Creane, Anthony; Jeitschko, Thomas D.

**Working Paper**

Shipping the Good Apples Out Under Asymmetric Information

EAG Discussion Paper, No. 12-4

**Provided in Cooperation with:**
Economic Analysis Group (EAG), Antitrust Division, United States
dDepartment of Justice

Suggested Citation: Creane, Anthony; Jeitschko, Thomas D. (2012) : Shipping the Good Apples Out Under Asymmetric Information, EAG Discussion Paper, No. 12-4, U.S. Department of Justice, Antitrust Division, Economic Analysis Group (EAG), Washington, DC

This Version is available at:
http://hdl.handle.net/10419/202397
Shipping the Good Apples Out Under Asymmetric Information

by
Anthony Creane†
and
Thomas D. Jeitschko‡

EAG 12-4, April 2012

EAG Discussion Papers are the primary vehicle used to disseminate research from economists in the Economic Analysis Group (EAG) of the Antitrust Division. These papers are intended to inform interested individuals and institutions of EAG’s research program and to stimulate comment and criticism on economic issues related to antitrust policy and regulation. The Antitrust Division encourages independent research by its economists. The views expressed herein are entirely those of the author and are not purported to reflect those of the United States Department of Justice.

Information on the EAG research program and discussion paper series may be obtained from Russell Pittman, Director of Economic Research, Economic Analysis Group, Antitrust Division, U.S. Department of Justice, LSB 9446, Washington, DC 20530, or by e-mail at russell.pittman@usdoj.gov. Comments on specific papers may be addressed directly to the authors at the same mailing address or at their e-mail address.

Recent EAG Discussion Paper and EAG Competition Advocacy Paper titles are available from the Social Science Research Network at www.ssrn.com. In addition, recent papers are now available on the Department of Justice website at http://www.usdoj.gov/atr/public/eag/discussion-papers.htm. To obtain a complete list of titles or to request single copies of individual papers, please write to Kathy Burt at the above mailing address or at kathy.burt@usdoj.gov or call (202) 307-5794.

†Department of Economics, Michigan State University, East Lansing, MI 48824, creane@msu.edu.
‡Antitrust Division, U.S. Department of Justice; Washington, D.C. 20530; thomas.jeitschko@usdoj.gov.
Abstract

The importance of institutions for economic growth has gathered considerable interest. For example, weak institutions can prevent firms from communicating their quality, which can lead to lower welfare. We explore how and whether exporting to markets with strong institutions may alleviate this when firms have high and low quality goods. Surprisingly, we find that access to developed markets can exacerbate the problems caused by weak institutions and harm home welfare. First, exporting can harm home welfare: the country is better off if all exporting were prevented. Second, any harm is increasing in the amount exported. Third, if not all high quality is exported, then home welfare can always be increased by restricting exports. Fourth, the opening of trade can reduce producer surplus and so in the long run lead to a reduction in the production of the export good. Fifth, welfare can decrease even if production of the exported good increases.

Keywords: adverse selection, moral hazard, asymmetric information, quality, trade, development, brain drain

JEL classifications: D82, F12, L15
1 Introduction

Recent work has shown the extent to which weak institutions can hamper economic growth (e.g., Acemoglu et al., 2001, Rodrik et al., 2004, Acemoglu et al., 2005). Among other problems, weak regulatory and judicial institutions hinder markets that are characterized by asymmetric information, as consumers cannot trust firms’ claims when regulatory scrutiny is scant or unreliable, and the ability to enforce contractual promises and warranties are very costly. Indeed, “regulatory quality” and “rule of law” are two of the six indicators used to appraise governance of countries and economic development (Kaufmann et al., 2009). As a result, government regulation of product quality may be sufficiently weak, or recourse to the judicial system is sufficiently costly and inefficacious to make quality promises or warranties meaningless. This leads to high quality products that are, at a minimum, not efficiently allocated and potentially driven out of the market by low quality products.1

International trade offers a possible means to alleviate this inefficiency as high quality producers can bypass the home market and sell their products to developed markets where strong institutions allow them to credibly communicate their quality. More broadly, trade may even have a feedback effect improving domestic institutions (Levchenko 2011).2 Many developing countries often see their high quality products being exported and only low quality products remaining—shipping the good apples out—even though seemingly there is home demand for high quality. For example, there are anecdotal stories of home residents only being able to buy high quality home products while they are abroad. Thus, there is a cost from this remedy: less high quality for home consumers. One question is if this only results because of a coordination problem: if no one expects high quality in the home market, then the market price is based on low quality and so firms export their high quality product.

At the same time there has been empirical interest in the general role of quality in international trade. In particular, Hummels and Skiba (2004) find support for the Alchain

1Weak regulatory and judicial institutions have also been used to motivate recent work (Acemoglu, et al., 2007, Levchenko 2007, Nunn 2007 and Constinot 2009) focusing on how weak institutions can lead to a comparative disadvantage in producing goods in the “institutionally dependent” sector because of weak contract enforcement.

2There is also a partly analogous story of financial capital outflows to bypass weak domestic institutions. Ju and Wei (2010) find that this has an ambiguous effect for the home country.
and Allen (1964) conjecture that high quality products are exported, while low quality ones remain at home. These studies do not however examine the role of information asymmetry in trade and in fact quality generally is not directly observed (with the notable exception of Crozet, et al. 2009), but rather inferred from price differences. Other work has examined the role Hong Kong and other re-exporters have in sorting quality.3

We develop a model with these characteristics: a competitive home market with downward sloping demand for the product, asymmetric information in the home country regarding quality, and a developed export market in which quality is observable so that firms can export their high quality product. Specifically, heterogeneous firms first choose when to enter a market and then a fraction of each firm’s output is of low quality, which consumers cannot observe. An alternative, behaviorist interpretation is that a fraction of firms have less innate ability and so must incur a small effort cost to provide high quality, while for the remainder effort is costless (for example, personal integrity) to provide high quality.4

We begin with autarky as the benchmark. The presence of low quality when quality is not observable lowers home welfare as firms choose not to sort their quality (or low ability firms do not incur the cost to provide high quality), putting both high and low quality together on the market. We then consider the economic effects of exporting to a large market where quality is verifiable considering both short-run (or unanticipated trade) and long-run (i.e., firms enter anticipating trade) effects.5 Not surprisingly, some or all of the high quality product is exported—good apples are shipped out. Surprisingly, all high quality can be exported even when the export price is below the home choke price and this is not

3See Young (1999) and Hanson and Feenstra (2001).

4This interpretation is closer in nature to the assumptions in Acemoglu et al., (2007), where a fraction of the contracts are nonverifiable and noncontractible; Levchenko (2007), where a fraction of the investment is relationship specific and unenforceable; Vogel (2007), where monitoring can be interpreted as probability of observing unproductive effort; and Constinot (2009), where only a fraction of contracts are enforced.

5A different interpretation of the model is the export of high quality workers: the brain drain. Talented workers may not be able to signal their quality either because verification methods are weak, or perhaps are blocked by other institutional, legal, sociological or cultural barriers. By immigrating to countries that can verify and reward quality, these workers reduce the average quality of the remaining workers. Even if signals are available (e.g., standardized test scores, or acceptances to universities), they may not be viewed as verifiable at home. While a benefit of the “brain drain” is that a significant portion of the higher income is repatriated home, our results suggest that even if all of their income is repatriated welfare may be reduced. Interpreting this in terms of the effect of weak institutions on input markets which others have examined (Acemoglu, et al. 2007, Levchenko 2007, Nunn 2007 and Constinot 2009), trade can result in export of high quality inputs to the detriment of more complex home production that depends on higher quality inputs.
a coordination failure. Rather, the export of high quality may drive down the home price as average quality is decreasing. This can lead to home firms’ profits decreasing when the weighted average of the export price and the home price (which has decreased) is less than the autarkic price. However, the home price need not always decrease as the export of high quality also reduces output on the home market, which can lead to an increase in the home price and it is possible that in equilibrium the home price equals the export price.

Turning to welfare analysis, we find that access to developed markets can harm home welfare (i.e., the sum of home producer and consumer surplus). First, there always exists an export price that induces adverse selection at home harming welfare. That is, there are export prices at which the country is better off if all exporting is prevented. This harm can occur if some or all the high quality products are exported. Second, the harm is increasing in the amount of high quality exported. The primary force behind this result is intuitive: home market prices are determined by the marginal consumer’s valuation of quality, but the loss to home welfare from high quality being exported is based on the average valuation of quality. Third, home welfare can always be increased by restricting exports if in equilibrium not all high quality is exported. Further, even if all high quality is exported and home welfare is greater with exporting, it is still possible that home welfare can be increased by restricting exports. Fourth, the harm to home welfare may be highly non-monotonic in export prices. We provide examples in which for low export prices home welfare decreases with exporting, and with higher prices home welfare increases with exporting, but with even higher prices home welfare decreases with exporting.

Comparing the long and short-run outcomes, since producer surplus can decrease with exporting, we find that the export industry can contract in the long-run. That is, a country that unexpectedly is able to export its product can find this export sector shrinking over time. Further, while one might expect that entry decisions should mitigate any harm found in the short-run, we find that welfare can decrease even more in the long-run. The reason for this is that the additional entry can result in even less high quality being sold on the home market.

---

6Levchenko (2007), when considering institutionally dependent sectors, finds a similarly flavored result; namely that the country with weak institutions can lose from “institutional comparative advantage driven trade.” The source of the results differ, as ours is a loss in consumer welfare and in Levchenko it springs from the loss of good jobs in the institutionally dependent sector which contracts with trade.
market. Thus, the range of export prices that harm home welfare can be greater in the long-run. Finally, it is possible that home welfare with unanticipated trade (i.e., firms entered expecting autarky but exporting unexpectedly is possible), could be greater than welfare with anticipated trade (i.e., firms correctly anticipated that export would be possible).

We then extend the model to allow for investments in quality improvements as well as the decision to sort. That is, firms choose whether to invest so as to increase the fraction of their output that is of high quality. We find first that the opening of the export market may have no effect on the decision to invest in quality and so the previous results still hold. This occurs when trade equilibrates the home price to the export price. As a result all quality is sold at the same price and so firms have no reason to invest in quality improvements. The other possibility is that all high quality is exported. While this resolves the moral hazard issue (all firms will make the Pareto improving investment which they did not in autarky), home welfare, and perhaps even more surprisingly, home producer surplus can still decrease. This can be true even when investments make an order of magnitude increase in the percentage of high quality output.

There is an extensive literature examining trade with asymmetric information including the seminal papers of Grossman and Horn (1988), Bagwell and Staiger (1989) and Bagwell (1991). However, the emphasis and approach here is different. In Grossman and Horn, as here, there is a home market in which institutions are not sufficiently developed so as to have quality verified. The key differences are that in Grossman and Horn (1988), 1) demand does not slope downward, which affects the results; 2) home firms can only sell in the home market in which they face competition from developed firms whose quality is known; 3) there is a second period of exchange so that high quality producers can earn reputational rents; 4) Grossman and Horn found no scope for protecting the home firms through temporary tariffs. Bagwell and Staiger (1989) using the same demand and timing as Grossman and Horn (1988), examine export policy, but for a monopolist of exogenous quality (so they do not consider moral hazard) that is not observable in the export market (the home market is irrelevant). They find that an export subsidy could help a high quality producer signal its quality, while in our model, this would (further) lower welfare. Finally, Bagwell (1991) considers a similar set-up in timing but with a monopolist that chooses quality and with
downward sloping demand. He finds that a subsidy can raise welfare by reducing the first period upward distortion in price that a high quality producer must make to signal its quality. A key modeling difference is that there is again a second period of sales to enable reputational rents. We view our modeling difference as an asset since the alternative assumption is subject to the criticism that a firm would want to reduce its quality in the second period to save cost. However, our basic results extended to the model of two periods of fixed quality.

In the next section we introduce the basic assumptions of the model where in the first stage firms make their entry decisions and in the second stage they make their exchange decisions. Section 3 characterizes the second stage – which is equivalent to an unanticipated opening of trade. Section 4 characterizes the first stage and in the penultimate section firms can also make investments in the first stage to increase quality. The final section concludes.

2 The Model

In this section we introduce the base model. There is a small, developing home country and a large developed foreign country or market such as the OECD. In the home market neither consumers nor the home government can observe product quality, $s$, of firms. The developed foreign country has the institutions to verify a product’s quality allowing home firms to receive a higher price for their high quality products.

For home consumer preferences, we adopt the standard structure (e.g., Bagwell and Riordan, 1991), but allow for non-linear demand. There is a continuum of consumers with mass normalized to one, with each consumer demanding exactly one unit of the good. The quality is either high or low: $s \in \{H, L\}$. There is a common reservation price for a low-quality product, normalized to zero. In contrast, consumers have heterogeneous reservation prices, $v$, for a high-quality product, distributed with a strictly positive density everywhere with the support normalized to [0,1]. Let $h(q)$ be the inverse demand for high quality generated by this distribution, i.e., the reservation price $v$ for the consumer with the $q^{th}$ highest reservation price, so $-\infty < h'(q) < 0$. To fix ideas we will sometimes give as an example the specific solutions when $v$ is uniformly distributed so that demand is linear.

On the production side there are now two stages: broadly, in the first stage firms choose
whether to enter and in the second stage it is revealed whether trade can occur and exchange occurs. Specifically, in the first stage there is a continuum of potential firms that simultaneously choose whether to enter the market. Firms are heterogeneous in entry costs, $e$, distributed with a strictly positive density everywhere on $[0, 1]$. From this distribution let $e(q)$ denote the entry cost for the firm with the $q^{th}$ lowest entry cost (so $0 < e'(q) < \infty$) with the most efficient firm’s entry cost normalized to zero ($e(0) = 0$). Later, in Section 5, firms can also invest so as to increase the percentage of their output that is of high quality.

In the second stage, each firm has a unit of output of which a percentage $\rho \in (0, 1)$ of each firm’s output is of high quality. Each firm has zero cost of production: high and low qualities have the same cost of production. While $\rho$ is common knowledge, quality itself is unobservable. However, a firm can sort (learn) its products’ quality by incurring a sorting cost $c \geq 0$, but this sorting is not observable to home consumers or government.

High quality is observable in the foreign country. It is assumed that home firms are price takers on the foreign market so as to abstract from the well-known market power reasons (terms-of-trade effects) for export (output) restrictions which would strengthen our results if it were included. That is, a firm does not internalize its export’s negative price externality on other exporters and so it is optimal for the home government to restrict exports.

The equilibrium concept is subgame perfection and so the second stage is characterized first. The second stage equilibrium, which corresponds to the classic environment analyzed by Akerlof (1970), can also be interpreted as the outcome when firms enter in the first stage anticipating autarky, but then trade is unexpectedly possible as then the first stage decisions are sunk. That is, the second stage equilibrium can be used to characterize what occurs when trade unexpectedly becomes possible (or impossible) for exporters and in this sense can be considered the short-run effects of unanticipated trade (or unanticipated autarky).\(^8\)

---

\(^7\)This assumption can alternatively (and commonly) be interpreted as with probability $\rho \in (0, 1)$ a given firm has high quality. As it is more natural in the context of our story, we use the first interpretation. The fraction of high quality could also be interpreted as being a group with a lower percentage of low quality.

\(^8\)This second-stage set-up is also used in empirical investigations on quality-differentiated trade (see, for instance, Hummels and Skiba, 2004).
3 Second stage (unanticipated trade)

In the second stage, there is a continuum of home firms that entered whose mass is denoted $q$ with $q = 1$ denoting the (normalized) saturation quantity (i.e., the maximum bought at a price of zero). Since there is a continuum of firms, $\bar{p} \bar{q} =: \bar{q}_H$ of the aggregate output is of high quality; and, $(1 - \bar{p}) \bar{q} =: \bar{q}_L$ is the low quality output, with $\bar{p} \equiv \frac{\bar{q}_H}{\bar{q}_H + \bar{q}_L}$ and $\bar{q}_H + \bar{q}_L \equiv \bar{q}$.

3.1 Autarky

We take autarky as the status quo, i.e., home firms are unable to export to the developed market because of trade barriers, either foreign or domestic. Since both types of quality have the same cost of production and quality is unobservable, all firms are willing to sell all of their output at any positive price and so consumers expect for any positive price that a product is of high quality with probability $\bar{p}$.

The marginal consumer’s valuation given $q$ units of supply is $\bar{p} h(q) + (1 - \bar{p})0$, so the Walrasian-market clearing price in the home market (under autarky) is $P_A \equiv \bar{p} h(q) > 0$. In equilibrium, then, all firms supply their output and the equilibrium price is $P_A$.

Home welfare ($W$) is consumer surplus and profits and so home welfare in autarky (denoted by $W_A$) is

$$W_A(\bar{q}) = \left\{ \int_0^{\bar{q}} \bar{p} h(x) dx - \bar{q} \bar{p} h(\bar{q}) \right\} + \bar{q} \bar{p} h(\bar{q}) = \frac{\bar{q}_H + \bar{q}_L}{\bar{q}_L + \bar{q}_H} h(x) dx. \quad (1)$$

It is straightforward to show this is the second-best welfare optimum, i.e., constrained Pareto optimal where the constraint is that there is incomplete information about product quality.

A useful benchmark is the first best outcome ($W_F$) under autarky, which is obtained with complete information as the qualities are separated (sorting costs are incurred). Low quality is sold at a price of zero with zero consumer benefit and so welfare is generated by high quality. Provided that sorting costs are not too great, welfare is greater than under the second best. To see this, note that when sorting cost are zero, $c = 0$, one obtains

$$W_F(\bar{q}) = \left\{ \int_0^{\bar{q}} h(x) dx - \bar{p} \bar{q} h(\bar{q}) \right\} + \bar{p} \bar{q} h(\bar{q}) = \int_0^{\bar{q}} h(x) dx > \bar{p} \int_0^{\bar{q}} h(x) dx = W_A(\bar{q})$$
where the inequality holds because \( \int_0^y h'(x)dx \) is positive and strictly concave in \( y \) (i.e., demand slopes downward \( h'(q) < 0 \)). Note that the home price is higher in the first best and, if \( h(q) \) is not too convex in \( q \), e.g., with linear demand, consumer surplus is lower.\(^9\) However, home welfare is greater because the allocative inefficiency is eliminated. That is, the high quality product is matched to those with the greater value (i.e., the consumer with the highest reservation value obtains the high quality production with probability 1 given complete information, but only with probability \( p \) under incomplete information).

### 3.2 Exporting

Consider now the effect of trade liberalization, e.g., either the developed foreign country dramatically lowering its tariffs or the home country lifts export restrictions. The home firms instead of selling in the home market can export their product at a per unit cost \( t > 0 \) to the developed foreign market. In addition, upon entering the foreign country the home firm may have to pay the cost of verification at \( v \geq 0 \) per unit, which can be zero.\(^10\)

The foreign market has a competitive price \( p_F \) for the high quality good and a price of zero for the low quality good. The home firm can export their high quality product and receive a price net of sorting, transportation and verification costs: \( p_X = p_F - c - t - v \). As the issue at hand is the welfare effect of exporting, it is assumed that \( p_X > P_A \), since otherwise exporting is not an attractive alternative to selling at home. Thus, starting from autarky, a firm would want to export its high quality product. Since the home country is relatively poor, there is not demand for the import of the foreign good, \( p_F + t > h(0) \). Alternatively, it can be assumed that since quality cannot be verified in the home country, it cannot be verified that the foreign import is actually of high quality.

Since there is zero cost of production, all the non-exported units are supplied to the home market at any positive home price.\(^11\) Denote the amount of high (low) quality supplied in

---

\(^9\)Consumer surplus, \( CS(q) := \int_0^q h(x)dx - h(q)q \), is strictly convex in \( q \) (and so \( \rho CS(q) > CS(\rho q) \) if \( -h'(q) - h''(q)q > 0 \)). For consumer surplus to be convex, \( h'' \) cannot be positive and too large in magnitude.

\(^10\)Since the home country’s institutions are weak even if the firm incurs the sort and verification cost at home it still cannot convince buyers that what it claims is its high quality portion is truly so. Further, this verification cost is not necessary for the results given transportation and sorting cost, but rather is included only for purposes of generality.

\(^11\)Implicitly we are ruling out equilibria with coordination failure. The efficiency implications of coordina-
the home market as \( q_H \) (since no low quality is exported, \( q_L = \overline{q}_L \)), so the amount exported is \( q^X_H = \overline{q}_H - q_H \). The equilibrium with the export is determined by i) the fraction of high quality output that firms choose to export over serving the home market and ii) the home consumers’ beliefs about the average quality of goods in the home market; where home consumers’ beliefs as to the average quality of the products on the home market must be consistent with the actual average quality. If consumers beliefs are consistent, then their belief of average quality (\( \rho \)) when the export market opens, must equal the actual average quality in equilibrium, i.e., given \( q_H \) and \( q_L \) units serving the home market in equilibrium, beliefs must be \( \rho(q_H, q_L) \equiv q_H/(q_H + q_L) \). Then, the Walrasian price is

\[
P(q_H, q_L) = \rho(q_H, q_L)h(q_H + q_L) = \rho((\overline{q}_H - q^X_H), \overline{q}_L)h((\overline{q}_H - q^X_H) + \overline{q}_L).
\]

For these quantities and prices to be an equilibrium requires that no firm would want to deviate from its choice as to where it supplies its high quality units. For an individual firm, exporting its high quality product is more profitable when the export price (net of sorting, transportation, and verification costs) is greater than the home price. Since it is assumed \( p_X > P_A \), there are two possible equilibria: either the export price is above the home price and all high quality is exported or the prices are equal and some units are exported. The former (export price greater than the home price) is an equilibrium as no firm would want to deviate to sell some of its high quality on the home market and cannot export any of its low quality. The latter is an equilibrium since no firm has an incentive to deviate in where it sold its high quality: its export units receive the same price as the non-exported units. In the latter case, the equilibrium quantity is implied by \( P(q_H, \overline{q}_L) \equiv p_X \), or

\[
P(q_H, \overline{q}_L) = \rho(q_H, \overline{q}_L)h(q_H + \overline{q}_L) = \frac{q_H}{q_H + \overline{q}_L}h(q_H + \overline{q}_L) = p_X.
\]

The second equilibrium (when the home price equals the export price) can occur because the home price can increase in high quality exports. The reason for this is that while high quality exports cause average quality on the home market to decrease \( (\partial \rho(q_H, q_L)/\partial q_H \equiv \rho_1(q_H, q_L) = q_L/(q_H + q_L)^2 > 0) \), output on the home market is also decreasing, so the marginal consumer’s value for high quality is increasing \( (h'(q) < 0) \). As a result the home

---

*Operation failures are well understood and would only reinforce our main findings.*
price can be decreasing in $q_H$ at $\bar{q}$ \((\partial P(\bar{q}_H, \bar{q}_L)/\partial q_H \equiv P_1(\bar{q}_H, \bar{q}_L) < 0)\) if the output effect is greater than the average quality effect:

$$P_1(q_H, q_L) = \rho_1(q_H, q_L)h(q_H + q_L) + \rho(q_H, q_L)h'(q_H + q_L)) < 0,$$

and so the home price is increasing in the export of high quality:

Since $\rho(q_H, q_L)$ and $h(q_H + \bar{q}_L)$ are continuous in $q_H$, then if such an equilibrium exists, there is at least one pair of $q_H$ that satisfy (2) and define a set of $q_H$ in which $P(q_H, \bar{q}_L) > p_X$. The smaller valued is an unstable equilibrium: an increase in exports would result in the home price decreasing (and vice versa), while for the greater it would result in it increasing (and vice versa). We will focus on the stable, greater valued of the pair of $q_H$ and denote this $q_H$ that satisfies (2) as $\hat{q}_H(\bar{q})$.

The first equilibrium can exist without a coordination failure (i.e., all consumers arbitrarily expect all high quality to be exported) if $P_1(q_H, \bar{q}_L) > 0$ for all $q_H \leq \bar{q}_H$: then for every level of export $p_X$ is greater than the home price. Even when $P_1(\bar{q}_H, \bar{q}_L) < 0$ the first equilibrium results if $p_X$ is greater than the upper bound on possible home prices. Specifically, let $q^*_H(q_L)$ denote the $q_H$ that maximizes $P(q_H, q_L)$. If $P(q^*_H(q_L), q_L) < p_X$, then while the home price is initially increasing in exports, its maximum is still below $p_X$ and so for all $q_H$ the export price is greater than the home price and all high quality is exported.

### 3.3 The Effects of Exporting on the Home Market

We now consider the implications of exporting on the home market, first examining the welfare implications and then giving a more detailed examination of the price effects.

#### 3.3.1 Welfare effects of exporting

Consider first consumer welfare, which decreases from the exporting of high quality, even when exporting causes the home price to decrease. The reason is that a price decrease reflects the marginal consumer’s decrease in expected value while all inframarginal consumers have

---

12 A coordination failure could arise, for example, even if the strictly positive export price is less than the autarkic price: all high quality units are exported if consumers arbitrarily believe that there will be no high quality units on the home market. Autarky in this case yields higher welfare, but as the inefficiencies from coordination failures are well understood this case is not particularly interesting.
a greater decrease in expected value as demand slopes downward. From the first term in (1) consumer welfare with \( q_H < \bar{q}_H \) units on the home market is

\[
CW(q_H) = \frac{q_H}{q_H + \bar{q}_L} \left[ \int_0^{q_H + \bar{q}_L} h(x) dx - (q_H + \bar{q}_L)h(q_H + \bar{q}_L) \right],
\]

where the first term captures gross utility and the second consumer expenditures. Differentiating with respect to high quality output (i.e., the negative of a unit exported) gives

\[
\frac{dCW(q_H)}{dq_H} = \frac{\bar{q}_L}{(q_H + \bar{q}_L)^2} \left[ \int_0^{q_H + \bar{q}_L} h(x) dx - (q_H + \bar{q}_L)h(q_H + \bar{q}_L) \right] - \bar{q}_H h'(q_H + \bar{q}_L) > 0.
\]

Turning to home welfare it is convenient to express it as the sum of welfare in the home market plus the repatriated profit from exporting. From (1) home welfare in terms of high quality output in the home market \((q_H, q_L)\) is

\[
W_H(q_H, q_L) = \int_0^{q_H + \bar{q}_L} \frac{q_H}{q_H + \bar{q}_L} h(x) dx.
\]

As \( q_H \) is the amount of high quality on the home market then export earnings are \( p_X(q_H - q_H) \). Home welfare with exporting and \( q_H \) in the home market, denoted \( W(q_H, \bar{q}_L) \) is

\[
W(q_H, \bar{q}_L) = W_H(q_H, \bar{q}_L) + p_X(q_H - q_H).
\]

The marginal effect of a high quality unit being exported \((q_H^X)\) in equilibrium on the home market welfare \( W_H(q_H, \bar{q}_L) \) is

\[
\frac{\partial W^H}{\partial q_H^X} = -\frac{\partial W^H(q_H, \bar{q}_L)}{\partial q_H} = - \left[ \frac{q_H}{q_H + \bar{q}_L} h(q_H + \bar{q}_L) + \int_0^{q_H + \bar{q}_L} \frac{\bar{q}_L}{(q_H + \bar{q}_L)^2} h(x) dx \right] < 0. \tag{3}
\]

The first term inside the brackets is the benefit of an additional unit being supplied to a market (the market price) and the second is the benefit from average quality increasing in the home market from another unit of high quality. The effect of a high quality unit being exported on the home market welfare then is negative.

The benefit of a unit being exported is the export price net of export costs, \( p_X \). Summing the two terms yields the marginal effect of a unit being exported when starting at autarky
\( q_H = \bar{q}_H \):

\[
\frac{\partial W}{\partial q_H} \bigg|_{q_H=\bar{q}_H} = -\left[ \frac{q_H}{q_H + \bar{q}_L} h(q_H + \bar{q}_L) + \int_0^{q_H+\bar{q}_L} \frac{\bar{q}_L}{(q_H + \bar{q}_L)^2} h(x) dx \right] + p_X
\]

\[
= -\bar{p} h(q) - \frac{\bar{q}_L}{(q_H + \bar{q}_L)^2} \int_0^\bar{q} h(x) dx + p_X.
\]

Starting from autarky, a high quality unit exported reduces home welfare if and only if

\[
p_X < \bar{p} h(q) + \frac{\bar{q}_L}{(q_H + \bar{q}_L)^2} \int_0^\bar{q} h(x) dx = \bar{p} h(q) + (1 - \bar{p}) \int_0^\bar{q} h(x) dx / \bar{q}.
\]  

(4)

Since the first term on the RHS of the inequality is the autarkic market price and the second term is strictly positive, there always exist some export prices such that the home welfare is harmed by the first unit exported, provided that \( \bar{p} < 1 \). If there were no asymmetric information about quality so \( \rho = 1 \), then the inequality in (4) would be violated, implying that there does not exist any price for which marginal exportation of goods lowers home welfare.

The intuition for why exporting can harm home welfare is straightforward. When a unit of high quality is exported, the marginal consumer no longer buys. The probability the high quality unit went to the marginal consumer is \( \bar{p} \) and so that consumer’s valuation was \( \bar{p} h(q) \), i.e., the market price (the first term on the RHS). However, with probability \( 1 - \bar{p} \) that high quality unit would have been randomly bought by some other consumer and so there is a second lost: the average value of high quality to all consumers who are purchasing weighted by the probability of one of them receiving it (the second term), all of whom have a higher value than the marginal consumer since demand slopes downward.\(^{13}\) At first glance it appears as if nothing is wrong (and actually better than autarky) with the exporting: home high quality units exported to a wealthier country for a higher price and low quality is sold at cost in the home market, good apples are exported, bad ones stay home. However, home welfare has been lowered even though—or rather because—the markets are segmented, which normally would increase welfare.

\(^{13}\)This intuition is related to Spence’s regarding monopoly quality choice under certainty (1975, 1976).
Consider next the effect of additional units being exported. Welfare is concave in \( q \):

\[
\frac{\partial^2 W(q_H, q_L)}{(\partial q_H)^2} = \frac{q_H}{q_H + \bar{q}_L} h'(q_H + \bar{q}_L) \frac{2\bar{q}_L}{(q_H + \bar{q}_L)^3} \int_0^{q_H + \bar{q}_L} h(x) dx + \frac{2\bar{q}_L}{(q_H + \bar{q}_L)^2} h(q_H + \bar{q}_L) \]

\[
= \frac{q_H}{q_H + \bar{q}_L} h'(q_H + \bar{q}_L) \frac{2\bar{q}_L}{(q_H + \bar{q}_L)^2} \left[ \int_0^{q_H + \bar{q}_L} \frac{h(x)}{q_H + \bar{q}_L} dx - h(q_H + \bar{q}_L) \right] < 0.
\]

The inequality is an implication of downward sloping demand, \( h'(\cdot) < 0 \). This makes the first term negative; but it also implies that the bracketed term is positive: the average value of high quality, \( \int_0^q h(x) dx / q \), is greater than the marginal buyer’s value of quality, \( h(q) \). Thus, the *harm to the home market* from a unit exported when (4) is true is *increasing* in the amount of exports.

There are two reasons why the harm is increasing in additional exports. First, fewer units bought in the home market means that the marginal consumer no longer buys so the average value for a given level of quality increases and hence the loss of another unit exported is greater. Second, the average quality is decreasing at an increasing rate as high quality exits the market. Thus, this condition (4) for home welfare to decrease from exports at autarky implies that home welfare decreases from a firm choosing to export independent of the amount of high quality exported. To summarize,

**Proposition 1** *In the second stage if \( p_X < \bar{p} h(\bar{q}) + (1 - \bar{p}) \int_0^q h(x) dx / \bar{q}, \) then home welfare is harmed by exports. Such a \( p_X \) always exists so long \( \bar{p} \neq 1 \).*\(^{14}\)

Interpreted in terms of the two-stage game, if the firms entered the market anticipating autarky, but unexpectedly firms could export their high quality products, then there always exist export prices such that home welfare is harmed (or equivalently there always exist export prices such that welfare is increased by unanticipated autarky).

As the harm to home welfare is increasing in exports while the benefit (\( p_X \)) is constant, even when the first unit exported increases home welfare (condition (4) does not hold) welfare may decrease from exporting. To put it differently, the harm from the marginal export when all is exported is greater than the harm from the first one:

\(^{14}\)This is because the RHS is larger than the autarkic price so long \( \bar{q}_L > 0 \), i.e., \( \bar{p} \neq 1 \).
\[
\int_0^{(1-\bar{p})q} h(x) \, dx = \frac{\int_0^{(1-\bar{p})q} h(x) \, dx}{(1-\bar{p})q} + (1-\bar{p}) \int_0^{(1-\bar{p})q} h(x) \, dx > \bar{p} h(q) + (1-\bar{p}) \int_0^q h(x) \, dx,
\]

since \( h'(q) < 0 \). We then have a weaker condition than (4) for exporting to harm home welfare: if the sum of benefits is less than the sum of harm. Comparing welfare with autarky, \( W_A(q) \), to when \( \bar{q}_H - q_H \) units are exported, \( W(q_H, \bar{q}_L) \), obtains

\[
W(q_H, \bar{q}_L) < W_A(q) \iff p_X < \frac{\bar{p} \int_{q_H+\bar{q}_L}^\bar{q} h(x) \, dx + (\bar{p} - \frac{q_H}{q_H+\bar{q}_L}) \int_0^{q_H+\bar{q}_L} h(x) \, dx}{\bar{q}_H - q_H}.
\]

Note that if in the limit as all high quality is exported, \( i.e., q_H \to \bar{q}_H \) one obtains the condition in Proposition 1 (for when only the first unit is exported). To summarize,

**Proposition 2** In the second stage, for a given level of high quality exports \( q_X^H = \bar{q}_H - q_H \), home welfare decreases with exporting when the export price \( p_X \) is less than the average home value for those units:

\[
p_X < \frac{\bar{p} \int_{q_H+\bar{q}_L}^\bar{q} h(x) \, dx + (\bar{p} - \frac{q_H}{q_H+\bar{q}_L}) \int_0^{q_H+\bar{q}_L} h(x) \, dx}{\bar{q}_H - q_H}. \tag{5}
\]

Further, \( \left[ \bar{p} \int_{q_H+\bar{q}_L}^\bar{q} h(x) \, dx + (\bar{p} - \frac{q_H}{q_H+\bar{q}_L}) \int_0^{q_H+\bar{q}_L} h(x) \, dx \right] / (\bar{q}_H - q_H) \) is increasing in exports.

Thus when all high quality units are exported the upper bound on the export price to reduce home welfare is the greatest: \( p_X < \bar{p} \int_0^\bar{q} h(x) \, dx / \bar{q}_H = \int_0^\bar{q} h(x) \, dx / \bar{q} \). The case of linear demand suggests that the range of prices under which the propositions hold are not trivial.

**Linear Demand.** With linear demand \( (h(q) = 1-q) \), \( P_A = \bar{p}(1-q) \) and the sufficient condition for exporting to harm home welfare (4) simplifies to

\[
p_X < \bar{p}(1-\bar{q}) + (1-\bar{p})(1-\bar{q}/2) = P_A + (1-\bar{p})(1-\bar{q}/2).
\]

For example if \( \bar{p} = 1/2 = \bar{q} \) then \( P_A = 2/3 \) and any \( p_X < 5/6 \) reduces home welfare (Proposition 1). With \( q_H \) on the home market home welfare is

\[
W(q_H, \bar{q}_L) = p_X(\bar{q}_H - q_H) + \frac{q_H}{q_H + \bar{q}_L} \left( q_H + \bar{q}_L - \frac{(q_H + \bar{q}_L)^2}{2} \right).
\]

The condition for home welfare to be harmed (Proposition 2) at \( q_H \) is
\[ p_X < 1 - (\bar{q}_H + q_H + \bar{q}_L)/2. \] (7)

When \( \bar{\rho} = 1/2 = \bar{q} \) and all high quality is exported \( (q_H = 0) \), any \( p_X < 3/4 = 3P_A \) (greater than the home choke-price of 1/2) reduces welfare. For example, if \( p_X = \frac{5.5}{8} \) welfare decreases even though the first unit exported increases welfare. □

We now consider the welfare effect of the marginal exporter: even if home welfare increases at \( q_H \) this does not guarantee that the marginal export unit exported benefited the home country and so, \( e.g. \), an export tax could raise home welfare. Begin with the case when not all high quality is exported \( (i.e., q_H > 0) \) so that the home price equals the export price. From (3) the harm from the unit exported is

\[
\frac{\partial W^H(q_H, \bar{q}_L)}{\partial q_H} = \frac{q_H}{q_H + \bar{q}_L} h(q_H + \bar{q}_L) + \frac{q_H + \bar{q}_L}{(q_H + \bar{q}_L)^2} \int_0^{q_H + \bar{q}_L} h(x)dx > \frac{q_H}{q_H + \bar{q}_L} h(q_H + \bar{q}_L) = p_X,
\]

where the inequality follows since the home price equals the export price. As this is clearly greater than the marginal benefit to home welfare (the export price) from a firm exporting, then so long as \( q_H > 0 \), home welfare always increases with a small restriction on exports. In the case when all units are exported, from (5) we already have the condition for the marginal exporter to harm welfare which is weaker than the condition for exporting to harm welfare:

\[
\int_0^{\bar{q}_L} h(x)dx/\bar{q}_L > \int_0^{\bar{q}} h(x)dx/\bar{q}.
\]

Summarizing we have,

**Proposition 3** In the second stage, home welfare is harmed by the marginal export if in equilibrium either

1. not all high quality is exported \( (i.e., q_H > 0) \) or
2. all high quality is exported and \( p_X < \int_0^{\bar{q}_L} h(x)dx/\bar{q}_L \).

The intuition for this result goes back to the central effect here: an additional high quality unit raises every consumer’s expected value and not just the marginal consumer, but the market price reflects the marginal consumer’s expected value.

**Linear Demand.** With linear demand \( \hat{q}_H \) must satisfy

\[
\frac{q_H}{q_H + \bar{q}_L} (1 - (q_H + \bar{q}_L)) = p_X.
\]

From this condition, the stable \( \hat{q}_H(\bar{q}) \) is
\[
\hat{q}_H(\bar{q}) = \frac{1}{2} \left[ 1 - \bar{q}_L - p_X + \sqrt{1 - 2\bar{q}_L - 2p_X + (\bar{q}_L - p_X)^2} \right].
\] (8)

Let \(\bar{\rho} = .9\) and \(\bar{q} = .9\) so \(P_A = .09\). From (6), if \(p_X \in (.22, .49)\), then home welfare is greater with exporting: \(W_A(\hat{q}_H(\bar{q}), \bar{q}_L) > W(\bar{q})\). Welfare is maximized at \(p_X \approx .47\), but the cost from the last unit exported is greater than the benefit: \(1 - \hat{q}_H - \bar{q}_L \approx \frac{2}{3} > .474 = p_X\). For when all high quality is exported let \(\bar{\rho} = .95 (P_A = .095)\). From (8), if \(p_X > .62\), all high quality is exported. However, the harm from the last unit exported \(1 - \bar{q}_L = .9975\). Thus, for \(p_X > 10P_A\) the marginal export harmed home welfare. \(\square\)

Home producer surplus can also decrease when all high quality is exported. Specifically, if the export price is sufficiently close to the autarkic price, then home producer surplus decreases. This is because with exporting only \(\bar{\rho}\bar{q}\) output sells at \(p_X = \rho(\bar{q}_H, \bar{q}_L) + \varepsilon\), while the remainder sells at cost. So, if \(\varepsilon\) is sufficiently small, then producer surplus with exporting is less than producer surplus in autarky: \([p(\bar{q}_H, \bar{q}_L) + \varepsilon] \rho\bar{q} < [p(\bar{q}_H, \bar{q}_L)] \bar{q}\). More precisely,

**Proposition 4** If the export price is less than the autarkic price when all home units are of high quality \((\bar{q}_H = \bar{q})\), i.e., \(p_X < P(\bar{q}, 0) \equiv h(\bar{q})\), then producer surplus decreases in the second stage when all high quality units are exported. Such an export price greater than the autarkic price \((\bar{\rho} h(\bar{q}))\) always exists.

**Proof.** \(p_X\bar{\rho}\bar{q} < [p(\bar{\rho}\bar{q}, (1 - \bar{\rho})\bar{q})] \bar{q} = \bar{\rho} h(\bar{q})\bar{q} \iff p_X < h(\bar{q})\). Existence follows as the autarkic price \(\bar{\rho} h(\bar{q})\) is less than the sufficient condition \(h(\bar{q})\). \(\square\)

**Linear Demand.** With linear demand the sufficient condition is \(p_X\bar{\rho}\bar{q} \leq \bar{\rho}(1 - \bar{q})\bar{q}\) or \(p_X < 1 - \bar{q}\). For example, let \(\bar{\rho} = 3/5\) and \(\bar{q} = 3/8\), then \(P_A = 3/8\) and for \(p_X < 5/8\) producer surplus decreases. \(\square\)

### 3.3.2 Effect of exporting on the home price

In this section we further examine the effect of exporting on the home price. First, more can be said about how changes in high quality output affects the home market price. Starting from no high quality \((q_H = 0)\) and \(q_L \in (0, 1)\), the home market price is increasing in \(q_H\) since positive value is being added: \(P(0, q_L) = 0\) and \(P_1(0, q_L) > 0\) (since \(\rho_1 > 0\) and \(h(q_L) > 0\)). The average quality effect dominates. However, the average quality effect is
decreasing in high quality ($\rho_{11} < 0$). Eventually the aggregate quantity effect dominates: as either $h(q) \to 0$, or $\rho_1 \to 0$, we have $P_1(q_H, q_L) < 0$. That is, for small $q_H$ the home price is increasing in $q_H$ and for large $q_H$ it is decreasing. Further, for common specifications of high quality demand $h(q)$, e.g., linear (or quadratic or square root) in price, or constant elasticity of demand, $P(q_H, q_L)$ is \textit{strictly unimodal} (single peaked) in $q_H$: the home market price is first strictly increasing and then decreasing in $q_H$.\textsuperscript{15} Indeed, for unimodality not to hold would require near horizontal or vertical segments in high quality demand.\textsuperscript{16} For ease of presentation we will explicitly assume unimodality for the remainder of this subsection although extending the analysis without unimodality is straightforward albeit tedious.

If the home inverse demand is unimodal in high quality, then it follows from the earlier discussion that in equilibrium has all high quality exported if either $P_1(q_H, q_L) > 0$ (equivalently, $\bar{q}_H > q^*_H(\bar{q}_L)$), or if $P_1(q_H, q_L) < 0$ and $P(q^*_H(q_L), q_L) < p_X$. This is because for any $q_H < \bar{q}_H$, $P(q_H, \bar{q}_L) < p_X$. Note that $P_1(q_H, \bar{q}_L) > 0$ is a sufficient condition for prices to exist such that all high quality is exported and producer surplus to decrease (Proposition 4).

With unimodal demand if neither $P_1(q_H, \bar{q}_L) > 0$ nor $P_1(q_H, \bar{q}_L) < 0$, then there exist $q_H$ such that the home price equals the export price and furthermore there is only one pair of $q_H$ that satisfy (2). In such an equilibrium we see from (2) that as the export price increases, the amount of high quality on the home market $\hat{q}_H(q)$ decreases:

$$\frac{\partial \hat{q}_H}{\partial p_X} = -\frac{1}{P_1(\hat{q}_H, (1 - \rho)q)} < 0,$$

since $P_1(\hat{q}_H, \bar{q}_L) < 0$. That is, since the home price is increasing in exports, when the export price increases the only way to maintain (2) is to have more high quality exported.

Consider next the range of export prices that harm home welfare. Surprisingly, welfare need not be increasing in the export price and the set of export prices that harm welfare need not be a convex set because while a higher export price increases exporters’ profit it also increases exports, which increases the harm to the home market. A way to see this is

---

\textsuperscript{15} A function $h(x)$ is a strictly unimodal function if for some value $m$, it is strictly increasing for $x \leq m$ and strictly decreasing for $x \geq m$. Though similar, this concept is distinct to the strict definition of unimodal for distributions, which admits multiple local maxima although those are usually referred to as being multimodal.

\textsuperscript{16} Strict unimodality is a weaker condition than the standard assumption of strategic substitutability (industry marginal revenue is decreasing in aggregate output) as strategic substitutability implies that, with constant elasticity demand for high quality, high quality demand $h(q)$ must be inelastic while with unimodality it can be elastic as well as inelastic. With unimodality there can be strictly convex segments.
that an export price induces the level of exports that equates the home price to the export price. In Figure 1, the home price as a function of exports is plotted $P(\bar{q}_H - q^X_H, \bar{q}_L)$ with

$$p^*_X(q^X_H)$$

$P(\bar{q}_H - q^X_H, \bar{q}_L)$

Figure 1: Non-Monotonicity in Export Price

unimodal demand. A higher export price induces more exports, which increases the harm requiring a higher export price to offset the harm. The minimum export price needed to offset the harm for a given level of exports is plotted as $p^*_X(\bar{q}_H - q^X_H)$, which comes from the RHS of (5):

$$p^*_X(\bar{q}_H - q^X_H) = p^*_X(q^X_H) \equiv \left[ \bar{p} \int_{q_H}^{\bar{q}_H} h(x)dx + (\bar{p} - \frac{q_H}{q_L}) \int_{q_H}^{q_H+\bar{q}_L} h(x)dx \right] / (\bar{q}_H - q_H)$$

That is, for any amount exported $q^X_H$, welfare decreases when the export price $p_X < p^*_X(q^X_H)$. In Figure 1, for low export prices (e.g., $p_1$), exporting harms home welfare, for higher export prices exporting helps home welfare (e.g., $p_2$) and for even higher export prices it harms home welfare again ($p_3$). As the following shows, this can be the case with linear demand.

**Linear Demand** Using $\hat{q}_H(\bar{q})$ from (8), so that the home price equals the export price in welfare (6) gives welfare at $\hat{q}_H(\bar{q})$ as a function of $p_X$:

$$W(p_X) = p_X \left[ \bar{q}_H - \frac{1}{2} \left[ 1 - \bar{q}_L - p_X + \left( 1 - 2\bar{q}_L - 2p_X + (\bar{q}_L - p_X)^2 \right)^{\frac{1}{2}} \right] \right]$$

$$+ \left[ 3 - \bar{q}_L + p_X - (1 - 2\bar{q}_L - 2p_X + (\bar{q}_L - p_X)^2)^{\frac{1}{2}} \right] \left[ 1 - \bar{q}_L - p_X + (1 - 2\bar{q}_L - 2p_X + (\bar{q}_L - p_X)^2)^{\frac{1}{2}} \right].$$
Let $\bar{p} = .9$ and $\bar{q} = .9$ (so $P_A = .09$). For $p_X \in (.22, .49)$, $W(p_X) > W_A(\bar{q})$: home welfare from exporting is greater than home welfare under autarky. However, for $p_X \in (.49, .55)$, all high quality is exported and $W(p_X) < W_A(\bar{q})$: home welfare is greater with autarky. For Figure 1 this would correspond to the second intersection occurring at an output greater than $q_{X*}^H$ (e.g., $P_{X*}^H(q_{X*}^H)$ being sufficiently flatter). For higher export prices home welfare is greater with exporting. □

To summarize

**Lemma 1** *In the second stage, the set of export prices that reduce home welfare need not be convex, e.g., for low export prices exporting harms home welfare, for high prices exporting increases home welfare, but for even higher prices exporting harms home welfare.*

### 3.3.3 Positive sorting costs and value for low quality

We briefly consider two extensions to our model that are related as they both make the equilibrium with incomplete information relatively more attractive. For ease of presentation, we had assumed that sorting costs where zero ($c = 0$). As is known, if the cost of acquiring the information is sufficiently large, then welfare could be greater with incomplete information, i.e., $W_F(\bar{q}) < W_A(\bar{q})$. Adding positive sorting cost makes the incomplete information equilibrium more attractive. Specifically, this means that the upper limit on the developed market price ($p_F$) such that exporting reduces home welfare is even greater: $p_X = p_F - c - t - v$.

That is, that exporting reduces home welfare is even more likely.

A second assumption was that low quality had value equal to marginal cost ($0$). The analysis, however, has shown that the basic cause of the inefficiency results from the home market price reflecting the marginal consumer’s value, while the welfare loss is from the weighted average of the marginal and the average consumer’s valuation for the high quality. So long as low quality does not have greater value than high quality this insight does not change. In particular, with positive value for low quality, the result would be modified to the average consumer’s additional valuation for high quality (over low quality) and so the basic insight remains. More specific to the model here, as there is common valuation for low quality, it is straightforward to consider what occurs if the value for low quality is greater than marginal cost. When comparing autarky to the trade equilibrium, the (now positive)
contribution of low quality sales to welfare is the same. Hence, having a positive value for
low quality works as a shift parameter.

4 First Stage Equilibrium (Long Run Response)

We now move to the first stage of the game which can be interpreted as the long-run effects
of the unanticipated opening of the export market characterized in the previous section.
Recall that in the first stage there is a continuum of heterogeneous firms that simultaneously
choose whether to enter the market.\footnote{Creane and Jeitschko (2009) consider an autarkic model of entry but with firms that are \textit{ex ante} identical, but \textit{ex post} differentiated in cost and quality.}

If home firms cannot trade in the second stage and the firms anticipate this, which will
be denoted as \textit{anticipated autarky}, the second stage of the game is the same as the autarky
game of the previous section. The second stage equilibrium has all firms selling and the price
given \(q\) firms is \(P_A = \overline{p}h(q)\). Given this second stage equilibrium, in the first stage firms enter
until the cost of entry equals the autarkic price given the entry level so the entry equilibrium
\(\overline{q}\) (which previously denoted the arbitrary fixed supply) is defined by \(\overline{p}h(\overline{q}) = e(\overline{q})\). Welfare
in autarky then is Equation (1) plus entry costs:

\[
W_A(\overline{q}) = \int_0^{\overline{q}} \overline{p}h(x)dx - \int_0^{\overline{q}} e(x)dx. \tag{9}
\]

It is straightforward to show that as previously this is the second best welfare optimum.
Specifically, a social welfare maximizer that cannot observe quality would choose the same
level of entry. As before we will use the case of linear demand and (now) cost to illustrate
the model.

\textbf{Linear Demand and Cost: Anticipated Autarky.} In addition to the uniform
distribution on consumer types in the previous section, firms’ type (cost) is also uniformly distributed on \([0,1]\): firm \(q\) has cost \(e(q) = \overline{c}q\) if it enters. With anticipated
autarky, the entry equilibrium \(\overline{q}\) is defined by \(\overline{p}(1 - \overline{q}) - \overline{c}\overline{q} = 0\), i.e., \(\overline{q} = \overline{p}/(\overline{p} + \overline{c})\)
and so the autarky price is \(P_A = \overline{p}\overline{c}/(\overline{p} + \overline{c})\). Welfare, which now includes entry cost, then
is \(W_A(\overline{q}) = \overline{p}^2/2(\overline{p} + \overline{c})\).
4.1 Anticipated Trade

We now characterize the equilibrium level of entry and exports when firms anticipate the ability to export their high quality goods. From the previous section we know that given some level of entry \( q \), there are two possible equilibria. The first equilibrium has all high quality units exported and all low quality units remaining at home. The home price \( P = 0 \). This is always a second stage equilibrium and with unimodal demand is a unique one if either \( P_1(q_H, q_L) > 0 \) or, \( P_1(q_H, q_L) \leq 0 \) and \( P(q_H^*(q_L), q_L) < p_X \). If firms expect all high quality to be exported, then in the first period firms enter until the marginal firm’s profit from exporting (\( \bar{p} \) of their output is exported) equals its entry cost:

\[
\bar{p}p_X = e(q_X). \tag{10}
\]

The two-stage equilibrium then has \( q_X \) firms entering in the first stage, and in the second stage all high quality units are exported. From the previous section we know that this second stage equilibrium of all high quality being exported always exist and may be unique. Thus, this two-stage equilibrium always exists and may be unique. Further, from (10), the entry level \( q_X \) is increasing in the export price \( p_X \).

The first thing we note is that it is possible that the equilibrium level of entry with anticipated trade may be less than that with anticipated autarky, i.e., it is possible that \( q_X < \bar{q} \). Specifically, if \( \bar{p}p_X < \bar{p}h(\bar{q}) = P_A \), then \( q_X < \bar{q} \). This corresponds to the case in the previous section when producer surplus decreases with all high quality exported and indeed producer surplus is lower in the anticipated trade equilibrium than in the anticipated autarky equilibrium as each firms earns higher profit with anticipated autarky. As this is the condition in Proposition 4 for producer surplus to decrease (and so home welfare) with the unanticipated opening of the export market, there is an implication in a dynamic setting. That is, imagine the two stage game is repeated over two periods, i.e., in Period 1 the two stage game is played (entry, then exchange), and then in Period 2 it is played again. In this setting imagine that in Period 1 firms anticipated autarky, but by an unanticipated policy shift (say by the importing countries), trade is allowed. In period two trade is anticipated. In this scenario, the country that unexpectedly sees its export market open has its welfare decrease initially (by Proposition 4) and over time sees its exports (and industry production)
decrease, \textit{i.e.}, its export industry \textit{contracting}.

From the previous section there is also potentially a second stage equilibrium in which some high quality output is not exported, remaining at home. For this to be an equilibrium the home price in the second stage equilibrium must equal $p_X$. If in the first stage firms expect this second stage equilibrium, then the marginal entrant is the firm whose entry cost equals the export price:

$$p_X = e(q_X). \quad (11)$$

The two-stage equilibrium in this case then has $q_X$ firms entering in the first stage, and in the second stage some high quality is sold on the home market and the home price is $p_X$. From (11), the entry level is increasing in the export price $p_X$. Thus, since $p_X > P_A$ it is necessarily true that $q_X > \bar{q}$.

As established earlier, for such an equilibrium to exist with unimodal demand, necessary and sufficient conditions are that the maximum home price with this entry level $P(q^*_H(q^X_L),q^X_L)$ is greater than the export price and that $q^*_H(q^X_L)$ is feasible:

$$P(q^*_H(q^X_L),q^X_L) \geq p_X, \quad (12)$$

$$P_1(q_H,q_L) < 0. \quad (13)$$

The stable equilibrium level of high quality on the home market such that the home price equaled the export price, Equation (2), was denoted by $\hat{q}_H$.

**Linear Demand and Cost: Anticipated Trade.** For the case of all high quality being exported with anticipated trade, from Equation (10), we have $q_X = p_X \bar{p}/\bar{c}$. For the case of some high quality staying in the home market, the entry level is defined by Equation (11), or $q_X = p_X/\bar{c}$. For this to be an equilibrium requires that the maximum home price is greater than the export price (12), $P(q^*_H(q^X_L),q^X_L) = (1 - \sqrt{(1 - \bar{p})p_X/\bar{c}})^2 \geq p_X$ and that it is feasible (13), $P_1(q^*_H(q^X_L),q^X_L) = (1 - \rho)\bar{c}/p_X - 1 < 0$. In this linear case Equation (2), the level of high quality in the home market that makes the home price equal to the export price, is

$$p_X = \frac{\hat{q}_H}{\hat{q}_H + (1 - \bar{p})p_X/\bar{c}}(1 - \hat{q}_H - (1 - \bar{p})P_X/\bar{c}). \quad (14)$$
4.1.1 Characteristics of entry and export levels

In this section we establish some comparative characteristics between entry levels when the second stage has some high quality sold on the home market, which will be useful in establishing comparisons between unanticipated and anticipated trade, as well determining which equilibria may arise for a given export price. We begin by noting that using the implicit function theorem on Equation (2) we have that for fixed $p_X$ as there is more entry ($q$ increases), $\hat{q}_H(q)$ decreases

$$\frac{\partial \hat{q}_H}{\partial q} = -\frac{P_2(\hat{q}_H, (1-\bar{p}))q(1-\bar{p})}{P_1(\hat{q}_H, (1-\bar{p}))} < 0.$$ 

This follows since $P_1(\hat{q}_H, q_L) < 0$ and $P_2 < 0$ (increasing low quality always reduces the price as it both increases aggregate quantity and lowers average quality). In other words, for a given export price, as the mass of home firms increases, the amount of high quality on the home market necessary to equate the home market price to the export price decreases. This is intuitive: a greater mass of firms means more low quality on the home market and the only way to maintain the export price is by reducing high quality on the home market as $P_1 < 0$. Note also that aggregate output on the home market must decrease to maintain the export price since average quality is lower.

Next we compare the home market price when there are equal levels of output on the home market and different levels of entry (hence different amounts of high quality are exported). Specifically, consider two levels of entry $q_1$ and $q_2$, and denote the amount of high quality on the home market with the first level of entry as $q_{1H}$. Define $\hat{q}_{2H}(q_{1H})$ as the level of high quality on the home market with the second level of entry such that total output on the home market is equal: $q_{1H} + (1-\bar{p})q_1 = \hat{q}_{2H}(q_{1H}) + (1-\bar{p})q_2$. With this there as an immediate lemma: for two different levels of entry, if the same amount is sold on the home market, then the market price is lower with the greater level of entry as average quality must be lower.

**Lemma 2** Consider two levels of entry $q_1$ and $q_2$, with $q_1 < q_2$, then for the same amount sold on home market the home price is always lower with the higher level of entry: $P(q_{1H}, (1-\bar{p})q_1) > P(\hat{q}_{2H}(q_{1H}), (1-\bar{p})q_2)$.

This proof and the remaining proofs are left to the appendix.
When there exist a unique $q_H$ on the home market that maximizes the home price (i.e., $q_H^*(q_L)$), then Lemma 2 implies an immediate and intuitive result: the maximum home price is decreasing in entry.\footnote{Note that the existence of $q_H^*(q_L)$ is a weaker assumption than unimodality, i.e., unimodality need not hold for for there to be a $q_H^*(q_L)$.} To see why, consider for the greater level of entry ($q_2$) the amount of high quality on the home market that maximizes the home price: $q_H^*((1-\bar{p})q_2)$. However, by Lemma 2, with less entry ($q_1$) the same aggregate output ($q_H + (1-\bar{p})q_1 = q_H^*((1-\bar{p})q_2) + (1-\bar{p})q_1$) on the home market yield a greater price, say $\hat{P}$. This latter price is by definition less than the maximum price given entry $q_1$: $\hat{P} \leq P(q_H^*((1-\bar{p})q_1), (1-\bar{p})q_1)$. Thus, we have the following corollary:

**Corollary 1** The maximum home price $P(q_H^*(q_L), q_L)$ is decreasing in entry.

Note that this price may not always be feasible. However, the continuity of $P_1(q_H, q_L)$, Corollary 1 and $P'(q) < 0$ imply that the maximum feasible price is also decreasing in entry. Let $P^F(q)$ denote the maximum feasible home price given entry level $q$: $P^F(q) = P(\text{Min} \{q_H^*(q_L), q_H\}, q_L)$.

**Lemma 3** The maximum feasible home price $P^F(q)$ is decreasing in entry $q$.

Finally, we have some results regarding the effects of the export price. First, for the case of all export equilibria there is only one comparative static and it is trivial: by Equation (10) as the export price increases, exports increase. Turning to the case of when some of the high quality remains on the home market, using (11) and (2) we have from the implicit function theorem that as the export price increases, the amount on the home market decreases:

$$\frac{\partial \hat{q}_H}{\partial p_X} = \frac{-(1-P_2(\hat{q}_H, (1-\bar{p})q_X(p_X))(1-\bar{p})q_X'(p_X))}{P_1(\hat{q}_H, (1-\bar{p})q_X(p_X))} \leq 0$$

This implies the seemingly obvious result that as the export price increases, exports increase (since entry increases). With these lemmas and corollaries we can now see how equilibria compare between anticipated and unanticipated trade.
4.2 Welfare

We begin the analysis with a simple observation: Propositions 1-4 can be applied to the equilibrium entry level $q_X$. That is, if the entry level $q_X$ meets the conditions in a proposition, then the proposition can be applied comparing welfare with anticipated trade to welfare if instead trade was unexpectedly banned in stage two (i.e., welfare with $q_X$ level of entry and no trade).\(^{19}\) This comparison allows for the establishing of a sufficient condition for welfare to decrease with exporting when there is entry, that is, when the full game is considered (with the proof in the appendix):

**Proposition 5** Welfare is greater with anticipated autarky than with anticipated trade when the condition in Proposition 2 holds for $q_X$.

Proposition 5 is only a sufficient condition: the condition in Proposition 2 may not be met for $q_X$ (i.e., given entry level $q_X$ trade has greater welfare (denoted $W_T(q_X)$) than autarky denoted $W_A(q_X)$) and welfare still can be greater with anticipated autarky ($W_A(q)$) than with anticipated trade ($W_T(q_X)$). As we will see in Example 1 below, this is easily possible.

We next consider whether there always exist export prices that reduce home welfare when there is entry. The welfare results of the previous section suggest this is so. We consider each type of possible equilibrium separately, beginning with when all high quality is exported. We first note that Proposition 4 still holds with entry: if $p_X \leq P_A/\bar{\rho}$, then producer surplus decreases when all high quality is exported. To see this, note that producer surplus with autarky less producer surplus with entry is

$$P_A\bar{q} - \int_0^\bar{q} e(x)dx - \left[ p_X \bar{q} - \int_0^{q_X} \right] = \left( P_A(q - q_X) - \int_{q_X}^{\bar{q}} e(x)dx \right) + \left[ P_A - p_X\bar{\rho} \right] q_X. \quad (15)$$

When $p_X \leq P_A/\bar{\rho}$, then $q_X \leq \bar{q}$, so the parenthetical term in (15) is positive since $P_A = e(\bar{q})$ by the entry condition, and so (15) is positive. Thus, Proposition 4 still holds with entry: producer surplus decreases when all high quality is exported if $p_X \leq P_A/\bar{\rho}$.

Turning to home welfare when all high quality is exported, welfare is

$$W_T(p_X) = p_X \bar{q} q_X(p_X) - \int_0^{q_X(\bar{p}p_X)} e(x)dx. \quad (16)$$

\(^{19}\)This comparison could also to be used to consider the welfare effects of *unanticipated* no trade.
Welfare with autarky, (9), less welfare with all exporting, (16), is

\[ W_A(q) - W_T(p_X) = \int_0^q \rho h(x)dx - \int_0^q e(x)dx - p_X q_X (\rho p_X) + \int_0^{q_X (\rho p_X)} e(x)dx. \]  

(17)

This simplifies to

\[ W_A(q) - W_T(p_X) = \int_0^q \rho h(x)dx - p_X q_X (\rho p_X) + \int_0^{q_X (\rho p_X)} e(x)dx. \]

With this we have an entry version of Proposition 1 for when all high quality is exported when \( p_X \to \frac{P_A}{\rho} \) (note that \( \frac{P_A}{\rho} > P_A \)):

**Lemma 4** There exist export prices such that home producer surplus (hence, welfare) is lower when all high quality is exported than with autarky.

The following example shows that \( p_X \) need not be arbitrarily close to \( \frac{P_A}{\rho} \) for the lemma to hold, that the condition in Proposition 5 need not hold for welfare to be greater with autarky and that welfare can decrease even though entry (home production) increases.

**Linear Demand and Cost: Welfare decreasing trade when all high quality is exported**

\[ q_X = \frac{p_X \rho}{c} \]

It is straightforward to derive welfare with anticipated trade \( W_T(q_X) = \frac{p_X^2 \rho^2}{2c} \). For when trade is unexpectedly blocked, since \( q_X = \frac{p_X \rho}{c} \) firms entered, welfare is \( W_A(q_X) = \frac{p_X^2 \rho^2 (2c - p_X (\rho + \rho))}{2c} \). Finally, autarky welfare is denoted \( W_N T(\bar{q}) \). In all of these examples \( W_T(p_X \rho/\bar{c}) > W_A(p_X \rho/\bar{c}) \) and yet \( \bar{q} = \rho/ (\rho + \bar{c}) \):

\[ W_A(\rho/(\rho + \bar{c})) > W_T(p_X \rho/\bar{c}). \]

**Example 1**

<table>
<thead>
<tr>
<th>( \bar{p} )</th>
<th>( \bar{c} )</th>
<th>( p_X )</th>
<th>( q_X )</th>
<th>( P_A )</th>
<th>( \bar{q} )</th>
<th>( W_T(q_X) )</th>
<th>( W_A(q_X) )</th>
<th>( W_A(\bar{q}) )</th>
<th>( P_A/\bar{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>2</td>
<td>0.82</td>
<td>.369</td>
<td>.621</td>
<td>.31</td>
<td>.136</td>
<td>.135</td>
<td>.139</td>
<td>.69</td>
</tr>
<tr>
<td>0.8</td>
<td>2</td>
<td>0.84</td>
<td>.336</td>
<td>.571</td>
<td>.286</td>
<td>.113</td>
<td>.111</td>
<td>.114</td>
<td>.714</td>
</tr>
<tr>
<td>0.7</td>
<td>2</td>
<td>0.86</td>
<td>.301</td>
<td>.519</td>
<td>.259</td>
<td>.090</td>
<td>.088</td>
<td>.091</td>
<td>.741</td>
</tr>
<tr>
<td>0.6</td>
<td>2</td>
<td>0.87</td>
<td>.261</td>
<td>.462</td>
<td>.231</td>
<td>.0681</td>
<td>.0680</td>
<td>.0692</td>
<td>.769</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>0.89</td>
<td>.223</td>
<td>.4</td>
<td>.2</td>
<td>.0495</td>
<td>.0494</td>
<td>.05</td>
<td>.8</td>
</tr>
</tbody>
</table>

\[20\text{In these examples, all high quality being exported is the unique equilibrium. That is, there does not exist a } q_X \text{ in these examples that satisfy conditions (12) and (13).}\]

\[21\text{Note that while there is a range of } p_X \text{ that gives } W_T(q_X) > W_A(q_X), \text{ for space consideration only a specific value is reported in the table.}\]
Further, for sufficiently lower export prices $p_X$, welfare is greater when exporting is unexpectedly blocked: $W_T(q_X) < W_A(q_X)$ and so by Proposition 5 $W_T(q_X) < W_A(\bar{q})$.

Turning to the case in which some high quality is sold on the home market, welfare is exporters’ profit, home consumer welfare and entry cost:

$$W_T(p_X) = p_X \left[ \overline{\rho} q_X(p_X) - \hat{q}_H(q_X(p_X)) \right]$$

\[+ \int_0^{\hat{q}_H(q_X(p_X))+(1-\overline{\rho})q_X(p_X)} \frac{\hat{q}_H(q_X(p_X))}{\hat{q}_H(q_X(p_X)) + (1-\overline{\rho})q_X(p_X)} h(x)dx \]

\[- \int_0^{q_X(p_X)} e(x)dx. \tag{18} \]

Differentiating $W_T(p_X)$ (with arguments suppressed for ease of reading) obtains

$$W'_T(p_X) = \left[ \overline{\rho} q_X - \hat{q}_H \right] + p_X \left[ \overline{\rho} q_X' - \hat{q}_H q_X' \right]$$

\[+ \left[ \frac{\hat{q}_H' + (1-\overline{\rho})q_X'}{\hat{q}_H + (1-\overline{\rho})q_X} \right] h(\hat{q}_H + (1-\overline{\rho})q_X) \]

\[+ \int_0^{\hat{q}_H + (1-\overline{\rho})q_X} \left( 1-\rho \right)(\hat{q}_H q_X' q_X - \hat{q}_H q_X h(x)dx - q_X' e(q_X). \tag{19} \]

The integrand is negative (average quality on the home market decreases) and also $[\hat{q}_H' + (1-\overline{\rho})]$; for the home price to increase given that there is more low quality (as $q_X' > 0$), aggregate output on the home market must decrease. With this we can obtain (see the appendix)

**Lemma 5** There exist export prices such that welfare is lower when some high quality is exported than it is with autarky.

The following example with linear demand and cost shows that in this case as well the condition in Proposition 5 need not hold for welfare to be greater with autarky and that welfare can decrease even though entry (home production) increases.

**Linear Demand and Cost: Welfare decreasing trade when some high quality is exported** From (11), the entry level is $q_X = p_X/\overline{\rho}$. For this equilibrium, conditions (12) and (13) must hold and then there exists an amount of high quality in the home market $\hat{q}_H$ that makes the home price equal to the export price (14). Welfare then is high quality export revenues, home consumer benefit less entry costs:

$$W_T(p_X/\overline{\rho}) = p_X(\overline{\rho} p_X/\overline{\rho} - \hat{q}_H) + \int_0^{(1-\overline{\rho})p_X/\overline{\rho} + \hat{q}_H} \frac{\hat{q}_H}{(1-\overline{\rho})p_X/\overline{\rho} + \hat{q}_H} (1-x)dx - \int_0^{p_X/\overline{\rho}} \overline{\rho} xdx. \tag{27}$$
With no trade \((q_X = p_X/\bar{c} \text{ entering})\) welfare is \(W_A(p_X/\bar{c}) = p_X(2\bar{c}\bar{p} - p_X(\bar{c} + \bar{p}))/2\bar{c}^2\).

For some high quality in the home market to be an equilibrium, the export price needs to be lower than in Example 1. To obtain an equilibrium with welfare greater with trade than unexpected autarky \((W_T(q_X) > W_A(q_X))\), production costs need to be reduced to \(\bar{c} = .2\) and even then this only occurs for \(\bar{p}\) roughly greater than .8. In the other cases, even if firms were anticipating trade, welfare is greater with unanticipated autarky \((W_T(q_X) < W_A(q_X))\) and so by Proposition 5 welfare is greater with autarky than with anticipated trade.\(^{22}\)

**Example 2**

<table>
<thead>
<tr>
<th>(\bar{p})</th>
<th>(\bar{c})</th>
<th>(p_X)</th>
<th>(q_X)</th>
<th>(P_A)</th>
<th>(\bar{q})</th>
<th>(W_T(q_X))</th>
<th>(W_A(q_X))</th>
<th>(W_A(\bar{q}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>.2</td>
<td>0.2</td>
<td>1</td>
<td>.164</td>
<td>.818</td>
<td>.356</td>
<td>.35</td>
<td>.368</td>
</tr>
<tr>
<td>0.8</td>
<td>.2</td>
<td>0.23</td>
<td>1.15</td>
<td>.16</td>
<td>.8</td>
<td>.264</td>
<td>.259</td>
<td>.32</td>
</tr>
</tbody>
</table>

\(\square\)

Combining Lemmas 4 and 5 we have

**Proposition 6** *In the two stage game with entry in the first stage (long-run), there exist export prices such that welfare is lower with trade than with autarky.*

We can also use Equations (9) and (18), along with the fact that aggregate output \((\hat{q}_H + (1 - \bar{p})q_X)\) must decrease on the home market for the home price to increase when some is sold on the home market, to obtain an entry version of Proposition 2

**Corollary 2** *Home welfare decreases with exporting when the export price \(p_X\) is less than*

\[
\left[ \bar{p} \int_{\hat{q}_H + q_X^L}^{\bar{q}} h(x)dx + (\bar{p} - \frac{\hat{q}_H}{\hat{q}_H + q_X^L}) \int_{0}^{\hat{q}_H + q_X^L} h(x)dx + \int_{\bar{q}}^{q_X} e(x)dx \right] / [(\bar{p}q_X - \hat{q}_H)]. \tag{20}
\]

There is also with entry a similar result to Proposition 3 with fixed supply: the marginal entrant reduces home welfare. However, this only holds when some high quality remains on

\(^{22}\)Note that because we are for ease assuming that the export price is fixed, caution must be used when lowering production costs. Recall from the previous section that relaxing the fixed export price assumption is likely to make trade more harmful. To see this consider the limit: if \(\bar{c} = 0\), then the home country would supply an infinite amount of the good as the export price does not change (and welfare would increase).
the home market as, even though welfare can be lower when all export, given that all are being exported the marginal entrant’s cost just equals its benefit \((p_X)\). Using the welfare expression (18) as a function of \(q\), we obtain (see the appendix for the proof):

**Proposition 7** *Home welfare is harmed by the marginal entrant if in equilibrium not all high quality is exported.*

A different interesting result is that an *increase* in the export price can reduce home welfare. This occurs when not all high quality is exported: as the export price increases, there is a maximum export price such that there is an amount of high quality on the home market to equate the home price to the export price. Denote this as \(\hat{p}_X\).

**Proposition 8** *When some high quality remains on the home market, home welfare decreases in the export price at \(\hat{p}_X\).*

Since we know that when some high quality is exported there always exist export prices sufficiently close to the autarkic price that harms welfare, as in the short-run it is possible that the effect of exporting on home welfare is non-monotonic in export prices.

### 4.2.1 Comparing autarky, anticipated & unanticipated trade welfare

We first examine if the maximum export price such that home welfare is harmed by trade could be higher with anticipated trade than anticipated trade. That is, could it be that there are export prices such that autarky welfare dominates anticipated trade, but autarky does not dominate unanticipated trade? This would be surprising as entry would *expand* the export prices such that trade harms home welfare. To put it differently, allowing for long run responses should normally increase welfare, but it may do the opposite.

Consider first the case when all high quality is exported. With unanticipated trade autarkic welfare less trade welfare yields

\[
\int_0^{\bar{q}} \bar{p}h(x)dx - p_X\bar{q}.
\]  

(21)

Let \(p_X^*\) denote the export price such that this difference (21) is zero. With anticipated trade, autarkic welfare less trade welfare (17) can be rearranged as

\[
\int_0^{\bar{q}} \bar{p}h(x)dx - \int_0^{\bar{q}} e(x)dx - \left[ p_X\bar{q} - \int_0^{\bar{q}} e(x)dx + p_X\bar{p}[q_X - \bar{q}] - \int_0^{q_X} e(x)dx \right].
\]
At $p_X^*$ this simplifies to

$$-p_X\bar{p}[q_X - \bar{q}] + \int_{\bar{q}}^{q_X} e(x)dx < 0,$$

since $p_X\bar{p} = e(q_X)$ by the entry condition (10) and $p_X^* > P_A/\rho$ so $q_X > \bar{q}$. That is, welfare is strictly greater with anticipated trade: with unanticipated trade there are export prices that harm home welfare that do not harm home welfare with anticipated trade.

Turning to when some high quality is sold on the home market with entry then, as $q_X > \bar{q}$, $\hat{q}^H$ with entry (20) is less than $\hat{q}^H$ with autarky (5) since $\partial \hat{q}^H/\partial q \leq 0$ and so less remains on the home market. Thus, the first two terms in the numerator of (20) are greater: there is greater loss to consumers with entry because even less high quality remains on the home market. The final term in the numerator is the cost of the additional entry and so the numerator is clearly greater in (20). However, the greater entry and less remaining in the home market means more export and so the denominator is also larger in (20). In addition, with anticipated trade the marginal entrant reduces home welfare (Proposition 7).

Unfortunately, a meaningful condition has not been obtained, even restricting the model to linear demand. However, with linear demand, no example was found in which when autarkic welfare was greater than unanticipated trade welfare, anticipated trade welfare was greater than autarkic welfare. That is, in all examples there are export prices with anticipated trade that harm welfare that do not harm home welfare with unanticipated trade.

**Conjecture 1** With linear demand and some high quality sold on the home market, when autarkic welfare is greater than unanticipated trade welfare, then autarkic welfare is greater than anticipated trade welfare.

There is a second way in which anticipated trade (i.e., the two stage game in this section) can be welfare dominated by unanticipated trade (i.e., trade when entry is based on anticipated autarky). This is because the second stage equilibrium – whether all or some of high quality is exported – can depend on the entry equilibrium, which in turn depends on whether trade is anticipated. Specifically, what can occur is that with unanticipated trade (i.e., trade based on the autarkic level of entry $\bar{q}$), the equilibrium has some high quality

---

23 $P_A/\bar{p}$ is the export price at which producer surplus in autarky equals producer surplus with unanticipated trade. Since the unanticipated trade results in loss of home consumer surplus at this export price, (21) is negative so $p_X^*$ must be greater than $P_A/\bar{p}$. 

30
sold on the home market. (This is true, e.g., if \( p_X = \hat{p}_X \) since \( \bar{q} < q_X(\hat{p}_X) \) and \( \partial \hat{q}_H / \partial q < 0 \).) However, once trade is anticipated, this results in more entry so long as the home price equals the export price. More entry creates a downward pressure on the home price and as a result, at the zero profit level of entry when the price is \( p_X > \hat{p}_X \), there may not exist an amount of high quality sold on the home market such that the home price equals the export price. As a result, the only equilibrium now is all high quality being exported. Recalling that the welfare harm to the home market is increasing in its loss of high quality output, it is possible then that welfare decreases. Specifically, a sufficient condition is

**Proposition 9** If \( \hat{p}_X < P_A/\bar{p} = h(\bar{q}) \), then there exist export prices such that home welfare with unanticipated trade is greater than home welfare with anticipated trade.

That such \( \hat{p}_H \) can exist and that welfare can be greater with unanticipated trade even when the condition in the proposition does not hold (i.e., when \( \hat{p}_X > P_A/\bar{p} \)) is demonstrated in the next examples.

**Linear Demand and Cost: Greater welfare with unanticipated trade than anticipated trade.** For these examples, there is an equilibrium with some high quality in the home market with unanticipated trade and the unique equilibrium when trade is anticipated has all high quality exported. Let \( W_T(\bar{q}) \) denote welfare with unanticipated trade in an equilibrium with some high quality in the home market defined by Equation (14). Recalling that \( \bar{q} = \bar{p}/(\bar{p} + \bar{c}) \), we have

\[
W_T \left( \frac{\bar{p}}{\bar{p} + \bar{c}} \right) = p_X \left( \frac{\bar{p}^2}{\bar{p} + \bar{c}} - \hat{q}_H \right) + \int_0^{(1-\bar{p})\frac{\bar{p}^2}{\bar{p} + \bar{c}} + \hat{q}_H} \hat{q}_H (1 - x) dx - \int_0^{\frac{\bar{p}}{\bar{p} + \bar{c}}} \bar{c} x dx.
\]

With anticipated trade and all high quality being exported as \( q_X = \bar{p}p_X/\bar{c} \) entering welfare is \( W_T(\bar{p}p_X/\bar{c}) = p_X^2 \bar{p}^2 / 2\bar{c} \). With this we have

**Example 3**

<table>
<thead>
<tr>
<th>( \bar{p} )</th>
<th>( \bar{c} )</th>
<th>( p_X )</th>
<th>( q_X )</th>
<th>( P_A )</th>
<th>( \bar{q} )</th>
<th>( \hat{p}_X )</th>
<th>( W_T(q_X) )</th>
<th>( W_T(\bar{q}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>1</td>
<td>0.6</td>
<td>.54</td>
<td>.47</td>
<td>.47</td>
<td>.58</td>
<td>.15</td>
<td>.20</td>
</tr>
<tr>
<td>0.8</td>
<td>.4</td>
<td>0.4</td>
<td>.8</td>
<td>.267</td>
<td>.6</td>
<td>.34</td>
<td>.13</td>
<td>.23</td>
</tr>
<tr>
<td>0.7</td>
<td>.1</td>
<td>0.2</td>
<td>1.4</td>
<td>.09</td>
<td>.875</td>
<td>.13</td>
<td>.09</td>
<td>.27</td>
</tr>
<tr>
<td>0.6</td>
<td>.5</td>
<td>0.28</td>
<td>.34</td>
<td>.27</td>
<td>.55</td>
<td>.28</td>
<td>.03</td>
<td>.15</td>
</tr>
<tr>
<td>0.5</td>
<td>.4</td>
<td>0.223</td>
<td>.28</td>
<td>.22</td>
<td>.55</td>
<td>.22</td>
<td>.02</td>
<td>.14</td>
</tr>
</tbody>
</table>
Note that when $\bar{p} = 0.7, .8, .9$, output increases, the export sector grows as in previous examples, and welfare still decreases and that the condition in proposition (9), $\hat{p}_X < P_A/\bar{p}$, is not met and still welfare is greater with unanticipated trade.

4.3 Comparison of Unanticipated to Anticipated Trade Equilibria

The possibility of the export equilibria depending on whether trade is anticipated raises the question of how the equilibria with unanticipated trade (essentially those derived in the previous section) map into possible equilibria when trade is anticipated. This is a case in which using linear demand could lead to an incorrect conjecture. With linear demand if $\bar{q} < 1 - \bar{p}$, then all export is the unique equilibrium. Thus, it might seem that if $q_X > 1 - \bar{p} > \bar{q}$, then there may exist an equilibrium with some high quality in the home market. However, this is incorrect: as we show below, with unimodal demand if the only equilibrium with unanticipated trade has all high quality exported, then the only equilibrium with anticipated trade has all high quality exported.

To intuitively see why there cannot be an anticipated trade equilibrium with some high quality on the home market when there is no such equilibrium with unanticipated trade, note that there would be greater entry in the first stage if such an equilibrium existed: $q_X > \bar{q}$ since $p_X > P_A$. This increased entry would lower the maximum feasible home price by Lemma 3 and given the maximum feasible home price in autarky (with $\bar{q}$ entrants) was less than the export price (since all export was the unique equilibrium), the maximum feasible home price with $q_X$ entrants must be less than the export price. In addition, as Example 3 (unanticipated trade having greater welfare than anticipated trade) demonstrates, an equilibrium with some high quality on the home market with unanticipated trade (i.e., starting from autarky output) is not a sufficient condition for the existence of such an equilibrium with anticipated trade.

**Proposition 10** If the unique autarkic equilibrium with unanticipated trade is all high quality exported, then the unique equilibrium with anticipated trade is all high quality exported.
The existence of an autarkic equilibrium with unanticipated trade with some high quality does not imply such an equilibrium exist with anticipated trade.

Finally, we note from the analysis of the previous section that with unimodal demand the equilibrium with all high quality exported, i.e., defined by Equation (10), is the unique equilibrium if \( p_X > P^F(q_X) \). That is, if all high quality is exported is the entry equilibrium and this is not a result of a coordination failure, then this is the unique equilibrium.

5 Investments in Quality Improvements

We now extend our model to allow firms to choose whether to invest to improve the fraction of high quality. That is, we allow for a second type of moral hazard given that firms also chose whether to sort their output. As is shown below, welfare can easily decrease with trade even when there is moral hazard in the investment for quality. The model is as in the previous section except that when a firm chooses whether to enter the market it also chooses whether to invest to increase the fraction of its output that is of high quality from \( \rho \) to \( \rho_I \), \( \bar{p} < \rho_I \). Such an investment increases an entrant \( q \)'s entry cost: \( e_I(q) > e(q) \).

It is useful to begin by considering what occurs if in the home market quality could be certified. In this case, an entrant \( q \) would choose to invest so long as

\[
\rho_I p - e_I(q) > \bar{p} p - e(q),
\]

where \( p \) is the price it anticipates receiving in the second stage. It is assumed that investing is efficient for all potential entrants: \( \rho_I/e_I(q) \geq \bar{p}/e(q) \).\(^{24}\) As this makes for the strongest form of moral hazard in investment, it is assumed true.

5.1 Autarky

With autarky the second stage does not change: all firms choose not to sort and all quality receives the same price. Thus, in the first stage it is more profitable to not invest:

\[
p - e_I(q) < p - e(q).
\]

\(^{24}\)As \( e(0) \) was normalized to zero this requires that \( e_I(0) \) also equals zero.
As a result, the autarkic equilibrium is the same as the previous section $\overline{\rho h}(\overline{q}) = e(\overline{q})$ and welfare is the same (Equation (9)).

5.2 Trade

As before, the first stage decision depends on the equilibrium anticipated in the second stage. We consider first what occurs when some high quality is sold on the home market and then when all high quality is exported.

5.2.1 Some high quality exported in the second stage

If the second stage equilibrium has some high quality exported, then in the second stage all firms – whether they export or sell on the home market – receive the same price for their high and low quality product. As a result, if such an equilibrium is anticipated in the first stage, and so it is anticipated that low and high quality receives the same price, then all firms would choose not to invest (Equation (22)). Thus, all the results from the previous section for when some high quality is exported still hold even with moral hazard in investment:

**Proposition 11** When in the first stage entrants can make a costly but efficient investment to increase the fraction of high quality and in the second stage some high quality is exported:

1. there exist export prices such that welfare decreases from exporting,

2. home welfare is harmed by the marginal entrant,

3. there exist an export price at which home welfare is decreasing in the export price

4. when autarkic welfare is greater than unanticipated trade welfare, then autarkic welfare can be greater than anticipated trade welfare, and

5. welfare can be greater with unanticipated trade than with anticipated trade.
5.2.2 All high quality exported in the second stage

When in the first stage firms expect all high quality to be exported, then by the efficiency assumption \( \rho_I/e_I(q) \geq \rho/e(q) \) investing is more profitable for any entrant:

\[
p_X \rho_I - e_I(q) > p_X \bar{p} - e(q).
\]

As a result, the entry level, denoted \( q_I \) is now defined by

\[
p_X \rho_I - e_I(q) = 0
\]

and so welfare is simply producer surplus:

\[
p_X \rho_I q_I - \int_0^{q_I} e(x)dx.
\]

Intuitively it should be clear from previous results that it is possible to have trade still reduce welfare as compared to autarky by following the proof to Lemma 4. For example as \( e(q) \to e_I(q) \) and \( \bar{p} \to \rho_I \), there can exist export prices such that welfare and producer surplus can decrease. However, this and the potential for this to occur is more clearly seen by considering the linear demand and cost case.

**Linear demand and cost** As before let \( h(q) = 1 - q \) and \( e(q) = \bar{c}q \), but now let \( e_I(q) = c_I q, \bar{c} < c_I \). Since no firm invests, autarkic producer surplus and welfare is as previously determined in Section 4: \( PS_A(\bar{p}) = \bar{c} \bar{p}^2/2(\bar{p} + \bar{c})^2 \) and \( W_A(\bar{p}) = \bar{p}^2/2(\bar{p} + \bar{c}) \).

When instead all high quality is exported in the second stage, firms enter until

\[
\rho_I p_X - c_I q = 0
\]

and so welfare is simply producer surplus

\[
W_I(\rho_I) = PS(\rho_I) = p_X \rho_I \frac{\rho_I p_X}{c_I} - \int_0^{\frac{\rho_I p_X}{c_I}} c_I x dx = p_X^2 \rho_I^2 / 2c_I.
\]

Producer surplus is greater with autarky \( (PS(\bar{p}) > PS(\rho_I)) \) so long \( p_X < p_X^{PS} \equiv \rho_I (\bar{c}c_I)^{1/2}/\rho_I (\bar{c} + \bar{p}) \). (The condition for welfare is \( p_X < p_X^W \equiv \rho_I (c_I)^{1/2}/\rho_I (\bar{c} + \bar{p})^{1/2} \). It is straightforward to show that \( p_X^{PS} > P_A(\bar{p}) \equiv \bar{p}\bar{c}/(\bar{p} + \bar{c}) \): there always exist export prices such that producer surplus (and so welfare) decreases with exporting. To get a sense of how much greater \( p_X \) can be than \( P_A \) and producer surplus or welfare decreases with trade, we provide the following examples.

35
We conclude with

**Proposition 12** If entrants can make a costly, but efficient investment to increase the fraction of high quality and all high quality is exported, then with linear demand and entry cost there exists export prices such that home producer surplus and welfare decrease with exporting.

### 6 Conclusion

International trade is rightfully considered welfare enhancing in most settings and an important means by which lesser developed economies can grow. For countries with weak institutions, trading partners with strong institutions can also offer a means for both to circumvent the barriers to growth associated with weak institutions.

We develop a model in which trade can allow home firms to bypass home institutions that hinder efficient exchange and find a caveat to these standard views regarding the welfare-enhancing aspect to international trade: when home markets are characterized by asymmetric information in product quality then the possibility of trade to markets not characterized by asymmetric information may harm the home country. This was found both for when trade was unexpected and when home exporters could respond through entry and investment. The intuition for this is simple: in markets with asymmetric information regarding product quality the price is determined by the *marginal* consumer’s expected value of the product. However, when a firm exports its high quality product, the loss in home welfare is the *average* consumer’s value of the high quality product since in the home market which consumer would have received the high quality product is random. With downward sloping demand this average valuation is strictly greater than the market price and so there

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\tau$</th>
<th>$\rho_t$</th>
<th>$c_I$</th>
<th>$P^P_{X}/P_A$</th>
<th>$P^{W}_{X}/P_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>0.9</td>
<td>1.0</td>
<td>1.57</td>
<td>1.72</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>0.9</td>
<td>2.0</td>
<td>2.22</td>
<td>2.43</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>0.9</td>
<td>3.0</td>
<td>2.72</td>
<td>2.98</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>0.9</td>
<td>4.0</td>
<td>3.14</td>
<td>3.44</td>
</tr>
</tbody>
</table>
always exist export prices that harm home welfare. Surprisingly, trade can lead to an expansion of the export sector and home welfare can still decrease, an *immiserizing* growth from asymmetric information.

Even home firms can be made worse-off from trade. This occurs when trade results in all high quality being exported. The intuition here is also simple: the home price declines when only low quality remains in the home market and only a fraction of home firms’ product is exported. Thus, if the autarkic home price is greater than the average price with export, then producer surplus decreases and the export industry contracts and total high quality output can *decline*.
Appendix

Lemma 2 Consider two levels of entry $q_1$ and $q_2$, with $q_1 < q_2$, then for the same amount sold on home market the home price is always lower with the higher level of entry: $P(q_1H, (1 - \bar{p})q_1) > P(\hat{q}_2H(q_1H), (1 - \bar{p})q_2)$.

Proof.

$$P(q_1H, (1 - \bar{p})q_1) = \frac{q_1H}{q_1H + (1 - \bar{p})q_1} h(q_{H1} + (1 - \bar{p})q_1)$$

$$> \frac{\hat{q}_{2H}(q_{1H})}{\hat{q}_{2H}(q_{1H}) + (1 - \bar{p})q_2} h(\hat{q}_{2H}(q_{1H}) + (1 - \bar{p})q_2) = P(\hat{q}_2H(q_1H), (1 - \bar{p})q_2).$$

as $q_1H + (1 - \bar{p})q_1 = \hat{q}_{2H}(q_{1H}) + (1 - \bar{p})q_2$, and $q_{1H} > \hat{q}_{2H}(q_{1H})$ since $(1 - \bar{p})q_1 < (1 - \bar{p})q_2$.

Lemma 3 The maximum feasible home price $P^F(q)$ is decreasing in entry $q$.

Proof. Consider the case when for some level of entry $\hat{q}$, $P_1(\rho\hat{q}, (1 - \rho)\hat{q}) > 0$. For the range of entry $q$ around $\hat{q}$ in which $q_{H}^*$ is not feasible the maximum feasible home price is the entry level, which is clearly decreasing in entry. If the range of entry that satisfies this is the entire range, the proof is complete. If not, then by continuity at the limit of the range there is a $q$, $\hat{q}$, such that $P_1(\rho\hat{q}, (1 - \rho)\hat{q}) = 0$ and at this limit point the maximum feasible home price is still decreasing in entry. If beyond $\hat{q}$, there is a range of $q$ such that $P_1(\rho\hat{q}, (1 - \rho)\hat{q}) < 0$, then by Corollary 1, the maximum feasible home price is decreasing in entry. The identical logic applies if there are any other levels of entry $q$ such that $P_1(\rho q, (1 - \rho)q) = 0$.

Lemma 4 There exist export prices such that welfare is lower when all high quality is exported than with autarky.

Proof. As $p_X \to P_A/\bar{p} > P_A$, (17) becomes

$$W_A(\bar{q}) - W_T(p_X) = \int_0^\eta \bar{p}h(x)dx - p_X\bar{p}q_X(P_A) > \int_0^\eta \bar{p}h(x)dx - P_Aq_X(P_A) > 0,$$

where the second inequality follows from $P_A = \bar{p}h(\eta)$. In addition, note that for any $p_X \in [P_A, P_A/\bar{p}]$, producer surplus is lower with all export and hence so is home welfare.

Lemma 5 There exist export prices such that welfare is lower when some high quality is exported than it is with autarky.
Proof. Evaluating (19) at \( p_X = P_A \) yields

\[
W'_T(P_A) = [\bar{q} - \bar{\rho}q] + P_A[q'q_X - \hat{q}_H q_X] \\
+ [\hat{q}_H + (1 - \bar{p})]q_X \bar{p} h(\bar{q}) \\
+ \int_0^q \frac{(1 - \rho)(\hat{q}_H q_X q_X - \hat{q}_H q_X)}{(\hat{q}_H + (1 - \rho)q_X)^2} h(x) dx - q'_X e(\bar{q}).
\]

Noting that \( \bar{p} h(\bar{q}) = P_A \), this simplifies to

\[
W'_T(p_X) = P_A q'_X + \int_0^q \frac{(1 - \rho)(\hat{q}_H q_X q_X - \hat{q}_H q_X)}{(\hat{q}_H + (1 - \rho)q_X)^2} h(x) dx - q'_X e(\bar{q}) < 0,
\]

since from the entry condition \( P_A = e(\bar{q}) \), \( q'_X > 0 \) and \( \hat{q}_H < 0 \).

Proposition 7 Home welfare is harmed by the marginal entrant if in equilibrium not all high quality is exported.

Proof. For when some high quality remains on the home market, home welfare as a function of entry \((q)\) is

\[
W_T(q) = p_X [\bar{q} - \hat{q}_H(q)] + \int_0^q \frac{\hat{q}_H(q)}{\hat{q}_H(q) + (1 - \bar{p})q} h(x) dx - \int_0^q e(x) dx.
\]

Differentiating we obtain

\[
W'_T(q) = p_X [\bar{q} - \hat{q}'_H(q)] + [\hat{q}'_H(q) + (1 - \bar{p})]p_X \\
+ \int_0^q \frac{(1 - \rho)(\hat{q}_H(q) q_X - \hat{q}_H q_X)}{(\hat{q}_H(q) + (1 - \rho)q_X)^2} h(x) dx - e(q).
\]

where the second \( p_X \) comes from the home price equaling the export when some high quality is sold on the home market. As the integral is negative (high quality exports reduces the average quality on the home market and at \( q_X, p_X = e(q_X) \), \( W'_T(q_X) < 0 \).

Proposition 8 When some high quality remains on the home market, home welfare decreases in the export price at \( \hat{p}_X \).

Proof. For a given export price, home welfare with all exporting (16) less home welfare some exporting (18) is

\[
p_X \hat{q}_H - \int_0^{\hat{q}_H} \frac{\hat{q}_X}{\hat{q}_H + q_X} h(x) dx < 0
\]

since by definition \( \frac{\hat{q}_H}{\hat{q}_H + q_X} h(\hat{q}_H + q_X) = p_X \). As \( d\hat{q}_H/dp_X < 0 \) there is a maximum export price \( \hat{p}_X \) such that there is an equilibrium with some high quality sold on the home market.
As \( P_1(0, q_X^L) > 0 \), \( \hat{q}_H(\hat{p}_X) > 0 \) so at \( \hat{p}_X \), and increase in \( p_X \) results in all high quality being exported and home welfare decreasing.

**Proposition 9** If \( \hat{p}_X < P_A/\bar{\rho} \), then there exist export prices such that home welfare with unanticipated trade is greater than home welfare with anticipated trade.

**Proof.** As \( p_X \to \hat{p}_X \) from above, with anticipated trade the only equilibrium has all high quality exported. Since \( \bar{q} < q_X(p_X^*) \) and \( \partial \hat{q}_H / \partial q < 0 \), then with unanticipated trade there is an equilibrium with some high quality on the home market for \( p_X \) sufficiently close to \( \hat{p}_X \). Since \( \hat{p}_X < P_A/\bar{\rho} \), for \( p_X \) sufficiently close to \( \hat{p}_X \), producer surplus is greater with unanticipated trade and hence home welfare is greater with unanticipated trade than with anticipated trade.

**Proposition 5** Welfare is greater with anticipated autarky than with anticipated trade when the condition in Proposition 2 holds for \( q_X \).

**Proof.** If proposition 2 holds for \( q_X \), then given entry level \( q_X \) welfare is greater with no trade (denoted \( W_A(q_X) \)) than with trade (subscript T for emphasis): \( W_A(q_X) > W_T(q_X) \). Since \( \bar{q} \) is the (information-constrained) optimal level of entry without trade then \( W_A(\bar{q}) \geq W_A(q_X) > W_T(q_X) \).

**Proposition 10** If the unique autarkic equilibrium with unanticipated trade is all high quality exported, then the unique equilibrium with anticipated trade is all high quality exported. The existence of an autarkic equilibrium with unanticipated trade with some high quality does not imply such an equilibrium exist with anticipated trade.

**Proof.** Assume there exist an entry equilibrium with anticipated trade in which some high quality sells on the home market. In this case the home price must equal the export price. Let \( q_X^L \) (defined by Equation 11) denote the amount of low quality associated with this entry equilibrium. By Lemma 3, since \( q_X^L > q_L^A \), then \( P^F(\bar{q}) > P^F(q_X) \). As \( p_X > P^F(\bar{q}) \), then there does not exist an entry equilibrium with some high quality on the home market.
References


