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Stochastic model specification in Markov switching vector error correction models

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 **WORKING PAPERS**
IN ECONOMICS

Stochastic model specification in Markov switching vector error correction models

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Abstract

This paper proposes a hierarchical modeling approach to perform stochastic model specification in Markov switching vector error correction models. We assume that a common distribution gives rise to the regime-specific regression coefficients. The mean as well as the variances of this distribution are treated as fully stochastic and suitable shrinkage priors are used. These shrinkage priors enable to assess which coefficients differ across regimes in a flexible manner. In the case of similar coefficients, our model pushes the respective regions of the parameter space towards the common distribution. This allows for selecting a parsimonious model while still maintaining sufficient flexibility to control for sudden shifts in the parameters, if necessary. In the empirical application, we apply our modeling approach to Euro area data and assume that transition probabilities between expansion and recession regimes are driven by the cointegration errors. Our findings suggest that lagged cointegration errors have predictive power for regime shifts and these movements between business cycle stages are mostly driven by differences in error variances.

Keywords: Non-linear vector error correction model, Markov switching, hierarchical modeling, variable selection, equilibrium credit level, Euro area

JEL Codes: C32, C11, E32, E44, E51

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1 Introduction

We propose a hierarchical multivariate regime switching model that allows for detecting which parameters differ across regimes. This model, a Markov switching vector error correction model (MS-VECM), explicitly discriminates between short- and long-run dynamics and potentially allows for time-varying transition probabilities that depend on the cointegration error. We assess whether parameters differ across regimes by using novel shrinkage priors that have recently been utilized in finite mixture modeling (see [Yau and Holmes, 2011](#); [Malsiner-Walli et al., 2016](#)).

The literature on Bayesian estimation of Markov switching (MS) models is voluminous (see, among many others, [Chib, 1996](#); [Kim and Nelson, 1998](#); [Kaufmann, 2000](#); [Sims et al., 2008](#); [Kaufmann, 2015](#); [Droumaguet et al., 2017](#)). By contrast, contributions that explicitly deal with the Bayesian estimation of MS-VECMs is comparatively sparse ([Martin, 2000](#); [Paap and Van Dijk, 2003](#); [Jochmann and Koop, 2015](#)). Most of these contributions use Bayesian shrinkage priors to enable reliable and efficient estimation. These priors are typically specified in the spirit of standard Minnesota priors ([Doan et al., 1984](#); [Sims and Zha, 1998](#)) and symmetric across regimes. One implication is that coefficients are pushed towards a stylized prior model (like a multivariate random walk), irrespective of the regime and what the (regime-specific) likelihood suggests. For instance, a regression parameter may be zero in one regime, but different in another. Such a situation is effectively ruled out as both coefficients are pushed to zero.

In this paper, we circumvent such issues by proposing a novel hierarchical modeling approach that has originally been proposed in the literature on finite mixture models (see [Yau and Holmes, 2011](#); [Malsiner-Walli et al., 2016](#)). We estimate an MS-VECM assuming that the regime-specific coefficients arise from a common distribution. The mean of this common distribution is treated as unknown and estimated from the data. Using Normal-Gamma shrinkage priors ([Griffin and Brown, 2010](#)) on the variance-covariance matrix of the common distribution enables us to gain an understanding on what covariates drive the corresponding regime allocation. When compared to the existing literature, our approach allows for flexibly testing which coefficients (or sets of them) should differ across regimes while pushing similar coefficients towards a common mean. In addition, we follow the literature on MS models with time-varying transition probabilities (see, among others, [Filardo, 1994](#); [Kim and Nelson, 1998](#); [Kaufmann, 2015](#)) and assume that the transition probability matrix is time-varying and depends on the lagged cointegration errors.

For an empirical illustration, we estimate a Euro area business cycle model that discriminates between business cycle expansions and contractions. An additional empirical novelty of our approach is that we test whether deviations of macroeconomic fundamentals from their long-run equilibrium values impact the transition probabilities between business cycle stages. The empirical results suggest that our model successfully replicates Euro area business cycle behavior. Moreover, we find that deviations of output and credit from their long-run fundamentals have predictive power for the transition probabilities. When considering differences in parameters across regimes, our findings are threefold. First, we find that short-run adjustment coefficients do not differ across regimes. This is evidenced by strong shrinkage towards the common mean, leaving only little room for regime-specific deviations. Second, the autoregressive coefficients also display relatively little variation across regimes. For some equations, however, we find that selected coefficients do differ between expansionary and recessionary stages. Third, and finally, we find that the variance-covariance matrix of the shocks differs markedly across regimes. This indicates that

the regime allocation is strongly driven by differences in error variances and, to some extent, changes in the VAR coefficients.

The remainder of the paper is organized as follows. Section 2 outlines the proposed idea by means of a simple switching regression model while Section 3 presents the MS-VECM model as well as the prior set-up. Section 4 first gives an overview of the dataset used and subsequently shows the empirical results. The last section summarizes and concludes the paper.

2 A simple hierarchical model

In this section we outline the main idea on how to determine whether coefficients differ across regimes within a simple regression model. Subsequently, we generalize the stylized example to a multivariate non-linear error correction model.

We set the stage by assuming that a scalar time series $\{y\}_{t=1}^T$ follows a switching regression model (Goldfeld and Quandt, 1973),¹

$$y_t = \beta_{1S_t}x_{1t} + \beta_{2S_t}x_{2t} + \sigma_{S_t}\eta_t, \quad \eta_t \sim \mathcal{N}(0, 1), \quad (1)$$

with x_{jt} being exogenous covariates and β_{jS_t} (for $j = 1, 2$) the associated regression coefficients while $\sigma_{S_t}^2$ denotes the error variance. We assume that S_t denotes an indicator that takes values $0, \dots, R$ and follows a first-order Markov process.²

The Bayesian literature typically uses Gaussian priors on β_{jS_t} ,

$$\beta_{jS_t} \sim \mathcal{N}(0, \tau_j), \quad (2)$$

where τ_j denotes a fixed prior scaling for coefficient j . Notice that τ_j is regime invariant and the prior is centered on zero. In time series applications and for non-stationary data, it is common practice that the prior on the first lag of y_t is centered on unity, while for higher lag orders the prior mean is set equal to zero.

The symmetric prior in Equation 2 translates into a prior on the standardized distance between coefficients. For illustration, we compute the prior distance between β_{jk} and β_{jl} for $k \neq l$,

$$\frac{\beta_{jk} - \beta_{jl}}{\sqrt{2}} \sim \mathcal{N}(0, \tau_j). \quad (3)$$

If τ_j is close to zero, the corresponding coefficients do not differ significantly across regimes. However, they are simultaneously strongly pushed to zero. Thus, while such a prior is able to push selected coefficients towards homogeneity, it is not capable of handling cases where coefficients are non-zero but at the same time appear to be the same across regimes.

As a solution, we follow Yau and Holmes (2011) and assume that the regime-specific coefficients arise from a common distribution,

$$\beta_{jS_t} \sim \mathcal{N}(\beta_j, \tau_j). \quad (4)$$

¹For textbook introductions, see Kim and Nelson (1999) and Frühwirth-Schnatter (2006).

²The arguments we provide below hold for any law of motion of S_t .

The common mean β_j is treated as unknown and estimated from the data. It is noteworthy that if τ_j is close to zero, β_{js_t} is pushed to β_j across all regimes. Computing the standardized distance between β_{jk} and β_{jl} yields Equation 3 and the same intuition applies. However, instead of pulling β_{js_t} to zero for all regimes, this specification shrinks towards parameter homogeneity. Equation 4 can be interpreted as a Gaussian hierarchical prior on β_{js_t} .

In what follows, we use shrinkage priors on τ_j to test the existence of significant differences across regimes. Specifically, we assume that τ_j arises from a Gamma distribution,

$$\tau_j \sim \mathcal{G}(d_0, d_1), \quad (5)$$

with d_0 and d_1 being hyperparameters. This specification has been introduced by [Griffin and Brown \(2010\)](#) within a regression context and subsequently adopted by [Malsiner-Walli *et al.* \(2016\)](#) to determine variable relevance in finite mixture models. Using a Gamma prior allows for shrinking τ_j to zero if necessary, and thus permits selecting whether parameters differ across regimes.

3 Econometric framework

In Section 3.1, we first discuss the multivariate MS error correction specification while Section 3.2 discusses the prior choice and Section 3.3 briefly summarizes the main steps necessary to perform posterior inference.

3.1 A non-linear vector error correction model

The simple model outlined in the previous section is now generalized to a MS-VECM with two regimes. We opt for two regimes since we are interested in developing a model of the Euro area that encompasses several stylized facts about business cycles, namely pronounced co-movement of macroeconomic quantities over the business cycle as well as the distinction between expansionary and recessionary stages ([Burns and Mitchell, 1946](#)). Moreover, our proposed framework explicitly aims at discriminating between short- and long-run dynamics.

We assume that the first difference of a m -dimensional vector of macroeconomic time series $\{\mathbf{y}_t\}_{t=1}^T$ follows a MS-VECM with P lags,

$$\Delta \mathbf{y}_t = \lambda_{S_t} \mathbf{b}' \mathbf{y}_{t-1} + \sum_{p=1}^P \mathbf{B}_{pS_t} \Delta \mathbf{y}_{t-p} + \mathbf{H}_{S_t} \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_m). \quad (6)$$

Here we let λ_{S_t} be an $m \times r$ matrix of short-run adjustment coefficients, \mathbf{b} is an $m \times r$ matrix of long-run relations and \mathbf{B}_{pS_t} are $m \times m$ coefficient matrices associated with the p th lag of $\Delta \mathbf{y}_t$. Furthermore, \mathbf{H}_{S_t} is the lower Cholesky factor of a regime-specific variance-covariance matrix $\boldsymbol{\Sigma}_{S_t} = \mathbf{H}_{S_t} \mathbf{H}_{S_t}'$ and S_t is a discrete Markov process that takes values zero or unity detailed below. In what follows we rewrite Equation 6 as

$$\Delta \mathbf{y}_t = \mathbf{A}_{S_t} \mathbf{x}_t + \mathbf{H}_{S_t} \boldsymbol{\eta}_t, \quad (7)$$

whereby $\mathbf{A}_{S_t} = (\boldsymbol{\lambda}_{S_t}, \mathbf{B}_{1S_t}, \dots, \mathbf{B}_{PS_t})$ is a $m \times K$ matrix of stacked coefficients with $K = r + mP$. Moreover, $\mathbf{x}_t = (\mathbf{w}'_t, \Delta \mathbf{y}'_{t-1}, \dots, \Delta \mathbf{y}'_{t-p})'$ and $\mathbf{w}_t = \mathbf{b}' \mathbf{y}_{t-1}$ denoting the r cointegration errors.³ Notice that Equation 7 is a standard multivariate regression model conditional on \mathbf{b} . We explicitly rule out the possibility of breaks in \mathbf{b} since they are assumed to be long-run fundamental relations and thus not subject to abrupt changes. However, there exist studies which allow for non-linearities in the cointegration relationship (see, for instance, [Martin, 2000](#); [Bec and Rahbek, 2004](#); [Koop et al., 2011](#); [Jochmann and Koop, 2015](#)).

The transition probabilities of S_t , $\text{Prob}(S_t = j | S_{t-1} = i, \boldsymbol{\gamma}, \mathbf{w}_t) = p_{ij,t}$, are specified to be time-varying and depend on the (lagged) cointegration error through a set of regression coefficients in \mathbf{w}_t . More precisely, the matrix of transition probabilities is given by

$$\mathbf{P}_t = \begin{pmatrix} p_{00,t} & p_{01,t} \\ p_{10,t} & p_{11,t} \end{pmatrix}. \quad (8)$$

The rows of Equation 8 sum to unity for all i and t . Following the literature on early warning Markov switching models ([Filardo, 1994](#); [Kim and Nelson, 1998](#); [Amisano and Fagan, 2013](#); [Huber and Fischer, 2018](#)), we assume that the transition probabilities are parameterized using a probit specification,⁴

$$p_{ij,t} = \Phi(c_{0i} + \boldsymbol{\gamma}' \mathbf{w}_t), \quad (9)$$

with c_{0i} denoting a regime-specific intercept term and Φ denotes the cumulative distribution function of the standard normal distribution. The j th element in $\boldsymbol{\gamma}$ measures the sensitivity of the transition probabilities with respect to the j th cointegration error, w_{jt} . Following [Amisano and Fagan \(2013\)](#), we postulate that $\boldsymbol{\gamma}$ is time-invariant while the intercept term depends on the prevailing regime. Using the latent variable representation of the probit model yields

$$z_t^* = c_{0i} + \boldsymbol{\gamma}' \mathbf{w}_t + u_t, \quad u_t \sim \mathcal{N}(0, 1). \quad (10)$$

For identification purposes, the error variance is set equal to unity.

Before proceeding to the prior specification it is worth noting that $\boldsymbol{\lambda}$ and \mathbf{b} are not identified since they enter Equation 6 as a product. We achieve identification by using the linear normalization $\mathbf{b} = (\mathbf{I}_r, \boldsymbol{\Xi})'$ with $\boldsymbol{\Xi}$ being a $(m-r) \times r$ matrix of coefficients. This choice is clearly not invariant to the ordering of the elements \mathbf{y}_t but ensures that the model is exactly identified.⁵

3.2 Prior specification

The model outlined in the previous section is heavily parameterized and we thus adopt a Bayesian approach to estimation and inference. Consistent with the discussion in Section 2 we assume that the common distribution that gives rise to $\mathbf{a}_{S_t} = \text{vec}(\mathbf{A}_{S_t})$ follows a multivariate Gaussian distribution,

$$\mathbf{a}_{S_t} \sim \mathcal{N}(\mathbf{a}, \boldsymbol{\Omega}), \quad (11)$$

³For notational simplicity we suppress the dependence of \mathbf{w}_t on \mathbf{b} .

⁴In the case of more than two regimes, a potential alternative would be a logit specification ([Kaufmann, 2015](#); [Billio et al., 2016](#)).

⁵Another potential choice would be to identify the space spanned by the cointegrating vectors and introduce a restriction on this space ([Strachan, 2003](#); [Koop et al., 2009](#)).

where $\mathbf{\Omega} = \text{diag}(\tau_1, \dots, \tau_K)$ is a diagonal variance-covariance matrix with variances τ_j . Analogous to Section 2, τ_j determines the similarity between elements in \mathbf{a}_0 and \mathbf{a}_1 . For instance, if only the short-run adjustment coefficients differ across regimes, the corresponding elements in $\mathbf{\Omega}$ will be rather large whereas for the remaining coefficients, the associated τ_j s will be close to zero.

Following Malsiner-Walli *et al.* (2016) we specify a Gaussian prior on \mathbf{a} and the Gamma prior outlined in Equation 5 for all j ,

$$\mathbf{a} \sim \mathcal{N}(\underline{\mathbf{a}}, \underline{\mathbf{\Omega}}), \quad (12)$$

$$\tau_j \sim \mathcal{G}(d_0, d_1). \quad (13)$$

Hereby, we let $\underline{\mathbf{a}}$ denote the K -dimensional prior mean vector and $\underline{\mathbf{\Omega}}$ is a $K \times K$ prior variance-covariance matrix. In what follows, we specify $\underline{\mathbf{a}} = \mathbf{0}$ and $\underline{\mathbf{\Omega}} = 10^3 \times \mathbf{I}_K$ to obtain a weakly informative prior on \mathbf{a} . For τ_j , it is worth emphasizing that if $d_0 = 1$, we obtain the Bayesian Lasso (Park and Casella, 2008) used in Yau and Holmes (2011) while if $d_0 < 1$ increasing prior mass is placed on zero while the tails of the marginal prior become heavier (Malsiner-Walli *et al.*, 2016). In our empirical application, we set $d_0 = d_1 = 0.1$ to strongly center the prior on zero and allow for heavy tails.

The prior set-up described in Equations 11 to 13 effectively allows for detecting what elements in \mathbf{a}_{S_t} differ across regimes and which of them appear to be homogeneous over distinct business cycle stages. Especially in light of a moderate to large number of time series in \mathbf{y}_t as well as a moderate number of lags P , the number of parameters per regime is large relative to the length of a typical dataset. In the presence of multiple regimes, however, this problem is even more severe and shrinkage is necessary to obtain reliable parameter estimates. Our flexible hierarchical model specification enables for flexible shrinkage towards homogeneity while at the same time provides sufficient flexibility to allow for differences in the state-specific coefficients.

For $\boldsymbol{\xi} = \text{vec}(\boldsymbol{\Xi})$, the $v = (m - r)r$ free elements of \mathbf{b} , we use a Gaussian prior,

$$\boldsymbol{\xi} \sim \mathcal{N}(\mathbf{0}, \zeta \times \mathbf{I}_v), \quad (14)$$

where ζ is a prior hyperparameter that controls the tightness of the prior. We set $\zeta = 1$, which is a fairly uninformative choice given the scale of our data. In principle, it would also be possible to elicit a prior directly on the cointegrating space (Strachan, 2003). Here, we follow the traditional approach since we are interested in directly interpreting the corresponding cointegration error.

Following Frühwirth-Schnatter (2006), Malsiner-Walli *et al.* (2016) and Huber and Zörner (2017), we use a hierarchical Wishart prior on $\boldsymbol{\Sigma}_{S_t}^{-1}$,

$$\boldsymbol{\Sigma}_{S_t}^{-1} \sim \mathcal{W}(\mathbf{S}, s), \quad (15)$$

$$\mathbf{S} \sim \mathcal{W}(\mathbf{Q}, q). \quad (16)$$

The prior hyperparameters are specified such that (see [Frühwirth-Schnatter, 2006](#); [Malsiner-Walli et al., 2016](#))

$$s = 2.5 + \frac{m-1}{2}, \quad (17)$$

$$q = 0.5 + \frac{m-1}{2}, \quad (18)$$

$$\mathbf{Q} = \frac{100s}{\underline{s}} \text{diag}(\hat{\sigma}_1^2, \dots, \hat{\sigma}_m^2). \quad (19)$$

Here we let $\hat{\sigma}_j^2$ denote the OLS variance obtained by running a univariate autoregressive model of order P . This choice implies that all of the regime-specific variance-covariance matrices stem from a common distribution, similar to the case for \mathbf{a}_{S_t} outlined above.

Finally, we use Gaussian priors on $\mathbf{\Gamma} = (c_{0i}, \gamma')'$,

$$\mathbf{\Gamma} \sim \mathcal{N}(\mathbf{0}, \mathbf{V}), \quad (20)$$

with $\mathbf{V} = 10 \times \mathbf{I}_{r+1}$ to introduce relatively little prior information on $\mathbf{\Gamma}$. Decreasing the prior variance would lead to a situation where the researcher suspects that transition probabilities do not depend on w_t and tend to be time-invariant.

3.3 Full conditional posterior simulation

The model outlined in the previous sections features a joint posterior density that is intractable. Fortunately, however, all full conditional posterior distributions take a well known form and are thus amenable to perform Gibbs sampling. In this section, we briefly summarize the steps involved in order to obtain a valid draw from the joint posterior, focusing attention on the non-standard parts while providing references for the full conditionals that are standard.

Conditional on a suitable set of starting values, our Markov chain Monte Carlo (MCMC) algorithm cycles through the following steps:

1. Sample \mathbf{a}_j ($j = 0, 1$) conditional on the remaining parameters and the states from a K -dimensional multivariate Gaussian distribution. The corresponding moments take a standard form ([Zellner, 1973](#)).
2. Simulate ξ from a Gaussian posterior distribution conditional on the remaining parameters, states and a set of identifying assumptions. The precise formulas can be found in [Villani \(2001\)](#) and [Huber and Zörner \(2017\)](#).
3. The common mean \mathbf{a} is simulated conditional on $\mathbf{a}_0, \mathbf{a}_1$ and $\underline{\Omega}$ from a Normal distribution, with \odot indicating point-wise multiplication,

$$\begin{aligned} \mathbf{a} | \mathbf{a}_0, \mathbf{a}_1, \underline{\Omega} &\sim \mathcal{N}(\bar{\mathbf{a}}, \underline{\Omega} \odot \bar{\underline{\Omega}}) \\ \bar{\underline{\Omega}} &= (2\mathbf{I}_K + \underline{\Omega}^{-1})^{-1}, \\ \bar{\mathbf{a}} &= \bar{\underline{\Omega}} (\mathbf{a}_0 + \mathbf{a}_1), \end{aligned}$$

4. Draws from the conditional posterior of τ_j are obtained by noting that $p(\tau_j|\mathbf{a}_0, \mathbf{a}_1)$ follows a generalized inverted Gaussian (GIG) distribution,

$$\tau_j|\mathbf{a}, \mathbf{a}_0, \mathbf{a}_1 \sim \mathcal{GIG}\left(d_0 - 1, \sum_{j=0}^1 (\mathbf{a}_j - \mathbf{a})^2, 2d_1\right). \quad (21)$$

5. Draw Σ_j^{-1} (for $j = 0, 1$) from a Wishart conditional posterior distribution given by

$$\begin{aligned} \Sigma_j^{-1}|\mathbf{a}_0, \mathbf{a}_1, S^T, \mathbf{S}, \mathcal{D} &\sim \mathcal{W}(\bar{\mathbf{S}}_j, \bar{s}_j) \\ \bar{\mathbf{S}}_j &= \mathbf{S} + \frac{1}{2} \sum_{t:S_t=j} (\Delta \mathbf{y}_t - A_j \mathbf{x}'_t)(\Delta \mathbf{y}_t - A_j \mathbf{x}'_t)', \\ \bar{s}_j &= s + N_j/2 \end{aligned}$$

whereby $S^T = (S_1, \dots, S_T)'$ denotes the full history of the states, \mathcal{D} the data and $N_j = \#(t : S_t = j)$, that is, the number of observations in regime j .

6. The common scaling matrix for the Wishart prior \mathbf{S} is simulated from its Wishart distributed conditional distribution,

$$\begin{aligned} \mathbf{S}|\mathbf{a}_0, \mathbf{a}_1 &\sim \mathcal{W}(\bar{\mathbf{S}}, \bar{q}) \\ \bar{\mathbf{S}} &= \mathbf{Q} + \sum_{j=0}^1 \Sigma_j^{-1}, \\ \bar{q} &= q + 2s. \end{aligned}$$

7. Simulate the full history of the states S^T as well as the transition probabilities using the algorithm outlined in [Kim and Nelson \(1998\)](#) and adopted in [Amisano and Fagan \(2013\)](#).
8. Estimate the full history of z_t^* and Γ using the algorithm proposed in [Albert and Chib \(1993\)](#).

We repeat this algorithm 85,000 times where the first 50,000 draws are discarded as burn-in. Convergence is assessed using standard trace plots as well as inefficiency factors and the [Raftery and Lewis \(1992\)](#) diagnostic. All measures point toward rapid convergence for most parts of the parameter space under scrutiny.

4 Empirical application

In this section, we estimate a medium-scale empirical model for the Euro area that explicitly discriminates between business cycle phases of expansion and recession. In the next section (Section 4.1) we briefly outline key model specification and identification issues as well as the dataset adopted. We then proceed by specifying the cointegration rank in Section 4.2. In Sections 4.3 and 4.4 key properties of our model are analyzed.

$r =$	1	2	3	4	5
DIC	-7877	-8086	-7905	-7953	-8033
SD	70	81	92	132	129

Table 1: Selecting the cointegration rank using the DIC and the numerical standard errors (SD), respectively.

4.1 Data overview and model specification

For the empirical application we use a subset of the data provided in [Gambetti and Musso \(2017\)](#). The dataset employed is comprised of six quarterly Euro area time series ranging from Q1:1985 to Q4:2011. Variables included are real gross domestic product (GDP), the growth rate of the harmonized index of consumer prices (CP), the non-financial private sector loan volume (outstanding amounts of loans granted by financial intermediaries to households and non-financial corporations, labeled CR), the GDP to loans ratio (Loans), a composite lending rate (weighted average of lending rates for loans to households and non-financial corporations, abbreviated as LR) and a short-term interest rate (three-month Euribor up to August 2007, afterwards the Euro Repo rate for secured interbank lending, abbreviated as STIR).

Due to the normalization assumption on \mathbf{b} , the ordering of the variables in \mathbf{y}_t plays a crucial role. Here, we use the following ordering

$$\mathbf{y}_t = (CR, GDP, DP, Loans, LR, STIR)'. \quad (22)$$

As is standard in the literature for quarterly data, we choose $P = 4$ lags in the empirical specification. This translates into five lags of \mathbf{y}_t if the model is written in terms of a MS-VAR in levels. Moreover, we include an intercept term that is consequently included in \mathbf{a} and thus also originates from the common distribution in Equation 11. Furthermore, notice that the model is not identified with respect to the labeling of the states. Here, we achieve identification by assuming that the conditional mean of the equation for GDP growth is higher for $S_t = 0$. This is achieved by implementing a rejection step in the MCMC algorithm.

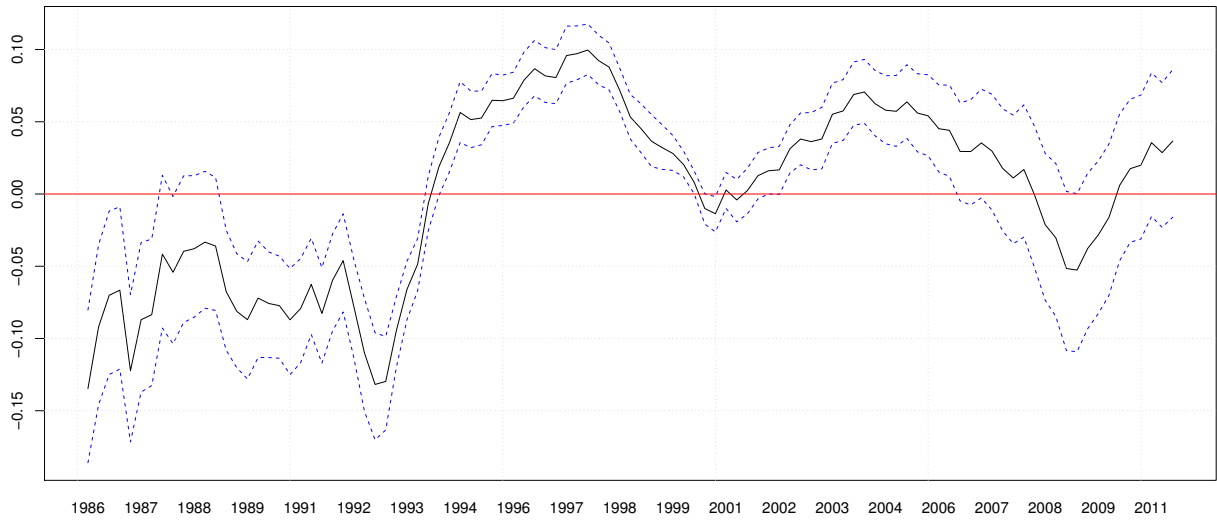
4.2 Selecting the cointegration rank

In this section we specify the cointegration rank r of the MS-VECM. For this purpose, we rely on the deviance information criterion (DIC, [Spiegelhalter et al., 2002](#)) and choose the r that minimizes the DIC. Since our model is a latent variable model, we need to decide on whether to use the conditional likelihood (i.e. conditional on the latent states) or integrate out the latent states to obtain the complete data likelihood. Here, we follow the latter approach because the complete data likelihood is readily available as a byproduct of the filtering recursions of our algorithm.

Based on the results reported in Table 1 that depicts the mean level as well as numerical standard errors of the DIC. These quantities are obtained by re-estimating the model for each cointegration rank 100 times and computing the DIC. The table indicates that $r = 2$ minimizes the DIC with differences for ranks higher than unity appearing to be rather small, especially when viewed in light of the numerical standard errors provided.

The posterior distributions of the two cointegration errors that are subsequently used to inform the transition distributions are depicted in Figure 1. Notice that the first error can be interpreted as the deviation of credit from its long-run equilibrium value determined by the remaining elements in \mathbf{y}_t . By contrast,

(a) First cointegration error



(b) Second cointegration error

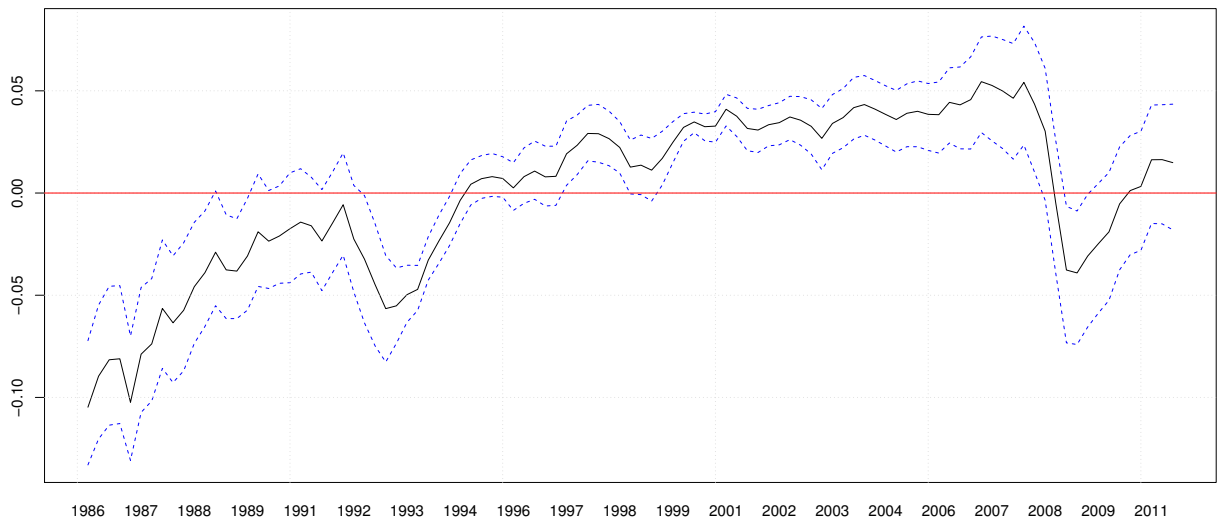


Fig. 1: Posterior distribution of cointegration errors.

the second error can be considered to be the deviation of output from a long-run equilibrium value and is thus related to the output gap. The first cointegration error, shown in Figure 1(a), indicates that credit was below its equilibrium value during the first third of the sample. From 1992 onwards, we observe that credit quickly reverted towards its long-run fundamental value and then subsequently overshoot it until the

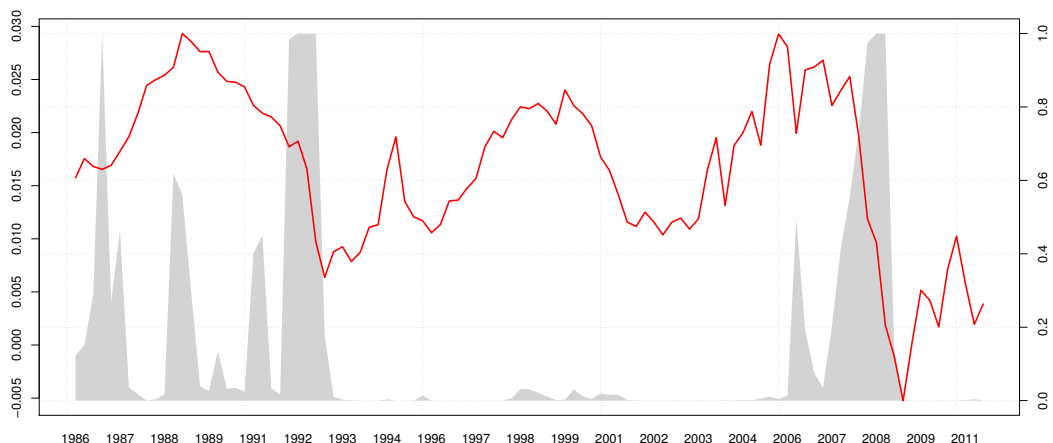


Fig. 2: Credit growth and filtered probabilities of being in the expansion or recession state.

Notes: The red line indicates the actual credit growth rate and the grey shaded area the posterior mean probability of the recession state.

beginning of the 2000s. From around 2001 onwards, the figure suggests that credit persistently remained above zero up to the beginning of the global financial crisis where it, again, dropped below zero.

Considering the second cointegration error in Figure 1(b) suggests that output was below its long-run equilibrium in the second half of the eighties while being approximately in equilibrium from 1988 to 1991. In 1992, we find that output dropped below its long-run equilibrium value. This is probably due to the crisis of the European Exchange Rate Mechanism (ERM) that led to a recession across a large number of EU member states. Afterwards, we find that output slightly remained above equilibrium until the global financial crisis led to a sharp decline in real activity. At this point, it is worth emphasizing that our goal is not to provide an accurate measure of the output gap. In order to obtain a more reliable estimate of the output gap, y_t needs to be augmented with additional macroeconomic and financial quantities.

4.3 Regime allocation and time-varying transition distributions

In this section, we first consider the corresponding posterior regime allocation and the transition probabilities. Figure 2 depicts the smoothed state probabilities of being in the recession state as a grey shaded area, while credit growth is indicated in red.

Our model tracks several periods of economic stress rather well. The two most dominant recessions in our sample, namely the area-wide recession due to the ERM crisis as well as the global financial crisis in 2008/2009, are captured by our model. We also find several periods during the midst of the 1980s that have been identified to be within a recessionary regime. These spikes in recession probabilities can be attributed to rather high levels of inflation in several member states, contractionary monetary policy, as well as the end of the Cold War. It is noteworthy that credit growth typically declines before economic downturns. This phenomenon is particularly evident in the years 1986, 1992 and the beginning of the Great Recession in 2007 and 2008. This is in line with [Borio and Lowe \(2004\)](#), who argue that the emergence and severity of crises is tightly linked to the availability of credit in a given economy.

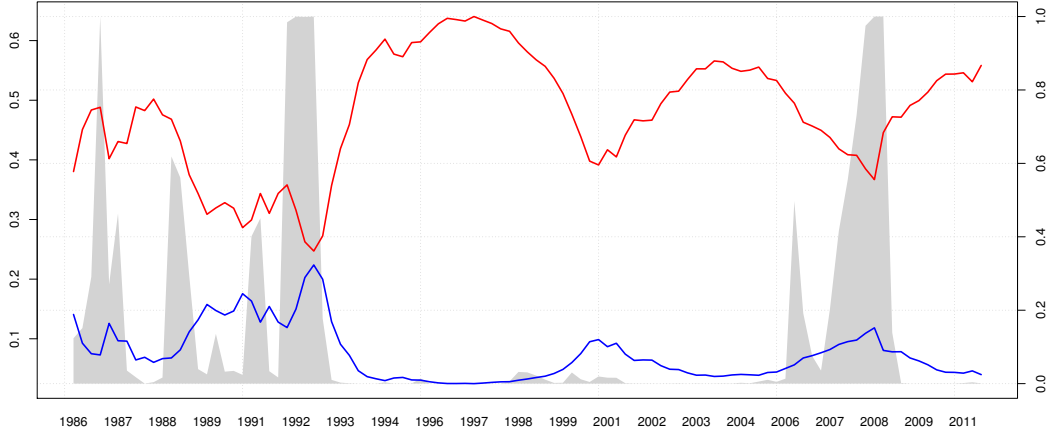


Fig. 3: Posterior mean of transition probabilities and filtered probabilities of being in the expansion or recession state.

Notes: The red line in the upper part indicates $Prob(S_t = 0 | S_{t-1} = 1)$, the blue line in the lower part denotes $Prob(S_t = 1 | S_{t-1} = 0)$. The grey shaded area indicates the posterior mean probability of the recession state.

Evidence for time variation in the posterior mean of the transition probabilities is reported in Figure 3, where the grey shaded area refers to the smoothed probability of being in the recession state. The red and blue lines are based on the time-varying off-diagonal elements of the transition probability matrix, which indicate the probability of entering a recession at time t when being in the expansion state in $t - 1$ (that is, $Prob(S_t = 1 | S_{t-1} = 0)$) and vice versa.

From 1993 onwards, a period of relative stability emerges, evidenced by a decline in the probability of moving into a recession when being in an expansion until the end of 1997. This period ends abruptly in 1998 with a surging probability of a recessionary phase. Arguably, this reflects the burst of the US-based dot-com bubble and the 9/11 terror attacks in 2001 and subsequent transmission to the economies of the Euro area. Interestingly, the smoothed probability of the recession state exhibits only a minor reaction, suggesting that a European recession was avoided. Subsequently, we again observe a period of comparatively low probability of entering a crisis when being in an expansion. The peak around the year 2008 marks the beginning of the Great Recession and the emerging European debt crisis.

4.4 Do parameters differ across regimes?

The key novelty of our proposed approach is that it allows for flexible testing what coefficients differ across regimes. In order to assess differences in parameters, we rely on two visual tools that enable assessing how much shrinkage is introduced as well as how large deviations of α_{S_t} from α are.

The first visual assessment is based on considering the posterior mean of the log posterior of τ_j . Figure 4 presents the scaling parameters associated with the short-run adjustment coefficients in λ_{S_t} and across equations. The left panel (a) of Figure 4 shows the variance parameters associated with the adjustment terms of the first cointegration error while the right panel (b) of the figure displays the scaling parameters related to the second column of λ_{S_t} .

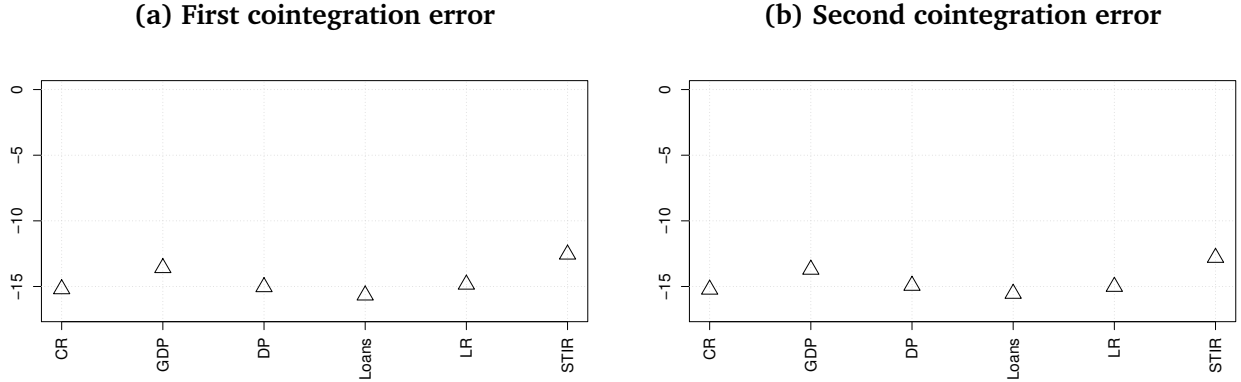


Fig. 4: Posterior mean of the log posterior of the scaling parameters across equations: Adjustment coefficients associated with the cointegration errors.

Across equations, we find that the scaling parameters are all close to zero and display relatively little variation. This finding holds true for both cointegration errors, suggesting that all elements in λ_{S_t} are strongly pushed towards the common mean. Thus, the figure clearly suggest that the way the economy adjusts to departures from long-run equilibrium values appears to be independent of the prevailing economic regime. From a modeling perspective, the key point to take away from Figure 4 is that a regime-invariant $\lambda = \lambda_0 = \lambda_1$ seems to suffice.

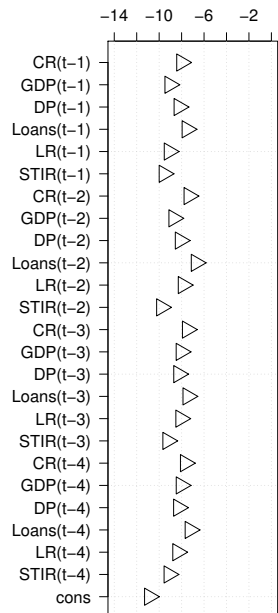
Next we investigate whether the coefficients associated with the lagged endogenous variables differ across regimes. Figure 5 shows the scaling parameters for each autoregressive coefficient as well as for the intercept term across equations.

For most equations, our results indicate that differences in coefficients are rather small. This overall conclusion stems from the fact that most log scalings are small, with some being almost equal to zero. For some equations, however, we find that the amount of shrinkage introduced on the standardized distances is considerably smaller for the first lag of credit (see panels (b), (c), and (e) of Figure 5), providing evidence that these macroeconomic quantities react differently to lagged changes in credit growth. Notice that differences of order two to four on the log-scale hint towards substantial differences in the amount of shrinkage introduced.

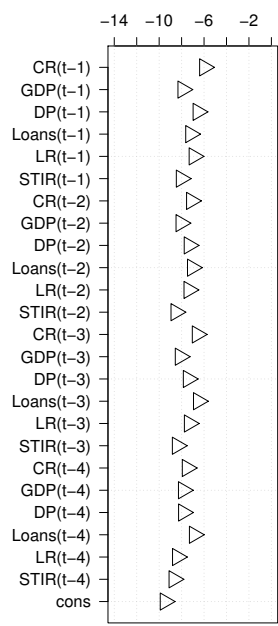
One shortcoming of the analysis presented in Figures 4 and 5 is that it is not invariant with respect to the scaling in y_t (and its changes). To provide some evidence on the quantitative differences in the autoregressive coefficients, we compute the posterior mean of the distance between regime-specific coefficients and the underlying common distribution. The results are reported in Figure 6 with values between -0.05 and 0.05 being zeroed out. We find comparatively large differences for the Loans and short-term interest rate equation. Minor deviations are also apparent in the case of GDP and CP while the lending rate and loan volume do not differ across regimes. Note that the two states closely mirror each other, and, for instance, positive deviations in the expansionary regime are typically accompanied by negative differences in the recession state.

Finally, we consider whether the variance-covariance matrices differ across regimes. To this end, Figure 7 presents a boxplot of the marginal posterior distributions of the log determinant (a), the log trace (b) and the log maximum eigenvalue (c) of the variance-covariance matrix for the recessionary as well as for the expansionary regime. Two findings are worth emphasizing. First, considering the posterior median

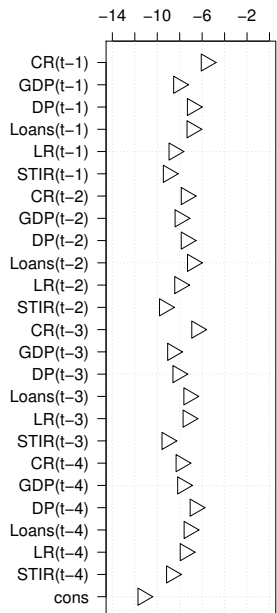
(a) Non-financial private sector loan volume



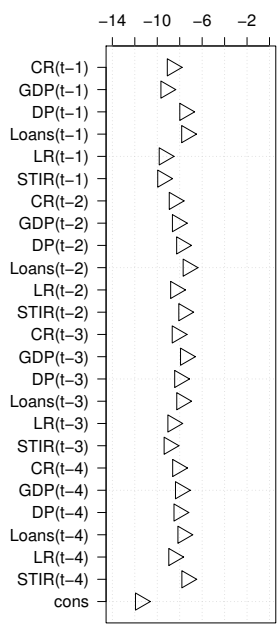
(b) Gross domestic product



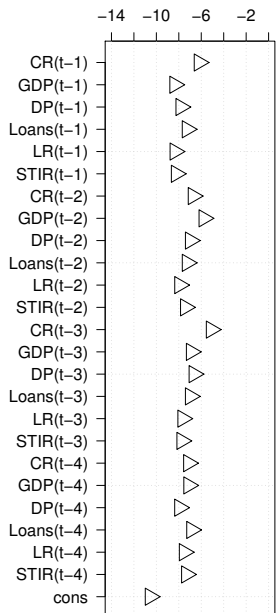
(c) Harmonized consumer price index (growth)



(d) GDP to loans ratio



(e) Lending rate



(f) Short-term interest rate

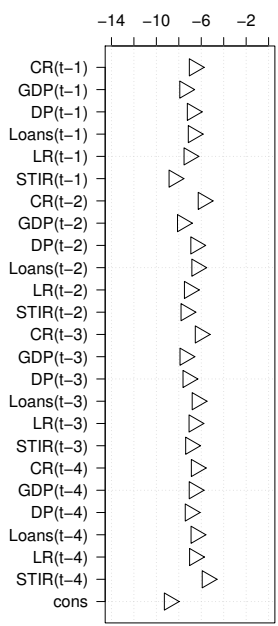


Fig. 5: Posterior mean of the log posterior of the scaling parameters across equations: autoregressive coefficients.

highlights that the trace of Σ_1 is much larger, pointing towards larger error variances in recessions. This is consistent with other findings in the literature that highlight the necessity to allow for heteroscedasticity if recessions are included in the sample (for forecasting evidence, see, [Clark, 2011](#)). Second, and finally, we find that posterior uncertainty sharply increases during recessionary episodes. This stems from the fact that the recessionary regime features considerably less observations.

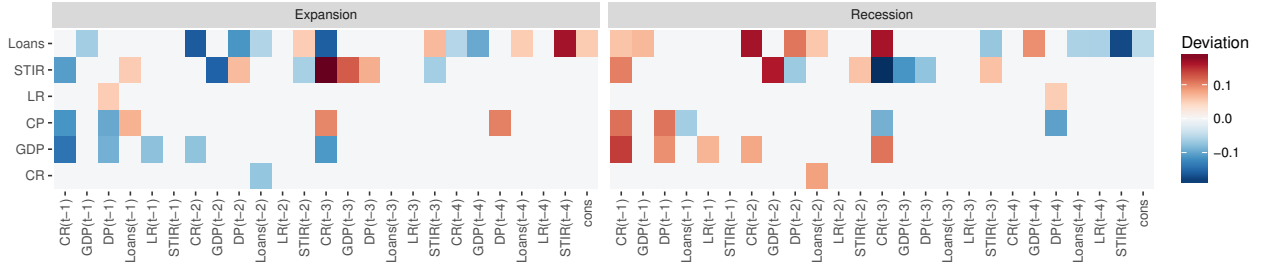


Fig. 6: Difference of posterior means between state-specific coefficients and the common distribution.

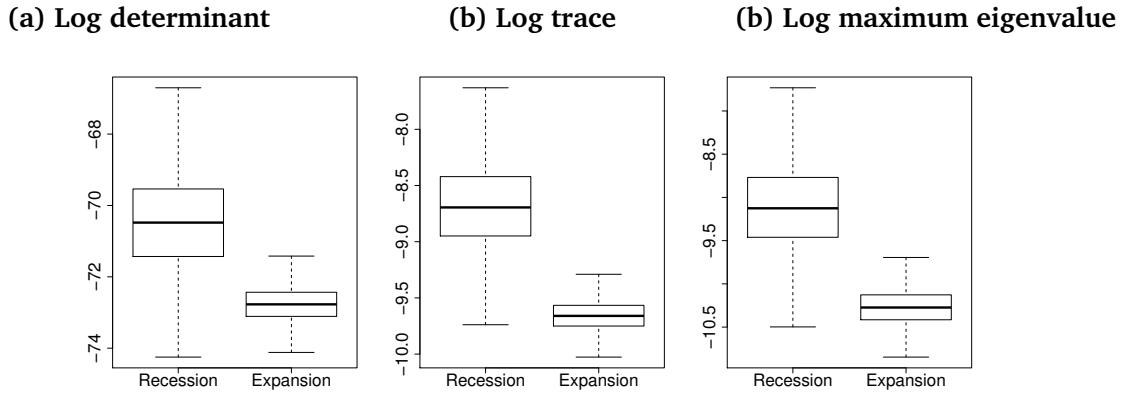


Fig. 7: Posterior distribution of the log determinant of the regime-specific variance-covariance matrices

To sum up, while we find that differences in the regression coefficients in α_0 and α_1 are rather small, our results provide strong evidence that error variance-covariance matrices differ sharply across regimes. Our modeling approach thus stochastically selects a model where only selected VAR coefficients differ while the short-run adjustment coefficients appear to be regime invariant. By contrast, our findings indicate that the regime allocation is mainly driven by differences in the variance-covariance matrices and this, to some extent, corroborates findings presented in [Sims and Zha \(2006\)](#).

5 Closing remarks

In this paper, we propose a hierarchical Markov switching model that allows for assessing what coefficients should differ across regimes. The specific model, a MS vector error correction model, discriminates between short- and long-run dynamics and assumes that the transition probability matrix of the underlying Markov process is time-varying. We assume that the autoregressive coefficients, the error variance-covariance matrices as well as the short-run adjustment coefficients differ across regimes and arise from a common distribution. Moreover, another novel feature of our model is that the transition distributions are parameterized using a simple binary probit model with the (lagged) cointegration errors included as covariates.

The modeling approach is then highlighted using a medium-scale Euro area dataset. Our empirical model discriminates between expansionary and recessionary business cycle stages and allows for assessing whether transition probabilities do vary over time. Considering the posterior mean estimates for the filtered

probabilities indicates that our model succeeds in replicating business cycle features of the Euro area. In addition, we investigate what coefficients differ across regimes and how this impacts the actual regime allocation.

Our findings suggest that using the proposed approach succeeds in reproducing dominant Euro area recessions and, moreover, show that deviations of output and credit from their long-run fundamentals drive the transition between regimes. Considering what parameters should differ across regimes, the proposed hierarchical model suggests that short-run adjustment coefficients can be assumed to be regime-invariant while selected VAR coefficients should differ across expansions and recessions. Error variances, however, tend to differ sharply and predominantly account for the corresponding regime allocation.

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