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ECONOMIC ANALYSIS GROUP
DISCUSSION PAPER

Dynamic Contract Breach
by
Fan Zhang*
EAG 08-3 March 2008

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Abstract

This paper studies the design of optimal, privately-stipulated damages when breach of contract is possible at more than one point in time. It offers an intuitive explanation for why cancellation fees for some services (e.g., hotel reservations) increase as the time for performance approaches. If the seller makes investments over time to improve her value from trade, she will protect the value of her investments by demanding a higher compensation when the buyer breaches their contract at a time closer to when contract performance is due.

Furthermore, it is shown that if the seller may be able to find an alternate buyer when breach occurs early but not when breach occurs late, the amount by which the damage for late breach exceeds the damage for early breach is increasing in the probability of finding an alternate buyer. (This result may explain why some hotels impose larger penalties for last-minute cancellations during the high season than during the low season.)

When the probability of finding an alternate buyer is endogenized, the seller's private incentive to mitigate breach damages is shown to be socially insufficient whenever she does not have complete bargaining power with the alternate buyer. Finally, if renegotiation is possible after the arrival of each perfectly competitive entrant, the efficient breach and investment decisions are shown to be implementable with the same efficient expectation damages that implement the efficient outcomes absent renegotiation.

1 Introduction

Contracts for the provision of services frequently have cancellation fees that penalize the party who backs out before the contract expires or before the date of performance of the contract. For example, vacation resorts often set two separate fees for cancellation of lodging reservations: an early cancellation fee if the reservation is cancelled with sufficient advanced notice, and a late cancellation fee, which is usually larger, if the reservation is cancelled "at the last minute." Furthermore, the difference between the fees for late cancellation and early cancellation is often larger during the high season, when demand is higher. What causes such variations in breach damages with respect to when a contract is signed and when it is breached? This paper proposes a possibile explanation by allowing for the possibility of contract breach and investment at multiple points in time.

Suppose that when the contract is signed, the buyer is uncertain about the value of his outside option at various future points in time and may therefore breach the contract before his performance (payment) is due. If the seller has multiple opportunities over time to make non-contractible, cost-reducing investments that improve her value from trade, she will want to protect the value of those investments by demanding a higher compensation for contract breach that occurs later, or closer in time to when contract performance is due. Therefore, the buyer's decision of whether to breach early or late involves a trade off between the option value of not breaching early (and waiting for a potentially cheaper supplier to arrive later) versus the higher penalty associated with potentially breaching late.

The law and economics literature on contract breach began by considering the efficiency of standard court-imposed damage measures in a setting where the buyer faces an alternate source of supply that is competitively priced. In particular, Shavell (1980) and Rogerson (1984) considered, respectively, the situations where the incumbent seller and buyer cannot and can renegotiate their initial contract. The common finding in both cases is that standard court-imposed damages generally induce socially excessive investment.

The efficiency of privately stipulated, or liquidated, damages for breach of contract has also been previously addressed, notably by Aghion and Bolton (1987) (assuming no investment or renegotiation), Chung (1992) (allowing for investments but not renegotiation), and Spier and Whinston (1995) (assuming both investments and renegotiation). The common focus of these papers is on the strategic stipulation of socially excessive breach damages when the entrant seller has market power, i.e., when the incumbent seller and buyer's original contract imposes externalities on third parties.[2]

In contrast, I assume that third parties have no bargaining power with the incumbent seller and buyer. Instead, the key innovation of this paper is the existence of multiple opportunities for breach of contract, which is due to the sequential arrival of two potential entrants. Section 2 introduces the rest of the model in detail, and Section 3 characterizes the ex-ante efficient breach and investment decisions.

In the event of breach, *expectation damages* compensate the breached-against party (in this case, the

[2]Most of the literature on contract damages, including this paper and those cited above, assumes investments are selfish in that they only directly affect the investing party's payoffs. Che and Chung (1999), however, assume cooperative investments, which directly affect the payoffs of the non-investing party. They show that the relative social desirability of expectation damages, liquidated damages, and reliance damages are different when investments are cooperative instead of selfish.

seller) for the profit that she would have made had breach not occurred, given her *actual* investment decision. By comparison, *efficient* expectation damages compensate the breach-against party for the profit she would have made absent breach – had she chosen the *efficient* investment level. First, absent externalities and assuming renegotiation is impossible, I demonstrate in Section 4 that the incumbent parties can implement the efficient breach and investment decisions in both periods by stipulating the efficient expectation damages in their contract. This result can be viewed as an extention to multiple periods of the well-known result that the efficient expectation damages is socially efficient when renegotiation is not possible.[3] Furthermore, I show that efficient expectation damages for late breach exceed those for early breach.

In a related paper, Chan and Chung (2005) also considers at a two-period model of contract breach with sequential investment opportunities. They focus on standard court-imposed breach rememdies and do not allow for renegotiation. In contrast, the main motivations of this paper are to provide explanations for why privately stipulated damages might increase over time as the date of performance approaches, and to examine the robustness of this result to the possibility of renegotiation. Another related paper is Triantis and Triantis (1998), which studies a continuous time model of contract breach and *assumes* that breach damages are increasing over time. The present paper can be viewed as providing a framework that justifies such an assumption when damages are privately stipulated.

Another novel feature of this model is the possibility that the seller may find an alternate buyer when the incumbent buyer breaches early but not when he breaches late.[4] In this case, contract law requires the seller to take reasonable measures to reduce, or mitigate, the damages that are owed to her for early breach. Since these damages are decreasing in the probability of trading with an alternate buyer, mitigation in this setting entails efforts to increase this probability of trading with the alternate buyer. Section 5 endogenizes this probability of trading with an alternate buyer and compares the private and social incentives for mitigation of damages. It is shown that unless the incumbent seller has complete bargaining power vis-a-vis the alternate buyer, her private incentives for mitigation are socially insufficient, leading to suboptimal mitigation efforts. However, this result crucially depends upon the implicit assumption that breach is defined as only a function of whether the incumbent buyer refuses trade, or delivery of the good (as opposed to being also a function of whether the incumbent seller is able to trade with an alternate buyer).

Next, I assume in Section 6 that the incumbent buyer and seller are able to renegotiate their original contract after the arrival of each perfectly competitive entrant. It is shown that if the incumbent seller has complete bargaining power with the alternate buyer (so that externalities are absent), socially efficient breach and investment decisions can still be implemented with the same contract that induces efficient decisions when renegotiation is not possible. Thus, this paper contributes to the literature on contract breach by demonstrating that, absent externalities, efficient expectation damages are socially optimal even if breach and renegotiation are possible at multiple points in time.

[3] See, for example, Chung (1992) and the references therein.

[4] For example, there may be insufficient time to find an alternate buyer if breach occurs late. The qualitative results would continue hold if the probability of finding an alternate buyer upon late breach is positive so long as it is less than the analogous probability given early breach.

Finally, Section 7 considers an application of the no-renegotiation version of the model to the lodging industry, and in particular, vacation resorts' policies regarding cancellation of lodging reservations. The model predicts that a resort's opportunity cost of honoring a reservation beyond the early cancellation opportunity is increasing in the likelihood of finding an alternate guest in case early cancellation occurs. Therefore, we should expect the amount by which the late cancellation fee exceeds the early cancellation fee to be larger during periods of high demand than during periods of low demand.

Section 8 briefly concludes.

2 A Model with Multiple Breach Opportunties

Consider a contract between a buyer and a seller to exchange one unit of an indivisible good or service. The buyer's value for the good, v, is commonly known to both parties.[5] The seller can make sequential cost-reducing investments of r_1 and r_2 to improve her value from trade with the buyer. After the original seller makes each investment r_i, another seller observes her own production cost c_{Ei} and announces a price p_{Ei} that she will charge the buyer if the buyer breaches his contract with the incumbent seller and buys from her, the entrant seller, instead.[6] I study the case where the buyer has all the bargaining power when dealing with the entrants, so that each entrant sets her price equal to her cost, $p_{\mathrm{Ei}} = c_{\mathrm{Ei}}$, and behaves as if she were perfectly competitive.[7]

The buyer has two opportunities to breach his contract with the incumbent seller: once after each entrant seller arrives and announces p_{Ei}. The entrant's price p_{Ei} and the incumbent's investments r_i are observable by all parties but not verifiable. For now, assume the incumbent seller and buyer cannot renegotiate their contract after each entrant's announcement of p_{Ei} (I examine the case where renegotiation is possible in Section 6). So the model is essentially the stage game of Spier and Whinston (1995) repeated twice, with perfectly competitive entrants and with the following additional modification. I assume that if the original buyer breaches early, i.e., immediately after the first entrant sets her price, then with probability θ the seller is able to find an alternate buyer who has the same value v for the good and is charged a price p' by the seller. (Except for the discussion on mitigation of damages in Section 5, I will assume throughout the rest of this paper that $p' = v$, so that the alternate buyer has no bargaining power with respect to the incumbent seller.) If the original buyer breaches late, i.e., after the second entrant announces her price, the seller cannot find an alternate buyer. For example, it may be the case that the incumbent seller requires sufficient time to have a chance of finding an alternate buyer.

Because the buyer will have two opportunities to breach, the seller specifies in the contract two liquidated damages, x_1 and x_2, where the buyer must pay x_i to the seller if he cancels the contract

[5] Stole (1992) argues that when the parties are asymmetrically informed, liquidated damages not only provide incentives for efficient breach, but also serve to efficiently screen among different types of buyers and sellers.

[6] Fixed costs of entry for the entrants are not explicitly modeled. Each of them simply observes her production cost and then costlessly shows up to announce a price.

[7] If an entrant has some bargaining power with respect to the buyer, the damage for breach that the buyer must incur if he were to buy from the entrant would still constrain the entrant's price choice. Since the entrant would make positive profits if she sells to the buyer in this case, the incumbent seller can use (socially excessive) stipulated breach damages to extract surplus from the entrant. See Spier and Whinston (1995).

after the seller has made her investment r_i. If the buyer never breaches the contract and buys from the incumbent seller, the only payment that he makes to the seller is a price p, which is paid when the contract is performed in the last period (when the buyer accepts delivery of the good from the seller). In this case, the seller's investment costs are $r_1 + r_2$ and her production cost is $c(r_1, r_2)$, where $c(\cdot,\cdot)$ is strictly decreasing and strictly convex in r_1 and r_2 for all $(r_1, r_2) \gg 0$.[8] I will refer to r_1 as the early investment and r_2 as the late investment. In the event that early breach occurs, $r_2 = 0$.

To summarize, the sequence of events, shown in Figure 1 for the case when renegotiation is impossible, is as follows.

t=0 Seller S offers a contract (p, x_1, x_2) to Buyer B. If B rejects, both parties receive a payoff of zero and the game ends. If B accepts, the game continues.

t=1.1 S makes a non-contractible *early investment* $r_1 \geq 0$ to reduce her production costs.

t=1.2 Nature draws Entrant seller E1's cost c_{E1} from a distribution $F(\cdot)$ with support $[0, v]$, and E1 chooses her price p_{E1}.

t=1.3 B decides whether to *breach early* and buy from E1. The cost of the first investment, r_1, is a sunk cost for S at this point, but if B breaches early, S incurs production costs $c(r_1, 0)$ only if she finds an alternate buyer (which occurs with probability θ). Therefore, payoffs for the incumbent buyer, incumbent seller, the first entrant, and the alternate buyer in the case of early breach are, respectively,

$$u_B = v - p_{E1} - x_1, \quad u_S = x_1 - r_1 + \theta\left[p' - c(r_1, 0)\right], \quad u_{E1} = p_{E1} - c_{E1}, \quad u_{AB} = \theta[v - p'].$$

The game ends after an early breach. If B does not breach early, $u_{E1} = u_{AB} = 0$ and the game continues.

t=2.1 S makes a non-contractible, relationship-specific *late investment* $r_2 \geq 0$ to further reduce her production costs.[9]

t=2.2 Nature draws Entrant seller E2's cost c_{E2} from $F(\cdot)$, independent of c_{E1}, and E2 chooses her price p_{E2}.[10]

t=2.3 B decides whether to *breach late* and buy from E2. Because I assume that S is unable to find an alternate buyer if breach occurs late, payoffs for the buyer, incumber seller, and second entrant in the case of B breaching late are, respectively,

$$u_B = v - p_{E2} - x_2, \quad u_S = x_2 - r_1 - r_2, \quad u_{E2} = p_{E2} - c_{E2}.$$

If B does not breach, payoffs are

$$u_B = v - p, \quad u_S = p - c(r_1, r_2) - r_1 - r_2, \quad u_{E2} = 0.$$

[8] While no functional form assumptions are made with respect to how the seller's production costs depend on her investments, it is assumed that these investments are selfish in the sense that they do not directly affect the *buyer's* payoff.

[9] The seller's late investment r_2 is relationship-specific because it does not improve the her payoff at all if the incumbent buyer breaches late. In contrast, S's early investment r_1 is not completely relationship-specific because it reduces her cost of selling to the alternative buyer, if one is found.

[10] The analysis would clearly be the same if we assumed that there is only one entrant who takes another independent draw of his cost if the buyer does not buy from her at time t=1.3.

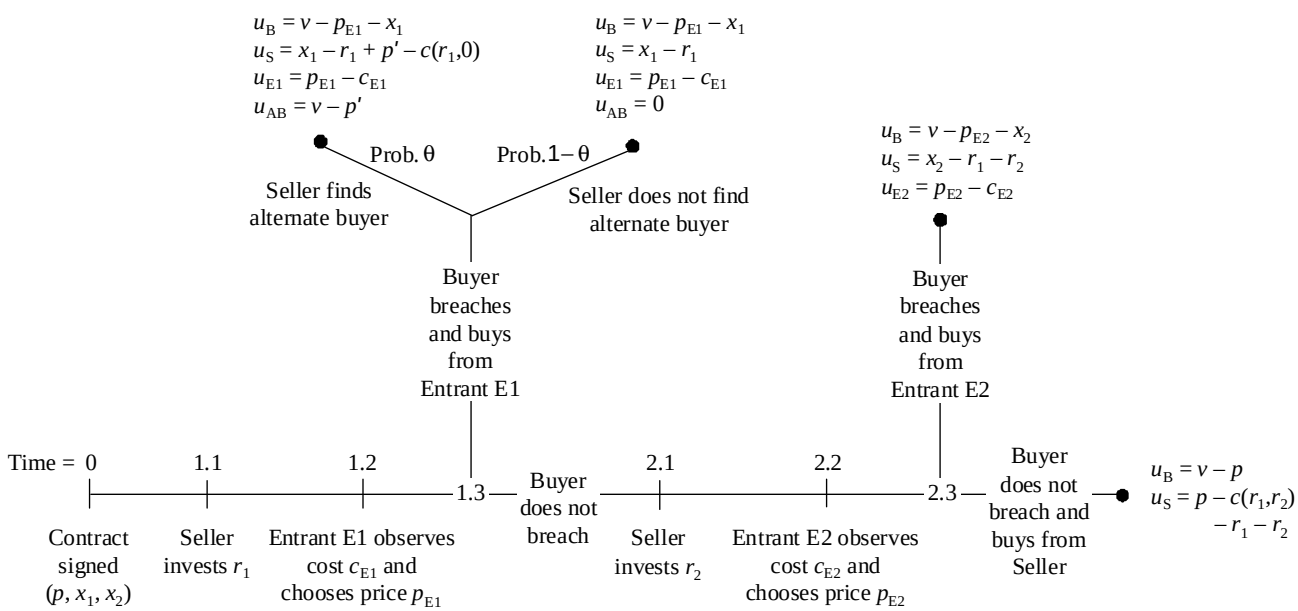

Figure 1: Timeline and payoffs when renegotiation is not possible.

3 Efficient Investment and Breach

As a benchmark, I identify the investment and breach decisions that maximize expected social surplus, or the sum of payoffs for all parties. Let r_1^* and $r_2^*(r_1^*)$ denote the (ex-ante) efficient investments for the seller.

Proceeding in reverse chronological order, I first characterize the buyer's efficient late breach decision. Assuming no early breach and investments r_1 and r_2, the social surplus (i.e., the sum of payoffs for B, S, and E2) is $v - c_{E2} - r_1 - r_2$ if B breaches and $v - c(r_1, r_2) - r_1 - r_2$ if B does not breach. Thus, given investment levels r_1 and r_2 and no early breach, social surplus is maximized when B breaches late if and only if potential entrant E2 can produce the good at a lower cost than the incumbent seller:

$$c_{E2} \leq c(r_1, r_2). \tag{1}$$

In particular, because all investment costs are sunk, they do not have any direct effect on the efficient late breach decision. However, investments indirectly affect the late breach decision through their effects on the seller's production costs.

Next, consider the seller's efficient late investment, $r_2^*(r_1)$, which by definition maximizes expected social surplus given early investment r_1, no early breach, and late breach occurring if and only if $c_{E2} \leq c(r_1, r_2)$. In other words, $r_2^*(r_1)$ is the solution to the problem

$$\max_{r_2 \geq 0} S(r_2|r_1), \text{ where}$$

$$S(r_2|r_1) \equiv \begin{cases} \int_0^{c(r_1,r_2)} [v - c_{E2} - r_1 - r_2] f(c_{E2}) dc_{E2} \\ + \int_{c(r_1,r_2)}^{v} [v - c(r_1, r_2) - r_1 - r_2] f(c_{E2}) dc_{E2}. \end{cases}$$

The seller's efficient late investment $r_2^*(r_1)$, assuming it is positive, is characterized by the first order condition

$$1 = -c_2(r_1, r_2^*(r_1))(1 - F[c(r_1, r_2^*(r_1))]). \tag{2}$$

This condition requires that, at its efficient level, the marginal cost of increasing r_2 should equal the expected marginal benefit of increasing r_2, which is the cost reduction from increasing r_2 multiplied by the probability that the cost reduction will be realized (i.e., the probability of late breach not occurring, conditional on early breach not occurring).

Now consider the efficient early breach decision. Social surplus from early breach is $v - c_{E1} - r_1 + \theta[v - c(r_1, 0)]$. Given that the late breach decision is efficient (follows (1)) and late investment is efficient (as characterized by (2)), expected social surplus from not breaching early is

$$\begin{aligned} & S(r_2^*(r_1)|r_1) \\ = \; & v - F[c(r_1, r_2^*(r_1))]E[c_{E2}|c_{E2} \leq c(r_1, r_2^*(r_1))] \\ & -(1 - F[c(r_1, r_2^*(r_1))])[c(r_1, r_2^*(r_1))] - r_1 - r_2^*(r_1). \end{aligned}$$

Thus, it is efficient for B to breach early if and only if $v - c_{E1} - r_1 + \theta[v - c(r_1, 0)] \geq S(r_2^*(r_1)|r_1)$, or

$$c_{E1} \leq c^*(r_1) + r_2^*(r_1) + \theta[v - c(r_1, 0)], \tag{3}$$

where

$$\begin{aligned} c^*(r_1) \quad \equiv \quad & F[c(r_1, r_2^*(r_1))]E[c_{E2}|c_{E2} \leq c(r_1, r_2^*(r_1))] \\ & +(1 - F[c(r_1, r_2^*(r_1))])c(r_1, r_2^*(r_1)) \end{aligned} \tag{4}$$

is the expected continuation production cost given r_1, and efficient late investment and efficient late breach. So breaching early is efficient if and only if the first entrant's cost, c_{E1}, is lower than the expected *social* cost of continuing with the incumbent seller, given efficient investments and efficient late breach. In other words, in order for the buyer's early breach decision to be efficient, his total expected continuation cost must include not only his private expected continuation cost $c^*(r_1)$, but also internalize the additional investment cost $r_2^*(r_1)$ that the seller will incur once early breach is foregone, as well as the lost expected surplus $\theta[v - c(r_1, 0)]$ that would have been realized had the seller been given the opportunity to find an alternate buyer.

Finally, given the seller's efficient late investment and the buyer's efficient breach decisions as described above, the seller's efficient early investment, r_1^*, should maximize the ex-ante expected social surplus:

$$\begin{aligned} & \max_{r_1 \geq 0} S(r_1) \\ = \; & \max_{r_1 \geq 0} \left\{ \begin{array}{c} \int_0^{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]} \{v - c_{E1} - r_1 + \theta[v - c(r_1, 0)]\} f(c_{E1}) dc_{E1} \\ + \int_{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]}^{v} \{v - c^*(r_1) - r_2^*(r_1) - r_1\} f(c_{E1}) dc_{E1} \end{array} \right\} \\ = \; & \max_{r_1 \geq 0} \left\{ \begin{array}{c} v - r_1 + \int_0^{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]} \{-c_{E1} + \theta[v - c(r_1, 0)]\} f(c_{E1}) dc_{E1} \\ + \int_{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]}^{v} \{-c^*(r_1) - r_2^*(r_1)\} f(c_{E1}) dc_{E1} \end{array} \right\} \end{aligned} \tag{5}$$

In the first version of this problem, the two integrals represent the expected social surpluses when early breach is efficient and when not breaching early is efficient, respectively. The seller's efficient early investment r_1^*, assuming it is positive, can be characterized by the first order condition

$$\begin{aligned} 1 &= -c_1(r_1^*,0)\theta F[c^*(r_1^*)+r_2^*(r_1^*)+\theta(v-c(r_1^*,0))] \\ &\quad -\frac{d}{dr_1}[c^*(r_1^*)+r_2^*(r_1^*)]\{1-F[c^*(r_1^*)+r_2^*(r_1^*)+\theta(v-c(r_1^*,0))]\}. \end{aligned} \tag{6}$$

Because (2) implies $\frac{d}{dr_1}[c^*(r_1^*)+r_2^*(r_1^*)] = c_1(r_1^*, r_2^*(r_1^*))\left\{1-F[c(r_1^*, r_2^*(r_1^*))]\right\}$, (6) can be rewritten as

$$\begin{aligned} 1 &= -c_1(r_1^*,0)\cdot\theta F[c^*(r_1^*)+r_2^*(r_1^*)+\theta(v-c(r_1^*,0))] \\ &\quad -c_1(r_1^*, r_2^*(r_1^*))\cdot\left\{1-F\left[\begin{array}{c} c^*(r_1^*)+r_2^*(r_1^*) \\ +\theta(v-c(r_1^*,0)) \end{array}\right]\right\}\{1-F[c(r_1^*, r_2^*(r_1^*))]\} \end{aligned} \tag{7}$$

Equation (7) states that in order for early investment r_1^* to be efficient, its marginal cost must equal its expected marginal benefit. When the buyer (efficiently) breaches early and an alternate buyer is found, an event which occurs with probability $\theta F[c^*(r_1^*)+r_2^*(r_1^*)+\theta(v-c(r_1^*,0))]$, the marginal benefit of early investment r_1^* is a reduction of the seller's production cost by the amount $-c_1(r_1^*,0)$. When the buyer (efficiently) never breaches and buys from the incumbent seller, which occurs with probability $(1-F[c^*(r_1^*)+r_2^*(r_1^*)+\theta(v-c(r_1^*,0))])\{1-F[c(r_1^*, r_2^*(r_1^*))]\}$, the marginal benefit of early investment r_1^* is a reduction of the production cost by the amount $-c_1(r_1^*, r_2^*(r_1^*))$. Note that when $\theta = 0$, so that there is no possibility of finding an alternate buyer even if early breach occurs, (7) reduces to

$$1 = -c_1(r_1^*, r_2^*(r_1^*))(1-F[c^*(r_1^*)+r_2^*(r_1^*)])\left\{1-F[c(r_1^*, r_2^*(r_1^*))]\right\},$$

where the right hand side is the reduction in production cost that results from investment r_1^*, multiplied by the probability that this benefit will actually be realized, i.e., the probability that breach never occurs.

Proposition 1 *The incumbent seller's efficient investments, r_1^* and $r_2^*(r_1^*)$, are characterized by (6) and (2), respectively. The buyer's efficient breach decision is to breach early if and only if (3) is satisfied and (conditional on not breaching early) to breach late if and only if (1) is satisfied.*

4 Private Contracts Induce Efficient Decisions

In this section, I show that if the incumbent parties' original contract imposes no externalities on third parties,[11] and if renegotiation is not possible, then the incumbent seller and buyer can implement the efficient investment and breach decisions in both periods by stipulating efficient expectation damages. This result has been demonstrated previously for the case of a single breach opportunity,[12] but not for multiple breach opportunities.

[11] That is, assume both entrant sellers are perfectly competitive, i.e., constrained to set price equal to cost, and that the incumbent seller has complete bargaining power with respect to the alternate buyer.

[12] See paragraph 4 on p. 186 of Spier and Whinston (1995) for references.

Suppose the buyer and seller agreed to a contract (p, x_1, x_2) where

$$x_1 = p - c(r_1^*, r_2^*(r_1^*)) - r_2^*(r_1^*) - \theta[v - c(r_1^*, 0)] \tag{8}$$

$$x_2 = p - c(r_1^*, r_2^*(r_1^*)) \tag{9}$$

Furthermore, assume each entrant Ei sets price equal to cost, $p_{\mathrm{Ei}} = c_{\mathrm{Ei}}$ for $i = 1, 2$, and that the incumbent seller can charge the alternate buyer his value for the good, i.e., $p' = v$. The following proposition states that this contract will induce the seller to invest efficiently and the buyer to make the efficient breach decision in each period. Note that if a contract satisfies (8) and (9), then whenever the buyer breaches, the damages that he pays makes the seller as well off as if the contract had been performed, *assuming the seller invested efficiently.* Hence these damages are the *efficient expectation damages.*

Proposition 2 *Assume that entrants are perfectly competitive, the alternate buyer has no bargaining power, and renegotiation is not possible. Then any contract (p, x_1, x_2) satisfying (8) and (9) induces the seller to always invest efficiently and the buyer to always breach efficiently.*

Proof. Using backwards induction to solve for the subgame perfect Nash equilibrium of the game, consider first B's private incentives for late breach. Given a contract (p, x_1, x_2) that satisfies (8) and (9), suppose early breach did not occur. B's equilibrium incentive is to breach late if and only if $v - c_{E2} - x_2 \geq v - p$, or $c_{E2} \leq p - x_2 = c(r_1^*, r_2^*(r_1^*))$. Thus, (1) implies that B's late breach decision is efficient if S's *equilibrium* investments r_1^e and r_2^e are efficient, i.e., if they equal r_1^* and $r_2^*(r_1^*)$, respectively.

Given this late breach decision by B, an early investment of r_1^e by S, and no early breach, (9) can be used to write S's late investment problem as choosing r_2 to maximize her expected continuation payoff:

$$\begin{aligned} r_2^e(r_1) &= \max_{r_2 \geq 0} \left\{ \begin{array}{c} \int_0^{c(r_1^*, r_2^*(r_1^*))} [x_2 - r_1^e - r_2] f(c_{E2}) dc_{E2} \\ + \int_{c(r_1^*, r_2^*(r_1^*))}^{v} [p - c(r_1^e, r_2) - r_1^e - r_2] f(c_{E2}) dc_{E2} \end{array} \right\} \\ &= \max_{r_2 \geq 0} \left\{ -r_2 - \int_{c(r_1^*, r_2^*(r_1^*))}^{v} c(r_1^e, r_2) f(c_{E2}) dc_{E2} \right\}. \end{aligned} \tag{10}$$

Then S's equilibrium choice of r_2^e is characterized by the first order condition

$$1 = -c_2(r_1^e, r_2^e(r_1^e))(1 - F[c(r_1^*, r_2^*(r_1^*))]). \tag{11}$$

Since $c_{22}(\cdot) > 0$, equations (2) and (11) imply that $r_2^e(r_1^e) = r_2^*(r_1^*)$ if $r_1^e = r_1^*$. Hence, S's late investment is indeed efficient if her early investment is efficient.

Anticipating the late investment and breach decisions characterized above, B's equilibrium incentive is to breach early if and only if

$$v - c_{E1} - x_1 \geq \int_0^{c(r_1^*, r_2^*(r_1^*))} [v - c_{E2} - x_2] f(c_{E2}) dc_{E2} + \int_{c(r_1^*, r_2^*(r_1^*))}^{v} [v - p] f(c_{E2}) dc_{E2}.$$

By using (8)-(9) and rearranging, this inequality can be shown to be equivalent to $c_{E1} \leq c^*(r_1^*) + r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)]$, which is the same as (3). Therefore, if $r_1^e = r_1^*$ so that S's early investment is efficient, B's early breach decision will be also efficient (as will be the late investment and late breach decisions).

So it remains to show that S's equilibrium early investment is efficient, i.e., $r_1^e = r_1^*$, when breach damages are specified by (8) and (9). Given that B breaches early if and only if $c_{E1} \leq c^*(r_1^*) + r_2^*(r_1^*) +$

$\theta[v - c(r_1^*, 0)]$, the probability of early breach only depends on the efficient early investment r_1^* and not S's equilibrium choice of r_1. Therefore, S chooses her early investment $r_1 \geq 0$ to maximize

$$\begin{aligned}&F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))](x_1 - r_1 + \theta[p' - c(r_1, 0)])\\&+\{1 - F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))]\}\Gamma(r_1),\end{aligned}$$

where $x_1 - r_1 + \theta[p' - c(r_1, 0)]$ is S's expected payoff conditional on early breach, and

$$\begin{aligned}\Gamma(r_1) \quad \equiv \quad & \int_0^{c(r_1^*, r_2^*(r_1^*))} [\underbrace{p - c(r_1^*, r_2^*(r_1^*))}_{x_2} - r_1 - r_2^e(r_1)] f(c_{E2}) dc_{E2}\\&+ \int_{c(r_1^*, r_2^*(r_1^*))}^{v} [p - c(r_1, r_2^e(r_1)) - r_1 - r_2^e(r_1)] f(c_{E2}) dc_{E2}\end{aligned}$$

is the maximized value of the first problem in (10) when x_1 and x_2 are given by (8) and (9). That is, $\Gamma(r_1)$ is the continuation payoff for S from choosing early investment r_1 when B does not breach early, S's chooses her late investment according to $r_2^e(\cdot)$, and B breaches late if and only if $c_{E2} \leq c(r_1^*, r_2^*(r_1^*))$. Note that $\Gamma(r_1)$ can be rewritten as

$$\begin{aligned}\Gamma(r_1) \quad = \quad & p - c(r_1^*, r_2^*(r_1^*)) F[c(r_1^*, r_2^*(r_1^*))]\\&-c(r_1, r_2^e(r_1)) \{1 - F[c(r_1^*, r_2^*(r_1^*))]\} - r_1 - r_2^e(r_1).\end{aligned}$$

The first order condition for S's equilibrium early investment r_1^e can be written as

$$\begin{aligned}0 \quad = \quad & -F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))](1 + \theta c_1(r_1^e, 0))\\&+\{1 - F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))]\}\Gamma'(r_1^e),\end{aligned} \tag{12}$$

where (11) implies that

$$\Gamma'(r_1^e) = -1 - c_1(r_1^e, r_2^e(r_1^e))(1 - F[c(r_1^*, r_2^*(r_1^*))]) \tag{13}$$

Substituting (13) into (12) and rearranging, (12) can be written as

$$\begin{aligned}1 \quad = \quad & -c_1(r_1^e, 0) \cdot \theta F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))]\\&-c_1(r_1^e, r_2^e(r_1^e)) \cdot \left\{1 - F\left[\begin{array}{c} c^*(r_1^*) + r_2^*(r_1^*) \\ +\theta(v - c(r_1^*, 0)) \end{array}\right]\right\} \{1 - F[c(r_1^*, r_2^*(r_1^*))]\}\end{aligned}$$

This equation, when compared with (7), implies that S's equilibrium early investment is indeed efficient: $r_1^e = r_1^*$ (recall $c_{11}(\cdot) > 0$). Therefore, by the calculations above, S's equilibrium late investment is also efficient ($r_2^e(r_1^e) = r_2^*(r_1^*)$), and both of B's breach decisions are efficient. ■

By Proposition 2, a contract satisfying (8) and (9) maximizes the joint expected payoffs of the seller and buyer. Therefore, such a contract must also maximize the seller's ex-ante expected payoff given that the buyer accepts the contract. Since the seller's original contract proposal is a take-it-or-leave-it offer, she will find it in her interest to offer a contract satisfying (8) and (9) and choose the price p so that the buyer is just indifferent inbetween accepting or rejecting the contract offer.

Because the alternate buyer and each competitive entrant seller always earn a payoff of zero, a contract satisfying (8) and (9) also maximizes social surplus. Therefore, assuming all of the assumptions of the

model are satisfied, standard court-imposed breach remedies cannot improve welfare. Note that this result crucially depends on the absence of externalities. When an entrant has market power (and the buyer and seller are able to renegotiate after entry), Spier and Whinston (1995) show in a one-period model that "privately stipulated damages are set at a socially excessive level to facilitate the extraction of the entrant's surplus." Presumably, this inefficiency result would continue to hold if entrants have market power and renegotiation is introduced into the above two-period framework.

Note that the intuition behind Proposition 2 can also be seen without resorting to first order conditions. Because the original contract imposes no externalities, the incumbent seller's investments are always efficient *given* the incumbent buyer's breach decisions. Therefore, since efficient expectation damages induce the buyer to make breach decisions that are efficient assuming the seller's investments are ex-ante efficient,[13] such damages will also induce the seller to make (ex-ante) efficient investment decisions.

Subtracting equation (8) from (9), the following observations are evident.

Corollary 3 *When the entrants are perfectly competitive, the breach damages is higher* after *the second investment has been made than* before *the second investment has been made:*

$$x_2 - x_1 = r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)] > 0.^{14}$$

Furthermore, this difference is increasing in the probability of finding an alternate buyer (if breach occurs early):

$$\frac{d}{d\theta}(x_2 - x_1) = v - c(r_1^*, 0) > 0.$$

The first part of this corollary says that the fee for cancelling the contract increases over time. The relationship $x_2 = x_1 + r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)]$ between the damages for late and early breach illustrates the intuition. If the buyer does not breach at his first opportunity to do so, the seller will make the investment $r_2^*(r_1^*)$ and forgo an expected surplus of $\theta[v - c(r_1^*, 0)]$ from possible trade with an alternate buyer. Therefore, the penalty for late breach must include the additional cost of the seller's second investment, as well as the lost expected surplus from potential trade with an alternate buyer, in order to induce the buyer to internalize these social opportunity costs of continuing with the contract when making his second breach decision.

Because the opportunity cost $\theta[v - c(r_1^*, 0)]$ of continuing with the contract is increasing in the probability of finding an alternate buyer in case of early breach, the second part of the corollary simply points out the fact that the difference in the penalties between late breach and early breach must also be increasing in this probability.

[13] To see why this is so with sequential breach decisions, first note that with efficient second period investment, the efficient expectation damage for late breach will induce the buyer to make his late breach decision efficiently. Thus, given efficient first period investment, the efficient expectation damage for early breach will also cause the buyer to make his early early breach decision efficiently (since his continuation payoff from not breaching early is based on efficient second period breach and investment decisions). This reasoning should also apply to the case in which there are $N > 2$ periods in which breach may occur.

[14] This assumes that trade with the alternate buyer is efficient, conditional on efficient early investment.

5 Mitigation of Damages

Corollary 3 shows that the amount by which the damages for late breach exceed the damages for early breach is increasing in θ, the probability of finding an alternate buyer. While so far it has been assumed that this probability is exogenous, in reality the incumbent seller frequently has some influence over the likelihood of recouping some of her initial investment, and therefore the damages owed her by the incumbent buyer. When this is the case, contract law stipulates that the seller (i.e., the breached-against party, or promisee) has the responsibility of undertaking (a reasonable amount of) effort to reduce, or mitigate, those damages.[15]

Mitigation usually involves effort costs or other opportunity costs, so I modify the previous model by introducing a cost of mitigation for the seller. I demonstrate that the seller's incentive to engage in such mitigation efforts is socially efficient only when she has complete bargaining power vis-a-vis the alternate buyer; otherwise, her mitigation effort is socially insufficient.

5.1 Binary Mitigation Decision

First I consider the case where the seller simply makes a binary decision (immediately after early breach occurs) regarding whether or not to mitigate the damages owed to her by the incumbent buyer. Choosing to mitigate implies, as before, encountering an alternate buyer with (fixed) probability θ, and not mitigating implies being unable to find an alternate buyer with certainty. Assume mitigation involves a disutility of $\gamma > 0$ for the incumbent seller.

Suppose that the incumbent seller's early investment is r_1 and that early breach has occurred. The seller's payoff from not mitigating is $x_1 - r_1$, and her payoff from mitigating is $x_1 - r_1 + \theta[p' - c(r_1, 0)] - \gamma$, where recall p' is the price paid by the alternate buyer. Therefore, if there is no legal requirements on the seller's mitigation decision, she will choose to mitigate if and only if

$$\theta[p' - c(r_1, 0)] > \gamma.$$

That is, effort is expended to search for an alternate buyer when the probability of, or gains from, trade with such a buyer is high, or when the search effort associated with mitigation is not too costly.

How does this compare with the socially efficient mitigation decision? The payoffs of the incumbent buyer and the first entrant seller are independent of whether the incumbent seller mitigates, so they do not influence the socially efficient mitigation decision. Summing the payoffs of the incumbent seller and the alternate buyer, it is straightforward to see that social surplus is maximized with the incumbent seller mitigating if and only if

$$\theta[v - c(r_1, 0)] > \gamma.$$

[15] According to Restatement (Second) of Contracts, §350 (p. 127), "As a general rule, a party cannot recover damages for loss that he could have avoided by reasonable efforts." Goetz and Scott (1983) provide a detailed discussion of the general theory of mitigation. Miceli, et al. (2001) consider a specific application to property leases with court imposed damages. They show that whether it is optimal for there to be a duty for the landlord to mitigate damages from tenant breach of contract depends on whether leases fall under the domain of contract law or property law.

By comparing the above two inequalities, it can be readily observed that the incumbent seller's private incentives for mitigation of damages is socially insufficient unless $p' = v$, in which case she has complete bargaining power when dealing with the alternate buyer.[16]

5.2 Continuous Mitigation Decision

Now consider the more general case where the seller's mitigation effort choice is continuous. Without loss of generality, suppose that the seller directly chooses the probability of finding an alternate buyer, $\theta \in [0, 1]$. In doing so, she incurs an effort cost of $\gamma(\theta)$, where $\gamma(\cdot)$ is strictly increasing and strictly convex in θ, with $\gamma(0) = 0$.

Given early investment r_1 by the incumbent seller, and early breach by the incumbent buyer, the seller chooses her mitigation effort level θ to maximize her expected payoff:

$$\max_{\theta \in [0,1]} \{x_1 - r_1 + \theta[p' - c(r_1, 0)] - \gamma(\theta)\}.$$

Assuming $p' - c(r_1, 0) > \gamma'(0)$, the first order condition characterizing the interior solution is

$$p' - c(r_1, 0) = \gamma'(\theta^e(r_1)),$$

where $\theta^e(r_1)$ represents the incumbent seller's *equilibrium* choice of mitigation effort. This expression simply states that the privately optimal mitigation effort level equates the marginal private benefit of increasing such effort with the marginal cost.

In contrast, the socially efficient mitigation effort level $\theta^*(r_1)$ satisfies

$$v - c(r_1, 0) = \gamma'(\theta^*(r_1))$$

because the marginal social benefit from increasing the probability of trade with an alternate buyer is the total surplus from such trade, or $v - c(r_1)$. Since this marginal social benefit exceeds the marginal private benefit whenever $v > p'$, or whenever the alternate buyer has some bargaining power, the incumbent seller will tend to choose a socially insufficient mitigation effort level (due to the convexity of her effort costs $\gamma(\cdot)$): $\theta^e(r_1) \leq \theta^*(r_1)$ for all r_1, with equality if and only if $v = p'$.[17]

5.3 Contractibility of the Mitigation Decision

Regardless of whether the mitigation choice involves a binary or continuous decision variable, the incumbent buyer usually exerts a socially insufficient amount of effort to mitigate breach damages, and her mitigation decision is socially efficient if and only if she is able to capture all of the gains from trade with the alternate buyer. The intuition for this inefficiency result is analogous to the intuition for inefficient (under-)investment in property rights models with separate ownership: here, unless the seller is able to charge the alternate buyer a price equal to the latter's willingness to pay for the good or service, she (the seller) does not appropriate all of the surplus from trade and therefore has inefficiently weak incentives for mitigation. (Recall that the seller always bears all of the mitigation costs.)

[16] When $p' = v$, the social efficiency of the incumbent seller's mitigation decision follows immediately from the observation that her decision to mitigation can be viewed as an example of a selfish investment.

[17] The same intuition as in footnote 16 above applies here as well.

Notice that the above analysis assumes the damages for early breach, x_1, is fixed and unaffected by the mitigation choice. This requires an implicit assumption that while the incumbent seller is able to commit to her choices of damages, she is unable to commit to her mitigation decision when the contract is first signed. This assumption is reasonable to the extent that mitigation effort cannot be contracted upon at the start of the game, and it seems justified as least in the model where the mitigation decision is continuous and assumed to be equivalent to the *probability* of finding an alternate buyer. In such an environment, it is difficult to conceive how the contracting parties may verify to a court the actual mitigation effort level, since it is *possible* that an alternate buyer is found ex-post even though the incumbent seller may have chosen a very small, but positive, mitigation effort level ex-ante. This case would be relevant, for example, when the mitigation effort decision is not publicly observable.[18]

On the other hand, if the mitigation decision is binary, and there really is no chance of finding an alternate buyer upon late breach, it is conceivable that the mitigation decision might be verifiable ex-post and hence contractible ex-ante.[19] The reason is that if, upon early breach, an alternate buyer is indeed found and trade occurs, then the incumbent seller necessarily chose to mitigate damages. However, this logic depends on the assumption that trade with the alternate buyer is verifable. Were this not the case, the incumbent seller would have an incentive to frabricate evidence of trade with an alternate buyer. Nevertheless, this issue is not problematic to the extent that (i) trade with the incumbent buyer is verifiable, so that the original contract is enforable; and (ii) verifiability of trade for the incumbent seller is correlated among buyers.

If the parties truly cannot contract upon the mitigation decision ex-ante, the incumbent seller would no longer have any contractual obligations towards the incumbent buyer once breach has occurred. She would then be free, in the event of early breach, to choose her mitigation decision in any manner she sees fit. In light of this consideration, the legal requirement that breached-against parties take reasonable efforts to mitigate their damages in the event of breach can be viewed as an attempt to ameliorate the social insufficiency of private mitigation incentives when contracts are incomplete.[20]

5.4 The Nature of the Breach Outcome

There is one final observation to make regarding the efficiency of the incumbent seller's mitigation effort. Assuming that she has full bargaining power vis-a-vis the alternate buyer, the preceeding analysis shows that the incumbent seller has socially efficient incentives for mitigation. This result relies on the implicit assumption that whether the contract is breached directly depends upon only the incumbent buyer's action and not the action of the incumbent seller. If whether breach occurs is a function of both party's

[18] If the mitigation effort decision is publicly observable, the question then becomes whether mitigation should be viewed as the mere exertion of effort to search for an alternate buyer, or actual discovery of such an opportunity *and* the consumation of trade with the alternate buyer.

[19] It would be interesting to analyze whether the incumbent seller has private incentives to write a contract that induces socially efficient mitigation effort when this decision is verifiable and included as a part of the original contract. If the incumbent seller has complete bargaining power with respect to both the incumbent and alternate buyers, it may be reasonable to expect that private mitigation efforts will be socially efficient.

[20] See Goetz and Scott (1983).

actions (as is the case in some tort models), the following analysis will show that the incumbent seller's action (mitigation decision) may be socially inefficient, even if she has full bargaining power with respect to the alternate buyer.

The duty to mitigate damages usually arises in situations where breach damages are imposed ex-post by the court, as opposed to being privately stipulated ex-ante. Therefore, to see the importance of the way in which breach is defined, consider the following example, where I assume court-imposed expectation damages.

Suppose there is just one period, with no investment, buyer value v, seller cost c, and a binary mitigation decision for the incumbent seller. Assume the entrant's cost c_E is either c_E^L or c_E^H with $c_E^L < c_E^H \leq v + \theta[v - c] - \gamma$, where γ is the seller's effort cost of mitigation. In particular, if she mitigates upon breach, there is probability θ that she will be able to find an alternate buyer with whom to trade at the price $p' = v$ and cost c. If the incumbent seller does not mitigate after breach, there is zero probability finding an alternate buyer.

First, suppose breach of contract is defined simply as the buyer's refusal to trade with the incumbent seller. As the previous subsection showed, the seller's mitigation decision will be efficient because upon breach, she receives all the expected surplus from trade with the alternate buyer and therefore will decide to mitigate if and only if $\theta[v - c] - \gamma > 0$, as required by efficiency.

Now suppose breach of contract is said to occur (and hence breach damages x due) if and only if the incumbent buyer refuses trade *and* the incumbent seller cannot find an alternate buyer.[21] Conditional on the incumbent buyer's refusal of trade, efficiency requires that the seller mitigates, i.e., exerts effort to find an alternate buyer, if and only if $v - c_E + \theta[v - c] - \gamma \geq 0 \iff c_E \leq v + \theta[v - c] - \gamma$.[22] Since $c_E \leq c_E^H \leq v + \theta[v - c] - \gamma$ by assumption, the efficient mitigation decision is to always mitigate (conditional on the incumbent buyer's refusal of trade). However, the seller will never exert mitigation effort. To see this, note that if she does not mitigate, then with probability 1 she does not find an alternate buyer to trade with, and hence by definition breach occurs. So the seller's payoff from not mitigating, given expectation damages, is $x = p' - c = v - c$.[23] The seller's payoff from mitigation is $\theta[v - c] + (1 - \theta)x - \gamma = v - c - \gamma$, which is less than her payoff of $v - c$ from not mitigating.[24,25] Thus,

[21] Because the seller's mitigation decision affects her probability of finding an alternate buyer, it also affects the probability that breach is said to occur.

[22] If S does not mitigate after B refuses trade, no surplus is realized because S would not be able to trade with either B or the alternate buyer.

[23] The expectation damage equates the seller's payoff from breach, x, to her payoff from no breach. Conditional on the incumbent buyer's refusal to trade, no breach corresponds to the case in which the seller is able to find an alternate buyer with whom to trade. In this case, the seller receives a payoff of $p' - c = v - c$.

[24] With probability θ, the seller finds and trades with an alternate buyer. In this case, there is no breach and the seller receives $v - c$ from trade with the alternate buyer. With probability $1 - \theta$, the seller is unable to find an alternate buyer, and so by definition breach occurs. The seller receives the breach damage x in this case. Regardless of whether an alternate buyer is found, the seller incurs the effort cost γ if she mitigates.

[25] Note that if the expectation damages were to compensate the seller for her disutility of mitigation effort, then $x = v - c + \gamma$. In this case, the seller's payoff from mitigation is $\theta[v - c] + (1 - \theta)x - \gamma = v - c + (1 - \theta)\gamma - \gamma = v - c - \theta\gamma$, which is still less than her payoff of $v - c$ from not mitigating. Therefore, as long as the court-imposed expectation damage does not grossly over-estimate the seller's disutility of mitigation, she will still prefer to not mitigate.

the seller will never choose to mitigate even though it is efficient for her to do so after the buyer's refusal to trade.

The intuition for this result is straightforward. When breach is equivalent to the incumbent buyer's refusal to trade, the seller's mitigation decision does not affect the incumbent buyer's payoff conditional on his refusal to trade. Instead, the mitigation decision only affects the seller's own payoff (recall the alternate buyer always earns zero by assumption), and so her mitigation decision will be efficient. In contrast, if the definition of breach requires not only the buyer's refusal to trade but also the seller's inability to find an alternate buyer, then the seller will not mitigate even when it is efficient to do so. To see this, note that expectation damages ensure that regardless of whether the seller mitigates, she will receive the same gross payoff (excluding any mitigation effort costs) of $v - c$ after the incumbent buyer refuses to trade. Therefore, because mitigation effort is costly, the seller will choose to not mitigate.[26] (This inefficiency result still obtains even if the seller is accurately compensated for her disutility of mitigation effort when no alternate buyer is found. The reason is that while the cost of mitigation is certain, finding an alternate buyer is not. See footnote 25.)

6 Renegotiation

I now examine the situation where the incumbent seller S and buyer B are able to renegotiate their original contract after each entrant seller announces its price p_{Ei} and prior to each breach opportunity. Once again, assume each entrant is perfectly competitive and sets price equal to cost, $p_{\mathrm{Ei}} = c_{\mathrm{Ei}}$, and suppose that S has complete bargaining power vis-a-vis the alternate buyer. Then S and B's contract imposes no externalities on other parties, and so they have joint incentives to induce efficient breach and investment decisions. As Proposition 4 below demonstrates, the efficient breach and investment decisions can in fact be implemented with the same efficient expectation damages as before, when renegotiation was impossible. The logic underlying this argument depends crucially on analyzing the parties' payoffs off the equilibrium path.

Assume Nash bargaining during each renegotation period, so that the renegotiation outcome maximizes the seller and buyer's joint payoffs. The renegotiation surplus, which is split between S and B in the proportions α and $1 - \alpha$, is defined as the difference in the sum of payoffs for S and B with and without renegotiation: $s_{reneg} \equiv (u_S + u_B)|_{w/reneg} - (u_S + u_B)|_{w/o\ reneg}$. Hence, the payoffs after each stage of renegotiation are $u_S|_{w/o\ reneg} + \alpha \cdot s_{reneg}$ for the seller and $u_B|_{w/o\ reneg} + (1 - \alpha) \cdot s_{reneg}$ for the buyer. If B is indifferent between buying from an entrant or S, assume B buys from the entrant, regardless of whether the indifference arises before or after renegotiation.

Suppose that early and late investment are complementary, i.e.,

$$c_{12}(r_1, r_2) \leq 0 \text{ for all } (r_1, r_2). \tag{14}$$

Then S's privately optimal, or equilibrium, late investment $r_2^e(r_1)$ is increasing in her early investment

[26] Alternatively, the intuition for the inefficiency result follows from the observation that when breach depends on both parties' actions, the incubment seller's mitigation decision has an externality on the incumbent seller (even though $p' = v$ implies no externality on the alternate buyer) and therefore will be inefficient.

r_1. Finally, assume

$$1 - \max\{F[c(r_1, r_2^e(r_1))], F[c(r_1^*, r_2^*(r_1^*))]\} \geq \theta \text{ for all } r_1, \tag{15}$$

which can be shown to imply that: (i) when r_1 is less than r_1^*, the private value of early investment for S exceeds its social value assuming early breach occurs; and (ii) when r_1 is greater than r_1^*, the private value of early investment for S is less than its social value assuming early breach does not occur.

Proposition 4 *Suppose S and B can renegotiate after each competitive entrant arrives and that (14) and (15) are satisfied. Then the ex-ante efficient breach and investment decisions (as characterized in Section 3) can be implemented by the same contract that implements the efficient outcome when renegotiation is not possible, i.e., any contract* (p, x_1, x_2) *where* x_1 *and* x_2 *are the effficient expectation damages and satisfy (8) and (9).*

The intuition for this result is as follows. When $r_1 < r_1^*$, early renegotiation causes early breach to occur (but not absent early renegotiation) for intermediate realizations of the early entrant's cost. In this case, S's private incentive to increase r_1 slightly exceeds the social marginal benefit of increasing r_1. (To see this, suppose no alternate buyer exists. Then a social planner would not value early investment at all given that early breach occurs. However, S obtains a share of the early renegotiation surplus, which is *increasing* in S's early investment.[27]) Similarly, when $r_1 > r_1^*$, early renegotiation causes early breach to not occur (but it does occur absent early renegotiation) for intermediate realizations of the early entrant's cost. Here, assumption (15) implies that S has a smaller private incentive to increase r_1 relative to the social marginal benefit. Together, these two observations will induce S to choose the efficient early investment r_1^*.

Given that S chooses the efficient early investment r_1^*, early renegotiation implies that B's early breach decision will be (ex-ante) efficient as well. It can also be shown that S's privately optimal late investment, $r_2^e(r_1)$, coincides with the efficient late investment $r_2^*(r_1)$ when $r_1 = r_1^*$. In other words, given that S's early investment is efficient, so is her late investment (see Lemma 6 below). Late renegotiation then leads to the efficient late breach decision. (These observations also imply that no renegotiation occurs on the equilibrium path.)

The rest of this section details the proof of this proposition.[28] Using backwards induction, I first look at B's late breach decision, then S's late investment decision, then B's early breach decision, and finally S's early investment decision.

6.1 Late Breach Decision

First consider B's late breach decision. Given there is no early breach and that x_2 satisfies (9), B has a private incentive to breach late absent renegotiation if and only if $v - c_{E2} - x_2 \geq v - p$, i.e.

$$c_{E2} \leq p - x_2 = c(r_1^*, r_2^*(r_1^*)).$$

[27] If trade with an alternate buyer is possible and $r_1 < r_1^*$, assumption (15) implies that S's private marginal benefit from increasing early investment continues to exceed the social marginal benefit, given that early breach occurs.

[28] Readers who are either uninterested in the technical details underlying Proposition 4 or more interested in a concrete application of this model may wish to skip ahead to Section 7.

On the other hand, conditional on S having actually chosen investment levels r_1 and r_2, renegotiation after the second entrant arrives (what I will sometimes refer to as "late renegotiation") leads to late breach if and only if $v - c_{E2} \geq v - c(r_1, r_2)$, i.e.,

$$c_{E2} \leq c(r_1, r_2).$$

Given (r_1, r_2), this is the ex-post efficient breach decision. Since ex-ante efficiency requires late breach to occur exactly when $c_{E2} \leq c(r_1^*, r_2^*(r_1^*))$, late renegotiation implies that B's late breach decision is ex-ante efficient *if* S's early and late investments are ex-ante efficient, i.e., if $(r_1, r_2) = (r_1^*, r_2^*(r_1^*))$.

6.2 Renegotiation Payoffs in the Second Period

Before examining S's late investment decision, we must first consider the (renegotiation-induced) payoffs of S (and B) for all possible realizations of the second entrant's price/cost c_{E2}, as well as for all possible early and late investments (r_1, r_2) that S might make (including those off the equilibrium path).

When $c_{E2} \leq \min\{p - x_2, c(r_1, r_2)\}$, B breaches late regardless of whether late renegotiation is possible, and so payoffs are $u_S = x_2 - r_1 - r_2$ and $u_B = v - c_{E2} - x_2$. On the other hand, when $c_{E2} > \max\{p - x_2, c(r_1, r_2)\}$, B does not breach late regardless of whether late renegotiation is possible, and so payoffs are $u_S = p - c(r_1, r_2) - r_1 - r_2$ and $u_B = v - p$.

If $p - x_2 < c_{E2} \leq c(r_1, r_2)$, B does not breach late absent late renegotiation because $p - x_2 < c_{E2}$. But since the second entrant can produce the good at a lower cost than S in this case, renegotiation will induce B to breach and allow the parties to share the renegotiation surplus $c(r_1, r_2) - c_{E2} \geq 0$. Disagreement payoffs are those associated with the no-breach outcome, i.e., $p - c(r_1, r_2) - r_1 - r_2$ for S and $v - p$ for B, and so the renegotiation payoffs are $u_S = p - c(r_1, r_2) - r_1 - r_2 + \alpha[c(r_1, r_2) - c_{E2}]$ and $u_B = v - p + (1 - \alpha)[c(r_1, r_2) - c_{E2}]$.

On the other hand, if $c(r_1, r_2) < c_{E2} \leq p - x_2$, B breaches late absent late renegotiation because $c_{E2} \leq p - x_2$. But late renegotiation will cause B to not breach and allow the parties to share the renegotiation surplus $c_{E2} - c(r_1, r_2) > 0$ (in this case, S has the lower cost). Disagreement payoffs are therefore those associated with the breach outcome, i.e., $x_2 - r_1 - r_2$ for S and $v - c_{E2} - x_2$ for B, and so the renegotiation payoffs are $u_S = x_2 - r_1 - r_2 + \alpha[c_{E2} - c(r_1, r_2)]$ and $u_B = v - c_{E2} - x_2 + (1 - \alpha)[c_{E2} - c(r_1, r_2)]$.

To summarize:

Lemma 5 *If early breach does not occur and S's investments are (r_1, r_2), payoffs after late renegotiation (excluding investment costs) for the incumbent seller S and buyer B, respectively, are given by:*

$$\begin{array}{ll}
\{x_2, v - c_{E2} - x_2\} & \text{if } c_{E2} \leq \min\{p - x_2, c(r_1, r_2)\}; \\
\{p - c(r_1, r_2), v - p\} & \text{if } c_{E2} > \max\{p - x_2, c(r_1, r_2)\}; \\
\left\{ \begin{array}{c} p - c(r_1, r_2) + \alpha[c(r_1, r_2) - c_{E2}], \\ v - p + (1 - \alpha)[c(r_1, r_2) - c_{E2}] \end{array} \right\} & \text{if } p - x_2 < c_{E2} \leq c(r_1, r_2); \\
\left\{ \begin{array}{c} x_2 + \alpha[c_{E2} - c(r_1, r_2)], \\ v - c_{E2} - x_2 + (1 - \alpha)[c_{E2} - c(r_1, r_2)] \end{array} \right\} & \text{if } c(r_1, r_2) < c_{E2} \leq p - x_2.
\end{array}$$

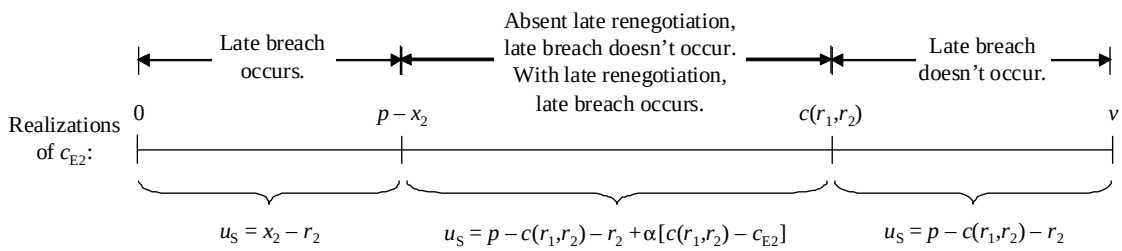

Figure 2: Seller S's ex-post payoffs after late renegotiation, for the case when $p - x_2 \leq c(r_1, r_2)$.

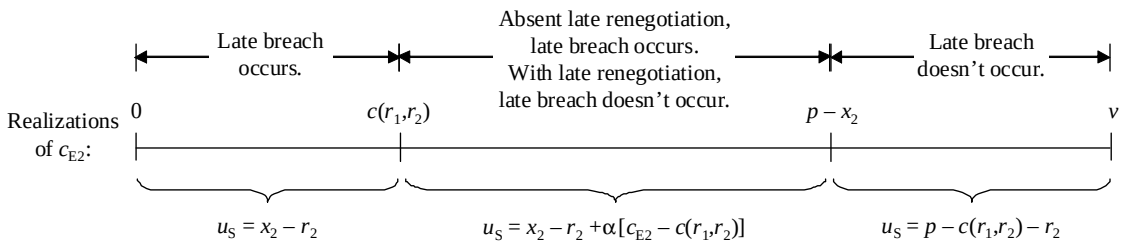

Figure 3: Seller S's ex-post payoffs after late renegotiation, for the case when $p - x_2 \geq c(r_1, r_2)$.

6.3 Late Investment Decision

Now consider S's late investment decision given that she chose r_1 in period 1. First, suppose S chooses r_2 such that $p - x_2 \leq c(r_1, r_2)$. Conditional on early breach not occurring, Figure 2 summarizes S's ex-post payoff after late renegotiation (from Lemma 5) as a function of the second entrant's price offer $p_{E2} = c_{E2}$.

In this case, S's expected payoff (exclusive of her early investment cost) is

$$\begin{aligned}\pi_L(r_1, r_2) &= F[p - x_2]x_2 + \int_{p-x_2}^{c(r_1 r_2)} \{p - c(r_1, r_2) + \alpha[c(r_1, r_2) - c_{E1}]\} f(c_{E1}) dc_{E1} \\ &\quad +(1 - F[c(r_1, r_2)])[p - c(r_1, r_2)] - r_2.\end{aligned}$$

On the other hand, if S chooses r_2 such that $p - x_2 \geq c(r_1, r_2)$, Figure 3 depicts her ex-post payoff after late renegotiation as a function of the second entrant's price.

For these values of r_1 and r_2, S's expected payoff (exclusive of early investment cost) is

$$\begin{aligned}\pi_H(r_1, r_2) &= F[c(r_1, r_2)]x_2 + \int_{c(r_1 r_2)}^{p-x_2} \{x_2 + \alpha[c_{E1} - c(r_1, r_2)]\} f(c_{E1}) dc_{E1} \\ &\quad +(1 - F[p - x_2])[p - c(r_1, r_2)] - r_2.\end{aligned}$$

Note that $\pi_H(r_1, r_2)$ can be rewritten as

$$\begin{aligned}\pi_H(r_1, r_2) &= F[p - x_2]x_2 + \int_{c(r_1 r_2)}^{p-x_2} \alpha[c_{E1} - c(r_1, r_2)] f(c_{E1}) dc_{E1} \\ &\quad +(1 - F[p - x_2])[p - c(r_1, r_2)] - r_2 \\ &= \pi_L(r_1, r_2),\end{aligned}$$

where the second inequality follows from (i) switching the bounds of integration in the second term and multiplying the integrand by -1; and (ii) writing $1 - F[p - x_2]$ in the third term as $(1 - F[c(r_1, r_2)]) + (F[c(r_1, r_2)] - F[p - x_2])$ and then rearranging. Thus, given r_1, simply denote S's expected payoff from chosing r_2 (exclusive of early investment cost) by

$$\pi(r_1, r_2) \equiv \pi_L(r_1, r_2) = \pi_H(r_1, r_2) \text{ for all } (r_1, r_2).$$

Let $r_2^e(r_1)$ denote S's privately optimal, or equilibrium, late investment choice, given that her early investment is r_1. It is characterized by the first order condition

$$\begin{aligned} 0 &= \pi_2(r_1, r_2^e(r_1)) \text{ for all } r_1 \\ &= -c_2(r_1, r_2^e(r_1))\{1 - \alpha F[c(r_1, r_2^e(r_1))] - (1-\alpha)F[p - x_2]\} - 1 \end{aligned} \tag{16}$$

Lemma 6 *If S's early investment is efficient, her late investment is efficient as well:*

$$r_2^e(r_1^*) = r_2^*(r_1^*).$$

Proof. To see this, observe that since $r_2^*(r_1^*)$ maximizes social surplus given r_1^*, $S'(r_2^*(r_1^*)|r_1^*) \gtreqless 0$ for all $r_2 \lesseqgtr r_2^*(r_1^*)$. Thus, if $p - x_2 = c(r_1^*, r_2^*(r_1^*)) \leq c(r_1^*, r_2)$, then $r_2 \leq r_2^*(r_1^*)$ (as $c_2 < 0$). In this case, (9) implies $\pi_2(r_1^*, r_2) \geq -c_2(r_1^*, r_2)\{1 - F[c(r_1^*, r_2)]\} - 1 = S'(r_2|r_1^*) \geq 0$. Similarly, $p - x_2 = c(r_1^*, r_2^*(r_1^*)) \geq c(r_1^*, r_2)$ implies $r_2 \geq r_2^*(r_1^*)$ and hence $\pi_2(r_1^*, r_2) \leq -c_2(r_1^*, r_2)\{1 - F[c(r_1^*, r_2)]\} - 1 = S'(r_2|r_1^*) \leq 0$. ■

This result is analogous to Proposition 1 in Spier and Whinston (1995), where efficient expectation damages lead the seller to invest efficiently. (As in their Proposition 1, I also assume renegotiation and a perfectly competitive (late) entrant.) The intuition is the same as well. When the seller's late investment is less than efficient (given r_1^*), late renegotiation allows her to capture a share of the return on her cost reduction for realizations of c_{E2} that ultimately lead to late breach (see the middle interval in Figure 2). Since a social planner only values late investment when S actually produces the good, the seller's incentive to increase her late investment exceeds that of a social planner when r_2 is less than efficient (given r_1^*). Similarly, when r_2 is more than efficient (given r_1^*), the seller's incentive to increase her late investment is less than that of a social planner. Hence, the seller chooses the efficient late investment (given early investment r_1^*).

Finally, assuming the second order condition is satisfied, (14) implies that $r_2^e(r_1)$ is increasing in r_1.[29] Hence, because $c_1 < 0, c_2 < 0$, we have $\frac{d}{dr_1} c(r_1, r_2^e(r_1)) < 0$. Therefore Lemma 6 implies that

$$r_1 \lesseqgtr r_1^* \iff p - x_2 = c(r_1^*, r_2^*(r_1^*)) \lesseqgtr c(r_1, r_2^e(r_1)) \tag{17}$$

with equality if and only if $r_1 = r_1^*$.

6.4 Early Breach Decision

Absent Early Renegotiation.

[29] A sufficient condition for the second order condition to be satisfied is that $\pi_{22} = -c_{22}(r_1, r_2)\{1 - \alpha F[c(r_1, r_2)] - (1 - \alpha)F[c(r_1^*, r_2^*(r_1^*))]\} + c_2(r_1, r_2)^2 f[c(r_1, r_2)] < 0$ at $r_2 = r_2^e(r_1)$ for all r_1. Given (14), $\pi_{21} = -c_{21}\{1 - \alpha F[c] - (1 - \alpha)F[p - x_2]\} + \alpha c_1 c_2 f(c) > 0$, and so $r_2^{e\prime}(r_1) = -\pi_{21}/\pi_{22} > 0$ at $(r_1, r_2^e(r_1))$.

Absent early renegotiation, the incumbent buyer B obtains a payoff of $v - c_{E1} - x_1$ if he breaches early to buy from the first entrant. Now consider B's expected payoff from not breaching early, with late renegotiation still possible.

Given S's early investment r_1, B will anticipate S's late investment choice of $r_2^e(r_1)$. First, suppose $r_1 \leq r_1^*$, which is equivalent to $p - x_2 \leq c(r_1, r_2^e(r_1))$ by (17). Lemma 5 and (9) imply that B's expected payoff from not breaching early is

$$\begin{aligned} & \int_0^{p-x_2} (v - c_{E2} - x_2) f(c_{E2}) dc_{E2} + (1 - F[c(r_1, r_2^e(r_1))])(v - p) \\ & + \int_{p-x_2}^{c(r_1, r_2^e(r_1))} (v - p + (1-\alpha)[c(r_1, r_2^e(r_1)) - c_{E2}]) f(c_{E2}) dc_{E2} \\ = \quad & v - p - \int_0^{p-x_2} (c_{E2} - c(r_1^*, r_2^*(r_1^*)) f(c_{E2}) dc_{E2} \\ & + \int_{p-x_2}^{c(r_1, r_2^e(r_1))} (1-\alpha)[c(r_1, r_2^e(r_1)) - c_{E2}] f(c_{E2}) dc_{E2}. \end{aligned}$$

Since (9) implies $c^*(r_1^*) = \int_0^{p-x_2} c_{E2} f(c_{E2}) dc_{E2} + \int_{p-x_2}^{v} c(r_1^*, r_2^*(r_1^*)) f(c_{E2}) dc_{E2}$ (recall (4), the definition of $c^*(r_1)$), B's expected payoff from not early early can be further rewritten as $v - \psi(r_1) - x_2$, where

$$\psi(r_1) \equiv c^*(r_1^*) - \int_{c(r_1^*, r_2^*(r_1^*))}^{c(r_1, r_2^e(r_1))} (1-\alpha)[c(r_1, r_2^e(r_1)) - c_{E2}] f(c_{E2}) dc_{E2}.$$

If $r_1 \geq r_1^*$ instead, i.e., $p - x_2 \geq c(r_1, r_2^e(r_1))$, B's expected payoff from not breaching early is

$$\begin{aligned} & \int_0^{c(r_1, r_2^e(r_1))} (v - c_{E2} - x_2) f(c_{E2}) dc_{E2} + (1 - F[p - x_2])(v - p) \\ & + \int_{c(r_1, r_2^e(r_1))}^{p-x_2} (v - c_{E2} - x_2 + (1-\alpha)[c_{E2} - c(r_1, r_2^e(r_1))]) f(c_{E2}) dc_{E2}. \end{aligned}$$

It turns out that this expression can also be written as $v - \psi(r_1) - x_2$.

So for any r_1, B breaches early absent early renegotiation if and only if $v - c_{E1} - x_1 \geq v - \psi(r_1) - x_2$, or equivalently,

$$c_{E1} \leq \psi(r_1) + x_2 - x_1. \tag{18}$$

Since $\psi'(r_1) = (1-\alpha) \frac{dc(r_1, r_2^e(r_1))}{dr_1} (F[c(r_1^*, r_2^*(r_1^*))] - F[c(r_1, r_2^e(r_1))])$, (17) implies that

$$\psi'(r_1) \gtreqless 0 \text{ for all } r_1 \lesseqgtr r_1^*, \tag{19}$$

with equality only at r_1^*.

Finally, $\psi(r_1^*) = c^*(r_1^*)$ follows from Lemma 6. So *if* S's early investment is efficient, (8) and (9) imply that B will breach early absent early renegotiation if and only if $c_{E1} \leq \psi(r_1^*) + x_2 - x_1 = c^*(r_1^*) + r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)]$, which is the efficient early breach decision.

With Early Renegotiation.

With early renegotiation, B will breach early to buy from the first entrant if and only if expected social surplus is higher from his breaching early. Absent early breach, surplus is $u_S + u_B = v - \psi(r_1) - x_2 + \pi(r_1, r_2^e(r_1)) - r_1$. With early breach, $u_S + u_B = v - c_{E1} + \theta[v - c(r_1, 0)] - r_1$. Thus, early renegotiation leads to early breach if and only if

$$\begin{aligned} c_{E1} \quad & \leq \quad \phi(r_1) + \theta[v - c(r_1, 0)], \\ \text{where } \phi(r_1) \quad & \equiv \quad \psi(r_1) + x_2 - \pi(r_1, r_2^e(r_1)), \end{aligned} \tag{20}$$

(which is the efficient breach decision given r_1).

Recall from Section 4 that when renegotiation is never possible, early breach is efficient given r_1 if and only if

$$c_{E1} \leq c^*(r_1) + r_2^*(r_1) + \theta[v - c(r_1, 0)] \tag{21}$$

(compare with (3) for the case $r_1 = r_1^*$). It can be verified that $c^*(r_1) + r_2^*(r_1)$ and $\phi(r_1)$, and hence the right hand sides of (20) and (21), are not equal unless $r_1 = r_1^*$. Therefore, the efficient early breach decisions when renegotiation is and is not possible do not coincide with each other unless S's early investment is efficient. *In other words, the possibility of renegotiation does not alter the efficient early breach decision on the equilibrium path but does affect it off the equilibrium path.*

Since B's early breach decision (with early renegotiation) is ex-ante efficient given r_1^*, it remains to show that S's early investment is indeed efficient.

6.5 Renegotiation Payoffs in the First Period

Before analyzing S's early investment decision, we first derive the payoffs of S (and B) after early renegotiation for all possible realizations of the first entrant's price/cost c_{E1} and all levels of S's early investment r_1. Recall that absent early renegotiation, B breaches early if and only if (18) holds, while with early renegotiation early breach occurs if and only if (20) is satisfied.

When $c_{E1} \leq \min\{\psi(r_1) + x_2 - x_1, \phi(r_1) + \theta[v - c(r_1, 0)]\}$, B breaches early regardless of whether early renegotiation is possible, and so payoffs are $u_S = x_1 + \theta[v - c(r_1, 0)] - r_1$ and $u_B = v - c_{E1} - x_1$. On the other hand, when $c_{E2} > \max\{\psi(r_1) + x_2 - x_1, \phi(r_1) + \theta[v - c(r_1, 0)]\}$, B does not breach early regardless of whether early renegotiation is possible, and so payoffs are $u_S = \pi(r_1, r_2^e(r_1)) - r_1$ and $u_B = v - \psi(r_1) - x_2$.

If $\psi(r_1) + x_2 - x_1 < c_{E1} \leq \phi(r_1) + \theta[v - c(r_1, 0)]$, B does not breach early absent early renegotiation because $\psi(r_1) + x_2 - x_1 < c_{E1}$. But early renegotiation induces B to breach early and allow the parties to share the renegotiation surplus $s_{reneg}^L \equiv \phi(r_1) + \theta[v - c(r_1, 0)] - c_{E1} \geq 0$. Disagreement payoffs are those associated with the no-early-breach outcome, i.e., $\pi(r_1, r_2^e(r_1)) - r_1$ for S and $v - \psi(r_1) - x_2$ for B, and so the renegotiation payoffs are $u_S = \pi(r_1, r_2^e(r_1)) - r_1 + \alpha \cdot s_{reneg}^L$ and $u_B = v - \psi(r_1) - x_2 + (1 - \alpha) \cdot s_{reneg}^L$.

If $\phi(r_1) + \theta[v - c(r_1, 0)] < c_{E1} \leq \psi(r_1) + x_2 - x_1$, B breaches early absent early renegotiation because $c_{E1} \leq \psi(r_1) + x_2 - x_1$. However, early renegotiation induces B to not breach early and allow the parties to share the renegotiation surplus $s_{reneg}^H \equiv c_{E1} - \phi(r_1) - \theta[v - c(r_1, 0)] \geq 0$. Disagreement payoffs are those associated with the early breach outcome, i.e., $x_1 + \theta[v - c(r_1, 0)] - r_1$ for S and $u_B = v - c_{E1} - x_1$ for B, and so the renegotiation payoffs are $u_S = x_1 + \theta[v - c(r_1, 0)] - r_1 + \alpha \cdot s_{reneg}^H$ and $u_B = v - c_{E1} - x_1 + (1 - \alpha) \cdot s_{reneg}^H$.

To summarize:

Lemma 7 *If S's early investment is r_1, the expected payoffs after early renegotiation (excluding early*

investment costs) for S and B, respectively, are given by:

$$\{x_1 + \theta[v - c(r_1, 0)], v - c_{E1} - x_1\} \qquad \text{if } c_{E1} \leq \min \left\{ \begin{array}{c} \psi(r_1) + x_2 - x_1, \\ \phi(r_1) + \theta[v - c(r_1, 0)] \end{array} \right\};$$

$$\{\pi(r_1, r_2^e(r_1)), v - \psi(r_1) - x_2\} \qquad \text{if } c_{E1} > \max \left\{ \begin{array}{c} \psi(r_1) + x_2 - x_1, \\ \phi(r_1) + \theta[v - c(r_1, 0)] \end{array} \right\};$$

$$\left\{ \begin{array}{c} \pi(r_1, r_2^e(r_1)) + \alpha \cdot s_{reneg}^L, \\ v - \psi(r_1) - x_2 + (1-\alpha) \cdot s_{reneg}^L] \end{array} \right\} \qquad \text{if } \psi(r_1) + x_2 - x_1 < c_{E2} \leq \phi(r_1) + \theta[v - c(r_1, 0)];$$

$$\left\{ \begin{array}{c} x_1 + \theta[v - c(r_1, 0)] + \alpha \cdot s_{reneg}^H, \\ v - c_{E1} - x_1 + (1-\alpha) \cdot s_{reneg}^H \end{array} \right\} \qquad \text{if } \phi(r_1) + \theta[v - c(r_1, 0)] < c_{E2} \leq \psi(r_1) + x_2 - x_1;$$

where $s_{reneg}^L \equiv -s_{reneg}^H \equiv \phi(r_1) + \theta[v - c(r_1, 0)] - c_{E1}$.

6.6 Early Investment Decision

Given the preceeding analysis, to prove Proposition 4 it suffices to show that S's privately optimal early investment is indeed at the efficient level r_1^*. Define $\Pi(r_1)$ to be S's ex-ante expected payoffs from choosing r_1. Recall that ex-ante expected social welfare given r_1 is denoted by $S(r_1)$ and, by definition, is maximized at r_1^*. We will show that

$$\begin{aligned} \Pi'(r_1) &\geq S'(r_1) \geq 0, \forall r_1 \leq r_1^*, \text{ and} \\ \Pi'(r_1) &\leq S'(r_1) \leq 0, \forall r_1 \geq r_1^*. \end{aligned}$$

It will then follow that S's privately optimal early investment (the value of r_1 that maximizes $\Pi(r_1)$) is indeed the efficient one, r_1^*. (Note that similar to the proof of Lemma 6 above, this part of the proof of Proposition 4 also follows the strategy of the proof of Proposition 1 in Spier and Whinston (1995). The complicating factor in this model is that because there is a second period if early breach does not occur, one must replace the (final) renegotiation payoffs derived in Spier and Whinston's Lemma 1 with the (interim) renegotiation payoffs given by Lemma 7 above.)

First of all, observe that given assumption (15),

$$r_1 \lesseqqgtr r_1^* \iff \psi(r_1) + x_2 - x_1 \lesseqqgtr \phi(r_1) + \theta[v - c(r_1, 0)],$$

with equality only at $r_1 = r_1^*$. To see this, note that $\psi(r_1) + x_2 - x_1 \lesseqqgtr \phi(r_1) + \theta[v - c(r_1, 0)]$ is equivalent to

$$0 \lesseqqgtr \theta[v - c(r_1, 0)] + x_1 - \pi(r_1, r_2^e(r_1)),$$

which is satisfied for all $r_1 \lesseqqgtr r_1^*$ because the right hand side of this expression is zero at r_1^* (by (8) and (9)) and strictly decreasing in r_1 for all r_1 (by assumption (15)).[30,31]

Case (A). Suppose $r_1 \leq r_1^*$, which implies $\psi(r_1) + x_2 - x_1 \leq \phi(r_1) + \theta[v - c(r_1, 0)]$. There are three subcases to consider for different realizations of c_{E1}, and Figure 4 shows S's payoffs in each subcase.

[30] $\frac{d}{dr_1}\left\{\theta[v - c(r_1, 0)] + x_1 - \pi(r_1, r_2^e(r_1))\right\} = -\theta c_1(r_1, 0) - \pi_1(r_1, r_2^e(r_1))$, which is negative for all r_1 by assumption (15).

[31] Recall that excluding early investment cost and absent early renegotiation, S's earns a payoff of $x_1 + \theta[v - c(r_1, 0)]$ from early breach occurring and $\pi(r_1, r_2^e(r_1))$ from early breach not occurring. Therefore, $\pi(r_1, r_2^e(r_1)) \lesseqqgtr x_1 + \theta[v - c(r_1, 0)]$ for all $r_1 \lesseqqgtr r_1^*$ implies that (i) when r_1 is less than r_1^*, early investment is more valuable to S if early breach occurs; and (ii) when r_1 is greater than r_1^*, early investment is more valuable to her if early breach does not occur.

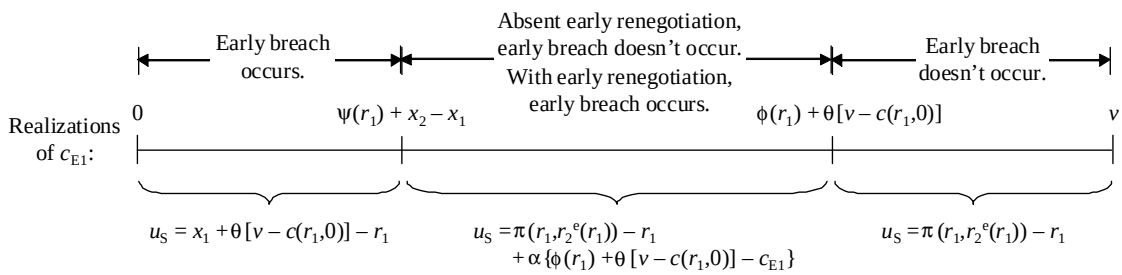

Figure 4: Seller's payoffs after early renegotiation in Case (A), where $r_1 \leq r_1^*$.

(i) If $c_{E1} \leq \psi(r_1) + x_2 - x_1$, early breach always occurs. Social surplus is $v - c_{E1} + \theta[v - c(r_1, 0)] - r_1$ for these realizations of c_{E1}, so the marginal net social return from increasing r_1 slightly is $-\theta c_1(r_1, 0) - 1$. Since S's private payoff is $x_1 + \theta[v - c(r_1, 0)] - r_1$ in this range, her marginal net private return from increasing r_1 corresponds to the net social return.

(ii) If $\psi(r_1) + x_2 - x_1 < c_{E1} \leq \phi(r_1) + \theta[v - c(r_1, 0)]$, early breach still occurs because of early renegotiation, and so the marginal social return from increasing r_1 is still $-\theta c_1(r_1, 0) - 1$. For these realizations of c_{E1}, however, S's private expected payoff given early renegotiation is $\pi(r_1, r_2^e(r_1)) - r_1 + \alpha\{\phi(r_1) + \theta[v - c(r_1, 0)] - c_{E1}\}$ (Lemma 7), and so her marginal private return is

$$\begin{aligned} & \pi_1(r_1, r_2^e(r_1)) + \alpha\{\psi'(r_1) - \pi_1(r_1, r_2^e(r_1)) - \theta c_1(r_1, 0)\} - 1 \\ = \quad & \alpha\{\psi'(r_1) - \theta c_1(r_1, 0)\} + (1 - \alpha)\pi_1(r_1, r_2^e(r_1)) - 1. \end{aligned}$$

The marginal private return of S from increasing r_1 exceeds the marginal social return, $-\theta c_1(r_1, 0) - 1$, if and only if $\alpha\psi'(r_1) + (1 - \alpha)\{\pi_1(r_1, r_2^e(r_1)) + \theta c_1(r_1, 0)\} \geq 0$, which is indeed satisfied because $r_1 \leq r_1^*$ and (19) imply $\psi'(r_1) \geq 0$ while $r_1 \leq r_1^*$ and footnote 30 imply $\pi_1 + \theta c_1 \geq 0$.

(iii) If $\phi(r_1) + \theta[v - c(r_1, 0)] < c_{E1}$, early breach never occurs. The continuation social surplus is $v - \psi(r_1) - x_2 + \pi(r_1, r_2^e(r_1)) - r_1$ from these realizations of c_{E1}, and the marginal social return from increasing r_1 is $\pi_1(r_1, r_2^e(r_1)) - \psi'(r_1) - 1$, which is less than $\pi_1(r_1, r_2^e(r_1)) - 1$, i.e. S's marginal private return (recall $r_1 \leq r_1^*$ and (19) implies $\psi'(r_1) \geq 0$).

So to summarize case (A), when $r_1 \leq r_1^*$, S's marginal net private return from increasing r_1 slightly is weakly greater than the marginal net social return for all realizations of c_{E1}. Hence $\Pi'(r_1) \geq S'(r_1) \geq 0$ when $r_1 \leq r_1^*$.

Case (B). If $r_1 \geq r_1^*$, then $\psi(r_1) + x_2 - x_1 \geq \phi(r_1) + \theta[v - c(r_1, 0)]$. S's payoffs for all possible realizations of c_{E1} are depicted in Figure 5.

Similary to the previous case, it can be shown that S's marginal net private return to increasing r_1 slightly is weakly less than the marginal net social return for all c_{E1}. Therefore $\Pi'(r_1) \leq S'(r_1) \leq 0$ for all $r_1 \geq r_1^*$.

Hence, given any a contract (p, x_1, x_2) where x_1 and x_2 are the efficient expectation damages (satisfying (8) and (9)), S's privately optimal early investment (the value of r_1 that maximizes $\Pi(r_1)$) is indeed the efficient one, r_1^*.

This concludes the proof of Proposition 4.

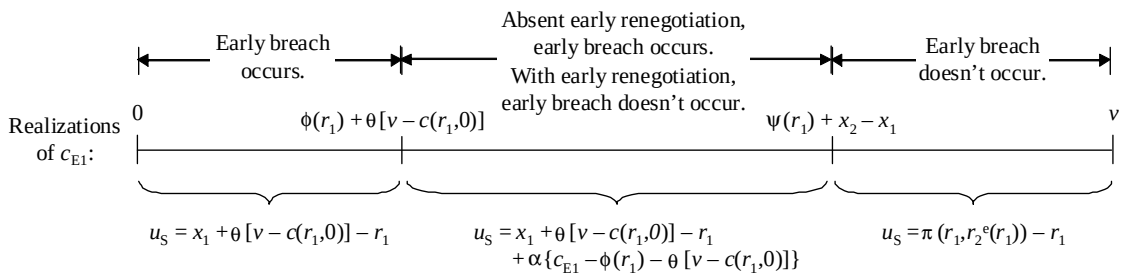

Figure 5: Seller's payoffs after early renegotiation in Case (B), where $r_1 \geq r_1^*$.

To summarize, any contract (p, x_1, x_2) where x_1 and x_2 are the efficient expectation damages specified in (8) and (9) will induce S to choose the ex-ante efficient early investment r_1^*. The work above shows that early renegotiation then leads to B making the efficient early breach decision, S making the efficient late investment, and B making the efficient late breach decision.

Proposition 4 says that the same contract that implements the efficient outcome when renegotiation is not possible also implements the efficient outcome when renegotiation is possible. Therefore, renegotiation will not occur on the equilibrium path. Nevertheless, it is crucial in establishing Proposition 4 to consider the payoffs of the parties from choices made off the equilibrium path.

7 An Application

Consider once again the model without renegotiation or mitigation effort (so that the probability of finding an alternate buyer is exogenous). One application of this model is to study the way in which hotels structure their fees for cancellation of a reservation. There are usually different cancellation policies for reservations during the high season versus the low season. For example, the following is a summary of the deposit and cancellation policies of The Lodge at Vail, a ski resort in Vail, Colorado.[32]

> *Deposit Policies*: In the winter season, a 50% deposit is due at the time of booking. The remaining balance is then due 45 days prior to the arrival. In spring, summer, and fall seasons, no deposit is required.
>
> *Cancellation Policies*: In the winter season, a full refund, less the first night's room and tax, will be given if reservations are cancelled more than 45 days prior to arrival. However, there will be a full forfeiture of the entire reservation value if cancelling within 45 days of arrival. In spring, summer, and fall seasons, one night's deposit will be forfeited if cancellation occurs within 24 hours of arrival.[33]

In the case of The Lodge at Vail, their penalities for breach of contract (cancelling the reservation) are increasing as one approaches the date of performance (start of the reserved stay), regardless of the time

[32] See http://lodgeatvail.rockresorts.com. For the cancellation policy, see http://lodgeatvail.rockresorts.com/info/rr.fees.asp.

[33] Even though no deposit it required at the time a reservation is made in the spring, summer, or fall season, the price of one night's stay is still charged to the guest if cancellation occurs within 24 hours of arrival.

of the year. Furthermore, presumably because of higher demand in the winter season for ski resorts, the difference between their penalties for cancelling late and cancelling early is larger during the winter than during other times of the year (ignoring the seasonal difference in the definitions of what constitutes a late breach). This choice of breach damages is consistent with the assumption that it is impossible (or in general, more difficult) to find an alternate buyer if breach occurs late, and the fact that it is easier (by definition) to find an alternate buyer in case of early breach during the high season than low season.

In order to precisely apply the model to this lodging industry example, the parameter θ should, strictly speaking, be interpreted as the probability of finding an alternate buyer/guest (upon early breach) to fill the *same* room that was vacated by the incumbent buyer/guest who breached the original contract. (For example, the seller/hotel may be booked to capacity at the time that the original contract is breached.) Otherwise, without a binding capacity constraint, the seller may be able to accommodate another buyer even if early breach does not occur.

Note that the seller/hotel is less likely to be booked to capacity during the low season than during the high season, which is consistent with θ being lower during the low season. Furthermore, whether breach is considered late or early in the low season depends on whether it occurs within 24 hours prior to arrival; whereas during the high season breach is considered late if it occurs within 45 days prior to arrival. The shorter prior notice requirement for early breach during the low season is also consistent with θ being lower during the low season.

To formalize the connection between the Lodge at Vail example and the model, suppose that the price of the entire reserved stay can be written as np^s, where p^s is the price per night, with $s \in \{H, L\}$ denoting the season, and n is the number of nights. Assume that the price is higher during the high season than during the low season, or $p^H > p^L$ (presumably, short-run supply in the lodging industry is fixed), and that the stay is for at least $n > \frac{p^L}{p^H} + 1$ nights. Then the Lodge at Vail's policy is such that during the high (winter) season, $x_2^H - x_1^H = np^H - p^H = (n-1)p^H$, which exceeds the analogous difference $x_2^L - x_1^L = p^L - 0 = p^L$ during the low season. Thus this example is consistent with the second inequality in Corollary 3. Note that the Lodge at Vail's policy also satisfies $x_2^H = np^H > p^L = x_2^L$, i.e., the penalty for cancelling a reservation at the last minute is larger in the high season than in the low season. If the model formally accounts for seasonal variations in the contract price, then this observation would again be consistent with the model's predicted efficient expectation damages for late breach. (This claim follows from replacing p with p^H and p^L in (9) and noting that $(r_1^*, r_2^*(r_1^*))$ do not depend on p).

Finally, observe that both results in Corollary 3 could have been obtained even if the seller does not make any investments, or if she only invests before the first breach decision. If the seller only invests before the first breach decision, efficient investment and breach decisions can be induced by $x_2 = p - c(r_1^*)$ and $x_1 = p - c(r_1^*) - \theta[v - c(r_1^*)]$ so that $x_2 - x_1 = \theta[v - c(r_1^*)] > 0$. Similarly, if the seller does not make any investments ($r_1^* \equiv 0$), efficient breach decisions can still be induced with (x_1, x_2) satisfying $x_2 - x_1 = \theta[v - c(0)] > 0$. Therefore, an empirical investigation is necessary to determine whether, and how, a seller's investments affect the difference in her chosen penalties for late breach versus early breach in reality. However, regardless of whether, and when, the seller makes investments, the models predict that the difference in the penalties for late breach versus early breach, $x_2 - x_1$, is increasing in θ, the

likelihood of finding an alternate buyer if breach occurs early.

8 Conclusion

This paper studies optimal liquidated damages when breach of contract is possible at multiple points in time. It suggests that when the potentially breached-against party makes sequential investment decisions, efficient breach damages should increase over time so as to make the potentially breaching party internalize those increasing opportunity costs. This provides an intuitive explanation for why fees for cancelling some service contracts, such as hotel reservations, tend to increase as the time for performance approaches.

Furthermore, when the investing party may be able to find an alternate trading partner when breach occurs early but not when breach occurs late, it is shown that the amount by which the damages for late breach exceeds the damages for early breach is increasing in the probability of finding an alternate trading partner. This provides one possible explanation for why hotels tend to charge larger penalties for late cancellation of high-season reservations than late cancellation of low-season reservations.

When an incumbent seller, as the potentially breached-against party, can affect the probability of finding an alternate buyer, her private incentives to mitigate breach damages are shown to be socially insufficient whenever she does not have full bargaining power vis-a-vis the alternate buyer. This is because while mitigation costs are always borne entirely by the incumbent seller, the benefits of mitigation are shared whenever the alternate buyer has some bargaining power. However, if breach is defined as not only a function of whether the incumbent buyer refuses trade, but also a function of whether the incumbent seller is able to trade with an alternate buyer, then the incumbent seller's mitigation incentives may be insufficient even if she has full bargaining power with the alternate buyer.

Finally, it is shown that when the incumbent buyer and seller are able to renegotiate their original contract after the arrival of each perfectly competitive entrant, the socially efficient breach and investment decisions can still be implemented with the same efficient expectation damages that implement the first best outcome absent renegotiation.

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