate state,  $s^t$ . We define  $y^t = (y_0, y_1, \dots, y_t)$  as the history of idiosyncratic shocks and, abusing notation, we let  $f(y^t)$  denote the density of the idiosyncratic state,  $y^t$ . In what follows, the argument of  $f$  will make it clear whether it refers to the density over histories or over current realizations of the idiosyncratic shock. A household's state in period  $t$  is  $(s^t, y^t)$ .

## 2.2 Households

Households in the home country have preferences given by:

$$\sum_{t=0}^{\infty} \int_{s^t} \int_{y^t} \beta^t U(C(s^t, y^t)) g(s^t) f(y^t) ds^t dy^t \quad (1)$$

where  $\beta \in (0, 1)$ ,  $U(C) = C^{1-\eta}/(1-\eta)$  with  $\eta > 0$ , and  $C(s^t, y^t)$  is the amount of a composite good consumed in state  $(s^t, y^t)$ . The composite good (equation (2)) is an aggregate of traded and non-traded consumption, i.e.,  $c_T(s^t, y^t)$  and  $c_N(s^t, y^t)$ , with constant elasticity of substitution  $\sigma > 0$  and weight  $a \in (0, 1)$  on tradables.

$$C(s^t, y^t) = \left[ a c_T(s^t, y^t)^{\frac{\sigma-1}{\sigma}} + (1-a) c_N(s^t, y^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (2)$$

We normalize the price of the traded good to 1, let  $p_N(s^t)$  be the price of the non-traded good in the home country, and let  $P(s^t)$  be the price index for one unit of home country composite consumption. Given a level  $C(s^t, y^t)$  of composite consumption and the non-traded goods price,  $p_N(s^t)$ , the demands for traded and non-traded goods, and the price index  $P(s^t)$  solve the static cost-minimization problem (P1).

$$\begin{aligned} P(s^t) C(s^t, y^t) &= \min_{c_T, c_N} \{ c_T(s^t, y^t) + p_N(s^t) c_N(s^t, y^t) \} \\ \text{s.t. } &\left[ a c_T(s^t, y^t)^{\frac{\sigma-1}{\sigma}} + (1-a) c_N(s^t, y^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \geq C(s^t, y^t) \end{aligned} \quad (\text{P1})$$

The solution to problem (P1) is given in equations (3), (4) and (5).

$$c_T(s^t, y^t) = [aP(s^t)]^\sigma C(s^t, y^t) \quad (3)$$

$$c_N(s^t, y^t) = \left[ \frac{1-a}{p_N(s^t)} P(s^t) \right]^\sigma C(s^t, y^t) \quad (4)$$

$$P(s^t) = \left[ a^\sigma + (1-a)^\sigma p_N(s^t)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (5)$$

Home households have access to a goods market account and an asset market account. The goods market account restricts a household's current consumption and savings by income and any asset market transfers, while the asset market account tracks the evolution of their asset balances. The goods market account is summarized by the budget constraint in (6).

$$P(s^t) C(s^t, y^t) \leq y_t [Y_{Tt} + p_N(s^t) Y_{Nt}] + z(s^t, y^t) \cdot [\tau(s^t, y^t) - P(s^t) \gamma] \quad (6)$$

Here  $z(s^t, y^t)$  is an indicator, equal to 1 or 0. If  $z(s^t, y^t) = 1$ , the household consumes more or less than its current income, and  $\tau(s^t, y^t)$  is the amount transferred into or out of the asset market account. If  $\tau > 0$ , the household withdraws resources from the asset market account and consumes more than current income, and if  $\tau < 0$ , the household saves some of its current income. Transferring to or from the goods market (i.e., choosing  $z(s^t, y^t) = 1$ ) requires the payment of a fixed amount  $\gamma$  of composite consumption goods out of asset balances. If  $z(s^t, y^t) = 0$ , the household consumes all of its income in the current period and doesn't change the amount of asset it holds. This is in contrast to the hand-to-mouth households considered in Kollmann (2012), who own no assets.

In the asset market, all households start in period 0 with  $b(s^0)$  initial assets. In any period, they can purchase a full set of one-period securities denominated in traded goods and with payoffs contingent on the aggregate and idiosyncratic state in the next period. These transactions are carried out with a competitive financial intermediary. Contingent on future shocks  $(s_{t+1}, y_{t+1})$  and the household's current state  $(s^t, y^t)$ , the price of a claim to one unit of traded goods is  $q(s^t, s_{t+1}, y^t, y_{t+1})$ , and a household purchases  $b(s^t, s_{t+1}, y^t, y_{t+1})$  amount of these securities. The asset market budget constraint is:

$$\begin{aligned} \int_{s_{t+1}} \int_{y_{t+1}} q(s^t, s_{t+1}, y^t, y_{t+1}) b(s^t, s_{t+1}, y^t, y_{t+1}) ds_{t+1} dy_{t+1} \\ + z(s^t, y^t) \tau(s^t, y^t) \leq b(s^t, y^t) \end{aligned} \quad (7)$$

where the current payoff from asset holdings,  $b(s^t, y^t)$ , is allocated toward purchases of new securities and transfers to the goods market account, if any.

In the foreign country, the price index  $P^*(s^t)$  and the consumption levels  $c_T^*(s^t, y^t)$  and  $c_N^*(s^t, y^t)$  are defined in the same way as in the home country, given composite consumption level  $C^*(s^t, y^t)$  and the non-traded goods price  $p_N^*(s^t)$ . A foreign household's goods market budget constraint in state  $(s^t, y^t)$  is:

$$P^*(s^t) C^*(s^t, y^t) \leq y_t [Y_{Tt}^* + p_N^*(s^t) Y_{Nt}^*] + z^*(s^t, y^t) [\tau^*(s^t, y^t) - P^*(s^t) \gamma] \quad (8)$$

while the asset market budget constraint is:

$$\begin{aligned} \int_{s_{t+1}} \int_{y_{t+1}} q(s^t, s_{t+1}, y^t, y_{t+1}) b^*(s^t, s_{t+1}, y^t, y_{t+1}) ds_{t+1} dy_{t+1} \\ + z^*(s^t, y^t) \tau^*(s^t, y^t) \leq b^*(s^t, y^t) \end{aligned} \quad (9)$$

### 2.3 Asset market

There is a world financial intermediary that buys and sells assets from households. The intermediary has no wealth of its own, so total purchases of assets from households must equal sales of assets to other households. Net revenues of the intermediary when the aggregate state is  $s^t$  are given by adding up the transactions of  $(s_{t+1}, y_{t+1})$ -contingent assets to households of all histories  $y^t$ :

$$\int_{y^t} \int_{y_{t+1}} \int_{s_{t+1}} q(s^t, s_{t+1}, y^t, y_{t+1}) [b(s^t, s_{t+1}, y^t, y_{t+1}) + b^*(s^t, s_{t+1}, y^t, y_{t+1})] f(y^t) ds_{t+1} dy_{t+1} dy^t$$

The intermediary maximizes these net revenues subject to the constraint that at all future states  $s^{t+1}$ , net payments on  $s^{t+1}$ -contingent claims must be zero. That is, adding up the payments made on  $y_{t+1}$ -contingent purchases across households that had histories  $y^t$  must equal zero.

$$\int_{y^t} \int_{y_{t+1}} [b(s^t, s_{t+1}, y^t, y_{t+1}) + b^*(s^t, s_{t+1}, y^t, y_{t+1})] f(y_{t+1}) f(y^t) dy_{t+1} dy^t = 0$$

The intermediary's problem yields the following no-arbitrage condition:

$$q(s^t, s_{t+1}, y^t, y_{t+1}) = q(s^t, s_{t+1}) f(y_{t+1}) \quad (10)$$

where  $q(s^t, s_{t+1}) > 0$  is the value of one unit of traded goods in state  $s^{t+1} = (s^t, s_{t+1})$  in units of traded goods in state  $s^t$ . This condition states that the value of one unit of traded goods for a household in state  $(s^{t+1}, y^{t+1})$  must equal the value of one unit of traded goods for any household in aggregate state  $s^{t+1}$ , weighted by the probability of receiving the idiosyncratic

shock  $y_{t+1}$  in period  $t + 1$ .

## 2.4 Market Clearing and Equilibrium

In the goods market, home households' consumption of traded goods plus foreign households' consumption of traded goods plus the traded portion of fixed costs of transferring between accounts equals the world endowment of traded goods:

$$\begin{aligned} & \int [c_T(s^t, y^t) + \gamma_T(s^t) z(s^t, y^t)] f(y^t) dy^t \\ & + \int [c_T^*(s^t, y^t) + \gamma_T^*(s^t) z^*(s^t, y^t)] f^*(y^t) dy^t \\ & = Y_{Tt} + Y_{Tt}^* \end{aligned}$$

The market clearing conditions for non-traded goods are:

$$\begin{aligned} & \int [c_N(s^t, y^t) + \gamma_N(s^t) z(s^t, y^t)] f(y^t) dy^t = Y_{Nt} \\ & \int [c_N^*(s^t, y^t) + \gamma_N^*(s^t) z(s^t, y^t)] f(y^t) dy^t = Y_{Nt}^* \end{aligned}$$

Here,  $\gamma_T(s^t) = [aP(s^t)]^\sigma \gamma$  and  $\gamma_N(s^t) = [(1-a)P(s^t)/p_N(s^t)]^\sigma \gamma$ , and analogously for the foreign country, are the cost-minimizing expenditures on traded and non-traded goods for the fixed cost  $\gamma$ .

In the asset market, at each aggregate state  $s^{t+1}$ , bond holdings summed across all households equal zero:

$$\int_{y^t} \int_{y_{t+1}} [b(s^t, s_{t+1}, y^t, y_{t+1}) + b^*(s^t, s_{t+1}, y^t, y_{t+1})] f(y_{t+1}) f(y^t) dy_{t+1} dy^t = 0$$

An equilibrium consists of goods prices and asset prices along with consumption quantities and asset holdings that solve households' problems and the financial intermediary's problem taking prices as given, and that satisfy the market clearing conditions.

## 2.5 Characterizing Equilibrium

We follow a procedure similar to Alvarez, Atkeson, and Kehoe (2002) to show that an equilibrium is characterized by a few simple, static conditions determining consumption allocations and asset market participation decisions. The set of households that is active in asset markets (i.e., those for whom  $z(s^t, y^t) = 1$ ) is characterized by a static threshold rule:

households with a current idiosyncratic income shock in a certain range transfer, and others do not. Active households pool their income within a period, and have equal consumption, while inactive households consume the value of their income.

With complete asset markets, a household's sequence of asset market budget constraints collapses to a single date-0 budget. Let  $Q(s^t) = q(s^0, s_1) q(s^1, s_2) \cdots q(s^{t-1}, s_t)$  denote the price of traded goods at state  $s^t$  in terms of traded goods at date 0. Using the no-arbitrage condition (10), the sequence of budget constraints for home households (7) can be written as:

$$\sum_{t=0}^{\infty} \int_{s^t} \int_{y^t} Q(s^t) f(y^t) z(s^t, y^t) \tau(s^t, y^t) ds^t dy^t \leq b(s^0) \quad (11)$$

The household's problem is then to choose consumption,  $C(s^t, y^t)$ , transfer decisions  $z(s^t, y^t)$ , and transfers  $\tau(s^t, y^t)$  to maximize expected utility (1) subject to the date-0 budget constraint (11) and the goods market budget constraint (6).

The first order conditions for  $C(s^t, y^t)$  and  $\tau(s^t, y^t)$ , in states for which  $z(s^t, y^t) = 1$ , yield:

$$\beta^t U'(C(s^t, y^t)) g(s^t) = P(s^t) \lambda Q(s^t) \quad (12)$$

where  $\lambda$  is the multiplier on the date-0 budget constraint. Equation (12) shows that  $C(s^t, y^t)$  is independent of  $y^t$  if  $z(s^t, y^t) = 1$ . That is, idiosyncratic risk is pooled among all active households, and they all consume the same level. We denote this consumption level  $C_A(s^t)$ , for active households' consumption.

Now, we consider the choice of whether to transfer into or out of the asset market account. We know that if  $z(s^t, y^t) = 1$ , then  $C(s^t, y^t) = C_A(s^t)$  and using the goods market budget constraint (8), the amount transferred is:  $\tau(s^t, y^t) = P(s^t) [C_A(s^t) + \gamma] - y_t [Y_{Tt} + p_N(s^t) Y_{Nt}]$ . We plug this into the date-0 budget constraint (11) and write the household's problem as:

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \int_{s^t} \int_{y^t} \beta^t [z(s^t, y^t) U(C_A(s^t)) + (1 - z(s^t, y^t)) U(C(s^t, y^t))] g(s^t) f(y^t) ds^t dy^t \\ & \text{subject to:} \\ & \sum_{t=0}^{\infty} \int_{s^t} \int_{y^t} Q(s^t) f(y^t) z(s^t, y^t) \{P(s^t) [C_A(s^t) + \gamma] - y_t [Y_{Tt} + p_N(s^t) Y_{Nt}]\} ds^t dy^t \leq b(s^0) \end{aligned}$$

If we consider the Lagrangian of this problem (again with multiplier  $\lambda$  on the date-0 budget

constraint), the value in state  $(s^t, y^t)$  of setting  $z(s^t, y^t) = 1$  is given in (13).

$$\beta^t U(C_A(s^t)) g(s^t) f(y^t) - \lambda Q(s^t) f(y^t) \{P(s^t) [C_A(s^t) + \gamma] - y_t [Y_{Tt} + p_N(s^t) Y_{Nt}]\} \quad (13)$$

And the value of setting  $z(s^t, y^t) = 0$ , using the fact that  $C(s^t, y^t) = \frac{y_t(Y_{Tt} + p_N(s^t) Y_{Nt})}{P(s^t)}$ , is:

$$\beta^t U\left(\frac{y_t(Y_{Tt} + p_N(s^t) Y_{Nt})}{P(s^t)}\right) g(s^t) f(y^t) \quad (14)$$

Lastly, the value of  $\lambda$  is given by the first order condition when  $z(s^t, y^t) = 1$ :

$$\lambda = \frac{\beta^t U'(C_A(s^t)) g(s^t)}{P(s^t) Q(s^t)} \quad (15)$$

So the net gain of setting  $z(s^t, y^t) = 1$  versus setting  $z(s^t, y^t) = 0$ —the difference between expressions (13) and (14)—is positive whenever:

$$U(C_A(s^t)) - U\left(\frac{y_t(Y_{Tt} + p_N(s^t) Y_{Nt})}{P(s^t)}\right) - \frac{U'(C_A(s^t))}{P(s^t)} \{P(s^t) [C_A(s^t) + \gamma] - y_t [Y_{Tt} + p_N(s^t) Y_{Nt}]\} > 0 \quad (16)$$

The first two terms in (16) give the increase in consumption for a household in state  $(s^t, y^t)$  that switches from being inactive to being active. The third term gives the net cost of the change in asset balances necessary to get to the active consumption level  $C_A(s^t)$ : an active household increases or reduces asset balances, which has an effect on future lifetime utility.

To characterize the transfer decision, we define the net gain from transferring as a function of the household's idiosyncratic shock  $y$ :

$$h(y; C_A, Y_T, Y_N, p_N, P) = U(C_A) - U\left(\frac{y(Y_T + p_N Y_N)}{P}\right) - \frac{U'(C_A)}{P} [P(C_A + \gamma) - y(Y_T + p_N Y_N)]$$

It is straightforward to verify that  $h$  has a minimum when  $y = \frac{PC_A}{Y_T + p_N Y_N}$ , is decreasing for  $y < \frac{PC_A}{Y_T + p_N Y_N}$  and increasing for  $y > \frac{PC_A}{Y_T + p_N Y_N}$ , and is convex. For CRRA utility,  $U(C) = C^{1-\eta}/(1-\eta)$  with  $\eta > 0$ ,  $\lim_{y \rightarrow 0} h = \lim_{y \rightarrow \infty} h = \infty$ , so that  $h$  is U-shaped, with two zeros. We refer to the two zeros of  $h(\cdot; C_A(s^t), Y_T(s^t), Y_N(s^t), p_N(s^t), P(s^t))$  as  $y_L(s^t)$  and  $y_H(s^t)$ , with  $y_L < y_H$ . For households with  $y_t \in [y_L(s^t), y_H(s^t)]$ , the cost of transferring outweighs the benefit, so they consume their current income in period  $t$ . For households with

$y_t < y_L(s^t)$  or  $y_t > y_H(s^t)$ , the benefit of being active and consuming  $C_A(s^t)$  outweighs the cost.

The characterization of this decision is analogous in the foreign country, where active households consume  $C_A^*(s^t)$ . Combining the first order condition (15) with its foreign analogue yields the following risk sharing condition:

$$\frac{P^*(s^t)}{P(s^t)} = \frac{\lambda^* U'(C_A(s^t))}{\lambda U'(C_A^*(s^t))} \quad (17)$$

This condition relates the ratio of marginal utilities to the real exchange rate, i.e., the ratio of consumption price indices in the two countries. The marginal utility of home country active households relative foreign country active households' rises in proportion to the appreciation of the home real exchange rate. Active households therefore share the risk associated with national endowment shocks internationally. In addition, in each country, equation (15) implies that the stochastic discount factor that prices assets is determined by the consumption of households who are active in different periods:

$$q(s^t, s_{t+1}) = \beta g(s^{t+1}|s^t) \left( \frac{C_A(s^{t+1})}{C_A(s^t)} \right)^{-\eta} \frac{P(s^t)}{P(s^{t+1})} = \beta g(s^{t+1}|s^t) \left( \frac{C_A^*(s^{t+1})}{C_A^*(s^t)} \right)^{-\eta} \frac{P^*(s^t)}{P^*(s^{t+1})}$$

In this way, international risk sharing as measured by active households' consumption or by asset prices will appear high. However, active households' consumption is a time-varying fraction of aggregate consumption in both countries. Aggregate consumption in the home country is given by:

$$C(s^t) = \frac{Y_T(s^t) + p_N(s^t) Y_N(s^t)}{P(s^t)} \int_{y_L(s^t)}^{y_H(s^t)} y f(y) dy + [F(y_L(s^t)) + 1 - F(y_H(s^t))] C_A(s^t)$$

where  $F$  is the cdf associated with the density  $f$ . The time-varying relationship between active and aggregate consumption implies that there is potential for high risk sharing among active households along with a low degree of risk sharing reflected in aggregate consumption. In the next section, we quantify how large is the difference in risk sharing measures and illustrate how risk sharing among active households is transmitted to aggregate consumption.

An equilibrium allocation is characterized by active consumption levels and cutoffs determining the set of active households in each country, along with the implied consumption

levels for traded and non-traded goods. The market clearing conditions can be written as:

$$\begin{aligned}
& Y_{Tt} + Y_{Tt}^* \tag{18} \\
&= \int_{y_L(s^t)}^{y_H(s^t)} c_T(s^t, y^t) f(y) dy + [F(y_L(s^t)) + 1 - F(y_H(s^t))] (c_{TA}(s^t) + \gamma_T(s^t)) \\
&\quad + \int_{y_L^*(s^t)}^{y_H^*(s^t)} c_T^*(s^t, y^t) f(y) dy + [F(y_L^*(s^t)) + 1 - F(y_H^*(s^t))] (c_{TA}^*(s^t) + \gamma_T^*(s^t)) \\
& Y_{Nt} = \int_{y_L(s^t)}^{y_H(s^t)} c_N(s^t, y^t) f(y) dy + [F(y_L(s^t)) + 1 - F(y_H(s^t))] (c_{NA}(s^t) + \gamma_N(s^t)) \\
& Y_{Nt}^* = \int_{y_L^*(s^t)}^{y_H^*(s^t)} c_N^*(s^t, y^t) f(y) dy + [F(y_L^*(s^t)) + 1 - F(y_H^*(s^t))] (c_{NA}^*(s^t) + \gamma_N^*(s^t))
\end{aligned}$$

where  $c_{TA}(s^t)$ ,  $c_{NA}(s^t)$ ,  $c_N(s^t, y^t)$ , and  $c_T(s^t, y^t)$  (and their foreign analogues) follow from the demand functions in (3) and (4).

All equilibrium variables depend only on the current realization of  $s_t = (Y_{Tt}, Y_{Nt}, Y_{Tt}^*, Y_{Nt}^*)$  and not on its history. For each  $s_t$ , we solve the three market clearing conditions along with the risk sharing condition (17) and the conditions  $h(y_L(s^t)) = h(y_H(s^t)) = h(y_L^*(s^t)) = h(y_H^*(s^t)) = 0$  for the active consumption levels, the thresholds for households to make transfers, and the equilibrium prices of non-traded goods,  $p_N(s^t)$  and  $p_N^*(s^t)$ . We solve for an equilibrium in which all home and foreign households are identical in period 0, so that  $\lambda = \lambda^*$  in (17).

### 3 Quantitative Analysis

We parameterize and simulate the endogenous segmented markets model and compare its predictions to a variant with no trade in goods and a frictionless, complete markets model. We show that only the model with trade in goods and segmented asset markets delivers different consumption-based and asset price-based risk sharing measures. Section 3.1 describes the parameters used in our quantitative experiments. Section 3.2 discusses the main results on the asset price-based and the consumption-based risk sharing measures and the Backus-Smith-Kollmann puzzle. We illustrate the risk sharing mechanisms in our model in Section 3.3. We then compute welfare gains in Section 3.4 and consider an extension with non-asset holders and suggestive evidence from micro data in Section 3.5.



### 3.1 Parameterization

A model period corresponds to one quarter. Since we focus on evaluating the implications of an asset market friction, we set the parameters governing preferences to values commonly used in international macro models. We set the discount factor  $\beta = 0.99$  and the coefficient of relative risk aversion  $\eta = 2$ . We set the elasticity of substitution  $\sigma$  between traded and non-traded goods to 0.5, and we set the share  $a$  on traded goods in consumption so that the fraction of expenditures on traded goods is 50%. These are both close to the estimates in Stockman and Tesar (1995).

We choose the distribution of idiosyncratic income shocks and the fixed cost of transferring income to match statistics on income and consumption inequality in the US. Since our model assumes a time-invariant cross-sectional variance of income, we pick average measures of inequality in the US Consumer Expenditure Survey (CEX) over 1980Q1 – 2003Q4. Using data from the CEX from Heathcote, Perri, and Violante (2010), we estimate residual variances of quarterly income and consumption unexplained by household characteristics. We regress income and consumption on the following characteristics of the reference person: sex, race, education, experience (proxied by age), interaction terms between experience and education, and dummies for region of residence, following Krueger and Perri (2006). From 1980Q1 to 2003Q4, these characteristics explain, on average, about 23 percent of the cross-sectional variance of income and consumption. The variance of the log residual income is 0.37, so we choose the distribution of idiosyncratic shocks in the model to be lognormal with mean 1 and variance of log income of 0.37.

Varying the fixed cost  $\gamma$  allows us to match a given cross-sectional variance of consumption: an arbitrarily low value of  $\gamma$  implies that all households are active, and hence the variance of consumption is zero, while an arbitrarily high value of  $\gamma$  means that no households are active, so that the variance of consumption equals the variance of income. In the CEX, the variance of consumption unexplained by household characteristics is, on average, 0.23. The calibrated value of  $\gamma$  implies that about 7.9% of households are active on average per quarter, so that about 28% of households are active each year.<sup>11</sup> Vissing-Jørgensen (2002b) calculates that between 29% and 53% of US households adjust their holdings of risky assets each year, so our number is at the low end of this range. We experiment with an alternative calibration in Section 3.5 below, choosing  $\gamma$  so that on average 40% of households are active each year.

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<sup>11</sup>A household is inactive for one year with probability  $0.921^4$ , so it is active at least once in the year with probability  $1 - .921^4 = 0.28$ .

We assume the stochastic process of shocks is given by:

$$\log \begin{bmatrix} Y_{Tt+1} \\ Y_{Nt+1} \\ Y_{Tt+1}^* \\ Y_{Nt+1}^* \end{bmatrix} = \begin{bmatrix} \rho_T \\ \rho_N \\ \rho_T \\ \rho_N \end{bmatrix} \log \begin{bmatrix} Y_{Tt} \\ Y_{Nt} \\ Y_{Tt}^* \\ Y_{Nt}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{Tt+1} \\ \varepsilon_{Nt+1} \\ \varepsilon_{Tt+1}^* \\ \varepsilon_{Nt+1}^* \end{bmatrix}$$

where the persistence parameters,  $\rho_T$  and  $\rho_N$ , and the covariance matrix of the  $\varepsilon$ 's are chosen to match second moments in growth rates of the US and a trade-weighted aggregate of 19 OECD countries. We use GDP of the manufacturing, mining, agriculture, and utilities sectors as a measure of the traded endowment  $Y_T$ , and the remainder (services and construction) as a measure of the non-traded endowment  $Y_N$ . These are annual data from the OECD STAN database for 1978 – 2008. We calculate annual parameters and then adjust the autocorrelations and standard deviations to make them quarterly following Edmond and Veldkamp (2009). We calculate annual persistence parameters  $\rho_T^A = 0.49$ , and  $\rho_N^A = 0.63$ . The annual standard deviation of growth rates of traded output is 3.6 percent, and of non-traded output is 1.6 percent. The cross-country correlations are 0.25 for  $Y_T$  and 0.23 for  $Y_N$ . The within-country correlation between  $Y_T$  and  $Y_N$  is 0.60, and the correlation between traded output in one country and non-traded output in the other is 0.01. For our quarterly model, we use  $\rho_T = (\rho_T^A)^{1/4}$  and  $\rho_N = (\rho_N^A)^{1/4}$  as the persistence parameters, and we multiply the standard deviations of the traded and non-traded shocks by the factors  $(1 + \rho_T + \rho_T^2 + \rho_T^3)^{-1}$  and  $(1 + \rho_N + \rho_N^2 + \rho_N^3)^{-1}$ , respectively.

### 3.2 Implications for International Risk Sharing

We show that the model generates high risk sharing when measured using asset prices and lower risk sharing when measured using aggregate consumption. In addition, the correlation between the real exchange rate and the ratio of consumption across countries is low.

Table 1 presents results for our benchmark model, and for two alternative asset market structures: a frictionless, complete markets model with all households active, and a segmented markets model with no trade in goods. We report risk sharing measures based on all households' consumption and on active households' consumption. In addition to the cross-country correlations of consumption and of intertemporal marginal rates of substitution, we report a risk sharing index developed by Brandt, Cochrane, and Santa-Clara (2006) given in equation (19) and labeled BCS risk sharing index in the table.

$$1 - \frac{\text{var}(m_{t,t+1} - m_{t,t+1}^*)}{\text{var}(m_{t,t+1}) + \text{var}(m_{t,t+1}^*)} \quad (19)$$

In equation (19),  $m_{t,t+1}$  is the intertemporal marginal rate of substitution (MRS) for active home households (i.e.,  $\beta^t (C_{At+1}/C_{At})^{-\eta}$ ), while  $m_{t,t+1}^*$  is the analogue for the foreign country. The numerator of the fraction in equation (19) measures—via differences in the MRS—how much risk is not shared across countries, while the denominator measures how much risk there is to share. The index lies between  $-1$  and  $1$ , and measures how far countries are from perfect risk sharing, corresponding to an index value of  $1$  when  $m_{t,t+1} = m_{t,t+1}^*$ .

The significance of constructing statistics with two different measures of consumption (active vs. all households) is that active households price assets, so their intertemporal marginal rate of substitution is reflected in asset prices. Hence, the analogue of Brandt, Cochrane, and Santa-Clara (2006)’s construction of stochastic discount factors from stock market data in our model is to use the marginal rate of substitution of active households. Statistics based on aggregate consumption in our model correspond to data measures of risk sharing based on aggregate consumption.

Table 1: Results from benchmark model and alternative environments

	Segmented markets		
	Benchmark model	No trade in goods	Frictionless complete markets
STANDARD DEVIATION (%)			
Real income	1.39	0.95	1.39
Real exchange rate	0.83	2.61	1.25
INTERNATIONAL CORRELATIONS			
Real income	0.17	0.17	0.17
<b>All households’:</b>			
consumption	0.53	0.17	0.84
intertemporal MRS	0.56	0.17	0.88
BCS risk sharing index	0.55	0.15	0.88
<b>Active households’:</b>			
consumption	0.93	0.17	0.84
intertemporal MRS	0.95	0.17	0.88
BCS risk sharing index	0.95	0.15	0.88
CORRELATION BETWEEN $\frac{C}{C^*}$ AND $\frac{P^*}{P}$	0.24	1.00	1.00

Note: The table reports averages of standard deviations and correlations of logged series over 1000 simulations of 400 quarters.

The first column of Table 1 shows that the model generates a substantial difference between asset price-based risk sharing measured from active households’ consumption and the measures based on aggregate consumption. For example, the correlation of aggregate

consumption is 0.53, while the correlation of active households' consumption is 0.93. The correlations of the intertemporal MRS and the BCS risk sharing index display a similar difference. Active households in each country trade a complete set of state-contingent assets, and thus are able to share country-specific risk with active households in the other country. Using data from the US, UK, and Japan, Brandt, Cochrane, and Santa-Clara (2006) show that their risk sharing index based on asset prices is around 0.99, while the analogue of the index constructed from aggregate consumption data is in the range 0.3 – 0.4, and we show in Table 2 that the average bilateral correlation of consumption across G7 countries is 0.28. Thus, our model is successful in generating high risk sharing based on asset prices and low risk sharing based on aggregate consumption.

Table 2: Aggregate consumption correlations and consumption-real exchange rate correlations in G7 data

	AGGREGATE CONSUMPTION CORRELATIONS, $\text{CORR}(C, C^*)$					
	France	Germany	Italy	UK	Canada	Japan
US	0.41	0.22	0.35	0.27	0.60	0.29
France		0.37	0.46	0.24	0.34	0.32
Germany			0.26	0.17	0.23	0.20
Italy				0.16	0.19	0.26
UK					0.17	0.11
Canada						0.23
Average consumption correlation = 0.28						
	CONSUMPTION-REAL EXCHANGE RATE CORRELATIONS, $\text{CORR}(\frac{C}{C^*}, RER)$					
	France	Germany	Italy	U.K.	Canada	Japan
US	-0.09	-0.06	-0.03	-0.03	-0.10	0.09
France		-0.04	0.19	0	-0.05	0.02
Germany			0.13	0.03	-0.07	0.04
Italy				-0.05	-0.06	0.06
UK					-0.14	0.02
Canada						-0.06
Average consumption-real exchange rate correlation = -0.01						

Note: The G7 data is from the OECD and spans 1960Q1 to 2015Q4. The consumption data are private final consumption expenditures from the OECD Quarterly National Accounts. The consumption-real exchange rate correlations are between bilateral real exchange rates and bilateral relative consumption for the seven countries. Real exchange rates (RER) are computed using nominal exchange rates and consumer price indices from OECD Main Economic Indicators. Consumption and real exchange rates are logged and first differenced.

Our model also partly resolves the Backus-Smith-Kollmann puzzle as the correlation between the real exchange rate and the cross-country ratio of aggregate consumption in the benchmark model is 0.24, significantly lower than 1 (see the last row of Table 1), but still above the average correlation of zero observed in G7 data (see Table 2). For consumption among active households, a high correlation between relative consumption and the real exchange rate is the result of equation (17). However, even for households that do not participate in asset markets, and therefore for whom an analogue of (17) does not hold, some degree of risk sharing is achieved; indeed, the table shows that aggregate consumption is significantly more correlated than output. In the next subsection, we explore the mechanisms that transmit international risk sharing to aggregate consumption and that explain why the ratio of aggregate consumption is positively correlated with the real exchange rate.

In the second column of Table 1, we consider an alternative goods market structure, with a single good in each country, and no international trade in goods. This is the goods market structure of the original model in Alvarez, Atkeson, and Kehoe (2002). With no trade in goods, there is essentially no risk sharing, even among active households, because consumption of all households within a country is constrained by domestic endowments.

In the last column of Table 1, we solve the model assuming that  $\gamma = 0$ . This model is consistent with a high degree of risk sharing implied by asset prices in the data, but inconsistent with a low degree of risk sharing implied by aggregate consumption data. Since all households are active in this model, the risk sharing equation (17) applies to aggregate consumption, and the correlation between the real exchange rate and the ratio of aggregate consumption across countries is equal to one.

### 3.3 Aggregate Consumption Risk Sharing: Mechanism

We illustrate the effects of endogenous asset market segmentation on risk sharing by isolating the impact of traded and non-traded shocks. The quantitative results in Table 1 show that active consumption is highly correlated across countries, and equation (17) implies that the ratio of active households' consumption across countries is perfectly correlated with the real exchange rate. In this section, we illustrate how traded and non-traded shocks interact with endogenous asset market segmentation to generate positive comovement in aggregate consumption and a low, but positive, consumption-real exchange rate correlation.

Aggregate consumption is defined as:

$$C(s^t) = \int_{y^t} C(s^t, y^t) f(y^t) dy^t$$

Using the equilibrium decision rules, for consumption in period  $t$ , we can write:

$$\begin{aligned} C_t &= m_{At}C_{At} + \int_{y_{Lt}}^{y_{Ht}} y \frac{Y_{Tt} + p_{Nt}Y_{Nt}}{P_t} f(y) dy \\ C_t &= m_{At}C_{At} + y_{It} \frac{Y_{Tt} + p_{Nt}Y_{Nt}}{P_t} \end{aligned} \quad (20)$$

where we define  $y_{It} = \int_{y_{Lt}}^{y_{Ht}} y f(y) dy$  as the fraction of income held by inactive households in period  $t$ , and  $m_{At} = F(y_{Lt}) + 1 - F(y_{Ht})$  as the mass of active households in period  $t$ . In the appendix, we show that, to a first-order approximation near a symmetric steady state, log changes in relative consumption across countries are given by:

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* &= \phi_I \left[ \omega_T(\hat{Y}_{Tt} - \hat{Y}_{Tt}^*) + \omega_N(\hat{Y}_{Nt} - \hat{Y}_{Nt}^*) + \hat{y}_{It} - \hat{y}_{It}^* \right] \\ &\quad + \phi_A \left[ \frac{1}{\eta}(\hat{P}_t^* - \hat{P}_t) + \hat{m}_{At} - \hat{m}_{At}^* \right] \end{aligned} \quad (21)$$

where all variables with hats are log deviations from the steady state, e.g.,  $\hat{C}_t = \log \frac{C_t}{\bar{C}}$ . We refer to the left hand side as relative aggregate consumption growth. Here,  $\phi_A = \frac{\bar{m}_A \bar{C}_A}{\bar{C}}$  is active households' steady state share of consumption and  $\phi_I = \frac{\bar{Y}_T + \bar{p}_N \bar{Y}_N}{\bar{P} \bar{C}} \bar{y}_I = 1 - \phi_A$  is inactive households' steady state share of consumption. The constant  $\omega_T = \frac{\bar{Y}_T}{\bar{Y}_T + \bar{p}_N \bar{Y}_N}$  is the steady state share of traded output and  $\omega_N = \frac{\bar{p}_N \bar{Y}_N}{\bar{Y}_T + \bar{p}_N \bar{Y}_N}$  is the steady state share of non-traded output. We use equation (21) to illustrate the effects of shocks to traded and non-traded output on relative aggregate consumption growth. The first term multiplied by  $\phi_I$  is the contribution of inactive households' consumption decisions to relative aggregate consumption growth. If all households were always inactive,  $\phi_I = 1$  and  $\hat{y}_{It} = \hat{y}_{It}^* = 0$ , and aggregate relative consumption growth would simply reflect the difference in traded and non-traded income growth, weighted by the appropriate consumption expenditure shares,  $\hat{C}_t - \hat{C}_t^* = \omega_T(\hat{Y}_{Tt} - \hat{Y}_{Tt}^*) + \omega_N(\hat{Y}_{Nt} - \hat{Y}_{Nt}^*)$ . On the other extreme, if all households were always active,  $\phi_A = 1$ , and  $\hat{m}_{At} = \hat{m}_{At}^* = 0$ , then aggregate relative consumption growth would be perfectly correlated with the exchange rate, reflecting perfect cross-country risk sharing,  $\hat{C}_t - \hat{C}_t^* = \frac{1}{\eta}(\hat{P}_t^* - \hat{P}_t)$ .

Endogenous asset market segmentation has two effects on consumption risk sharing relative to either of these two extremes. First, aggregate relative consumption growth is a weighted average of contributions from active and inactive households. Second, the movements in the fraction of active households have additional effects that reinforce risk sharing. We illustrate these effects through impulse responses to 1% shocks to home country traded and non-traded output, respectively. The impulse responses are plotted in Figures 1 and 2.

First, consider the responses to a positive shock to home country traded output,  $Y_T$ , in

Figure 1. The upper-left panel shows home country active and aggregate consumption,  $C_{At}$  and  $C_t$ . In response to the shock to home country traded output, all households receive higher income, but active households smooth out some of the shock by saving, so their consumption rises less than aggregate consumption. In the foreign country, active households' consumption increases by borrowing from home households, and inactive households' consumption doesn't move, so that aggregate consumption rises less than active households' consumption (upper-right panel). The important feature of these impulse responses is that relative aggregate consumption growth and relative active consumption growth move in opposite directions (lower-left panel). The relative growth of active households' consumption across countries satisfies the risk sharing condition  $\hat{C}_{At} - \hat{C}_{At}^* = \frac{1}{\eta}(\hat{P}_t^* - \hat{P}_t)$ . Since home non-traded consumption is complementary with traded consumption, the home price of non-traded goods (and therefore the price index for consumption) rises, so that  $P_t^* - P_t$  falls, and  $C_{At}^*$  rises more than  $C_{At}$ . By contrast,  $C_t$  rises more than  $C_t^*$ , since inactive foreign households' consumption doesn't change. Therefore, a shock to traded output generates a negative correlation in relative aggregate consumption growth and the real exchange rate,  $\text{corr}\left(\frac{C}{C^*}, \frac{P^*}{P}\right) < 0$ , but cross-country comovement in aggregate consumption that is similar to the comovement of active consumption.

In the lower-right panel of Figure 1, we examine the contribution of changes in the set of active and inactive households. The fraction of households active in the foreign country rises more than in the home country, because foreign households have the most to gain from becoming active and taking advantage of the shock to home country output. In addition, the income of inactive households goes down in the home country relative to foreign country, because high-income domestic households who want to save their higher income become active, while poor foreign households become active to borrow. These two movements reinforce the risk sharing apparent in aggregate consumption, in the sense that the relative movements in the fraction active and in inactive households' income shares go in the same direction as relative active consumption, pulling relative aggregate consumption in the same direction as relative active consumption. Overall however, the impact of a shock to traded income on the consumption of inactive households dominates these risk sharing effects: a large fraction of home households must consume their higher income, at the same time that the relative price of non-traded goods is rising, so  $\hat{C}_t - \hat{C}_t^*$  rises while  $\hat{P}_t^* - \hat{P}_t$  falls, inducing a negative correlation between relative consumption and the real exchange rate.

Next, in Figure 2 we plot the responses to a positive shock to home country non-traded output,  $Y_N$ . The mechanisms described above work differently in response to a non-traded shock. In the home country, both active and inactive households increase their consumption, but the movements are similar. Active households would like to smooth by lending to foreign

households, but because the increased endowment is not actually traded, it must all be consumed domestically. The price of non-traded goods in the home country therefore falls, so that the increased  $Y_N$  is all consumed domestically, and active and aggregate consumption move closely together (upper-left panel). Foreign consumption barely moves (upper-right panel), so that relative aggregate and relative active consumption growth is positive, at the same time the real exchange rate goes up (lower-left panel). Therefore, a shock to non-traded output generates a positive correlation between relative aggregate consumption growth and the real exchange rate, but very little comovement in aggregate consumption measures across countries. Finally, in response to the shock to  $Y_N$ , there is a larger incentive to become active in the home country relative to the foreign country, since  $C_A$  rises much more than  $C_A^*$ , so  $m_A/m_A^*$  increases (lower-right panel). The income of inactive households moves little, because there are greater numbers of both borrowers and lenders. The increase in income makes rich home households want to save, so inactive households tend to become poorer on average, and  $y_I$  goes down relative to  $y_I^*$ . Here again, movements in the relative fractions of active households in each country, i.e.,  $m_A/m_A^*$ , reinforce risk sharing. Overall, a shock to non-traded income generates little cross-country comovement in consumption, and positive comovement in relative aggregate consumption and the real exchange rate.

The decomposition in equation (21) and the impulse responses in the lower-right panels of Figures 1 and 2 demonstrate that endogenous asset market segmentation has two offsetting effects on international risk sharing. While the presence of inactive households disconnects aggregate consumption from asset market conditions, the endogenous equilibrium movements in the fraction of active households and the average income of inactive households reinforce international risk sharing in aggregate consumption. That is, fluctuations in the fraction of active households contributes to greater risk sharing apparent in aggregate consumption, and a stronger relationship between aggregate consumption and the real exchange rate. This result contrasts with Kollmann (2012), where exogenous fluctuations in the fraction of hand-to-mouth households contribute to weakening the relationship between relative aggregate consumption and real exchange rates.

### 3.4 Welfare Gains from International Risk Sharing

In this section, we show that our model is consistent with the finding in Lewis (2000) that welfare gains from international financial markets measured using asset price data are higher than estimates based on aggregate consumption data. We use our model to construct three alternative measure of welfare gains: (i) *actual gains* based on the ex ante utility of a representative household, (ii) *asset price-based gains* based on the utility of a hypothetical



household who is active every period, and (iii) *consumption-based gains* based on the utility of a hypothetical household who consumes aggregate consumption each period. We find that asset price-based welfare gains are significantly larger than consumption-based gains, and the asset price-based gains give a more accurate picture of the actual welfare gains from access to international financial markets than the consumption-based gains.

We consider lifetime consumption-equivalent measures of the gains from international asset trade relative to financial autarky. The ex-ante expected utility for the representative household (all households are ex-ante identical) is:

$$U = \sum_{t=0}^{\infty} \beta^t \int_{s^t} \int_{y^t} \frac{C(s^t, y_t)^{1-\eta}}{1-\eta} f(y_t) g(s^t) dy_t ds^t$$

To define the actual welfare gain, let  $\bar{C}(s^t, y_t)$  and  $\bar{U}$  be the consumption allocation and utility level under international financial autarky – that is, imposing the two conditions:

$$\begin{aligned} Y_{Tt} &= \int_{\bar{y}_L(s^t)}^{\bar{y}_H(s^t)} \bar{c}_T(s^t, y^t) f(y) dy + [F(\bar{y}_L(s^t)) + 1 - F(\bar{y}_H(s^t))] (\bar{c}_{TA}(s^t) + \bar{\gamma}_T(s^t)) \\ Y_{Tt}^* &= \int_{\bar{y}_L^*(s^t)}^{\bar{y}_H^*(s^t)} \bar{c}_T^*(s^t, y^t) f(y) dy + [F(\bar{y}_L^*(s^t)) + 1 - F(\bar{y}_H^*(s^t))] (\bar{c}_{TA}^*(s^t) + \bar{\gamma}_T^*(s^t)) \end{aligned}$$

instead of the single market-clearing condition (18). Then, the actual gains from international financial markets is given by the factor  $\Gamma$  that solves:

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \int \frac{C(s^t, y_t)^{1-\eta}}{1-\eta} f(y_t) dy_t = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \int \frac{[\Gamma \bar{C}(s^t, y_t)]^{1-\eta}}{1-\eta} f(y_t) dy_t$$

Equivalently,  $\Gamma = (U/\bar{U})^{1/(1-\eta)}$ , so that  $100 \times (\Gamma - 1)$  is the percentage increase in consumption each period that a representative household in financial autarky would need to attain the same utility level as in the benchmark equilibrium with international asset trade.

The asset-price based gains are calculated from the utility of a hypothetical household who is active every period:

$$U_A = \sum_{t=0}^{\infty} \beta^t \int_{s^t} \frac{C_A(s^t)^{1-\eta}}{1-\eta} g(s^t) ds^t$$

Similarly, the consumption-based gains are calculated from the utility of a hypothetical

household who consumes aggregate consumption every period:

$$U_C = \sum_{t=0}^{\infty} \beta^t \int_{s^t} \frac{C(s^t)^{1-\eta}}{1-\eta} g(s^t) ds^t$$

Denoting  $\bar{U}_C$  and  $\bar{U}_A$  the analogues of these two utility levels in financial autarky, the asset-priced based gains are given by  $\Gamma_A = (U_A/\bar{U}_A)^{1/(1-\eta)}$  and the consumption-based gains are given by  $\Gamma_C = (U_C/\bar{U}_C)^{1/(1-\eta)}$ . The measure  $\Gamma_C$  gives the factor by which consumption in autarky would have to be increased in order for a hypothetical household consuming the aggregate consumption to attain the same utility as in the equilibrium with international asset trade, and  $\Gamma_A$  is the corresponding measure for a hypothetical household who is always active. These measures of welfare gains are analogous to the welfare gains calculated from data on aggregate consumption and asset prices by Lewis (2000). Comparing these two measures illustrates that our model is consistent with the empirical finding in Lewis (2000), that observed asset prices imply higher welfare gains from international financial markets than observed aggregate consumption.

Table 3: Welfare gains in model: actual, asset price-based, and consumption-based

	Benchmark Model		Frictionless complete markets	
	$\eta = 2$	$\eta = 5$	$\eta = 2$	$\eta = 5$
WELFARE GAINS (in percent)				
Actual gains	0.0085	0.0294	0.0124	0.0234
Asset price-based	0.0132	0.0411	0.0124	0.0234
Aggregate consumption-based	0.0043	0.0063	0.0124	0.0234

Note: Welfare gains are computed using 1000 simulations of 400 quarters.

Table 3 reports the welfare gains in both the benchmark model and in the frictionless, complete markets model for different measures of risk aversion. In the frictionless, complete markets model, all the measures are the same, and indicate that, if  $\eta = 2$ , international asset trade raises welfare by the equivalent of a 0.0124 percent increase in consumption in every quarter. In the benchmark model, the actual welfare gains are a bit smaller, at 0.0085 percent. Within the benchmark model, we find that welfare gains implied by asset prices are indeed larger than welfare gains implied by aggregate consumption, by a factor of three (0.0132% vs. 0.0043%). In addition, these results indicate that the asset price-based measure of welfare gains gives a more accurate picture of the actual welfare gains from international asset trade than welfare gains based on aggregate consumption. Table 3 also reports results for a higher value of risk aversion,  $\eta = 5$ , which magnifies welfare gains and the difference

between the aggregate consumption measure and the asset price measure. With this higher level of risk aversion, asset prices imply welfare gains from international asset trade that are more than six times larger than implied by aggregate consumption.

### 3.5 Extension with non-asset holders

In this subsection, we consider an extension of the basic model in which a fraction of households hold no assets and simply consume their income each period. Previous empirical studies have shown that only a fraction of households hold stocks, bonds, and other assets. For example, Vissing-Jørgensen (2002a) shows that only 20 percent of households hold stocks, while 30 percent hold bonds. Taking into account indirect stock holding through pension funds and IRAs, Haliassos and Bertaut (1995) calculate that only 37 percent of households hold stocks.<sup>12</sup> In this section, we show that introducing non-asset holders in our model reduces the consumption-real exchange rate correlation, but only modestly when we calibrate the model to the same cross-sectional moments. In the model, asset holders' consumption displays better international risk sharing than non-asset holders' consumption, and we show that this implication is consistent with survey data from the US and UK.

We modify the benchmark model to assume that a fraction  $\omega$  hold assets and a fraction  $1 - \omega$  have no assets, each subject to the same idiosyncratic income shocks as above. Asset holders behave as described above, while non-asset holders consume their income every period, like the traditional hand-to-mouth households in Kollmann (2012). We compute statistics on asset holding and non-asset holding households in the data from the US CEX dataset of Krueger and Perri (2006). We choose a value of  $\omega = 0.83$ , which is equal to the average fraction of households that report positive financial wealth in the CEX data for 1980-2003. Given this value of  $\omega$ , we recalibrate the fixed cost  $\gamma$  so that, across all households, the variance of log consumption is 0.23, as in the benchmark model; since a fraction of households consume exactly their income every period, the new value of  $\gamma$  is lower than in the benchmark model, to yield less dispersed consumption among asset holders to compensate for the high dispersion of consumption among non-asset holders.

Table 4 shows the model results for the cross-country correlations of consumption among all households, asset holders, and non-asset holders. In the model with non-asset holders, the cross-country correlation of consumption among asset holders (0.61) is higher than the correlation of consumption among non-asset holders; the latter simply reflects the correlation of aggregate income (0.17), since non-asset holders consume their income every period. The correlation of overall aggregate consumption is in between, and equal to the correlation of

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<sup>12</sup>Vissing-Jørgensen (2002a) uses the US Consumer Expenditure Survey for 1980-1996, and Haliassos and Bertaut (1995) use the 1983 Survey of Consumer Finances.

Table 4: Cross-country correlations of consumption and the consumption-real exchange rate correlation in the model with non-asset holders

	Benchmark model	17% non-asset holders
CALIBRATED VALUE OF $\gamma$	1.012	0.851
CORRELATION OF HOME AND FOREIGN:		
aggregate consumption	0.53	0.53
asset holders' consumption	0.53	0.61
non-asset holders' consumption	n/a	0.17
CORRELATION BETWEEN $\frac{C}{C^*}$ AND $\frac{P^*}{P}$	0.24	0.23

Note: The table reports averages of standard deviations and correlations of logged series over 1000 simulations of 400 quarters.

aggregate consumption in the benchmark model. The last row of the table shows that, compared with the benchmark model, introducing non-asset holders lowers the correlation of relative aggregate consumption and the real exchange rate slightly, from 0.24 to 0.23.

Our share of non-asset holders of 17% is small relative to the values often used for non-stockholders in the literature cited at the beginning of Section 3.5. One reason is that the CEX measure of financial wealth includes all financial assets, not just stocks. In Figure 3, we show how the statistics in Table 4 change as we increase the fraction of non-asset holders from 0% to 60% to match the asset holder fraction in Haliassos and Bertaut (1995). The top panel shows that the cross-country correlation of asset holders' consumption rises. This is because with more non-asset holders the calibrated fixed cost required to match the dispersion in consumption reduces, as shown in the lower-right panel, so risk sharing improves among asset holders. There is a trade-off between the diminished aggregate risk sharing from households who do not own any assets and the increased risk sharing among asset holding households, who are active more often because of a lower fixed cost. The lower-left panel shows that the correlation between the real exchange rate and relative aggregate consumption declines very little as we increase the fraction of non-asset holders.

We consider an alternative calibration procedure, choosing  $\gamma$  to target the fraction of households active rather than the cross-sectional dispersion of consumption. Table 5 shows the results for three different fractions of non-asset holders—0%, 17%, and 60%—when we target an average of 40% of asset-holders active each year, which is in the middle of the range estimated in Vissing-Jørgensen (2002b) for the US. We find that changing the calibration target to the average fraction active preserves the discrepancy between the asset price-based

and the consumption-based measures of risk sharing, but it has an impact on the level of the consumption-real exchange rate correlation. For example, with 0% non-asset holders, the correlation increases from 0.24 to 0.58. Moreover, in the alternative calibration, the consumption-real exchange rate correlation declines drastically as the fraction of non-asset holders increases. We conclude that even under this alternative calibration our model is successful in accounting for the discrepancy between consumption-based and asset price-based risk sharing measures.

Table 5: Results from benchmark and alternative calibration<sup>†</sup>

	Benchmark calibration			Alternate calibration		
Fraction of exogenous non-asset holders	0%	17%	60%	0%	17%	60%
Average fraction active, annualized	28%	36%	98%	<b>40%</b>	<b>40%</b>	<b>40%</b>
Variance of log consumption	<b>0.23</b>	<b>0.23</b>	<b>0.23</b>	0.19	0.22	0.30
Calibrated value of $\gamma$	1.012	0.851	0.086	0.789	0.789	0.789
Correlation of home and foreign aggregate consumption	0.53	0.53	0.56	0.61	0.55	0.37
active consumption	0.93	0.93	0.93	0.93	0.93	0.89
Correlation between $\frac{C}{C^*}$ and $\frac{P^*}{P}$	0.24	0.23	0.18	0.58	0.32	-0.40

<sup>†</sup>Our benchmark calibration targets a variance of log consumption of 0.23, while the alternate calibration targets an annualized average fraction of active households equal to 40%. Results presented are averages of logged series from 1000 simulations of 100 periods each. Bold numbers indicate statistics targeted.

### 3.5.1 Suggestive evidence from micro data

We provide suggestive evidence on our model's mechanism from micro data. We show that the prediction that international risk sharing for asset holders is better than for non-asset holders is borne out in US and UK survey data over the period 1980-2003. To fully test the cross-country risk sharing implications of endogenously segmented asset markets, we would need to identify households *actively accessing assets*, which would require panel data at a high frequency on asset holders' consumption, income, and asset holdings for two or more countries. Since we do not have such data, we instead compare the model's predictions for aggregate consumption and *average* consumption among asset holders and non-asset holders to analogous averages constructed from micro data. The data are described in the appendix.

Table 6 reports statistics from the micro data for two consumption measures: nondurables and total expenditures. The first two columns of the table show that the correlation between aggregate nondurable consumption in the US and UK is 0.075, and for total consumption expenditures it is similar, 0.072. Asset holders' consumption comoves more strongly across the US and UK than the aggregate, with a correlation of 0.316 for non-durables and 0.214 for total expenditures. Non-asset holders' consumption is essentially uncorrelated. The ranking of these correlations is consistent with the model results: asset holders' consumption is more highly correlated than non-asset holders', as shown in the last column of the table.

Table 6: Cross-country consumption correlations in US and UK survey data

	US and UK Survey Data		Model with 17% non-asset holders
	Non-durables	Total consumption	
Aggregate consumption	0.075	0.072	0.53
Asset holders' consumption	0.316	0.214	0.61
Non-asset holders' consumption	-0.011	0.012	0.17

Note: Table shows log first-differences of annual averages for 1980 – 2003. US data are based on Consumer Expenditure Survey (CEX) and UK data are based on Family Expenditure Survey (FES). See Appendix 5.2 for further description of data. Model results are the same as in Table 4.

The aggregate consumption correlation from micro data provided in Table 6 is significantly lower than the one computed from national accounts data. Indeed, in Table 2, we show that over the period 1960Q1 to 2015Q4, US and UK aggregate consumption from the OECD have a correlation of 0.27 in log first differences. If we restrict the time period to 1980 – 2003 as in the survey data we use, the correlation changes to 0.44. The discrepancy between the measures of consumption from survey data and from national accounts for the US and the UK has been previously acknowledged in the literature. Krueger, Perri, Pistaferri, and Violante (2010) document that in both the US CEX as well as the UK FES per capita consumption growth is significantly slower than the corresponding national accounts measures. Our focus on US and UK data is driven solely by the availability of financial assets variables which allow us to measure consumption for asset holders and non-asset holders. Our findings for the US and UK suggest that documenting differences in international risk sharing among asset holders and non-asset holders across more countries is a promising avenue for future empirical work.

## 4 Conclusions

We extend the segmented asset markets model of Alvarez, Atkeson, and Kehoe (2002) to incorporate trade in goods, and show that it is successful in resolving the puzzling observation that asset prices imply a high degree of international risk sharing, while aggregate consumption suggests a low degree of risk sharing (as documented in Brandt, Cochrane, and Santa-Clara (2006)). While asset market segmentation in principle breaks the link between aggregate consumption and asset prices, our contribution is in quantitatively evaluating this mechanism. First, we calibrate the degree of asset market segmentation to match features of the cross-sectional distribution of consumption and income. Second, we illustrate how traded shocks move the ratios of relative aggregate consumption and relative active consumption in opposite directions, but non-traded shocks move the ratios of relative consumption in the same direction, so it is not obvious that segmented asset markets will lead to poor observed risk sharing for aggregate consumption. In addition, we show that trade in goods as well as asset market segmentation is necessary for asset prices to imply better risk sharing than aggregate consumption. Moreover, a model with these features partly resolves the puzzle highlighted by Backus and Smith (1993) and Kollmann (1995), generating a low correlation between the cross-country ratio of aggregate consumption and the real exchange rate. Lastly, an extension of the model in which a fraction households do not hold assets is consistent with microdata from the US and the UK showing that asset holders share risk across countries more than non-asset holders.

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## 5 Appendix

### 5.1 Aggregate Consumption Risk Sharing Equation

In this section, we log-linearize the model's equations around a symmetric steady state, in order to derive a first order approximation of the cross country differences in the growth rates of aggregate consumption, i.e.,  $\hat{C}_t - \hat{C}_t^*$ , where  $\hat{C}_t \equiv \log(C_t) - \log(\bar{C})$  and  $\bar{C}$  denotes the steady state level of aggregate consumption in the home country.

Log-linearizing the aggregate consumption in the home country, equation (20), we find:

$$\bar{C} \cdot \hat{C}_t = \frac{\bar{Y}_T}{\bar{P}} \bar{y}_I \hat{Y}_{Tt} + \frac{\bar{p}_N \bar{Y}_N}{\bar{P}} \bar{y}_I (\hat{p}_{Nt} + \hat{Y}_{Nt}) + \frac{\bar{Y}_T + \bar{p}_N \bar{Y}_N}{\bar{P}} \bar{y}_I (\hat{y}_{It} - \hat{P}_t) + \bar{m}_A \bar{C}_A (\hat{m}_{At} + \hat{C}_{At})$$

We define the steady state traded fraction of income as  $\omega_T \equiv \frac{\bar{Y}_T}{\bar{Y}_T + \bar{p}_N \bar{Y}_N}$ , the steady state non-traded fraction of income as  $\omega_N \equiv \frac{\bar{p}_N \bar{Y}_N}{\bar{Y}_T + \bar{p}_N \bar{Y}_N}$ , the steady state fraction of aggregate consumption attributed to inactive households as  $\phi_I \equiv \frac{\bar{Y}_T + \bar{p}_N \bar{Y}_N}{\bar{P} \bar{C}} \bar{y}_I$ , and the fraction attributed to active households as  $\phi_A \equiv \frac{\bar{m}_A \bar{C}_A}{\bar{C}}$ . The log-linear equation becomes:

$$\hat{C}_t = \phi_I \left[ \omega_T \hat{Y}_{Tt} + \omega_N (\hat{p}_{Nt} + \hat{Y}_{Nt}) + \hat{y}_{It} - \hat{P}_t \right] + \phi_A (\hat{m}_{At} + \hat{C}_{At})$$

Similarly, we can derive  $\hat{C}_t^*$ . Then,  $\hat{C}_t - \hat{C}_t^*$  is given in (22).

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* &= \phi_I \left[ \omega_T (\hat{Y}_{Tt} - \hat{Y}_{Tt}^*) + \omega_N (\hat{Y}_{Nt} - \hat{Y}_{Nt}^*) + \hat{y}_{It} - \hat{y}_{It}^* - (\hat{P}_t - \hat{P}_t^*) \right] \\ &\quad + \phi_I \omega_N (\hat{p}_{Nt} - \hat{p}_{Nt}^*) + \phi_A [\hat{m}_{At} - \hat{m}_{At}^* + (\hat{C}_{At} - \hat{C}_{At}^*)] \end{aligned} \quad (22)$$

Next, we simplify the left-hand side of equation (22).

First, log-linearizing  $\left(\frac{C_{At}^*}{C_{At}}\right)^{-\eta} = \frac{P_t^*}{P_t}$ , we find  $\hat{C}_{At}^* - \hat{C}_{At} = -\frac{1}{\eta} (\hat{P}_t^* - \hat{P}_t)$ .

Second, log-linearizing the price index, i.e.,  $P_t = (a^\sigma + (1-a)^\sigma p_{Nt}^{1-\sigma})^{\frac{1}{1-\sigma}}$ , we find  $\hat{P}_t = (1-a)^\sigma \left(\frac{\bar{p}_N}{\bar{P}}\right)^{1-\sigma} \cdot \hat{p}_{Nt}$ . Then, it can be easily shown that the steady state fraction of consumption expenditures on non-traded goods, i.e.,  $\psi_N \equiv \frac{\bar{p}_N \cdot \bar{C}_N}{\bar{P} \cdot \bar{C}}$ , equals  $(1-a)^\sigma \left(\frac{\bar{p}_N}{\bar{P}}\right)^{1-\sigma}$ . Hence,  $\hat{P}_t = \psi_N \cdot \hat{p}_{Nt}$ . Moreover,  $\hat{P}_t^* = \psi_N \cdot \hat{p}_{Nt}^*$ , where  $\psi_N = \psi_N^*$  in a symmetric steady state. Thus,  $\hat{P}_t - \hat{P}_t^* = \psi_N \cdot (\hat{p}_{Nt} - \hat{p}_{Nt}^*)$ .

Lastly, we show that  $\omega_N = \psi_N$ . In a symmetric steady state, the goods market clearing conditions are given below.

$$\begin{aligned} \bar{Y}_T &= a^\sigma \bar{P}^{\sigma-1} \cdot \{ (\bar{Y}_T + \bar{p}_N \bar{Y}_N) \bar{y}_I + \bar{m}_A \bar{P} [\bar{C}_A + \gamma] \} \\ \bar{Y}_N &= \left( \frac{\bar{p}_N}{1-a} \right)^{-\sigma} \bar{P}^{\sigma-1} \cdot \{ (\bar{Y}_T + \bar{p}_N \bar{Y}_N) \bar{y}_I + \bar{m}_A \bar{P} (\bar{C}_A + \gamma) \} \end{aligned}$$

Using the expression for the price index,  $P_t$ , total income is then given by  $\bar{Y}_T + \bar{p}_N \bar{Y}_N = (\bar{Y}_T + \bar{p}_N \bar{Y}_N) \bar{y}_I + \bar{m}_A \bar{P} [\bar{C}_A + \gamma]$ . The steady state non-traded fraction of income, i.e.,  $\omega_N$ , is then equal to  $\psi_N$ .

$$\omega_N = \frac{\bar{p}_N \bar{Y}_N}{\bar{Y}_T + \bar{p}_N \bar{Y}_N} = (1 - a)^\sigma \left( \frac{\bar{p}_N}{\bar{P}} \right)^{1-\sigma} = \psi_N$$

Using the expressions derived for  $\hat{C}_{At}^* - \hat{C}_{At}$  and  $\hat{P}_t - \hat{P}_t^*$ , as well as the result that  $\omega_N = \psi_N$ , equation (22) reduces to equation (21) from the main text.

## 5.2 Micro Data

We use data from the US Consumer Expenditure Survey provided by Heathcote, Perri, and Violante (2010)—henceforth HPV—and from the UK Family Expenditure Survey provided by Blundell and Etheridge (2010). Both datasets are available for download at: <https://www.economicdynamics.org/si-cross-facts/>. For the calibration, we estimate residual variances of income and consumption unexplained by household characteristics, as described in Section 3.1, using the “CEX sample b” provided by HPV.

For the statistics in Table 6, for the US we instead use the CEX data from Krueger and Perri (2006), since it contains information on financial asset holdings. We classify a household as an asset holder if their financial wealth is positive. We construct aggregate average consumption per capita (i.e. per adult equivalent as defined by OECD equivalence scale) and average consumption per capita for non-asset holders (i.e. households with zero financial wealth) and asset holders (i.e. households with positive financial wealth). We use two measures of consumption: nondurables and total expenditures. The UK survey data is annual, so for the US, we aggregate quarterly consumption into an annual average. We then compute the US–UK correlations in the first difference in log consumption for three groups: aggregate, asset holders and non-asset holders.

Figure 1: Impulse responses to 1% increase in home country traded output,  $Y_T$

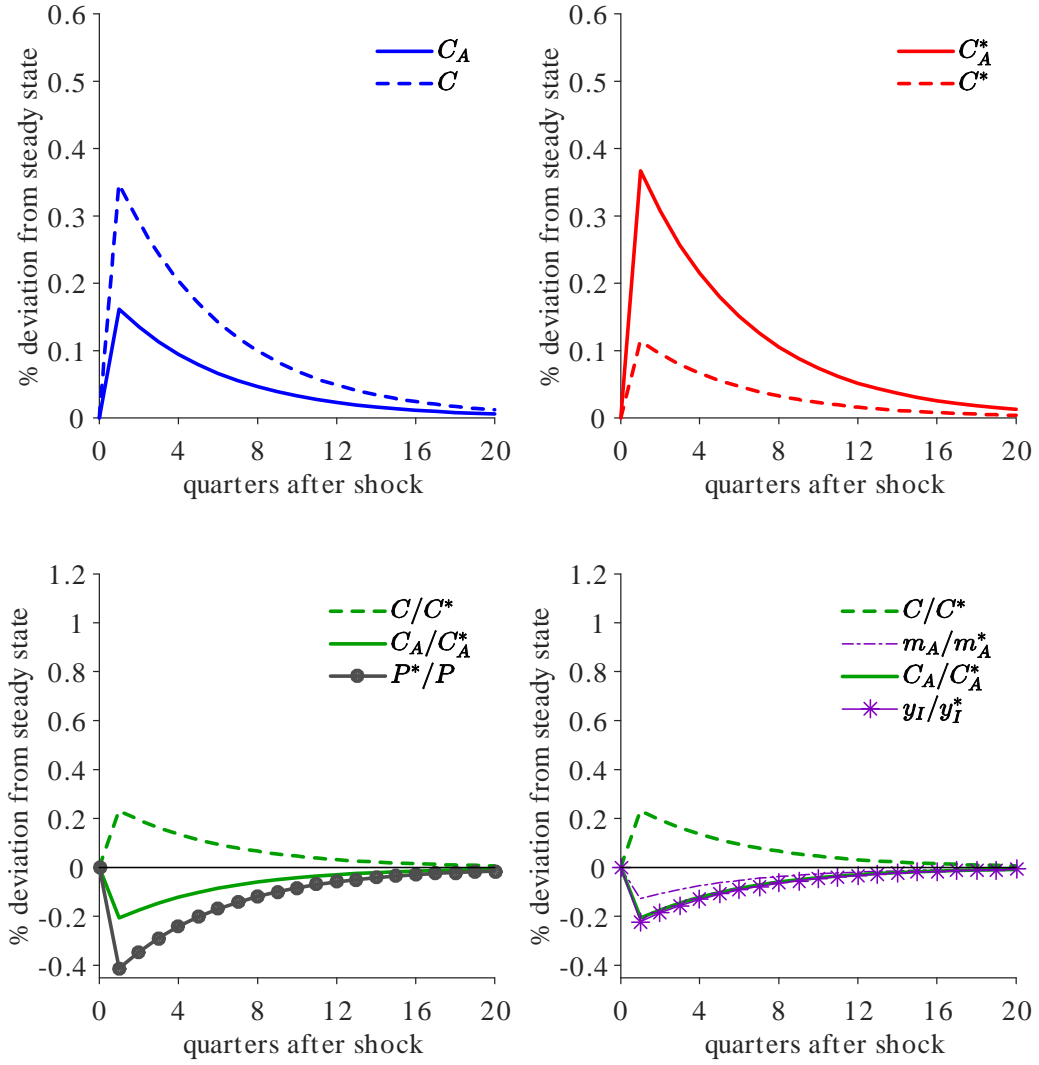


Figure 2: Impulse responses to 1% increase in home country non-traded output,  $Y_N$

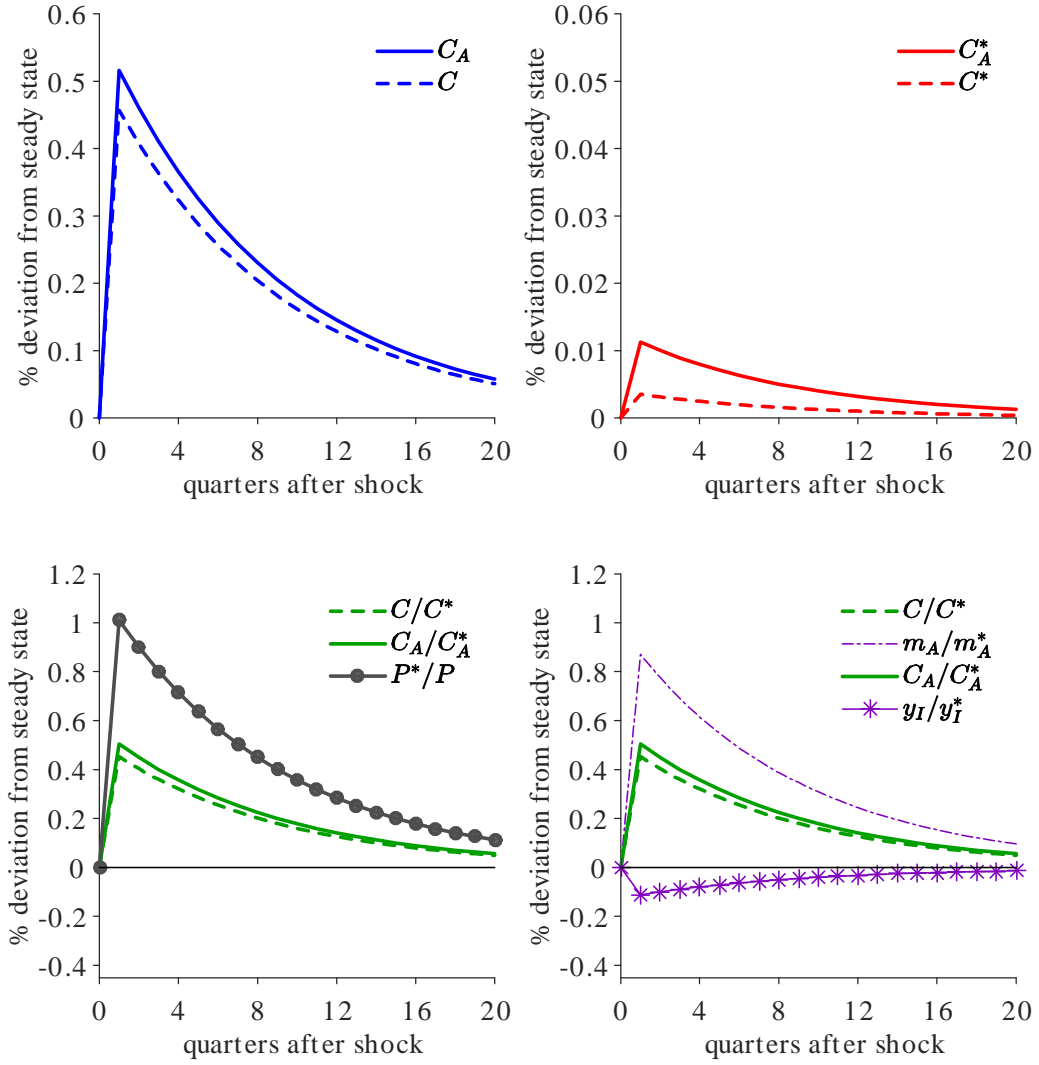


Figure 3: Cross-country consumption correlations and the consumption-real exchange rate correlation as the fraction of non-asset holders varies

