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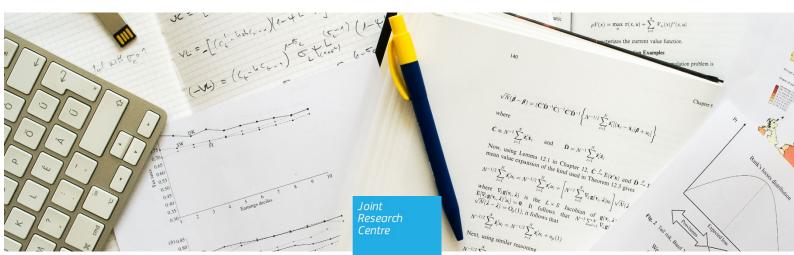
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Change and convergence of income distributions in the European Union during 2007-2014

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Abstract

Mainstream monitoring of income dynamics and inequality is based on summary measures that can miss important phenomena prevalent in income distributions. Relying on quantile functions and the adapted statistical framework suggested by Székely and Rizzo (2004), we characterize the change and convergence of net equivalized income distributions among European Union countries. We exploit the scale-independence property of proper inequality metrics to evaluate not only the total but also the inequality-affecting (shape-influenced) convergence of distributions.

Keywords: convergence, European Union, income distribution, inequality.

JEL Codes: D31, D63, O15.

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1 Introduction

Sustainability of the European Union (EU) is hardly possible without adequate economic and social cohesion. According to the Five Presidents' report, the 'notion of convergence is at the heart of our Economic Union' (Juncker et al., 2015, p.7). The harmonization of institutions with the defined structural convergence objectives aim at the development of the similar potential of countries and, together with the cohesion policy and structural funds, are supposed to create adequate capacity, mechanisms, and means fostering economic convergence of EU regions lagging in terms of income.

Empirical evidence shows that economic convergence, as measured by the beta-convergence, is indeed taking place among European countries whenever more extended periods are considered (see, e.g. Hoyo et al., 2017). At the same time, the sigma-convergence that took place before the global financial crisis became weaker or even turned into divergence in the aftermath of it (see ibidem and Alcidi et al., 2018). Potentially, this indicates either the emergence of different (conditional) steady states or the increasing dispersion of shocks (see Young et al., 2008) hitting the EU countries, which in any case overwhelms the observed beta-convergence effect. It is important, therefore, to understand which regions of EU were affected and how during the analyzed period featured by the financial crisis and recovery afterwards.

The approach and findings mentioned above rely on average data (aggregate gross domestic product per capita levels at country or regional level) and are therefore not informative about the situation at different parts of income distribution, and thus about the *inclusiveness* of convergence. If in converging countries, the poorer part of the population were left behind with increasing disparity within a country, substantial social tension might build up, potentially affecting the prospects of economic growth. If the more affluent part of the population is endangered to incur significant relative losses, threatening their socioeconomic status, they might become active in lobbying for the change, which will be shown to be potentially relevant in the case of UK. Given that a sufficiently large part of the politically active population would feel as being left behind in absolute or relative terms, this might also influence (or even determine) their political choice both at the national and EU level. It is crucial, therefore, to consider the convergence by taking into account the whole income distribution, also evaluating where convergence fails.

Besides the characterization at which parts of income distribution the income level increases or decreases, the analysis of the whole distribution opens up the possibility to consider additional aspects of convergence. In particular, it becomes feasible to identify how much of convergence takes place because of a common income scaling of all population relative to some benchmark distribution (e.g., the EU-wide income distribution) and how much is caused because of scale-independent changes of income, i.e., after removing the effects of such scaling. The latter part is directly linked to the convergence of income distributions in terms of *income inequality*. One of the critical properties of proper metrics of income inequality is their scale-independence or, in other words, their invariance to a common rescaling of everyone's income. One might be tempted to use for such purpose the difference in (or ratio of) averages or medians (as, e.g. Handcock et al., 1996, as well as Handcock and Morris, 1998, and Handcock and Morris, 1999), which is however arbitrary. We propose to use a general rescaling approach based on the minimization of the distance between the analyzed income distributions.

Concentrating on economic convergence in terms of equivalized net income distributions during 2007-2014, this paper characterizes the changes and convergence of income at several aggregation levels of the European Union (EU). The EU-wide, region-to-EU, country-to-EU as well as convergence within and between various geographic EU macroregions are under consideration describing separately patterns observed in the *total convergence* and the *scale-independent convergence*. The former one characterizes changes in the similarity of income distributions both in terms of scale and shape, whereas the latter is scale-independent and therefore connected to the convergence in terms of income inequality. The underlying distributions of annual income are derived from the harmonized European Union Statistics on Income and Living Conditions (EU-SILC) survey database.

There is a general (and statistically significant) tendency for the EU-wide equivalized net personal income to converge, but we reveal substantial variations over different income quantiles and various geographic entities. Economic convergence is mostly understood as a process of catching up in terms of (real) income of those living in the most disadvantaged areas. We show that this is only one side of the income convergence process observed in the EU during the 2007-2014 period. After the financial crisis, the situation in Southern Europe has changed dramatically, and part of the income convergence is driven not only by the catching-up Central and Eastern Europe but also by a substantial reduction of income in Southern Europe, disproportionally affecting the lower tail of income distribution. Thus the experience of poorer and richer individuals differs here substantially, which reinforces the need for analyses that reveal the potential heterogeneity of changes at different parts of the income distribution.

The paper is structured as follows. Section 2 defines the applied methodology discussing central concepts used for the pairwise and multi-country comparison of distributions and the evaluation of their convergence due to the changes in their scale and shape differences. The empirical analysis is presented afterwards in the order of increasing complexity of the methodology used. First, Section 3 characterizes income changes observed at different aggregation levels that underlie the convergence process. Second, we explore the bilateral convergence of income relative to the EU-wide income distribution in Section 4, looking at the pairs of distributions. Third, Section 5 evaluates the multi-country convergence of income distributions using the modified Székely and Rizzo (2004) framework. Section 6 concludes.

2 Methodology

Because our interest lies in the evaluation of income distributions at very different aggregation levels ranging from various multi-country to bilateral comparisons, the suitable and most informative tools will differ. After reviewing some basic concepts in Subsection 2.1, we discuss the employed toolbox starting from the usage of relative quantiles for pairwise comparison of income distributions as discussed in Subsection 2.2. Afterwards, in Subsection 2.3, we propose a methodology to single-out the income changes affecting and non-affecting income inequality, i.e., the total convergence and scale-independent income convergence. The multi-country convergence evaluation and testing methodology are defined in Subsection 2.4. Subsection 2.5 discusses the semantic aspects of the employed notions of convergence and divergence of income distributions. Finally, some Monte Carlo simulations-based illustrations of the functionality of our convergence evaluation and testing approach are presented in Subsection 2.6.

2.1 Underlying basics

We will base the comparison of income distributions on several functions of random variables (r.v.). As an underlying object the usual cumulative distribution function (CDF) of a univariate real-valued continuous r.v. X will be used as denoted by $F_X(x) = P(X \le x), x \in \mathcal{X} \subset \mathcal{R}$, where P is the probability measure. Under absolute continuity of F_X , the probability density function (PDF) will be defined by $f_X(x) = \frac{dF_X(x)}{dx}$. The quantile function, given by $Q_X(s) = \inf\{x \in \mathcal{R} : s \le F_X(x)\}$ in the general case, will be given by the inverse function of CDF, with $Q_X(s) = F_X^{-1}(s), s \in \mathcal{S} \subset [0, 1]$, whenever it exists. The corresponding quantile density function (QDF) is given by $q_X(s) = \frac{dQ_X(s)}{ds}$. In the sequel, we will assume that the underlying distribution functions are also monotonically strictly increasing and twice differentiable. It should be pointed out that, while $Q_X(s)$ will represent the (value of) *income quantile*, the argument s of the quantile function $Q_X(s)$ represents a *quantile level* under consideration.¹

For notational brevity, we will use hereafter for a sequence of univariate random variables $\{X_i\}_{i=1}^N$ the respective shorter notations $F_i := F_{X_i}$, $f_i := f_{X_i}$, and $Q_i := Q_{X_i}$, $i \in \{1, 2, ..., N\}$. In the empirical applications, we shall use consistent nonparametric estimators of these functions. Since a large number of observations (usually tens or hundreds of thousands) will be used for estimations, the corresponding confidence or variability bounds are quite narrow² and therefore not plotted apart from a few cases to be discussed explicitly.

¹Whenever one looks at distributions of income, it also corresponds to the share of the respective poorest population. Hence, in the empirical applications, we shall call it also 'Population share'.

²Considering both the asymptotic and bootstrap-based variability or confidence bands.

2.2 Pairwise comparison of income quantiles and convergence

In many cases, a bilateral comparison of income distributions is of interest, for instance, in order to see how the distribution of a particular country relates to that of the whole EU. The multi-country methodology to be defined in Subsection 2.4 could also be employed for the bilateral evaluation, but it would not reveal if convergence/divergence takes places in a 'beneficial direction'. For instance, it matters if a distribution is converging to the reference from above or from below (and at which parts of the distribution, if differently). Ideally, we would hope that income convergence happens due to the increasing income of the poorer population and not because of dropping income of richer persons.

In the following we, therefore, introduce an *income quantile scaling function* (or, in short, *relative quantiles*) defined by the ratio of the corresponding income quantiles in a country under consideration and the reference country or region (which might be the same country in a different period). We will use two versions of such ratios of income quantiles: for comparison between different countries/regions (change in space) and for comparison of quantiles in the same country/region over time (change in time). Formally, their definition is the same, but they have some specific aspects to consider (especially in terms of economic interpretation); therefore, we discuss them separately in the sequel.

2.2.1 Quantile ratios in space (over countries/regions)

Let $\mathcal{S} \subset [0, 1]$. Given the income quantile functions $Q_{k,t} : \mathcal{S} \to \mathcal{R}_+, k \in \{i, j\}$, we shall use a *quantile scaling function* to compare quantiles in two different countries/regions, as defined by

$$\psi_{i,t}^{(j)}(s) = \frac{Q_{i,t}(s)}{Q_{j,t}(s)}, \ s \in \mathcal{S} \subset [0,1],$$
(1)

with the corresponding function $\psi_{i,t}^{(j)} : S \to \mathbb{R}_+$. As a shorthand, $\psi_{i,t}^{(j)}$ will also be referred to as *relative quantiles*. The subset S of [0, 1] can be used to omit the potential cases of zero income, which can be important in the general case, while it is less relevant in our case and therefore is ignored hereafter.

A couple of characteristic features of this function are of interest and exploitable for

further analysis. First, under equality of distributions $(||F_i - F_j|| = 0)$, it holds $\psi_{i,t}^{(j)}(s) = 1, \forall s \in [0, 1]$. Letting the function $i : [0, 1] \to 1$, we define a natural metric of dissimilarity in the L^1 norm, the integrated absolute deviation (IAD) as

$$d_{i,t}^{(j)} = \|\psi_{i,t}^{(j)} - \mathbf{i}\| = \int_0^1 \left|\psi_{i,t}^{(j)}(s) - 1\right| \mathrm{d}s.$$
(2)

The IAD is additively decomposable, which we use to characterise the dissimilarity connected with the parts above and below selected quantiles of the reference distribution:

$$d_{i,t}^{(j)} = d_{i,t}^{(j+)} + d_{i,t}^{(j-)},$$
(3)

where $d_{i,t}^{(j+)} = \int_0^1 \mathbb{1}_{\psi_{i,t}^{(j)}(s)>1} \cdot |\psi_{i,t}^{(j)}(s)-1| ds$ and $d_{i,t}^{(j-)} = \int_0^1 \mathbb{1}_{\psi_{i,t}^{(j)}(s)\leq 1} \cdot |\psi_{i,t}^{(j)}(s)-1| ds$, respectively. Here the indicator function $\mathbb{1}_{\bullet}$ takes value one whenever the condition is satisfied and zero otherwise. We shall use the IAD metric defined in eq. (2) together with its decomposition in eq. (3) not only to quantify the total dissimilarity between two distributions, but also to indicate which part of it exceeds and is below the quantiles of the reference distribution, i.e., where $\psi_{i,t}^{(j)}$ is greater or less than one. Furthermore, besides integrating over the whole range [0, 1], we will also consider by defining $d_{i,t}^{(j)}[a, b] = \int_a^b |\psi_{i,t}^{(j)}(s) - 1| ds$ the convergence/divergence in terms of the lower, middle and the upper thirds of income distribution by integrating over the respective ranges [0, 1/3), [1/3, 2/3), and [2/3, 1] of the support of the quantile function, correspondingly.

Second, if nonzero $\psi_{i,t}^{(j)}(s)$ does not vary with s (is constant), the distributions under comparison differ only in scale and not in shape. Therefore any scale-independent inequality metric would give the same inequality level in both distributions under comparison. This feature allows easily to identify distributions similar in shape up to the scaling constant, because $\psi_{i,t}^{(j)}$ becomes a (horizontal) line parallel to that of one, which itself marks the full equality of distributions. Furthermore, given that such a feature holds, one can rescale the initial distribution with any quantile ratio without affecting the value of any scaling invariant (scale-independent) inequality metric. Thus we will also consider identification of shape-dissimilarity-driven divergence after a certain rescaling of quantile functions that will be discussed in more details in Section 2.3.

2.2.2 Quantile ratios over time

One can use the quantile ratios also for comparing changes of distributions over time, for the same country or region. The ratio of quantiles from a time-varying distribution is given by

$$\bar{\psi}_{t,t_0}^{(i)}(s) = \frac{Q_{i,t}(s)}{Q_{i,t_0}(s)}, \ s \in [0,1].$$
(4)

In the empirical application, we shall use the growth rate $(\bar{\psi}_{t,t_0}(s) - 1), s \in [0, 1]$ instead of the gross growth factor given by eq. (4), revealing respectively the variation and tendency of income growth along different quantile levels, if any.

Similarly to the relative quantiles of different countries discussed in Subsection 2.2.1, we can define a metric of income convergence/divergence and their decompositions in relation to the relative quantiles over-time $\bar{\psi}_{t,t_0}^{(i)}(s)$:

$$\begin{split} \bar{d}_{i,t}^{(j)} &= \|\bar{\psi}_{i,t}^{(j)} - \mathbf{i}\|,\\ \bar{d}_{i,t}^{(j+)} &= \int_0^1 \mathbbm{1}_{\bar{\psi}_{i,t}^{(j)}(s) > 1} \cdot \left|\bar{\psi}_{i,t}^{(j)}(s) - 1\right| \mathrm{d}s,\\ \bar{d}_{i,t}^{(j-)} &= \int_0^1 \mathbbm{1}_{\bar{\psi}_{i,t}^{(j)}(s) \le 1} \cdot \left|\bar{\psi}_{i,t}^{(j)}(s) - 1\right| \mathrm{d}s,\\ \bar{d}_{i,t}^{(j)}[a,b] &= \int_a^b \left|\bar{\psi}_{i,t}^{(j)}(s) - 1\right| \mathrm{d}s. \end{split}$$

2.3 Scale-independence, inequality and income convergence

A usual requirement for an income inequality metric is its scale-independence so that multiplication of income of all persons by a positive constant would not change the inequality level. In this case, any rescaled quantile function $\widetilde{Q}_{i,t}^{(b)} = b \cdot Q_{i,t}, \ b \in \mathbb{R}_+$, and the original one with b = 1 would share the same inequality level. To concentrate on the essence, we ignore for the time being the potential variation of b in space and time. Any b from this point of view is admissible, but we are interested in the one that minimizes the distance between the new function $\widetilde{Q}_{i,t}^{(b)}: [0,1] \times \mathbb{R}_+ \to \mathbb{R}_+$ and the reference $Q_{j,t}: [0,1] \to \mathbb{R}_+$, i.e.,

$$\|\widetilde{Q}_{i,t}^{(b)} - Q_{j,t}\| \xrightarrow{b} \min.$$
(5)

In this way the rescaled quantile function becomes as much similar to the reference function as possible subject to a common re-scaling of all quantiles³. The remaining difference after such a transformation between the rescaled $\widetilde{Q}_{i,t}^{(\tilde{b})}$, with the optimal $\tilde{b} = \arg\min_{b} ||\widetilde{Q}_{i,t}^{(b)} - Q_{j,t}||$, and the reference quantile function $Q_{j,t}$ cannot be removed without affecting the value of scale-independent inequality metric of income.

Notice that utilizing eqs. (1) and (5) yields

$$\|\widetilde{Q}_{i,t}^{(b)} - Q_{j,t}\| = \|b \cdot Q_{i,t} - Q_{j,t}\|$$

$$= \|b \cdot \psi_{i,t}^{(j)} Q_{j,t} - Q_{j,t}\| |_{\forall s: \ \psi_{i,t}^{(j)}(s) = \psi_{i,t}^{(0,j)} = const. \notin s}$$

$$= \|b \cdot \psi_{i,t}^{(0,j)} - 1| \cdot \|Q_{j,t}\|.$$
(6)
(7)

Whenever the whole difference between the distributions stems from the scaling difference, the scaling function $\psi_{i,t}^{(j)}$ is constant and thus the minimization is achieved straightforwardly by setting $\tilde{b} = 1/\psi_{i,t}^{(j)}(s)$, for any $s \in [0, 1]$. This follows from eq. (7) and also implies that the two quantile and distribution functions will be the same after the rescaling (distance is zero). In the general case, minimization (5) relies on equality (6). The L^1 norm will be used hereafter both to be consistent with the choice made previously and because it is less sensitive to the potential presence of outlying spikes in the difference of quantile functions.

After the rescaling of initial quantiles with the \tilde{b} that solves eq. (5), one cannot get closer to the reference $Q_{j,t}$ without affecting inequality. We therefore can define the *inequality*affecting rescaling function (or scale-independent relative quantiles) by

$$\widetilde{\psi}_{i,t}^{(j)}(s) = \frac{\widetilde{Q}_{i,t}^{(b)}(s)}{Q_{j,t}(s)} = \frac{\widetilde{b}Q_{i,t}(s)}{Q_{j,t}(s)}, \ s \in [0,1].$$
(8)

³It is important to notice that the usage of quantile functions and not the distribution functions is very convenient here: the support of quantile functions is always the same and bounded, i.e., [0, 1].

From eqs. (1) and (8) it follows that the (total) scaling function $\psi_{i,t}^{(j)}(s) = \frac{1}{\tilde{b}} \widetilde{\psi}_{i,t}^{(j)}(s)$. Hence, in log-terms, we can decompose the total difference between income quantiles of the analyzed and the reference distributions at a particular quantile level into the scaling- and shape-linked components.

The discussion up until now was based on the bilateral evaluation at fixed time point. Let us now allow the rescaling constant b to vary over space and time. Then, the similar arguments apply also for the multi-country evaluation of similarity of distributions. The difference is that, instead of evaluating the original/initial quantile functions, one can consider the pair-wisely rescaled functions $\tilde{Q}_{i,t}^{(\tilde{b}_{i,t})}$ for a collection of countries indexed by $i \in \{1, 2, \ldots, N\}$ with the number of countries $N \geq 2$, where $\tilde{b}_{i,t}$ is derived individually for each country (and period) under consideration from the optimization as in eq. (5) using the pooled distribution as the reference. Section 2.6 will illustrate the good performance of such an approach using Monte Carlo simulations.

Comparison of $\{\tilde{b}_{i,t}\}$ or $\{1/\tilde{b}_{i,t}\}$ over time reveals the changes in the scale (dis)similarity of income. Changes in $\tilde{\psi}_{i,t}^{(j)}$ represent inequality-affecting (dis)similarity that is independent of scale. It can also be noted that $1/\tilde{b}_{i,t}$ is similar to the usual country-to-region (average) income level comparisons, e.g., that of country-to-EU-wide income per capita.

2.4 Multi-country convergence: evaluation and testing

To evaluate the multi-country convergence (divergence) of distributions we rely on the Székely and Rizzo (2004) approach of testing for equal distributions in high dimension. Namely, let $\{\mathcal{A}_i\}_{i=1}^k$ denote independent random samples of random vectors (in \mathcal{R}^d) from the corresponding distributions $\{F_i\}_{i=1}^k$. Let the respective sample sizes be denoted by $\{n_i\}_{i=1}^k$ and $n := \sum_{i=1}^k n_i$. The k-sample test statistic is given by

$$\xi_n = \sum_{1 \le i < j \le k} e(\mathcal{A}_i, \mathcal{A}_j), \tag{9}$$

where for any pair $\mathcal{A} = \{a_1, a_2, ..., a_{n_1}\}$ and $\mathcal{B} = \{b_1, b_2, ..., b_{n_2}\},\$

$$e(\mathcal{A},\mathcal{B}) = \frac{n_1 n_2}{n_1 + n_2} \left(\frac{2}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \|a_i - b_j\| - \frac{1}{n_1^2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_1} \|a_i - a_j\| - \frac{1}{n_2^2} \sum_{i=1}^{n_2} \sum_{j=1}^{n_2} \|b_i - b_j\| \right).$$
(10)

Eq. (10) satisfies the triangle inequality and therefore $\xi_n = 0$ if all elements in the samples coincide and $\xi_n > 0$ positive otherwise. Székely and Rizzo (2004) show that, under the null hypothesis,

$$H_0: F_1 = F_2 = \dots = F_k,$$

 ξ_n has a well defined limiting distribution, whereas under the composite alternative

$$H_1: \exists i, j, F_i \neq F_j,$$

 $E[\xi_n]$ is asymptotically a positive constant times n, provided that $n_i/n \to c_i \in (0, 1)$. Since the limiting distribution under the H_0 depends on $\{F_i\}_{i=1}^k$, Székely and Rizzo (2004) suggest bootstrapping from the pooled sample to derive the critical values.

Let $\kappa_{1-\alpha}^{(b)}$ stand for the α -level critical value from the bootstrap. Since $\xi_n > \kappa_{1-\alpha}^{(b)}$ would imply that H_0 is rejected, we use $\tau_{1-\alpha} = \xi_n / \kappa_{1-\alpha}^{(b)}$ as an indicator of how significantly the actual situation deviates from the equality of distributions as evaluated at the α significance level. Namely, for $\tau_{1-\alpha} \leq 1$, one would infer from this sample that the difference in distributions is insignificant at the α significance level, whereas $\tau_{1-\alpha} > 1$ would indicate a significant deviation from equality of distributions⁴.

It should be noted that empirically the computational costs of eq. (9) are quite prohibitive for very large n faced in practice (implying millions of pairs of persons under comparison despite the use of survey-based samples instead of the total country population), therefore we apply the following approach which at the same time is useful for the calculation of the empirical analogue of $E(\tau_{1-\alpha})$ denoted⁵ by $\bar{\tau}_{1-\alpha}$ and also avoids the potentially changing

⁴To simplify the presentation we use a generic notation without the time and cross-sectional indexes, i.e., $\tau_{1-\alpha}$ is used instead of $\tau_{1-\alpha}(i,t)$. Were it becomes important, we discuss explicitly this aspect of the potential change in time and/or variation over cross-section.

⁵Note that, in order to avoid heavy notation, we drop here and later on the bootstrap size index and use $\bar{\tau}_{1-\alpha}$ instead of $\bar{\tau}_{1-\alpha}^{(b)}$.

number of population in different years. First, after the non-parametric estimation of the distribution functions and obtaining its inverse (the quantile function), we draw random samples of smaller size (e.g. 500 observations) for each country using the procedure suggested by Hutson and Ernst (2000). This procedure entails first generating a random sample from the uniform distribution on (0,1) and then mapping it through a quantile function to the respective independent and identically distributed random sample of the variable of interest (income). Then, after implementing the earlier described bootstrap procedure that delivers the critical value $\kappa_{1-\alpha}^{(b)}$, we use the obtained repetitive samples to estimate $E[\xi_n/\kappa_{1-\alpha}^{(b)}]$ by averaging $\xi_n^{(r)}/\kappa_{1-\alpha}^{(b)}$ over the outcomes of repeated samples (thus yielding $\bar{\tau}_{1-\alpha}$). It should be further pointed out that, under the H_0 , the probability⁶

$$P\{\xi_n^{(r)} < \kappa_{1-\alpha}^{(\infty)}\} = P\{\xi_n^{(r)} / \kappa_{1-\alpha}^{(\infty)} < 1\} \to 1 - \alpha, \text{ as } n \to \infty.$$
(11)

Therefore, in cases where the equality of income distributions under evaluation cannot be rejected, the distribution of $\xi_n^{(r)}/\kappa_{1-\alpha}^{(b)}$, connected with the generated samples as described above, should (approximately) satisfy the condition (11) for not too small values n (as well as sufficiently large number of bootstrap replications). In the sequel we, therefore, will plot not only the mean value of $\xi_n^{(r)}/\kappa_{1-\alpha}^{(b)}$, but also its 95% quantile.

We will point out again that the usage of the discussed multi-sampling-based testing separately for each year not only makes the testing computationally feasible in our case but also has a couple of additional benefits. First, based on the draws of independent samples we estimate the empirical analogue $\bar{\tau}_{1-\alpha}$ of $E[\xi_n/\kappa_{1-\alpha}^{(b)}]$ together with its confidence bands, which enables to draw inference about the statistical significance of convergence/divergence. Second, it allows to fix the size n of draws (in fact, to use the same $n_i = n_0$, i = 1, 2, ..., k) and therefore makes the results of different years comparable with each other, as otherwise the population number in countries vary over time also affecting $E[\xi_n/\kappa_{1-\alpha}^{(b)}]$ through the number of subjects under comparison whenever the distributions differ (as the test statistic diverges with n).

⁶Eq. (11) holds for all $\alpha \in (0, 1)$ implying the uniform distribution of p-values of the test statistic under the null hypothesis that could be tested further, but, due to computational intensity, we restrict our attention hereafter only to $\alpha = 0.05$.

Although in this study we concentrate on the convergence of only net householdequivalized income, it should also be noted that the approach can be beneficial for the potential extensions to further multidimensional evaluation of convergence, where the particular (type of) income is only one of many dimensions of interest.

2.5 On the employed semantics of *divergence* and *convergence*

Because instead of established aggregates-based economic convergence concepts we use the distributions-based methods to evaluate the convergence of income, we feel the need to explain the meaning of the employed notions of *divergence* and *convergence*.

First, we point out that convergence and divergence are symmetric concepts that are connected to the dynamic behavior of dissimilarity, which itself represents a certain amount of difference between two distributions of different geographic entities at a fixed moment of time (or period). Such dissimilarity is captured by $d_{i,t}^{(j)}$ defined in eq. (2) for pairwise comparisons and by $E(\xi_n)$ that relies on eq. (9) for comparisons between multiple entities. We understand convergence as a process of reducing differences between distributions over time, while divergence as increases in such differences. Hence, decreasing values of $d_{i,t}^{(j)}$ and $E(\xi_n)$ (or its normalized analogue $E[\xi_n/\kappa_{1-\alpha}^{(\infty)}]$, see Subsection 2.4) would point to the presence of convergence, and the other way round.

Since the convergence evaluation statistics are directly unobservable, we will use their respective estimates $\hat{d}_{i,t}^{(j)}$ and $\bar{\tau}_{1-\alpha}$. It should be pointed out that we cannot construct the proper (even bootstrap) confidence bands of $\hat{d}_{i,t}^{(j)}$ because of the complicated survey designs underlying the observations, and therefore we just rely on the consistency of the (nonparametric) estimators. However, the approach proposed in Subsection 2.4 allows to get approximate confidence bands for $\bar{\tau}_{1-\alpha}$.

Notice that $\bar{\tau}_{1-\alpha}$ associated with different years will be evaluated using the same number of observations. In this way the increase in $\bar{\tau}_{1-\alpha}$ would be associated with some changes of the underlying (empirical) data generating process –increasing difference between the underlying distributions under consideration⁷, and not the increasing *n*. Hence an increase

⁷We have to note that the concept of divergence appears in our discussion also in the statistical sense: as *n* increases, the $\bar{\tau}_{1-\alpha}$ statistic diverges under the non-equality of distributions. It is important, however,

of $\bar{\tau}_{1-\alpha}$ over the years, observed using empirical data with some fixed n_0 (and hence n) applied across all studied years for comparison purpose, will be associated with *divergence* of distributions, whereas a decrease will be understood as *convergence* of distributions. Under the null hypothesis of equal distributions, $\bar{\tau}_{1-\alpha}$ would become a non-zero constant. It is clear that to have empirical power against the null hypothesis, n cannot be very small⁸.

Since $\bar{\tau}_{1-\alpha}$ is a random variable, we will be discussing the statistical significance of the change of $\bar{\tau}_{1-\alpha}$ (either convergence or divergence) by considering its (95%) confidence bounds, i.e., the confidence bounds of the average of the underlying realizations $\xi_n/\kappa_{1-\alpha}^{(b)}$. A shift that goes beyond these confidence bounds will be called as *significant* convergence or divergence.

We shall use the notion of *full convergence* to characterize the state of equality of distributions, and not the potential shift in their difference. Such full convergence will be evaluated relying on eq. (11) by considering the 95% quantile of $\xi_n^{(r)}/\kappa_{1-\alpha}^{(b)}$.

Finally, we should stress that *total convergence* – which is not connected with the employed notion of *full convergence* – aims at the evaluation of convergence without any potential adjustment for the scale differences of income distributions. The total here means that both the scale and shape (inequality-affecting) parts of distributions are implicitly under consideration (allowed for). Whenever only the scale-independent, but shape-dependent part of distributions will be under comparison after the proper rescaling, the notion of *scale-independent convergence* (or *inequality-affecting convergence*) will be employed.

2.6 Monte Carlo illustration of convergence testing

To demonstrate the characteristics of the discussed methodology, we next present a few Monte Carlo (MC) simulations using a couple of Data Generating Processes (DGPs) that illustrate the dynamics of convergence of (income) distributions. Let Lg stand for the distribution function of income to be simulated. For a reasonably close replication of empirical data, we use the lognormal approximation of the EU-wide income distribution with its estimated mean and standard deviation being 9.5 and 0.7, respectively. In the DGP,

not to confuse the two sources of divergence.

 $^{^{8} \}mathrm{One}$ can increase it as well, but uniformly across all countries and years to retain the discussed interpretation.

for all countries but the deviant, each year income are given by random realizations from Lg, i.e., $y_{i,t} \sim Lg(9.5, 0.7)$ for all periods indexed by t. All countries having the same income distribution are indexed by $i \neq i^*$, where i^* is reserved for a country having a potentially different distribution of income.

For the deviant country indexed by i^* , we consider two types of deviations from Lg. First, an additive deviation from the underlying distribution Lg is considered that cannot be removed using the common re-scaling of all quantiles using the approach described in Subsection 2.3. Letting the median of Lg be denoted by $q_{0.5}$, the realizations of DGP with the additive deviation (DGP-A) are generated by

DGP-A:
$$y_{i^*,t}^{(a)} \sim \begin{cases} Lg(9.5, 0.7) + q_{0.5} \cdot \frac{6-t}{10}, & t \in \{1, 2, \dots, 5\}, \\ Lg(9.5, 0.7), & t \ge 6. \end{cases}$$
 (12)

Consequently, during the first five years⁹ the absolute deviation, given by the non-zero second term on the top-right part of eq. (12), is decreasing linearly and proportionally to the median of Lg (the scaling factor decreases each year by 0.1 from 0.5 to 0.1 of the median $q_{0.5}$), while during the last three years the distribution of $y_{i^*,t}^{(a)}$ is the same as for the rest of countries.

Second, we allow for the multiplicative perturbation of realizations of the deviant country, which can be eliminated by the common re-scaling if, applied. Now the DGP with the multiplicative deviation (DGP-M) takes the following form

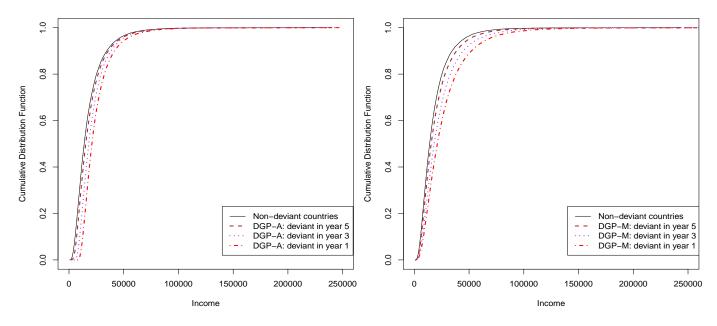
DGP-M:
$$y_{i^*,t}^{(m)} \sim \begin{cases} Lg(9.5, 0.7) \cdot \left(1 + \frac{6-t}{10}\right), & t \in \{1, 2, \dots, 5\}, \\ Lg(9.5, 0.7), & t \ge 6. \end{cases}$$
 (13)

Hence, during the first five years the multiplicative factor is decreasing proportionally to values of Lg (each year becoming smaller by 0.1, namely, from 1.5 in the first year to 1.1 in the fifth). As in the case of DGP-A, during the last three years the distribution of $y_{i^*,t}^{(m)}$ remains the same as for the rest of countries.

Figure 1 plots the (empirical) CDFs of the simulated data restricting the plotting

⁹In simulations we use eight periods which coincides with the number of years to be considered in the empirical application.

range to 250000 for better visibility of the differences emerging over the years, while still reflecting large income variation at the top of the income distribution. Both the additive



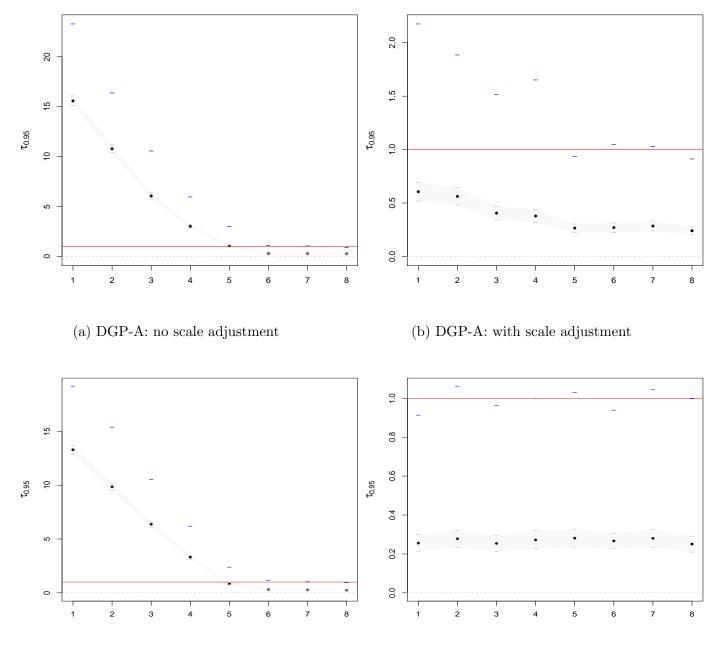
(a) Non-deviant countries vs. DGP-A
 (b) Non-deviant countries vs. DGP-M
 Figure (1) Empirical CDFs of DGP-based realizations of income.

and multiplicative type of deviations is quite clearly visible in the plots with the whole distribution function shifting rightwards in DGP-A, whereas with a proportional increase of income in DGP-M.

Figure 2 plots the results of the implemented convergence evaluation and testing procedure using the above described DGPs. Black dots represent the multi-country convergence test statistic $\bar{\tau}_{0.95}$ with its 95% confidence bounds around it in gray color. The upper dashes in blue color stand for the 95% quantile of $\tau_{0.95}$ (see the discussion in connection with eq. (11) in Subsection 2.4) that allows inferring about the acceptability of the null hypothesis of equal income distributions. The top and bottom panels of the figure correspond to DGP-A and DGP-M, respectively, whereas the left and right panels represent the testing without and with the rescaling adjustment¹⁰, correspondingly.

In both the additive and multiplicative cases, there is a clearly identifiable pattern of

¹⁰Namely, for each separate period and country using the pooled income distribution as the reference, the rescaled income $\tilde{x}_{i,t} = \tilde{b}_{i,t}x_{i,t}$, where $x_{i,t}$ denotes the original income observations and $\tilde{b}_{i,t}$ the derived scale factor satisfying eq. (5) for a fixed period and country.



(c) DGP-M: no scale adjustment
 (d) DGP-M: with scale adjustment
 Figure (2) Monte Carlo simulation of convergence evaluation.

convergence observed with non-rescaled data (see Figures 2 (a) and (c)), just as it was embedded into the underlying DGPs. During each of the first five years, there is a statistically significant reduction in the divergence statistic of $\bar{\tau}_{0.95}$. After the fifth year, there is a clear structural break with no statistically significant changes in $\bar{\tau}_{0.95}$ anymore. Furthermore, as far as one can judge from these few realizations, the limiting condition in eq. (11) seems to work reasonably well with the 95% quantile of $\tau_{0.95}$ (blue dashes) being very close to one (despite the small number of observations to be discussed shortly).

The results of the evaluation using the scale-adjusted data seem to be also quite reasonable (see the right panel of Figure 2). First, the statistically significant changes in the test statistic $\bar{\tau}_{0.95}$ are observed only in the case of DGP-A. This result is expected a priori because the common rescaling of the quantile function cannot eliminate the additive shift present in DGP-A and Figure 2 (b), but effectively removes the scale difference present in DGP-M, as seen in Figure 2 (d). At the same time, the reduction of the deviation in DGP-A over the years implies a closer distribution when the shift is smaller, which is also reflected in lower values of $\bar{\tau}_{0.95}$ statistic over the years. However, the performance of the limiting condition given by eq. 11 seems to be less precise in the DGP-A case for smaller values of deviation observed in the fourth and fifth year. It is connected with the employed low values of n and b used in the simulations (to be discussed shortly), leading to a more substantial variance of potential realizations, but also might be implicitly caused by the optimization-based preestimation of the rescaling constant. The fifth-year realization in DGP-A also shows no statistically significant difference from the Lg distribution in Figure 2 (b) which potentially hints that small absolute deviations are harder to detect after the rescaling of original data. On the other hand, the performance of testing in the DGP-M case (as presented in Figure 2 (d)) seems to be barely affected. Hence, if the single source of the difference between income distributions would stem from different scales, while the shape of inequality-affecting distributions was the same in all countries, one could reasonably expect to get picture of flat $\bar{\tau}_{0.95}$ with the 95% quantile of $\tau_{0.95}$ being close to one in all years, just as in Figure 2 (d).

The MC results are reasonably good, despite the moderate number of MC iterations (500) as well as relatively few sampling observations (250) and bootstrap replications (200) within

each MC iteration¹¹. The ability to detect dissimilarity of distributions and the presence of convergence appears to be good in the simulations in spite of the relatively small size of the population in the deviating country: the proportion is the same as that of the population of Spain relative to the whole EU.

3 Income changes in the European Union

In this section, we characterize the change in the distribution of annual net equivalized household income in EU over 2007-2014, that underlies the convergence process to be evaluated later. The characterization of distributions relies mainly on income quantiles, also utilizing the relative distribution concept (see Handcock and Morris, 1998). We consider three geographic aggregation levels of EU: the EU as a whole, three geographic EU macroregions, and individual countries. The macroregions are North-West Europe (NWE), Southern Europe (SoE) and Central and Eastern Europe (CEE).¹²

3.1 Data

Our primary data source is the Joint Research Center elaboration of the cross-sectional microdata files of the European Union Statistics on Income and Living Conditions survey. Here we give only essential pieces of information, referring to Benczur et al. (2017) for a complete discussion. We use the years from 2007 to 2014, determined by the availability of data: since 2007 we have data for all 27 countries under consideration, while at the moment of constructing the research database, the latest available year was 2014. 27 EU member states are included in the analysis as data were absent for Croatia for too many years.

The cross-sectional EU-SILC microdata of 27 EU countries were appended, corrected, harmonized, and weighted to yield a genuinely EU-wide database of individuals. Data

¹¹The same parametrization will be applied later in the empirical part and therefore the chosen values are relatively low, because of significant computational costs in terms of time needed to get the output, especially whenever all EU countries are under consideration instead of some restricted set.

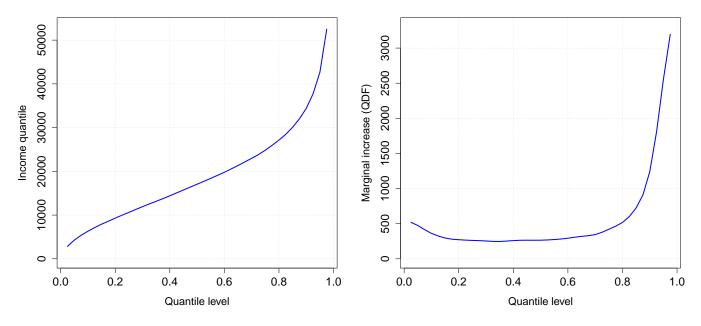
¹²The North-West comprises Austria (AT), Belgium (BE), Denmark (DK), France (FR), Finland (FI), Germany (DE), Ireland (IE), Luxembourg (LU), The Netherlands (NL), Sweden (SE), The United Kingdom (UK); Southern Europe comprises Cyprus (CY), Greece (EL), Italy (IT), Malta (MT), Portugal (PT), Spain (ES); Central and Eastern Europe comprises Bulgaria (BG), the Czech Republic (CZ), Estonia (EE), Hungary (HU), Latvia (LV), Lithuania (LT), Poland (PL), Romania (RO), Slovakia (SK), Slovenia (SI).

are trimmed; thus we do not consider records with income below 2.5% and above 97.5% income quantiles because of less reliable sampling statistics connected with answer biases and non-response rates at the tails of the income distributions. The unit of observation is the individual, persons older than fifteen years.

Calculations rely on equivalized and purchasing-power parity adjusted net household income. Household net income in the EU-SILC equals disposable household income less the balance of intra-household transfers. Household disposable income includes all income types, such as labor income, those from transfers, and those from assets. Equivalization takes into account economies of scale within the household, thus bringing it closer to its final use as an expenditure (it is based on the modified OECD scale). Purchasing power parity correction expresses monetary values at 2015 prices, taking into account the differences in relative prices across countries. Both corrections shift countries' distributions, while equivalization affects within-country distributions too – see Benczur et al. (2017) for details.

3.2 The EU-wide income distribution

Figure 3 plots the quantile function (part (a) on the left side) and the related quantile density function (part (b) on the right side) of EU-wide income at the beginning of the analyzed period, i.e., 2007. To facilitate the presentation, let us recoup in simple terms the key concepts we have defined formally in Section 2.1. The quantile function reveals the (lowest) upper bound of income of the population share corresponding to the considered quantile level – here the population is ordered increasingly in terms of income. For instance, it indicates in Figure 3 (a) that the poorest fifth of the EU population has annual income somewhat below ten thousand. Interpreting these monetary values, one has to keep in mind the data harmonization procedure described in Section 2.3, including adjustments for household size, purchasing power parity, inflation, etc. It will be therefore used hereafter mostly for relative comparison, including the differences in terms of time and between countries. Nevertheless, the steepness of the quantile function is informative about local (in terms of quantile level) income inequality and indicative about where the potential risk and opportunity are lower or higher if a person would shift locally on the EU-wide income ladder.

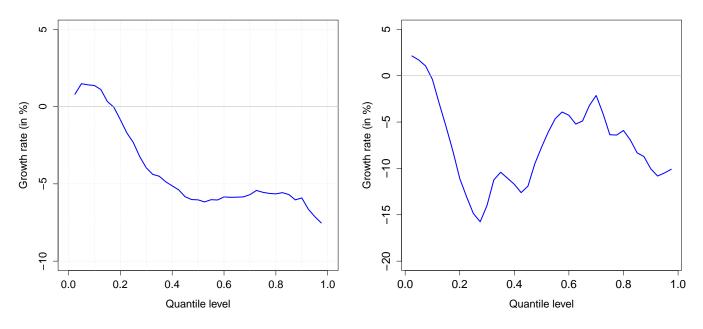


(a) EU-wide income quantiles.(b) Marginal increase of income quantiles.Figure (3) EU-wide income quantile and quantile density functions in 2007.

Marginal changes in income quantiles – the quantile density function – plotted in Figure 3 (b) help to evaluate these patterns more clearly. Note that zero Gini coefficient would be associated with the uniform distribution of income (affine quantile function and constant quantile density function), whereas under the perfect equality of income of all persons, the income quantile function would be flat, whereas the quantile density function (marginal income increase over quantiles) would take zero values everywhere.

Much larger values of marginal increase at the tails show that inequality between income in the poorest and richest fifth of EU population is much larger than that that between any fifth of population standing between them on the EU-wide income ladder. This situation also suggests the presence of greater costs of slipping down and potential gains of moving up for persons in the bottom and top fifths of the population ranked according to income size. At the same time, it is also indicative that the total income inequality is more heavily affected by the situation at the bottom and top thirds of the EU-wide income distribution than by its middle range, where the distribution is almost uniform and thus such inequality metric as the Gini coefficient would be much closer to zero. It is therefore of interest to evaluate what changes took place during 2007-2014.

Figure 4 plots the changes of the quantile function (on the left) and the quantile density function (on the right) during the considered period. Figure 4 (a) shows that income quantiles



(a) Of income quantiles.(b) Of marginal increase (QDF).Figure (4) Growth rates of EU-wide income quantiles and quantile density functions from 2007 to 2014.

increased nearly for the 20% of the EU-wide (poorest) population, whereas income decreased for the rest. This pattern suggests the convergence of income levels within the EU as a whole and a decrease in its overall income inequality.

At the same time, the change in the marginal increase of income over 2007-2014 plotted in Figure 4 (b) reveals that the steepness and thus also income inequality at the bottom of the distribution (approximately for the poorest 10% of the population) has even increased. The two processes taken together imply that the income of the poorest fifth of population increases along with the increasing dispersion of income for the poorest 10% – this latter process leads to even higher income inequality between persons within this group in 2014 than it was in 2007. However, the EU-wide convergence in income levels does not necessarily imply that the same change is taking place in all regions, nor that poor benefited everywhere, as we shall show it.

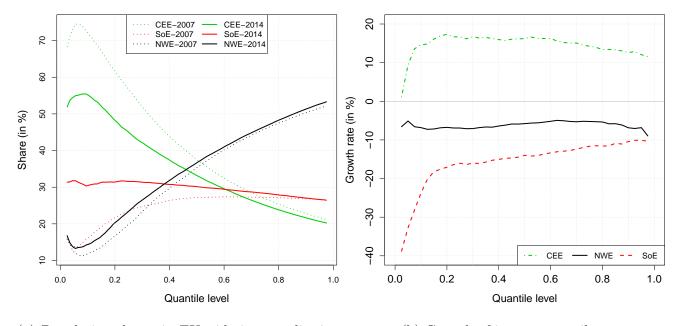
3.3 Distributional changes in EU macroregions

The change in the EU-wide income quantiles illustrated in Figure 4 (a) is determined by the initial share of macroregions (NWE, SoE, and CEE) within the EU population (at particular quantile levels) and their underlying income changes within these geographic entities. Figure 5 (a) shows first the regions' population shares in the EU at different quantile levels in the initial 2007 and the final year 2014, the latter being an outcome of the underlying income changes over the period. The figure shows¹³ for instance that in 2007 around 75% of EU population with income below the first EU-wide income decile was living in CEE (compared to around 55% remaining in 2014). The most substantial part of persons with income above the EU-wide median income reside in NWE, followed then by SoE. However, the most significant shifts in the regional population shares take place below the EU median, where the CEE share is substituted by the substantially and modestly increasing shares of SoE and NWE, respectively. In 2014, the SoE share at lower income quantiles had reached levels above 30% overshooting its total population share in the EU (being around 27% in 2014).

These changes in the regional composition of the EU population were induced by income changes in the regions – see Figure 5 (b) showing the rates of income growth at different quantile levels. CEE improved its income position in general, while NWE worsened it modestly and SoE suffered quite substantially. The previously discussed large initial population share of CEE at the bottom of EU-wide income distribution in 2007, coupled with the substantial income gains in this region, produced the improvement at the lower end of the EU-wide income distribution witnessed in Figure 4 (a).

Next to the discussed regional disparities, the income change is not uniform at different parts of income distribution within the SoE and CEE regions. Figure 5 (b) reveals that the income drop in SoE is most notably tilted: the lower is the considered quantile level, the larger reduction of income in percentage terms one observes with the poorer experiencing (much) higher costs of income reduction. In a similar way, the very poorest population of

¹³Note also that the shares at the top quantile level (i.e. one) would indicate the overall distribution of the total EU population among the regions.



(a) Population shares in EU-wide income distribution (b) Growth of income quantiles

Figure (5) Distribution changes in EU macroregions from 2007 to 2014.

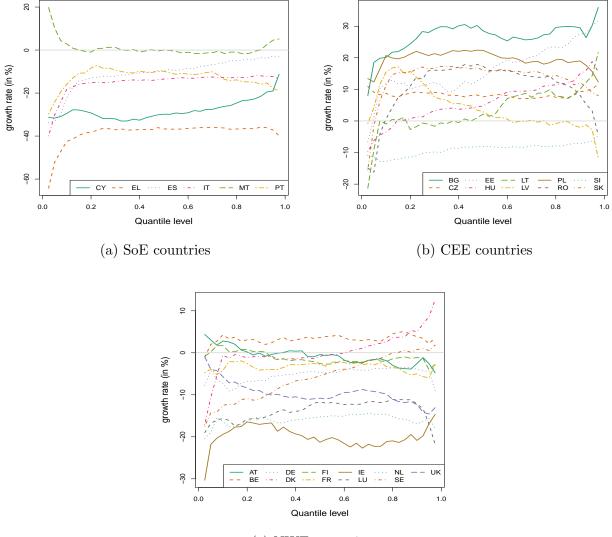
CEE experienced the slowest income rise despite that there was an increase in income in this region at all considered quantile levels¹⁴. The lowest part of income distribution is likely to be often associated with persons whose family members are in the state of unemployment or out of the labor force. Therefore, the sharp decrease in SoE and a relatively minor increase in CEE of income quantiles can be indicative about the higher vulnerability and social insecurity of the poorest in these regions: they experienced either absolute or relative income loss.

Summing up, the analysis thus reveals that it is the catching-up of the CEE that mostly drives the improvement at the lower part of EU-wide income distribution observed in Figure 4 (a). The poor in other regions practically experienced neither absolute nor relative income gain over the analyzed period. On the contrary, the poorer population in SoE experienced a much larger income reduction. As will be shown in Subsection 4, this income drop moved them from being substantially above the EU-wide income level in 2007 to ending below it in 2014.

 $^{^{14}}$ Income below and above the 2.5% and 97.5% quantiles are omitted because of less reliable sampling statistics connected with answer biases and non-response rates.

3.4 Country-level features

The patterns revealed at the regional level in Figure 5 (b) of a tilted income increase/decrease at lower quantiles might be an artifact of a few countries facing specific economic situation – in particular, Greece in SoE experienced severe financial difficulties and Romania in CEE is the country with the lowest income levels in the EU27. To reveal the underlying situation, Figure 6 plots the same type of income changes also for different countries within the respective regions.



(c) NWE countries

Figure (6) Country-level growth rates of income quantiles by EU macroregions (2007-2014).

Because at the regional level, the change in NWE was more homogeneous across various quantiles, we start from the two other regions which experienced the most drastic changes over time. Figure 6 (a) plots the growth rates of income quantiles in SoE countries. Apart from Malta, which is a definite positive outlier both in terms of the general direction of income change and the situation of poorest population, as well as some qualification regarding Cyprus, the pattern of unbalanced income adjustment over quantile levels, which was seen at the regional level, is observed for the rest of SoE countries. Greece experienced the most substantial reduction of income over the considered period not only in SoE but the whole EU, while Cyprus had the second largest in SoE quite close to that of Greece. Malta performed exceptionally well in the region experiencing even some increase at the lowest and highest quantiles.

The finding of smaller increases of income for the poorest population at the CEE level is observable also in case of individual CEE member states – see Figure 6 (b). However, at the country level, there is a more visible heterogeneity at higher quantile levels than was noticeable at the regional level. Income convergence is quite clearly tilted towards more upper quantiles in Estonia, Hungary, Lithuania, Slovenia (which is the only that experienced a total decrease in income among the CEE countries), and, partially, Bulgaria. There are also a few countries where the middle-class persons seem to have benefited most over the analyzed period: Bulgaria (with some qualifications), Poland, Romania, and the Slovak Republic. The Czech Republic achieved a quite uniform increase in income, while Latvia is an interesting outlier where the rise in income is concentrated at the lower-to-middle quantile levels with a decrease in the income of the richer population.

The relatively uniform reduction of income over all quantiles observed for NWE at the regional level in Figure 5 (b) varies at the country-level. Figure 6 (c) shows that Denmark and Ireland have a similar pattern that the largest drop of income hits the poorest population that was typical for CEE and SoE regions. Denmark and Sweden are standing out by the sign-separated income change: the upper quantiles have even increased while the lower ones dropped, thus undoubtedly increasing inequality between rich and poor. In Austria, Finland, and the UK the situation is the other way round: poorer here benefited more or were affected less by the reduction of income. In the rest of the countries, one observes a more uniform change of income, similar to what was established at the regional level.

Given the current sociopolitical attitude of UK towards EU, it is of interest to point out its distinctive tilted pattern of income reduction: it is much larger at the top income levels than at the lower part of quantile levels. In absolute terms, the total income drop experienced by the UK is not the largest among NWE countries and is surpassed by Ireland, the Netherlands, and Luxembourg, but the reduction of income in these countries is less tilted. Hence, it seems that the richer part of the UK population had more motivation to argue and lobby for the change needed to retain their relative income and socioeconomic status.

Next, we turn to the evaluation of convergence of income distributions between various geographic entities as implied by the discussed individual income changes.

4 Bilateral convergence to the EU-wide income distribution

We start the convergence analysis from bilateral comparisons of regional and country income distributions with the EU-wide income distribution using the methodology explicated in Subsection 2.2.1.

4.1 Region-to-EU convergence patterns

Although Central and Eastern Europe experienced improvements in income levels, they remain far below EU-wide one. Considering the income of someone at the same position on the income ladder in the three regions and the EU as a whole, Figure 7 (a) plots the relative region-to-EU income quantiles (the scaling functions), for 2007 and 2014 – the EU-wide income benchmark is the horizontal line at 1. It shows for instance that in 2007, the income of 20% poorest in NWE was about 50% higher while in CEE about 50% lower than that of the EU, whereas the SoE income quantiles were closest to the EU-wide ones. Looking at the differences between regions along different quantile levels it is obvious that income variation is much greater at the bottom of the income distribution: the income gap between poorer population in different regions is much larger than between richer ones.

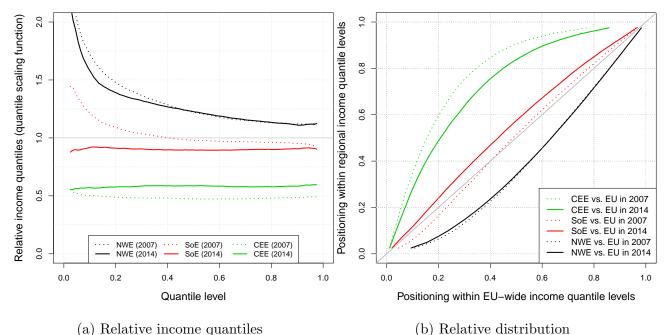


Figure (7) The CEE, NWE, and SoE income relative to the EU-wide income.

Despite the drop of income in absolute terms seen previously in Figure 5 (b), the relative position of NWE remains mostly favorable compared to the EU-wide in both 2007 and 2014 – there is a small relative reduction only below the 0.4 quantile level, i.e., for the poorest 40% of the population. This outcome is due partly to its substantial weight in the aggregate (the share of NWE population under consideration makes up around 53% in EU in 2014), but also to the fact that the EU-wide improvement was at the lower end only, as was seen in Section 3. The gain of CEE has been significant over time, but still not enough to bring its overall income close to EU-wide levels. Finally, the dramatic change for the SoE poor brought them from substantially above the EU-wide level in 2007 to below it in 2014.

The almost constant ratios of income quantiles of CEE and SoE vis-à-vis the EU in 2014 (see the solid red and green flat lines in Figure 7 (a)) indicate that, in terms of income inequality, their income distributions became very similar to each other and to that of the EU as a whole. Because of this, the level of inequality which would be captured by using any top-to-bottom percentile ratios would be approximately the same in CEE and SoE (and also the EU as a whole) in 2014. Any scale-independent income inequality metric, such as the Gini coefficient, Theil index, etc., would have revealed the more similar inequality levels

between these regions in 2014 than in 2007.

In 2014 the situation in SoE became much more alike to that in CEE, whereas, in 2007, inequality in SoE resembled much more that of NWE. As there is a disproportionally larger reduction of income at lower quantile levels, the SoE became more unequal in 2014 than it was in 2007. At the same time, inequality in CEE has also increased over the period because of a relatively slower increase in the income of the poorer. Despite this CEE and SoE convergence in terms of inequality¹⁵, the gap in terms of income levels between those regions remains sizable.

Figure 7 (b) maps quantile levels (the respective shares of population) between the analyzed region with that of the EU using relative distribution $approach^{16}$. It shows that, in 2014, a person with the median EU-wide income – i.e., at 0.5 quantile level – surpasses income of above 80% of CEE, nearly 60% of SoE, and somewhat more than 30% of NWE population. At the same time, the median income in CEE barely passes over the income threshold of the poorest 20% in the EU taken as a whole. Therefore there is a substantial room for and need of further catching-up here.

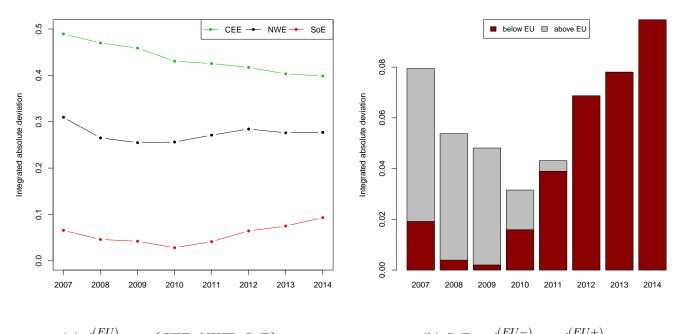
Turning back to Figure 7 (a), let us recall that the potential equality of regional and EU income is represented by the horizontal line at one. The area between the particular scaling function (relative quantiles) and the horizontal line at one represents the amount of dissimilarity¹⁷ from the ideal situation of equal distributions. It corresponds to the integrated absolute deviation metric $(d_{i,t}^{(EU)})$ defined in Subsection 2.2, whereas $d_{i,t}^{(EU+)}$ and $d_{i,t}^{(EU-)}$ stand for the respective areas above and below the line at one. Such a split is informative about the parts of the distribution, where the differences are concentrated.

Figure 8 (a) plots the dynamics of $d_{i,t}^{(EU)}$ on a common scale for $i \in \{\text{CEE}, \text{NWE}, \text{SoE}\}$ and $t \in \{2007, 2008, \dots, 2014\}$. Although being the furthermost away, the CEE converges

 $^{^{15}}$ Section 5.2 will reveal that there is a full convergence between the inequality-driven parts of income distributions in CEE and SoE.

¹⁶Figure 7 (b) can be interpreted as comparison of the relative position of a person with some given income, positioned in terms of quantile level, within each of the two distributions.

¹⁷In a similar way, in Figure 7 (b), the distance between the relative distribution function and the diagonal line could be considered. However here some additional transformations might be needed because the range of relative distribution function values is squeezed differently at different quantile levels as given by the diagonal line, thus the distance at various quantile levels have different ranges and might not be directly comparable.



(a) $d_{i,t}^{(EU)}$, $i \in \{\text{CEE, NWE, SoE}\}$ (b) SoE: $d_{SoE,t}^{(EU-)} + d_{SoE,t}^{(EU+)}$ Figure (8) Dynamics of integrated absolute deviation (IAD) and its decomposition for SoE.

towards EU income levels quite steadily over time. There is much less clear tendency for NWE; although some convergence to the EU-wide levels took place during the financial crisis, there is a slight rebound during later years of recovery. The largest variation in terms of convergence took place in SoE, which was converging towards the EU levels until 2010 but diverged afterwards.

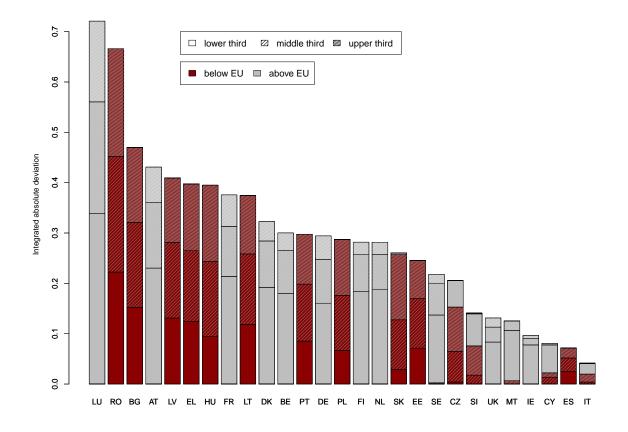
For SoE, Figure 8 (b) shows furthermore the decomposition of the integrated absolute deviation into parts below and above the EU reference line at one; $d_{i,t}^{(EU-)}$ and $d_{i,t}^{(EU+)}$ are depicted in brown and gray colors, respectively. This additional figure is presented only for SoE, because income quantiles in CEE are below those of EU at all levels and $d_{i,t}^{(EU)} = d_{i,t}^{(EU-)}$, whereas in NWE the situation is the other way round with $d_{i,t}^{(EU)} = d_{i,t}^{(EU+)}$. As the figure reveals, the divergence in SoE took place with a downwards adjustment of income: the income part above the EU levels, which was dominating the divergence until 2009, has been overthrown since 2010 with that below the EU levels; only partially for a few years and completely by 2012.

4.2 Country-to-EU convergence

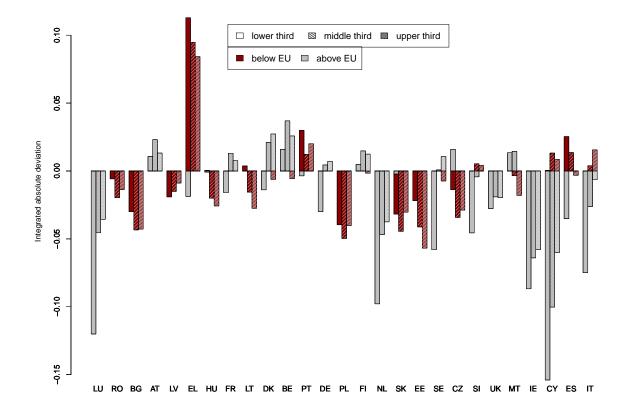
To be able to present the results of divergence and convergence concisely for all EU countries under consideration, we now employ somewhat aggregated results without dwelling into the yearly dynamics. Figure 9 (a) first characterizes the dissimilarity of individual EU countries as compared with the EU income quantiles for a fixed year using the IAD metric $d_{i,2014}^{(EU)}$, i.e., in 2014 (see Figure 9 (a)). It also decomposes the IAD in two additional ways. First, as for SoE previously, the deviations above and below the EU quantiles are revealed (in correspondence with $d_{i,2014}^{(EU+)}$ and $d_{i,2014}^{(EU-)}$) by the gray and brown colors, respectively. Second, it identifies the range of quantile levels where the deviations emerge, i.e., at the lower third, the middle third, or the upper third.

Looking at the IAD in 2014, the three countries with the largest upwards deviation are Luxembourg, Austria, and France, whereas the three countries with largest downwards deviation are Romania, Bulgaria, and Latvia. The three countries being least distant from the EU income quantiles are Cyprus, Spain, and Italy. In all countries where the upwards deviation dominates (only in gray color at all ranges), the income quantiles at the lower third contribute most to the total dissimilarity, i.e., there is the most substantial difference between income quantiles in a country relative to EU at the lower part of quantile levels.

In countries that are substantially below the EU quantiles (only in brown color), the contribution is more equally distributed among the thirds indicating about the similar distance between income quantiles in a considered country and EU along with all quantile levels. As most of the countries below the EU levels are from CEE, while those with above-EU levels come predominantly from NWE, these individual country-based findings correlate well with the picture observed in Figure 7 earlier. There, CEE was quite uniformly distant from EU-wide income quantiles, whereas NWE deviation was much higher at the bottom. The picture is less clear-cut for countries that are closer to EU income quantiles. In some of them, there is both upwards, and downwards deviations (like in Cyprus, Malta, Italy, Slovenia, Sweden) with different split and contribution from the considered three ranges of quantile levels. For instance, in Slovenia, the dominant part of the difference is associated with the upper third (being below the EU levels), while the lower third is close to the EU with



(a) Deviations from EU in 2014



(b) Change from 2007 to 2014 Figure (9) IAD distance from the EU-wide quantiles and its change.

even some positive overshoot. In Sweden, the most significant contribution to the difference stems from the lowest third (being above EU levels), while quantiles at the top third are close to those of EU, even partially falling below the EU levels. Very similar situation to Sweden is also in Malta, whereas in other countries the variation is even more considerable.

The discussed features characterize the static picture of income dissimilarity in individual countries relative to EU-wide levels in 2014. Next, we turn to the change of IAD observed over the 2007-2014 period that is presented in Figure 9 (b). Its negative values (reduction) implies convergence towards EU-wide quantiles, whereas values above zero imply a divergence from the EU-wide quantiles.

The largest divergence dominated by downwards shift relative to the EU took place in Greece, placing it in terms of the level of dissimilarity next to Latvia and Hungary in 2014. The most significant adjustment in Greece took place at the lower third¹⁸, while the smallest at the upper third. Belgium stands out as a country with the dominant upwards divergence with the largest increase in the middle third of quantiles and also a substantial increase at the top third, moving the Belgian quantiles above the EU levels also there. Cyprus experienced the largest downwards convergence losing its part of the above-EU-levels position at all ranges. A somewhat similar downwards convergence towards EU levels is apparent also in Luxembourg, the Netherlands, and Ireland, but without falling below the EU levels. In several countries, e.g., France, Denmark, Germany, Sweden, the convergence/divergence takes even different directions at different quantile levels: the lower third converged downwards towards EU levels, while at least some of the upper two diverged upwards.

Looking across the country experiences, we see again that the convergence process dominates among EU countries: there are more and more substantial shifts below the zero line in Figure 9 (b) as compared with the divergence represented by the positive part. The largest upwards convergence has been achieved by Bulgaria, Estonia, Slovakia, and Poland. In Hungary and Lithuania, the situation of the poor relative to that of EU-wide even worsened, while there is an upwards convergence of the middle and upper quantiles in

 $^{^{18}\}mathrm{With}$ the vanishing part above EU levels that Greece had in 2007.

these countries. However, the total EU-wide convergence is not only driven by the catchingup by the CEE countries. There is a lot of downwards convergence observed among both the NWE and, especially, SoE countries but concentrated more at lower quantile levels. Some countries that experienced the largest reduction of relative income had a highly tilted income reduction already within the country (like Luxembourg). However, the catching-up by the CEE countries situated at the lower part of EU-wide income distribution augmented this relative income change of poorer population in NWE and SoE for many other richer countries, such as the Netherlands, Cyprus, and Ireland (to give just a few examples).

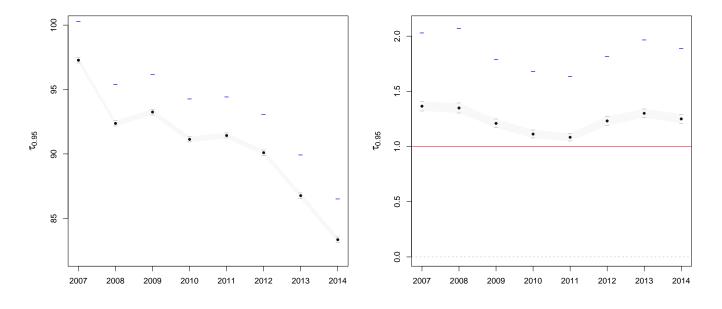
The country-level analysis thus again reiterates that the EU convergence during 2007-2014 was neither only the catching-up nor typically beneficial for the poorer part of society within a given country either in absolute or relative terms. The latter situation was an exception among countries, rather than the rule.

5 Multi-country evaluation of the convergence of income distributions

In the previous section, we have characterized the similarity of regional and country income distributions to that of the EU-wide. Now we turn to the analysis of income convergence between all countries within the EU, as well as the convergence of countries within and between the macroregions, namely, the CEE, NWE, and SoE. This type of analysis requires a multi-country approach. Applying the convergence evaluation methodology described in Subsection 2.4, we now aim at the characterization of convergence patterns among the income distributions within and between the discussed groups of EU countries.

5.1 EU-wide convergence between countries

In the analysis of EU-wide country-level convergence, we evaluate the changes and (dis)similarity of income distributions among the considered 27 EU countries. Figure 10 plots the divergence evaluation statistic $\bar{\tau}_{0.95}$ (black dots with the corresponding 95% confidence bands) and the 95% quantile of the realized $\tau_{0.95}$ (dashes in blue at the top), which under the null hypothesis of the same income distributions should take value one that is highlighted by the red horizontal line (the left part of the figure omits it because of huge scale differences). Figure 10 (a) presents the results with the initial data (without any re-scaling), whereas Figure 10 (b) plots the results after the scale-adjustment of income as described in Subsection 2.3.



(a) Total: scale and inequality-affecting.(b) Scale-independent, inequality-affecting.Figure (10) Dynamics of divergence of income distributions among EU member states.

Previous analysis already revealed the substantial income level differences (see Figure 7); thus there is no surprise that the hypothesis of the same income distribution in all EU countries is strongly rejected for all years. The blue dashes being far above one, where they are expected to converge under the null hypothesis of equal income distributions, indicate the rejection of the null hypothesis of equal income distributions (see Figure 10 (a), drawn with unadjusted data). Nevertheless, there is a substantial reduction in divergence: $\bar{\tau}_{0.95}$ became much smaller over the analyzed period. This finding is fully consistent with the previous results provided in Section 3.2. The convergence of distributions among the EU countries from 2007 to 2014 is statistically significant but experienced certain speed variation within this period. The largest convergence within a single year took place just after the financial crisis. In the aftermath of the financial crisis, the convergence became seemingly slower for

several years and accelerated again after 2012, i.e., during the latest years of recovery.

To assess convergence in terms of scale-independent (inequality affecting) part of income, we use the rescaled income¹⁹ exploiting the results of Section 2.3. The evidence on convergence is much weaker, as can be seen in Figure 10 (b). The divergence evaluation metric has an initial downswing, but an upswing afterward that could be potentially (and inversely) connected with the economic situation during this period (we will show later that such dynamics is driven mainly by the CEE countries). Whenever economies experienced the hardest time during the initial years after the crisis, there was a significant convergence of income distributions also in terms of inequality. However, the bouncing back was almost complete later, whenever the situation initiated to normalize. Nevertheless, considering the whole period from 2007 to 2014, there is a marginally significant convergence, although it is still far from the full convergence. Even at the lowest levels of divergence in 2010 and 2011, the hypothesis of equal scale-adjusted income distributions is still firmly rejected at the 5% significance level (blue dashes are quite far above one).

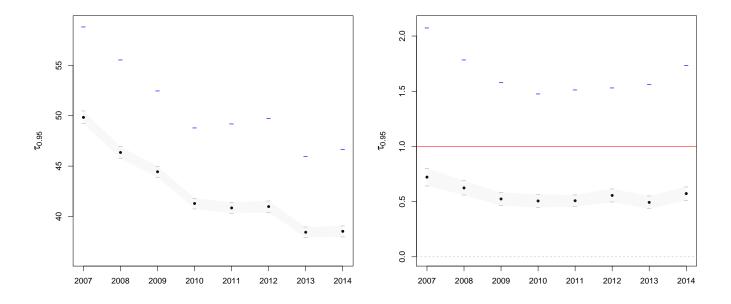
5.2 Convergence between EU macroregions

For the convergence analysis of distributions between the three EU macroregions, the country observations are pooled into the respective regional ones, i.e., the CEE, NWE, and SoE. The scale-adjusted evaluation is based on the regional re-scaling coefficients derived from the pool of regional income distributions, i.e., the EU-wide distribution.

Figure 11 plots again the statistic $\bar{\tau}_{0.95}$ (black dots) with its 95% confidence bounds for the evaluation of convergence/divergence dynamics, and the 95% quantile of $\tau_{0.95}$ (the upper dashes in blue) for the evaluation of the full convergence of distributions.

In terms of almost continuous decrease over years and its significance, the convergence between EU macroregions observed in Figure 11 (a) is similar to that between all countrylevel income distributions of EU seen in Figure 10 (a). The middle part of the period is again characterized by a deceleration of convergence. Hence, a part of the total convergence

¹⁹Namely, for each separate year and country using the EU-wide income distribution as the reference, the rescaled income $\tilde{x}_{i,t} = \tilde{b}_{i,t}x_{i,t}$, where $x_{i,t}$ denotes the original income observations and $\tilde{b}_{i,t}$ the derived scale factor satisfying eq. (5) for a fixed year and country.

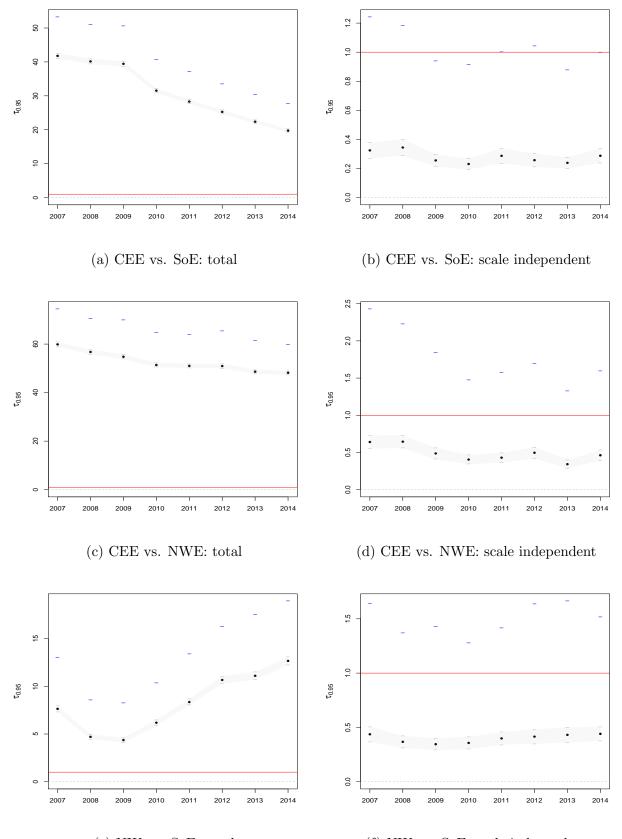


(a) Total: scale and inequality-affecting.(b) Scale-independent, inequality-affecting.Figure (11) Dynamics of divergence of income distributions between EU macroregions.

between EU countries is connected with the inter-regional convergence.

The scale-adjusted (inequality-affecting) convergence between regions seen in Figure 11 (b) is also marginally significant from 2007 to 2014, and more steady over time than that established previously at the country level considering the whole EU. Hence the variation of divergence in the latter case is likely to be associated with some variation within the regions and not between them (this is indeed the case, as will be illustrated shortly). The divergence is also somewhat smaller in terms of size, but still far from the case of full convergence. These findings seem to be in line with those derived in Subsection 4.1 using the simple approach of relative quantiles, despite that in Figure 11 (a) the convergence of all three regions is evaluated together and not bilaterally. For better comparability, we can though measure also the changes of divergence for each pair of macroregions using the same methodology. The results are presented in Figure 12.

When comparing only the change from 2007 to 2014 of the total convergence, the results confirm the patterns discovered in Figure 7 (a). First, due to the increasing income in CEE, there is a clear convergence both between CEE and NWE and between CEE and SoE, as confirmed by Figures 12 (a) and (c). However, a swifter decrease in income in SoE than



(e) NW vs. SoE: total(f) NW vs. SoE: scale independentFigure (12) Pairwise dynamics of divergence between EU macroregions.

in NWE caused the divergence between these two regions. Figures 12 (e) shows that this process initiated after 2009 and continued until the very last year.

The pieces of evidence from Figure 7 (a) and Figure 12 are mutually consistent. However, the former, based on relative quantiles allows seeing changes over all of the quantile levels, whereas the latter reveals the yearly dynamics together with the evaluation of the statistical significance of both convergence/divergence dynamics and the full convergence (equality of distributions).

We have established earlier that the CEE and SoE quantile functions became almost scaled versions of each other (see the solid green and red lines in Figure 7 (a)), thus leading to very similar income distributions, if scaling differences were removed. Figure 12 (b) confirms this intuition, because in 2014, one cannot reject anymore even full convergence: blue dashes of the 95% quantile of $\tau_{0.95}$ are very close to one. Already since 2009, the regional income distributions in CEE and SoE became very similar in terms of inequality. There is no analogous similarity in terms of scale-independent and inequality-affecting parts of income distributions neither between CEE and NWE nor between NWE and SoE. Nevertheless, the scale-adjusted income distributions in CEE and NWE also became more similar (and statistically significantly).

Given the similarity of findings of this subsection with those derived in Subsection 4.1, it is of interest to point out that the evaluation there is based on the (relative) quantile functions, whereas in the 'multi-country' evaluation methodology, the random samples are drawn directly from the distribution functions of geographic entities. Despite these methodological differences, potentially complicated further by the nonparametric estimation of different objects of analysis, the correspondence of findings is quite reassuring.

5.3 Convergence within EU macroregions

The previous subsection evaluated the convergence of income distributions between EU macroregions. Now we turn to the convergence of income distributions in countries within each region. Figure 13 plots for each region comparable figures as previously, just using the corresponding country-level data within each region. The scale adjustment here is obtained

separately for each region using the pooled regional distribution as the reference for re-scaling.

Despite considering a more similar set of countries, as compared with the total EU, the hypothesis of equal income distributions (full convergence) is rejected in all the regions (see Figures 13 (a), (c), and (e)). It was previously established in Figure 8 (a) that the CEE is quite steadily converging towards the EU levels. Figure 13 (a) furthermore shows that there is also a statistically significant convergence within the CEE; with some acceleration during the financial crisis in 2008, and 2012 and 2013. There was an increase in similarity of income distributions within the NWE region between countries after the financial crisis, which slightly bounced back since 2013. Within SoE, countries experienced income divergence after 2009 (caused by the jumps of divergence in 2010 and 2011), notoriously led by the substantial reduction of income in Greece (and Cyprus, too). The direction has changed again since 2012 towards more convergence, mainly due to the decrease in income also in most other countries of the region.

There is no full convergence either among the scale-independent parts of distributions in any of the regions. Nevertheless, in CEE and NWE, countries became somewhat more similar also in terms of inequality from 2007 to 2014. In NWE the convergence stopped after 2009, whereas that in CEE is wave-like, just as it was observed for the whole EU in Figure (7).

6 Conclusions

There is a significant EU-wide convergence of net equivalized income distributions from 2007 to 2014 driven mainly by the catching-up of CEE countries and more significant income reduction in SoE. These income adjustments from below and above caused substantial convergence between income distributions in CEE and SoE regions. The difference between these two EU macroregions remains substantial in terms of income levels, but scale-adjusted (inequality affecting) income distributions became practically the same in both regions (and similar to the EU-wide). There is furthermore a significant convergence of country-level income distributions within the CEE and NWE regions, but not within SoE.

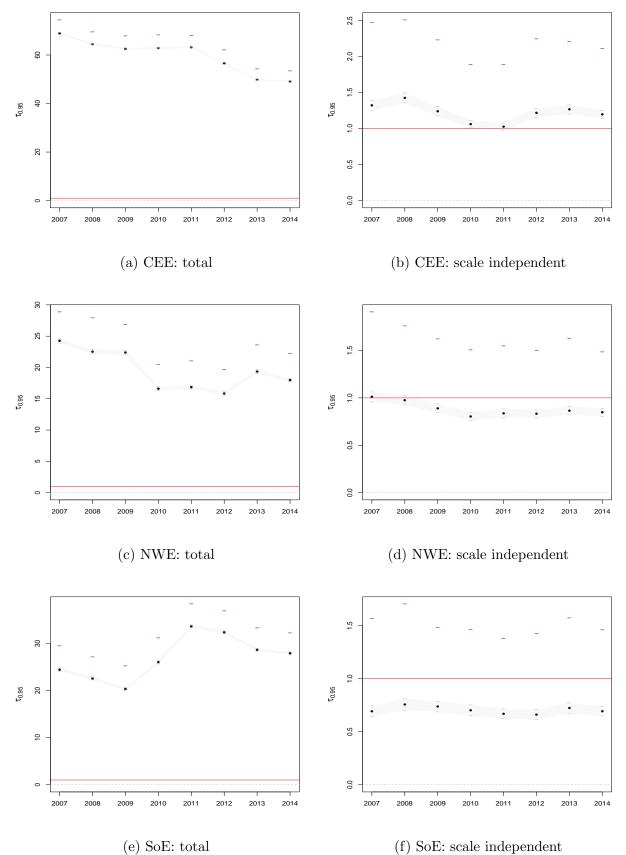


Figure (13) Dynamics of divergence of income distributions within EU macroregions.

There is substantial heterogeneity of income distributions both in terms of income levels and their changes over the analyzed period. NWE has the largest income at all quantiles while CEE has the smallest. SoE stands quite close to the EU-wide income distribution along almost all quantile levels. More importantly, income variation among EU macroregions (and countries) is tilted: the relative income gap between the poorer population in different regions is much wider than the one between richer ones. The income change over the period is not uniform either, especially within the SoE and CEE regions. In SoE, the income change is very unbalanced, the poorer experiencing (much) higher income reduction, which places them below the EU-wide income levels in 2014. Similarly, the very poorest population of CEE experienced the slowest income rise despite that there was an increase in income in this region at all considered quantile levels. Since the poorest part of the population in these regions (especially SoE) experienced either absolute or relative income loss, this can be indicative about higher levels of their vulnerability and social insecurity in 2014 than in 2007 and a potential augmentation of social tensions.

The situation of individual countries varies. Looking at the (static) picture in 2014, the three countries with largest upwards deviation from EU-wide income levels are Luxembourg, Austria, and France, whereas the three countries with largest downwards deviation are Romania, Bulgaria, and Latvia. The three countries being least distant from the EU income levels are Cyprus, Spain, and Italy. In all countries where the upwards deviation dominates (mostly from NWE), the lower third of income quantiles contribute most, i.e., the most significant difference between income quantiles in these countries relative to EU are at the bottom.

Looking at the change from 2007 to 2014, the largest divergence (as a process), dominated by downwards shift relative to EU-wide levels, took place in Greece, placing it in 2014 next to Latvia and Hungary in terms of distance from the EU-wide income levels. Cyprus experienced the largest downwards convergence, which put part of its income quantiles even below the EU levels. Luxembourg, the Netherlands, and Ireland suffered a somewhat similar downwards convergence towards the EU levels, but without falling below them. In several countries, e.g., France, Denmark, Germany, Sweden, the convergence/divergence takes even different directions at different quantile levels: the lower third converged downwards towards EU levels, while the upper ones diverged upwards. Bulgaria, Estonia, Slovakia, and Poland achieved the largest upwards convergence over the period. In Hungary and Lithuania, the situation of the poor relative to that of EU-wide even worsened, while middle and upper quantiles show upwards convergence in these countries.

The analysis at various levels of geographic aggregation is concordant and shows that the EU convergence during 2007-2014 was neither only the catching-up nor typically beneficial for the poorer part of the society within a given country or macroregion either in absolute or relative terms. Countries, where the poor benefited more during the period, were the exception rather than the rule.

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