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## An Economic Model of Child Custody

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August 2003

# An Economic Model of Child Custody

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Discussion Paper No. 857

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## ABSTRACT

### **An Economic Model of Child Custody\***

This paper develops a model of child custody based on an incomplete-contract approach to the allocation of property rights. Because of the presence of transaction costs in marriage, altruistic parents cannot contract upon the investments they make into their children, but can reduce the resulting inefficiencies by determining *ex ante* the parent who would be allocated custody in case they divorce. We show that: (i) the optimal allocation of custodial rights depends on both preferences and technological factors; (ii) custodial rights can be allocated either to the parent who values the benefits from child welfare more or, viceversa, to the parent with the lowest valuation; (iii) if one parent's investment is significantly more important than the other parent's investment, then sole custody is preferred to joint custody and it should be allocated to the parent whose investment is relatively more important; and (iv) if the importance of the parents' investments is sufficiently similar and if the differences in parents' valuations of child quality are large, then joint custody is optimal with the low-valuation parent receiving a relatively greater share, because the other parent would invest in the child anyway while the low-valuation parent would be endowed with greater bargaining power. The implications of these results are then interpreted in the context of current custody laws, discussed in relation to empirical estimates of some of the parameters underlying the optimal custody rule, and used to question the skepticism surrounding prenuptial contracts.

JEL Classification: C78, D23, D64, J12, J13, K10

Keywords: child custody, incomplete contracts, terms of divorce, parental altruism, parental investments, negotiations

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“When by birth a child is subject to a father, it is for the general interest of families, and for the general interest of children, and really for the interest of the particular infant, that the Court should not, except in very extreme cases, interfere with the discretion of the father.” *Re Agar-Ellis* (1883) [cited in *Maidment* (1984, p. 98)].

“Since wives will, under most circumstances, be awarded custody regardless of the statutory standard, and since it seems wise to discourage traumatic custody contests whenever it is possible to do so, the [Uniform Marriage and Divorce] act should discourage those few husbands who might wish to contest by establishing a presumption that the wife is entitled custody. ... [I]t may well be true that because of the presumption some fathers who would be better custodians than their wives will either fail to seek custody or will be denied custody following a contest, but that disadvantage has a lower ‘social cost’ than the disadvantages of any alternative statutory formulation.” *Ellsworth and Levy* (1969, p. 203)

## 1. INTRODUCTION

The custody of children after their parents separate or divorce is a complex issue. It is a matter of concern not only for the parties directly involved (i.e., mothers, fathers, and children) but also for society at large.<sup>1</sup> Although newspapers describe the bitter and protracted divorces of the rich and famous, and television and cinema dramatize courtroom battling, conflict over the terms of divorce — that involves such issues as child custody, visitation rights and child support — is very common (*Sarat and Felstiner 1989*).<sup>2</sup> With more than 50 percent of children in the United States and 40 percent in Britain expected to live apart from at least one of their parents before reaching adulthood

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<sup>1</sup>In many countries, there has recently been an increasing emphasis on the importance of public investments in services for children in general, and for children of divorced and lone parents in particular. Examples are the Earned Income Tax Credit, Head Start, and the Temporary Assistance for Needy Families in the United States, and the Child Tax Credit, the Working Tax Credit, and the New Deal for Lone Parents in Britain.

<sup>2</sup>Using data on a sample of about 1100 California families who filed for divorce between September 1984 and April 1985, *Maccoby and Mnookin (1997)* find that approximately 50 percent of these families had some form of custody conflict. However, only 1 in 4 of the families in this sample report high-conflict resolutions, with settlements that require court-annexed mediations or court-ordered evaluations.

(Bumpass, Raley, and Sweet 1995; Ermisch and Francesconi 2000), and with many of these children facing an array of adverse consequences (McLanahan and Sandefur 1994; Haveman and Wolfe 1995), child custody is clearly a key issue for a variety of agents, including policy makers, social workers, lawyers, and social analysts.

The problem of which parent should have custody is, in practice, discussed and resolved only when the parents have already separated or when they are in the process of getting divorced. That is, custody is determined *ex post* (Maccoby 1999). In line with this practice, most of the policy interest has often centered on the question of what can be done to help divorcing parents resolve the custody issue effectively (Garfinkel et al. 1998). Child custody could, alternatively, be discussed and determined *ex ante*, while the parents are married, and well before divorce becomes an issue. It may be argued that discussing the terms of divorce at the outset of marriage is unnecessary or even counterproductive to the success of the marriage itself. This argument may partly explain why binding and legally enforceable prenuptial contracts are either not in use in a number of countries, such as Britain (Leech 2000), or largely under-regulated in others — including the United States, Canada, and Australia — where, in any case, custody rights are not covered by prenuptial agreements (Nasheri 1998; Fehlberg and Smyth 2002).

A fundamental notion that underlies this paper is that, given the presence of “transaction costs” (such as the costs of coordinating and enforcing an agreement about partners’ interdependent actions), it is *optimal* to determine the terms of divorce at the outset of marriage. Optimality here is defined in terms of maximizing children’s welfare *and* marital surplus, which in turn would help minimize the likelihood of divorce.<sup>3</sup> The terms of divorce — that among other things stipulate which of the two parents would have custody — determine the parents’ respective outside options. These influence the parents’ bargaining powers over bargaining situations that they encounter during their marriage, and hence influence the *distribution* of the marital surplus. This, in turn, affects each partner’s incentives to contribute to the generation of the surplus in the first place. Hence, the terms of divorce have also *efficiency* consequences. The levels of various relationship-specific investments (such as investments in children’s human capital) made by the parents are key determinants of both children’s welfare and marital surplus. If such investments can be contracted upon, then

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<sup>3</sup>As explained later, this optimality criterion is also consistent with the prevailing legal principle of allocating custody “in the best interest of the child”.

— as would be implied by an application of the Coase theorem (Coase 1960) — efficient levels of such investments would be implementable, irrespective of when the terms of divorce are determined and irrespective of what those terms are. In such a frictionless world, the terms of divorce would have *no* efficiency consequences; they would however continue to affect the distribution of the efficient level of the surplus. But, when the investments cannot be contracted upon — due to the presence of transaction costs — the terms of divorce will also matter for efficiency (Grossman and Hart 1986; Hart and Moore 1990).

In this paper, we are concerned with intact families that are comprised of married or cohabiting partners,<sup>4</sup> who either already have or are planning to have children. We are not concerned with the situation faced by divorcing couples or by individuals who already went through a marital dissolution. By looking at intact families, the entire course of actions of the parents during marriage (e.g., their marriage-specific investments) is affected. The equilibrium terms of divorce are then those that induce the best optimizing actions from parents. Given that it is optimal to establish child custody *ex ante*, the key issue is to determine the parent who should optimally be allocated sole custody or the shares of joint custody that each parent would receive upon divorce. To analyze this issue we develop a simple three-stage model. In the first stage, the two partners negotiate and determine the terms of divorce and stipulate them in a contract that allocates custodial rights in the eventuality of divorce.<sup>5</sup> In the second stage, the partners independently and non-cooperatively choose their respective levels of investments in their children.<sup>6</sup> Finally, in the third stage, they negotiate over the partition of the resultant marital surplus in the shadow of divorce. The optimal custody rule is underpinned by two sets of key

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<sup>4</sup>Throughout the paper, the terms “marriage” and “cohabitation” are synonymous. Similarly, the terms “husband/wife” and “partner” are used interchangeably. It is worth noticing, however, that in a number of countries (including the United States and Britain) children are by default allocated to their mother after the break-up of a cohabiting union and, thus, child custody *per se* is not an issue legally. But in this paper we are concerned with the behavioral implications of a rule that is likely to affect the economic decisions of both partners in any durable union. In this respect, the legal distinctions between marriage and cohabitation are inconsequential.

<sup>5</sup>Although other aspects of the post-divorce relationship (e.g., child-rearing, child and spousal support, and visitation rights) are relevant to the determination of the terms of divorce, they are not considered here for simplicity.

<sup>6</sup>The non-cooperative nature of such investments is related to the fact that they are not contractable and thus cannot be verified by third parties (such as the court or the state).

parameters. The first is given by parents' child-preference parameters, which capture the degrees of altruism of each parent towards their children in each of the three possible regimes (or marital states), that is: (i) the parents remain married, (ii) the parents divorce and the father is allocated custody, and (iii) the parents divorce and the mother is allocated custody (these regimes will be operative also in the case of joint custody). The second set is represented by technological parameters that capture the productivities (or importance) of each parent's investment in their children in each of the three marital states.

Our analysis shows several interesting results. Here we draw attention to four. First, the optimal allocation of custodial rights depends on both parental preferences and technology structure, regardless of whether joint custody is ruled out or not. Even though there are situations in which the fact that a parent is the key investor and other aspects of the technology do not matter, both comparative degrees of parental altruism and comparative parental productivities in generating child quality are relevant to the determination of custodial rights. Second, sole custodial rights or greater shares of joint custody can be allocated either to the parent who values the benefits from child quality more or, viceversa, to the parent with the lowest valuation. The actual allocation must take account of who the key investor is and the productivity of each parent's investment. A fundamental principle that governs the allocation of custodial rights is the "balancing out" of parents' bargaining power and investment incentives. In this perspective, giving a share of joint custody (or perhaps even sole custody) to the parent with the lowest valuation is consistent with the aim of endowing this parent with a greater bargaining power and thus inducing him/her to provide the optimal level of child investment. Third, if one parent's investment is significantly more important than the other parent's investment, then sole custody is preferred to joint custody, and it should be allocated to the parent whose investment is relatively more important. In these circumstances, parents' preferences are irrelevant. Fourth, joint custody is optimal when the productivities of parents' investments are sufficiently similar and the differences in their valuations of child quality are sufficiently large. In these cases, a greater share of joint custody should go to the parent with the lowest valuation precisely because the parent with the highest valuation would invest in the child anyway, and the low-valuation parent would be endowed with greater bargaining power.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 lays down a simple model that characterizes the equilibrium implications of alternative allocations of sole



custodial rights. Section 4 analyzes the optimal custody decision and derives our main results on sole custody. Section 5 extends the model to the analysis of joint custody. Section 6 discusses the implications of our main results in the context of current custody laws and links them to the empirical knowledge of the key parameters in our model. Section 7 concludes. For ease of exposition all technical proofs are relegated to the Appendix.

## 2. RELATED LITERATURE

Ever since the contributions of Becker (1974, 1981/1991), economists have developed a wide range of family behavior models in which partners are assumed to have either common preferences or separate preferences (for comprehensive surveys, see Bergstrom [1997], Behrman [1997], Weiss [1997] and Ermisch [2003]). Within the non-unitary models, an increasing number of studies explore the implications of a couple's inability to make binding legally-enforceable commitments about future behavior, inability that typically leads to inefficient allocations of household resources (Mazzocco 2001; Basu 2001; Lich-Tyler 2001; Lundberg and Pollak 2001). This strand of research on the family has been preceded by another large and rich theoretical literature on incomplete contracts and hold-up problems, which has shed light on many issues ranging from vertical and lateral integration to ownership rights and delegation of power within firms (Grossman and Hart 1986; Hart and Moore 1990; Hart 1995). Rarely have these two classes of research (i.e., family interactions and incomplete contracts) been joined.<sup>7</sup> The aim of our paper is to bridge this gap by exploiting the insights of the incomplete-contract literature to improve our understanding of the allocation of custodial rights upon divorce.

The incomplete contracting approach emphasizes the presence of transaction costs, which could be relevant to many household decisions. Examples of such costs are the costs of coordinating an agreement about partners' interdependent actions, the costs of monitoring and enforcing such an agreement, and the costs of renegotiation (Pollak 1985; Lundberg and Pollak 2001).<sup>8</sup> The presence of these costs implies that an efficient allocation of resources cannot be guaranteed, even in a world of complete information (Muthoo 1999). Although our model

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<sup>7</sup>An example of application that combines elements of the incomplete-contract approach with some aspects of the economic theory of the family is Rasul (2002). Rainer (2003) also applies this approach to explore the optimal allocation of household property rights upon divorce.

<sup>8</sup>For a clear and critical exposition of the hypotheses underlying such costs within the broader realm of the incomplete-contract literature, see Tirole (1999).

is embedded in an incomplete contracting framework, its key features are shared by most of the economic analyses of the family, namely: (i) parents' *altruism* towards their children (Becker 1981; Ermisch 2003); (ii) the assumption that the distribution of the marital surplus is determined by *negotiations* between couples (McElroy and Horney 1981; Chiappori 1988); and (iii) parents' *investments* in their children (Becker and Tomes 1986; Mulligan 1997).

What we currently know on child custody issues comes primarily from legal studies. These either investigate the implications of the legal norms behind customary custody rules, such as the rule based on the "best interest of the child" (Goldstein, Freud, and Solnit 1979; Mason 1994), or describe the implementation of such rules and how they affect the parties involved (Kurki-Suonio 2000). From this literature we build, in particular, on the work by Mnookin and Kornhauser (1979). Their study argues that the primary role of divorce law is to provide the framework within which divorcing parents are allowed to reach an agreement over, among other things, the allocation of custodial rights through private negotiations (in the shadow of the law). Our model takes this notion of private ordering one step further, by considering custody negotiations between partners well before divorce becomes an issue.

Economists, in contrast, have largely neglected the study of child custody.<sup>9</sup> One exception is given by the study of Weiss and Willis (1985). Both our and their models consider an intact family, in which the allocation of child custodial rights upon divorce are determined upfront, that is, when the parents (and children) are still living together. In our model, however, the allocation of custodial rights have efficiency consequences *within* marriage, and the optimal rule is the one which induces the smallest amount of inefficiency. This is because our model is based on an incomplete-contract framework. Conversely, in the Weiss-Willis model, custodial rights have no efficiency consequences because

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<sup>9</sup>In most of the existing empirical studies, this neglect is partly justified by the fact that maternal legal and physical custody is still the predominant arrangement among divorcing couples (see, among others, Del Boca and Flinn [1995] and Garfinkel et al. [1998]). Joint (or shared) custody, however, has increasingly become more common. For example, in Britain and the United States, at the end of the twentieth century, joint custody accounts for about one-quarter of post-divorce living arrangements for children. Less than 10 percent of cases are awarded to the father, while the remaining two-thirds are still awarded to the mother. At the beginning of the 1970s, joint custody was virtually inexistent, with at least 90 percent of cases awarded to the mother.

partners are assumed to be able to write complete contracts on intrahousehold allocations of resources.<sup>10</sup> Another point of departure is that parents' investments in the child during marriage and negotiations over the marital surplus in the shadow of divorce play a key role in our analysis. But they do not in the Weiss-Willis model, which instead focuses on the set of Pareto-efficient allocations within marriage, and on the sets of privately optimal (but inefficient) allocations in the case of divorce.

Another exception is the paper by Rasul (2003), who has independently developed a model of child custody that shares some of the basic ideas of this paper. There are, however, three important differences between this paper and Rasul's. First, in this paper, we emphasize that parental altruism may vary depending on whether a parent is (or is not) allocated custody to. Such a link will turn out to be of primary importance for some of our results. Second, this paper explores in greater detail the effect of parental technology in producing child quality on the optimal custody rule. Parents' differential abilities in producing child quality will be another critical determinant of our optimal rule. While Rasul (2003) finds that the parent who values the benefits from child quality more should be allocated either sole custody or a higher share of joint custody (thus extending the results of Besley and Ghatak [2001] to the analysis of child custody), our model offers a more complex set of results. In our world, in fact, sole custody or greater shares of joint custody can be allocated either to the high-valuation parent or to the low-valuation parent depending on who the key investor is, as well as on the specific structure of the technology parents use to produce child quality. A third difference is that we specify a model in which divorce cannot occur *in equilibrium*. Rasul (2003) instead, in line with Weiss and Willis (1985), proposes a model in which divorce occurs with positive probability and is endogenous to the parents' investment decisions.<sup>11</sup> Given these differences and the different results that these two papers deliver, we view them as complementary to each other. Combining their insights in a unified framework would surely

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<sup>10</sup>In their model custodial rights matter, in the sense that they have efficiency consequences, but only *after* the partnership breaks down. Furthermore, divorce occurs with a probability that depends on the selected efficient intrahousehold allocation. Hence, the optimal custodial rule is the one which is efficient *overall*, internalizing the fact that divorce occurs with nonzero probability and an inefficient allocation is instead implemented.

<sup>11</sup>The possibility of divorce however comes at the cost of introducing a number of hard-to-test assumptions on the distribution of the gains from marriage and its properties (see, for instance, Assumptions A2-A5 in Rasul [2003]).

enhance our understanding of the allocation of custodial rights and is left for future research.<sup>12</sup>

As we mentioned above, our study draws from the large literature on incomplete contracting, which has typically examined the optimal allocation of property rights for physical assets that are private goods (see the comprehensive survey in Hart [1995]).<sup>13</sup> The analysis in Besley and Ghatak (2001), however, has extended that literature to the case of physical public goods (e.g., social services, investment projects, agricultural extensions, and microlending in developing countries). They show that if contracts are incomplete the ownership of such goods should lie with the party that values the benefit generated by them relatively more. This is true regardless of whether this party is the key investor and regardless of technological (dis)advantages that may characterise this agent. In their model, however, it is implicitly assumed that the value attached to the asset by each party is independent of who actually owns the asset. When interpreted in the context of the allocation of child custodial rights, this means that each parent's degree of altruism is the same regardless of which parent is allocated custody to. In contrast, we will explicitly allow for the possibility that the degree of parental altruism depends on which parent gets custody. Indeed, it may be the case that the custodial parent places a higher value on the child than the noncustodial parent does, because, for example, the latter suffers a loss of control over the allocative decisions of the former (Weiss and Willis 1985).

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<sup>12</sup>Brown and Flinn (2002) present a model of the relationship between parental investment decisions under a given divorce policy regime and outcomes realized by children. In their paper, divorce occurs endogenously (as in the Weiss-Willis and Rasul's models) and, as in our paper, parental investments can be specified *ex ante*. In contrast to our model, however, parents can only choose investments and not custody, even though custody arrangements can be endogenous to the parental investments themselves within marriage or to the choice of divorce.

<sup>13</sup>Exactly because of the private nature of the goods involved, the main results of this literature cannot be directly applied to the allocation of child custodial rights. The reason is that, if parents split up, the noncustodial parent would obtain *no* utility from the child after divorce. This is in contrast with the basic notion that children are public or collective consumption goods from the point of view of the mother and the father (Becker 1974), and continue to be seen as such even after divorce by both parents, despite the loss of proximity and contact faced by the noncustodial parent (Becker, Landes, and Michael 1977; Weiss and Willis 1985 and 1993).

### 3. THE MODEL

Consider a family that is comprised of a mother  $m$ , a father  $f$  and a child.<sup>14</sup> Our model is a three-stage game between  $m$  and  $f$ . In the background, besides the child, there is also another passive player, namely the legal institutions (e.g., courts), which are assumed to have the powers to enforce the terms of any (incomplete) contract that the couple might have voluntarily negotiated and agreed upon.

In this section and the next we illustrate the case in which only sole custody allocations (whereby one *or* the other parent is allocated legal and physical custody to) are possible.<sup>15</sup> We do this for two reasons. First, excluding joint custody makes the modeling easier and sharpens our insights into the economic forces driving custody allocation decisions. Second, and perhaps more importantly, the results that we obtain in a world in which there is only sole custody are interesting in their own right, and are likely to be of general interest within the latest developments of both the incomplete contracting literature and the economic literature on family decisions in the presence of transaction costs (Besley and Ghatak 2001; Lundberg and Pollak 2001; Rasul 2003). In section 5 we will extend this model by including the possibility of joint custody, and thus explore the circumstances under which joint custody gives rise to higher equilibrium marital surplus (and child welfare) than either of the two sole custodial allocations.

**3.1. Preferences and Technology.** The child utility (or welfare) depends on the levels of investments  $I_f$  and  $I_m$  made respectively by the father and mother, and on whether or not the parents remain married.<sup>16</sup> Specifically, the child utility is  $W(I_f, I_m)$  if the parents remain married, and  $w(I_f, I_m; i)$  if they divorce and  $i$  ( $i = f, m$ ) is allocated custody. The three functions  $W(\cdot)$ ,  $w(\cdot; f)$  and  $w(\cdot; m)$  (as functions of  $I_f$  and  $I_m$ ) are strictly concave, strictly increasing and satisfy the Inada endpoint conditions. Given the overwhelming stock of evidence

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<sup>14</sup>For simplicity and without loss of generality, the discussion will be carried out as though the couple produces only one child. Since splitting siblings after divorce has been resisted by courts in most countries, on the grounds that it entails further erosion of the original family unit (Heatherington 1993), the one-child assumption is an adequate first approximation.

<sup>15</sup>Although the law distinguishes between physical and legal custody, our treatment of custody here combines the two concepts.

<sup>16</sup>Such investments (in child quality or human capital) may involve parental time, effort and financial inputs. For the purposes of this paper such distinctions are generally unimportant, although in the interpretation of the results derived below, we sometimes may refer to one input rather than another.

according to which children fare better off when their parents remain married than when they divorce (McLanahan and Sandefur 1994; Havenman and Wolfe 1995), we assume that both the child's total utility and his/her marginal utility of each parent's investment is higher in the former regime. That is:

**Assumption 1.** *For each  $i = f, m$  and for any  $I_f \geq 0$  and  $I_m \geq 0$ ,  $W(I_f, I_m) > w(I_f, I_m; i)$  and  $W_j(I_f, I_m) > w_j(I_f, I_m; i)$  (where  $j = f, m$ ).<sup>17</sup>*

If parents remain married, the payoff to  $i$  is  $\mu_i W(I_f, I_m)$ , where  $\mu_i > 0$  is a parameter that represents the degree of altruism (or value) that  $i$  attaches to child welfare in the regime where the child is brought up in an intact household. However, if they divorce and  $i$  has custody, then the payoff to  $j$  is  $\lambda_j(i)w(I_f, I_m; i)$ , where  $\lambda_j(i) > 0$  is a parameter that represents the degree of altruism that  $j$  attaches to child welfare in the regime where a separation has occurred and  $i$  has custody. We therefore allow for the possibility that a parent's degree of altruism towards the child varies across the three possible marital states.

Marriage may be seen in part as a relationship of trust and proximity (Becker 1991; Dasgupta 1988), while in the divorce state, there is no available mechanism which will induce the custodial parent to internalise the effects of his/her actions on the absent parent (Weiss and Willis 1985). This reasoning underpins the assumption that parents' degree of altruism is higher in the regime when they are married, that is:

**Assumption 2.**  $\mu_i > \lambda_i(j)$  for all  $i, j = f, m$ .

Besides the child-preference parameters denoted by  $\mu_f, \mu_m, \lambda_f(f), \lambda_m(f), \lambda_f(m)$  and  $\lambda_m(m)$ , the other core parameters of the model are embedded in the technology that parents use to invest in their child. These are captured by the productivities of the parental investments and are given by the properties of the functions  $W(\cdot)$ ,  $w(\cdot; f)$  and  $w(\cdot; m)$ , as functions of the parents' investments. Our main objective is to analyze the impact of these parameters on the optimal custody rule. To do so parsimoniously, in what follows we abstract from other elements that may be relevant and play a part in custody decisions, e.g. private consumption and parental income.<sup>18</sup>

<sup>17</sup>The terms  $W_j$  and  $w_j$  respectively denote the first-order derivatives of  $W$  and  $w$  with respect to  $I_j$ .

<sup>18</sup>The model developed by Weiss and Willis (1985) does consider parents' incomes, but these are taken as exogenous to the key parental choices, namely personal consumption and child expenditures. A fully strategic representation of family

**3.2. The Strategic Environment.** At date 0, parents are married and agree upon a contract that determines the parent who is to have custody if they divorce (at date 2). The contract is costlessly enforceable by the court. At date 1, the father and the mother (who continue to be married) simultaneously and non-cooperatively choose the levels of investments on their child,  $I_f$  and  $I_m$ , respectively.<sup>19</sup> Such investments are sunk at this date. The cost to  $i$  ( $i = f, m$ ) of investing  $I_i$  equals  $k_i I_i$ , where  $k_i > 0$  denotes  $i$ 's constant marginal cost of investment.

For any arbitrary set of choices made at dates 0 and 1, it follows from Assumptions 1 and 2, that

$$(\mu_f + \mu_m)W(I_f, I_m) > [\lambda_f(i) + \lambda_m(i)]w(I_f, I_m; i).$$

The left-hand side of this inequality is the size of the marital surplus that the couple can generate by remaining married, while the right-hand side is the sum of their divorce payoffs if at date 0 it is decided that  $i$  will have custody upon divorce. Although it is efficient for the mother and father to remain married, they will need to reach an agreement over the partition of this marital surplus before they are in a position to generate and consume it. Thus, at date 2 the couple engages in bilateral negotiations. If agreement is reached with the father making a transfer  $T$  to the mother, then the utility payoffs to the father and the mother are respectively:<sup>20</sup>

$$\begin{aligned} V_f &= \mu_f W(I_f, I_m) - T \\ V_m &= \mu_m W(I_f, I_m) + T. \end{aligned}$$

It should be noticed that, although (given Assumptions 1 and 2) efficient bargaining implies that in equilibrium the parents will remain married, the size of the marital surplus depends on the equilibrium investments made at date 1. The latter, it will be shown below, depend on the equilibrium distribution of the surplus between the couple, which, in turn, depends on the parents' relative bargaining powers that are in general influenced by their respective divorce payoffs. To describe the outcome of the negotiations at date 2, we adopt the Nash Bargaining

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interactions would require married partners to choose their optimal labour supplies (Chiappori 1992), which in turn leads to endogenous parents' incomes. This extension is left for future research.

<sup>19</sup>Since these investments are "non-contractable" — because, for example, they can be verified neither by the court nor by the other partner — this necessarily implies that they will be chosen "non-cooperatively".

<sup>20</sup>If  $T$  is positive the transfer is from the father to the mother, but if  $T$  is negative then the opposite is true.

Solution (NBS), in which the threat point is given by divorce.<sup>21</sup> We assume that the above three-stage game is one with *complete information*. In particular, the values of all the parameters (such as each parent's degrees of altruism in the three regimes) are common knowledge between the two partners.

**3.3. Equilibrium Bargaining Outcome.** Before addressing the question as to which of the two parents should be allocated custody at date 0, we need to derive the equilibrium implications of each of the two custody arrangements (i.e., paternal custody and maternal custody). At date 0, the two partners agree that if they divorce at date 2 custody will be allocated to  $i$ , where  $i = f$  or  $i = m$ . At date 1 the mother's investment choice is  $I_m$  (where  $I_m \geq 0$ ) and the father's investment choice is  $I_f$  (where  $I_f \geq 0$ ). With the NBS, the utility payoffs to the parents can be described by the *split-the-difference* principle:<sup>22</sup> each parent's Nash-bargained utility payoff equals her/his divorce payoff *plus* one-half of the marital surplus. Hence, after simplifying and rearranging terms, it follows that the Nash-bargained utility payoffs to

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<sup>21</sup>Muthoo (1999) discusses the strategic non-cooperative foundations of the NBS, and, using a number of versions of Rubinstein's alternating-offers model, shows when and how to use this bargaining solution concept. In the context of marital bargaining situations, there are circumstances under which the divorce payoffs (i.e., the parties' outside options) can be identified with the threat point in the NBS, as we assume in this paper; for example, when the risk of divorce occurring is sufficiently large. In other circumstances — when such a risk is sufficiently small — the divorce payoffs should not be identified with the threat point, but instead affect the bargaining outcome in the manner described by the so-called *outside option principle* (see Muthoo [1999, chapter 5]). It may be noted that in such circumstances the threat point would be more convincingly identified with the payoffs obtained from non-cooperation within marriage, along the lines modelled in Lundberg and Pollak (1993). This is another interesting extension which we leave for future analysis.

<sup>22</sup>We adopt the symmetric NBS which leads the parties to split equally the net marital surplus. This, however, does not imply that husband and wife will have equal bargaining powers because such powers depend crucially also on their respective divorce payoffs. Adopting an asymmetric NBS would provide us with an additional parameter that captures sources of bargaining power that are not already embodied into the disagreement payoffs. But doing so would not alter our main conclusions.



the father and the mother are respectively:

$$(1) \quad V_f^N(i) = \frac{1}{2} \left[ (\mu_f + \mu_m)W(I_f, I_m) + [\lambda_f(i) - \lambda_m(i)]w(I_f, I_m; i) \right]$$

$$(2) \quad V_m^N(i) = \frac{1}{2} \left[ (\mu_f + \mu_m)W(I_f, I_m) + [\lambda_m(i) - \lambda_f(i)]w(I_f, I_m; i) \right].$$

Notice that if  $\lambda_m(i) = \lambda_f(i)$  then the parents' payoffs are identical, i.e., they split equally the marital surplus. However, if the parents' degrees of altruism towards the child differ (in the hypothesized regime that custody upon divorce would be allocated to  $i$ ), then the parent with the relatively higher degree of altruism obtains more than one-half of the marital surplus, and hence, has a relatively larger payoff. Perhaps not surprisingly, this indicates that the difference between the parents' bargaining powers is proportional to the difference between their respective degrees of altruism. Interestingly, the parents' degrees of altruism when they remain married ( $\mu_f$  and  $\mu_m$ ) have no impact on their respective bargaining powers. These only play a role in determining the size of the marital surplus. However, the Nash-bargained transfer (which underlies the utility payoffs) will depend on the relative magnitudes of  $\mu_f$  and  $\mu_m$ . In fact, it is easy to show that the transfer is given by

$$(3) \quad T^N = \frac{1}{2} \left[ (\mu_f - \mu_m)W(I_f, I_m) - [\lambda_f(i) - \lambda_m(i)]w(I_f, I_m; i) \right].$$

The following example illustrates the role of parents' degrees of altruism on the bargaining outcome. Suppose that  $i = f$ ,  $\lambda_f(f) > \lambda_m(f)$  and  $\mu_m > \mu_f = 0$ . That is, the father is to be allocated custody upon divorce, and his degree of altruism in that regime exceeds that of his wife, but the exact opposite is the case in the regime where they are married. Under these assumptions, the father has more bargaining power than the mother, and hence obtains more than one-half of the marital surplus. This is so, even though he makes no contribution to that surplus ( $\mu_f = 0$ ). In this case,  $T_N < 0$ , that is, the mother makes a transfer to the father in the Nash bargain. Now change the above example by assuming that  $\mu_f > \mu_m = 0$ . In this case, while the father continues to obtain more than one-half of the surplus, it is possible that he has to make a transfer to the mother. This is because (since  $\mu_m = 0$ ) she receives no direct utility, and hence, in order to strike a bargain, she will have to receive some transfer from her husband.

**3.4. Equilibrium Underinvestments.** Having characterized the outcome of the negotiations at date 2 over the partition of the marital surplus (for an arbitrary set of decisions made at dates 0 and 1), we now proceed in the standard backward induction fashion and characterize the equilibrium investment levels chosen by the parents at date 1. We begin by characterizing the *first-best* investment levels.

The first-best (or Pareto-efficient) investment levels  $I^e = (I_f^e, I_m^e)$  maximize the difference between the marital surplus and the total cost of the investments,  $(\mu_f + \mu_m)W(I_f, I_m) - k_f I_f - k_m I_m$ . As such,  $I^e = (I_f^e, I_m^e)$  is the unique solution to the following first-order conditions:

$$(4) \quad (\mu_f + \mu_m)W_f(I_f, I_m) = k_f$$

$$(5) \quad (\mu_f + \mu_m)W_m(I_f, I_m) = k_m.$$

The left-hand sides of equations (4) and (5) are respectively the social (or aggregate) marginal benefits of the father's and the mother's investments, while the right-hand sides denote their respective private (which are identical to the social) marginal costs.

The Nash equilibrium investment levels  $I_f^i$  and  $I_m^i$  chosen by the father and the mother (given the assumption that  $i$  will have custody upon divorce) are the unique solutions to the following first-order conditions:

$$(6) \quad \frac{(\mu_f + \mu_m)W_f(I_f, I_m)}{2} + \frac{[\lambda_f(i) - \lambda_m(i)]w_f(I_f, I_m; i)}{2} = k_f$$

$$(7) \quad \frac{(\mu_f + \mu_m)W_m(I_f, I_m)}{2} + \frac{[\lambda_m(i) - \lambda_f(i)]w_m(I_f, I_m; i)}{2} = k_m.$$

The left-hand sides of equations (6) and (7) are respectively the private marginal benefits of the father's and the mother's investments, while the right-hand sides denote their respective private marginal costs. Under Assumptions 1 and 2, the social marginal benefit from each parent's investment strictly exceeds her/his private marginal benefit. It then follows that the parents underinvest relative to their respective first-best investment levels:

**Lemma 1.** *Regardless of which parent receives custody of the child, the equilibrium investment levels chosen at date 1 are strictly less than the (corresponding) first-best levels.*

This underinvestment result comes about because neither parent is able to obtain — in the bargaining equilibrium — the full, social (or aggregate) marginal benefit from her/his respective investment. Each parent is “held-up” *ex post*, after investments are sunk, by the other parent. This hold-up problem arises in many other contexts (see Hart

[1995] and the references therein). It is precisely for this phenomenon that the allocation of custodial rights has efficiency consequences.

#### 4. THE OPTIMAL SOLE CUSTODY RULE

We now analyze the issue of what comprises the optimal custody rule.<sup>23</sup> By *optimal* we mean the rule that maximizes the net surplus, which for an arbitrary pair of investment levels  $(I_f, I_m)$  is given by<sup>24</sup>

$$(8) \quad S(I_f, I_m) \equiv (\mu_f + \mu_m)W(I_f, I_m) - k_f I_f - k_m I_m.$$

The first-best net surplus — i.e., the net surplus with the first-best levels of investments — is  $S^e \equiv S(I_f^e, I_m^e)$ , where the first-best investment levels are characterized in subsection 3.4. In the same subsection, we also characterized the equilibrium investment pair  $(I_f^i, I_m^i)$  that yields the equilibrium net surplus under  $i$ -custody,  $S^i \equiv S(I_f^i, I_m^i)$ ,  $i = f, m$ . From Lemma 1 (and the assumption that  $W$  is concave) it follows that  $S^e > S^i$  ( $i = f, m$ ). If  $S^f > S^m$  it is optimal to allocate custody to the father, and if  $S^f < S^m$  it is optimal to allocate custody to the mother. If  $S^f = S^m$  it does not matter as to which parent is allocated custody. From (8) it is clear that the optimal custody rule will also provide relatively higher equilibrium child utility and equilibrium marital surplus, and relatively smaller distortions of the equilibrium investment levels from their respective first-best levels.<sup>25</sup> In this sense, therefore, the optimal custody rule is consistent with the best-interest-of-the-child principle (Goldstein, Freud, and Solnit 1979).

**4.1. The Case With A Sole Investor.** For simplicity, we first analyze the case in which only one of the two parents makes an investment decision at date 1. Without loss of generality, we assume that the sole investor is the father. This is equivalent to the assumption that the

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<sup>23</sup>We extend this analysis in section 5 by allowing for the possibility of joint custody as an alternative feasible option.

<sup>24</sup>In what follows, we say, for convenience, “ $i$ -custody” to indicate that child custody is allocated to parent  $i$  ( $i = f, m$ ).

<sup>25</sup>Notice that if each parent’s equilibrium investment level is higher under  $i$ -custody than under  $j$ -custody ( $i \neq j$ ), then  $S^i > S^j$ , and hence  $i$ -custody is unambiguously the optimal custody rule. In this case, therefore, the magnitudes of the equilibrium net surplus in the two possible custody regimes do not have to be computed and compared in order to determine which parent should be optimally allocated custody. But if one parent’s equilibrium investment is relatively higher under  $i$ -custody while the other parent’s equilibrium investment is relatively higher under  $j$ -custody ( $i \neq j$ ), then in order to determine which parent should receive custody we need to evaluate the relative magnitudes of the two equilibrium net surpluses.

mother's constant marginal cost of investment,  $k_m$ , is sufficiently large. This situation is applicable, for example, when parental investments involve significant monetary expenditures, and the father is the main (or the only) wage earner in the family. Clearly, even if the mother makes no monetary investments, her presence and control over the father's investment might be as relevant to the child utility as the father's investment itself. Indeed, the comparative importance (productivities) of the two parents for the generation of the returns from the father's investment is a key force determining the optimal custody rule.

4.1.1. *Characterization and Preliminary Results.* Because the father is the only investor, we focus solely on the father's investment incentives.<sup>26</sup> It follows immediately after differentiating the expression in (1) with respect to  $I_f$  that:

$$(9) \quad \frac{\partial V_f^N(f)}{\partial I_f} - \frac{\partial V_f^N(m)}{\partial I_f} = \frac{1}{2}[\lambda_f(f) - \lambda_m(f)]w_f(\cdot; f) - \frac{1}{2}[\lambda_f(m) - \lambda_m(m)]w_f(\cdot; m).$$

If the right-hand side of (9) is strictly positive, then it is optimal to allocate custody to the father. But if the right-hand side of (9) is strictly negative, then it is optimal to allocate custody to the mother.<sup>27</sup> The sign of the right-hand side of (9) depends on: (i) the parents' respective degrees of altruism towards their children upon divorce, and (ii) the child's marginal utilities of the father's (the sole investor) investment when the father is allocated custody and when the mother is allocated custody.<sup>28</sup>

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<sup>26</sup>From (8) maximizing the net surplus when only the father invests is equivalent to minimizing the distortion of the father's equilibrium investment from his first-best level. Given Lemma 1 and the assumption that  $W$  is concave, this means maximizing his investment incentives (or equilibrium marginal returns to his investment).

<sup>27</sup>If the right-hand side of (9) is zero, then it does not matter whether it is the father or the mother who receives custody. In this case, custody could be determined by some other rule that departs from the "winner-take-all" principle illustrated in this section. Examples of such compromise solutions are discussed, among others, by Elster (1989), and include random selection of the custodial parent and joint custody. We shall return to the joint custody solution in section 5.

<sup>28</sup>If custody is awarded to the mother, the father will continue to invest in the child not only because he is the only parent with an income (or the mother's income is too low), but also because courts (or other legal institutions) will guarantee the implementation of the terms of divorce. In this context, we abstract away from problems of enforcement (such as compliance with child support orders), which have

Optimal custody is independent of the parents' respective degrees of altruism when they are married,  $\mu_f$  and  $\mu_m$ . This conclusion would be obvious if child custody were to be determined *ex post* after divorce, because  $\mu_f$  and  $\mu_m$  would no longer be relevant. However, this result is obtained in the context of a model in which custody is established *ex ante*, in a prenuptial contract, when the values of both  $\mu_f$  and  $\mu_m$  are still potentially relevant. Indeed, here the optimal custody rule provides the sole investor (father) with the best incentives to invest while the parents live together. Even from that *ex-ante* standpoint, therefore, the determination of child custody should only be influenced by the parents' degrees of altruism in the two divorce regimes.<sup>29</sup>

Another implication of (9) is that in order for custody to matter — by which we mean that custody should optimally be allocated to one parent rather than the other — it must not be the case that in each divorce regime the parents' degrees of altruism are identical. In fact, if the mother and father had the same degree of altruism, then they would have equal bargaining power regardless of whom is allocated custody to, and hence, the father's marginal returns from his investment (which determine his investment incentives) are the same whether he gets custody or not. We summarize these results in the following:

**Lemma 2.** *If joint custody is not a possibility, the optimal custody rule is independent of the parents' degrees of altruism when they are married. Custody matters only if in at least one of the two divorce regimes the parents' degrees of altruism differ from each other.*

4.1.2. *Parental Altruism is Independent of the Divorce Regime.* We now derive and discuss a set of results that concern the exact nature of optimal custody. They apply to the case in which each parent's degree of altruism is the *same* across the two divorce regimes.

**Proposition 1.** *Assume that for each  $i = f, m$ ,  $\lambda_i(f) = \lambda_i(m) = \lambda_i$ . (a) If the child's marginal utility of the father's investment is higher under  $f$ -custody than under  $m$ -custody, then it is optimal to allocate custody to the parent who has the relatively greater degree of altruism in the two divorce regimes.*

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been addressed in other studies (e.g., Weiss and Willis 1993; Seltzer, McLanahan, and Hanson 1998; Flinn 2000).

<sup>29</sup>It should be emphasized, however, that a different conclusion will be reached in section 5 when joint custody is explicitly modeled. In that world both  $\mu_f$  and  $\mu_m$  affect the optimal custody rule through the equilibrium value of the joint custody allocation.

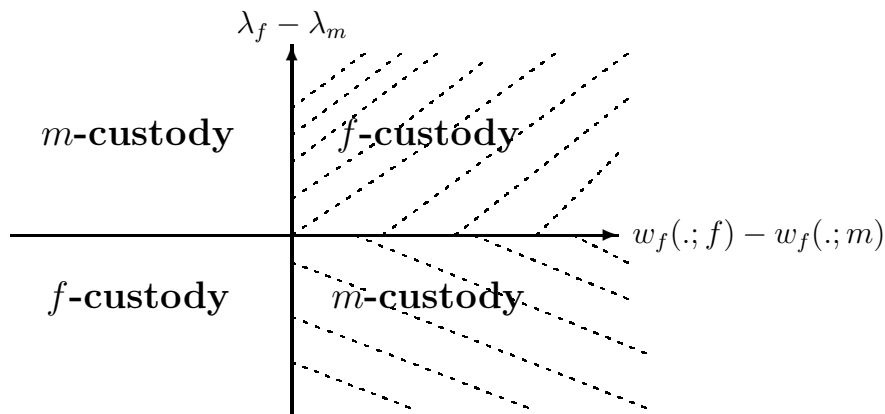


FIGURE 1. An illustration of Proposition 1.

(b) *If the child's marginal utility of the father's investment is higher under  $m$ -custody than under  $f$ -custody, then it is optimal to allocate custody to the parent who has the relatively smaller degree of altruism in the two divorce regimes.*

Figure 1 illustrates these results. It plots differences in parents' valuations during divorce,  $\lambda_f - \lambda_m$ , against differences in the relevant technological parameters,  $w_f(\cdot; f) - w_f(\cdot; m)$ . The figure clearly shows that, for a given technological structure, a slight variation in parental preferences may deliver opposite custody allocations in equilibrium. Similarly, for given parental valuations, small changes in technology may imply different custody rules. We shall return to this point later in this section as well as in sections 5 and 6.

Contrary to what is implied by this proposition, casual observation and introspection may suggest that in order to provide the sole investor (father) with the best investment incentives, custody should be allocated to him. Indeed, such intuition lies at the heart of the incomplete contracting literature. But the assets considered in that literature are private goods, while the asset in our model (i.e., the child) is a public consumption good. In an environment in which agents deal with private goods, the non-owning party gets no benefit from the asset when the party who owns the asset chooses not to cooperate. In contrast, in the case of children, the noncustodial parent's utility is likely to be always affected by the child's welfare (as assumed, for instance, by Del

Boca and Flinn [1995]), although perhaps in a somewhat reduced way because of the loss of proximity and control or less frequent contact (Weiss and Willis 1985).

As in Besley and Ghatak (2001) and Rasul (2003), Proposition 1(a) shows that custody should go to the parent who cares most about the child (the shaded first and fourth quadrants in Figure 1), even if the other parent is the sole investor (fourth quadrant). This conclusion is also consistent with the results of Weiss and Willis (1985). With Proposition 1(b) we reach however a different conclusion. The reason is because other studies derive their result under the assumption that the marginal utility of investment is higher for the sole investing party when it has ownership of the asset, which is the underlying hypothesis of Proposition 1(a). This assumption has some appeal in the context of the ownership of physical private (or public) assets. But it may not always be appropriate in the context of the allocation of child custody rights after divorce. In fact, Proposition 1(b) identifies the possibility that the father’s (financial) investment in the child is far more valuable if it is the mother who implements or has control over the investment itself (second quadrant of Figure 1). Put it differently, even if the father is the sole investor and the mother’s valuation of the child after divorce is lower than the father’s, it is optimal (in terms of generating utility benefits for the child) to assign custody to her if the father’s investment is more productive “in her hands” rather than his.<sup>30</sup>

Proposition 1 emphasizes the importance of both the two parents’ valuations of the child in the divorce regimes and the technologies that the parents use to produce child services as major determinants of custody allocations. The model does not point to the organizational configurations or institutions that are best positioned to provide the father with an incentive to continue to invest in the child (e.g., child support orders, presumptive guidelines, visitation rights). It does however identify the mechanism through which the optimal custody rule operates. The central force at work when there is only one investor (the

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<sup>30</sup>Interestingly, this line of reasoning gives, at least in part, a rational foundation to the Solomonic judgement in a well-known custody dispute (Elster 1989). To two women who both claimed custody of a child, King Solomon ruled to cut the disputed child in two, so that each woman could enjoy half of it. The woman who was willing to have the child cut in two revealed herself ineligible for custody, whereas the woman who was willing to give the child up (and, therefore, arguably the one who valued the child less) received the custody (1 Kings 3:16-28). Solomon reached this conclusion on the basis of the two women’s behaviour during the custody dispute, while in our model legal institutions do not play any role. Proposition 1(b) reaches a similar conclusion but hinges, instead, on the parents’ differences in post-divorce preferences and investment productivities.

father in our example) concerns his bargaining power *relative* to that of the mother during their negotiations. This determines his marginal returns on his investment, which, in turn, determine his investment incentives. Hence, given any configuration of parental degrees of altruism and the father's investment productivities across the two divorce regimes, the optimal custody rule aims to maximize the father's relative bargaining power. The consideration of bargaining powers between partners will turn out to be of primary importance also when we allow for the possibility of joint custody (see section 5).

4.1.3. *Parental Altruism is Sensitive to the Divorce Regime.* Proposition 1 examines the case in which each parent's degree of altruism is the same across both divorce regimes. We now analyze the consequences of relaxing this assumption. It is plausible, in fact, that such a parameter is regime specific (i.e., for each parent, it may differ depending on whether it is the father or the mother who has custody): for example, the father's valuation of the child can be greater when he has custody. The following proposition considers the case in which in each divorce regime the custodial parent's degree of altruism is no less than that of the noncustodial parent, *and* in at least one of these regimes it is strictly greater.

**Proposition 2.** *If  $\lambda_f(f) \geq \lambda_m(f)$  and  $\lambda_m(m) \geq \lambda_f(m)$ , with at least one of these inequalities being strict, then it is optimal to allocate custody to the father.*

In contrast to the result stated in Proposition 1, Proposition 2 implies that the optimal custody rule is independent of the technological parameters. That is, whether the father's investment is more productive when he has custody or when the mother has custody does not matter. The irrelevance of the parents' productivities should be contrasted with the results in the incomplete-contract literature, where such factors are the major determinant for an optimal allocation of property rights over physical assets. The reason for this difference lies once again in the differing nature of the assets under consideration (i.e., children versus private goods and services).

When taken in conjunction with Proposition 1, the result of Proposition 2 has a substantial implication. Suppose each parent's degree of altruism is virtually the same across both divorce regimes, with the mother's being slightly greater. That is,  $\lambda_f(f) = \lambda_f(m) = \alpha$  and  $\lambda_m(f) = \lambda_m(m) = \alpha + \epsilon$ , where  $\epsilon$  is positive and arbitrarily close to 0. Suppose also that the child's marginal utility of the father's investment is higher under  $f$ -custody than under  $m$ -custody. Proposition



1(a) is applicable, and implies that it is optimal to allocate custody to the mother. Now suppose, instead, that the mother's degree of altruism varies slightly across the two divorce regimes. So, while it is still the case that  $\lambda_m(m) = \alpha + \epsilon$ , assume that  $\lambda_m(f) = \alpha - \epsilon$ . Proposition 1 is no longer applicable. But Proposition 2 is, and implies that custody should be allocated to the father. Therefore, when parental valuations differ across divorce regimes, the optimal custody rule rests on such valuations as well as on the parent who invest in the child. As in Besley and Ghatak (2001), the allocation of custody in this case depends solely on the comparative altruism of the two parents. But, differently from previous models, Proposition 2 underlines also the importance of which parent makes the investment, although the (differential) efficiencies with which this investment is converted into child welfare are inconsequential.<sup>31</sup>

Our next result considers the case in which one parent's productivity is more important than the other parent's for generating the returns on the father's investment, namely:

**Proposition 3.** *Assume that at least one parent's degree of altruism varies across the two divorce regimes.*

(a) *If the child's marginal utility of the father's investment under m-custody is sufficiently larger than under f-custody, then it is optimal to allocate custody to the parent who has the relatively smaller degree of altruism in the divorce regime when the mother has custody.*

(b) *If the child's marginal utility of the father's investment under f-custody is sufficiently larger than under m-custody, then it is optimal to allocate custody to the parent who has the relatively greater degree of altruism in the divorce regime when the father has custody.*<sup>32</sup>

As with our previous results, the implications of this proposition can be appreciated in terms of the impact on the father's relative bargaining power. Suppose that the mother's productivity is greater than the father's in terms of generating the returns from his investment in the child. By making the mother the custodian, one maximizes the *aggregate* returns of any given level of investment. However, one ought to

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<sup>31</sup>That a small change in parents' degrees of altruism could lead to a substantial change in custody allocations may be of some concern. This is driven by the fact that joint custody is excluded. We shall return to this point in sections 5 and 6.

<sup>32</sup>It is worthwhile noticing that in Proposition 3(a) the parents' degrees of altruism in the divorce regime when the father has custody (i.e.,  $\lambda_f(f)$  and  $\lambda_m(f)$ ) are irrelevant, whereas  $\lambda_f(m)$  and  $\lambda_m(m)$  play no role in Proposition 3(b).

account for the level of investment that the father will choose in the first place. We need to ensure that the father obtains most of such returns. Proposition 3(a) shows that custody should be optimally allocated to the mother as long as her degree of altruism in that divorce regime is smaller than that of the father. This would provide the father with a relatively greater bargaining power.

Proposition 3, therefore, conflates the insights of all the results illustrated so far. That is, the optimal custody rule is underpinned by the comparative degrees of parental altruism, the differential investment productivities with which parents produce child welfare, and which parent invests in the child. This last ingredient is obviously relevant here because we have been concerned with the case of only one investor. We now extend our analysis and examine the case in which both parents can invest in their child at date 1.

**4.2. Optimal Custody When Both Parents Invest.** The analysis in this subsection concerns the case when parental altruism is independent of the divorce regime. This makes our analysis similar to that presented in 4.1.2 above.

The following expression, which is analogous to expression (9), is obtained by differentiating (2) with respect to  $I_m$ :

$$(10) \quad \frac{\partial V_m^N(f)}{\partial I_m} - \frac{\partial V_m^N(m)}{\partial I_m} = \frac{1}{2}[\lambda_m(f) - \lambda_f(f)]w_m(\cdot; f) - \frac{1}{2}[\lambda_m(m) - \lambda_f(m)]w_m(\cdot; m).$$

If both the right-hand side of (9) *and* the right-hand side of (10) are strictly positive, then it is optimal — in terms of providing both the father and the mother with best investment incentives — to allocate child custody to the father. If both the right-hand side of (9) *and* the right-hand side of (10) are strictly negative, then it is optimal to give the child to the mother. But if the signs of the right-hand sides of (9) and (10) differ — i.e., one is strictly positive and the other strictly negative — then the optimal custody allocations can be determined only by comparing the equilibrium net surpluses across the two divorce regimes. These allocations will be derived in section 5.<sup>33</sup> Here instead

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<sup>33</sup>Under, for example, the hypothesis on parental altruism of Proposition 2, the optimal custody allocation cannot be determined when both parents make investment decisions by analyzing expressions (9) and (10). In general, optimal custody will involve some *compromise* in the provision of investment incentives to the parents, in the sense that the parents will end up with less than their respective best investment incentives. To determine how this compromise looks like in equilibrium, we ought to examine the equilibrium levels of the net surpluses under each of the

we state the result that applies to the case when the degrees of parental altruism do not depend on the divorce regime:

**Proposition 4.** *Assume that for each  $i = f, m$ ,  $\lambda_i(f) = \lambda_i(m) = \lambda_i$ .*

*(a) If the child's marginal utility of a parent's investment is higher when custody is allocated to that parent, then it is optimal to allocate custody to the parent who has the relatively greater degree of altruism in the two divorce regimes.*

*(b) If the child's marginal utility of a parent's investment is higher when custody is allocated to the other parent, then it is optimal to allocate custody to the parent who has the relatively smaller degree of altruism in the two divorce regimes.*

This result generalizes the conclusions of Proposition 1 to the case when both parents invest in the child under the hypothesis that each parent's degree of altruism is the *same* across divorce regimes. Therefore, the parents' valuations of the child after separation and the technology that they use (while married) to generate child welfare are both equally important to determine the optimal custody allocation. In particular, in terms of preference parameters, there are conditions under which custody should optimally be allocated to the parent who values the child the most in the two divorce regimes, and other conditions under which the exact opposite conclusion holds (i.e., in equilibrium sole custody should be allocated to the parent who values the child the least in the two divorce regimes).

## 5. AN EXTENSION TO JOINT CUSTODY

In order to develop our main results so far, we excluded the possibility of joint custody from our analysis; our attention was restricted to the two sole custody allocations (in which one *or* the other parent is allocated custody). In this section we extend our model and explore the circumstances under which joint custody gives rise to a higher equilibrium marital surplus (and child welfare) than either of the two sole custodial allocations. This requires a slightly different model.

**5.1. The Extended Model: Notation and Assumptions.** As before, decisions are taken sequentially at three dates. At date 0 the parents agree and commit to the child custody allocation that would

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two possible sole custody regimes. This requires a slightly different and more articulated model than that developed so far. The analysis of that model is deferred until section 5.

be implemented if they divorce at date 2. Investments are undertaken (and sunk) at date 1, and negotiations over the marital surplus (in the shadow of divorce) occur at date 2. The new aspect of this extended model is that, at date 0, the set of feasible custodial allocations now include joint custodial allocations besides the two sole custodial allocations.

We denote by  $\pi$  ( $\pi \in [0, 1]$ ) the fraction of time spent by the child in the father's custody if the parents divorce at date 2; hence  $1 - \pi$  is the fraction of time spent in the mother's custody. By definition, the father's share is increasing in  $\pi$ , while mother's share is decreasing in  $\pi$ . This formulation allows for a continuum of possible custodial allocations ranging from the mother having sole custody ( $\pi = 0$ ) to the father having sole custody ( $\pi = 1$ ), with intermediate values of  $\pi$  defining the set of all possible joint custodial allocations. The optimal value of  $\pi$  (determined at date 0) is one which maximizes the equilibrium value of the net marital surplus and hence the equilibrium child welfare. We make three relatively mild assumptions which, while restricting to some extent the class of situations under study, will allow us to reach a few additional salient results about the optimal custody rule when joint custody is a feasible option. All three assumptions have been introduced for algebraic convenience and, perhaps with the exception of Assumption 4, could be easily relaxed without altering the gist of our arguments. The first assumption is that the constant marginal costs of investments to the father and the mother ( $k_f$  and  $k_m$ ) are identical, that is:

**Assumption 3.**  $k_f = k_m = k$ .

Our second assumption is that the child's (flow) utility payoff during the state in which the parents are divorced and the child is with parent  $i$  ( $i = f, m$ ) is some fraction of the child's (flow) utility payoff when the parents are married.<sup>34</sup> While limiting, this assumption simplifies the analysis of and discussion on the determination of the optimal value of  $\pi$ . Formally, we assume:

**Assumption 4.** For each  $i = f, m$  and for any  $I_f \geq 0$  and  $I_m \geq 0$ ,  $w(I_f, I_m; i) = \gamma_i W(I_f, I_m)$ , where  $0 < \gamma_i < 1$ .

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<sup>34</sup>In a framework in which custody can be shared by parents, it is instructive to interpret the players' payoffs as being "flow" payoffs (or payoffs per unit time). Thus, given an arbitrary value of  $\pi \in [0, 1]$ , the child's (aggregate) payoff if the parents divorce is  $\pi w(I_f, I_m; f) + (1 - \pi)w(I_f, I_m; m)$ .

The child utility function  $W$  could then be interpreted as a mapping that defines the transformation of parental investments into child quality and child utility when the parents remain married. Similarly, the parameter  $\gamma_i$  — which captures the properties of the technology that parents use in their investments — could be viewed as a measure of the importance of parent  $j$  ( $j \neq i$ ) in the transformation of parental investments into child utility when the parents are divorced and the child is with parent  $i$ . Clearly parent  $j$ 's importance is decreasing in  $\gamma_i$ . Our third assumption is the hypothesis underlying Proposition 2, namely, that in each sole custody regime the custodial parent's degree of altruism is no less than that of the non-custodial parent, *and* in at least one of these regimes it is strictly greater. That is:

**Assumption 5.**  $\Delta_f \equiv \lambda_f(f) - \lambda_m(f) \geq 0$  and  $\Delta_m \equiv \lambda_m(m) - \lambda_f(m) \geq 0$ , with at least one of these inequalities being strict.

**5.2. Equilibrium Bargaining Outcome and Investments.** Given an arbitrary value of  $\pi$  (determined at date 0) and an arbitrary pair of investment levels chosen at date 1),  $f$  and  $m$  bargain at date 2 over the marital surplus in the shadow of divorce. The (aggregate) payoffs to the father and the mother respectively from divorce are as follows (making use in particular of Assumption 4):

$$(11) \quad d_f \equiv \left[ \pi \lambda_f(f) \gamma_f + (1 - \pi) \lambda_f(m) \gamma_m \right] W(I_f, I_m)$$

$$(12) \quad d_m \equiv \left[ \pi \lambda_m(f) \gamma_f + (1 - \pi) \lambda_m(m) \gamma_m \right] W(I_f, I_m).$$

Assumptions 2 and 4 imply that there exist gains from staying married. Hence, applying the NBS in which the threat points are given by (11) and (12), we find that the Nash-bargained utility payoffs to the father and the mother are respectively

$$(13) \quad V_f^N = \Omega_f W(I_f, I_m) \quad \text{and} \quad V_m^N = \Omega_m W(I_f, I_m),$$

where

$$(14) \quad \Omega_f = \frac{1}{2} \left[ (\mu_f + \mu_m) + \pi \gamma_f \Delta_f - (1 - \pi) \gamma_m \Delta_m \right] \quad \text{and}$$

$$(15) \quad \Omega_m = \frac{1}{2} \left[ (\mu_f + \mu_m) - \pi \gamma_f \Delta_f + (1 - \pi) \gamma_m \Delta_m \right].$$

As expected, the sum of the Nash-bargained utility payoffs equals the value of the gross marital surplus  $(\mu_f + \mu_m)W(I_f, I_m)$ . Notice that  $i$ 's ( $i = f, m$ ) payoff is a fraction,  $\Omega_i/(\mu_f + \mu_m)$ , of that gross marital surplus. Thus,  $\Omega_i$  defines  $i$ 's bargaining power. By Assumption 5, the father's bargaining power  $\Omega_f$  is increasing in  $\pi$ , while the mother's bargaining power  $\Omega_m$  is decreasing in  $\pi$ .

We now characterize the equilibrium investment levels chosen at date 1. Given an arbitrary value of  $\pi$ , the equilibrium investment levels, denoted by  $I_f^*$  and  $I_m^*$ , comprise the unique solution to the following first-order conditions:

$$(16) \quad \Omega_f W_f(I_f, I_m) = k \quad \text{and} \quad \Omega_m W_m(I_f, I_m) = k.$$

It is straightforward to verify that the conclusion of Lemma 1 is robust to the inclusion of joint custody: for any  $\pi \in [0, 1]$ , the equilibrium investment levels will be strictly less than the corresponding first-best levels. However, different custodial allocations (i.e., different values of  $\pi$ ) will induce different equilibrium investment levels, and hence different equilibrium marital surplus and equilibrium child utility.

Totally differentiating the first-order conditions in (16), we find that

$$(17) \quad \frac{\partial I_f^*}{\partial \pi} = \frac{-1}{\Sigma} \left( \frac{\gamma_f \Delta_f + \gamma_m \Delta_m}{2} \right) \left[ \frac{W_{fm} W_m}{\Omega_m} + \frac{W_{mm} W_f}{\Omega_f} \right] \quad \text{and}$$

$$(18) \quad \frac{\partial I_m^*}{\partial \pi} = \frac{1}{\Sigma} \left( \frac{\gamma_f \Delta_f + \gamma_m \Delta_m}{2} \right) \left[ \frac{W_{fm} W_f}{\Omega_f} + \frac{W_{ff} W_m}{\Omega_m} \right],$$

where  $\Sigma = W_{ff} W_{mm} - (W_{fm})^2$  — with all the first-order and second-order partial derivatives evaluated at the equilibrium investment levels  $I_f^*$  and  $I_m^*$ . If the investments are weak substitutes ( $W_{fm} \leq 0$ ), then  $I_f^*$  is strictly increasing in  $\pi$  and  $I_m^*$  is strictly decreasing in  $\pi$ . But if the investments are complements ( $W_{fm} > 0$ ), then the effects of a marginal change in  $\pi$  on equilibrium investments are in general ambiguous. This is because a marginal change in  $\pi$  has two effects: a direct effect and an indirect (or strategic) effect. The direct effect, which is captured by the second terms in the square brackets in (17) and (18), entails that  $I_f^*$  increases in  $\pi$  and  $I_m^*$  decreases in  $\pi$ : a marginal increase in  $\pi$  increases  $\Omega_f$  and decreases  $\Omega_m$  leading  $f$  to increase his investment level and  $m$  to decrease her investment level. There is however also an indirect, strategic effect that is captured by the first terms in the square brackets in (17) and (18): if investments are substitutes then the indirect effects are in the same direction as the direct effects, but if they are complements then the indirect effects are in the opposite direction.

**5.3. Optimal Custody Allocations.** Moving backwards to date 0, we derive the value of  $\pi$  which maximizes the equilibrium net surplus  $S(I_f^*, I_m^*)$ , where  $S(\cdot, \cdot)$  is defined in (8). The equilibrium net surplus depends on the value of  $\pi$  indirectly, via its influence on the equilibrium investment levels. We write it as  $S^*(\pi)$ . So, the optimal custody allocation, denoted by  $\pi^*$ , is the value of  $\pi$  which maximizes  $S^*(\pi)$  over all  $\pi \in [0, 1]$ . If  $\pi^* = 1$  then  $f$ -custody is optimal, and if  $\pi^* = 0$  then  $m$ -custody is optimal. If instead  $0 < \pi^* < 1$  then joint custody is optimal, with  $\pi^*$  and  $1 - \pi^*$  being the shares in that joint custody allocated to the father and the mother respectively.

The following result is useful in developing our subsequent analysis of optimal custody allocations:

**Lemma 3.** *For any  $\pi \in [0, 1]$ ,*

$$(19) \quad \frac{\partial S^*(\pi)}{\partial \pi} \begin{matrix} \geq \\ < \end{matrix} 0 \iff W_{ff}(\Omega_f)^4 - W_{mm}(\Omega_m)^4 \begin{matrix} \geq \\ < \end{matrix} 0,$$

where  $W_{ff}$  and  $W_{mm}$  are evaluated at the equilibrium investment levels.

Without imposing any further restrictions on the child utility function  $W$ , it is evident from Lemma 3 that not much can be said about  $\pi^*$ . In what follows, therefore, we derive a number of results about  $\pi^*$  in the context of some additional parametric restrictions on  $W$ . We begin with the case in which the third partial derivatives of  $W$  are zero, which means that  $W_{ff}$  and  $W_{mm}$  are identical and equal some strictly negative constant, but we do not impose any restriction on the sign of  $W_{fm}$ .

**Proposition 5.** *If the third partial derivatives of  $W$  equal zero, then the optimal custody allocation is a joint custodial allocation, with the shares in that joint custody to the father and the mother being  $\pi^*$  and  $1 - \pi^*$  respectively, where*

$$\pi^* = \frac{\gamma_m \Delta_m}{\gamma_m \Delta_m + \gamma_f \Delta_f}.$$

While the hypothesis of this proposition restricts the class of applicable functions  $W$  (e.g., quadratic), its implications are powerful, as they reappear in the context of a larger class of utility functions (see below) and offer a significant benchmark. The optimal value  $\pi^*$  from Proposition 5 is the unique value of  $\pi$  such that  $\Omega_f = \Omega_m$ , i.e., it is the unique custody allocation that ensures that parents have *equal* bargaining powers. As in the case when only sole custody is possible (see

Propositions 1 and 4), this balancing out of bargaining powers is relevant, especially in relation to the provision of investment incentives to the two parties. Of course, the parents' equilibrium investments may differ as they also depend on the structure of the function  $W$ .

Turning to the optimal custody allocation itself, we note that it is increasing in  $\gamma_m/\gamma_f$ . This implies that the less important the father is in the transformation of parental investments into child utility when the parents are divorced and the child is with the mother, the larger should be his share in the optimal joint custody allocation. Although unappealing at first, this relationship is consistent with the idea that a key force underlying the determination of the optimal custody allocation concerns the tendency to equalize parental bargaining powers. This, in turn, would help induce an optimal compromise in the provision of investment incentives to the two parents.<sup>35</sup> Hence, an increase in  $\gamma_m$ , which increases the mother's bargaining power, is (partially) offset by an increase in  $\pi$ .

The dependence of  $\pi^*$  on the parental degrees of altruism also merits some discussion. If  $\Delta_f = 0$  ( $\Delta_m = 0$ ) then  $\pi^* = 1$  ( $\pi^* = 0$ ). Thus, if the degrees of parental altruism are identical when parents are divorced and the child is with parent  $i$ , it is optimal to allocate sole custody to parent  $i$  — even if parent  $j$  ( $j \neq i$ ) values the child significantly more when custody is solely allocated to parent  $j$ . Once again, this must be viewed within the logic of allocating child custody rights optimally so as to equalize or balance out parental bargaining powers. Therefore, the general result from Proposition 5 is that, regardless of productivity or technology considerations (as captured by  $W$ ,  $\gamma_f$  and  $\gamma_m$ ), the parent who values the child relatively more receives a relatively *lower* share in the optimal joint custodial allocation. Technology parameters however matter by determining the exact value of  $\pi^*$ .

We now examine another widely used class of functions of the Cobb-Douglas type, namely,  $W = A(I_f)^\eta(I_m)^\xi$ . These functions are strictly increasing in each of their two arguments, and are strictly concave. Furthermore, since  $W_{fm} > 0$ , the investments are complements. Our main results are summarized in the following:

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<sup>35</sup>On this point, the increasing labor supply of mothers, which can be seen as an indication of mothers' greater bargaining power (Blau 1998; Basu 2001), has been accompanied by the demise of maternal presumption in custody disputes and by increasingly important legal changes promoting fathers' custodial rights (Maccoby and Mnookin 1997).



**Proposition 6.** *Suppose that  $W = A(I_f)^\eta(I_m)^\xi$ , where  $A > 0$ ,  $0 < \eta < 1$ ,  $0 < \xi < 1$  and  $\eta + \xi < 1$ . Define*

$$\bar{\Delta}_f \equiv \frac{(1-\theta)(\mu_f + \mu_m)}{(1+\theta)\gamma_f} \quad \text{and} \quad \bar{\Delta}_m \equiv \frac{(\theta-1)(\mu_f + \mu_m)}{(\theta+1)\gamma_m},$$

$$\text{where } \theta = \sqrt{\frac{(1-\eta)\xi}{(1-\xi)\eta}}.$$

- (a) *If  $\xi < \eta$  and  $\Delta_f \leq \bar{\Delta}_f$ , then the optimal custody allocation is  $\pi^* = 1$  (sole custody to the father).*  
 (b) *If  $\xi > \eta$  and  $\Delta_m \leq \bar{\Delta}_m$ , then the optimal custody allocation is  $\pi^* = 0$  (sole custody to the mother).*  
 (c) *Otherwise — i.e., if either  $\xi = \eta$ , or  $\xi < \eta$  and  $\Delta_f > \bar{\Delta}_f$ , or  $\xi > \eta$  and  $\Delta_m > \bar{\Delta}_m$  — then the optimal custody allocation is a joint custodial allocation, with the shares in that joint custody to the father and the mother being  $\pi^*$  and  $1 - \pi^*$  respectively, where*

$$\pi^* = \frac{\gamma_m \Delta_m}{\gamma_m \Delta_m + \gamma_f \Delta_f} + \frac{(1-\theta)(\mu_f + \mu_m)}{(1+\theta)(\gamma_m \Delta_m + \gamma_f \Delta_f)}.$$

An immediate result of Proposition 6 is that the optimal custody rule is not independent of the parents' degrees of altruism when they are married,  $\mu_f$  and  $\mu_m$ . This is in stark contrast to the results of Lemma 2 and to those reported in Rasul (2003). Such parameters determine the critical bounds  $\bar{\Delta}_f$  and  $\bar{\Delta}_m$  as well as the equilibrium share of joint custody,  $\pi^*$ . If  $\xi < \eta$  (hence  $\theta < 1$ ), then  $\pi^*$  increases as  $\mu_f + \mu_m$  increases. So if the father's investment is more important than the mother's investment, his share of joint custody increases *ceteris paribus* with the size of the marital surplus generated when the couple is married. The opposite is true if  $\xi > \eta$ . Thus, parents' altruism and their investments during marriage do affect the optimal custody rule by affecting the size of the marital surplus. Differently from the case in section 4, the possibility of joint custody gives each parent certain bargaining endowments in the shadow of the law, which are related to the parents' degrees of altruism while married.

Consider now the case in which  $\eta = \xi$ , that is, the function  $W$  is symmetric in parental investments. Proposition 6(c) implies that the optimal custody allocation is identical to the allocation identified in Proposition 5 (indeed, if  $\eta = \xi$ , then  $\theta = 1$  and the second term in  $\pi^*$  disappears). The earlier discussion on Proposition 5 therefore applies

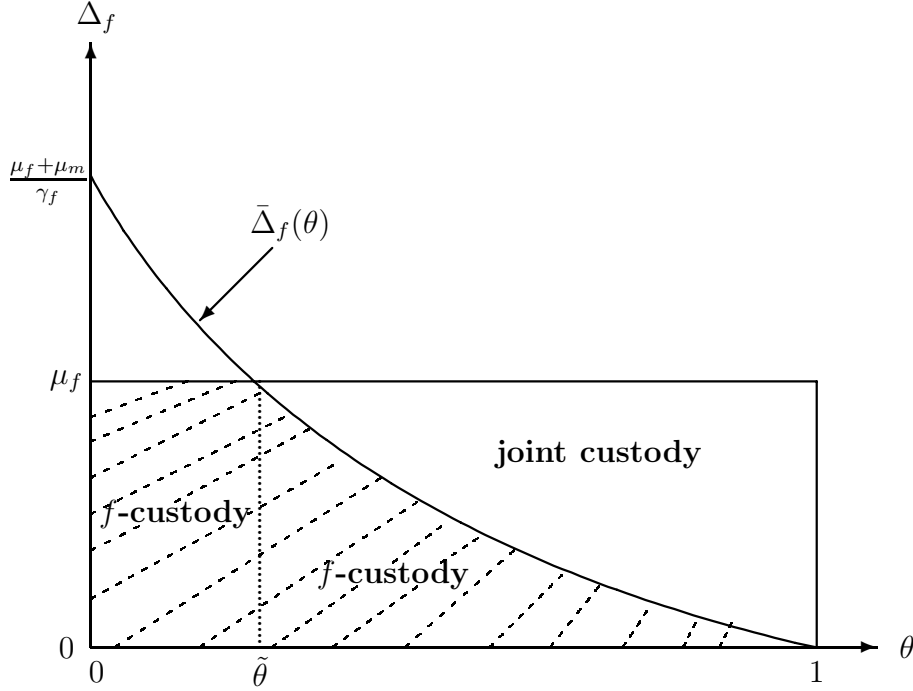


FIGURE 2. An illustration of Proposition 6 for the case when  $\xi < \eta$ .

to this case as well, even though the third partial derivatives of  $W$  here are not zero.<sup>36</sup>

Figures 2 and 3 illustrate the main insights of Proposition 6 when  $\xi < \eta$  and  $\xi > \eta$ , respectively. We focus our discussion on Figure 2 (opposite points hold for the symmetric case of Figure 3). Since  $\xi < \eta$ ,  $\theta$  must be less than one, while Assumption 2 implies that  $\Delta_f < \mu_f$  (by that assumption,  $\lambda_f(f)$  is bounded above by  $\mu_{f_2}$ , while  $\lambda_m(f)$  is by definition bounded below by 0). Figure 2 plots  $\bar{\Delta}_f$  as a function of  $\theta$ , which divides the  $(\theta, \Delta_f)$  space into two regions. It is easily checked that  $\bar{\Delta}_f$  is strictly decreasing and strictly convex in  $\theta$ , and that  $\bar{\Delta}_f(0) = (\mu_f + \mu_m)/\gamma_f$  and  $\bar{\Delta}_f(1) = 0$ . Because  $\bar{\Delta}_f(0) > \mu_f$ , there exists a critical, strictly positive value of  $\theta$  (namely,  $\tilde{\theta} = [(1 - \gamma_f)\mu_f + \mu_m]/[(1 + \gamma_f)\mu_f + \mu_m]$ ) such that for any  $\theta < \tilde{\theta}$ , custody is always allocated to the father. So if the father's investment is substantially more important than the mother's investment, then it is optimal to

<sup>36</sup>Note that the efficiency parameter  $A$  has no impact on the optimal custodial allocation. This is because it has an identical impact on the parents' investment incentives.

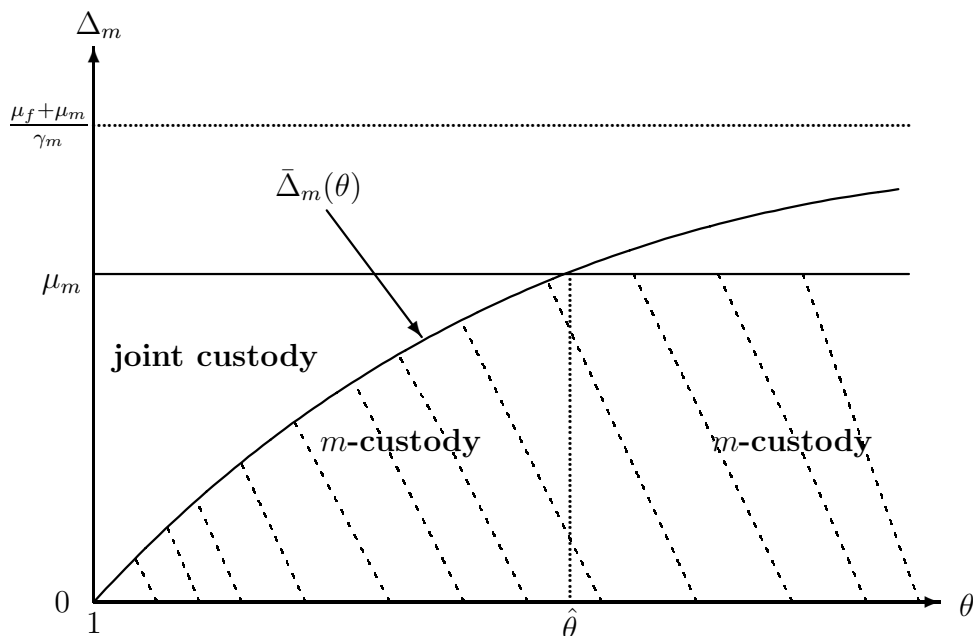


FIGURE 3. An illustration of Proposition 6 for the case when  $\xi > \eta$ .

give the father sole custody regardless of parents' preferences. This result generalizes Proposition 2, which applies when  $\theta = 0$  (that is, when the father is the sole investor). As  $\theta$  increases beyond  $\hat{\theta}$  (thus, the importance of the father's investment progressively decreases until it equals that of the mother's at  $\theta = 1$ ), there are two possible regimes of custody allocations, which do depend on parents' valuations. If  $\Delta_f \leq \bar{\Delta}_f$ , i.e., the father's degree of altruism is not too greater than the mother's when he has sole custody, then it is again optimal to allocate sole custody to the father (Proposition 6(a)).<sup>37</sup> But once  $\Delta_f$  exceeds  $\bar{\Delta}_f$ , the relatively greater bargaining power of the father must be offset by allocating some positive share of custody to the mother (and in this case, both  $\Delta_f$  and  $\Delta_m$  are relevant to the determination of  $\pi^*$ ). Joint custody (the non-shaded areas in Figures 2 and 3 below  $\mu_f$  and  $\mu_m$  respectively) arises in equilibrium precisely because it maintains a balance in the provision of investment incentives between the two parents (Proposition 6(c)).

As  $\theta$  approaches 1 (from below), the parents' valuations contained in  $\Delta_f$  are such that the region of  $f$ -custody gradually vanishes. At

<sup>37</sup>Interestingly, this allocation is independent of the difference in parents' valuations if custody is given to the mother,  $\Delta_m$ .

$\theta = 1$ , sole custody cannot arise in equilibrium. At that point in fact we return to the case in which  $\eta = \xi$ , with the optimal share of joint custody coinciding with that described by Proposition 5. As  $\theta$  increases beyond 1 ( $\xi > \eta$ ), Figure 3 shows how the symmetric case operates. At first there is a mixture of joint custody and  $m$ -custody. In this region, the exact optimal rule depends on the values of  $\Delta_m$  being greater or smaller than  $\bar{\Delta}_m$ . But for all values of  $\theta$  greater than the critical value  $\hat{\theta} = [(1 + \gamma_m)\mu_m + \mu_f]/[(1 - \gamma_m)\mu_m + \mu_f]$ , the optimal rule is  $m$ -custody.

The exact paths of the optimal custody rule crucially depend on the degree to which the father's investment is more or less important than the mother's (i.e., it depends on how large  $\xi$  is relative to  $\eta$ ), while in general the equilibrium value of  $\pi^*$  depends both on  $\Delta_f$  and  $\Delta_m$ , as well as on both  $\mu_f$  and  $\mu_m$ . Figure 4 shows, for given values of  $\Delta_f$  and  $\Delta_m$ , the relationship between  $\pi^*$  and  $\theta$ . It visually displays what we have just discussed. For values of  $\theta$  between 0 and  $\tilde{\theta}$ ,  $\pi^* = 1$ , that is, the optimal rule is  $f$ -custody. If  $\theta > \tilde{\theta}$ ,  $f$ -custody is still optimal, but only up to the point in which  $\theta = \theta_f$ , where  $\theta_f$  is such that  $\bar{\Delta}_f(\theta_f) = \Delta_f$ . Beyond  $\theta_f$ , joint custody is optimal, with the father's share monotonically declining in  $\theta$ . At  $\theta = 1$ ,  $W$  is symmetric in parental investments and joint custody is allocated according to the rule of Proposition 5. The share of father's custody continues to decline as  $\theta$  further increases, until it reaches  $\theta_m$ , where the optimal rule assigns sole custody to the mother, i.e.,  $\pi^* = 0$ . Figure 4 also illustrates the cases of a reduction in the relative degrees of parental altruism under  $f$ -custody ( $\Delta'_f < \Delta_f$ ) and under  $m$ -custody ( $\Delta'_m < \Delta_m$ ). For a given technology (that is, for a given value of  $\theta$ ), a reduction, say, in  $\Delta_f$  is associated with an increase of the region of  $f$ -custody and an increase in  $\pi^*$ . Such changes are consistent with the notion of giving a greater custody responsibility (in the form of either greater chances of sole custody or greater shares of joint custody) to the parent with a relatively more important investment. This is exactly because this parent's valuation declines relative to the other parent's: the greater custody responsibility is meant to provide the higher-productivity party with sufficient incentives to invest in the child despite his/her relatively lower valuation. As both  $\Delta_f$  and  $\Delta_m$  continue to decline, the curve connecting  $\theta_f$  (at  $\pi^* = 1$  in Figure 4) to  $\theta_m$  (at  $\pi^* = 0$ ) shifts towards the vertical line at  $\theta = 1$  (although it will never reach this line because of Assumption 5). In the limit, as both  $\Delta_f$  and  $\Delta_m$  become arbitrarily small, the optimal custody rule is  $f$ -custody if  $\theta < 1$ ,  $m$ -custody if  $\theta > 1$ , and joint custody for  $\theta = 1$  (where  $\pi^*(1)$  depends on the ratio

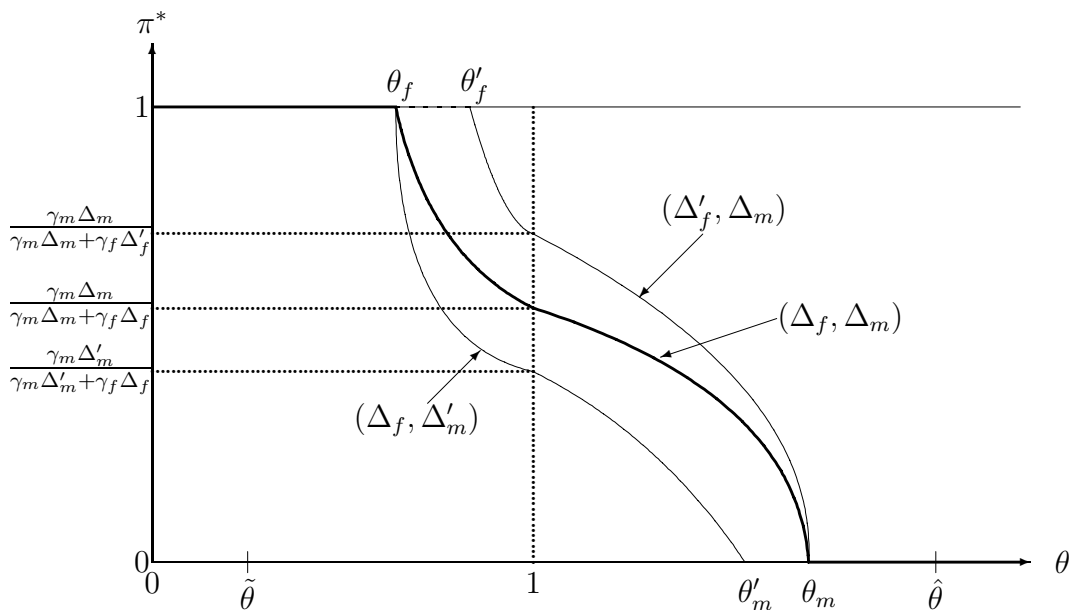


FIGURE 4. An illustration of Proposition 6 for three arbitrary and feasible pairs:  $(\Delta_f, \Delta_m)$ ,  $(\Delta'_f, \Delta_m)$  and  $(\Delta_f, \Delta'_m)$ , where  $\Delta'_i < \Delta_i$  ( $i = f, m$ ). Notice that for each  $i = f, m$ ,  $\theta_i$  is the unique value of  $\theta$  such that  $\bar{\Delta}_i(\theta) = \Delta_i$  and  $\theta'_i$  is the unique value of  $\theta$  such that  $\bar{\Delta}_i(\theta) = \Delta'_i$ .

$\Delta_f/\Delta_m$ ). In this limiting case, therefore, Proposition 6 generalizes the results of Proposition 1.

In sum, the main message of Proposition 6 is that when one parent's investment is significantly more important than the other parent's *or* when differences in the degrees of parental altruism are sufficiently small, technological factors dominate the equilibrium allocation of custodial rights. In these cases, *sole* custody should optimally be allocated to the parent whose investment is relatively more important. However, when the importance of the parents' investments are sufficiently similar *and* differences in the degrees of parental altruism are large, *joint* custody allocations are optimal as long as both parents contribute to the production of child quality. Joint allocations are optimal also when the importance of the father's investment is equivalent to that of the mother's, regardless of the preference structure. For a given technology with which parents produce child quality (and welfare), the optimal share of joint custody is systematically pegged to the parents' comparative degrees of altruism in such a way that their respective

bargaining powers are weighed against each other (and thus their joint investment incentives are maximized). In these circumstances, their degrees of altruism while married matter too for the determination of the equilibrium share of joint custody.

We finally consider a simple parametric case in which investments are neither complements nor substitutes ( $W_{fm} = 0$ ). The results for this case, which are virtually identical to those discussed in Proposition 5, are formalized in the following:

**Proposition 7.** *Suppose  $W = 2\sqrt{\tau_f I_f} + 2\sqrt{\tau_m I_m}$ , where  $\tau_f > 0$  and  $\tau_m > 0$ . Then the optimal custody allocation is a joint custodial allocation, with the shares in that joint custody to the father and the mother being  $\pi^*$  and  $1 - \pi^*$  respectively, where*

$$\pi^* = \frac{\gamma_m \Delta_m}{\gamma_m \Delta_m + \gamma_f \Delta_f}.$$

The analysis in this section does not offer a general characterization of the optimal custody allocation when both parents invest in the child and joint custody is possible. Additional research on this issue remains to be done. However, in spite of the specific parametrizations of  $W$ , three insights of Propositions 5-7 are worth emphasizing. First, as in the world in which joint custody is excluded (section 4), both preference parameters and technological factors matter in determining the optimal custody rule. This ties in the findings of both strands of the incomplete contracting literature, that with private goods (Grossman and Hart 1986) as well as that with physical public goods (Besley and Ghatak 2001). Second, joint custody is not always optimal, which may help explain why sole custody is still the predominant arrangement among divorcing couples in many countries. Third, in equilibrium, child custody is allocated (jointly or otherwise) so as to balance out the bargaining powers of both parents and their investment incentives. Under the right technological conditions, this can be achieved by trading off greater child valuations with smaller shares of custody. This strategy is likely to be more appealing than that implied by Propositions 1-4, where in the absence of joint custody small perturbations in parental preferences are sufficient to lead to opposite custody allocations (see Figure 1).

## 6. DISCUSSION

**6.1. The Optimal Custody Rule in the Context of Child Custody Law.** Almost everywhere, legal design and judicial interpretation of child custody norms have historically changed according to social,

cultural and religious values and beliefs about the basic institutions of the family and of marriage (Elster 1989).<sup>38</sup> In particular, in most Western countries, there has been a shift from rules (e.g., paternal preference or maternal preference) to a discretionary principle based on the best interest of the child.

The absolute *paternal preference rule*, which dominated British legislation up to the end of the nineteenth century (and possibly well into the twentieth century), was based on the father's unconditional rights in all family issues (as an example, see the quote from Maidment [1984] at the outset of the paper). This implied that the court could not adjudicate a custody dispute in favor of the mother, even if an abusive father might lose his legal rights to child custody. In nineteenth-century America, instead, the paternal preference rule did not take hold, with courts generally awarding custody to either parent on the basis of a *fault-based presumption* — according to which children are best taken care of by the ‘innocent’ party, who, in the majority of cases, turned out to be the mother (Mnookin 1975). Besides other considerations, this principle was primarily motivated by a compensatory and retributive reasoning. In the twentieth century, a *maternal preference rule* gradually emerged as the dominant standard in most countries (see, for instance, the quote from Ellsworth and Levy [1969]). This rule was based on the presumption that children, and particularly children of tender ages, were “entitled to have such care, love, and discipline as only a good and devoted mother can usually give” (Ullman vs Ullman [1912], cited in Mnookin [1975, p. 235]). But by the beginning of the 1970s, most countries had replaced all such rules with the principle that custody ought to be decided according to what is *in the best interest of the child*.<sup>39</sup>

Custody allocations under the three early rules mentioned above (paternal, fault-based, and maternal presumption) are simple and precise in that they require a single factual determination to be implemented (e.g., whether one of the claimants is the biological father of the child, or a showing of fault on the part of a spouse).<sup>40</sup> The best-interest

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<sup>38</sup>Social and historical analyses of child custody law are, among others, in Maidment (1984) on English common law, Maccoby and Mnookin (1997) on the American development, and Pearl and Menski (1998) on Muslim family law.

<sup>39</sup>An analysis of the legislative reasons for the emergence of this principle is beyond the scope of this paper. On this issue, see Mnookin (1975), Maidment (1984), Elster (1989), Buehler and Gerard (1995).

<sup>40</sup>Elster (1989) discusses the possibility of justifying such rules on the grounds of the best interest of the child. They could also be interpreted within our optimal rule. For example, the conditions underlying either Proposition 1(a) or Proposition 6(a)

principle, instead, has been severely attacked by legal scholars essentially because it is indeterminate. That is, it gives substantial discretion to judges to make their own implicit predictions and impose their own value judgements about which parent might better serve the child's interests (Maidment 1984; Maccoby and Mnookin 1997). Because all custody rules create bargaining endowments for the parents (what Mnookin and Kornhauser called "bargaining chips" [1979, p. 968]), an important consequence of the indeterminacy of the best-interest principle is that parents are likely to engage in strategic behavior that affects not only resource allocations after divorce but also *pre-divorce* intrahousehold decisions (including the investments on the child). This therefore may have detrimental effects on child welfare if parents strategically underinvest or it may be inefficient if parents strategically overinvest.

The *optimal custody rule* that we have developed in sections 4 and 5 is embedded in an economic (optimizing) model of household behaviour.<sup>41</sup> As such, our rule is consistent with the best-interest principle, in that it maximizes child welfare by providing the best incentives to both parents to invest in their child even after divorce. But, while retaining this principle, our rule: (i) is underpinned by a well-defined set of behavioral parameters that govern family decisions (namely, parents' preferences and technologies), and (ii) relies on parents' private ordering and prenuptial agreements. To the extent that both (i) and (ii) can be gauged, verified and enforced, the optimal custody rule provides us with precise guidelines as to how custody allocations should be achieved. In this sense, the optimal rule is determined, or at least it is not confronted by the same high degree of indeterminacy that an otherwise unqualified rule based on the best-interest principle must face. In the next two subsections, we elaborate on some of the issues that lie behind points (i) and (ii).

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(i.e., the father's investment is decisively more important than the mother's and his degree of altruism is not less than the mother's) would be sufficient to give rise to a paternal presumption. Conversely, times in which the mother's valuation is greater than the father's and his investment is more important than hers (Proposition 1(a)) or times in which the mother's investment is perceived as being essential (Proposition 6(b)) would be characterized by a maternal presumption.

<sup>41</sup>The interpretation of the best-interest principle has been usually driven by hypotheses that, in the words of Elster (1989) "have sometimes acquired the status of 'legislative facts' " (p. 133). One of such facts rests on the psychoanalytic view that continuity of the parent-child relationship is crucial (Goldstein, Freud, and Solnit 1979). In practice, this amounts to award custody to the parent with whom the child is living (or has been living for some time) at the time of the dispute or to the parent who has the role of the primary caretaker or caregiver.



**6.2. Theory Ahead of Measurement Again?** As shown in sections 4 and 5, the optimal custody rule depends crucially on two sets of parameters, i.e., parents' degrees of altruism towards the child both before and after divorce (in different custody regimes) and productivities of the parents' investments in the child after divorce. Before having any judiciary function, these parameters need to be measured. Admittedly, it is hard to assess them for each mother and father (and not only for those who are going through a divorce), particularly because they refer to a state (divorce) and a custody regime ( $m$ -custody vs  $f$ -custody) that are, in most cases, hypothetical. Formal econometric analyses of family behavior may help identify such parameters, but at present only a limited number of studies provide us with the information we need.

In the case of the parental preference parameters, we refer to the work of Del Boca and Flinn (1995), Flinn (2000), and Del Boca and Ribero (2001). They all use data on separated and divorced individuals with one child, whose physical custody has been assigned to the mother ( $m$ -custody).<sup>42</sup> So, at best, they inform us only on  $\lambda_f(m)$  and  $\lambda_m(m)$ . In all these studies, parental preferences over private consumption and child consumption (or, as in Del Boca-Ribero, time spent with the child) are assumed to be represented by Cobb-Douglas utility functions, with each parent's parameter being contained in the open unit interval. Del Boca and Flinn (1995) can only recover the private consumption weight of the father ( $\lambda_f(m)$ ) but not that of the mother. Flinn (2000) estimates both  $\lambda_f(m)$  and  $\lambda_m(m)$ . His results reveal that the mean weight on child consumption is between 0.225 and 0.250 for fathers and between 0.077 and 0.249 for mothers (see his Table 3, p. 566), suggesting that divorced mothers tend to be less altruistic than divorced fathers (i.e.,  $\Delta_m \equiv \lambda_m(m) - \lambda_f(m) < 0$ ).<sup>43</sup> Finally, Del Boca and Ribero (2001) find that, on average, mothers value the time spent with their child much more highly than fathers (i.e.,  $\Delta_m > 0$ ): their estimates are 0.689 and 0.253 for custodial mothers and noncustodial fathers respectively (see their Table 2, p. 133). In sum, our knowledge about parental

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<sup>42</sup>If the evaluation of child welfare by a divorced parent is a function of the amount of time spent with the child, then all noncustodial parents will weight child's welfare less heavily than they would if they had custody of the child (see our Proposition 2, and Flinn and Del Boca [1995]). Therefore, the estimates reported in such studies can be seen as lower bounds of weights given by *all* parents on child utility.

<sup>43</sup>However, Flinn (2000) argues that, with his data, the identification of the mother's preference parameter distribution is weak, and its estimates should then be taken with caution.

preferences is limited. More research is clearly needed to establish how robust the existing estimates are, and, especially, to recover the *relative* concern for child welfare on the part of all parents (including those who are married).

Our knowledge is perhaps even more limited in the case of the technology parameters. Three points are in order. First, there are no estimates of the productivities of individual parental investments in children. Neoclassical growth models have been augmented with home production (see, among others, Parente *et al.* [2000]). But such models are uninformative about the parameters of interest here, because they do not identify child investments separately from either home or market investments and do not distinguish the father's investment from that of the mother (these are in fact representative-agent models). Second, we have some understanding of the relationships that link a number of parents' behaviors and characteristics (such as education, market work, family structure, and income) to various child outcomes, e.g., birth weight, schooling, economic activity, and teenage childbearing (see the comprehensive surveys by McLanahan and Sandefur [1994], Haveman and Wolfe [1995], Duncan and Brooks-Gunn [1997], and Mayer [1997]). Although these relationships are potentially relevant for the identification of our key technology parameters, they are not enough as they typically do not single out the contributions of fathers from those of mothers. In the studies that do (such as Lazear and Michael [1988] and Thomas [1990]), it is hard to separate parental investments from parental endowments, which include preferences.<sup>44</sup> Third, we also have some idea about the time that parents allocate to their children, and how that has changed over time. For example, Sandberg and Hofferth (2001) find that the time American children aged 3-12 spent with their parents (both fathers and mothers) did not decrease between 1981 and 1997, and in two-parent families it increased substantially.<sup>45</sup> Yet these findings say very little about how productive parental investments actually are and whether the parents' productivities differ depending on

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<sup>44</sup>In a related empirical paper, we show that a few parental activities (for instance, childcare and caring for ill children) are important determinants of actual custody allocations for a sample of divorced British families in the 1990s. Although father's custody is observed in only 13 percent of cases, fathers who have the main responsibility for childcare see their odds of getting custody increased by about 70 percent, and for those who are mainly responsible for the care of ill children, the odds more than double (Francesconi and Muthoo 2003).

<sup>45</sup>Gershuny (2000) reports similar results for about 20 other countries since the early 1960s up until the early 1990s.

whether it is the father or the mother or both parents who invest. Again, more empirical research on this issue is needed.

**6.3. Is There a Case for Prenuptial Contracts on Child Custody?** Our study has revolved around the notion that husbands and wives are empowered to create their own legally binding commitments in the shadow of the law as a form of private ordering (Mnookin and Kornhauser 1979).<sup>46</sup> This notion stresses the importance of prenuptial agreements on a number of aspects of the marital relationship, including child custody in case of divorce. Unfortunately, one difficulty with prenuptial contracts is that they are hard to enforce. This could be at least partially addressed by improving the clarity of the statutes and norms governing premarital law and custody adjudications.

Legal scholars and professionals however point to other, more fundamental complications posed by prenuptial contracts, which ought to be weighed against the advantages that we have illustrated so far (Nasheri 1998). We briefly discuss three of such problems. First, marriage is arguably a more intimate and complex relationship than a business partnership and, as noted by the Supreme Court of the United States, has “more to do with the morals and civilization of a people than any other institution” (Maynard vs Hill [1888, p. 205]). Although marriage in the United States and elsewhere does not operate today as it did in 1888 at the time of the Maynard decision, this is the reason why many countries are opposed to the idea of treating marriage as just another business contract and place it under the direct control of state courts and legislatures.<sup>47</sup> Second, small perturbations in the deep parameters that underlie the optimal custody rule described in section 4 may often result in dramatically different custody allocations. This problem, which is less severe in the world described in section 5, may be magnified by the fact that some of those parameters are not fully observed at the time of the custody decision (because they refer to future

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<sup>46</sup>In our model parents are able to contract only on custody allocations. Arguably these are easily verifiable by third parties (or courts). Parents however cannot legally bind themselves on child investment levels. Typically, such investments are complex requiring several hard-to-measure parental inputs, and thus they cannot be easily verified by third parties. Parent-child relationships are naturally characterized by unforeseeable and *indescribable contingencies* (Tirole 1999): this aspect provides an additional reason as to why parents cannot make binding commitments on their child investments.

<sup>47</sup>The fact that marital relationships are very personal matters may actually lead to the opposite conclusion, whereby it is natural to allow marriage to be treated as an issue of private agreements.

and potential custody regimes), and so parents may have a dangerous incentive for strategic behavior (Elster 1989).

Third, prenuptial contracts may be embroiled with unequal bargaining powers held by future spouses. This has been seen as a manifest form of abuse (Brod 1994), which can be most effectively prevented by having instead a uniform legislation that applies to marriage and divorce and provides all family members with greater economic and legal predictability. Perhaps more importantly, this inequality could also fail to reflect possible changes in the balance of power between partners during the course of their marriage and after its dissolution (Basu 2001). Such concerns seem to be even more cogent in the case of child custody, which, at present, has not yet been considered a matter that prenuptial contracts can cover.

## 7. CONCLUSIONS

The terms of divorce, in general, and the allocation of child custodial rights, in particular, do matter. They have not only distributive but also efficiency consequences within intact families, by influencing each parent's incentives to make relationship-specific investments, including investments in their children. An *optimal* allocation of child custody rights cannot disregard such considerations. In this paper we have formally explored the issue of child custody from this perspective. We show that the optimal custody rule is systematically related to the comparative degrees of parental altruism, the comparative parental productivities in generating the returns to the child from parents' investments, and to which parents actually invests in the child. In particular, four results deserve special mention. First, the optimal allocation of custodial rights depends on both preferences and technological factors, regardless of whether joint custody is ruled out or not. This reconciles the findings of the incomplete contracting literature with private goods (Grossman and Hart 1986) and those of the same literature with physical public goods (Besley and Ghatak 2001). Second, and in contrast to Besley and Ghatak (2001) and Rasul (2003), custodial rights can be allocated either to the parent who values the benefits from child quality more or, viceversa, to the parent with the lowest valuation. Custody can go to the low-valuation parent if this endows the parent with a greater bargaining power and this, in turn, induces him/her to provide optimal levels of child investment. Third, if one parent's investment is significantly more important than the other parent's investment, then sole custody is preferred to joint custody and it should be allocated to the parent whose investment is relatively more

important. In these circumstances, parents' preferences are irrelevant. Fourth, joint custody is optimal when the importance of the parents' investments are sufficiently similar and the differences in their valuations of child quality are sufficiently large. In these cases, a greater share of joint custody should go to the parent with the lowest valuation precisely because the parent with the highest valuation would invest in the child anyway, and the low-valuation parent would be endowed with greater bargaining power.

We have only scratched the surface of the several and complex matters that impinge on the allocation of child custody rights. More theoretical and empirical work needs to be done to improve our understanding of the optimal custody rule. One possible direction of the theoretical analysis is the formulation of a dynamic game in which parents (and children) interact repeatedly over time, learn about their investments and quality, and build up some trust. In that environment, marriage could move closer to first-best equilibria even in the presence of transaction costs. Likewise, endogenous divorce could arise naturally not only after a particularly bad marriage innovation but also as a result of a persistent deterioration of parental investments. In general, on the grounds that transaction costs affect spouses' decisions, an incomplete-contracts approach could be used to study many other family issues that are likely to be influenced by such costs, including family formation, fertility, household labor supply, retirement, and intergenerational links. Even few of these other extensions make up a rather full agenda for future research.

#### APPENDIX

##### **Proof of Proposition 1**

After substituting the hypothesis of this proposition — namely, that for each  $i = f, m$ ,  $\lambda_i(f) = \lambda_i(m)$ , which, in what follows, we denote simply by  $\lambda_i$  — into the right-hand side of (9), we obtain, after re-arranging terms, that:

$$(A.1) \quad \frac{\partial V_f^N(f)}{\partial I_f} - \frac{\partial V_f^N(m)}{\partial I_f} = \frac{1}{2}(\lambda_f - \lambda_m)[w_f(., f) - w_f(., m)].$$

The proposition follows almost immediately from an examination of this expression. Part (a) follows by noting that if, by the additional hypothesis of part (a),  $w_f(., f) > w_f(., m)$ , then the sign of the left-hand side of (A.1) is the same as the sign of the difference  $\lambda_f - \lambda_m$ ; which, in turn, means that the father's investment incentives are relatively higher under  $f$ -custody [ $m$ -custody] if  $\lambda_f > \lambda_m$  [if  $\lambda_f < \lambda_m$ ]. Part (b) follows by noting that if, by the additional hypothesis of part (b),  $w_f(., f) < w_f(., m)$ , then the sign of the left-hand side of (A.1) is the same as the sign of the difference  $\lambda_m - \lambda_f$ ;

which, in turn, means that the father's investment incentives are relatively higher under  $f$ -custody [ $m$ -custody] if  $\lambda_f < \lambda_m$  [if  $\lambda_f > \lambda_m$ ].

### Proof of Proposition 2

Given the hypothesis of this proposition, we can define  $\lambda_f(f) = \lambda_m(f) + \alpha$  and  $\lambda_m(m) = \lambda_f(m) + \beta$ , where  $\alpha \geq 0$ ,  $\beta \geq 0$  and  $\alpha + \beta > 0$ . After substituting for  $\lambda_f(f)$  and  $\lambda_m(m)$  in the right-hand side of (9), we obtain that:

$$(A.2) \quad \frac{\partial V_f^N(f)}{\partial I_f} - \frac{\partial V_f^N(m)}{\partial I_f} = \frac{1}{2}[\alpha w_f(:, f) + \beta w_f(:, m)].$$

The proposition follows immediately by noting that since the right-hand side of (A.2) is strictly positive, this implies that the father's investment incentives are relatively higher under  $f$ -custody.

### Proof of Proposition 3

If  $w_f(:, m)$  is sufficiently larger than  $w_f(:, f)$ , then the sign of the left-hand side of (9) is the same as the sign of the difference  $\lambda_m(m) - \lambda_f(m)$ . This means that  $f$ -custody [ $m$ -custody] is optimal if  $\lambda_f(m) < [>]\lambda_m(m)$ . This establishes part (a). If, on the other hand,  $w_f(:, f)$  is sufficiently larger than  $w_f(:, m)$ , then the sign of the left-hand side of (9) is the same as the sign of the difference  $\lambda_f(f) - \lambda_m(f)$ . This means that  $f$ -custody [ $m$ -custody] is optimal if  $\lambda_m(f) < [>]\lambda_f(f)$ , which establishes part (b).

### Proof of Proposition 4

After substituting the hypothesis of this proposition — namely, that for each  $i = f, m$ ,  $\lambda_i(f) = \lambda_i(m)$ , which, in what follows, we denote simply by  $\lambda_i$  — into the right-hand sides of (9) and (10), we obtain, after re-arranging terms, that:

$$(A.3) \quad \frac{\partial V_f^N(f)}{\partial I_f} - \frac{\partial V_f^N(m)}{\partial I_f} = \frac{1}{2}(\lambda_f - \lambda_m)[w_f(:, f) - w_f(:, m)],$$

and

$$(A.4) \quad \frac{\partial V_m^N(f)}{\partial I_m} - \frac{\partial V_m^N(m)}{\partial I_m} = \frac{1}{2}(\lambda_m - \lambda_f)[w_m(:, f) - w_m(:, m)].$$

The proposition follows almost immediately from an examination of these expressions. Part (a) follows by noting that if, by the additional hypothesis of part (a),  $w_f(:, f) > w_f(:, m)$  and  $w_m(:, m) > w_m(:, f)$ , then both the sign of the left-hand side of (A.3) and the sign of the left-hand side of (A.4) is the same as the sign of the difference  $\lambda_f - \lambda_m$ ; which, in turn, means that both the father's and the mother's investment incentives are relatively higher under  $f$ -custody [ $m$ -custody] if  $\lambda_f > \lambda_m$  [if  $\lambda_f < \lambda_m$ ]. Part (b) follows by noting that if, by the additional hypothesis of part (b),  $w_f(:, f) < w_f(:, m)$  and  $w_m(:, m) < w_m(:, f)$ , then the signs of the left-hand sides of (A.3) and (A.4) are the same as the sign of the difference  $\lambda_m - \lambda_f$ ; which, in turn,

means that both the father's and the mother's investment incentives are relatively higher under  $f$ -custody [ $m$ -custody] if  $\lambda_f < \lambda_m$  [if  $\lambda_f > \lambda_m$ ].

### Proof of Lemma 3

Differentiating  $S^*$  w.r.t.  $\pi$ , we obtain that:

$$\frac{\partial S^*}{\partial \pi} = (\mu_f + \mu_m) \left[ W_f \frac{\partial I_f^*}{\partial \pi} + W_m \frac{\partial I_m^*}{\partial \pi} \right] - k_f \frac{\partial I_f^*}{\partial \pi} - k_m \frac{\partial I_m^*}{\partial \pi}.$$

Since, by (14) and (15),  $\mu_f + \mu_m = \Omega_f + \Omega_m$ , it follows using (16) that

$$(A.5) \quad \frac{\partial S^*}{\partial \pi} = \Omega_m W_f \frac{\partial I_f^*}{\partial \pi} + \Omega_f W_m \frac{\partial I_m^*}{\partial \pi}.$$

After substituting for the derivatives of the equilibrium investments (using (17) and (18)) into the right-hand side of (A.5), simplifying, re-arranging terms, using the result [which follows from (14) and (15)] that

$$\frac{\partial \Omega_f}{\partial \pi} + \frac{\partial \Omega_m}{\partial \pi} = 0,$$

and then the result that

$$\frac{\partial \Omega_f}{\partial \pi} = \frac{\gamma_f \Delta_f + \gamma_m \Delta_m}{2},$$

and finally using the first-order conditions in (16) to substitute for  $W_f$  and  $W_m$ , we obtain that:

$$\frac{\partial S^*(\pi)}{\partial \pi} = \frac{k[\gamma_f \Delta_f + \gamma_m \Delta_m]}{2\Sigma(\Omega_f \Omega_m)^3} \left[ W_{ff}(\Omega_f)^4 - W_{mm}(\Omega_m)^4 \right].$$

The lemma follows immediately since  $\Sigma > 0$ ,  $\Omega_f > 0$ ,  $\Omega_m > 0$ , and given Assumptions 4 and 5.

### Proof of Proposition 5

Under the (additional) hypothesis of this proposition, it follows from Lemma 3 that for any  $\pi \in [0, 1]$ ,

$$\frac{\partial S^*(\pi)}{\partial \pi} \begin{matrix} \geq \\ \leq \end{matrix} 0 \iff \Omega_m - \Omega_f \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

Thus, using (14) and (15), we obtain that

$$\frac{\partial S^*(\pi)}{\partial \pi} \begin{matrix} \geq \\ \leq \end{matrix} 0 \iff (1 - \pi)\gamma_m \Delta_m - \pi\gamma_f \Delta_f \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

Hence, it follows that  $S^*$  has a unique stationary (or turning) point, namely at  $\pi = \pi^*$  (which is stated in the proposition), and that

$$\frac{\partial S^*(0)}{\partial \pi} > 0 \quad \text{and} \quad \frac{\partial S^*(1)}{\partial \pi} < 0.$$

Hence, we have established that the unique stationary point  $\pi = \pi^*$  is the point at which  $S^*$  achieves its maximum value over the interval  $[0, 1]$ .

**Proof of Proposition 6**

From the first-order conditions in (16) one obtains the following relationship between the equilibrium investment levels:

$$I_f^* = \left[ \frac{\Omega_f \eta}{\Omega_m \xi} \right] I_m^*.$$

It follows from an application of Lemma 3 — after substituting for the equilibrium values of  $W_{mm}$  and  $W_{ff}$ , using the above relationship between the equilibrium investment levels, some simplification, and finally substituting for  $\Omega_f$  and  $\Omega_m$  — that

$$\frac{\partial S^*(\pi)}{\partial \pi} \underset{\leq}{\geq} 0 \iff \phi(\pi) \underset{\leq}{\geq} 0, \quad \text{where}$$

$$\phi \equiv (1 - \theta) \left[ \frac{\mu_f + \mu_m}{2} \right] + \left[ \frac{1 + \theta}{2} \right] \left[ (1 - \pi) \gamma_m \Delta_m - \pi \gamma_f \Delta_f \right].$$

We then note that

$$\phi(\pi) \underset{\leq}{\geq} 0 \iff \pi^* \underset{\leq}{\geq} \pi, \quad \text{where}$$

$\pi^*$  is as stated in Proposition 6(c). This implies that  $\pi^*$  is the unique stationary (or turning) point of the function  $S^*$ . Proposition 6 follows immediately from the above results and the following two results:

$$\pi^* \underset{\leq}{\geq} 0 \iff \Delta_m \underset{\leq}{\geq} \bar{\Delta}_m \quad \text{and} \quad \pi^* \underset{\leq}{\geq} 1 \iff \bar{\Delta}_f \underset{\leq}{\geq} \Delta_f.$$

**Proof of Proposition 7**

From the first-order conditions in (16) one obtains the following relationship between the equilibrium investment levels:

$$I_f^* = \frac{\tau_m I_m^* (\Omega_f)^2}{\tau_f (\Omega_m)^2}.$$

It follows from an application of Lemma 3 — after substituting for the equilibrium values of  $W_{mm}$  and  $W_{ff}$ , using the above relationship between the equilibrium investment levels, some simplification, and finally substituting for  $\Omega_f$  and  $\Omega_m$  — that

$$\frac{\partial S^*(\pi)}{\partial \pi} \underset{\leq}{\geq} 0 \iff (1 - \pi) \gamma_m \Delta_m - \pi \gamma_f \Delta_f \underset{\leq}{\geq} 0.$$

Hence, it follows that  $S^*$  has a unique stationary (or turning) point, namely at  $\pi = \pi^*$  (which is stated in the proposition), and that

$$\frac{\partial S^*(0)}{\partial \pi} > 0 \quad \text{and} \quad \frac{\partial S^*(1)}{\partial \pi} < 0.$$

Hence, we have established that the unique stationary point  $\pi = \pi^*$  is the point at which  $S^*$  achieves its maximum value over the interval  $[0, 1]$ .



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