Yoo, Yoonha

Research Report
Economics of Private Tutoring: In Search for Its Causes and Effective Cures

KDI Research Monograph, No. 2002-01

Provided in Cooperation with:
Korea Development Institute (KDI), Sejong


This Version is available at:
http://hdl.handle.net/10419/200939

Standard-Nutzungsbedingungen:
Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden. Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen. Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

www.EconStor.eu
Economics of Private Tutoring

- In Search for Its Causes and Effective Cures -

Yoon Ha Yoo
Foreword

No one would deny that private tutoring is one of the most vexing educational problems in Korea. The list of anti-private-tutoring measures taken by the Korean government during the past several decades is impressively long and complicated. Reform after reform, new innovative measures were taken, ranging from a simple admission policy reform to a drastic equalization of all secondary schools in the country. A few years ago, even an outright ban of all tutoring activities was instituted, which was later abandoned as the Supreme Court adjudicated it to be unconstitutional. In spite of all these attempts, however, there seem to be no signs of improvement yet. Rather many people find the situation is worsening, to the point that some even talk about the disintegration of the school education system in Korea.

A tremendous amount of energy and resources have been poured into searching for the real causes and effective cures of this social disease. Understandably private tutoring and education in general is a complicated process that would defy any simple characterization. It has multiple facets, and a successful analysis would require interdisciplinary efforts from many diverse fields, not only pedagogy, but also sociology, psychology, economics, and so on. Professor Yoo endeavors in this book to fill the gap from the economics side.

With unusual persistence, the author strenuously tries to put things related to private tutoring in an economic perspective and then to dig out practical implications. Some of propositions the author establishes are quite surprising because he turns some seemingly plausible arguments upside down and proves them to be economically non-sensical. As an example, the following is fairly thought-provoking. From the common, widely held belief that college education is the most effective and surest means to improve one’s future income and social status, the author forcibly draws the conclusion that the statement only proves that the college education market is regulated and as a result, inefficient. According to him, the statement is true only because entry to college education is restricted and regulated. Without artificial restriction, nothing can remain as a surest means to improve one’s fortune in a free open market. Moreover, he argues that tuition control is the prime culprit that perpetuates the private tutoring disease in our society.

Based on a comprehensive analysis, the author presents several policy suggestions including a complete deregulation of college tuitions and enroll-
ment quota and the liberalization of the student selection system. And more than anything else, prof. Yoo argues for a complete revocation of the Secondary School Equalization measure, which has been in force since late 1970s.

Certainly, not everybody would agree with what the author says in this book. But no doubt that the author is laying a foundation from which more informed discussions and debates can be based. It is hoped that this book is used as a fruitful point of departure for validating or rejecting the elements of beliefs that give rise to various policy proposals concerning private tutoring and thereby facilitating systematic comparison and evaluation of policy alternatives.

**Choong Soo Kim**  
President  
Korea Development Institute  

December 2002  
Seoul
## Contents

**Foreword**
**Summary** .................................................. 1

**Chapter 1. Introduction** .................................. 4

**Chapter 2. College Education as a Good** ............... 12
   2-1. Supply and Demand in the College Education Market 12
   2-2. Excess Demand for College Education and Private Tutoring 16
   2-3. Possible Solutions to Private Tutoring Problem ....... 18
      A. Ration Coupons ........................................ 19
      B. Allowing Tuition to Rise .............................. 21
         1) General Subsidy for College Education .......... 22
            a) Externalities and General Subsidy for College Education 22
            b) Income Distribution and General Subsidy for College Education 23
      2) Selective Aid to Needy Students ..................... 24
      3) Deregulation of Tuition and Student Quality ........ 26
      C. Expansion of Enrollment Quota ..................... 27
         1) Social Welfare Loss Due to the Enrollment Quota Control 29
         2) Possible Reasons for Imposing Enrollment Quota .... 30
            a) Concern for the Oversupply of College Graduates 30
            b) Quality Control of College Graduates ........... 31

**Chapter 3. Special Features of Schooling, Entrance Exams, and Private Tutoring** ................. 34
   3-1. Special Features of Education ..................... 34
   3-2. Economic Role of Entrance Exam and Self-Selection 39
      A. Economic Role of Entrance Exam .................... 39
      B. Which School Is a Good School? .................... 43
      C. Self-Selection and Competition to Enter a Better School 44
   3-3. The Middle and High School Equalization Policy .... 45
   3-4. First-Class School Disease and Cliquism ........... 52
      A. An Obsessive Pursuit of First-Class Schools ...... 52
      B. Economics of Privilege: Its Creation and Exclusive Sharing 57

**Chapter 4. Reforms in Entrance Exam and Private Tutoring** ........................................ 62
   4-1. Introduction .......................................... 62
   4-2. Basic Model .......................................... 64
### Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. The Effects of Varying Valuations Among Individuals</td>
<td>68</td>
</tr>
<tr>
<td>B. Changing the Cut-Off Score of the Test</td>
<td>70</td>
</tr>
<tr>
<td>C. Reducing the Effectiveness of the Private Tutoring</td>
<td>73</td>
</tr>
<tr>
<td>D. Increasing the Noise of the Test</td>
<td>76</td>
</tr>
<tr>
<td>E. Increasing the Cost of Private Tutoring</td>
<td>80</td>
</tr>
<tr>
<td>4-3. Rent Seeking Under the Quota System</td>
<td>80</td>
</tr>
<tr>
<td>A. Increasing the Admission Quota</td>
<td>82</td>
</tr>
<tr>
<td>B. Increasing the Cost of Private Tutoring</td>
<td>84</td>
</tr>
<tr>
<td>C. Lowering the Marginal Returns to Private Tutoring</td>
<td>92</td>
</tr>
<tr>
<td>D. Increasing the Noise of the Test</td>
<td>94</td>
</tr>
<tr>
<td>4-4. Rent Seeking Under Tuition Control</td>
<td>96</td>
</tr>
<tr>
<td>4-5. Summary and Conclusion</td>
<td>101</td>
</tr>
<tr>
<td><strong>Chapter 5. Competition to Enter a Better School</strong></td>
<td>104</td>
</tr>
<tr>
<td>5-1. Introduction</td>
<td>104</td>
</tr>
<tr>
<td>5-2. The Model</td>
<td>106</td>
</tr>
<tr>
<td>A. Problem of School Choice</td>
<td>108</td>
</tr>
<tr>
<td>B. Problem of Choosing Optimum Private Tutoring</td>
<td>114</td>
</tr>
<tr>
<td>5-3. Comparative Statics and Properties of the Equilibrium</td>
<td>122</td>
</tr>
<tr>
<td>5-4. Effects of School Equalization Measure</td>
<td>129</td>
</tr>
<tr>
<td><strong>Chapter 6. Summary and Conclusion</strong></td>
<td>134</td>
</tr>
<tr>
<td><strong>References</strong></td>
<td>146</td>
</tr>
</tbody>
</table>
Summary

This book tries to uncover the economic forces at work under the phenomenon known as private tutoring, a social headache the Korean society has long grappled to cope with. Finding the real economic causes behind private tutoring would facilitate formulating effective cures for the disease, if it can be called a disease at all. Broadly speaking, students take private tutoring for two distinct purposes: for successfully advancing to the next stage of the hierarchical ladder of education, and for entering a better quality school. This book explores the economic incentives working at these two fronts.

Most high school students expect to advance to college when they graduate, as most middle school graduates expect to attend high school as a natural course of matters. When these expectations are somehow threatened and, as a result, when students cannot be sure of the advancement they had long planned as a natural sequence, they take private tutoring, to keep their plans as intact as possible. They buy whatever they believe would help improve their odds of advancing to the next stage of education. They study harder at home, hire a tutor or enroll at private academic institutions to help them, to get ahead of others who are equally desperate in achieving the same goal. This implies that if every student is assured of the thing he/she desires to have, no one would take private tutoring to have the upper hands over others. This is the first proposition that has been established in the thesis. With no excess demand for higher education, there would be no private tutoring.

We can think of two different ways of distributing something valuable, say a diamond, to people. One conventional way of distribution is to ask the contenders to pay the price. The price will be determined in the manner of auction. And the highest bidder will get the prize. To earn the money with which to buy the diamond, the demander has to provide something valuable to others. That something valuable to others is production. Higher productivity is gained this way in a free capitalistic society. Nothing is lost in the distribution process.

There is another way to distribute the diamond. The diamond can be given for free, but with a proviso that the contender does something that the diamond owner asks him/her to do. As an example, suppose that the owner asks the contenders to stand in line on one foot for several hours. The one who can tolerate the hardship for the longest hour will get the prize. People stand in line, wriggling on one foot, sweating and frowning from pain, until there is only one person left. In the end, the diamond is handed over to the longest suf-
In both cases, the social task of distributing a diamond has been successfully executed. The second method, however, is non-sensical if one assumes that watching people suffering on one foot while in line pleases nobody. There is a social loss, keeping people suffering for several hours for no apparent social good. Each individual in the line is behaving rationally, but from the standpoint of the society as a whole, the second method of distribution is a sheer absurdity. Private tutoring has a similar element in it: students suffer for nothing from a social point of view. This is the second proposition of the thesis.

Why do we then keep this social absurdity? Surprisingly, explanations abound, although none of them stand scrutiny, and some of them are even self-contradictory. The real tragedy is that there is no shortage of reasons, half-baked as they may be. So in the end, this only results in a perpetuation of the deplorable situation. Some people argue that an excess demand in the higher education market is inevitable or even desirable. They contend that education is different from other ordinary goods; it has the property of public goods or positive externalities; and, therefore, it is argued that tuition must be kept below the market clearing equilibrium level. Basic education aside, the proposal to subsidize college education whose benefit would exclusively accrue to the student him/herself, however, doesn’t make sense at all.

Others contend that college education has to be subsidized for the purpose of enhancing equity because college education is the most effective means to improve one’s future income. If this is true, however, public aid must be given to other non-college students because college students have already secured one of the surest means to become rich. Furthermore, the statement cannot be true unless the higher education market is somehow artificially regulated. In spite of this, if one keeps tuition low to help the students, it would only end up with helping the high and middle income groups of the society because most of the college students come from those groups, thus aggravating the income distribution.

If sum, either from an efficiency or from an equity point of view, keeping college tuition low, and thus maintaining a chronic excess demand for college education, cannot be justified. The argument for imposing a college enrollment quota cannot stand either. More than anything else, controlling the total number of college students below the market equilibrium level cannot go hand in hand with the view that college education has to be subsidized to promote its consumption.

Despite all these negative conclusions, educational authorities have been playing with some innovative ideas to reform student selection methods, but still keeping the overall imbalance in the higher education market intact. These measures include earmarking some slots for special students such as from rural areas or abroad, making the Academic Aptitude Test easier, asking more essay type questions than multiple choice questions, making private tutoring
more costly to take, including an outright ban of all tutoring activities outside school, etc. Unfortunately, the effects of all of these measures are again dubious as some students are expected to take more tutoring and others less, depending on the elasticity of demand for tutoring, and the current state of competitiveness of college admittance, etc. Some of these measures would only deteriorate the average quality of the students body as they make the selection process contaminated with more noises.

Turning our attention to the second reason for private tutoring, i.e., tutoring for the purpose of entering a better school, we can once again prove that some of the measure taken have been ungainly as far as the possible effects are concerned. First of all, it must be noted that the competition to go to a better school occurs because of price control and resultant imperfect pricing of the quality differentials among schools. Without resolving this fundamental cause of the problem, any off-hand allopathic measures are not likely to generate the desired effects. In particular, the Secondary School Equalization measure taken in late 1960s and early 1970s is very unlikely to have positive effects on reducing private tutoring. Rather it appears to have done more harm than good. The intuition for this theoretical conclusion is easy to grasp. Suppose there are two alternative routes to take to expedite students’ choice for better schools; one is school education, and the other private education. Then it can be easily concluded that the Equalization measure has simply rendered one of the routes, school education, to be less effective and less rewarding, and thus caused the students to stampede to another alternative route, private education.

All in all, the analysis in this book strongly suggests that economic forces are always well in force under the surface, and the phenomenon of private tutoring is the result of these natural forces working within a misguided and mismanaged education system. One sure way to cure the social disease is, therefore, to put the education system back in order and let the economizing forces make their way toward a socially desirable direction.
CHAPTER 1

Introduction

When it comes to matters related to higher education in Korea, nothing seems to be more troublesome than the problem of overheated competition to enter college and the rampant private tutoring. In the past several decades, the majority of educational policies were directed towards alleviating the social distress associated with private tutoring. But in spite of all these attempts, no signs of improvement seem to be in sight yet. Whenever the issue of college entrance comes up, newspapers still churn out expressions such as “examination hell” or “private tutoring disease ruinous to the State”. Lately, even expressions as horrifying as “the total disintegration of school education” are being used with little hesitation. The recent adjudication by the Supreme Court, which has declared the prohibition of private tutoring as unconstitutional, appears to have given rise to another round of apprehension of spreading private tutoring.

Despite the profound importance of the matter and the grave concern poured thereon, the diagnosis about the causes of the disease and prescriptions for effective cures are still lacking. Discussions are unfocused, poorly reasoned and discouragingly simplistic or ad hoc. Suggested solutions are fragmented and scrappy, yielding no prospect for improvement. Here are some examples:

When the problem of private tutoring became a social issue, one of the arguments we heard most frequently was that the cost of private education in Korea was unbearably high. People argued that the nation as a whole was annually spending about 20 trillion won on private education (1997 estimate by Korea Consumer Protection Board). They went on to show that it amounted to 3% of Korea’s GNP and exceeded the whole budget of the Ministry of Education. These numerical figures were then followed by some dramatic testimonies and evidences on the financial difficulties that some of real world families had to undergo to make ends meet. The flagrant “excessiveness” of Korea’s private educational expenditures is often demonstrated even through international comparison, saying that other counties have such and such figures compared to Korea’s such and such. In short, the argument goes that too much money is being spent on private education in Korea, and the financial strains bestowed upon the households with school children are enormous.

Do these arguments make any sense at all, at least from an economist’s point of view? Probably not. No matter how pathetic it may sound, it lacks a
substantive logic for us to give any credit to. We can safely presume that education, be it private or public, is a good that is produced and consumed within a society, just as all other goods. If this proposition is true, it would make no sense at all to argue that the price is excessively high or low, or the volume of trade is too big or small, merely by observing the actual amount money that a consumer or the whole society bears to pay and buy. The reason is all too simple: we do not have a criterion to make such a sweeping judgement for an appropriate price or volume of trade. The percentage of a nation's GNP used to purchase the good, or a comparison with other nations cannot be a valid criterion because a consumer's preferences may differ by individual, and a nation's technical conditions to produce a specific good may differ by nation.

Let's take another equally familiar argument that not only is the expenditure extravagant, it is also unnecessary and wasteful. Private education, some people argue, basically pursues the same goals as public education. Thus spending money on the former is an unnecessary duplicative investment. It is apparent, they claim, that to make an investment of such an extraordinary amount to achieve one and the same goal is a waste for a nation as a whole.

A little bit more logical than the first claim it may appear, however, this second claim does not make economic sense, either. The reason is again simple. If there is a certain demand for education, the optimum share between public and private investments to bring it into existence must be based on the respective marginal costs of production. If in terms of marginal cost of production, private education proves to be more efficient, the investment in private education should increase, and consequently its share must increase correspondingly. If the opposite is true, priority should be given to the public sector.

At the root of our society's antipathy towards private tutoring, therefore, nothing significant appears to lie except the ungrounded judgement that private education is something somehow unnecessary and improper. From this unproven, hardly-justifiable perspective, every factor that has even the slightest likelihood to bring about an increase in private tutoring is couched as a social evil: the over-zealousness of parents to educate their offsprings, an obsessive pursuit for first class top-notch schools, the hierarchical rank of universities, the entrance assessment test, which depends solely on the written examination, and so on. Then a social rush follows in which an enormous amount of public efforts and resources are poured to eliminate or curtail such evils. As a matter of fact, the long series of measures that the government has introduced during the past several decades to cope with the private tutoring problem seem to be all based upon such a perspective. The equalization or standardization of all middle schools in 1968, the equalization of high schools in 1974, the illegalization of private tutoring in 1980, the introduction of the AAT(Academic Aptitude Test), the introduction of a performance report system, and various other attempts to reform the college admission system all fall into this category. The recently proposed recommendations, such as diversifi-
cation of selection methods, undoing of university ranking, or more explicit equalization of universities, seem to still be on the same line of logic.

The objective of this book is to examine the various issues surrounding private tutoring from a pure economic point of view. To be more specific, this thesis attempts to analyze the issue of “excessiveness” of private tutoring in terms of economic efficiency. With a given amount of resources invested in education — be it public or private — if one can show that we could produce more or better output than we are producing now, that would mean that the current allocation of resources is not efficient\(^1\). Also, the question on how much resource should be invested in education, and how it should be allocated will be addressed in a context of social welfare maximization.

Needless to say, educational policies are not exclusively implemented on the basis of economic efficiency alone; other socio-political goals are surely to be taken into consideration. In this regard, one might say that a so-called economic approach is too narrow to have any practical relevance: it has the tendency to view all matters solely in terms of optimum allocation of resources and efficiency. Granted, the approach adopted here is an analysis based on one single perspective; economic efficiency among many possible alternatives, and as a result, it may provide only a partial diagnosis and prescriptions on the issues of private tutoring. Nevertheless, an inquiry on the matter from an economic efficiency point of view is a prerequisite for any sensible public policies because it can provide us a valuable guide in choosing the minimum cost solution to achieve a given goal. Once we are equipped with this minimum cost solution, we may then proceed to consider various tradeoffs among competing social objectives.

Now let me start with the definition of the private tutoring. In this thesis I will define private tutoring as an act of purchasing educational services outside the formal school, with a purpose of increasing the probability of advancing into the next level of school or entering a more prestigious school. Receiving a private lesson on piano, art, or other hobbies is not included. In most cases, this would be receiving education by employing a private tutor or registering at a private academic institute on the formal curriculum subjects that would appear in the entrance examination or other forms of admission test. Also, I will make no distinction between the terms private education and private tutoring. Private education would normally have a distinct meaning of its own in other countries, different from private tutoring. But in Korea the two terms have been conventionally used interchangeably, and therefore, I follow the convention.

On an outward appearance, the competition to enter the next level of higher educational institution, say from middle to high school, or from secondary

\(^{1}\) Or if one can show that we could produce the same amount of output with a lesser amount of investment.
school to college, cannot be distinguished from the competition to enter a better, more prestigious school because these decisions are made simultaneously. When we try to look for the fundamental factors that cause competition, however, it is useful to treat the two competitive activities separately. For example, if we assume that the schools in a given hierarchical level are all homogeneous, the latter competition to seek a better school would by definition vanish because there is no better or worse school. Given this, the competition to enter an advanced level school (in this thesis, advanced level school mostly refers to college; henceforth the term college will be used in substitution for advanced level school) is a matter of advancing or not advancing to the next level of education. Consequently, the difference in the quality of education provided by each college is presumed to be non-existent.

In Chapter 2, I develop my argument under the assumption that there is no difference in the quality of education that each student receives from schools. In other words, it will be assumed that all colleges provide exactly the same quality of education. Thus the prospective student needs only to decide whether or not to purchase this commodity. He doesn’t need to worry about which college to attend. On the other hand, colleges need only to decide how much education they should supply.

Once we simplify the matter this way, the college education market is no different from any other markets of homogeneous quality, like a market for rice or gasoline. In this case, the issue of resource allocation comes down to an issue of the optimum supply of educational service at the college level in a society. Under the assumption that each student wants to purchase just one unit of college education, this in turn boils down to the question of optimum number of college students in any given society. This will be discussed in Chapter 2.

In this setting, the demand for private tutoring is generated when there is a price control in the college educational market. Under the binding tuition control, excess demand for college education will develop, and college entrance examination will be used as a means to rationing the limited supply among a large number of contenders. In this market, each student makes investments to get the upper hand relative to others in the contest. In other words, there is a rent seeking competition among students under tuition control. The immediate conclusion here is that one sure way of eliminating private tutoring is lifting the price control imposed on the college education market. If everybody is allowed to purchase as much education as he wants at the prevailing market clearing price, nobody will try to make a extra investment for increasing his/her “probability” of successfully purchasing that commodity.

I examine some of the policy measures adopted or proposed to curb the private tutoring in this context. I will show that many of the measures are just variants of typical rationing devices that one can find in basic microeconomics text books. At the same time, it will be shown that many of the suggested solutions are quite crude, with some unlikely to generate desired effects, or simply
nonsensical. Why would a price control on school tuition be employed in the first place? Various arguments offered are examined and evaluated, with a special attention to the possible effects.

In Chapter 3, the assumption that each student receives exactly the same quality of education by purchasing one unit of education will be lifted, allowing product differentiation, or equivalently school differentiation. Once one sets out to relax the assumption of homogeneity of schools and the educational service they sell, however, a large number of unexpected questions loom before us.

To get a taste of the complications it may create, ask yourself: what you really mean by the different quality of educational service or different school quality. Which school is a good school anyway? Many people presume that they know the answer to this question, immediately bringing into their mind the names of a few universities so prestigious and well known that everybody yearns to get into them. What exactly makes these schools “better”, however? Is it the quality and reputation of the faculty they retain, or the quality of peer students that an individual student would mingle with once he/she is admitted? More attentive care they provide? The quality of knowledge and information they mete out?

These and many other probable questions necessarily bring us to a more fundamental question of what education, or a bit more precisely, school education, is really about. Firstly, it is not simply a transaction that concludes with a one-time exchange of money and good between a seller and a buyer, as is the case of most ordinary goods that are traded in the market places. In the case of education, the final product that the student will eventually consume is produced jointly by both seller and buyer. The school injects a certain amount of inputs in the production process. At the same time, the student also provides his own input to the process. As a consequence, it is not clear what the student is really purchasing or paying for, the input that college is pouring into the production process, or the final output itself. In terms of the homogeneity of schools, it is also not clear, either, whether equality of college education refers to equality of the college’s input or to equality of the final product.

Secondly, because of the joint nature of this production process, the quality and quantity of education that each individual student receives would differ from student to student. In other words, even if exactly the same content and quality of education were provided by every college, each student’s received content and quality of education would differ according to each individual.

---

2) Why should I care about who else is purchasing the good that I am purchasing if the commodity in question is something like gasoline or rice?

3) Books also transmit knowledge and information as school education does. But in the case of purchasing a copy of a book, we do not have this ambivalence, because it is clear from the outset what a buyer is paying for. A reader purchases a book, and the knowledge production process takes place at his/her own home. Here he/she purchases just an input.
That is, a perfect homogeneity among schools does not guarantee that each student will get exactly the same final output. The quality of input that a student carries with him/herself into the production process, such as how diligently he/she studies, or how intelligent he/she is, how much of inventory knowledge he/she is equipped with initially, is so crucial in determining what he/she would eventually get, that there is no simple way to define, not to mention secure, a homogeneity of final outputs.

Thirdly, school education normally involves simultaneous participation of many students. Therefore how an individual student behaves in the production process or how well a student is equipped with exert some influences on what and how much other students in the class would get from the school. In short, there exist externalities among students. Some externalities come directly from interactions among students. This is so called “peer effects”. Even in the case where there is no direct peer effects, however, the extent of heterogeneity among students would still inflict some influences indirectly on the final output each students can get if the school authority’s input vector depends on the distribution of students’ input vector. This takes special importance when we try to analyze the effects of School Equalization Measures introduced in 1968 and 1974 on the demand for private tutoring. This is because the equalization of secondary schools inevitably increases the variance of the quality distribution of students’ inputs to the education production process.

Section 1 of Chapter 3 deals with the aforementioned general characteristics of educational goods. Using some of the results obtained there, Section 2 discusses the economic role of entrance exam or any other forms of pre-assessment of student performance or ability. This is followed by Section 3, in which I develop a simple informal framework in which the effects of Secondary School Equalization Measures are analyzed. The conclusion reached there is that it is very unlikely that the equalization measure has had a positive effect on reducing the private tutoring. On the contrary, it might have prompted a more pervasive tutoring. Therefore, if one wants to reduce the social costs associated with the private tutoring, it is advisable to abolish the School Equalization Measure. Allowing a free grouping of students with relative homogeneous inputs in the production process would reduce the demand for private tutoring by strengthening the efficacy of school educations. To put it more bluntly, one might say that the equalization measure is the prime culprit responsible for the current disintegration of school education.

Any proposal asking for the revocation of the equalization measure and the reestablishment of a ranked secondary school system, however, invariably evokes the spectre of the so called “the first-class school disease”, — the obsessive pursuit of first-class schools and favoritism among alumni of some presti-
gious secondary schools. Given the fact that the main reason why equalization was implemented in those days was to get rid of the “first-class obsession disease”, such concerns are understandable. Section 4 thus picks up on this issue and looks into the relationship between the equalization and the so-called disease.

In the first place, it would be extremely difficult to find any justification for putting the blame on such natural pursuit of prestigious schools. So some excursions on the subject is provided in this Section. Secondly, favoritism, loosely speaking, means a social mechanism through which a special social privilege is shared exclusively among a small group of people. In this regard, the competition to enter a prestigious school may be linked to the competition to acquire the economic rents associated with a mere title of specific schools. The top ranking schools, however, are not the only organizations that help such exclusive indulgence of privileges. All sorts of formal and informal organizations in which the comprising members are emotionally or functionally attached to each other, through, say, shared hometowns or family ties, can serve the purpose. Therefore abolishing top ranking schools will not in itself dissolve favoritism. In order to destroy favoritism or nepotism, a more fundamental solution must be sought which attacks directly the very source of social rents. This is the theme of Section 4 of Chapter 3.

The ensuing two chapters are the most rigorous and technically involved in this thesis. In Chapter 4, I attempt to make a theoretical evaluation on some of the policy measures that have been proposed or adopted with regard to entrance exams until recently, with the goal of eradicating private tutoring. In doing this, I first set up a formal model and develop various comparative static analyses. The overall framework is the same as the one set up in Chapter 2. I examine the following questions: 1) whether making the college entrance examination or the Academic Aptitude test easier would help reduce the demand for private tutoring. 2) whether devising a test method that would render private tutoring less potent in improving test scores would help, 3) whether increasing the essay type questions in the test would help. Surprisingly, the answers to all these questions are not determinate. They may sometimes help in reducing private tutoring, but not at other times. Therefore, we must be extremely careful in implementing these seemingly effective and often recommended measures.

Chapter 5 deals with the competition to enter a better school. The general nature of the competition to get into a better quality school is analyzed. Therefore, the overall setting is comparable to the one presented in Chapter 3.

---

5) Indeed, is there anything wrong with the pursuit of top class quality of any goods so as to be called a disease? Is there any mortal who would not yearn for the best, if it is available at a reasonable price? If it is a matter of degree, as some people would argue, where should we draw the dividing line between “reasonable” pursuit of the best and an “excessive” one?
formally introduce the quality differences among schools and ability differ-
ences among students, and derive the equilibrium distribution of school quali-
ty hierarchies. It will show how a school quality hierarchy will emerge in equi-
librium. The issue of the effects of school equalization measure is picked up
again to be more formally analyzed in this context, and the necessary and suffi-
cient conditions for the measure to have the desired effects are derived.
Unfortunately, it turns out that the conditions are very restrictive to have any
practical significance. Rather, the overall conclusion is disheartening, suggest-
ing that the equalization may have caused the exactly opposite effects to that
intended.

As mentioned earlier, these two chapters are a bit technical. For those read-
ers who do not want to get bogged down in mathematical subtleties, therefore,
I have provided a detailed verbal explanation on what has been discussed in
these chapters in Chapter 6 which summarizes and concludes the whole thesis.
So those readers who are more interested in conclusions and their policy impli-
cations can skip Chapters 4 and 5 and may directly jump to the last concluding
chapter.
2-1. Supply and Demand in the College Education Market

I will start from a rather strong assumption, or a standpoint that educational goods are no different from other, more mundane goods such as automobiles or gasoline. Of course, such an assumption may take us far away from reality, sometimes leading to implausible, or even unreasonable conclusions. Starting with a set of strong assumptions, however, is very convenient because in many cases it allows us to tackle the very core of otherwise extremely complicated issues, by making things much simpler. It is especially useful in dealing with the phenomenon of private tutoring because so many factors appear to be intractably interwound there. Looking things first in a rather simplified setting, we will be able to sort out the essential elements that cause the vexing social problem. Then by relaxing the assumptions step by step, we can take a note of the implications of each assumption until we get a better understanding of the whole.

First of all, let’s start with a definition of school education. In this thesis, education is meant to be something a student purchases from an educational institution by enrolling in it. To be more specific, a student is assumed to be endowed with some human capital whose stock at a given point in time is denoted by $H_t^6)$. He/she can increase his/her human capital to $H_{t+1}$ by attending school for a certain length of time. This increment in human capital a student can obtain by attending a school is a school education, and is denoted by $E_t$. Therefore, the following relationship holds.

$$E_t = H_{t+1} - H_t$$ (1)

Next, I assume that the education provided by colleges are all identical. In

---

6) The terminology, “human capital” may give an impression that a student can effectively increase his productivity by attending a school. This is the view taken by the so-called “human-capital theory” of education. Viewed from the standpoint of the “signaling theory” of education, however, it can be argued that the primary function of education is just to certify one’s inherent ability rather than to increase one’s productivity. The term, human capital, in this thesis, however, is in no way meant to take sides with a particular point of view about the role of education. It simply refers to what the student wishes to receive from the educational organization, and for which he/she is willing to give up a certain amount of other things.
reality, colleges are more or less different from each other, providing different educational contents and environments, thus establishing different ranks in students’ perceptions and aspirations to choose a specific school\(^7\). Also even within a college, the content of a curriculum differs according to the fields or majors students choose to study, making it unrealistic to assume homogeneity. But for the sake of facilitating the argument, I will assume that the educational products of all colleges are homogeneous\(^8\).

The upshot of the assumption of homogeneity is that consumers do not have to choose among colleges. The distinction between a good or better school no longer exists. The only meaningful decision for a student in this case is, therefore, just whether or not to advance to college, not which school to choose.

Let’s now assume the homogeneity of not only the product provided by the seller but also of the product attained by the consuming student. This assumption may seem redundant since in most cases if the product provided by the seller is homogeneous, then the same product attained by the consumer would naturally be homogeneous too. For instance, if all the apples sold in the market were homogeneous, then every consumer would have the same apples. But in the case of educational products, even if all colleges provide exactly the same education, that doesn’t necessarily mean that all students were receiving homogeneous education. For instance, imagine a professor who goes around several colleges and deliver the same lectures. In this case, the contents of his/her lectures would be all the same at every college. So as far as this lecture is concerned, every college is providing its students with the same education. But this does not mean that every student is making the same academic achievements. Some students may be getting a lot of new information or technique out of this lecture, while others may be getting only the minimal.

Of course, in the case of apples too, the utility that each individual gets from consumption of an apple may differ according to his/her own digestive ability. And as in many other cases, this individual difference in utility is not the concern of the apple vendor. It is up to the consumer to consume as much as he/she wants and in the way he/she likes. But in the case of educational products, the seller implicitly guarantees the consumer the content of the educational product that students will ultimately acquire, and usually takes some actions to help the knowledge absorption process. In addition, the transaction of educational products does not conclude with a one-time payment of money and delivery of the final product. The transaction is more like a long-term contract, in which both the school and the student agree to jointly participate in the pro-

\(^7\) We will deal with the issue of different qualities in the next chapter.

\(^8\) In fact, we have to admit that the statement that a product is homogeneous is bound to be vague. Unless a product is exactly cloned, every product is different in some way. In its strictest sense, one cannot say apples, rice, or even manufactured goods, such as automobiles or sheets of paper, are homogeneous. It is considered as homogeneous simply because consumers conceive or analysts assume it as so.
duction process of something the student finally obtains. Considering all these, the assumption that the educational products that students receive are homogeneous is a very strong one. This assumption will be modified later in the following chapters when we look into the fundamental characteristics of education in more detail.

Still another assumption, which is not essential but could add a little resemblance to reality, is that every potential consumer buys only one unit of education. In other words, a college education is a whole four year process that cannot be bought in smaller pieces, such as one or two year units, nor will a consumer go to college twice or thrice. In short, every college sells a homogeneous product, $E$, and every consumer buys only one unit of $E$. The value that any student attaches to the education, measured by the amount of other goods he/she is willing to forego to acquire one unit of educational product, is, however, assumed to be different by individual.

Under these assumptions, we can now derive the market demand for college education, simply by matching the pair of information, i.e., the value each student puts on the education with the aggregate number of students who want to purchase the product at that price. This demand curve has a negative slope as usual, which implies that the cheaper the good, the higher the demand.

Next, let us define the supply function of college education. A school, as the supplier of the good, has to make an outlay for school building, facilities, remuneration of professors and staffs, educational equipment and materials in order to provide educational services. I assume that the more students it wants to serve, the higher the cost it has to bear. I further assume that the marginal cost of college education is upward sloping. The cost function of individual schools may differ. By horizontally adding each college’s marginal cost function, we can derive the market supply function of college education, which would have, by the assumption we made, a rising slope.

Once we define the demand and the supply functions as above, it is straightforward to explain what will happen in this market. The equilibrium price (school tuition) is determined at the cross of these two curves, and so is the equilibrium quantity traded. In the diagram below, the market price is given by $P^*$ and the total number of students admitted by $N^*$. It can easily be proven that, under certain conditions (no externalities, etc.), this price and quantity pair is the social optimum, the best outcome a society can achieve and thus, nobody can improve upon further.

---

9) The negativity of the slope comes from our earlier assumption that the values attached by students to an education are all different. At a given price, only those students who have a higher value than the price will want to buy the good. This implies that the more students will buy education, the lower the price.

10) That is, I assume that $C=C(N), C'>0, C''>0$.

11) This is “Pareto efficient”. 
Not only is it optimum to the whole society, but it is also satisfactory to every individual involved. All participants are completely satisfied with the outcome in the sense that no supplier will have unsold left-overs that he/she still wants to sell at the prevailing price and no customer will want to buy more (or less) than he is currently purchasing at the given price\(^{12}\). Furthermore, this desirable state of matters is easily attainable. No extraordinary efforts or devices are needed to achieve the result. The market will automatically work out the final outcome once open and free competition is guaranteed and is left alone. If the amount demanded at a given price exceeds the amount supplied, the price will rise, causing reduced demand and increased supply until the market is cleared. If instead there is a surplus in the market, the price will fall, and the market will again be cleared at a new equilibrium point. In economists’ jargon, the equilibrium is stable.

Another important attribute of this market is that no sellers need know anything about a buyer’s characteristics other than the price he/she is willing to pay. For instance, one is not required to have, say, a high school diploma, nor to be over a certain age in order to be able to buy an automobile. There would be no evaluation on the qualifications of the buyers and no restrictions of any sort whatsoever. The price the buyer is willing and prepared to pay is the only criterion to be used to execute a transaction. I will call the fact that one does not need any other information about the attributes of the buyer other than the price he/she is offering in a market “the principle of anonymity”.

In most educational markets, this principle of anonymity does not hold. In\(^{12}\) Everyone who wants to go to college will go to one. There will be no student who is left out as long as he is willing to pay the tuition. Satisfied as they are, no one will take issues with the college admission policies. And as a result, we will rarely read newspaper articles commenting on college education market, let alone demanding a complete overhaul of educational system.

---

\(^{12}\) Everyone who wants to go to college will go to one. There will be no student who is left out as long as he is willing to pay the tuition. Satisfied as they are, no one will take issues with the college admission policies. And as a result, we will rarely read newspaper articles commenting on college education market, let alone demanding a complete overhaul of educational system.
almost every case in advancing to the next higher level of education, there is a procedure of screening and testing students to determine whether they meet the requirements of the school, besides the price they are willing to pay. Nowhere in the world do schools distribute their admission tickets solely based on the student’s willingness to pay, as in an auction market.

But if we assume that the quality or quantity of the educational service each student receives from their school is identical, as we have already assumed, it is evident that there is no need for the seller to be concerned with the qualities or any other attributes of the buyers. In other words, there is no reason why the principle of anonymity should not be in force. This demonstrates that under the current assumptions, it is possible and also efficient to get rid of all types of examinations and to distribute college admissions solely on the basis of price.

2-2. Excess Demand for College Education and Private Tutoring

Even in a market where the homogeneity of a supplied product is completely guaranteed, there are occasions when some particular attributes of the buyers play a pivotal role in determining who gets the product. To demonstrate this, let us suppose a market in which a price regulation has been imposed, fixing the price below the market equilibrium price (at $P_0$ in the above diagram). At that price, the quantity demanded, $N_d$, exceeds the quantity supplied, $N_s$. In this case, the supplier must introduce some kinds of rationing device in order to parcel out the limited supply among a large number of contending demanders.

The seller may give preferential privileges to those whom he/she is well acquainted with through blood, school or provincial connections. Or he/she may distribute the goods selectively among buyers based on the buyer’s income level, race, or age. In other words, all sorts of personal attributes can, and will be utilized as a rationing criterion in the distribution process. The seller has to discriminate buyers on basis other than the price they are willing to pay, because he/she is now prohibited to use price as an allocation device.

Once a particular non-price criterion is adopted for the allocation of the controlled product, buyers then compete with one another through that particular mode of contest to win the commodity. The acquisition of the good, which will bestow a windfall gain to the fortunate winner, now hinges on how much the buyers can satisfy the seller on that particular criterion adopted. A rational buyer will make an investment in activities which he/she perceives will help enhance his/her probability of winning it up to the level where his/her expected marginal gain is equated with the marginal cost. Since the value that a buyer attaches on the product is given by $P_1$ at that margin, it can be expected
that he/she would be willing to invest the amount of $P_1 - P_0$ in the contest.

In the educational market, schools administer entrance examination to select more qualified students. This is certainly a form of non-price devices of allocating admission tickets. Students then devote non-negligible amount of resources and efforts, including private tutoring, to raise their chances of admittance. Therefore, the simple analysis given above shows that the phenomenon of private tutoring has a very close resemblance to the rent-seeking activities in other markets where prices are controlled. In many cases, the price for college education, tuition, is regulated. And at this regulated price, more students want to enter colleges than the pre-set quota or the existing capacity allows. So there exists an excess demand for college education, and therefore, a need to resort to some sort of rationing mechanism. One apparatus adopted by most schools is the entrance examination, through which students have to obtain a good score by whatever means. Getting help from others via private tutoring is certainly an effective means to increase the chance of hitting high marks in the examination. The maximum investment they will make towards this means is $P_1 - P_0$, discounted by the probability of the ultimate winning, as explained earlier.

The following article in a weekly periodical (Economist Weekly, 29 July, 1997) forcibly attests to this.

“Mr. L has a 16 year old son, now a freshman in high school, and a 7 year old daughter in kindergarten. … He spends 1,810,000 won, 48.2% of his salary, every month on private tutoring expenses. But Mr. L has no intention to cut down the expenses of private tutoring. He thinks this may be his ‘last investment’ for his children. He is expecting a loan of 10,000,000 won from a bank during the second half of this year, or the beginning of next year at the latest. He needs this loan in order to pay for two years of private tutoring in subjects such as Korean, English, Math and Essay. If his son can enter a “good” university by an investment of 20 million won during the remaining two years of his son’s high school years, his investment will be, he believes, a great success.”

Of course, in the above article, the desired goal is to enter a “good” university. So it doesn’t exactly fit with the current assumption that all colleges are identical. But even in the case of simply advancing onto college and not aim-

13) In 1999, there were 310,000 spaces available in colleges whereas the total number of applicants for the Academic Aptitude Test were 869,000, including students who are on their second trial for successful admission. This brings the competition rate to 2.8:1. Ordinarily it is rather difficult for us to imagine a market where unsatisfied customers outnumbers the satisfied ones almost twice at the on-going price. A businessman would quickly open a new venture in that market and garnish a huge fortune if a similar situation develops in other goods market. Assuming the average annual tuition is about 5 million won, a simple arithmetic shows that the volume of the pent-up demand amounts to an appalling 11 trillion won.

14) If every applicant spends similar amount of money (about 10 million won) each year, the total volume of expenditure by high school students would reach a whopping 30 trillion won, assuming the number of applicants to be around 1 million.
ing for a top-class university, we can easily imagine that almost the similar kind of calculation would apply to many of the decision-making processes of students and their parents.

In short, private tutoring and entrance competition in general is a kind of non-price competition resulting from an artificial restriction that blocks the free movement of the market price\(^{16}\). This implies that unless the excess demand in the college education market is somehow resolved, or certain drastic counter-measures are adopted, private tutoring will never disappear. In particular, strengthening public education, which has been often suggested as an ultimate solution to the aggravating private tutoring problem, would have minimal effect as long as there exist a huge, insatiable demand for college education. Better educated high school graduates through the improved public education still have to compete to enter colleges where supply falls far short of demand. What is crucial in this market is not how well a student is qualified in absolute terms but how far he/she is ahead of other students. Improving the overall quality of secondary schools, therefore, will have no effects on the keen competition to take the upper hand over others.

In addition, if we borrow from the textbook analysis of price control, the regulated education market, which gives rise to private tutoring as a natural consequence, is inefficient and results in a substantial welfare loss. The excess demand caused by price control creates a social loss of the following types\(^{17}\): 1) the unrealized gains from trade due to the reduced supply\(^{18}\), 2) wasteful rent-seeking activities in the regulated markets such as standing in a long line to buy the good\(^{19}\).

2-3. Possible Solutions to Private Tutoring Problem

In light of this, the solutions to private tutoring are also easy to figure out

---

\(^{15}\) Actually the assumption of homogeneity among colleges has only been introduced to facilitate the definition of a meaningful market demand curve. It is not essential to the analysis above. We frequently speak of the demand for cars and the demand for food, but strictly speaking, there are no such products as cars or food. There are only specific kinds of cars with specific features such as a Tico or a Grandeur. Therefore, one cannot define the market demand for a good, unless one specifies which commodity in particular he is referring to. Nevertheless, we keep on speaking of the demand and supply of such vague products as cars and food for the sake of convenience by supposing an imaginary existence of such a good. By the same token, an analysis on aggregate demand and supply of college education on a similar level of abstraction can be performed as a meaningful intellectual experiment.

\(^{16}\) Although their emphasis is a bit different, Rothschild and White (1993, 1995), Winston (1999) view the pre-screening of student ability from a similar point of view.

\(^{17}\) A bit detailed explanation on this is provided in Figure 1-2 below.

\(^{18}\) A binding price control combined with a rising supply curve results in reduced supply.

\(^{19}\) Of course we can't say all the private tutoring is totally unproductive. Private tutoring may enhance a student's academic abilities and his/her human capital. But conceptually the part of the resources consumed in the process of competition to get something ahead of others is wasteful because no value added is created in return.
from elementary economics text books. First, coupons can be handed out to prevent unproductive rent seeking activities. Second, lifting the price regulation would help the market be cleared at the new equilibrium price, even under the limited short run supply. Third, in the long run, supply restriction should be eliminated to allow the socially optimum amount of education services to be provided. Let’s take a closer look into each of these respective solutions.

A. Ration Coupons

One simple scheme to distribute a limited amount of goods to a large number of competing demanders is randomization. This can be done by meting out a given number of ration coupons to the contenders. The coupon winners will be chosen by lottery. Since no human endeavor can affect the outcome of this lottery-type selection, no efforts in rent seeking will be made. And some of the aforementioned wastes of non-price competition can be avoided by this per-capita rationing.

On first impression, the idea of distributing the right to advance to colleges by means of randomized coupons would strike many as an absolute absurdity. If we take a closer look, however, we can realize that many of the policy measures actually adopted or proposed for future implementation do not fall very far from this category.

Let us start with the most frequently made suggestion that the government should make the Academic Aptitude Test, one of the most important and widely used indicators in selecting students by colleges, much easier. This proposal is based on the belief that an easier test will deprive the private tutoring of its determining power of the selection outcomes. More explicitly, one government official who is in charge of college admission policies is reported to have said that “the test shall be composed of such questions as answers to which cannot be acquired by private tutoring.” In most cases, however, a set of easier questions will simply let most students get high scores. It will, therefore, deprive the test of its evaluative or discriminating power, and thereby increase the scope of random elements by which the destiny of the students are determined. This proposal, therefore, recommends nothing but a lottery in a disguised form.

The policy recommendation ardently pushed by the Ministry of Education that colleges should put more weights on the student’s performance in the Essay Test, than those in the English or Math Tests follows the same line of logic. Apparently, it is more difficult to grade essay type questions accurately than the multiple choice questions or any other forms of questions that require simple, straightforward answers. This will necessarily increase the variance of individual student’s test outcomes, again increasing the randomness of the selection results.
In fact, the list does not stop here. Some of the recently adopted solutions to discourage private tutoring, under the name of diversifying the means of student selection, contains such esoteric systems as a system of a principal’s recommendation, handing out preferential quotas to students from remote agricultural or fishing provinces, and setting up a separate screening machinery for students who have recently moved in from abroad, all of which are, in nature, very close to distributing rationing coupons. As a matter of fact, these solutions are just a few of the classic examples of the old truth that the best way to mete out privileges is to randomize the distribution, thus making any systematic efforts and competition unavailing (Hirshleifer, 1980).

We can go one step further. We occasionally meet the following radical suggestion that one has to completely overhaul and abolish the existing rank-oder structure among universities if one really wants to reduce the private tutoring in any significant amount. This is the wrong way to tackle the problem, as will be explained in a later chapter. But the underlying logic behind this suggestion is quite similar to the idea of rationing coupon. Of course, equalization of all the universities was proposed with the intention of resolving the overheated competition to enter so called ‘top rank universities’. Let’s assume that this idea was actually implemented by the government and every university is now identical in every respect. Students will now choose randomly among many available universities. Competition to enter ‘better’ schools will disappear but we still have the competition to enter universities at all, whether they are better or not, because the total supply is far smaller than the demand. And if the decision as to which school a student is to study at can be determined by lottery, there is no reason why we couldn’t just decide by lot the question of who shall have the privilege to study at college and who shall not.

Ration coupons were originally designed to get rid of the unproductive competition among demanders in a market with excess demand (assuming that there is no competition to get the coupon itself). So if private tutoring should be considered as an unproductive rent-seeking activity, the use of coupons or a similar means can be an effective way to reduce private tutoring. Surely, rent-seeking competition will be incapacitated by this randomizing allotment. This is a social gain. However, the inefficiency inherent to price regulation cannot be avoided by mere introduction of ration coupons. First, the dead-weight loss, the unrealized social welfare that otherwise would have been fruitfully materialized is inherent to any binding price ceiling. Secondly, there is a mis-allocation of goods among varying demanders. This comes from the fact that not every student puts the same marginal values on college education. Therefore, higher-valuing demanders may be left out and low-valuing consumer may luckily succeed in getting it. Sure, if the coupon is salable, this mis-allocation can be avoided. But it is very much unlikely that the policy authority would ever allow students to make a transfer of their right to another student, much less sell it for a positive price.
B. Allowing Tuition to Rise

A free movement of price in the market assures a socially optimal amount of supply to be provided in the long run. So it would be advisable not to impose any price or quantity restrictions on the market. But even in the situation where the price can make no effect on the quantity supplied, allocation by means of the price mechanism still helps to enhance the social welfare. A free movement of price in this case contributes to efficiency by insuring the good to end up in the hands of those who will realize its highest value. In addition, it contributes to efficiency by also putting an end to unnecessary and inefficient non-price competitions.

Now let us assume that there are no entrance exams or any other assessment of student qualities in deciding admissions. So only price competition determines college entrance. Colleges ask no information whatsoever on the qualifications of the buyers (not even a high school diploma). And assume that tuition is determined at the point where demand and supply meet. In other words, we are assuming that college education service is just auctioned out.

In this case, those wanting to advance to college have only to pay the price, and need not concern themselves with meeting any other requirements. Therefore, private tutoring will, of course, disappear. If a student desperately wants to go to college, now he/she has to bid a higher price, instead of hiring a tutor to mark a better score in the entrance exam. This seemingly innocuous and highly effective solution will, however, raise the following concerns.

One of the immediate concerns is whether it is really appropriate to distribute college education solely on the basis of the prices that students are willing and able to pay. In reality, educational products have never been distributed this way, so a proposal to let the highest bidder take the prize may sound extremely radical and unrealistic.20) The first criticism to such a proposal would be that an educational service is different from other ordinary goods and therefore, deserves a special treatment, including price subsidy. The second possible criticism would be that only the wealthy will afford an education, and the poor will be deprived of any educational opportunities. Lastly, some concerns may arise about the quality of the student body when the student selection is based on price only, completely independent of any academic achievement that a student demonstrates. I will examine these three potential criticisms in turn below.

The first two concern the issue of educational subsidy. We can, therefore,
examine this issue from the following two perspectives. The first is the question of whether or not the government should financially subsidize college education in general, and the second relates to the question of the need for selective support of poor students and its methods. The first issue is about indiscriminate aid to anyone wanting college education while the second is about discriminate aid to only those with particular characteristics.

1) General Subsidy for College Education

Some people argue that education has a property of public goods. Consequently it is prone to be under-produced and under-consumed if left alone to the market. Therefore, they argue, either the government has to provide it directly or the government has to subsidize its production or consumption. This is one argument for a general subsidy for education. Another argument for a subsidized education is that education, especially the college education, is one of the most effective and surest means by which to improve one’s future income, and therefore, it should be made available to as many students as possible if the government cares to improve the overall income distribution. One way to maximize the number of beneficiaries is to provide again a general subsidy.

a) Externalities and General Subsidy for College Education

As is already well known, the need for general government aid for college education is directly linked to the issue of whether there exist external effects in the process of both production and consumption of college education. That is, if with positive externalities, the government is advised to subsidize the production or consumption of the commodity to boost up the amount supplied and consumed. Otherwise, consumption will fall short of the social optimum when left to the decision of individuals. On the other hand, with negative externalities, the government should impose taxes or some other form of penalties to curb the over-production and/or consumption.

On the issue of externalities of educational goods, the general consensus among economists is that there ought to be a distinction between general elementary education and college education. A general elementary education equips students with the basic knowledge and common understanding needed to get along well with others as a normal citizen. The more equipped citizens are with these basics, the higher the marginal utility of education for each indi-

---


22) Temporarily putting aside the general subsidy for the purpose of improving the income distribution, to which I will come back later.

23) This primarily refers to elementary school, but in a wider sense this may encompass the secondary schools as well.
individual will become. It is a kind of external economies existing in the form of a network effect. Therefore, the need for governmental aid on elementary education is justified24).

On the other hand, college education is, to a great extent, a product that enhances a student’s earning power, the benefit from which accrues solely to the student himself/herself25). Therefore, there should be no harm with sticking to the general principle that one must pay for what one gets26). There is no need for the government to subsidize college students who will apparently benefit from high salaries and other social benefits upon graduation27). Let us imagine that a natural hot spring of great economic value has been discovered in a farm. The owner of the farmland will be able to garner a great profit in the future by developing that hot spring. In that regard, he/she is relatively wealthier than those who do not have the luck of bumping into a similar fortune. No one will argue for a governmental subsidy to the land owner in developing his/her hot spring. In the same sense, college students are people who already have a hot spring of a great potential value within themselves compared to those who haven’t advanced to college28).

b) Income Distribution and General Subsidy for College Education

The same logic can be applied to the argument that the government should aid college education in order to improve income distribution. It is often claimed that college tuition should be kept low so that educational opportunities may become available to a wider range of people because college education is one of the surest ways to enhance one’s future income and social status. This is a slippery argument. Certainly, college education is a very effective means to improve one’s future income. This implies that college students, who have already grabbed the surest means to get rich, are guaranteed higher income, compared to those who have not advanced to college. If aid is needed at all to promote a better income distribution, that aid should be given to those who do not advance to college rather than to college students. If in spite of this,
aid is given to college education, that would simply worsen the situation.

There is another channel through which the government-aided college education aggravates the distribution of income. Students that advance to college come from relatively wealthier families than those who do not. That is, most college students are from middle class families or higher. If this is the case, then the government is giving aid not only to those who will have promisingly high incomes but to those who already have wealthy backgrounds. It doesn’t seem appropriate that taxes paid by parents who can’t afford to send their children to college should be used to aid the college education of those with already wealthy backgrounds29).

What we have discussed above suggests that governmental aid to college education is unnecessary in terms of efficiency and income distribution. In other words, those wanting college education should pay its full price.

2) Selective Aid to Needy Students

Even if an indiscriminate aid in the form of keeping tuition low for everybody is unnecessary, wouldn’t a selective aid to needy students be necessary? The fact that most college students are from the middle class or higher just attests that only the rich can afford college education. Therefore, it would be all the more necessary, some may argue, to have selective aid to needy students for the sake of equal educational opportunities.

The answer to this question can be found again from our previous example of the owner with a hot spring. When the college student is likened to this owner, he/she is not needy at all in terms of his/her expected lifetime income. Suppose the landowner happens to have been destitute. He/she is literally penniless, let’s assume, having nothing other than the ownership of the small piece of land, so small that it has little economic value in itself. Then the news has arrived that his/her land holds an enormously valuable hot spring underneath the surface30). Even if the owner is currently at the point of bankruptcy, he/she is still richer than his/her friends who do now own a hot spring at all. He/she may get a bank loan to develop his/her hot spring by putting up his/her expected future income as collateral. The potential college student may do the same. Still better than that, he/she doesn’t need to wait for the arrival of the good news. The opportunity is lying there for everybody, ready to be picked up by anybody who are intelligent and quick enough just to pick it up.

29) In Korea, it is hard to show a specific quantitative evidence due to lack of data. But in the USA, children from relatively wealthier backgrounds show a higher college entrance rate, and the income gap seems to get wider as the schools are more prestigious. See Bishop(1977), Winston(1999). In the case of other developing countries, refer to Psacharopoulos(1986).

In such a situation, government aid to college education becomes an aid to the relatively wealthy. Johnson(1984), Creedy and Francois(1990), Lott(1990), Fernandez and Rogerson(1995). They view such reverse subsidies as well-established stylized facts and attempt to explain them in the context of political economy.

30) Note that the hot spring is economically valuable only when the cost of developing it into a commercial spa is outweighed by the expected revenue from it. Otherwise, it would be valueless.
We have assumed that college education is something effective and worthwhile to enhance one’s future income. This implies the expected future benefit exceeds the cost of buying and developing. So everything goes as stated above by assumption. He/she may lay his/her future income as security and get a bank loan to pay for his/her education. As more and more students are picking up the golden opportunities, the expected future income stream will decline, and at the same time, the cost of education will increase, making the college education less and less valuable. At the point where cost exceeds the expected future income, no one will, or should pick it up, because the product is no longer socially worth producing.\(^{31}\)

Therefore, the argument for selective subsidization to the needy does not stand either. Of course it is harder for a college student to get a loan than it is for a hot spring owner. If things turn out for the worst, the hot spring owner has some tangible assets which the lending bank can seize while the college student only has human capital, which cannot be seized (unless the slavery system is brought back). The capital market is imperfect as it is. All the more imperfect is a capital market that takes human capital as a collateral. This, however, should not be interpreted as a final death blow to our proposal to deregulate school tuition, because the government can take a lot of complementary measures to make a bank loan available for “needy” students.\(^{32}\)

In sum, deregulation of college tuition in order to eliminate the excess demand for college education seems reasonable as far as economic efficiency and income distribution is concerned. This is especially so if it is complemented with a loan system for needy students. In this case, deregulation will help in reducing the demand for private tutoring because excess demand for college education will disappear. In some sense, the proposed liberalization of college tuition is just a way of channeling towards schools the costs that demanders already bear. The only difference is that in this case, the money goes to the schools instead of private tutors and private academic institutes. If this money flows to schools, it may help to increase the supply of college education, which we will later discuss. On the other hand, if this money were to keep on going to private tutors and institutes, then that would only result in the continued flourishing of the private tutoring industry.

---

31) It is really interesting to examine the common belief that college education is the most effective and surest way to improve one’s future income and social status, for it cannot be true for most people if we have a free open education market. In a free market, any such opportunities for profit must be completely exploited and dissipated. A complete exhaustion of profitable opportunities is indeed a proposition so firmly established in economics that no one would take issue with it. In short, there is no free lunch, a proposition that we are all too familiar with. If the above statement is true, nonetheless, it must then imply that there exist some sorts of rigidities in the market that hinder the flexible and smooth working of the market. Strangely enough, though, not many turn their attention to the implied imperfections of the market when they make such a statement, nor to the logical inconsistency imbedded in their remarks.

32) Kodde and Ritzen(1985). In the USA, a guaranteed loan system is well in force.
3) Deregulation of Tuition and Student Quality

The remaining problem that one may relate to the proposal to deregulate college tuition in order to resolve excess demand and allow the market to settle itself, is that of controlling the quality of college students. In other words, when students are admitted solely on the basis of tuition, we may face the problem of unsatisfactory educational achievement due to the heterogeneity of the students.

Some time ago, there was a suggestion to introduce a new system of the “admission for donative contributors” in a limited scale. It came to nothing, however, due mainly to the tremendous social criticism poured upon it. If the partial adoption of a flexible college tuition system, such as the contributory admissions received so much a criticism, we can easily imagine how much opposition there will be against a complete liberalization of tuition. As we have already examined, the primary basis on which such opposition stands (the concern that educational opportunities will only be available to the wealthy), is indeed ill founded. Keeping tuition low because of concerns that the wealthy will monopolize educational opportunities is in effect, the same as giving aid to the wealthy.

The second basis of opposition against contributory admissions is the concern that the quality of the student body may deteriorate due to the admission of wealthy but less qualified students. In this chapter, we have deliberately put aside the issue of school quality and differences in student ability by assuming that all schools are homogeneous and that so are the students. We will later pick up this issue of heterogeneity in the next chapter, when we discuss the economic role of various student selection criteria, such as entrance exams. Nevertheless, if we lay out the fundamental principle beforehand, it must be that colleges as a whole should open the door to everybody who is willing to pay (there shall be absolutely no excess demand on an aggregate level), but the allocation of individual students among different schools should be determined in accordance to the ability level of each student to secure the relative homogeneity of student body in each school. This is the fundamental principle to be kept to achieve the social optimum in the higher education market. Moreover, this social optimum will be achieved as a natural equilibrium outcome under no tuition control and no enrollment quota control. There will also arise a natural hierarchy in school qualities where both schools and students are satisfied.

Another related issue is whether higher ranking schools should have higher tuition than lower ranking schools in a deregulated market, for instance, 100 million won for first-class schools, 80 million won for second-class schools, and so forth. On first impression, this may sound facetious or, at least, strange. If the qualities of goods provided are indeed different from each other, however, this is the correct principle to be kept, and to be established in equilibrium in unfettered markets. No one in his right mind would argue that a Grandeur and
a Tico should be sold at the same price. If the quality of the product differs, it is only natural that the price should also differ. But again, this is a subject closely related to heterogeneity, which inevitably involves complications, and is therefore, not appropriate to discuss here.

One of the complications that come along with different quality education amongst colleges is that it is difficult to pinpoint the reason for the differences in quality between, say, the first-class schools and second-class schools. For instance, is Seoul National University good because of the excellence of its faculty and facilities or because of the excellence of its students? We may easily say that it is because of both. However, if we acknowledge that the students themselves contribute even in the slightest manner to the excellence of the University, the matter becomes rather complicated. This is because the quality of a product being traded relies on the characteristics of the buyer and not solely on the product’s own intrinsic attributes. In such a case, it is not an easy task to decide how to price a product’s attribute that relies on the characteristics of its buyers. Therefore, we will discuss this issue after we have taken a closer look at the essential characteristics of schooling. Until then, we will proceed to take a look at another way of resolving the problem of excess demand: the expansion of the total college enrollment quota.

C. Expansion of Enrollment Quota

Another way to resolve the excess demand for college education is to increase the supply. We have seen that university entrance competition rate was 2.8:1 in 1999. The table below shows the movements of the recent competition rate since 1995. With the lowest being 2.8:1 and the highest being 3.2:1, the number of available spaces in universities has always been considerably smaller than the number of demanders, with the gap sometimes widening to over 580 thousand. There would probably be no other markets with so many unsatisfied demanders. It would be improbable to expect the overheated competition to somehow subside in such a market where there are so many unsatisfied customers craving to buy at the current on-going price.

Needless to say, such an insatiable hunger for higher education is created and maintained by the price and enrollment control in the market. Whether we like the expected outcome or not, it is clear that excess demand will not resolve itself completely unless price is liberalized. In this case, we can easily predict that in a market with such a huge excess demand, price will instantly soar to an


34) Perhaps the market for newly built apartments may be another possible example. No wonder that we have also price control in that market. One would naturally wonder what real policy objectives the relevant authority has in that apartment market: an increase in the total supply of housing or a perpetuation of housing shortages.
enormously high level. Therefore it seems inevitable that in the long run there must be an increase in the amount of supply. Of course, due to a rapid decrease of the population in that age group, the excess demand is expected to drop significantly in just a few more years. Some colleges, especially junior colleges in the provincial areas, may soon find it difficult to secure students. Nevertheless, if we confine ourselves to the four-year college, it appears that there still remains a substantial amount of excess demand.

Then by how much should we increase the total enrollment? The answer to this question, an economically sensible one, is unbelievably simple and clear: the supply of college education must be expanded up to the point where the marginal value that demanders of college education are willing to give up to get one more unit of education, is equated to the marginal cost, or the amount of other resources that a society has to give up in order to produce one more unit of college education\(^{35}\). The tragedy is that this simple truth is all too often forgotten or ignored in most of the discussions on college education and private tutoring\(^{36}\).

<table>
<thead>
<tr>
<th>Year</th>
<th>A/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>3.1</td>
</tr>
<tr>
<td>1996</td>
<td>3.2</td>
</tr>
<tr>
<td>1997</td>
<td>2.9</td>
</tr>
<tr>
<td>1998</td>
<td>2.9</td>
</tr>
<tr>
<td>1999</td>
<td>2.8</td>
</tr>
<tr>
<td>2000</td>
<td>2.9</td>
</tr>
<tr>
<td>2001</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Source: Korea Educational Development Institute

\(^{35}\) Let the utility function of an individual, \(i\), be given as \(U_i(X_i,E_i)\). Here, \(X\) is all other goods and \(E\) is the college education. Let’s assume that social production function of educational services is given by \(G(X,E)=0\). A social planner wanting to maximize the total sum of the individual utilities must solve the following optimization problem,

\[
\text{Max} \quad \sum U_i(X_i,E_i) \quad \text{subject to} \quad G(X,E)=0 \quad \text{where} \quad X=\sum X_i \quad \text{and} \quad E=\sum E_i.
\]

The first order condition is then,

\[
\frac{U_i}{U_j} = \frac{G_i}{G_j} \quad \text{for all} \quad i.
\]

In other words, the marginal rate of substitution between \(X\) and \(E\) must be equal to the marginal rate of technical substitution between the two for every individual.

\(^{36}\) Of course, this argument is made under the tacit assumption that there are no externality effects resulting from the consumption and production of college education. We have already discussed the externalities in consumption, concluding that practically no externalities are found. We will assume that there are no externalities in production, either, since it is hard to presume any particular externalities associated with the production of college education. If there do exist externalities in production, we may have to assume that the government can correct them.
1) Social Welfare Loss Due to the Enrollment Quota Control

Now let’s take a look at the social loss our society is bearing by imposing an inappropriate regulation on the total number college students that our society can have. Because we are not sure of where the current situation stands, we will explore three different possible cases, as illustrated in the graphs below.

In the figures, \( N_r \) stands for the regulated level of college enrollment. The panel (a) of <Figure 2-2> is simply a reproduction of <Figure 2-1>. It depicts a market where there is no regulation on the total number of students, but tuition is regulated below the market equilibrium level. In this case, the deadweight loss due to the regulation is represented by the triangle abc. This is the social loss resulting from the unrealized transaction. It is very likely that an additional loss will develop, a loss due to a non-price competition to grab the underpriced goods ahead of other people, which will differ depending on the methods of how this limited amount of goods is distributed among contending demanders.

If the selection of winners is done in a completely randomized manner, so that a buyer can affect his winning odds in no way whatsoever, the additional loss will be zero. On the other hand, if the buyers engage themselves in unproductive competition to appropriate the potential individual gains, the loss can reach the total of rectangle \( P_1P_0ba \). This is a topic to be discussed in more detail in Chapter 3. In the meantime, we can easily infer that under this situation, colleges will keep on petitioning the government to allow them to increase the tuition. It is also clear that a tuition increase in this case will surely contribute to reducing the social loss incurred.

Next, diagram (b) depicts a market with regulations on both tuition and enrollment quota. Note that \( N_r \) is smaller than \( N_s \). Therefore, the tuition controlled is higher than the marginal cost at the current level of enrollment quota. The deadweight loss due to frustrated transaction in this case increases further from abc to dec. The incentive for non-price competition also increases because the gap between the marginal value from a college education and the regulated price widens from ab to df. Since the tuition that colleges can charge exceeds the marginal cost, colleges have an incentive to increase its enrollment. Consequently, colleges will request for an increase in quota. Colleges would also strongly request permission to increase the tuition because they would have no problem securing applicants even after a small increase in tuition.

The panel (c) also represents the case where both college tuition and quota are regulated but where the quota has been set at such a high level that it cannot meet the college’s production costs. In this case, colleges would need government aid in order to cover its deficit. Colleges will thus request a raise in tuition while having an incentive to cut down the number of students they admit.

In all, diagram (b) seems closest to reality. Although aid has been given to colleges, it is quite negligible in most cases. Furthermore, most supports have
been earmarked for special purposes, mostly for scientific researches and are not a general financial subsidy to cover the cost of providing the college education itself. Another circumstantial evidence is that colleges have been continuously requesting an increase in enrollment quota. If this is the case, which seems most likely, then it implies that we have the worst case in terms of both social welfare loss and the size of incentives for unproductive non-price competition. This is because we are adding more excess demand by enrollment quota to the already existing one due to tuition control.

2) Possible Reasons for Imposing Enrollment Quota

a) Concern for the Oversupply of College Graduates

Why is then such an enrollment quota being imposed? One of the reasons may probably come from the concern of an oversupply of college graduates. When there are too many college graduates churning out of schools, it will cause, some may argue, massive unemployment, and thus creating a social problem. Authorities are often very much sensitive to such a criticism that they are mass-producing over-qualified bums by indiscriminately increasing the quota. Actually there have been repeated claims that the overall number of college

<Figure 2-2> The Welfare Loss due to Enrollment Quota Regulation
graduates in Korea is not that small. Compared to other countries, Korea’s college advancement rate out of the proper age-group is one of the highest in the world, they claim.

It is apparent, however, that such an argument is hardly tenable at least in terms of economic efficiency. Unless we assume that authorities know better than the individual economic agents about what is best for them, there is no standard for us to use in determining the social optimum other than the marginal value that is expressed in monetary value by the agents, and the marginal cost of production. For a social optimum, the basic proposition “demander’s marginal value = price = producer’s marginal cost” must hold in every commodity. To produce what, and as much as the buyers want, is the social optimum.

Another baffling thing is that the logic behind the concern about the over-supply of college graduates stands in a stark contradiction to the logic behind the claim for governmental subsidy to college education. The latter argues of the possibility that when people are left on their own, they will not consume enough education because they do not fully perceive its benefits, whether individual or social ones. Therefore, the government should give subsidies, the argument goes, in order to prop up consumption to the socially optimal level. On the other hand, the former argues that people will purchase too much education when left on their own. If the former is the case, then the government should impose a tax on educational consumption rather than subsidizing it. The logic behind restricting the total number of college graduates is, therefore, utterly contradictory to the logic for the necessity of subsidization of college education.

Contradictory as they may be, people never stop emitting two mutually incompatible statements: we need to regulate the total number of college graduates within a “reasonable” limit, and at the same time, we need keep the tuition at some low level, so that college education is made available to as many students as possible, or available just at a relatively cheap price where the number of students is not the major concern. Furthermore, these two antithetical propositions are sometimes combined mysteriously into one and inadvertently disclose another hidden idea, so common and so powerful that people often do not care where it comes from and how it fits the general logical framework they have. It is the idea of “educating a small minority of elites with a concentrated public investment”. It is certainly an enchanting idea which deserves a scrutiny in its own right. What is clear and worth commenting at this particular context, however, is that one cannot stretch the cause for a concentrated investment to the case of general college education today, which is so universal that one can hardly call it a small minority elite education.

b) Quality Control of College Graduates

The second reason for regulating the number of enrolled students may be
the concern that an increase of quotas may deteriorate the quality of education services and students. Again, it concerns the quality of educational products which is not a proper subject to be discussed in the current chapter. It is worth noting here, however, in reality, it may constitute a valid concern. We can easily guess that the average quality of college graduates will deteriorate as the number of college students increases. However, the problem of guaranteeing a certain level of quality should, in principle, be solved through competition in the market. If consumers want to have a high quality education, then producers will provide it for the best possible terms via competition.

It is true that, in a certain sense, the quality competition in the education market is likely to be more limited compared to other markets. For example, in general terms, educational goods are experience goods in the sense that its value cannot be ascertained before having actually consumed the goods. In addition, the majority of the demanders will buy a certain level of education only once in their lifetime. It is very unlikely that one will attend the same secondary school twice, once he/she is graduated from it. Therefore, one may argue that the enforcement mechanism to supply a high quality education is rather weak in the education market. The punishment that a dissatisfied customer can impose on the unfaithful supplier, thereby taming and disciplining the producer into continuing to provide good quality goods, is very limited.

There are, however, even stronger countervailing forces in the education market which make quality competition feasible. Firstly, the education industry is a sort of equipment industry. And the assets invested in the education industry have specificities that cannot easily be transferred to other industries. In other words, the salvage value of its assets is quite small. Therefore, if a school loses its reputation by deceptively providing a low quality education, it will have to bear a tremendous amount of loss. Secondly, most school education involves putting a large number of students together into a common educational process for a relatively long period of time. This makes news about quality deterioration easy to spread by word of mouth, thus imposing an effective constraint on deviant behaviors. The fact that education involves a relatively long-time interaction between the producer and consumers also put a very effective pressure on suppliers about their implicit quality standard. Actually, the reason why people are not greatly concerned about the shirking behavior of schools may be because of this long-term interaction, specificity of its productive inputs and the word of mouth effect.

This will allow us to exclude the possibility of a one-time glut of low quality products. The remaining concern about deterioration of college education would then be a concern about the wide distribution of quality spectrum. The market would have various products whose quality varies, from top quality to bottom. Some may view it as a failure of market, and argue for a more strict quality control. In fact, we have often heard such arguments, especially when it comes to the matter of expanding the enrollment quota of some vocational...
schools such as medical schools and law schools. We are fully aware that most of the vocational licensing system are indeed introduced on this seemingly persuasive ground of quality control.

If such is the concern, however, it must be said that the basis is really unacceptable. The top quality products, which producer groups or relevant authorities are anxious to provide, are not the only products consumers want to have. They also want to have low quality products. They want, at least, a wider freedom in their choices. In the automobile market, a variety of passenger cars are provided, ranging from the Ticos to the Grandeurs. Surely, the more expensive a car, the better it will be in terms of its functions and safety characteristics. There are, however, millions of consumers who cannot afford the Grandeurs. One should not force consumers into consuming only the safest, top-quality car, no matter how good and genuine an intention he/she may have. Not only in automobiles, the same principle must be applied to every product that varies in quality, if one is really concerned with individual welfare. There is no reason that a uniform quality standard should be imposed on educational goods.37

In some sense, the attempt to regulate the quantity supplied in order to maintain the quality is very similar in structure to the attempt to limit the quantity in order to raise the monopoly rent of those engaged in that industry. As mentioned above, the majority of vocational licensing systems are all alike in this regard. It is, therefore, really worthwhile to ask for ourselves whether there is a tacit or unconscious plot to maintain the high monopoly rent that college graduates are now enjoying when one tries to put a shackle on the supply of college graduates.38

37) Note also that there is no serious informational asymmetry between schools and students about the quality being provided. Students are quite quick in figuring out what kind of quality product is provided by which school. Recall what kind of quality hierarchy amongst high schools used to have before the introduction of the secondary school equalization measure. Everybody had then a fairly clear idea what kind of education he/she would get, and what the ultimate outcome of that education would turn out.

38) The Economist magazine (1995) once featured a long analytical article on the expansion of the governmental sector in western countries. The gist of argument is as follows. Although capitalist countries have in theory accepted the proposition that a “small government” is desirable, the government sector in reality has continuously grown in most of these countries during the past 50 years. Such an expansion can be viewed as a natural outcome that comes from each government’s efforts to cope with the income distribution problem which the market cannot solve on its own. If we look into who the real recipients of the ever expanding public expenditure have been, however, it turns out that, surprisingly, the middle class families rather than the low class families have gained the most in almost every country. Many scholars these days conjecture that much of these retrogressive effects of financial expenditure has its roots in educational subsidy and quantity regulation on higher education. See footnote (25) and the literature cited therein.
Up until now, we have analyzed the education market and the phenomenon of private tutoring under the assumption that educational goods are no different from other ordinary goods such as automobiles, gasoline, etc. In doing so, we have been dealing with a world where there is no entrance exams nor requirements, such as high school graduation, in order to advance to college. Admittedly, such an assumption is very much unrealistic. In investigating the question of a social optimum, such as how much of college education service should be provided in aggregate, however, this simplifying assumption has been very useful. It is because the assumption allows us to see in a clear-cut manner the fundamental relationship between the marginal value, the price and the marginal cost necessary to achieve a social welfare maximization.

In this chapter, I will relax the assumption of the previous chapter and look into how college education (or education in general) differs from other products. I will also try to relate the newly found perspectives gained through the investigation to private tutoring problem. To be more specific, I will examine the character and role of entrance exams being used by the majority of educational organizations. Secondly, I will discuss the significance of the equalization policy currently in effect at the junior middle and high school level, and the problems frequently referred to as “the obsessive pursuit of the first class schools (一流病)” and “the hierarchical ranking of colleges (大学序列化)”.

3-1. Special Features of Education

There is no college or any other public education organization in the world that selects its students solely by the price the students are willing to pay. Students usually have to pass through some sort of screening process before they get admittance to a school. If so, what is the role of this pre-assessment or the entrance exams adopted by some schools as one form of this assessment? If we look at education as a kind of commercial product, an exam is a means of limiting the qualifications of its buyer. Why and when is such a restriction on the buyer necessary in commercial transaction? Of course in the case of educational organizations, one simple answer is that it is conducive to maximizing the educational outcome to have a homogeneous student body. Why is it then
necessary to insure the homogeneity of the student body? Are there any instances of a similar buyer selection in other commercial transactions?

In order to answer these questions we need to examine the attributes of the educational good a little bit more closely. Until now we have assumed that the buyer gains Et amount of educational service upon purchase. We have also assumed that all provided educational services are homogeneous, and, therefore, there is no difference among buyers in the quality or quantity of the education that they receive. Such assumptions are unrealistic in the following two senses: Firstly, as mentioned earlier, the content and quality of education provided by different educational organizations vary substantially. Secondly, the actual benefit of educational services that an individual student can enjoy may be all different depending on who the student is and how actively he/she participates in the educational process. Some may gain a lot, others little. So it appears that there is no fixed Et that every student can enjoy invariably.

Actually, the good called education is unique in many ways. In the case of other regular goods, the power of the goods themselves to increase the utility of a human being is transferred to the buyer at the moment of purchase. In the case of education, however, there is nothing transferred upon the payment of tuition, no tangible product nor increased utility. In this sense, an educational good is not a good that one can purchase, and then freely dispose of at one’s own will in order to increase his/her own satisfaction later. To be closer to reality, an educational good is a product jointly produced by both supplier and demander, each of whom puts one’s own input into the production process, and where ultimately, the demander gets what is being produced. We can summarize this as below.

**Characteristic 1**: Educational products are special goods produced jointly through a continuous interaction of both the selling school and the buying student.

The fact that the education process is a cooperative productive activity which involves the participation of many inputs, especially those by the school and students, inevitably creates many complications. Just to take a few examples, how are we to measure the respective contributions each productive input has made? How are we to control the quality of each input and/or output? Who is going to have the final claim on the products produced and how? In short, most of the issues associated with the well-known, fundamental question, “what is a firm or a team production, and why does it exist? will arise.

---

39) We have said that what a student acquires from college education is an increase in his/her human capital. But to be more accurate, it may be just "the right to occupy a seat in the lecture room" that he/she actually acquires. The school has no control nor can it guarantee what the student accomplishes in his/her given space.

40) Rothschild and White(1995) refers to such a characteristic of education as a "transaction that has customers as inputs."
All these issues roughly come down to the problem of how the transaction outside of a firm is different from the transaction within. In light of such a dichotomized framework, the representative characteristic of market transactions is the aforementioned “principle of anonymity”. In other words, a market transaction is a standardized transaction that accurately defines and measures who gave whom how much of a product under what conditions. In such a transaction, it is not the business of the seller to gain the knowledge of the personal attributes of the buyer. He/she only needs to hand the product over to the highest bidder. This is because everything that has to be known in trade is already and accurately reflected in the market price.

The transaction within an organization involving co-participation of many inputs is quite different. First of all, the owner of the enterprise cannot accurately control nor measure the amount of inputs invested (the more so in the case of human capital input). Moreover, it is equally hard to measure the contributions of each input. Therefore, it is difficult to have a standardized trade that is strictly based on voluntary agreement at every instant of transaction. Instead, a rule of command and control takes the place of price as the practical means to secure coordination amongst many participants. That’s why enterprises screen job applicants and select only those workers who they believe can easily adapt to the working conditions of their firms. In other words, no enterprises hire a worker on the sole basis of how little salary that worker is willing to work for. They will invent various qualification requirements to see how well the worker is going to get along with other co-workers, how compliant he/she would be at his/her boss’s beck and call, how much he/she is motivated to work in a particular line of tasks, etc. That is, a worker’s personal characteristics and its correct assessment takes a special meaning of great significance in this transaction.

So we can conclude that an examination process of the students’ personal quality is inevitable in educational markets because the educational good is a product jointly produced by the school and the student42).

Now let’s take a look at the second characteristic of educational goods. In order to do this, I will dig one step further down on on the characteristic of the joint production. The production function in which both the seller and the buyer injects their inputs can be expressed in the following way.

\[ E^t = f(S, B^t) \]  \hspace{1cm} (1)

41) Coase(1937), Alchian and Demsetz(1972).

42) There are many other examples of pre-assessment, similar to those in the labor market. Membership restaurants limit its customers because there also exists an element of joint production. A particular ambience of the restaurant is jointly produced and consumed by the participating members. Another mundane instance is the careful choosing of one’s spouse in a marriage. No one chooses a spouse in an auction market, marrying whoever turns in the highest bid.
Here, $E_i$ denotes the amount of educational service student $i$ gets through education. $S$ is the vector of the various inputs the schools brings forth into the production process and $B$ the input vector a student brings forth with him. $S$ would include school buildings, facilities and equipments, educational materials, lectures, etc., while $B$ would include the student’s class attendance, his personal effort, and his level of ability.  

Now let us look at how the above education production function will generally look like. Although the form would vary depending on what variables are included in $S$ and $B$, and how those variables are measured, we can easily assume the following relations.

1) $f_S > 0, f_B > 0$, 
2) $f = 0$ if either $S = 0$ or $B = 0$, 
3) $f_{SB}$ = ?

Expression 1) implies that the educational achievement increases as the inputs of the school and the student increase. Expression 2) says that if there are no inputs, then there are no outputs. Mathematically, it only determines where the origin is located and has no significance in itself. Expression 3) is the cross effects between the inputs by school and students. Although it seems most probable that we will have $f_{SB} > 0$, it may not always be the case as explained below.

Let’s consider the following two possible cases. The first case is when we have $f_{SB} > 0$. In other words, the marginal product of the student’s input is an increasing function of the school’s input. A good example of this would be the conscious effort of a teacher and a student. With a given amount of the student's effort, the achievement will increase as the teacher exerts more effort to teach his/her students.

[43] We will hereafter omit the subscript of $E_i$ whenever little confusion is worried.

[44] In most cases education is conducted collectively with a simultaneous participation of many students. In such a case, the amount of educational achievement by student $i$ may also be influenced by the inputs which other students bring with them. If $B_j$ is the input elements of those other students besides student $i$, we will have

$$E_i = f(S, B_i, B_j).$$

In this case, the $B$ term will represent the so-called “peer group effects”. Summers and Wolfe (1977), Epple and Romano (1998).

[45] One may say that what a student is in fact buying in $E = f(S,B)$ is not $E$, but $S$, the school’s input, which is partly true. As a matter of fact, it is not really clear who buys what in this formulation. In the case of hiring a private tutor, it is apparent that the student is buying the tutor’s input $S$ rather than the final product $E$. On the other hand, in the case of public education such as schooling, it appears more like the student is purchasing the final product $E$. The reason for such a difference in perception may be due to the difference of who initiates, controls, and manages the overall educational process. This question of who the initiator is will of course determine who should assume the responsibility for the quality of the final product. In any case, we will ignore such distinctions in this thesis.
There may be a case in which \( f_{SB} > 0 \) does not hold. Take for example the level of the teacher’s lecture and the level of the student’s intellectual capacity. When the student’s intellectual capacity is yet at the level of kindergarten, but the teacher’s lecture is tuned to the college level, the resultant educational output would be close to zero. Vice versa, lecturing a college student on an elementary school arithmetic would render the net output virtually to zero. In other words, not the absolute level of the inputs that the school or a student brings in, but the relative gap between the two may exert a decisive impact on the educational attainment students can fruitfully enjoy.

The fact that educational achievement can be maximized when the level of the student input is exactly matched by the level of school input may be due to another unique characteristic of education or the human capital that grows through education. Let me elaborate more on this.

We have formerly defined an increase of the human capital obtained through education as,

\[
H_{t+1} = H_t + E_t. \tag{2}
\]

This suggests that the human capital is something that accumulates over time\(^{46}\). The human capital, especially that part of which is directly linked with education has another facet that cannot be explained by this simple accumulation equation. More specifically, although a human capital is accumulated through education, there is an order or a sequence of accumulation that must be strictly followed. For instance, let both the increase in human capital that can be gained through kindergarten education and the increase that can be gained through college education be denoted by \( E_t \). Although the magnitude of increase may mathematically be the same, the order of the two increases cannot be switched. One cannot receive college education before receiving kindergarten education. We will summarize this as below.

**Characteristic 2**: For human capital accumulation, a certain level of precedent stages must be completed before advancing to the next stage because there is a certain order to be followed.

This means that \( E_t \), the achievement a student can garner through education, is greatly influenced by \( H_t \), the level of human capital the student already has with him/her at the moment he/she is about to advance into the next stage of educational processes. In other words, \( H_t \) must be included as one of the key variables in \( B \), the set of input variables brought by the students. Lastly, through a casual observation of the real-world education processes, I deduce

\(^{46}\) Assuming that there is no decay due to oblivion or deterioration.
the following two more characteristics.

**Characteristic 3**: A school education typically involves a simultaneous participation of a large number of students.

A collective education of a large number of students certainly has something to do with the technical conditions of education production function or cost function. For instance, if the schooling cost function has the conventional U shape in terms of the size of the student body, we can certainly find a cost-minimizing class size. One simplest function with that property is \( C(n) = F + V(n), \ V', V'' > 0 \) where \( F \) is the fixed cost and \( n \) the size of the student body. Then depending on how the marginal revenue behaves with respect to \( n \), an optimal class size can be pinned down in the neighborhood of the cost-minimizing point.

**Characteristic 4**: Students are heterogeneous in their \( H_0 \) or their learning capability or preparedness.

In other words, students with various levels of human capital are simultaneously participating in the education process. Let the distribution of the student’s human capital be given by \( g(H) \). We will assume that \( H \) lies in between \([H_0, H]\).

### 3-2. Economic Role of Entrance Exam and Self-Selection

#### A. Economic Role of Entrance Exam

Combining the first two special features of education, a joint product by both school and student and its dependence on the stock of the individual’s human capital at the time of entry, I specify an education achievement function as the following:

\[
E_i = f(\theta, H^i) = aE - \beta(H^i - \theta)^2, \quad a, \beta > 0
\]

(3)

In the above, \( \theta \) represents the level of education a school provides to every student who is enrolled. It may be interpreted as an index representing the quality of a particular school. \( H^i \) represents the human capital stock a student i

---


48) In the long run, if the marginal revenue curve is flat, the optimum class size would coincide with the cost-minimizing one. If instead, the marginal revenue curve is downward sloping, the optimum would be smaller than the cost-minimizing level, resulting in an over-capacity.

49) I assume that all the other variables that influence educational achievement are fixed.
carries with him/her. Given this, whenever there is a mismatch between the level of individual human capital stock and the level of school education, the student has to pay some penalty in the form of reduced achievement. \( \alpha E \) represents the amount of education a student \( i \) can get when there is a perfect match between \( H \) and \( \theta \).

With a given set of student body, let’s assume a school tries to maximize the total sum of educational achievement of the participating students by setting its \( \theta \) at the optimum level\(^{50} \). The school’s objective function is then given by\(^{50} \),

\[
\text{Max} \quad \int_H \left[ \alpha E - \beta (H - \theta)^2 \right] g(H) \, dH. \tag{4}
\]

If we solve (4), the optimum level of education, \( \theta^* \), is simply \( \mu_H \), the mean value of the student distribution. In other words, in order to maximize the aggregate educational outcome of the whole student body, the school has to attune its education level to the average of the student ability distribution.

\[
\theta^* = \mu_H \tag{5}
\]

In this case, those students whose \( H \) value is far from the mean have to bear a loss in the form of a smaller educational attainment. The total sum of the loss to the whole student body is

\[
\int_H \beta(H - \mu_H)^2 g(H) \, dH = \beta \sigma_H^2, \tag{6}
\]

where \( \sigma_H^2 \) is the variance of the distribution. That is, the aggregate loss is proportional to the variance of the student distribution. This makes sense. If there is only one student who hires a tutor to get a private education, the discrepancy between his/her capacity and the tutor’s lecture would be zero, and therefore, the loss would be zero. Reducing the class size would have similar effect: a lower variance and thus a smaller loss\(^{53} \).

\(^{40} \) Economics of Private Tutoring

\(^{50} \) Arnott and Rowse(1987) have a similar education production function. Theirs is different from mine in that they have the mean and variance of the student ability distribution as independent variables. In other words, the educational achievement is not influenced by how far the student’s individual ability is from the school mean but by the mean itself. For a general introduction to the form of educational production function, see Hanushek(1979, 1986).

\(^{51} \) This may sound innocuous, but not necessarily so. If a school is concerned with the number of students who are successfully advancing into a prestigious university, it may choose to concentrate its efforts only on those students with high ability. Or, if a school is run with an egalitarian spirit, it may pour its resources on caring for those students with low abilities.

\(^{52} \) Assuming that there is no cost for the school to select the optimum education level.

\(^{53} \) Rooser(1995) and Krugger(1999) demonstrate that the class size has a significant impact on the educational achievement of students.
If we can divide the student body into several smaller classes, how should this be done and how should we determine the level of education for each class? For simplicity, let’s assume we have only two classes. One possible way of dividing the students is a randomization. The middle and high school equalization policy adopted in early 1970s, which is still in effect, is a good example of this. A better way, however, would be to divide the students into two groups in accordance with the individual student’s ability and to differentiate the education level to the mean of each class.

It is straightforward to prove the above contention. When two different levels of education, \( \theta_1 \) and \( \theta_2 \) are feasible, the way to maximize the aggregate educational achievement (ignoring the fixed) or to minimize the aggregate loss, is obtained by solving

\[
\int_{-H}^{H} \frac{\theta_1 + \theta_2}{2} g(H) dH + \int_{H}^{\infty} \frac{\theta_1 + \theta_2}{2} g(H) dH.
\]

The solution is as we had expected. To divide the classes according to the level of the students and then to adjust the level of education in each class to the mean of each:

\[
\theta_1 = \mu_1, \quad \theta_2 = \mu_2
\]

where

\[
\mu_1 = \frac{\int_{-H}^{H} \frac{\theta_1 + \theta_2}{2} g(H) dH}{\int_{-H}^{H} g(H) dH}, \quad \mu_2 = \frac{\int_{-H}^{H} \frac{\theta_1 + \theta_2}{2} g(H) dH}{\int_{-H}^{H} g(H) dH}.
\]

So natural and straightforward is the conclusion that we wonder if any other claims can ever stand against it: it is optimal to rank students according to their level of \( H_1 \) and then to divide the students into two separate classes.

Obviously, the above result is not limited in applicability only to the case of dividing classes within a single school. This indeed explains why we have such a hierarchical division in the educational system (as preschools, elementary schools, secondary schools, etc). Also, the division of grades within a school, such as by first graders, and second graders, can be explained by the same terms in principle. As long as the proposition that the additional \( E_t \) that a student can get through education is dependent on his/her human capital level \( H_t \) stands, it is best to conduct education in accordance with that level of \( H_t \). It can be concluded that this is the fundamental reason lying behind any hierarchical division of the student population according to individual characteristics.
The above result also explains why some goods are not distributed solely by price but by additional selection devices such as exams. Exams are a means of measuring the level of the students’ preparedness, and thereby, they minimize the dispersion of the students’ absorption capacity within a class. This will help to increase the aggregate educational attainment in a most efficient way. Of course, this does not necessarily mean that every school should have an entrance exam. Be it a report on earlier academic performance or any other attestations bearing on his/her future prospect, schools need some form of sorting machinery in order to ensure the homogeneity of its students.

It must be noted that the sorting here is different from the former chapter’s screening, which was adopted to ration out products in excess demand to a limited number of people. Here, the sorting is practiced to ensure the homogeneity of inputs in a production process, not to distribute goods that have already been produced. In the former, an access is completely denied to those who have failed the selection test. That’s the sole purpose of the test, a denial. In the latter, however, the prime objective of a test is not a denial but an optimal deployment. The student who turns out to have a little bit less preparation than is appropriate for a particular school will simply be sent to the next school with a little bit lower level, or a slower pace of teaching. Anyway, that next school would have been the very school that he/she would voluntarily have chosen because that is the best choice he/she can make in an ideal situation54).

In a sense, the difference between the test for denial and the test for an efficient deployment is similar to the difference between a vocational license and a certificate. Under the licensing system, anyone who does not have a license is strictly prohibited from practicing that particular business. On the other hand, no one is denied practicing the particular line of business under a certificate system. The certificate is simply a means of demonstrating his/her level of qualification to potential customers. He/she can display his/her certificate if he/she wants to do so, or withhold the information if he/she does not want to reveal it55). Take, for instance, the KS (Korean Standard) or GD (Good Design) mark. A manufacturer of a KS marked product will willingly advertize that his product has acquired the KS mark. This doesn’t mean, however, that a similar product with no KS mark should not be offered for sale. You can sell any products, whether it has the KS mark or not in the market, and customers will become the final judge on which is the best for their individual need. Those who are wealthier and, therefore, can afford an expensive and higher quality good may purchase the KS marked one, and those who cannot afford (or have a better knowledge or skill of how to complement the deficiency by their own effort) may choose to buy the unmarked one.

The policy implication that such a difference carries with it can be summa-
rized as the following: there is no need to impose a test or any form of pre-assessment in determining the total number of college students a society should have. As long as students are willing to cover the cost that society has to bear to supply educational products, college entrance should be made available to everybody who wants it. Once the total number is determined this way, then students’ qualification should be taken into consideration to allocate them among different schools.

B. Which School Is a Good School?

The above analysis also provides us with a stepping stone to another issue mentioned earlier: the question of what makes a school good and prestigious. Is it because of the excellence of the enrolled students or because of its excellent facilities and faculty? Here we assumed that all schools provide the same amount of educational services $E$. (disregarding the individual difference in qualification or readiness). In this sense, we can say all schools are homogeneous. But when we observe the average quality level of students in each school, there apparently exists a ranking among schools. One school can enroll students with the highest $H$, while another school can enroll mainly students with the next highest $H$, and so on.

In light of this, is a school that has a higher average of $H$ better? Probably, in a conventional layman’s sense, but not so in a strictly economic sense. To simplify, let’s assume that there are only two types of students: one with $H^\text{low} = 100$, the other with $H^\text{high} = 110$. Each group attends a different school. Let $E^\text{t}=20$ in both schools. After a successful completion of all course work, they will each come out of school with $H^\text{t+1}_{\text{low}}=120$ and $H^\text{t+1}_{\text{high}}=130$. The latter school has a higher mean of $H$, and thus higher $H^\text{t+1}$. Consequently, the graduates from the latter school will obtain better paid jobs or advance to more prominent positions after graduation. Most people will call the latter school better because its graduates are on average registering more splendid and eye-catching achievements. This is a superficial observation, though, because it does not take into account the initial $H$ at the point of entrance, and only focuses on the final $H^\text{t+1}$ at the point of graduation. The schools’ contribution to the students’ human capital is not the whole $H^\text{t+1}$ but only its increasement, $E_t$, which is the same by assumption.

Economically speaking, both schools are the same. To make the argument more striking, let $E^\text{t}_{\text{low}}=20$ and $E^\text{t}_{\text{high}}=15$. At the point of graduation, therefore, we will have $H^\text{t+1}_{\text{low}}=120$ and $H^\text{t+1}_{\text{high}}=125$. The latter school has still better qualified graduates, and consequently more conspicuous achievements by its graduates. Nevertheless, if the cost of education is the same in both schools, the former school is more efficient in generating the value added in education. Many people, however, will still continue to call the latter a better school, which is nonsensical from an economic point of view. Productive efficiency must not be measured by how high the quality of a final
product is but by how low its per unit cost is.

C. Self-Selection and Competition to Enter a Better School

Until now we have tacitly assumed that it is the schools that initiate and conduct the sorting and screening, and then allocate the students to appropriate schools. However, this does not necessarily have to be the case. The result would be the same if students were asked to choose their own best schools or classes. Students will choose a school that are closest to their $H_i$ level because attending a higher quality school whose curriculum they find difficult to keep pace with, would not help improve their academic achievement. In fact, that is what we observe in the private tutoring market. When students register for a private academic institute, they usually exercise such self selection. Private academic institutes have rankings of their own, and even within an institute, there are usually several classes of different level for each subject. Students then choose the institute, the textbooks and the class closest to their own level.

In short, no one would purposely put away the textbook that best fits their intellectual level, and instead, choose the most difficult to read. Or no freshman would sit in a sophomore class, knowing that he/she would have difficulty in understanding the subject matter discussed in the class or follow the pace of progress. Suppose that there are 100 students all different in ability, and four teachers. Imagine that the students are then asked to break into four different groups on a strictly voluntary basis. In this case, we can easily conjecture that in the final equilibrium, each group would have exactly 25 students, with the quality being distributed in an orderly ascending (or descending) manner. It is in their interest to follow this efficient rule.

However, the above explanation leaves us with the question of why we need examinations at all if students can self-select the school that best suits them. One easy answer would be that it is hard for the student to evaluate his/her own overall abilities. Therefore, the schools provides such evaluating service in lieu of students. The preference for top ranking schools in this case can be explained as a simple desire to “confirm one’s own ability in an objective manner.” However, this doesn’t seem to fully explain the fierce competition to enter top-ranking schools. As we have seen before, students do not prefer top-ranking private academic institute when they choose an institute. They self-evaluate their own preparedness and select what is best for them. As a consequence, seldom does a private institute administer an entrance exam. If

---

56) The subject matter of this section will be discussed more rigorously in Chapter 5.
57) I am assuming that there is no economic rents associated with a particular name of school, especially top rank schools. I will elaborate on this later.
58) The student at the margins will be indifferent in joining either a higher quality class or a next lower one. Note, however, that I am completely ignoring the psychological factors: some may find a psychological satisfaction from being a head rather than a tail.
the difficulty of self-evaluation is the sole reason for having entrance exams in schools, then private institutes should render the same service but it appears they seldom do59).

Another possible answer would be that it is profitable to enter better quality schools, so the students swarm to these schools, and schools can apply various selection mechanisms. There can be a variety of cases in which an attendance at a particular school becomes profitable: the tuition may be particularly cheap, the quality is better for a same price, or there may be a significant economic rent bestowed upon the title of top-ranking schools. But we will deal with this issue more systematically later in the section when we discuss “the first-class school diploma disease (一流病)”.

Before we go on to the next section, however, let us just look at a few more educational markets that do not have additional selection criterion other than price because the students’ self-selection mechanism is well in force. One instance would be the book market. One may call a book a typical educational good. Nonetheless, there is no ongoing interaction between the seller and the buyers in the production of education. It is no different from general goods in the sense that the buyer purchases the product that best suits him/her and consumes it as he/she sees fit. No selection of proper customers is needed in this market. Another interesting example is the distance education market. The suppliers in that market are commonly called schools, and more often than not, ordinary schools provide the business as an extension of their regular services. In terms of the depth and width of the interaction between the students and the schools, however, distance education is not comparable to the ordinary school education60). Also there is virtually no limit to the number of acceptable students because the education is not conducted in a specific geographical area. In view of this, it would be unnecessary to have an entrance exam in the distance education market. Of course, there is the problem of controlling the quality of its graduates, but this can be achieved by imposing strict requirements for graduation instead of limiting admission.

3-3. The Middle and High School Equalization Policy

We have been examining the special features of educational goods compared to other ordinary goods. One of those aspects has been that the transac-

59) Sometimes, even those schools that cannot fill up their classes due to a shortage of applicants, still select their students through exams. This may be because of the tacit warranty on the final product of public educational organizations. Unlike private institute certificates, a college diploma is usually considered as a warranty of the quality of its graduates or a certificate. Therefore, there is a stronger demand for quality control in schools.

60) Of course, a pre-selection of only qualified students is not essential to guarantee quality. A strict graduation exam will do the same. The idea of a flexible enrollment combined with a stricter control of the total number of graduates, once popular among policy-makers, is in the same line of thought.

60) Not to mention the peer group effect.
tion of educational goods is not concluded just by an one-time transfer of goods in return for payment. It is a good that requires an on-going involvement of both seller and the buyers in the production process. We have also pointed out that this partnership relation necessitates a process of selective input assortment. A second characteristic of educational goods is that there is an order that must be strictly followed in the accumulation process. Therefore, the amount of educational service one gains from the education process is greatly dependent upon the current level of human capital he/she already possesses. In view of these two characteristics, combined with the other two less salient ones, we have examined why it is inevitable to measure the level of the human capital of each student and to group them accordingly.

In light of this, it can be concluded that the middle and high school equalization policy introduced in 1968 and 1974 along with the arrangement to close down several “top-class high schools” is a catastrophic confrontation to the principle of efficient allocation of educational resources. In terms of the harm it has inflicted upon the society, nothing seems to be more disastrous than this, as we hear some testimonies made by teachers later. No wonder that after three decades of its implementation, we now dismayingly observe the whole educational system falling apart.

Sure, there remains a more practical question of how minutely the classes or schools should be broken down into. There also remains the task of actually testing and measuring the effects that such an ability-based subdivision of classes may bring forth. A more fundamental criticism might be that educational achievement cannot be solely measured by a single index called human capital nor can we ignore the side effects that ability tracking may accompany, such as the stigma effect. These are all important issues. But an examination of all these issues may bring us well beyond the boundaries of this thesis. Therefore, we will limit ourselves here to an investigation of how the aforementioned equalization measure may have effected the students’ demand for private tutoring.

What we are about to prove here is a matter of a mere common sense. We will show that if there is no ability-based division of classes, the students at the two ends of the ability distribution will have a high demand for private tutoring. Indeed, such a proposition is so intuitive and trivial that we may not need a formal proof. However, because the earlier researchers have paid little attention to this, I will venture a bit of a rigorous approach here.

Let’s assume that students or their parents consume two kinds of goods: educational good (E) and all other goods (X). Therefore, the utility function is

---

61) In the case of the US, there have been reports that show that an ability-based division of classes, or “ability tracking” does not bring a significant increase in educational achievement. However, this may be in a large measure due to the inadequate measurement of the differences. For a survey, see Slavin (1990).
62) The frequent controversies over the educational philosophy demonstrate this. Refer to the Ministry of Education (1996).
given by $U(X, E)$. Educational good can be obtained through two sources: through the public education($E_1$) and through private education($E_2$).

Although some earlier papers have labored to establish clear definitions about these two, here I will simply call private education what is generally regarded as private tutoring. Therefore, public education here refers to the official school education provided at either public or private schools, and private education to the education the student purchases in the market from private institutes (Hakwon in Korean) or private tutors.

What the consumer cares is the total of educational attainment ($E$). So we have $E = E_1 + E_2$. This also implies that the two are perfect substitutes. In view of the fact that in most cases, education at the secondary school level, is pursued with a single purpose of entering a college, or a better college, there should be no problem in treating the two to be mutually substitutable. Public education is assumed to be free. This is a subsidy in kind, financed by taxes. Private education, in contrast, has to be purchased in the market for a price $P_E$.

Finally, I will assume that there is no quality-ability mismatch problem in the private education market. This makes sense for the following two reasons. Firstly, when a student hires a private tutor and gets help from him/her on an one-to-one basis, there is no gap between the student’s ability and tutor’s lecture level because the tutor is supposed to fit himself into the student’s need. Secondly, if the student were to register in a private institute, he/she will elect the class that best suits his/her own capacity, therefore, the loss due to the discrepancy between the level of the class lecture and his/her own ability could be minimized. Or simply we may assume a more individualized care is provided in the private market. In public education, however, especially after the equalization measure, a student cannot choose his/her own school, but is forced to attend the school assigned by authorities. This necessarily creates the possibility of a wide gap between the student’s individual ability and the lecture level provided at the school. Therefore, the amount of actual educational attainment a student gets from school varies greatly depending on where he/she is positioned in the whole ability distribution of a particular school or class.

Putting all these together, we can set up the following utility maximization problem.

---

63) It seems that the many controversies are due to the peculiar usage of these terms in Korea. Internationally, private education refers to the education conducted in private schools, in contrast to the education provided by the state or local government. In Korea, however, the distinction between public schools and private schools in this sense is meaningless. Especially, ever since the equalization measure was put into effect, students have been compulsorily allocated across all school, whether public or not. Therefore, a more meaningful distinction would be the distinction between the official education received from schools and the unofficial education received from private institutes or tutors. For a more detailed conceptual distinction between public and private education, refer to Kho et al. (1998).

64) Students pay a small amount of tuition. But the tuition charged is negligible compared to the actual cost borne by the government.
Since $E_1$ is a gift in kind given by the government, it is not an object of optimization but an exogenously given endowment. Therefore, the consumer allocates his/her income ($Y$) onto private education ($E_2$) and other goods ($X$). This is the budget constraint, given in the third line above, in which the price of $X$, the numeraire good, is normalized to be 1. The inequalities in the fourth line indicates that the students cannot sell the subsidy in kind freely given and convert it into other goods. In other words, the public education cannot be transferred to another person.

One thing to note here is that the amount of public education given by the government, $E_1$, is not fixed, but varies depending on who the student is, or in particular, on how much of human capital he/she carries with him/her at the starting point. A student whose human capital level is equal to the mean of the whole distribution of the class will get the largest amount of education because he/she does not incur any loss or mismatch. On the other hand, students whose human capital level is positioned near the upper or lower tail of the distribution will get a smaller education because they have to incur a large amount of loss. This difference in the amount of public education will in turn induce each student to choose different amounts of private tutoring, $E_2$, to make up for the loss incurred within school. Without going through the formal solution process, I provide below a simple graphical illustration of the results\(^{65}\).

In the diagram above, KL represents the budget line when no free public education is provided. When the government starts to provide a free education as a subsidy in kind, the budget line shifts to the right, giving rise to the new budget lines, K’L’ and K”L” respectively, depending on the amount of the free gift. The horizontal distance, KK’(or 0a) and KK”(or 0b), represent the respective amount of the gift received. In the graph, therefore, the line K’L’ represents the budget line of a student who receives a relatively small amount of education, while the line K”L” represents the budget line of a student who receives a relatively large amount of education. In the case of the student who receives a relatively smaller amount of gift, the optimum consumption is achieved at point A where the budget line and one of its indifference curves become tangent to each other. The total amount of his education consumption is 0a’, out of which 0a is the public education and aa’ the private tutoring, the additional

\[\begin{align*}
\text{Max} & \quad U(X, E) \quad \text{s.t.} \quad E = E_1 + E_2, \quad \text{and} \\
& \quad E_1 = a \cdot (\bar{H} - \mu_H)^2 \\
& \quad Y = X + P_E E_2 \\
& \quad E_1, E_2 \geq 0.
\end{align*}\]

\[\text{(9)}\]

\(^{65}\) For a more formal derivation of the final equilibrium in a slightly expanded setting, see Chapter 5 below.
amount he/she purchases in the private education market. In the same way, B is the optimum point for the student that receives a relatively larger amount of free education. His total consumption of education is 0b', which is again subdivided into 0b and bb', respectively, with each representing the amount of public and private education.

Now if we compare the amount of private education these two students purchase, we can see that aa' is larger than bb'. In other words, the student who has been assigned to a school that does not fit his/her individual ability level has a greater demand for private tutoring. He/she cannot get an appropriate amount of education within the school, so he/she is, in a sense, forced to seek an additional education in the private market to make up for the deficiency. This conclusion holds as long as X is not an inferior good. In order to verify this on the graph, let us draw a horizontal line from the point A to the budget line KK'', and call the crossing point as B'. Then this point B' will become the threshold point, indicating whether X is an inferior good or not. If the point B is located to the north-west of B', X is a normal good. On the other hand, if the point B is located to the south-east of B' along the budget line K'L'', then X is an inferior good. If one draws a vertical line from the point B' down to the horizontal axis and calls the intercept as point b'', then bb'' would have exactly the same length as aa', or the amount of private tutoring the student with a relatively smaller amount of public education purchases, because bb'' is simply the horizontal shift of aa'. Therefore, as long as the equilibrium point B of the student, who has received a relatively larger amount of free gift, is located to the north-west of the point B', his/her demand for private tutoring is smaller than that of the other student who has received a smaller amount of the public education.

The above result demonstrates how the secondary school equalization mea-
sure can increase the demand for private tutoring, not decrease it as is hoped for. The equalization measure undoubtedly widens the variance of the student ability distribution within a school because it dictates a random assignment of students ignoring each student’s individual ability completely. How much of the actual demand for private tutoring can be attributed to the equalization measure? It is difficult to give a specific numerical answer to the question. First of all, we have not yet established, even conceptually, to what degree we should implement the ability-based division of schools and classes. Some may argue that since we have had already such a hierarchical division in the whole school system (into elementary, middle and high schools, and furthermore a division of grades in each of these schools), the gains from an additional gradation would not be that significant. In other words, some may argue, the loss of educational achievement due to the equalization policy may be negligible. For an accurate assessment, we may need an overall survey and careful measurement over the whole schools. The following anecdotal evidences, witnessed by some teachers at the front line of the battle fields, however, give us some clues on how serious the current situation is. 

“According to a second grade teacher in a middle school, the math level of the second graders within that school ranges from that of a fourth or fifth grader in elementary school to that of a first or second grader in high school. The classes are, however, conducted with no proper consideration to such enormous differences among students. It is impossible to provide any quality education in a class in which both the highest and lowest ability students are placed together indiscriminately. ... Students with a strong background sit there bored because there is nothing to learn in the class while students who struggle just gaze at the chalkboard having lost all the aspiration to learn because they do not understand what is going on in the class.” (Cho et. al (1998), p.18; requoted from Nho (1999))

“When I meet with parents of students that either have quite good or bad grades, I sometimes forget that I am a teacher and advise them to go out and try private tutoring. They (the government) have put all the kids of different abilities together in a school, and don’t allow us to regroup them according to their individual abilities nor do they allow students to select the classes that match their own level. A class has so many students that I can in no way teach each student in accordance with his/her ability.” (Commission for Prevention of Corruption (1994), p.62; requoted from Nho (1999))

We are not sure whether the pictures described by the testimonies above

---

66) In other words, as long as the income expansion path connecting point A and point B is upward sloping, the demand for private tutoring increases as the students' abilities move farther away from the class average.
are just a handful of exceptions scattered here and there among many well behaving schools or whether they illustrate a phenomenon epidemic in most classes of every school. If the latter is the case, it is a really serious matter because the above testimonies leave us the grim impression that the educational function of schools is almost becoming paralyzed. Today’s newspaper articles do not hesitate to put on such expressions as “the total collapse of schools”, or “the disintegration of school systems.” If the present situation is similar to the ones above, such expressions may not be an exaggeration or a misrepresentation.

Whenever the issue of private tutoring erupts, an all-too-familiar ready-made prescription is beckoned to stand by: we have to strengthen the public education. By solidifying the internal competitiveness of the public education system, many (mostly policymakers) claim, sometimes convincingly, we may be able to turn the tide away from schools to within schools. But how? The suggested prescription then jumps to another ready-made conclusion that there must be a major increase in public investment in education. Proposals asking a substantial increase in educational budget, say up to 5-6% of the GNP, or a significant reduction in class size in every school all belong to this category. Strangely enough, not many ask for a revocation of the equalization measure, the prime culprit that has caused a virtual dissolution of the whole secondary educational system, and furthermore, the abolition of which could be implemented immediately without having to increase the budget.

Reducing class size substantially in every school would certainly help to resolve the quality-ability mismatch problem under the current equalization measure. As the educational authorities have repeatedly stated, however, this will require an enormous amount of the government budget, which cannot possibly happen overnight. There are also suggestions of partially allowing or

---

67) Such a phenomenon may not be ours alone. At the time of the Cultural Revolution, China also adopted an equalization policy in order to cope with the overheated entrance exams and the obsessive pursuit of college diploma. Dore(1997) recollects the situation at that time as the following. “What only became clear to me later ... was the collapse of real learning in the primary and secondary schools; ... most of China’s urban student simply stopped paying attention in class.” China revoked the equalization measure later in 1977 and reinstated the exam system.

68) We have to keep in mind that a government policy measure that involves a substantial reshuffling of its budget appropriation necessarily create a new group of winners and loser in terms of benefits they can preferentially get. It is also worthwhile to note that in the US, a large number of literature is witnessing to the effect that an increase in educational budget has so far generated little improvement in students’ educational achievement. See Hanushek(1986, 1999). Consequently, the voice asking an overhaul of the entire incentive system within the school including its governance structure is growing in the US these days.

69) Curiously enough, some people propose an increase in the number of high schools that can stand on their own feet financially and are privileged to select their own students through competitive pre-assessment. This is the idea of the so-called “self-supporting private high schools (自立型 私立高)”. They do not, however, explain why a certain group of students should be denied a governmental education subsidy, nor do they provide a good explanation why an overall reinstatement of the student ability screening must be blocked. They may have gotten the idea from the current US system, but we have to keep in mind that the US government is actively seeking these days to dismantle this rigid public/private segregation in their secondary school system. The so-called “choice” or “voucher” proposal provides a good example of this movement.
even encouraging the formation of honor classes within a school. However, this doesn’t appear to make an effective cure to the disease we are currently suffering. Firstly, the gap between the highest and the lowest ability within a class appears to be so wide that a division of all this mixture of students into just two groups doesn’t seem to be capable of mopping the dirt away from the floor. Secondly, if we once accept the idea of ability-based grouping, it is not clear why we should stop at only two, when we can subdivide the schools and classes into several thousand of ability groups without incurring an additional cost.

Attempts to maximize educational achievement by guaranteeing students the right to choose their best school, stimulating competition among schools, and establishing the ability-based grouping of students can also be found in other countries. Several years ago the UK abolished the system of allocating students to a given school districts and allowed parents to choose the school they want for their children. In addition, the UK also changed their system of aiding schools into aiding students, thus forcing the schools to close down that fail to secure a sufficient number of students. A similar movement is also well underway in the US. As a means of incorporating more competitive forces into the education system, an open enrollment system supplemented with a voucher has been given a serious thought and is partially experimented within some states. As mentioned earlier, China, which once tried an equalization policy to cope with the overheated entrance competition problem, has long abandoned the policy after having seen its ill effects, and is now stimulating free competition among schools.

3-4. First-Class School Disease and Cliquism

A. An Obsessive Pursuit of First-Class Schools

Not many deny that the current situation of the public schooling in Korea is grave and that the ill effects of the equalization measure is felt more and more keenly everyday. When it comes to policy suggestions to dismantle the equalization policy, however, many shy away and start to waver. Understandably, people are apprehensive of the possible surge in private tutoring that the revocation may bring back. Without giving a deep thought to the question whether the equalization measure has been, or can ever be effective in dampening the private tutoring, they have a tendency to identify the revocation with

---

70) As of 1997, there are 1,892 high schools and each school has several classes in each grade.
72) Although many have pointed out the ill effects of the policy, few are enthusiastic of its abolition. Some propose its expanded enforcement even to the college level thus abolishing “college rankings”. Refer to Kho 1998).
a revival of private tutoring. Well, an annihilation of the private tutoring has been the prime objective of the measure in the first place, they reason, that any sign of a retreat would be interpreted as a public endorsement of private tutoring.

The analyses in the previous sections (to be further scrutinized in the following Chapter 5), however, clearly show that we can’t be sure that the equalization measure would have had such desired effects at all. First of all, much of the private tutoring in the past has been induced by the supply-and-demand imbalance and has little to do with school qualities. Secondly, even if there is a competition to enter a better school, the net effect we can get by abolishing the school ranks are not clear because it can increase the demand for private tutoring by increasing the variance of the student’s ability in a class. Surely, a reinstallation of a hierarchical rankings among secondary schools will revive the competition to enter higher ranking schools and may aggravate the situation. It is, however, apparent that an equalization measure cannot achieve much unless it is applied in its extreme form to every stage of the conceivable games of life.

For example, unless we have an equalization of colleges, students will continue to take private tutoring to go into a better colleges. If we go one step further and put all colleges under a similar equalization measure, some may say that no one will take private tutoring to go to a better college. That is true by definition. But it doesn’t mean a private tutoring in general will cease. Far from that. If a diploma from a particular university loses its property of quality assurance, then the employers may have to invent some devices to screen for student ability. To get a better job, students will take another form of private tutoring. Take for instance the bar exam. How many students are in queue there to grab the prestigious license to practice a law? How rampant is private tutoring there? In short, no one or no policy can successfully keep students away from exerting their efforts to get something they believe will benefit them unless a perfect randomization is instituted in every aspect of life. Education is, in a nutshell, a process of increasing one’s income-earning power. Wherever that opportunity lies, whether in better schools or better jobs, the niche will eventually be exploited and exhausted.

What is then a first-class, top-notch school, and why do people so enthusiastically try to enter top-ranked schools? A simple and fairly reasonable answer would be “because it is expected to be profitable.” No student will choose a top-ranking school for its own sake. The top-class schools are mere instruments to achieve their ultimate goals. This implies that top-class schools

73) No one would suggest that we abolish the bar exam or the State exam, and select lawyers or government officials randomly.

74) We have discussed earlier the meaning of “a good school” from a producer’s perspective. Here I am referring to it from a consumer’s perspective.
can be done away with as long as there exists a substitute that plays a similar role.

From the viewpoint of a student, a school could be profitable if its tuition fee is cheap. Everyone would prefer a school that offers the same service at a lower price. As a matter of fact, many of the so-called top-class schools in Korea are public schools whose tuition fees are much cheaper than those of private schools, especially at the university level. This may, therefore, explain why students prefer these schools.

If the preference is indeed based on price differences, the solution to the problem of excessive pursuit of top-class schools is very simple. Make all tuitions the same. Since there is no particular reason why tuition should be kept lower in the public schools than in private schools, there should be no problem with this solution.

However, such a preference does not seem to be based on price differences alone. Many of the so-called top-class colleges also include private universities whose tuitions are much higher than those of public universities. As well, at the secondary school level, there also used to be some prestigious middle and high schools that many people sought to enter. The price difference amongst the secondary schools at that time were minimal. Of course, it can be said that an entrance into a prestigious high school may guarantee entrance into a prestigious college and since the tuition of such colleges are cheaper, price difference may have still provided the incentive to go to a better high school. But still this appears too weak to explain the whole situation.

Secondly, it may be because of the difference in the quality of educational service provided by the schools. The preference for prestigious high schools that have long been known to guarantee its students a sure entrance into top-class colleges supports this. Also at the college level, it is generally accepted that the average student ability of the top-class colleges is higher than that of other colleges. This fact becomes glaringly apparent when we look at the human capital level at the point of graduation, \( H_{t+1} \). Strictly speaking, it is \( E_t \) that the student gains through the education process, as has been repeatedly emphasized earlier, but one may still argue that the reason why the \( H_{t+1} \) is higher is because a larger \( E_t \) has been provided at a particular school. In this sense, a top-class school is that provides exceptionally good products at a given price.

In order to look into this, let’s modify our previous assumption that all school have identical \( E \). Instead, we allow it to vary depending on what quality index a school has. We use the education level provided by an individual school, \( \mu \), to denote the school quality ranking\(^{75}\).

\(^{75}\) Assuming that there is a continuum of school quality which is represented by \( \mu \).
Note that $E' > 0$ now implies that a student can get a larger amount of education by attending a school with a higher value of $\mu$. In this case, if the tuitions of all schools are the same, a student will choose the $\mu$ that will maximize his $E^i$. Differentiating the above equation and collecting terms, we have,

$$E' = f(\mu, H) = a \bar{E}(\mu) - \beta (H^i - \mu)^2. \quad \bar{E}' > 0. \quad (10)$$

In other words, the optimum would be to choose a school that is slightly higher than one’s current ability level ($H^i$). The higher the $\alpha$, and/or the lower the $\beta$, the greater the gap between the school quality and the individual’s ability. That is, the greater the benefit you can get by attending a good quality school, and/or the smaller penalty you have to pay for the mismatch between the school’s quality and your own ability, the greater quality school you will aspire to attend\(^\text{76}\). In this case, self selection on the part of the students alone would not work out a complete stratification of students because every student has an incentive to go to a higher quality school. Schools must actively intervene in the allocation of students unless they charge different tuitions\(^\text{77}\).

This implies that the obsessive pursuit of a top-quality school can once again be attributed to imperfect pricing. If you sell goods with different qualities at a uniform price, the higher quality good will be overly sought, and consequently must be rationed out\(^\text{78}\). The cure is, then again, simple. Just allow the prices of different quality goods to differ. Then, the “irrational obsession” would disappear\(^\text{79}\). This can be done by simply stopping regulating the tuition at a uniform level. The market will accurately calculate and bring about the necessary adjustments in prices just to cancel out the $E'$ effect.

Until now we have labored with a bit of a lengthy and tedious discussion to explain the phenomenon of the obsessive pursuit of top-class schools in terms of price and quality differences. One may, however, justifiably argue that this is not exactly what the obsession is all about. We suspect that this first-class fervor would still persist even under an appropriate pricing of aforementioned quality and quantity differentials.

I believe we have a reason for such a suspicion. To put the conclusion first,

\[^{76}\] Of course, if there is no difference in the quality of education gained, $E'$ would be 0, rendering it again to be best to choose a school whose quality index is just equal to one’s ability level.

\[^{77}\] Strictly speaking, this statement is true only when the school quality and each school’s capacity are exogenously given. Here we have the so-called “rat race,” and the resulting equilibrium is inefficient. See Akerlof (1970).

\[^{78}\] If you sell a Grandeur at the price of a Tico, everyone will want to have a Grandeur, and there will develop an excess demand for it.

\[^{79}\] This is not irrational at all in economic sense. People is bound to seek a better deal.
people are eager to get into the first-class schools because there exists an economic rent associated with the so-called top-class schools, which cannot be explained away by a simple quality or quantity differential. Let us go back to the example of private academic institutes. Generally speaking, the private education market is one where the free competition principle is comparatively well applied. Although government regulations on fees and other things do exist, they are not strictly enforced. Even if they were, there is still more freedom to fine-tune prices in accordance with the quantity and quality, compared to public education. In other words, the possibility of imperfect pricing is relatively low.

In this private education market, students choose for themselves the institute, the class, and the peer student group that matches their own level. There is no overheated competition to register into an institute that is beyond one’s own ability. No one resorts to additional private tutoring to attend a private institute and no one reattempts twice or three times to sign up for a particular institute. Occasionally, there are preferences displayed for a certain lecturer, but this is hardly a preference for a certain institute since students would follow the lecturer should he/she move to another institute. In short, there is no such thing as a “top-class institute” that every student is eager to get in.

The above phenomenon may be explained as the most natural one to be observed in a market in which prices accurately reflect the degree of scarcity in quantity and quality. In fact, this does explain a considerable part of the phenomenon, but the essence of the problem lies in the fact that it can also be explained by another reason than a mere price adjustment. The reason might be the absence of social rents attached to a particular institute. I will elaborate on this since this is what I want to emphasize most in this thesis.

College or high school graduates seldom receive a social acknowledgment for having studied at a certain private institute. No one writes in their resume what private institutes they have gone through. In the case of school, however, we have a different story. The fact that one is a graduate of a certain prestigious school, say, Seoul National University, is expected to carry a highly acknowledged social value and is sure to be on the resume. It seems that society accords a great amount of socio-economic benefit to the graduates of the so-called top-class schools.

This means that with two graduates from two different colleges but with the same amount of human capital, $H_{t+1}$, the student from the more prestigious college will get a higher rewards than the other. In other words, it implies that the social reward function contains as its argument not only the individual’s human capital level, i.e., his/her productivity, but also the title of a particular school he/she is from. In mathematical terms, the reward one can get, $W$, (both monetary and non-monetary), is not a function of only one variable $H_{t+1}$, but also of another variable, $R$, in this case the title of the school he attended. So, we will have a social reward function of the following type,
\[ W = W(H_{t+1}, R), W_{tt} > 0, W_{tt} > 0. \] (12)

Of great significance is the term \( R \) and the sign of \( W_{tt} \), which is positive. Note that \( R \) doesn’t include the titles of the private institutes he/she attended, but the titles of the socially renowned top-notch schools he/she is graduated from. If this is the case, the problem of obsession with top-class schools will not be resolved even if an appropriate pricing of \( E_t \) were achieved in the education market. This is because, in addition to the competition for \( E_t \), a separate competition for \( R \) will arise. In this regard, the more compelling reason for the non-existence of top-quality fervor in the private education market may be the absence of social rents associated with the private institute, not its relatively flexible pricing.

Indeed, the quintessence of what many people call an obsessive infatuation with the top-class schools may lie in the existence of this social rent. That is, the fact that we have \( \partial W / \partial R > 0 \). To state it differently, we may view this phenomenon as a serious deviation from the principle of “ability-based reward,” which can here be conveniently characterized as \( \partial W / \partial R = 0 \). Although not always and everywhere is this principle embraced whole-heartedly, it is safe for us to assume that it is one of the fundamental fabrics with which a sound society is and should be woven. In general, people hate to see a deviation from this invaluable principle unless there is a redeeming virtue for the deviation. A social antipathy against this kind of hardly justifiable preferential rewarding may have given rise to such a gruesome name, likened to an ailment, as “a disease of first-class fervor” (一流病).

B. Economics of Privilege: Its Creation and Exclusive Sharing

Then how and why does such a phenomenon of endowing social rents to the top-class schools arise? Here I will attempt to generalize the phenomenon and make a more comprehensive observation of it.

Note that \( R \) in the reward function (12) may include any variables in addition to the one already identified — the name of first-class schools — that would affect the individual award in such a way that hinders the “pay-accord-ing-to-productivity principle.” It may therefore include the family ties or a particular provincial area one is from, etc. In fact, the list may include all sorts of variables that are connected with what we call factionalism or cliquism. No matter which variable some may name, however, it is evident that they share one common characteristic: they all depict the particular attributes of the participating individual in a transaction. For instance, in addition to how competent he/she is, one may sometimes ask which school or region he/she is from, who his/her father is, in which year he/she has passed the state examinations to become a government high official or a lawyer, who he/she or his/her spouse has as close friends, who else were in his/her graduating class, and so
When and why do they try to obtain the information about the personal attributes of their trading partner? To put the same thing in slightly different words, when is the “principle of anonymity” in the marketplace, mentioned in the earlier chapter, abandoned or replaced with something else? Taking the risk of an over-generalization, we may conveniently define the “ability-based rewarding” as an establishment of the anonymity principle, and the factionalism as an abandonment of the principle.

When does the principle of anonymity give way, then? The explanation in the previous chapter gives us a clue. We have seen that when there is no price or quantity regulation, a market settles itself at an equilibrium price and in this case there is no need for the seller to know the personal attributes of the buyers. But as soon as a binding regulation on prices or quantities is introduced, the need for identification of buyer attributes arises because the regulation necessarily creates an imbalance between the quantities demanded and supplied, and because the goods must be rationed out according to some criteria. One of the criteria that can be easily adopted for rationing is the personal attributes of buyers. Surely, not all the rationing mechanisms require a discriminatory evaluation of personal attributes. It cannot be denied, however, that a discrimination based on individual attributes is one of the most frequently used devices in rationing. In other words, a regulation leaves a lot of room for school, or regional or blood connections among people to come into play.

Another case we have identified as requiring an information on personal attributes of the participants in a transaction is when people are involved in a joint or team production. In a setting of a team production, people pre-screen the membership and monitors the behavior of the participating members in order to achieve efficient outcomes of whatever they set as objectives. As a matter of fact, the aforementioned rationing, especially the one that is based on personal attributes, can be considered just as another example of the joint pro-

80) In a slightly different but a closely connected context, there is another name for a similar phenomenon. When we say that “economic principle or logic” is being pushed away by “political logic or consideration”, this political consideration in effect means in most cases the abandonment of the anonymity principle. The only difference is that the population being considered is larger in this case. Since political support is more dependent upon the vote of a group, one tends to care more about the characteristic of the group than the attributes of an individual.

81) The converse is also true in a market where the homogeneity of products is perfectly guaranteed. Note that this time it is not the seller but the buyer that does not need to know the attributes of the seller. In most cases, however, the information about the seller is necessary to have the quality of a product guaranteed or to receive after-services. This is what a brand name capital is all about.

The reason why such an asymmetry exists in the informational requirement between sellers and buyers lies in the difference of the objects being exchanged. While a seller delivers a product whose quality may not be easily and perfectly detectable, a buyer renders the seller money over the quality of which we seldom dispute. Money is indeed a special product about whose quality there is no worry of being cheated (unless it is counterfeited) and which has no need for continued after-services for maintenance. Historically, currency has been limited to the objects whose quality and characteristics are easily distinguishable by everybody, and the governments have gone to great lengths to guarantee its quality.
duction: a joint creation of a social privilege and its exclusive indulgence amongst its closely-knitted members only.

To see this, let’s briefly review what has been discussed in the previous chapter. In a regulated market, there exists a gap between the price a buyer is willing to pay to have the product and the price he/she has to pay. That is, \( P_1 - P_0 > 0 \). If one is lucky enough to get that product, he/she can enjoy the windfall gain. If he/she fails to get one, he/she is a loser. This is true to any commodity in which there is a gap between the marginal value a consumer attaches to and the price he/she has to pay. So the commodity whose price is regulated becomes a good which bestows on its lucky owner a preferential privilege which everybody wants to have if he/she possibly can.

Privileges, whatever they may be, have a tendency to be exclusively distributed and shared within a small group of people, especially those who have close connections among them\(^\text{82}\). This joint creation of privilege and exclusive sharing amongst a preferential minority has its own economic rationale when viewed from an individual standpoint. Suppose I am in a position to give away a certain privilege. How should I mete it out to best promote my own interests? If I randomly select the recipients, there is no guarantee that a similar favor will return to me in the future. It is, therefore, in my interests to select one who has the highest probability to repay. One sure way to secure the repayment is to write an explicit contract. Unfortunately, however, it is impossible to explicitly state such terms in a contract. In most regulated markets, such a contract to get around the regulation is very much likely to be declared illegal\(^\text{83}\). So it becomes crucial to secure a means to control the future behavior of the beneficiaries in an informal but effective way.

Firstly, I have to make it sure that he/she knows who I am; the benefactor. He/she shouldn’t have any confusion about to whom his/her returning mail should be addressed later. But if he/she knows me, I know him/her too, in most cases. He/she must be someone within a close circle of mine. Secondly, there must not be much difficulty in enforcing this tacit agreement in an informal way. A formal enforcement through the judicial system is, again, not available. Until he/she makes a final return, he/she must continuously feel morally or psychologically obliged whenever he encounters me. He/she must be a man/woman of decency. Thirdly, he must be someone who has an enough wherewithal to make the repayment, either being rich or being empowered in a position to parcel out favors of a similar magnitude in return.

Sifting through the list of all the possible candidates, I finally get down onto a few names, all of whom share the following characteristics: they are members of a small group each of whom I maintain a frequent personal contact with, and the ties I have with whom are expected to continue well into the future, and

\(^{82}\) If it is universally available, who would call it a privilege?

\(^{83}\) Otherwise, there would be no regulation in the first place.
and they are certainly a who’s who, reasonably wealthy and powerful. I believe the picture that I draw here is a fairly close description of what many people call the “preferentialism (総合主義)”84).

The disparity between the value and the price, and consequently the privilege, is created undoubtedly by artificial market interventions85). In an unfettered free open market, everything will always be traded at the point where the marginal use value = market price86). An open market never breeds and nurtures a privilege. It is trivial to prove the proposition. If the value is greater than the price, people would try to get it. As more get it, the prestige associated with it declines and the marginal use value also diminishes as a consumer gets more of it. At the same time, the marginal cost of producing it increases as more of it is produced. Eventually a privilege will be completely dissipated unless there is an artificial barrier against the natural tendency. QED! One cannot keep a privilege for good any more than he can keep a full-blown balloon underneath the water. It follows then that the privilege or the economic rent bestowed on the so-called first-class schools must have been created and maintained somehow by some artificial constraints.

True, first-class fervor or elitism is not unique to Korea. It exists almost everywhere in the world. But it is undeniably more rampant and pervasive in a country where artificial regulations abound, whether explicit or implicit. We can also easily guess that such problems are more frequent in underdeveloped countries than in developed countries. The competition to enter colleges, especially top-class colleges in developing countries such as China and India is no less severe than in Korea. Among developed countries, France and Japan are notorious for its first-class fervor. It is hardly a sheer coincidence that these countries are the countries that are known to have many market interventions, a serious degree of cliquism and corruption.

This suggests that a cure to the disease of the first-class obsession or favoritism has to be found in a removal of various regulations and interventions in the markets that cause a disparity between the value and the price. It is to prevent the creation of the privilege itself. Without stamping out the formation of privileges themselves, it is not likely to be effective in dampening the favoritism to just shake some of the groups that feed on privileges. A privilege by definition cannot be fairly distributed and never has it been throughout history. If we remove a set of top-class schools, another form of small groups will form to take its place. Like it or not, a privilege, once created, is bound to find

---

84) It is easy to enforce an implicit contract within a small group closely knit by frequent personal contacts because breaching a promise within such a group brings about a stigma effect.

85) Of course, not all regulations create a divergence between the value and the price. Some regulations, especially those imposed to correct externality effects or to cope with the public good problem, help equate the value and the price.

86) If the market is open and competitive, we can add one more equality such that we have “value = price = cost.” In sum, a free open market assures the allocative efficiency, whereas a competitive market assures the productive efficiency.
its own way. If one path is blocked, another route will be cut through almost immediately\(^8\). Furthermore, there are always more ways than we can possibly imagine for getting around and realizing the economic value associated with a privilege. A removal of a handful of top-class schools or equalization of secondary schools would not make any significant dent on preferentialism.

\(^8\) We all very well remember the funny(?) idea that the format of the resume must be revised in such a way that nobody can put the school names from which one is graduated from. How can one compulsorily impose such a restriction on private person’s or organization’s behavior, though? Another equally facetious suggestion was to create a new demarcation of provincial districts based on the longitudes and latitudes to destroy the firmly established regional cliquism.
CHAPTER 4
Reforms in Entrance Exam and Private Tutoring

4-1. Introduction

In Chapter 2, under the assumption that an education good is no different from other ordinary goods, we have looked into the college education market and discussed various issues related to private tutoring: entrance exams, tuition controls and enrollment quota controls. In this chapter, we try to dig a little bit deeper into the issues concerned with entrance exam reforms or other pre-assessment procedure renovations. The overall settings are similar to those in chapter 2: education good offered in the market is all homogeneous. But there is a tuition control in the market below the market clearing level so that we have an excess demand for college education. The school authority, constrained not to raise the tuition to clear the market, therefore, has to adopt a certain kind of rationing device in allocating the limited slots to a large number of contending students. Administering various forms of admission tests is the most conventional way of selecting these students89). Students then compete to increase their chances of getting in the school, mainly due to the fact that their demand price is higher than the regulated one.

Obtaining admission in this situation is just like winning a prize in a contest. If one student succeeds in getting in, he/she can enjoy a windfall gain, so he/she becomes a prize winner. If he/she fails, he/she is just a loser in the contest. The prize, the gap between his/her demand price and the controlled price, therefore, drives students and their parents to compete in various ways; they may study or practice harder at home, attend private educational institutions, hire private tutors to get high scores in the entrance examination or other forms of admission tests. In short, they will do whatever will help to raise the chances of their being admitted90).

Seen from this perspective, private tutoring can be regarded as one form of rent seeking activity. Parents and students compete for grabbing the rent artifi-

89) Even in the absence of tuition control, some schools may choose not to raise the tuition and keep it below the market clearing level on purpose. Instead, they would rather have admission tests to select good quality students and thereby maintain the school’s reputation. Even though schools are usually not considered as profit maximizing entities, this kind of practice can still be explained in a profit maximization framework under the assumption that there exists an informational asymmetry between schools and students, and that a good reputation has a pecuniary value. See Rothschild and White (1995) and Stiglitz and Weiss(1981).

90) Some may even try illegal activities such as forging application documents, bribing the person in charge to get tips on test questions or fabricate the results.
cially created by the market intervention. So in this chapter, I build a simple
contest model in which private tutoring as an rent seeking can be systemati-

cally analyzed. In doing so, I introduce a small innovation to the existing contest
models most of which limit themselves to the cases where there are only two
contestants91. These models do not fit squarely with the situation described
above where a large number of students compete to get the coveted items and
multiple winners are selected92. The innovation I introduce here is to allow a
continuum of contestants. In a pair-wise competition model, if one wins, the
other necessarily loses. In a model of continuum of players, the number of win-
ners and losers can be freely controlled.

Selecting a small number of winners out of a large number of contestants
requires a kind of formal or informal test or criterion according to which the selec-
tion is made. Selection criterion can be either a standard or a quota. In a standard
system, the ones whose qualification are determined to lie above the pre-set value
will be chosen, whereas in a quota system, those whose relative ranks are equal to
or higher than the pre-set number will be chosen. Depending on which system is
used, the incentives faced by the contestants become different. A continuous agent
model allows us to develop a model which can consistently handle these two dis-
tinctive selection criteria in a coherent way.

Since private tutoring as rent seeking is inherently unproductive and waste-
ful from a social standpoint, governments in many countries have implement-
ed various policy measures to stem or reduce it, ranging from an outright ban
of private tutoring (in China and Korea) to designing a new test format that
will help select qualified students while eliminating the cost in private tutoring.
In this chapter, we specifically ask following questions: 1) whether making the
entrance exam easier would help reduce private tutoring, 2) whether asking
questions the answers to which cannot easily gained through private tutoring
would help, 3) whether putting more of essay type questions would help, and
lastly 4) whether making private tutoring more costly to take would help. As
we can easily see, all of these are the measures actually taken or suggested to
be taken by authorities with a purpose of suppressing the private tutoring fer-
vor.

One of the interesting results we obtain from our analysis is that the effec-
tiveness of policy measures to suppress the rent seeking activities crucially
depends on the current degree of competitiveness in the market, and in a
somewhat surprising way. For example, if the current competition to get in the

91) For contest and tournament models, see Lazear and Rosen(1981), Nalebuff and Stiglitz (1983),
McLaughlin(1988), and Nitzan(1994). In many cases, the tournament models are not directly concerned with rent
seeking activities. They are more concerned with the problem of eliciting an optimum amount of productive
efforts by setting up an incentive payment system. A slight modification and reinterpretation, however, will
allow it to be transformed into a contest model, most of which directly aims to explain rent seeking activities.

92) A tournament model with multiple contestants has been developed formally in Green and Stoekey(1983)
and touched briefly in Nalebuff and Stiglitz(1983) and O'keefie, Viscusi and Zeckhauser(1984). All are, however,
mainly concerned with the optimum payment scheme rather than rent seeking.
college is highly competitive, making it easier to get in either by lowering the cut-off score or by increasing the quota in the hope that a reduced competition will induce less tutoring, will not bring the desired result. Instead, making it even harder to get admission is a good way to discourage private tutoring. Easing the competition slightly in this situation will only encourage a large number of students to seek more private tutoring by reviving their hope, and will aggravate the situation. In short, even under quite general and plausible environments, there is no simple linear relationship between the extent of competitiveness and the aggregate amount of rent seeking, which is somewhat counter-intuitive on first impression.

Devising a test procedure which will render private tutoring to be less reliable means to improve the score is not a good way to curb private tutoring, either. Increasing the random elements in the test will have a differential impact on students’ chances of winning, depending on where their expected scores are located. As a result, the average quality of students admitted under this system will deteriorate. Furthermore, the total amount of private tutoring aggregated over the whole population varies differently, depending again on the current state of competitiveness in the college’s admission.

Under the quota system, things get further complicated because a fixed quota necessarily introduces negative externalities among students. At the margin, one student’s admission necessitates another student’s rejection. When there is a policy shift, therefore, some students will gain, and others will get hurt. In certain cases, schools may end up with undesirable outcomes, undesirable in the sense that students with stronger aspiration for study, and thus higher value on education may suffer from diminished odds for admission, while students with lower value enjoy increased chances.

This chapter unfolds as follows. In the next section, I set up a basic model in which private tutoring as rent seeking is analyzed in a setting of a pre-set quality standard. The effects of various policy measures to suppress private tutoring are also discussed. In Section 3, the model is expanded to cover the cases where the admission quota is fixed. The effects of policy measures are again taken up in this expanded setting. Section 4 discusses the effects of easing tuition control on rent seeking activities. Conclusions and summaries are given in Section 5.

### 4-2. Basic Model

Let’s consider a college education market. The demand is given as $D(X)$, and the supply as $S(X)$, where $X$ stands for the tuition that a student has to pay. As usual, the demand schedule is assumed to be downward sloping and the supply schedule upward sloping, $D'(X)<0$ and $S'(X)>0$. Let’s assume that each student wants to purchase just one unit of education. Thus, the horizontal axis...
measures the number of students who want to purchase the educational service from colleges. The value that each student attaches to college education is given by $X$, the height from the horizontal axis to the demand schedule. The maximum value, the intercept of demand curve with the vertical axis, is assumed to be $x$.

If there is no regulation on tuition in the college education market, the equilibrium tuition will be determined at $x^*$, and the market will be cleared. If the government imposes a ceiling on the level of tuition below $x^*$, the market will be in disequilibrium. Let’s assume the tuition ceiling is set at $x$ (<$x^*$). Students whose personal valuation on education is greater than $x$ will want to get in the college and would be willing to spend on something which can help increase their chance for getting admitted to college. Once they are admitted to school, the surplus they can enjoy is, therefore, given by $X$-$x$. The total number of students who are willing to participate in this selection contest is given $D(x)$ which is obviously greater than $D(x^*)$. Under the regulated scheme, the supply of educational service is constrained to be $S(x)$ (<$S(x^*)$). Then, the admission rate, which is the inverse of the competition rate, is given by $\phi(x) = S(x)/D(x)$.

![Tuition Control Diagram](image)

If there is no regulation on tuition in the college education market, the equilibrium tuition will be determined at $x^*$, and the market will be cleared. If the government imposes a ceiling on the level of tuition below $x^*$, the market will be in disequilibrium. Let’s assume the tuition ceiling is set at $x$ (<$x^*$). Students whose personal valuation on education is greater than $x$ will want to get in the college and would be willing to spend on something which can help increase their chance for getting admitted to college. Once they are admitted to school, the surplus they can enjoy is, therefore, given by $X$-$x$. The total number of students who are willing to participate in this selection contest is given $D(x)$ which is obviously greater than $D(x^*)$. Under the regulated scheme, the supply of educational service is constrained to be $S(x)$ (<$S(x^*)$). Then, the admission rate, which is the inverse of the competition rate, is given by $\phi(x) = S(x)/D(x)$.

I start with a simplest model, and then will try to zero in step by step by adding more constraints until it would appear to fit the general picture described above. Let’s start with a simple contest model in which the contes-

---

93) The model developed below is quite similar to Kim(2000) although there are several differences. Kim considers the problem of optimum test design assuming that students have different intrinsic abilities. His main concern was to design a test structure which can select the most able people to assign to two different tasks so that total social welfare is maximized. He doesn’t relate his model with the issue of tuition control. In this paper, in contrast, the contestants differ not in their ability but in their valuations of the prize they will get when they become winners. And the individual gains are directly affected by tuition control. Furthermore, tuition control also has an effect on the number of people who are willing to participate in the selection contest and on the number of people who will be selected for the prize.
tants have varying valuations for the objects that they are competing for\(^94\). It is
assumed that the valuations of the prize, \(X\), \((X \geq 0)\) which is distributed in the
range of \([x, x']\), has the cumulative density function, \(H(X)\).

There will be an admission test to select the qualified students. Students
will put forth effort, denoted as \(e\), to obtain high scores in the test. The effort
can be anything which can improve their odds of winning the prize, so it will
depend on the characteristics of the test adopted. More specifically, the effort in
this paper is interpreted to represent the amount of private tutoring a student
purchases outside of regular schools. It is assumed that the tutoring has no
intrinsic value or productivity in itself other than improving their score in the
admission test.

The test score will be distributed as
\[
T = ae + \varepsilon, \quad a > 0, \tag{1}
\]

The noise term, \(\varepsilon\), has a mean 0, and variance \(\sigma^2\), and has density function \(f\), and
cumulative distribution function \(F\). It is assumed that \(f\) is symmetric around 0,
and \(f' > 0\) when \(\varepsilon < 0\) and \(f' < 0\) when \(\varepsilon > 0\). Given \(e\), an individual student’s expec-
ted test score is \(\alpha e\), i.e., \(E(T) = \alpha e\).

Taking effort or purchasing private tutoring is costly, and the cost function
is given by \(\beta c'(e), \beta > 0\), where \(\beta\) is a parameter affecting the marginal cost of pri-
vate tutoring. About the general shape of the cost function, it is assumed that
\[
c' > 0, \quad c'' > 0, \quad c'(0) = 0. \tag{2}
\]

Students whose score is greater than the cut-off level, \(t^*\), will be admitted.
Later, this cut-off score will be determined endogenously in such a way that it
would be compatible with a given number of students who will be admitted.
In the meantime, the cut-off score is assumed to be fixed. It should be noted at
this juncture, however, that the fixity of the cut-off score is just a matter of
degree. Even when the total number of admissions is strictly controlled by the
relevant authority, schools occasionally apply a kind of implicit quality stan-
dard to maintain the overall quality of the students, sometimes even risking
the possibility of having unoccupied slots in the school, especially when the
overall quality of the applicants turns out to be poor. In this sense, the assump-
tion of a fixed cut-off score still has some practical relevance.

Given the cut-off level of score, \(t^*\), the probability of being admitted is given as

\(^94\) According to O’keefe, Viscusi and Zeckhauser(1984)’s terminology, the model described here is an
“uneven” but “fair” contest.
The last equality comes from the assumption of symmetry of \( F \) which assures the relationship, \( 1-F(z)=F(-z) \). This transformation allows us a more intuitive and straightforward interpretation of some of the results below. Under this transformation, \( F \) has the meaning of probability of success which is increasing in effort level, \( e \), and \( f \) the marginal probability of success.

The students with the given value \( X \) will choose their private effort level optimally to maximize their expected payoff. Their optimization problem can be represented as,

\[
\max_e \quad X \left( F(\alpha e - t^*) - \beta c(e) \right). \tag{4}
\]

The FOC is then,

\[
X f(\alpha e - t^*) \left( a - \beta c'(e) \right) = 0 \quad \text{for} \quad e > 0,
\]

\[
X f(\alpha e - t^*) \left( a - \beta c'(e) \right) \leq 0 \quad \text{for} \quad e = 0. \tag{5}
\]

The FOC simply says that the marginal gain from additional tutoring must be equated with the marginal cost for its interior solution. Otherwise, the equilibrium effort will be set at 0.

The corresponding SOC is

\[
X f'(\alpha e - t^*) a^2 - \beta c''(e) = \mu < 0. \tag{6}
\]

Since \( f > 0 \) when \( c < 0 \), the SOC may not hold with an arbitrary density function, especially when the distribution is highly concentrated and/or the marginal cost function is quite flat. In the following, it is assumed that the variance of the distribution, \( \sigma^2 \), is sufficiently large, and/or the marginal cost function is sufficiently steep that the SOC is always satisfied\(^95\).

From the FOC, it is evident that for a given cost function, the amount of private effort will increase as \( X \) or \( f \) increases. That is, the higher the value a student attaches to the prize, or the higher the marginal probability to win, the larger the effort will be.

\(^{95}\) The possibility that the SOC condition may not be met appears in most of the tournament models. See Lazear and Rosen(1981), Nalebuff and Stiglitz(1983) and McLaughlin(1988).
With regard to \( f \), the density function, it should be emphasized that what is important for the students’ decision is not the absolute level of probability of being admitted, \( F \), but the marginal improvement of their chance of being admitted by putting forth one more unit of effort, \( f \). Whatever the probability of being admitted may be, if this probability cannot be affected by their effort, nobody will ever try to make an investment in effort.

More formally, under the assumption that the objective function is twice differentiable with respect to its argument and parameters, we can express the optimum amount of private efforts as a function of parameters of the model, \( X, t^*, \alpha, \beta, \text{and} \sigma \).

\[
e^* = e^*(X, t^*, \alpha, \beta, \sigma),
\]

and we can perform the following comparative statics. Since we are assuming that private tutoring is one form of unproductive rent seeking activity, it would be desirable to minimize the aggregate expenditures on private tutoring from a social point of view. In evaluating the efficacy of various policy measures to curb private tutoring, therefore, we will use the following social welfare function which should be minimized as much as possible.

\[
W(t^*, \alpha, \beta, \sigma) = \int_x^X \beta \cdot c[e^*(t^*, \alpha, \beta, \sigma)] \, dH(X).
\]

A. The Effects of Varying Valuations Among Individuals

Differentiating the FOC with respect to \( X \) and rearranging the terms yields,

\[
\frac{\partial e^*}{\partial X} = -\frac{f \, \alpha}{\mu} > 0.
\]

The denominator is negative from the SOC, and since \( f > 0 \) everywhere, the whole expression is positive. In other words, a student who puts a higher value on education will purchase more tutoring to improve his/her test score. If \( c'(0) > 0 \), some students who are located near the lower end of the distribution, \( x \), may choose 0 effort and thus drop out of the contest. In order to ensure that every student with a positive surplus value participates in the game and exerts a certain positive amount of effort, I have imposed the constraint, \( c'(0)=0 \), in (2) above. If \( x=0 \), the very person at the end of the distribution will, of course, choose 0 effort. In this case, the equilibrium effort level will be monotonically increasing with \( X \) starting from 0.

\footnote{This accords with the general results in tournament models that the equilibrium effort is increasing in the prize spread between the winning and losing prize. In the current model, the loser gets 0 payoff.}
One interesting thing is that for a student with a given $X$ will have his/her equilibrium effort level at its highest when his/her expected score is near the cut-off score. This comes from the fact that $f$ is symmetric around 0 and has the largest value at $\varepsilon = 0$. From the FOC, therefore, $f$ will have the highest value at $\alpha e^* = t^*$. Since $\alpha e^*(., X, ..)$ is just the expected score of the student with value $X$, the student whose expected score is near the cut-off point will exert the largest efforts. The student whose expected score is far below the cut-off point does not work hard because his/her marginal gain from the incremental effort is expected to be small. By a similar logic, the student whose expected score is way above the cut-off point does not work hard, either, because his/her chance of being admitted is not affected that much by his/her additional effort.

A diagram is helpful for understanding the relationship explained above. Since $\delta e^*/\delta X$ is just the slope of the effort curve, $e^* = e^*(X)$, which is a function of $f$, and we know something about the shape of the density function, $f$, we can be a little bit more specific about the general shape of this effort function. Note that $f'>0$ when $\varepsilon < 0$, and $f'<0$ when $\varepsilon > 0$. Consequently, the value of $f(\alpha e^*-t^*)$ increases monotonically when $\alpha e^*(X)<t^*$, and reaches peak at $\alpha e^*(X)=t^*$, and then decreases when $\alpha e^*(X)>t^*$. The slope of the function $e^* = e^*(X)$ should, therefore, increase up until $\alpha e^*$ reaches $t^*$ and then decrease beyond that point, yielding a S shaped curve like in <Figure 4-2>. Note also that as $t^*$ moves upward (to $t^{**}$ in the <Figure 4-2>), more students fall into the range in which each student’s $f'$ at their $\alpha e^*-t^*$ takes a positive value. If $t^*$ moves downward (to $t^{***}$ in the <Figure 4-2>), the opposite result occurs.

<Figure 4-2> The Relationship between Test Scores and Values

The shaded area represents the probability of winning.
With this result, it is easy to show that the student with a higher value on education will have a higher probability of being selected. By differentiating \( F(\alpha e^* - t^*) \), the winning odds, with respect to \( X \), we obtain the following.

\[
\frac{dF}{dX} = f \alpha \frac{\partial e^*}{\partial X}
\]

\[
= - \frac{f^2 \alpha^2}{\mu} > 0. \tag{10}
\]

The fact that the student who has a higher value on college education will exert more effort guarantees the whole expression above to be positive, implying students with greater values on education will have higher probability of getting admitted. The selection process is, therefore, conducive to social efficiency in the sense that the opportunities for higher education are allocated to those who attach relatively higher values on education.

Rent seeking in this case, as are in most other cases, thus serves social efficiency even though the activities are in and of themselves unproductive. It may, however, create a social dilemma; if you make the testing procedure less effective, you can reduce the wasteful rent seeking activities, but it will increase the probability of allocating the valuable opportunities to the wrong students. On the other hand, if you make the testing procedure more effective, it will encourage more rent seeking.

Of course, this story presumes the existence of tuition control. If there is no tuition control in the first place, there will be no unsatisfied students and no rent seeking. The opportunities to enter the colleges will be allocated in the order of the values they attach to the college education in the free market. That is the beauty of the free market system emphasized so frequently in elementary economics textbooks.

**B. Changing the Cut-Off Score of the Test**

The effect of changing the cut-off level of the test score on the effort level can be obtained in a similar way. By differentiating the FOC with respect to \( t^* \), we have

\[
\frac{\partial e^*}{\partial t^*} = \frac{X f' \alpha}{\mu}. \tag{11}
\]

The direction of change in the above expression depends on the sign of \( f' \). It is positive when \( f' < 0 \), that is, when \( \alpha e^*(X) > t^* \). The students whose expected scores are higher than the cut-off point will work harder because their stakes are now threatened by the increased cut-off point, and their marginal returns
to efforts are made higher. The direction of change is negative when \( f' > 0 \), in other words, when \( \alpha e^{*}(X) < t^{*} \). It implies that the students whose expected test scores are below the cut-off point will decrease their effort level in response to the increased cut-off point. They will take more relaxed attitude because their chances of being admitted have gone farther away by the upward movement of the cut-off point. For them, additional efforts are no longer worth as much as they used to be. (see Figure 4-2.)

Even though each student is making adjustment in different directions depending on their own relative location in the distribution, the odds of success will invariably decrease for every student by the hike of cut-off point. Differentiation of \( F(\alpha e^{*}-t^{*}) \) with respect to \( t^{*} \) yields

\[
\frac{dF(\alpha e^{*}-t^{*})}{dt^{*}} = f \left( \alpha e^{*} \frac{\partial e^{*}}{\partial t^{*}} - 1 \right) = \frac{f \beta e^{''}}{\mu} < 0.
\]

For an obvious reason, the student who has chosen to work less in response to the increased cut-off point will have a smaller chance of being admitted. The student who has chosen to work harder will not be able to improve his/her odds, either, because he/she cannot fully recover the loss of odds due to the cut-off increase by working harder. Surely, as he/she works harder, his/her score will improve, but not as much as the whole amount of the change in the cut-off point. As a result, part of the gap between his/her expected score and the cut-off point, \( \alpha e^{*}-t^{*} \), will remain shrunk. This will lower his/her winning probability, \( F \), too. The reason why he/she cannot fully recover the gap between his/her expected score and the cut-off point is that he/she has already taken large enough effort (Note that he/she is the one whose expected score exceeds the cut-off score.) and increasing his/her effort further entails a cumulative cost increase.

One important implication of the above result is that the total number of students who will be admitted will invariably decrease when the cut-off point is raised, and will increase when the cut-off score is lowered, regardless of whether students work harder or not in response to the hike or cut. The expression given in (12) will reappear repeatedly in the next section when we deal with the quota system.

---

97) We use the notation \( dF/dt^{*} \) to distinguish it from the direct effect, \( \partial F/\partial t^{*} \), which is just \( f \), even though there are still some other parameters held constant. In the following, the same rule about the notation on derivatives will be applied; whenever the effect of a parameter change is evaluated in such a way that it contains not only the direct effect but also the indirect effect via the endogenous variables of the model, it is denoted as total derivatives, despite the fact there remain some fixed parameters. Needless to say, it is a bit confusing. But there appears to be no other good alternatives since we do not have yet a good notation for partial total derivatives.
There is one more implication which is equally important. In aggregate, if the test is highly competitive in the sense that the cut-off score is set at such a high level that only those students with very high X's can pass, most students will fall in the range of $\ell > 0^{98}$. In this case, making the examination more difficult by raising the cut-off score will have an effect of discouraging a larger number of students and making them work less. (see Figure 4-2). On the other hand, making an easy test a bit more difficult — easy in the sense that the cut-off point is set at a low level so that a large number of students can pass the exam — will make a large number of students, who used to be above the cut-off line, become alert and push them to work harder.

In short, making a hard test harder will make many students discouraged and work less. Whereas, making an easy test harder will induce many students to work more. This implies that if the policy objective is to minimize rent-seeking activities, it would be desirable to make a hard test harder, or an easy test easier because it can thereby reduce the aggregate amount of rent seeking activity on the whole population. Making a hard test easier or an easy test harder will aggravate the situation by increasing the aggregate volume of rent seeking expenditures.

In other words, an effective way of reducing the aggregate expenditures on rent seeking is to push the cut-off point toward either the upper or lower extreme end. It will make most of the students either discouraged or relaxed, thereby making additional efforts unworthy. On the other hand, moving the cut-off score toward the mid-point will make most of the people either newly alerted or revived, and the additional rent seeking will appear more attractive.

More formally, differentiating the social objective function (8) with respect to $t^*$ gives,

$$\frac{dW}{dt^*} = \int_x^\infty \beta c^* \frac{\partial e^*}{\partial t^*} dH(X).$$  \hspace{1cm} (13)

Therefore, the sign depends on the sign of $\frac{\partial e^*}{\partial t^*}$, which in turn depends on the location of $t^*$. If it is located near the upper end of the score distribution, then in a wide range of $X$, the partial derivative inside of the integral will take negative value, rendering the sign of the whole expression to be negative. If the cut-off point is located near the lower end, the opposite conclusion would ensue.

In an attempt to reduce private tutoring, policy authority may try to make the entrance examinations easier, believing that an easier test will induce students to take less tutoring. For the reasons explained above, however, the attempt may backfire. If the present cut-off score is set at a very high level, then

---

$^{98}$ In the extreme, if we set the cut-off score at a very high level, say at $\alpha e^*(x)$ so that the student with the highest value, $x$, can have 50% chance of winning, then everybody else will have $\ell > 0$. 

making the test easier by lowering the cut-off level will revive the otherwise discouraged student and will encourage them to take more efforts. On the other hand, if government makes an easy test even easier, then a lot of students will think that effort is not worthwhile, and they will reduce efforts\(^99\).

C. Reducing the Effectiveness of the Private Tutoring

Another measure that may be contemplated by the public authority which intends to discourage private tutoring is to design the test format in such a way that private tutoring becomes less effective in raising the test scores\(^100\). Technically it would be quite difficult to implement the idea in actual testing procedure, but incorporating it into the present model is quite simple, because all we need to do is just to make the size of \(\alpha\) smaller.

Differentiating the FOC with respect to \(\alpha\) and rearranging gives,

\[
\frac{\partial e^*}{\partial \alpha} = -\frac{Xf}{\mu} + \frac{-Xf' e^*}{\mu}.
\]

The first term is positive and the sign of the second term depends on the sign of \(f'\). The first term captures the pure effect of increasing the marginal product to effort, \(\alpha\), on the marginal revenue, holding the marginal probability of winning constant. A one unit increase in \(\alpha\) pushes up the marginal revenue by \(Xf\). This enhanced marginal returns to effort then induces everybody to increase their effort.

The second term captures the effects of an increase in \(\alpha\) that flows through the change in the marginal probability of winning. As the expected score gets higher due to an increase in \(\alpha\), the marginal probability of winning is affected differently depending on where the student is located. For a student whose expected score is below the cut-off line, \(\alpha e^*(X)<t^*\), a higher score moves him/her closer to the cut-off line, and raises his/her marginal probability of winning. Consequently, the student works harder. In the above expression, it means \(f'>0\), which makes the second term positive, thereby rendering the whole expression to be positive. In short, those students whose expected scores fall short of the cut-off point (who put a relatively lower value on college edu-

---

\(^{99}\) For example, in Korea, the college entrance examination has been highly competitive, with the competition ratio standing somewhere around 3:1. In this situation, making the exam easy is likely to bring forth the opposite effects to what the Korean government is intending for. That is, it will further encourage the private tutoring activities among students, especially those who otherwise would have given up in some sense.

\(^{100}\) A high ranking Korean government official was reported to have made the remark that in coming years, the test will be designed in such a way that having private tutoring will turn out to be totally worthless.
cation) will study harder as the returns to study get higher.

Those students whose expected scores are already higher than the cut-off line, \( \alpha e^*(X) > t^* \), however, work less as far as the effect of the second term is concerned when the marginal returns to effort gets higher. For them, a higher expected score moves their whole test score distribution upward, pushing further away from the cut-off point and making the marginal gain in their success probability smaller, that is, \( f' < 0 \). As a consequence, the second term becomes negative for them.

Of course, this doesn’t mean that every student with a higher than cut-off score will reduce his/her effort level, for the positive effect of the first term can still dominate the negative effects of the second term. Generally speaking, only those whose test scores are very high will reduce their efforts when the marginal returns to the effort are made higher.

Since it is impossible to proceed further with an arbitrary density function, I will assume here that the density \( f \) is normal and check the validity of the above assertion under this assumption. With a normal density and letting \( y = \alpha e^*-t^* \), we have

\[
 f'(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{y^2}{2\sigma^2}\right] \left( -\frac{y}{\sigma} \right)
 = \left( -\frac{y}{\sigma} \right) f
\]

The combined effect will be negative only when \(-Xf+Xf' = -X[f+f\alpha e^*] > 0\), alternatively, when \(f+f\alpha e^* < 0\). The following diagram shows the relationship between \( f \) and \( f\alpha e^* \). (See <Figure 4-3>). The sum of these two values will be negative only when the negative value of \( f\alpha e^* \) dominates the positive value of \( f \), which can happen when \( \alpha e^*-t^* \) is very large.

Substituting the above expression for \( f' \) gives

\[
 f + f\alpha e^* = f \left[ 1 - \frac{X e^*}{\sigma^2} \right] < 0.
\]

Solving this for \( \alpha e^* \), we obtain,

\[
 \alpha e^* > \frac{t^*}{2} + \frac{\sqrt{t^{*2} + \sigma^2}}{2}.
\]

Those students whose expected scores are higher than the value in the right hand side of the above inequality, which is a function of \( t^* \) and \( \sigma \), will reduce private tutoring when it is made more effective. The intuitive reasoning is that, for those students who have already put a large amount of efforts and secured
high scores, the increased returns to the existing efforts are large enough that they don’t feel like increasing their efforts further. They would rather choose to reduce part of their efforts.

Although some students with high enough scores choose to reduce their effort level, their odds to win the prize do not decrease. In fact, every student’s odds, regardless of his/her initial score, will improve as a result of this increased returns to efforts. Differentiating $F(\alpha e^* - t^*)$ with respect to $\alpha$, we have

$$\frac{d}{d \alpha} F = f(e^* + a\frac{\partial e^*}{\partial \alpha})$$

$$= f(e^* + a \frac{-X f - X f' \alpha e^*}{X f' a^2 - \beta c''})$$

$$= -f \frac{\beta (c'' e^* + c')}{\mu} > 0.$$  \hspace{1cm} (18)

Thus the odds improves for every student. In the above, $Xf\alpha$ is replaced with $\beta c'$ using the FOC. The reason why the odds improves even for those stu-
dents whose effort levels were lowered is that the increased marginal returns to per unit of effort outweigh the reduced effort effects. Thus the expected score improves for every student. Again, the halt in effort reduction comes mostly from the effect of the reduced effort on the cost of effort. As students lower their effort level, the marginal cost of efforts gets cheaper, creating the opposite effects to buy more.

This overall increase in scores will necessarily increase the number of students being admitted under a fixed cut-off score, $t^*$. This implies that the school authority must raise the cut-off point if it wants to maintain the fixed admission tickets.

The above results imply that an examination test which would cripple the marginal returns to private tutoring would be in general effective in reducing the expenditures on private tutoring. However, if the current $t^*$ is very low, there is a possibility that a reduction in $\alpha$ may stimulate a more tutoring in aggregate terms.

D. Increasing the Noise of the Test

Still another conceivable measure to abate the incentive for private tutoring might be to construct a test such that the test score should be relatively unpredictable. If private tutoring becomes an unreliable means to improve test scores, less people will be, on first impression, induced to make the investment in tutoring. In order to incorporate this idea into the model, I experiment with the model by increasing the variance of the test scores101). It turns out, however, that the final result is indeterminate because some students increase their effort while others decrease it and the overall effect is uncertain.

Taking the differentiation of the FOC with respect to $\sigma$, and collecting terms, we obtain the following equation,

$$\frac{\partial e^*}{\partial \sigma} = -\frac{aX}{\mu} f_\sigma, \quad \text{where} \quad f_\sigma = \frac{\partial f}{\partial \sigma}. \quad (19)$$

Therefore, the overall sign depends on the sign of $f_\sigma$. Again by resorting to the assumption of normality, we have,

$$f_\sigma(y) = \frac{y^2 - \sigma^2}{\sigma^3} f(y). \quad (20)$$

Plugging it into (19) yields,

101) One good example to do that would be to put more essay type questions than multiple choice questions in the test.
Implying that the sign of the change depends on the relative size of $y^2$ and $\sigma^2$; it is negative in the range of $t^*-\sigma < \alpha < t^*+\sigma$, and positive outside of the range. This suggests that students whose expected scores are in the middle range around the cut-off point will decrease their efforts whereas students who are located near both tails of the distribution will increase their efforts.

This result is quite intuitive if we look at the changes made in the shape of the density function by the increased dispersion. Augmenting the variance decreases the density in the middle range and makes the tails of both ends of the distribution fatter. These changes in marginal probability of success drive the results obtained above. Or, to put the same story in a slightly different way, the students in the upper tail work harder to guard against the possibility that their chances will be marred by the increased randomness of the test outcomes. The students in the lower tail also work harder but for a slightly different reason. They make a bet in an anticipation that they may get admitted, this time just by sheer chance.

The winning odds for each student moves in a bit of a complicated way. For some students, the odds improve as a result of the increased dispersion, and for some others, they deteriorate. And still for some others, the signs of changes are indeterminate. To see this, let us take a differentiation of $F$ with respect to $\sigma$. The result is,

$$\frac{\partial e^*}{\partial \sigma} = -\frac{a X f}{\mu} \left( \frac{y^2 - \sigma^2}{\sigma^2} \right).$$  \hfill (21) \]

The winning odds for each student moves in a bit of a complicated way. For some students, the odds improve as a result of the increased dispersion, and for some others, they deteriorate. And still for some others, the signs of changes are indeterminate. To see this, let us take a differentiation of $F$ with respect to $\sigma$. The result is,

$$\frac{dF}{d\sigma} \bigg|_{e'=\text{const}} = F_\sigma + \frac{\partial e^*}{\partial \sigma} \bigg|_{e'=\text{const}} \bigg(22\bigg)$$

where $F_\sigma = \partial F / \partial \sigma$. The first term captures the pure effect of the $\sigma$ change on winning odds, holding the effort level constant, and then the second term captures the effect of the effort level change due to the increased noises in the test.

Let’s take the second term first. It contains the expression, $\partial e^*/\partial \sigma$, which has appeared in (19), representing the direct effect of the $\sigma$ change on effort. As explained earlier, the sign of this term is indeterminate as students whose expected scores are near the cut-off score work less whereas those students far away in either direction from the cut-off score work harder. For later comparison, however, it is worthwhile to emphasize that what is crucial here is not the student’s location relative to the cut-off point itself, $t^*$, but his/her location relative to the $t^* - \sigma$.

The sign of the first term also depends on the location of student’s expected score, but this time relative to the cut-off point itself, $t^*$. For any density function which has a unimodal and symmetric around the zero mean, as assumed at the outset, $F_\sigma > 0$ when $\varepsilon < 0$, and $F_\sigma < 0$ when $\varepsilon > 0$. In the current setting, therefore, the first term is positive for those students whose expected score is below the cut-off score. It is negative for those students whose score is above
the cut-off score. More formally, assuming normality of the distribution, we have the following relationship.

\[
\frac{\partial F(y)}{\partial \sigma} = \int f(m) \frac{m^2 - \sigma^2}{\sigma^3} \, dm.
\]

(23)

Note that \(\frac{\partial F(y)}{\partial \sigma}\) is positive when \(m<0\), and negative when \(m>0\).

The above diagram, <Figure 4-4>, explains the \(F_{\sigma}\) effect. Note that the diagram represents the distribution of the actual test score, not its distance from the cut-off score. For a student whose expected test score is greater than the cut-off point, therefore, the cut-off line is located to the left of the \(\alpha e^*\). In this diagram, the winning probability is represented by the area to the right of the cut-off point and below the density curve. The density curve “flattens,” as the variance of the distribution increases. As a result, for every student whose expected score is greater than the cut-off line, the shaded area shrinks as the dispersion increases. Conversely, the area increases for every students whose expected score is below the cut-off line.

<Figure 4-4> The Direct Effect of Variance Change on Winning Odds

Now, we have to combine these two effects to get the overall picture about the effects on the winning odds from increased noises of the test. Since the effects appear a bit complicated, a graphical illustration is used to summarize the final results (see <Figure 4-5> below). The upper panel represents the \(f_{\sigma}\) effects, the middle panel the \(F_{\sigma}\) effects, and the bottom panel the combined total effects. The dark colored range represents where the winning odd effects are negative, the gray where they are indeterminate, and the white where they are positive. As shown in the diagram, the odds improves for those students
located near the lower end of the distribution. And those students who are near and above the cut-off point suffer from aggravated odds. For others, the signs are indeterminate.

Note that some students decide to work harder and improve their odds, and others decide to work less and suffer reduced odds. For these two groups of students, the world may appear to be fair enough. They are getting what they deserve; a hard work and a high payoff or the opposite. There are, however, still other students who may enjoy increased odds even with lowered efforts, or who may suffer declined odds with stepped-up efforts. We don't know why God favors a specific group of students, especially those who are just below the cut-off line but not very far from it, disfavors others, and applies what appears to be humanely fair standard to still others.

The above result implies that the quality of the student body is most likely to be deteriorated by an increase in the variance of the test: the low valuers have increased odds of being admitted while the high valuers have reduced or uncertain odds.
E. Increasing the Cost of Private Tutoring

Finally, still another measure that can be employed for the purpose of reducing private tutoring is to make private tutoring more costly. In the present model, increasing the cost means to raise the value of $\beta$.

Differentiation of the FOC with respect to $\beta$ renders the following expression.

$$\frac{\partial e^*}{\partial \beta} = \frac{c'}{\mu} < 0,$$

which invariably insures that the efforts will decrease in response to the increased cost. Whether this will reduce the aggregate social cost of private tutoring, however, is uncertain because the per-unit marginal cost of tutoring has been increased even though the total amount of private tutoring is clearly reduced. Differentiating the social objective function, (8) with respect to $\beta$ gives,

$$\frac{dW}{d\beta} = \int_{x}^{\infty} [c + \beta c' \frac{\partial e^*}{\partial \beta}] dH(x).$$

The second term inside the integral is negative from the above, but the first term is positive, and therefore, the net result is indeterminate.

As every student cuts down his/her effort level in response to the increased cost, the winning odds decreases for every student for a trivial reason.

$$\frac{dF}{d\beta} = f a \frac{\partial e^*}{\partial \beta} = \frac{f a c'}{\mu} < 0.$$ (26)

4-3. Rent Seeking Under the Quota System

So far, I have assumed that the cut-off score, $t^*$, is exogenously given. So the model described above pertains to cases where there are some kinds of pre-set

---

102) Legally banning private tutoring altogether is certainly one option, which can be interpreted as making the cost prohibitively high. After all, private tutoring is a voluntary transaction between students and tutors. So it is supposed to be mutually beneficial to the both parties. Criminalizing a voluntary exchange is extremely difficult to enforce and has never been completely successful in human history, because there is neither an offender nor a victim. Private tutoring is one of those “victimless crime” which includes drug trafficking and prostitution. Both sellers and buyers work together to thwart the authority’s attempt to catch the wrongdoers. As a consequence, illegalization of a transaction usually ends up with a mere increase in cost of executing the transaction, not elimination of the trade itself.

103) Apparently, it depends on the elasticity of the total cost with respect to $\beta$. Rewriting the inside of the integral of equation (25) gives $c[1+(\beta/c)(dc/d\beta)] = c[1+\eta c]$ where $\eta c$ is the elasticity of the cost with respect to $\beta$, which can be rewritten as $\eta c = \eta c \eta c$ product of two elasticities.
standards for passing. There is a variety of tests which fit this category; most
certificates or licenses are issued only to those who pass the minimum quality
standard, and some government administrative and judiciary officials are
selected this way.

There are, however, other tests where the total number of winners, not the
cut-off score, is pre-set. In this system, therefore, whatever the distribution of
scores may be, the selection criterion is not the absolute individual score but his/her relative ranking, and those people whose rank is above the pre-set
number will be selected. We can call the former the “Quality Standard System”
and the latter the “Quota System”. In most cases, student selection for admis-
sion is administered by the quota system described above, more so in the case
when the total number of admission tickets is controlled by an education
authority, as is done in many underdeveloped countries. In the following I try
to incorporate this new mode of selection method into the model and analyze
its implications.

The model is basically the same as before except that we need one additional
constraint that assures that the cut-off score now has to be set at the level that
would be compatible with the pre-set quota. The system works in the follow-
ing way: initially, ex ante, students form expectations about the cut-off score
that will be chosen by the test authority given the fixed number restriction, n.
And on the basis of this expected cut-off score, they choose their optimum level
of efforts, i.e. the amount of their tutoring purchase. Then the ex post cut-off
score will be determined by the school authority at the level of the nth highest
score. In the final equilibrium, students’ expectations are realized in the sense
that the ex ante expected cut-off score is equal to the ex post cut-off level cho-
sen by the school authority.

More formally, from the FOC and equation (7), we can see that for a stu-
dent with value \( X \), the probability that his/her score falls short of an arbitrary
level of score, \( t \), is given as

\[
\Pr \left[ \alpha e^*(t^*, X, \ldots) + \varepsilon < t \right] = \Pr \left[ \varepsilon < t - \alpha e^*(t^*, X, \ldots) \right] \\
= F \left[ t - \alpha e^*(t^*, X, \ldots) \right] \\
= 1 - F[\alpha e^*(t^*, X, \ldots) - t].
\]

(27)

In the above function, the relationship of \( F(z) = 1 - F(-z) \) has been utilized in
deriving the third equality. By taking the weighted average of this individual
distribution of test scores over \( X \), we can get ex ante population distribution of
test scores, \( G(t) \).

\[
G(t) = 1 - \int_{t}^{\infty} F[\alpha e^*(t^*, X, \ldots) - t] \, dH(X).
\]

(28)
In equilibrium, the fraction with \( t \) greater than \( t^* \) in \( G(t) \) must be equal to the admission rate, \( \phi \), where \( \phi = n/N \), with \( N \) denoting the total number of applicants\(^{104}\). Alternatively, \( 1-G(t^*) = \phi \) or simply, \( \phi = \int \mathbb{F}[a e^* - t^*] dH(X) \) must hold. Note that \( t^* \) determined in this way is no longer a function of \( X \), because the \( X \) effects are already integrated out. The cut-off point is not a function of a particular value of \( X \), but of the whole distribution of \( X \).

Under this reformulation, the whole system is now represented by the following two equations with two endogenous variables, \( e^* \) and \( t^* \).

\[
\begin{align*}
1) \quad & X \int f(a e^* - t^*) \, d\phi = 0, \\
2) \quad & \phi = \int X \mathbb{F}[a e^* - t^*] dH(X).
\end{align*}
\]

The equation 1) is just a replication of the FOC given in (5) in the previous section\(^{105}\). The equation 2) is the rational expectation perfect foresight condition.

Now that we have set the stage for this expanded model, we can proceed to examine some of the effects of policy changes as we have done in the previous section.

A. Increasing the Admission Quota

First, let’s take a look at the effects of changing \( \phi \), the new parameter which represents the admission rate, on \( t^* \) and \( e^* \). Differentiating equations 1) and 2) in (29) with respect to \( \phi \) generates the following.

\[
\begin{align*}
\int & f(a \frac{de^*}{d\phi} - \frac{dt^*}{d\phi}) - \beta c' \frac{de^*}{d\phi} = 0 \quad (30) \\
\int & X f(a \frac{de^*}{d\phi} - \frac{dt^*}{d\phi}) dH(X) = 1. \quad (31)
\end{align*}
\]

\(^{104}\) In a strict sense, the fixed number system is different from the fixed ratio system, especially when the total number of applicants varies every time. In this section, I am assuming the total number of applicants is fixed. Therefore, both are the same.

\(^{105}\) This is not innocuous though. In a strict sense, the environment and thus incentives faced by the students under the quota system are different from those under the quality standard system. Under the fixed quota system, how other students will behave has a direct impact on the probability of being selected. For example, at the margin, if I get admitted, then another student must be dropped out to keep the fixed quota. Partly because I have here a continuum of students and for the sake of simplicity, I rule out such strategic considerations and use the same FOC as is used in the earlier section. In other words, I am assuming that every student plays “against the field.” For a model which attempts to explicitly consider the strategic aspects, see Dixit(1981).
Solving (30) for $\frac{de^*}{d\phi}$ generates,

$$\frac{de^*}{d\phi} = \frac{Xt^*}{\mu} \frac{dt^*}{d\phi}. \tag{32}$$

Note that the coefficient of $\frac{dt^*}{d\phi}$ above is just the same as $\frac{\partial e^*}{\partial t^*}$ given in (11). Therefore, the following result holds,

$$\frac{de^*}{d\phi} = \frac{\partial e^*}{\partial t^*} \frac{dt^*}{d\phi}. \tag{33}$$

which is self-explanatory. That is, the question of how students will respond to an increase in admission rate depends on how much of the cut-off point will be adjusted in response to the admission rate change, and how students will react to that cut-off point change.

Of course, the required change in $t^*$ to make the new $\phi$ effective, in turn, hinges on how students will behave in response to the $\phi$ change. So, the logics are circular, and we have to solve them simultaneously.

Inserting (32) into (31), and solving for $\frac{dt^*}{d\phi}$ yields,

$$\frac{dt^*}{d\phi} = \int \frac{1}{\mu} \frac{\partial e^*}{\partial c^*} \frac{dt^*}{d\phi} dH(X) < 0. \tag{34}$$

Thus, the cut-off score must be cut down. This is, in a sense, a trivial result because in order to increase the number of students admitted, you have to lower the cut-off score with other things being fixed. As you will see below, however, it is not that automatic but is conditioned on some of the earlier results. Furthermore, this term plays an important role in many of the following comparative statics, acting as a unit price of $t^*$ in the sense that it gives the size of the necessary change in $t^*$ that will exactly cancel out the effects of the various parameter changes on the admission rate $\phi$. (See equations (39), (56), (61) below for example.)

Note that the expression inside of the integral sign above is just $dF/dt^*$, the change in winning odds of each student due to the change in the cut-off point, which can easily be confirmed by referring to equation (12) above\(^{106}\). This is always negative for every student despite the fact that some students choose to work harder in response to the cut-off score increase. As explained earlier, the cost constraint hinders every student from recovering the diminished winning

\(^{106}\) Strictly speaking, it doesn’t make sense to take a derivative with respect to one of the endogenous variables of the model in the current setting. In this hypothetical experiment, therefore, we are just temporarily holding $t^*$ as if exogenously fixed.
odds. Consequently every student will get hurt by a cut-off point hike. This drives the conclusion reached above\textsuperscript{107}).

By definition, an integral of individual student’s winning odds over the whole population gives the fraction of students who are admitted, the admission rate, $\phi$. Therefore, the whole expression in the denominator of equation (34) is simply $d\phi/dt^*$, the resulting change in admission rate when $t^*$ is one unit increased. This can be again easily checked by directly differentiating equation 2) in (29) with respect to $t^*$. Since the sign of this expression is negative, it gives simply how many students will additionally fail if you increase the cut-off score by one point.

It is now clear how the expression (34) works as a unit price of $t^*$. Suppose a certain number of students is expected to additionally fail as a result of a parameter change, say, an increase in tutoring cost. In order to maintain a fixed admission quota, the school authority has to lower the cut-off score. But by how much? The expression in (34) provides the answer: the new $t^*$ should be set at the level which can exactly save those students who are adversely affected by the parameter shift.

Plugging (34) back into (32) gives the effects of change in $\phi$ on individual student’s effort level. Since the coefficient of $dt^*/d\phi$ is the same as the one given in (11) and $dt^*/d\phi$ is negative, it implies that those students whose expected scores are in excess of the cut-off score will work less, and students with low scores will work more as the admission rate increases. As it gets easier to gain entry into a college due to the admission rate increase, those students with relatively high scores now work less hard, because it is no longer necessary to put that much effort to get in. Whereas, those students with lower scores work harder because their chance of being admitted has now been improved.

The fact that students will show different responses to the change in admission rate depending on his/her relative position to the cut-off score gives rise to similar implications on the aggregate tutoring discussed earlier. That is, the aggregate volume of private tutoring will crucially depend on the current state of competitiveness in college admissions. If it is already highly competitive, relaxing it by increasing the admission quota a little bit will not help reduce private tutoring, but aggravate the situation further.

### B. Increasing the Cost of Private Tutoring

The effect of increasing the cost of private tutoring can be analyzed in a similar way. Differentiating equations 1) and 2) in (29) with respect to $\beta$, gives,

\textsuperscript{107}If a student can improve his/her odds by working harder in response to the cut-off point increase, the above condition will not hold.
Solving (35) for \( \frac{de^*}{d\beta} \) yields,

\[
\int_{x} x f \left( a \frac{de^*}{d\beta} - \frac{dt^*}{d\beta} \right) dH(X) = 0. \tag{36}
\]

Solving (35) for \( \frac{de^*/d\beta}{y} \) yields,

\[
\frac{de^*}{d\beta} = \frac{Xf' \alpha}{\mu} \frac{dt^*}{d\beta} + \frac{c'}{\mu}. \tag{37}
\]

Note that the second term in (37) is exactly the same as that in (24) above. So this is the direct effect of raising the cost of private tutoring on students' effort level, the sign of which is negative. The first term then takes into account the effect of cost increase on \( t^* \) and each student’s subsequent adjustment in effort level due to the change in \( t^* \). Simply put, the equation (37) can be rephrased as,

\[
\frac{de^*}{d\beta} = \frac{\partial e^*}{\partial t^*} \frac{dt^*}{d\beta} + \frac{\partial e^*}{\partial \beta}. \tag{38}
\]

Inserting (37) into (36) and solving for \( \frac{dt^*/d\beta}{y} \) renders,

\[
\frac{dt^*}{d\beta} = -\frac{\int \frac{atc'}{\mu} dH(X)}{\int f\beta c''/\mu dH(X)} < 0. \tag{39}
\]

The denominator is what we have already seen in (34) and is negative. Overall, an increase in tutoring cost will put a downward pressure on the cut-off score. The intuitive reason behind this result is straightforward. An increase in tutoring cost induces every student to take less tutoring, and this lowers the overall distribution of test scores. Under this situation, the school authority has to lower the cut-off point of the test in order to continue to secure a given fraction of students.

It is interesting to note that the numerator in (39) is just \( d\phi/d\beta \) evaluated under the assumption of fixed \( t^* \). That is, differentiating the equation 2) in (29) with respect to holding \( t^* \) constant, yields,

\[
\frac{d\phi}{d\beta} \bigg|_{t^* = 0} = \int \frac{atc'}{\mu} dH(X) < 0. \tag{40}
\]
So it captures the effect of changing $\beta$ on the admission rate under the fixed $t^*$, and the sign is negative. As mentioned earlier, an increase in tutoring cost causes every student to take less tutoring. This lowers the expected score of every student. Consequently, if the school keeps the cut-off point at the old level ($dt^*=0$), the fraction of students who are admitted will be curtailed.

Cancelling this negative effect on the admission rate of increased cost and restoring the original admission rate requires a corresponding adjustment in the cut-off score in an opposite direction. The size of the latter adjustment necessary is obtained by dividing this expression by the denominator in (39) which happens to be equal to $d\phi/dt^*$. In short, the following relationship holds.

$$\frac{dt^*}{d\beta} = -\frac{\frac{d\phi}{d\beta} \big|_{t^*-\text{exog}}}{\frac{d\phi}{dt^*} \big|_{t^*-\text{exog}}} \quad (41)$$

Note that this is the very relationship between $t^*$ and $\beta$ (or any other parameters for that matter) that should hold when you want to keep $\phi$ constant in equation 2 in (29).

Generalizing the above result, we obtain the following “fundamental relationship” between the cut-off score and every exogenous parameter of the model for maintaining a fixed admission rate. Denoting the representative exogenous parameter as $z$, we have,

$$\frac{dt^*}{dz} = -\frac{\frac{d\phi}{dz} \big|_{t^*-\text{exog}}}{\frac{d\phi}{dt^*} \big|_{t^*-\text{exog}}} = -\frac{\int \frac{dF}{dz} \big|_{t^*-\text{exog}} dH(X)}{\int \frac{dF}{dt^*} \big|_{t^*-\text{exog}} dH(X)} \quad (42)$$

with an understanding that both numerator and denominator are to be evaluated at equilibrium point treating $t^*$ as if exogenous\(^{108}\). Note that the denominator contains no expressions related to exogenous parameters. Consequently the denominator is not associated with any particular parameter, but is common to every exogenous parametric change.

---

\(^{108}\) This explains why the expression, $\frac{d\phi}{dz}$, in the numerator has the partial derivative sign, and the expression, $\frac{d\phi}{dt^*}$, can be defined and have any meaning.
Basically, what we have here can be restated in the following way: firstly, by taking a total differentiation of $\phi$ with respect to two endogenous variables, $e^*$ and $t^*$, and an exogenous parameter whose effects we are interested in looking at, we get,

$$d \phi = \phi_e \, de^* + \phi_t \, dt^* + \phi_z \, dz. \quad (43)$$

Secondly, noting that $de^*$ and $dt^*$ cannot move independently because they are constrained to move together through the endogenicity constraint, we have the following relationship by differentiating $e^* = e^*(t^*, z)$,

$$de^* = \frac{\partial e^*}{\partial t^*} \, dt^* + \frac{\partial e^*}{\partial z} \, dz. \quad (44)$$

Plugging this into (43) gives,

$$d \phi = (\phi_e \frac{\partial e^*}{\partial t^*} + \phi_t) \, dt^* + (\phi_e \frac{\partial e^*}{\partial z} + \phi_t) \, dz. \quad (45)$$

Lastly, letting $d\phi = 0$, and solving for $dt^*/dz$ gives rise to (42). There is still another way to obtain the same result directly from the whole system of (29). This is given in the Appendix. Since the same relationship holds for every parametric change in the model, we will utilize this formula whenever deemed convenient in the following, without going through the tedious substitution process.

The effects of cost change on each student’s equilibrium effort level is now obtained by plugging (39) into (37). The overall effects on individual efforts are likely to be negative for those students whose expected scores are greater than the cut-off score since both the last term and $dt^*/d$ in the first term in equation (37) are negative, and the coefficient of $dt^*/d$ is positive for them. For those students whose expected scores are below the cut-off point, however, the sign is indeterminate because the whole first term is positive but the second term is negative.

In the final equilibrium after the adjustments in both students’ effort levels and the school’s cut-off score have been made, the change in individual student’s final winning probability depends crucially on his/her position relative to the whole distribution. Some students’ odds will improve, and others’ will not, depending on the criterion to be discussed shortly. This is one of the most important differences between the quality standard system and the quota system. In the earlier section where the cut-off point is exogenously set, the directions of changes in individual student’s odds have been in most cases uniform across the board, as are given in (17), (18) and (26)109.
Under the current quota system, however, if some students get extra benefits due to a policy change in terms of their winning odds, then some others must get hurt. This is simply because of the definition of the quota system. If someone gains, others must lose in order to keep the level flat\(^{109}\). The new cut-off score will be set exactly at the point that will accomplish this leveling task, somewhere in the middle that will balance off the opposing forces. And the resulting changes in individual student’s odds should occur in such a way that the aggregate admission rate will not be affected. This is, of course, trivial in the sense we have already imposed such a condition from the beginning, but worthwhile to check whether it really holds and how.

Since the above claim applies to every parametric change, I will use the generalized version given in (42) in deriving the proof. Let the inside of the integral sign in the denominator be denoted as \(\xi\). And then, by dividing and multiplying the inside of the integral of the numerator by \(\xi\), we have the following relationship,

\[
\frac{dt^*}{dz} = \frac{\int -\frac{dF}{dz} \xi e^{-x_{\text{ave}}} \xi dH(X)}{\int \xi dH(X)}.
\]  \hspace{1cm} (46)

Let the first term inside of the integral of the numerator be denoted by still another term \(K\), so that we have,

\[
\frac{dt^*}{dz} = \frac{\int K \xi dH(X)}{\int \xi dH(X)} = \frac{K}{\xi}.
\]  \hspace{1cm} (47)

Under this new representation, \(dt^*/dz\), the change in equilibrium cut-off point due to a parametric change, can be interpreted as a weighted average of \(K\), with the weights being given by \(\xi\). Correspondingly, the generalized version of the equation (37), which represents the change in equilibrium effort due to a parametric variation, becomes\(^{111}\),

---

109) One exception has been the case in which the dispersion of the test scores has been increased. In this case, there some students get hurt while others benefit in terms of winning odds.

110) Although I have introduced the assumption that every student is playing against the field, some remaining negative externalities are unavoidable, because they are inherent in the quota system.

111) See Appendix for actual derivation of the generalized version.
Now, differentiating the winning odds function, $F$, with respect to $z$, and substituting (48) for $dt^*/dz$ yields$^{112}$,

\[
\frac{d\, e^*}{dz} = \frac{\partial \, e^*}{\partial \, t^*} \frac{dt^*}{dz} + \frac{\partial \, e^*}{\partial \, z}.
\]

Collecting terms and further simplifying generates,

\[
\frac{dF}{dz} = \frac{\partial F}{\partial e^*} \frac{de^*}{dz} + \frac{\partial F}{\partial t^*} \frac{dt^*}{dz} + \frac{\partial F}{\partial z}
\]

\[
= \frac{\partial F}{\partial e^*} \left[ \frac{\partial e^*}{\partial t^*} K + \frac{\partial e^*}{\partial z} \right] + \frac{\partial F}{\partial t^*} K + \frac{\partial F}{\partial z}. \tag{50}
\]

Equation (50) says that change in individual student’s winning odds in the final equilibrium in the wake of a parametric change solely depends on his/her position relative to the population mean of $K$. Since $\xi$ is negative, if his/her $K$ is greater than the whole population mean, his chance of winning will improve, whereas if his/her $K$ is smaller than the mean, his chance will deteriorate.

As mentioned earlier, it is redundant to check whether these improvements and deteriorations of individual student’s winning odds will indeed average out among students, and, thus, consequently result in an unchanged admission rate, because we have solved everything so far under that condition. But it is still illuminating to confirm the result because it reassures us that all the necessary adjustments should be made in such a way that in the final equilibrium the admission rate should not be affected at all.

Differentiating $\phi$ with respect to $z$ gives,
where the last equality follows from the very definition of $K$, given in (47). This proves the claim asserted above.

Now, we have to explicate what the $K$ above really stands for. Looking back at the expression in the second equality of equation (42) is helpful in this regard. The inside of the denominator given there is simply $\xi$. Therefore, $K$ is just a ratio of the following two derivatives,

$$K = -\frac{dF}{dz} \bigg|_{t^\text{exog}}.$$

(52)

Given this, $K$ is simply the Marginal Rate of Substitution (MRS) between $z$ and $t^*$, holding $F$ constant. Thus it gives the rate of change in $t^*$ that is necessary to keep the winning odds of each student constant when there is a change in a parameter. If a student works harder in response to a particular parameter change, his/her odds will probably increase. To keep his/her odds at the old level, the school authority has to adjust the cut-off point a bit upward. If another student works less, then the school authority has to lower the cut-off point correspondingly to him/her.

Note that the adjustments of cut-off point in this hypothetical experiment are all individualized and targeted for each specific student. The school, however, cannot have multiple cut-off points, custom-tailored for each student. It has to set just one cut-off point applicable to every student somewhere in the middle considering the whole distribution of the required adjustments. So it must take a weighted average of those necessary adjustments. This is exactly what drives the results we have had in (47) and (50) above. This also explains why we should take $t^*$ exogenous while evaluating every term in (42) and (52).

The weights used to compute the average is $\xi=\frac{dF}{dt^*}$, the sensitivity of $F$ to the changes in $t^*$. Thus, $K$ is the weighted average of the individual student’s MRS between an exogenous parameter, $z$, and the cut-off point, $t^*$, with the weights being the sensitivity of $F$ with respect to the cut-off point, $t^*$.

How is this MRS affected by $X$, the individual index in our model? And how is the individual student’s winning odds affected at the final equilibrium?
Note that $K$ differs depending on which parameter we are considering\(^{113}\). So we cannot continue to work with this generalized version any longer. I will, therefore, switch here to a concrete example.

In the current setting, we have from (39),

$$K_\beta(X) = -\frac{\alpha^2 e^*}{\beta e^*} = -\frac{\alpha}{\beta} \frac{c^*}{c} < 0.\quad (53)$$

In this case, the MRS is negative for every student, which is again intuitive. A higher tutoring cost induces a student to take less tutoring. This depresses his/her score, and pushes down his/her odds. To put him/her back to the earlier position thus requires a slash in the cut-off point. Therefore an increase in $\beta$ should be matched with a decrease in $t^*$, which makes the sign of the whole expression negative. The terms of this trade-off is increasing in $\alpha$ because a given change in $e^*$ generates the larger change in score and winning odds, the larger $\alpha$ is. The slope is decreasing in $\beta$. In its extreme, a very high $\beta$ will make the slope almost flat. This is because the larger $\beta$ is, the larger drop in $F$ an increase in $t^*$ causes, either by lowering $e^*$ or compressing the size of $e^*$. Finally, it is a function of the relative convexity of the cost function, which makes the whole expression similar to the familiar absolute risk aversion parameter in the expected utility theory\(^{115})\(^{116}\).

Letting $c'(e^*)/c''(e^*)$ be denoted as $R(e^*)$, and differentiating $K$ with respect to $X$ gives,

$$\frac{dK^\beta}{dX} = -\frac{\alpha}{\beta} R'(e^*) \frac{\partial e^*}{\partial X}. \quad (54)$$

Since $\partial e^*/\partial X$ is positive, the sign of the whole expression hinges on how the cost function behaves as $e$ varies, that is, it depends on the sign of $R'(e^*)$\(^{117}\). If it is positive, the sign of the whole expression in (54) will be negative, implying that $K$ is decreasing in $X$. In this case, the student with a high $X$ will have a smaller MRS (greater in terms of absolute value because in this case the MRS is negative) than the average, so $[K-K]>0$, and his/her odds given in (50) will deteriorate.

---

\(^{113}\) And each individual student has a different value of $K$.

\(^{114}\) Note that this $K$ is specifically associated with the parameter. Therefore, it is denoted as $K_\beta$, to make it clear which parameter we are dealing.

\(^{115}\) This is quite intuitive since each student’s decision on whether to work harder or not necessarily depends on where he/she is currently located along the cost curve, and how much it will additionally cost him to take an extra work.

\(^{116}\) It is also interesting that the MRS does not depend on any particular density, $f$.

\(^{117}\) If the cost function is of a quadratic type, like $c(e) = e^2 + b e$, then $dR/de = 1>0$. If it is of an exponential type, $c(e) = \exp(e)$, then $dR/de = 0$. If.
Of course, if $R'(e^*)<0$, the opposite conclusion would hold. If $R'(e^*)=0$, $K$ is independent of $X$. The required change in $t^*$ will be the same for every student.

It is interesting to note that the sign of $R'(e^*)$ has quite striking policy implications for measures intended to reduce the private tutoring. If $R'>0$, the above result implies that increasing the cost of private tutoring (raising $\beta$) will hurt economic efficiency in the sense that it will lower the probability of the high valuers being selected and increase the low valuers’ probability. Consequently, the average quality of students, with the quality being measured by the values they attach to college education, will be lowered. Conversely, if $R'<0$, increasing the cost of tutoring will improve the economic efficiency by generating the opposite outcome.

Therefore, if a public authority has a concern for both allocative efficiency (increasing the probability of the high valuers’ obtaining the commodity) and reduction of rent seeking, it has to weigh the relative gains from the reduction of rent seeking against the resultant loss of allocative efficiency in the case of $R'>0$. If $R'<0$, the policy authority does not need to worry because it can achieve both objectives with a single stroke, just by raising the cost of rent seeking.

The above result also implies that if the public authority somehow has a control over the cost function, choosing a cost function of the type which will generate $R'<0$ is desirable when it consider raising the cost of rent seeking as a means of suppressing it.

C. Lowering the Marginal Returns to Private Tutoring

It is now straightforward to analyze the effect of a change in $\alpha$. All we need to do is to follow the path that has been cleared above. Differentiating equation 1) in (29) with respect to $\alpha$ and simplifying gives,

$$\frac{de^*}{d\alpha} = \frac{aXf'}{\mu} \frac{dt^*}{d\alpha} + \frac{-Xf - aXf' e^*}{\mu}. \tag{55}$$

Again, the second term in the right hand side is the direct effect of an increase in $\alpha$ on the effort level which has appeared in equation (14). As explained there, most students, except those with very high expected scores, will increase their efforts in response to the raised marginal returns to private tutoring. The first term is the indirect effects induced by the change in the cutoff score, the direction of which will be determined shortly.

Utilizing the results obtained above, taking partial total derivatives of $\phi$ in equation 2) in (29) with respect to $t^*$ and $\alpha$, respectively, treating $t^*$ exogenous, and lastly taking the negative ratio between the two gives,
Enhanced marginal returns to private tutoring thus causes the cut-off score to move upward. The intuition behind this results is again fairly simple. An improvement in marginal returns to tutoring raises the expected score, \( \alpha e^* \), of every student, given \( e^* \). Furthermore, almost every student chooses to take more effort, \( e^* \), in response to the \( \alpha \) increase. The combined effect of this two upward pressures causes the cut-off score to rise.

Plugging this result back into (55) will give the final equilibrium solution for \( de^*/d\alpha \).

From (56), it is obvious that the K associated with the \( \alpha \) change is given simply by,

\[
K_\alpha = \frac{\int \frac{f\beta (c' + c'e^*)}{\mu} dH(X)}{\int \frac{f\beta c''}{\mu} dH(X)} > 0.
\]  

(56)

implying that the cut-off score must be raised for every student if the winning odds for each student is to be kept at the old level. Although both (56) and (57) above appear to have the same information about the required adjustment in cut-off score, they are different in a strict sense. Recall that (56) above is nothing but \( K_\alpha \), the weighted average of \( K_\alpha \). Thus (56) applies to the whole population, whereas (57) applies to each individual. Needless to say, an average over the whole population can be positive even when some individuals have negative values. The fact that both (56) and (57) have the same sign is, therefore, a kind of coincidence, not a necessity. In this case, it just happens that every student has the same positive sign.

Even though the signs of the necessary adjustment are all the same for every student, the sizes of adjustment may be different among students. In general, the required size must be different because every student chooses different level of effort and thus locates him/herself on a different point on the given cost function. Note once more we have the expression that captures the effect of the relative convexity of the cost function. Differentiating it with respect to \( X \) gives
implying that if $R' > -1$, $K_\alpha$ is increasing in $X$. In this case, the winning odds will improve for those students with high $X$ and thus high $e^*$, and will worsen for those with low $X$ and low $e^*$.

When the marginal returns to effort is raised, most students work harder. At the same time, the increased $\alpha$ amplifies the expected scores further, the more so, the higher the old $e^*$ is. This means that the high valuers now have much higher scores than the low valuers, even those who have decided to reduce their efforts. It improves the high valuers’ odds. In fact, this amplifying effects are so great that it does not require $R'$ to be greater than zero in order for $K_\alpha$ to be increasing in $X$. Just $R' > -1$ is enough. Only when $R' < -1$, $K_\alpha$ becomes decreasing in $X$, and the opposite results ensue.

In a similar way explained earlier, lowering $\alpha$ to reduce rent seeking will have disparate effects on allocative efficiency depending on whether $R' > -1$ or not. If $R' > -1$, a diminution of the effectiveness of private tutoring (lowering $\alpha$) would be effective for dampening rent seeking but be detrimental to allocative economic efficiency.

More generally, any policy measure, $z$, which guarantees $(dF/dz)/dX > 0$ is to be preferred because it provides the higher valuers the greater odds of being selected, thus contributing to the allocative efficiency of the economy. In (50), $\xi$ is always negative. Therefore, the sign of the derivative depends on the sign of $dK_\alpha/dX$, which will, in turn, depend on the changes in relative convexity of the cost function.

### D. Increasing the Noise of the Test

Differentiating the equation 1) in (29) with respect to $\sigma$ yields,

$$\frac{dK_\alpha}{dX} = \left[ R'(e^*) + 1 \right] \frac{\partial e^*}{\partial X}, \quad (58)$$

We are familiar now with every term in the RHS of (59). The second term is the direct effect of $\sigma$ change on the effort level, which we saw in (19). The first term is the indirect effect that comes through its effect on $t^*$, which we will examine below.

In the previous section, we examined the effects of the increased noise of the distribution on the winning odds of each student at some length. There we

---

118) Of course, we should take the direction of policy change into consideration. In this regard, therefore, the $z$ should be interpreted as what includes the direction of change already in it. Otherwise, it may give rise to a misleading conclusion because our convention always assumes the change to be in the positive direction.
Reforms in Entrance Exam and Private Tutoring

have seen that the students can be grouped into four different categories; one group has improved winning odds with enhanced efforts, the second group has indeterminate odds with higher efforts. The third group has deteriorated odds with lowered efforts and the last group has indeterminate odds with decreased efforts. Taking an integral of these changes, which are represented as $\frac{dF}{d\sigma}$ in (22) but now has to be interpreted as $\frac{dF}{d\sigma}\vert_{t^*=\text{exog}}$ over the whole population gives the change in admission rate that arises due to the change in the variance of the density. This will work as a numerator in (42).

\[
\frac{dF}{d\sigma}\vert_{t^*=\text{exog}} = \int \frac{dF}{d\sigma}\vert_{t^*=\text{exog}} dH(X)
\]

\[
= \int \left[ F_\sigma - \frac{\sigma^2 f(X) \mu}{\mu} \right] dH(X).
\]  

(60)

Taking the negative of this, and dividing it by $\frac{d\phi}{dt^*}$, which is just the integral of (12) or, equivalently, the denominator of (42) itself, renders our desired solution for $\frac{dt^*}{d\sigma}$, the size of the required adjustment in $t^*$ that would keep the admission rate constant in the wake of the increased dispersion119),

\[
\frac{dt^*}{d\sigma} = \frac{\int \left[ -F_\sigma + \frac{\sigma^2 \mu f}{\mu} \right] dH(X)}{\int \frac{\mu^2 c^*}{\mu} dH(X)}.
\]

(61)

For the reasons explained already in the previous section, its sign is indeterminate. Depending on the location of the cut-off point, or, alternatively, the degree of competition in college admissions, the sign of the whole expression may be positive or negative. Probably, we can be a little bit more specific about the directions of the change. The relationship depicted in Figure 6 suggests that if the current cut-off point is set at a very high level, that is, if the test is highly competitive, a greater share of students will be in the range where the overall effect on winning odds are positive. In this case, admission rate will increase as more poorly qualified gain entry into college if there is no cut-off point adjustment. Consequently, the school authority has to raise the cut-off point if it want to keep the admission rate at the pre-set level.

If the test is not competitive, or alternatively if the cut-off point is set at a very low level, a larger portion of students will be either in the black area or in the shaded area. It implies that those students who are relatively well qualified, in the sense that their expected scores are higher than the cut-off point, have to face an increased probability of failure due to the increased random-

119) Or, we can take the negative of (60) and then multiply it with (34) to get the same result.
ness of the test outcome. In this case, if the school does not take any action on
the cut-off point, it is very likely that the admissions rate will be slashed. In
order to make up for this loss of students, the school has to lower the cut-off
point correspondingly.

Overall, the above analysis suggests that increasing the variance of the test is
not really a good policy measure in order to reduce private tutoring activities.
Either it increases the probability of ill prepared students being admitted or the
probability of relatively well qualified student being left out by increasing the
noise in the selection process. Thereby, it is likely to lower the average quality of
the student body, where quality is measured in terms of the value they attach to
the education, or their aspiration to get a higher education.

\section*{4-4. Rent Seeking Under Tuition Control}

Finally, we have arrived at the point where we can analyze the effects of
tuition control on students’ rent seeking efforts by putting together everything
we have obtained in earlier sections. Before proceeding, though, let’s briefly
review the general setting presented at the beginning once more. There is a
regulation in the college education market which sets the school tuition below
the market clearing level. $X$ is the price that students are willing to pay to get
one unit of college education, or to get admitted into college. The demand for
education is given as $D(X)$, $D'<0$, and supply as $S(X)$, $S'>0$.

Tuition ceiling is imposed at $x < x^*$, where $x^*$ is the market clearing tuition,
$D(x^*) = S(x^*)$. Those students whose demand price is less than $x$ will not pursue
to enter college. So there is practically a limit on the lower end of $X$, as far as
rent seeking contest is concerned. The upper limit of $X$, the intercept with the
vertical axis of the demand curve is denoted as $x$. Consequently, $X$ is distrib-
uted in the range of $[x, x^*]$. The surplus value which a student can enjoy from
the right to get in college is given by $X - x$, the value he/she attaches to the right
minus the tuition he/she must pay. The student who is at the bottom end of
the distribution gets just 0 surplus value.

The total number of students who want to go to college under the regulated
tuition is given by $D(x)$ and the total number of students that can be admitted
by $S(x)$. Obviously, $D(x) > S(x)$, since $x < x^*$, and $D'<0$ and $S'>0$. The admission
rate is defined as $\phi(x) = S(x)/D(x)$.

Given $\phi(x)$, the ex ante cut-off score, $t^*$, is determined in the same way as in
the previous section. All other things are also exactly the same as before. Under
the assumptions and definitions stated above, the whole system can now be
represented by the following set of three equations with three endogenous
variables, $e^*$, $t^*$ and $\phi^*$.
Let us first check a few of the basic facts about this new expanded system. First, differentiating 1) in (62) with respect to $e^*$, we get the following new SOC under the new specifications.

$$
\phi^* = \int_x^* F(a \ e^* - t^*) \ dH(X)
$$

(63)

To save notations, I will continue to use $\mu$ to denote this new SOC since there appears to be little possibility of confusion.

Second, efforts decreases as $x$ is raised for every student, with all other things being equal. This can be easily checked again by differentiating 1) in (62) with respect to $x$.

$$
\frac{\partial e^*}{\partial x} = \frac{1}{\mu} < 0.
$$

(64)

Mathematically, this is just the mirror image of equation (9) which says that effort is increasing in $X$. Economically, however, it has a bit deeper meaning than meets the eye. Under the current system, the net surplus value that a student can get from education is $X-x$, not $X$, and, therefore, any increase in $x$ reduces every student’s surplus value across the board. If this $x$ is raised sufficiently high to be equal to $x^*$, the remaining surplus $X-x^*$ would be equal to the consumer surplus which would accrue to each student under the unregulated open market\(^{120}\).

As usual, the solution for the three endogenous variables can be obtained by differentiating each of the three equations with respect to $x$, and solving for each variable. However, since the current system is highly recursive, we can start with the equation that is most loosely linked to the system, the equation 3), and then substitute it back to the remaining equations.

Differentiating 3) in (62) gives,

---

\(^{120}\) Raising the controlled tuition level up to $x^*$ does not automatically guarantee that the good will be sold as in ordinary markets with a linear pricing. If education is provided by a monopolist, he/she may try to practice price discrimination by maintaining the contest scheme. Note that the seller here does not need perfect information about the individual demand schedule or utility function because the increasing cost function provides an effective way of differentiating the customers’ demand intensity.
Trivially, if you increase the regulated tuition level, the admission rate increases. In other words, the competition among students for college admission is reduced with the reduction coming both from increased supply and reduced demand. Transforming the expression into elasticity form results in

\[
\frac{d\phi^*}{dx} = \frac{SD - SD'}{D'} > 0
\]

(65)

where \( \eta_z \) stands for the elasticity of \( z \) with respect to its argument. The change in admission rate depends on the price elasticities of supply and demand. Later it will be substituted into the same expression obtained from 2) in (62).

Differentiating the equation 1) in (62) with respect to \( x \) yields,

\[
\frac{d\phi^*}{dx} = \frac{\phi^*}{x} (\eta_S - \eta_D) \quad \text{or} \quad \eta_d = \eta_S - \eta_D
\]

(66)

The admission rate declines as \( x \) is raised under a given \( t^* \). The first term, whose sign is negative, is the winning odds effect, which in turn comes from the decreased effort effect. As \( x \) gets higher, every student’s surplus value declines, and thus every student works less. Consequently, every student’s expected score declines. Given \( t^* \), it lowers every student’s winning odds,

\[
\frac{d(e^*)}{dx} = \frac{a(X - x)}{\mu} \frac{dt^*}{dx} + \frac{a_i}{\mu}.
\]

(67)

We can get another \( \phi d^*/dx \) from equation 2) in (62) by differentiating it with respect to \( x \), holding \( t^* \) constant\(^{121}\),

\[
\frac{\partial \phi^*}{\partial x} = \int_x^\infty a \cdot \frac{\partial e^*}{\partial x} dH(X) - F(-t^*)
\]

(68)

\[
= \int_x^\infty \frac{a^2 t^2}{\mu} dH(X) - F(-t^*) < 0.
\]

\(^{121}\) Note that we can just as well take a total differentiation of 2) in (62), and substitute (66) and (67) to solve directly for \( dt^*/dx \), the solution to which is given (70) below. But this method tends to cram all the underlying adjustment processes into a black box and then churns out just the final outcome, which in most cases looks mystic, rendering it difficult to interpret. Instead, I take here the method expounded in the previous section because it provides much more detailed explanation in an illuminating way.
resulting in a lower admission rate.

The second term is the drop-out effects, which is also negative. As more students give up a higher education, the admission rate is lowered. As explained earlier, those students whose surplus value, \( X-x \), is negative have no incentive to participate in the admission contest. Therefore, as \( x \) gets higher, more students will drop out of the game. Note that \( F(-t^*) \) is just the winning odds for the student who puts forth 0 effort. Therefore, it is the probability of getting admitted by sheer chance, just by participating in the game but with 0 effort. As the student with 0 surplus value is withdrawing from the contest, that much of the admission rate is lowered.

By equating those two \( \partial \phi^*/\partial x \)'s, we now successfully eliminate one endogenous variable, thereby leaving only two endogenous variables and two equations.

Let's proceed to solve the next endogenous variable, \( dt^*/dx \). It requires the computation of \( d\phi^*/dx \) and \( dt^*/d\phi^* \), to be multiplied with each other. Fortunately, the latter is already given in (34). So what we newly need is \( d\phi^*/dx \). One thing to note at this point is, however, that we can no longer use the simple formula given in equation (42) once \( \phi \) is endogenized. Simply put, we do not have the \( d\phi=0 \) condition which enabled us to use the formula (42). Instead, we have here \( d\phi = (\eta_{s} - \eta_{D}) \phi/x \ dz = z \ dz \). Inserting this into the LHS of equation (45) and collecting terms gives,

\[
\frac{d t^*}{dz} = -\frac{\phi_x \frac{\partial e^*}{\partial z} + \phi_z - \phi_{t^*}}{\phi_x \frac{\partial e^*}{\partial t^*} + \phi_{t^*}}. \tag{69}
\]

Applying this new formula generates,

\[
\frac{d t^*}{dx} = \frac{\int_{s}^{x} - \frac{\sigma^2}{\mu} dH(X) + F(-t^*) + \frac{\phi}{x} (\eta_{s} - \eta_{D})}{\int_{s}^{x} \frac{1}{\mu} dH(X)} < 0. \tag{70}\]

What it says is that the cut-off score of the test must be lowered when tuition control is eased, in order to maintain the admission rate consistent with the new higher tuition level. The first two terms have been already explained even though the signs are opposite to what we had earlier. The reversal of sign was needed to generate the offsetting movement in \( t^* \) to net out the possible effect on the admission rate, \( \phi \), of \( x \) change in the absence of \( t^* \) adjustment. The

122) Or we need \( d\phi/d\phi \) to divide \( d\phi/dx \) by.
123) In this particular instance, \( z \) happens to be \( x \).
third term is the supply and demand effect which appeared afresh here. It has also negative effect on $t^*$ change.

Let’s examine the implications of each term one by one. There are basically three channels through which a change in the level of regulated tuition, $x$, affects the equilibrium cut-off point. One channel, which is represented by the first term above, is through the reduction of $X-x$, the surplus value each student attaches to the education. As tuition control is eased, the rent the students can enjoy just by obtaining the right to get in the college is not as high as it used to be. Now more students can enter college simply by paying the new, increased tuition. It is not that prestigious to become a college graduate any more. Since this erosion of prestige applies to every potential applicant, everybody now has a smaller incentive to exert effort in gaining entry. Consequently, they buy less tutoring, get lower scores, and therefore, the school authority has to lower the cut-off point.

Another channel is the drop-out effect, which has already been explained. As more students are dropping out of the contest, those empty slots must be filled with new batch of students. In order to attract students, the school has to lower the cut-off point.

Still another channel is the adjustment in the quantity supplied and demanded that is caused by the tuition increase. A hike in tuition causes the demanded to contract. People choose to buy less education because it is more expensive than before. At the same time, supply is expanded in response to the tuition increase. A reduction in the number of students who pursue higher education coupled with an increase in the supply, results in an increase in the admissions rate, the ratio between the number of students admitted to the number of applicants\(^{124}\). To accommodate this change in the admission rate, the school authority has to make an adjustment in the cut-off score in the downward direction.

Finally, plugging (70) back into (67) gives final equilibrium change in $e^*$. This, together with the results explained above, suggests that a raise in the level of regulated tuition would be an effective way for reducing rent seeking activities in the educational market. Needless to say, dismantling tuition control itself would be the surest way to wipe out rent seeking. However, if a complete elimination of rent seeking in one shot appears too radical and politically infeasible, raising the controlled tuition by steps over time would be a useful way to abate the harms caused by the social pest.

\(^{124}\) Technically speaking, this third channel can be handled independently by ignoring the supply constraint imposed by tuition control. If we assume that supply is not affected by the regulation of tuition, then the school authority can have one more degree of freedom in selecting the admissions rate because the authority in this case can set the admissions rate at an arbitrary level. Of course, maintaining a given level of admissions rate in this case implies that the school should necessarily increase or decrease the number of admitted students proportionately with the increase in applicants induced by the tuition change. But this is something that takes place outside of the current model and we don’t need to worry about it.
4-5. Summary and Conclusion

Rent seeking is pervasive in many controlled markets. In the education market, where tuition control is routinely practiced, this rent seeking is more serious than in other market. The divergence between the value a student is willing to pay and the price he/she has to pay to get one unit of education creates an exploitable rent, and thus incentives for every potential students to fiercely compete for an admission ticket.

The form and volume of rent seeking will crucially depend on the selection criterion adopted, its effectiveness and precision in measurement and the policy measures taken to cope with them. Two general forms of selection criteria have been examined in this chapter: a quality standard system under which those with higher test scores than a preset cut-off value are admitted and a quota system under which those with higher rank are admitted.

Under the quality standard system, the overall effectiveness of policy measures aimed at reducing rent seeking, say, reducing the marginal efficacy of private tutoring, increasing the cost of tutoring, or increasing noise elements in assessment will critically hinge on the current degree of competitiveness of the market. If it is highly competitive, a reduction of the cut-off score will not reduce rent seeking, but will encourage the rent seeking further. Any measures to increase the dispersion of test scores, thus making it a less reliable means to improve his/her test scores will not contribute to the reduction of the rent seeking activity, either. Rather, making the market more competitive by raising the cut-off standard or by reducing the quota may sometimes generate better results as far as a reduction in private tutoring is concerned.

Under the quota system, almost the same conclusions would hold, with regard to the competition rate in the market. However, in addition to this, the quota system inevitably creates negative externalities among students in the sense that an improvement in one contender’s winning odds must be purchased at the cost of another contender’s loss of his/her odds. And because of this, some one’s gain should be matched by somebody else’s loss. Depending on who becomes the new winner and who becomes a induced loser in the wake of policy shift, allocative efficiency can be improved or deteriorated. The relative convexity of the cost function faced by rent seekers will play a crucial role in determining which policy measure will generate socially desirable outcomes.
We can start from the two equations given in (29) which comprises the whole system. Rewriting them in a slightly modified notation gives,

\[ X F_e(e^*, t^*, z) - C_e(e^*, z) = 0 \]  

\[ \phi = \int F(e^*, t^*, z) \, dH(X). \]  

Differentiating (1) with respect to \( z \), and solving for \( \frac{de^*/dz}{dz} \) yields,

\[ \frac{de^*}{dz} = -\frac{XF_e}{\mu} \frac{dt^*}{dz} - \frac{XF_{e*} - C_{e*}}{\mu}. \]  

The coefficient of \( \frac{dt^*/dz}{dz} \) in the first term, and the second term itself can be replaced with some other expressions by making use of the following relationships. Differentiate (1) with respect to \( t^* \), treating it as if exogenous and we will get,

\[ \frac{\partial e^*}{\partial t^*} = -\frac{XF_e}{\mu}. \]  

Similarly, by differentiating (1) with respect to \( z \), again holding \( t^* \) as if exogenous, we obtain,

\[ \frac{\partial e^*}{\partial z} = -\frac{XF_{e*} - C_{e*}}{\mu}. \]  

Inserting both (4) and (5) back into (3) renders,

\[ \frac{de^*}{dz} = \frac{\partial e^*}{\partial t^*} \frac{dt^*}{dz} + \frac{\partial e^*}{\partial z}. \]  

This is one of the two relationships that should hold at equilibrium between the two endogenous variables. Another relationship can be found by differentiating (2) with respect to \( z \), which yields,

\[ \int [\frac{\partial F}{\partial e^*} \frac{de^*}{dz} + \frac{\partial F}{\partial t^*} \frac{dt^*}{dz} + \frac{\partial F}{\partial z}] \, dH(X) = 0. \]
Substituting (6) into (7) and solving for $\frac{dt^*}{dz}$ yields,

$$\frac{dt^*}{dz} = -\frac{\int \left[ \frac{\partial F}{\partial e^*} \frac{\partial e^*}{\partial z} + \frac{\partial F}{\partial z} \right] dH(X)}{\int \left[ \frac{\partial F}{\partial e^*} \frac{\partial e^*}{\partial t^*} + \frac{\partial F}{\partial t^*} \right] dH(X)}$$

which is the desired result given in (42).
1-1. Introduction

In the previous chapter, we have looked the private tutoring problem under the assumption that there is an excess demand for college education. A large number of students want to advance to colleges, but the total number of admission is limited. Entrance exams are conducted to select more qualified students, and students take private tutoring to increase their odds of admission. Various policy reforms on the selection methods are analyzed with regard to their possible impacts on the overall magnitude of private tutoring.

In this chapter, the assumption is slightly altered so that there is no excess demand in the higher education market. Every student who wants to go to college can enter at least one school of his choice. But there is a competition among students to enter a better school. So the decision is not about advancing to college or not, but about which quality school to enter. Under this new setting, we analyze how the demand for private tutoring is determined and affected.

Many students aspire to go to a better school, and study hard to win in the competition to get into better known, renowned schools\(^\text{125}\). So common a phenomenon is the aspiration to have a better quality good, whether the good in question is a house or an education, that there appears nothing special about the students’ craving for better schools. There exists, however, one significant difference between the competition to get into more prestigious schools and the competition to get higher quality ordinary goods. All you need to have to fulfill your long-cherished dream to live in a fancy apartment is more money. Better schools, however, cannot be bought with mere money in most cases. You need something more, or at the least, you need to convert the money you already have into something that is acceptable to the school you are eager to enter. That something else is, in most cases, a student’s academic achievement. Students are therefore, induced to study hard because studying hard is the only, or the most effective means to satisfy the school’s admission criterion.

\(^{125}\) In every admission season, schools with a good reputation have more applicants than they can possibly accommodate. Consequently, those schools can be very selective in their admission decision, and, in the end successfully net out only those students with highest quality. This process repeats itself every year, as much to be called as a stable equilibrium in economists’ jargon.
This raises a number of interesting questions. Why do schools refuse to accept money outright in their sale of educational goods, and instead asking students to transform it into some other form before they are allowed to be a rightful customer of the good? What would schools do to maintain their reputation, and thus continue to secure high quality students? How hard will students study to get into the desired school? Or, to put the same thing in slightly different words, how many resources would a student or his parents put into the required conversion process, etc.? In this chapter, I focus on the last two questions and build a simple choice-theoretic model which can shed some light on them.

As a point of departure, I allow quality variation among schools and also ability differences among students. The quality difference in schools then creates an incentive to go to better schools among students, because the quality differential is not, in most cases, correspondingly priced. On the other hand, the quality distribution of students creates the problem of securing homogeneity among students within a group who participate in a given educational process because an individualized care is, again in most cases, difficult and costly. In these circumstances, students do their best to maximize their utility, sometimes taking advantage of the under-priced quality differentials, and sometimes trying to make up for the loss they incur in the school by taking more private tutoring.

In this chapter, a special attention has been paid to an examination of the possible effects of the equalization measure taken in Korea in the 1970’s. This is of particular interest especially because there is a close resemblance between what the Korean government has done 30 years ago and what the US government is currently trying to do under the name of “choice”. In fact, the equalization measure is exactly the opposite to what the US government attempts to introduce these days: to equalize schools to reduce the ‘excessive’ competition among schools and students on the one hand vs., to allow more choices to promote and intensify the competition on the other hand. So, a close examination of the equalization measure may provide some hints on what will happen, in a negative way, if the choice program is successfully implemented in the US.

A growing number of papers are dealing with this quality and ability issue. Epple and Romano is one of them. Recently, in a quite influential paper, Epple and Romano (1988) develop a model in which school quality and student ability interact to generate many interesting reputable hypotheses on school choices, equilibrium distribution of school qualities, etc... In their model, however, there is nothing a student can do to increase his/her chance to enter a better school other than being born with high income and/or ability. Schools then

---

126) A little bit less formal a version has been presented in Chapter 3. Here we want to develop a more formal and rigorous analysis.
select students based on their ability and income. In this chapter, I explicitly incorporate the efforts that must be exerted by students or their parents. Secondly, there is a penalty a student has to pay when there is a mismatch between school quality and student ability. Having a too high quality school can be bad not good.

The overall conclusion we derive from the model is that much of the “excessive private tutoring” is the natural outcome of the distorted pricing system in the educational market. The much touted “diploma disease” (Dore, 1997), or “obsessive pursuit of top-rank schools” are not the cause of the extortionate private tutoring but just symptoms of a misguided system. The disease, if it is a disease at all, is not in the people’s preferences but in the badly managed educational system. Another surprising result is that the equalization measure adopted with the sole purpose of reducing private tutoring is not likely to have had the desired effects. Rather, it may have aggravated the situation by inducing students to take more tutoring.

This chapter unfolds as follows. In section 2, a formal model is constructed and the comparative static results are presented. Section 3 examines the effects of school equalization policy. Section 4 offers a summary and conclusion.

5-2. The Model

Students have a utility function, \( U(x,e) \), \( U_x, U_e > 0 \), \( U_{ii} < 0 \), \( U_{ij} > 0 \) (\( i \neq j \)) where \( e \) represents the amount of education they consume, and \( x \) all other goods. There are two different sources for educational attainment; one from formal school, \( e_1 \), and the other from private tutoring, \( e_2 \), and they are perfectly substitutable.

\[
e = e_1 + e_2. \tag{1}
\]

The school education is assumed to be provided for free by the government. This is a subsidy in kind. Since \( e_2 \) refers to the learning activity pursued outside of the schools, I do not have the problem of mutual exclusivity that appears in some of the literature on the relationship between public and private education (Pelzman, 1973 for example). In the model, all school education, whether public or private, is represented by \( e_1 \) and is assumed to be provided for zero price. The amount of education that an individual student can get from school is given by the following function:

\[
e_1 = \hat{e}_1 + \alpha \hat{e}_1 (a - q)^{\beta}, \quad a, \beta > 0, \quad \hat{e}_1 (0) = 0, \quad \hat{e}_1 > 0, \quad \hat{e}_1^{'} \leq 0 \tag{2}
\]

\(127) If \( e_1 \) and \( e_2 \) represent two different ‘school’ educations each, a student who is enrolled in one school cannot be enrolled at the other school simultaneously.
That is, the school education is composed of two parts; one is $e_1$ which every student gets invariably from school, and the other, which is related with the school quality, $q$, and student’s individual ability, $a$. A student who chooses a high-quality school gets a larger amount of education, but has to pay a penalty if his/her ability is far from the quality level of the school he/she attends. There exists a lower limit of $q$, $q_{\text{min}} \leq q$, which is assumed to be zero for convenience. The minimum quality education is guaranteed for every student.

Getting into a high quality school is costly. The cost is paid in the form of $e_2$. Studying hard outside the school will improve his chance of getting into a high quality school.

\[ q = \phi(e_2), \quad \phi' > 0, \quad q_{\text{min}} = \phi(0) \]  

(3)

The price of $e_2$ is $p$, ($p > 0$). In this sense, the school quality is not free at all. In the other sense which will become clearer shortly, however, it is sometimes free.

As far as major building-blocks are concerned, these complete the model. Before formally tackling the maximization problem with which a student is faced, however, there remains one thing to note.

Note that private tutoring in the above setting has a dual function. On the one hand, it is consumed in itself. Purchasing more $e_2$ therefore implies just having more education. On the other hand, it can be used to increase the probability of getting into a better school, and thereby to secure the chance of increasing his/her education still further. Note also that use of $e_2$ in either one of the purposes does not exclude the usage of the same thing for the other purpose. In this sense, private tutoring in the model has the typical property of public goods: the amount of $e_2$ used for one purpose does not reduce the amount of $e_2$ left for the other purpose.

Furthermore, it is freely disposable in each usage. Even if a very pre-
igious top-quality school offers admission to a student, he/she can reject it if he/she considers the school unfit compared to his/her own ability. A student has to buy if he/she wants more of school quality than he/she currently has, but he/she can dispose of with no additional cost if he/she has enough of it already.

This dual nature of private tutoring will require us to reformulate the variables and model in a slightly different way in the following. But before doing that, I will first examine the problem of choosing the optimum level of school quality in the current setting, and then later discuss the problem of choosing the optimum amount of private tutoring in a slightly altered setting.

A. Problem of School Choice

Let’s first take note of a special characteristic of the school education function given above in (2).

**Proposition 1:** For a given level of school quality, there exist a lower bound and an upper bound of student ability in such a way that students whose ability level lies outside of these bounds will not attend the school at all even if the school education is freely provided.

**Proof:** The school education function has a quadratic term in $a$ and $q$, implying that $e_1$ can be negative if the gap between $a$ and $q$ is large. Since a negative education cannot be imposed upon students, this implies that some students will opt or drop out of school. More concretely, for $e_1$ to be non-negative, we need,

$$q - \frac{\hat{e}_1 + \hat{a}_1(q)}{\hat{\beta}} \leq a \leq q + \frac{\hat{e}_1 + \hat{a}_1(q)}{\hat{\beta}}.$$  \hspace{1cm} (4)

Let these bounds be denoted as $a(q)$, and $a(q)$, each. Students outside of this range quit attending the school because the penalty they have to pay for the mismatch between the quality and ability is too high. For them, school education is not a good but a bad.

In short, a school with a given quality level can serve only a limited range of students. Probably it may explain why we have a hierarchical school system in every society, in which schools are graduated into some sort of ascending order, such as kinder-garten, elementary, secondary schools, and colleges. In addition to that, for those students who are extraordinarily talented or slow, we provide separate educational institutions specifically targeted for them only. In that regards, the parametric values in the above expression must be

---

133) This does not necessarily mean that students will actually stay out of school outright. Some students may continue to show up at school under the familial or societal pressure, but doing nothing or other things in the class.
interpreted to pertain only to a particular hierarchical school level.

For a later reference, note that the inequality in (4) can be restated in terms of $q$, implying that there is a lower and an upper limit of school quality outside of which a student with a given ability level will never choose.

Earlier it was assumed that school quality can be increased only by taking private tutoring. In order to prepare ourselves for the ensuing analysis, however, let's first consider a hypothetical case where students can directly choose school quality and look into the relationship between the school quality and student ability. Here, it is temporarily assumed that the price of quality is given by $\delta(\delta \geq 0)$, which implies that a student has to incur a proportionately higher cost as he/she chooses a better quality school. At the same time, it is also assumed that students are willing and able to pay $\delta$.

Under these assumptions, we can immediately establish the following propositions.

**Proposition 2:** there exists a unique school quality level that a student with the ability index $a$ will choose for the price $\delta$,

$$q^*(a, \delta) = a + \frac{\alpha \hat{e}_1'(q)}{2\beta} - \frac{\delta}{2\beta}$$  \hspace{1cm} (5)

with $q^*, \geq 0$ and $q^* < 0$.

**Proof:** The proof follows by itself when we apply the standard maximization method to the problem of

$$\max_q \ e_1(q) - \delta q$$

Let $d\xi/dq = \alpha e_1' + 2\beta(a-q) = \xi(q)$. Then the FOC is $\xi = \delta$. Since $\xi$ is monotonically decreasing ($\xi < 0$ because $e_1'' < 0$, and $\delta$ is constant, a unique solution, $q^*(a, \delta)$ exists. Differentiating the FOC with respect to $a$ and $\delta$, respectively, we have $q^* = \xi_0 = \xi > 0$, $q^* = 1/\xi < 0$.

Above, the first term of $\xi$ captures the effect of school quality improvement due to an increased $e_2$, and is thus positive. The second term captures the penalty effect, and is positive for those students whose ability is greater than the school quality, and negative for those students whose ability is lower than the school quality. Combining these two effects, we can see that $\xi(q)$ function declines as $q$ increases, cutting the horizontal axis at $a + \alpha e_1'/2\beta$ and then sinking into the negative range.

134) Given $q = \varphi(a)$, the price of $q$ will be equal to $\delta = \varphi'/p$.

135) Obviously, willingness and ability to pay is determined by the interaction of the marginal rate of substitution, income and price, which will be incorporated later.

136) Strictly speaking, we need a condition, $a + \alpha e_1(0)/2\beta > 0$ to validate this statement, which we assume in the paper.
The expression \( q^* > 0 \) says that, with other things being equal, a better-abled student will choose a higher quality school, and the expression \( q^*_\delta < 0 \) implies that students will buy less quality, the higher the price is.

Let the quality level to be chosen under the zero price be denoted as

\[
q^{-}(a) = \{q | \xi(q) = 0 \} = q^*(a, 0).
\]  

(6)

Since this is the level of school quality that a student will choose under zero price, it can be called a saturation quality level. Later, it will be shown that some students will pay virtually a zero price for school quality and become saturated, and others pay a positive price and be constrained by it. In the meantime, it is easy to check from (4) that, under zero price, a student will pick a school whose quality level is slightly higher than his/her ability level\(^{137}\). This optimum level of school quality will be higher, the larger \( \alpha \) is and the smaller \( \beta \) is. That is, the more effective school quality is in increasing the amount of education a student can get, the more he/she will purchase. Similarly, the smaller the penalty that a student has to pay for the mismatch between quality and ability is, the higher quality he/she will choose.

Since \( q^* \) is monotonic in \( a \) (\( q^* > 0 \)), we can invert this function to have

\[
a^*(q) = q - \frac{\alpha \hat{e}_{1}^{'}}{2\beta} + \frac{\delta}{2\beta}.
\]  

(7)

This can be interpreted as the ability level of those students who will choose a particular level of \( q \) as their optimum. In the following, I will use \( a^*(q) \) to denote the ability level when \( \delta > 0 \), and \( a^{-}(q) \) when \( \delta = 0 \). A similar notational rule will also be applied to \( q \) and \( e_2 \) when there is a need to make that distinction.

With a positive price for school quality, the optimum quality level chosen will always fall short of the corresponding saturation level, i.e., \( q^*-q^{-}<0 \), with the gap between the two ever widening as \( \delta \) gets higher. In an extreme case in which the price of private tutoring is extremely high, the desired \( q^* \) may be too low for most of the students, way below the lower limit of the quality that a given ability student would ever choose. This is obviously a least interesting case, which I rule out in the following\(^{138}\).

Now I introduce the school of minimum quality freely provided to every student into the picture.

\(^{137}\) Through education, individual ability will improve eventually. Here, it is assumed to be given at the time when he/she makes a choice on the level of school quality.

\(^{138}\) Since private tutoring in our model contains all the learning activities that take place out of school, including self-study at home, the price may not possibly go that high because of the close substitutability among the variety of activities that compete with each other. Furthermore, some schools may offer scholarships to highly talented students, thus lowering in effect their \( \delta \). The present model, however, does not allow this kind of price discrimination because schools cannot control the price of private tutoring directly.
Proposition 3: Students whose desired school quality, $q^*(a)$, is lower than the freely provided minimum, $q_{\text{min}}$, will attend the minimum quality school with no additional investment to enhance the quality. Students whose desired quality is higher than the minimum will make a positive investment to improve the quality and, therefore, incur a positive amount of cost.

Or, equivalently, students whose ability index lies within the range of $a(q_{\text{min}}) \leq a \leq a^*(q_{\text{min}})$ will attend the school of minimum quality with no investment. Students with $a > a^*(q_{\text{min}})$ will make a positive investment to improve the quality.

Proof: Note that the optimum quality $q^*(a)$ that a student would voluntarily choose can be lower than $q_{\text{min}}$, the minimum quality provided freely by the government. There are two distinct cases under this category. In one case, the minimum quality freely provided is genuinely too high for some of the students whose ability is low, i.e., for those students whose ability lies within the range of $a(q_{\text{min}}) \leq a \leq a^-(q_{\text{min}})$. They would have chosen a still lower quality school if such a school were available. Unfortunately, no such schools are available. Therefore, they have no choice but to attend a school whose quality level is undesirably high from the individual standpoint. The excess quality in this case is not freely disposable because of the absence of alternatives. Thus they attend, with some grudge, the school with $q_{\text{min}}$. The second group comprises of those students whose ability index falls within the range of $a^-(q_{\text{min}}) < a < a^*(q_{\text{min}})$. For them a school quality higher than the minimum is preferable, but they cannot afford to buy any additional quality because the price is too high for them. So, they also stay at the minimum quality school. Lastly, for the students with $a > a^*(q_{\text{min}})$, the minimum quality is not binding, compared to both their ability and price, so they will choose $q^*(a)$ with some positive investment.

Let's assume, again temporarily, that not only the lower bound, but also an upper bound of school quality exists below which a student can freely choose any quality but must incur the capacity expansion cost if he/she wants to have a quality beyond that level. In this case, we can establish the following proposition which states that within a certain range, school quality becomes freely disposable.

Proposition 4: (Free disposability) In the case where there is a quality capacity limit, $q$, which can be expanded by paying a positive price, $\delta$, a student will either choose $q^-(a)$ if he/she is not bound by the limit, or will optimally expand his/her $q$, with a positive payment if he/she is bound by the capacity constraint.
Proof: Again this is the standard result we would get when solving the following maximization problem,

\[
\max_{q, \bar{q}} e_1(q) - \delta \bar{q} \quad \text{subject to} \quad q \leq \bar{q}
\]

by setting the following Lagrangian equation,

\[
L = e_1(q) - \delta \bar{q} + \lambda(\bar{q} - q).
\]

The FOC is:

1) \(L_q = e_1' - \lambda = 0\)
2) \(L_a = \lambda - \delta = 0\)
3) \(L_\lambda = \bar{q} - q \geq 0, \lambda \geq 0\) and \(\lambda(\bar{q} - q) = 0\)

From 3), if \(q \leq \bar{q}\), then \(\lambda = 0\). Plugging into 1), we have \(e_1' = \xi = 0\), yielding \(q = q^\sim\). If, on the other hand, \(\lambda > 0\), then \(q = q\). In this case, from 1) and 2), \(e_1' = \xi = \lambda = \delta\). Therefore, we have \(q = q^\sim = q^*(a, \delta)\).

The following graph (Figure 5-1), which shows the relationship between school quality, \(q\), and individual ability, \(a\), sums up the major conclusions of the above discussion in an \((a, q)\) plane.

Note that the slope of the indifference curve associated with a given amount of \(e_1\) is given by the following equation,

\[
\frac{dq}{da} = \frac{2\beta(a - q)}{\xi} = \frac{1}{\frac{a}{2\beta} e_1''(q) + 1}
\]

Therefore, the slope is zero along the line, \(q = a\), implying that for a given \(q\), \(e_1\) is maximized when a student’s ability is exactly matched with the school quality level, and any deviation from it entails a penalty.

On the other hand, along the line \(q^\sim = a + \frac{2\beta}{\xi} e_1'(q^\sim)\), the slope of the indifference curve is infinite, implying that for a student with a given ability level,

\[\text{In a general setting, it is not a line but a curve with a slope,}\]

\[
\frac{dq}{da} = \frac{1}{1 - \frac{\beta}{\xi} e_1''}. \]

As \(q\) gets higher, the slope approaches 1 starting from some value less than 1. Of course, it can be 1 everywhere if \(e_1''(q) = 0\).
When the minimum quality is given at $q_{\text{min}}$, a student with ability $a_4$ ($a_4=q_{\text{min}}=0$) will get the highest $e_1$ among all students who stay at that school if he/she, too, decides to stay there. He/She will not, however, remain there, but move to a school with $q^\sim(a_4)$ to increase his school education if school quality is free ($\delta=0$). If the school quality is not free, but has to be bought for a positive price ($\delta>0$), he/she can not go all the way to $q^\sim(a_4)$, but has to stop at $q^\ast(a_4)$. In either case, if a school with a quality higher than $q^\sim(a_4)$ offers him/her an admission for free, he/she will simply reject it. Consequently, the area comprised of the points above the line and another line $q^\sim=a+\frac{\delta}{2} e_1'(q)$ and another line $q=q_{\text{min}}$ will become “the area of free disposability.”

Note also that for a student with ability $a_2(=a^\sim(q_{\text{min}}))$, the minimum quality provided is identical to the quality level he/she would have voluntarily chosen. This implies that any students whose ability is lower than $a_2$ (but higher than $a_1$, about which I will explain shortly) will attend the minimum quality school somewhat reluctantly. As explained earlier, they would have chosen a still lower quality school than $q_{\text{min}}$, if such a school exists. With no such school in existence, they stay at the lowest available school out of necessity, not out of free choice. The excessive quality in this case is represented by the vertical distance between the two lines, $q^\sim$ and $q_{\text{min}}$. 

$e_1$ is maximized when he chooses a school with its quality level slightly higher than his individual ability.
For those students who fall within the range of \( a_2 \) and \( a_3 \) (where \( q^* \) line meets with \( q_{\text{min}} \) line), any quality higher than \( q_{\text{min}} \) but lower than \( q^* \) is preferred, but unaffordable. Thus, these students will also hang around at the minimum quality school with no purchase of an additional quality.

Every student whose ability is greater than \( a_3 \) purchases a positive amount of private tutoring and chooses his/her own quality school. The amount of quality purchased is represented by the vertical distance between the \( q^* \) line and the horizontal line \( q_{\text{min}} \). Since \( q^* \) is strictly increasing in \( a \), no students with different abilities share the same quality school here. In this sense, there is a perfect sorting of student abilities across the schools\(^{142} \). However, this is because we have implicitly assumed in this section that students are not bound by their income and preferences. All students, therefore, behave as if they have exactly the same income and the same marginal rate of substitution between education and other goods. They differ only in their abilities from each other. Consequently, this difference in ability has been monolithically mapped into a different demand for school quality. This conclusion will be altered if we introduce a utility function and income in our formal analysis, as we do in the ensuing section.

Students whose ability is lower than \( a_1 \) will choose not to attend any school at all. For them, attending school generates sheer pain, not benefits. For that matter, students with ability higher than \( a_5 \) would have also chosen not to attend school if the school with \( q^* \) were the only school available. Fortunately, however, they have higher quality schools to choose and don’t need to worry, unless they are really hard pressed by the price constraint\(^{143} \). The thick solid indifference curve passes through the origin and represents the amount of school education which is just equal to \( e^1 \).

**B. Problem of Choosing Optimum Private Tutoring**

So far, we have analyzed how a different level of school quality is chosen by students, assuming, first, that the quality level itself is a control variable students can directly maneuver, and second, that all the students have the same income and preferences. But neither of them is true in our model, and we have

\(^{142} \) If there is a perfect sorting of student ability across schools, the penalty term in our school education function will lose much of its significance since each school in this case can adjust its teaching level, say, the curriculum or the pace of progress, to the given ability level of the perfectly homogeneous student body, unless the adjustment itself is very costly. We will not have a perfect sorting, however, for the following two reasons. First, in the real world, we do not have a continuum of school qualities. Thus, the sorting will necessarily be imperfect. The second reason is explained in the text.

\(^{143} \) As \( \delta \) gets higher, the \( q^* \) line shifts down continuously. This will make the \( a_3 \) point (the intersection between \( q^* \) and \( q_{\text{min}} \)) move to the right. If \( \delta \) is sufficiently high, \( a_3 \) will coincide with \( a_5 \) forcing every student below \( a_5 \) to stay at the lowest quality school. Outside of \( a_5 \), some students may or may not purchase some \( q \) (and attend a school higher than the minimum) depending on whether \( q^* \) line cuts the indifference curve from below or from above. This in turn depends on the curvature condition of the \( e^1(q) \) function. All these things, however, are logically possible, but quite unlikely for the reasons explained earlier. See footnote (136).
to correct the situation in this section. This can be done by explicitly introducing our second variable of education, $e_2$, and the utility function both of which have so far been put aside temporarily.

A formal introduction of private tutoring variable at this juncture, however, requires us to simultaneously consider the dual effect of private education aforementioned, i.e., a freely disposable public good within the capacity. In order to do this, I reformulate the school choice problem a student is faced with in the following way. First, imagine a situation where a student is endowed with a certain amount of $e_2$, denoted as $e_2$, which will play the role of capacity limit in subsequent decisions. The question of how this endowment is determined will be discussed later. The student then uses this $e_2$ for two different purposes: one for changing the school quality and the other for increasing his/her own consumption. Let the amount of $e_2$ used for the former purpose be denoted as $e_{21}$, and the amount for the latter purpose as $e_{22}$. As long as $e_{2i} (i=1,2)$, he/she can freely assign any amount of $e_2$ for either purpose. Now go back to the initial question of how this endowment is pinned down. Simply put, it has to be purchased for a price. When he/she makes a decision on $e_2$, however, he/she correctly foresees all these embedded features in advance, and sets it at the optimal level\[144].

We are now ready to set up the full-blown maximization problem a representative student has.

\[
\begin{align*}
\text{Maximize} & \quad U(x,e) \\
\text{subject to} & \quad e = e_{11} + e_{22} \\
& \quad e_{11} = e_{1} + a \hat{e}_{1}(q) - \beta(a - q)^d \\
& \quad q = \gamma(e_{2i}) \\
& \quad y = x + p \hat{e}_{2} \\
& \quad e_{11} \geq 0, \quad e_{2i} \geq 0 \\
& \quad 0 \leq e_{2i} \leq e_{2}, \quad 0 \leq e_{2i} \leq e_{2} .
\end{align*}
\]

Above, the fourth equation is the ordinary budget constraint where the price of $x$, the numeraire good, is normalized to be 1. The last two inequalities say that the student cannot sell his/her $e_1$ and convert it into more of $x$. Nonetheless, a student can control the actual amount of $e_1$ he/she consumes by choice of $e_2$, unlike the conventional subsidy-in-kind model where $e_1$, once given, is fixed.

This set of equations gives rise to the following Lagrangian function,

\[144\) This is similar to the peak-load pricing model. See Steiner (1964).
Here \( \mu_i \) has the meaning of the marginal utility of capacity expansion for either purpose. Note that the student has three control variables to optimize, \( e_2, e_1 \) and \( e_2 \), all of which are the derivatives from \( e_2 \).

The Kuhn-Tucker first order conditions are then,

\[
Z = U(x, e_1 + a \tilde{e}_1, e_2) - a(a - \Phi(e_2)) = e_{21}^2 + e_{22} - \lambda(y - x - p \tilde{e}_2) + \mu_1(\tilde{e}_2 - e_{21}) + \mu_2(\tilde{e}_2 - e_{22}).
\]

(12)

where \( \eta = \frac{\partial \Phi}{\partial e_{2i}} = [\alpha e_i \gamma + 2 \beta(a-q)] \gamma^\prime = \bar{\gamma} \gamma^\prime \), which yields the following series of lemmas.

**Lemma 1:** Private tutoring has the property of public goods.

**Proof:** From (13) and (16), we can immediately establish

\[
\lambda = 0, \quad e_{21} = 0, \quad e_{22} = 0
\]

(13)

If \( \frac{\mu_1 + \mu_2}{p} > U_x \), then \( e_2 > 0 \).

Otherwise, students will purchase a zero amount of \( e_2 \).

That is, marginal utilities in two different uses are summed together to be compared with its marginal cost.

Lemma 1 holds for any public goods, whether freely disposable or not. Thanks to the free disposability assumption, however, we can establish a much stronger lemma.

**Lemma 2:** Private tutoring is worthwhile to take if it is worthwhile for only either one of the purposes.

**Proof:** \( \frac{\mu_1 + \mu_2}{p} \geq \frac{\mu_1}{p} \) (i=1 or 2) because \( i = 0 \).

Conceptually, we may have four different possible combinations of \( \mu_i \) and
\( \mu_2 \) depending on which one takes 0 or a positive value. Out of those four, we can rule out the two cases in which \( \mu_2 = 0 \).

**Lemma 3:** \( \mu_2 > 0 \) always, i.e., the consumptive demand for \( e_2 \), \( e_{22} \) is always bound by the capacity limit, \( e_2 \).

**Proof:** From equation (15) and our assumption \( U_e > 0 \), \( \mu_2 > U_e > 0 \). Then, from (19), \( e_{22} = e_2 \) because \( \mu_2 > 0 \).

Unlike \( \mu_2 \), \( \mu_1 \) can be either 0 or positive, as shown below.

**Lemma 4:** Quality upgrading demand for \( e_2 \), \( e_{21} \) is either bound by \( e_2 \) or unbound. When it is bound, \( e_{21}^* = e_2 \). When it is unbound, the optimum amount of purchase will be either \( e_{21} \) or zero, \( e_{21} = [e_{21} | \eta(e_{21}) = 0] = [e_{21} | \xi(e_{21}) = 0] \).

**Proof:** From equation (18), if \( \mu_1 > 0 \), then \( e_2 \). From equation (14), if \( U_e \eta - \mu_1 < 0 \), then \( e_{21} = 0 \). If \( e_{21} > 0 \), then \( U_e \eta - \mu_1 = 0 \). If \( \mu_1 = 0 \), \( e_{21} < e_2 \). This implies \( U_e \eta = 0 \), which requires \( \eta = 0 \). Therefore \( e_{21} = e_{21}^* \).

So far we have checked whether \( e_{21} \) and \( e_{22} \) are bound by \( e_2 \) and found that \( e_{22} \) is always bound, and \( e_{21} \) may or may not be bound. \( e_2 \) can, however, still be equal or greater than 0. This implies that we can have the following two cases: one in which both \( \mu_1 \) and \( \mu_2 \) are binding, and the other in which only \( \mu_2 \) is binding.

**Case 1) Both \( \mu_1 \) and \( \mu_2 \) are binding.**

In this case, we have interior solutions for both \( e_{21} \) and \( e_{22} \) such that \( e_{21} > 0 \) and \( e_{22} > 0 \). Then equations (14) and (15) will hold with equality, yielding the following lemma.

**Lemma 5:**

\[
\text{If } \frac{U_e}{U_x} < \frac{p}{\eta + 1}, \text{ then } e_{21}^* = 0. \tag{20}
\]

\[
\text{Otherwise } e_{21}^* > 0 \text{ and at equilibrium, } \frac{U_e}{U_x} < \frac{p}{\eta + 1}. \tag{21}
\]

**Proof:** Similar to the proof to lemma 4, thus omitted.

Equation (20) says that a student whose marginal rate of substitution (MRS) between \( x \) and \( e \) at \( e = 0 \) is smaller than \( p/(\eta + 1) \) will opt to have zero amount of \( e_2 \), and, therefore, all \( e_2 \)'s are zero.

---

145) Caveat! \( \mu_2 > 0 \) doesn't mean that \( e_{22} \) is worthwhile to take. What the lemma 2 says is that if \( \mu_2/p \) is greater than \( U_e, \) then \( (\mu_2 + \mu_3)/p > U_e. \) A mere positive value of \( \mu_2 \) does not, however, guarantee that it will be greater than \( U_e. \) Theoretically speaking, conditions for having interior solution and conditions for free disposability are two different things. \( \mu_2 > 0 \) implies no slacks or complete exhaustion of existing capacity and the statement that something is worthwhile to take implies interior solutions.

146) \( \eta(e_2) = 0 \) implies \( \xi(e_2) = 0 \) since \( \eta \) and \( \xi \) are always positive.\( \eta(e_2) = 0 \) implies \( \xi(e_2) = 0 \) since \( \eta \) and \( \xi \) are always positive.

147) From our implicit assumption, we have \( \mu_1 > 0, \eta > 0 \) already. Therefore we have equations (15) and (17) with equality.

---

**Competition to Enter a Better School**
Case 2) Only $\mu_2$ is binding.

**Lemma 6:** When only $\mu_2$ is binding, $e_2 > 0$ always. In this case, $e_{21}=0$ or $e_{21}=\ast$. The optimum $e_2$ is achieved at $U_e=U_s$.

**Proof:** On the one hand, the fact that $\mu_1=0$ implies $e_{21}=e_2$ in (18) and $U_e\eta=0$ in (14). There are two cases; either $\eta<0$ or $\eta=0$. If $\eta<0$, then $e_{21}=0$. If $e_{21}>0$, then it must be $\eta=0$ to have $U_e\eta=0$. It implies $e_{21}=e_{21}=\ast$. In either case, $e_2>0$.

On the other hand, $\mu_2>0$ implies $e_{22}=e_2$, which is positive, and thereby establishes $U_e=\mu_2$ in (15) and $\mu_2=U_s$ in (16). Putting together, $U_e=U_s$.

In this range, students have already a high enough school quality and any addition to it will generate a pain. Thus, they stay at their best school with the saturation level of quality, without being forced to pay the penalty associated with the un-wanted higher quality school. In short, they will freely dispose of whatever redundant quality. They will, however, purchase whatever amount of $e_{22}$ without worrying about the bads it would generate because of this free disposability.

Putting all these lemmas together gives rise to the following summary proposition.

**Proposition 5:** The maximization problem posed in (11) has the following three solutions: one corner solution and two interior solutions.

1) Corner solution:

$$\bar{e}_2 = e_{21} = e_{22} = 0 \quad \text{if} \quad \frac{U_s}{U_s} < \frac{-p}{\eta+1} \quad \text{at} \quad \bar{e}_2 = 0$$

(with the constraint $\eta>0$)

2) Interior solution I:

$$\bar{e}_2 = e_{21} = e_{22} = e_2^{\ast} > 0 \quad \text{if} \quad \frac{-p}{\eta+1} \leq \frac{U_s}{U_s} < p \quad \text{at} \quad \bar{e}_2 = 0$$

where $e_2^{\ast} = \{e_2 \mid \frac{U_s}{U_s} [\eta(e_2) + 1] = p\}$

3) Interior solution II:

$$\bar{e}_2 = e_{21} = e_2^{\ast} \quad \text{and} \quad e_{22} = e_2^{\ast} \quad \text{if} \quad \frac{U_s}{U_s} \geq p \quad \text{at} \quad \bar{e}_2 = e_2^{\ast}$$

where $e_2^{\ast} = \{e_2 \mid \frac{U_s}{U_s} (e_2) = p\}$ and $e_2^{\ast} = \{e_2 \mid \eta(e_2) = 0\}$.

The following graph gives us an intuitive illustration on the two interior solutions (Figure 2). As explained, a given $e_2$ is used for two different purposes
at the same time. When it is used for direct consumption, the MRS is given by \( \frac{U_e}{U_x} \). When it is used for quality improvement, the corresponding MRS is given by \( \eta \frac{U_e}{U_x} \). The total demand for private education is then the vertical sum of these two curves, with the proviso that the negative part of \( \eta \frac{U_e}{U_x} \) curve must be discarded and replaced with 0 value. The thick solid curve in each panel represents this sum. At equilibrium, marginal value of private education must be equated to its price, and equilibrium is found at \( e_2^* \), in each panel.

Note that in panel (a) which depicts the Interior Solution I, \( e_2^* \) is located to the left of \( e_2^- \), the satiation point, which is the solution to \( \eta(e_2) = 0 \). Here, both \( \frac{U_e}{U_x} \) and \( \eta \frac{U_e}{U_x} \) are binding, and, therefore, we have \( e_2 = e_2^1 = e_2^2 = e_2^* = \{e_2 \mid \frac{U_e}{U_x}\frac{U_e}{U_x} = p/(\eta + 1)\} \). In panel (b) which represents the Interior Solution II, however, the equilibrium solution \( e_2^* \) is located to the right of \( e_2^- \). This implies that only \( \frac{U_e}{U_x} \) is binding, and, therefore, we have two different solution values for \( e_2 \): one for quality improvement, \( e_2^1 = e_2^- \) and the other for direct consumption, \( e_2^2 = e_2^* = \{e_2 \mid \frac{U_e}{U_x}\frac{U_e}{U_x} = p\} \).

Note that in panel (a), the availability of a better quality school has caused the student to have more private tutoring. With no school choice, he/she would have chosen an \( e_2' \) amount of private education. He/She is, however, now choosing \( e_2^* \) because he/she can upgrade the quality of the school with

---

148) Or, to put the same thing in different words, \( e_2^* \) is located in the range where \( \eta \frac{U_e}{U_x} \) is positive.
149) Or in the range where \( \eta \frac{U_e}{U_x} \) is negative, and thus the redundant quality is freely disposed of.
private tutoring. In contrast, the availability of quality school in panel (b) does not affect the student’s demand for private education because his/her consumptive demand for education already exceeds the demand for school quality. In short, the equilibrium value of $e_2$ is invariant to the location of the $\eta U_e/U_s$ curve.

Analogously we can depict the corner solution in a similar way. (See Figure 3 below). We already know that there are two different cases in which students decide not to purchase private tutoring: they do not buy either because the price is too high, or because the minimum they have is already high enough. Panel (a) represent the first case, and panel (b) the second. Note that a zero purchase of $e_2$ corresponds to the minimum school quality $q_{\text{min}}$. In panel (a), compared to the price, the sum of $U_e/U_s$ and $\eta U_e/U_s$ is so low that the student chooses not to purchase any private tutoring, even though he/she still wants to have more ($e_2^-$ is positive).

In panel (b), $e_2^-$ is negative, implying that his/her preferred level of school quality is lower than the minimum provided. Therefore, some of the quality is redundant for him/her, but unlike all other cases discussed so far, he/she cannot throw it away, because it is not freely disposable any longer. Graphically, it means that the $U_e/U_s$ and $\eta U_e/U_s$ curves now have to be vertically summed as they are without the negative part of $\eta U_e/U_s$ being slashed off. In the positive range, however, the free disposability of $\eta U_e/U_s$ gets back to work and the negative part must be cut off. This implies that at the vertical intercept, there arises a discontinuity in $U_e/U_s + \eta U_e/U_s$ curve because it is switched from the non-free disposal range to free disposal range when it crosses the vertical axis.

<Figure 5-3> Corner Solutions
The following proposition concerns the effective price that a student is paying in the first interior solution.

**Proposition 6:** The “effective shadow price” for \( e_2, p/(\eta+1) \), is lower than its nominal price, \( p \), for those students whose \( \eta \) is greater than 0 at equilibrium (or equivalently \( \eta U_e/U_x \) is binding at equilibrium).

**Proof:** This is because a student who has purchased a positive amount of \( e_2 \) can enjoy a still larger amount of education from the better school he/she is made able to attend without incurring additional cost. The purchase of one unit of \( e_2 \) generates its own utility and, in addition to that, an extra utility through its effects on the improved school quality. Therefore, those students with \( \eta > 0 \) purchase more \( e_2 \) to take advantage of the lowered “effective price” differential.

In other words, students compete to enter better schools since all school education is uniformly priced (at 0 nominal price) regardless of the quality differentials among schools. In this sense, school quality is a free gift bestowed upon the \( e_2 \) purchaser. Consequently, students have an incentive to exploit the quality differential to their maximum benefit. If, instead, the quality premium is properly and separately priced so that a higher quality education is purchasable only for a higher price, the competition to get into a better school would have not taken place. This implies that a part of the demand for private tutoring comes from the imperfect pricing of school quality differentials.

This raises the following question. Do we have similar competition to obtain a higher quality in the private tutoring market? Obviously not, at least in our model because our model does not allow any competition for quality to take place in the private education market\(^{150}\). Is this then a mere modeling artifact or more or less an accurate description of the reality? Probably the latter might be closer to the truth.

In the private tutoring market, competition to obtain a better quality education does not arise because there is a free competition among suppliers and demanders in terms of both price and quality. Firstly, competition among students to buy higher quality education will drive up the price of the higher quality good, until the price differential disappears. Therefore, all the quality differential in equilibrium will be reflected in the price, and consequently, competition for a higher quality will be transformed into a competition for a merely larger quantity. In this regard, the \( p \) term refers to the effective quality adjusted unit price of private education. Of course, this is an implicit assumption exogenously imposed on the model, not a conclusion derived within the model.

Nonetheless, it appears to provide a quite plausible and reasonable interpretation to the results we have. Secondly, in the private education market, the homogeneity of the student body may be more easily secured because more individual-

---

\(^{150}\) Private tutoring is assumed to be all homogeneous in our model. Is it because there is no quality difference in private education?
ized care by tutors is possible. In contrast, the nominal price in the public education market is uniform, and as a result, the real effective prices are all different. Furthermore, individually tailored education is more difficult in school education. This implies that in the school education market, there may exist some rigidity which hinders a smooth adjustment of the real effective price, even though it is not clear at this stage why and how such rigidity persists.

5-3. Comparative Statics and Properties of the Equilibrium

The second order conditions for the two interior solutions to be utility maximizing are

$$|\mathbb{H}|_1 = U_{xx}p^2 - 2pU_{xe}(\eta + 1) + U_{ee}(\eta + 1)^2 + U_{e} \eta' < 0,$$

(22)

for the first interior solution, and,

$$|\mathbb{H}|_2 = \begin{vmatrix} U_{ee} - 2pU_{xe} + p^2U_{xx} & 0 \\ 0 & \eta' \end{vmatrix} = (U_{ee} - 2pU_{xe} + p^2U_{xx}) \eta' > 0$$

(23)

for the second interior solution. These conditions are met under the current assumptions.\(^{151}\) Note that we have only one control variable, \(e_2 (= e_{21} = e_{22} = e_2)\), for the former, and two control variables, \(e_{22}\) and \(e_{21}\), for the latter. This means that both the amount of private tutoring and school quality are jointly determined in the former, whereas, in the latter, they are separately determined.\(^{152}\) With these at our hands, we can straightforwardly perform the following comparative statics.

---

\(^{151}\) As usual, these second order conditions are merely conditions for the indifference curves to be convex toward origin.

\(^{152}\) Note that the off-diagonal elements in (23) are all zero, implying \(e_{22}\) and \(e_{21}\) are independent each other. For comparison, (22) can be rewritten as

$$|\mathbb{H}|_1 = \begin{vmatrix} U_{xx} - 2pU_{xe} + p^2U_{xx} & (U_{ee} - pU_{xe}) \eta & 1 \\ (U_{ee} - pU_{xe}) \eta & U_{ee} \eta' + U_{e} \eta' & -1 \\ 1 & -1 & 0 \end{vmatrix}$$

for \(e_{22}, e_{21}\) and for some constraint \(k\) which assures \(e_{22} = e_{21}\). From this, it is evident that the condition (23) is obtained by imposing the condition on (22) that \(\eta = 0\) and dropping the condition \(e_{22} = e_{21}\).
1) $y$ effect
\[
\frac{\partial e_2}{\partial y} = \frac{p U_{xx} - U_{xx} (\eta + 1)}{|H|_1} > 0
\]
\[
\frac{\partial e_{21}}{\partial y} = \frac{(p U_{xx} - U_{xx}) \eta'}{|H|_2} > 0
\]
\[
\frac{\partial e_{21}}{\partial y} = 0
\]

2) $p$ effect
\[
\frac{\partial e_2}{\partial p} = \frac{e_2 (U_{xx} (\eta + 1) - p U_{xx}) + U_x}{|H|_1} < 0
\]
\[
\frac{\partial e_{21}}{\partial p} = \frac{[e_2 (U_{xx} - p U_{xx}) + U_x] \eta'}{|H|_2} < 0
\]
\[
\frac{\partial e_{21}}{\partial p} = 0
\]

3) $e_2$ effect
\[
\frac{\partial e_2}{\partial e_1} = \frac{p U_{xx} - U_{xx} (\eta + 1)}{|H|_1} < 0
\]
\[
\frac{\partial e_{21}}{\partial e_1} = \frac{(p U_{xx} - U_{xx}) \eta'}{|H|_2} < 0
\]
\[
\frac{\partial e_{21}}{\partial e_1} = 0
\]

4) $a$ effect
\[
\frac{\partial e_2}{\partial a} = \frac{2 \beta (a - q) [U_{xx} (\eta + 1) - p U_{xx}]}{|H|_1} - \frac{2 \beta U_x \eta'}{|H|_1} = ?
\]
\[
\frac{\partial e_{21}}{\partial a} = \frac{2 \beta (a - q) [U_{xx} - p U_{xx}] \eta'}{|H|_2} = ?
\]
\[
\frac{\partial e_{21}}{\partial a} = - \frac{2 \beta \eta'}{\eta} > 0
\]

5) $\alpha$ effect
\[
\frac{\partial e_2}{\partial a} = \frac{\tilde{e}_1 [p U_{xx} - U_{xx} (\eta + 1)]}{|H|_1} - \frac{U_x \tilde{e}_1 \eta'}{|H|_1} = ?
\]
\[
\frac{\partial e_{21}}{\partial a} = \frac{\tilde{e}_1 [p U_{xx} - U_{xx}] \eta'}{|H|_2} < 0
\]
Some explanations about the results are in order. As long as the amount of private tutoring is concerned, the first two are the standard results under the assumption of normality of educational good in both interior solutions: the demand for private tutoring increases as the income rises, and decreases as the price of private tutoring increases. What is peculiar here is the equilibrium school quality chosen by students in the second interior solution, which is invariant to changes in income, price and even $e_1$. In the second interior solution, students’ demand for school quality is saturated, and, therefore, is not affected by a marginal change in budget constraint\(^{153}\).

Third, the demand for private tutoring decreases as $e_1$ increases. As assumed in the model, $e_1$ represents an exogenously-given school education. Thus, an increase in $e_1$ stands for an improvement in the school education system. As more education is freely given in the school, the demand for the out-of-school education declines\(^{154}\).

Starting from the fourth to the last, all the results above have two terms in the first interior solution, one representing the demand for private tutoring that arises from student’s need to supplement his/her school education in a given quality school, and the other the demand for private tutoring for improving the quality level itself. The latter works through the induced change in the marginal effectiveness of private tutoring in improving the school quality of a student’s choice. In other words, the latter represents $\frac{\partial e_{21}}{\partial z}$ where $\partial z$ stands for a parametric change of the model. Note that $\eta$ represents the marginal effectiveness of $e_{21}$ in increasing educational achievement via quality improvement. In the second interior solution, these two terms are separated, because once the school quality is set at the saturation level, the demand for private tutoring is solely determined by the former alone. In this

\[ \frac{\partial e_{21}}{\partial a} = -\frac{\hat{e}_1 \psi}{\eta} > 0 \]

6) $\beta$ effect

\[ \frac{\partial e_{21}}{\partial \beta} = \frac{(a-q)^2[U_{ee}(\eta+1)-pU_{se}]}{|H|} - \frac{2(a-q)U_{ee} \psi}{|H|} = \gamma \]

\[ \frac{\partial e_{21}}{\partial \beta} = \frac{(a-q)^2[U_{ee} - pU_{se} \eta]}{|H|} \geq 0 \]

\[ \frac{\partial e_{21}}{\partial \beta} = -\frac{2(a-q) \psi}{\eta} = \gamma \]

---

153) Of course, the boundary locus will be changed accordingly.
154) Strictly speaking, this requires the normality of $x$, which is true under the current assumption of $U_{ii} < 0$ and $U_{ij} > 0$. As the quantity of educational good, $e$, increases, a student wants to consume more of $x$, because the marginal utility of education diminishes.
case, the second term does not represent the demand for private tutoring, but only the change in the level of saturated quality.

The fourth one shows how students with differing ability behave with regard to their choice of private tutoring. As explained above, it has two components. Note, however, that the expressions in the first equation are basically the sum of the second and third equations, which shows the two separate effects in the second interior solution. We may, therefore, look at the two distinct effects in the second interior solution first, and then interpret the results as happening simultaneously in the first solution.

In the second interior solution, the demand for private tutoring crucially depends on the sign of \((a-q)\) gap. In the range of \(a>q\), the demand for private tutoring increases as student's ability index increases, implying that the higher the ability, the larger is the demand. In the range of \(a<q\), however, the sign is opposite: the demand for private tutoring decreases as the ability index increases. In this range, students buy less, the higher their ability is. Appearing a bit complicated, it in effect says that students who are far away from the quality level of their own school will buy more private tutoring, which is logically sensible. They do not get as much education as those who are close to the school quality do, so they go out to private education market to make up for the deficiency.

The demand for a better school is, however, monotonically increasing in \(a\), implying that the greater the ability, the higher quality school he/she would want to go, and thus purchase the more private tutoring. This is also an intuitively appealing result. The underlying logic is that the higher the student ability, the smaller is the penalty that he/she has to pay for the mismatch when he/she advances to a higher quality school. Going back to the first interior solution where these two effects work together simultaneously, it implies that the higher-ability students would definitely buy more private tutoring because these tow effects operate in the same direction. For the lower-ability students, however, the combined effects are uncertain, because the two effects move towards opposite directions each other, one for increasing the demand for private tutoring, the other decreasing.

The fifth one presents the effect of change in \(\alpha\), the parameter representing the amount of school education one can get by attending a given level of quality school. In the second interior solution, the demand for private tutoring decreases as \(\alpha\) increases. This is because an increase in \(\alpha\) causes the amount of school education at a given quality school to rise. Therefore, demand for additional education in the private market decreases. The effect of an \(\alpha\) increase on the quality improving demand for private tutoring is, however, positive. As \(\alpha\) increases, the effectiveness of private tutoring in raising the total educational attainment through quality improvement is enhanced. This induces students to make more investment in private tutoring. The saturation level of school quality is also heightened because a student now can enjoy a higher quality educa-
tion without incurring an additional penalty. Here, these two effects, therefore, operate toward two opposite directions for everybody, one for decreasing the demand, and the other increasing. When combined into one in the first interior solution, the net effect is, therefore, indeterminate. Those students who are already attending the highest possible quality school at the saturation point, however, will buy definitely less private tutoring, because the latter effect is not in force for them.\textsuperscript{155)

Sixth, as far as the effect on private tutoring is concerned, the effect of $\beta$ change works in the opposite way to that of an $\alpha$ change. It first increases the demand for private tutoring for most of the students by reducing the school education one can get through the magnified penalty. Of course, this effect crucially depends on the size of discrepancy between the ability and quality. For those students who have established a perfect match, this effect is zero. Those students whose ability is far from the school quality, the penalty is proportionately higher as the gap widens, and, therefore, choose to buy proportionately more private tutoring. This suggests that the equalization measure (to be discussed more fully later) might have prompted more tutoring, not less as desired, because it inevitably increases the number of students who are away from the school quality in every school.

The demand for private tutoring for the purpose of entering a better school also hinges on the gap between quality and ability, but this time, on the direction of the disparity, not the size. For those students whose ability is higher than the school quality, improving the school quality through an additional investment in private tutoring is desirable. They used to attend a school whose quality is lower than their own ability because the quality is costly. Now that the mismatch penalty is higher, they have a stronger incentive to move up to a higher quality school even if it entails an additional cost. Therefore, high ability students purchase more tutoring. For those students whose ability is lower than the school quality, however, a further upgrading of school quality is the last thing they want to have. An increase in $\beta$ in this situation merely increases the penalty. As a result, it induces students to cut down their demand for school quality. Again, in the first interior solution, these two mutually conflicting forces must be summed, and therefore, the aggregate effect is indeterminate.

One interesting implication we can draw from the above results is that a school with a given quality cannot sort its students' ability just by looking at their parents' income, the crucial assumption that drives the main results in

\textsuperscript{155) It may sounds a bit counter-intuitive, at least on the first impression. An increase in $\alpha$ which will make $e_2$ more effective in raising the educational outcome has indeterminate effects on $e_2$? Of course, this reasoning has been taken care by our second term, the indirect effect. Somehow, one tends to forget the first direct effect: having a larger amount of school education will induce students to choose less of out-of-school education, its direct substitute. In fact, we tend to forget the fact that there are also other goods to consume, other than education, and an exogenous increase in school education will prompt one to cut the out-of-school education and instead to choose more of other goods.
Proposition 7: There is no simple trade-off relationship between income and student ability at a given quality school. For some lower value of \( a \), income decreases as \( a \) increases, and then after passing a certain threshold value of \( a \), \( y \) increases as \( a \) increases in the first interior solution. In the second interior solution, there exist no relationship at all between income and ability.

Proof: From 1) and 6) above, we have the following trade-off between \( y \) and \( a \) to have a given level of \( q \) in the first interior solution.

\[
\frac{dy}{da} \bigg|_{a_0 = a_0} = \frac{2\beta (a-q) p[U_{xx} - U_{xx}(q+1)] + U_x \phi'}{pU_{xx} - U_{xx}(q+1)} = ? \quad (24)
\]

In the second interior solution, this slope is not defined because the denominator is zero.

Note that, as explained earlier, a higher income family has a higher MRS for education, and as a result, a larger purchase of \( e_2 \). This is what Epple and Romano (1998) call the “single crossing property” in income, and holds, in general, when the commodity in question is a normal good. Unlike income, however, there is no simple, increasing or decreasing relationship between the student ability and the equilibrium amount of private tutoring in the first interior solution. In the second interior solution, there does exist a clear cut relationship between quality and income, and between income and the amount of private tutoring. Unfortunately, however, the quality in this case has nothing to do with income, thus again generating the indeterminacy.

It is clear that this indeterminacy in both cases originates from the penalty effect we have introduced in our model\(^{156}\). In the second interior solution, if we do not have the penalty for mismatch, we would not have the saturation point, either. In the first interior solution, the prime culprit for this inconclusiveness is the \((a-q)\) term in the numerator\(^{157}\), which again comes from the penalty effect.

Graphically speaking, when a increases, \( U_x / U_x \) curve can either shift up or down depending on whether the student ability is higher or lower than the school quality at the equilibrium. If it is higher, i.e., \( a > q \), then the MRS curve shifts up as the student ability increases, and if it is lower, i.e., \( a < q \), then the curve shifts down.

A student whose ability is higher than his/her school quality\(^{158}\) finds it beneficial to improve the school quality further. So he/she is willing to give up
other goods to buy one marginal unit of school quality, and this willingness increases as the student’s ability index gets higher because the marginal returns to quality improvement gets bigger. For a student who has already chosen a quality higher than his/her ability, however, a further increase in quality is not something desired for in itself unless a redeeming compensation is provided for\textsuperscript{159}. The compensation, the additional gain they can get from quality improvement, however, gets smaller as the ability index moves closer to the $a=q$ point. As a result, students become less inclined to sacrifice other goods to have a higher quality as they gets a higher ability, which causes the $U_e/U_x$ curve to shift down.

In the range where there is a negative trade-off between $y$ and $a$, a very high value of $y$ is associated with a very low value of $a$ at a given level of $q$, and vice versa. Therefore, a given level of quality is purchased either by families with a high income but a low $a$, or families with a low $y$ but a high $a$, or families in between the two, implying that the student body of a given quality school will be composed of the students who satisfy this particular trade-off relationship. On the individual family level, this implies that a family which does not put a high value on education would buy their kids a small amount of out-of-school education, or refuse to buy at all, unless their kid demonstrates an exceptionally high ability. On the other hand, the family which values education very highly will spend lavishly on private tutoring for their kids even when the kid shows mediocre ability\textsuperscript{160}.

For the reason already explained, however, this type of stratification of students is bound to be incomplete. In the first solution, some students whose ability is located close to the lower end purchase less and less quality as their ability increase toward $a=q$ point (or, slightly lower than this). In order to induce them to purchase the same amount of quality, therefore, you have to give them a correspondingly higher income. This creates a positive relationship between ability and income in this range. In the second solution, the situation is worse, having no relationship at all\textsuperscript{161}.

If schools are interested in securing students with high intrinsic ability, this implies that schools will have difficulty in achieving their goal because, in the current setting, schools have no way of knowing the true ability of a student.

\textsuperscript{159} This may be a bit confusing. We know there are three different ranges of quality a student with a given ability to choose: $q>q^-(-a)$, $q^-(-a)>q>a$, and $q<a$. In the range of $q>q^-(-a)$, every student will choose $q^-(-a)$, because any quality higher than $q^-(-a)$ is a real pain. This is the case of the second interior solution. The first interior solution covers the remaining range. Here, if $q>a$, an increase in $q$ is a bit painful, but still beneficial. If $q<a$, a gain in $q$ is wholly beneficial.

\textsuperscript{160} This may open the possibility for the schools to practice some sort of price discrimination of the type discussed in Epple and Roman (1998), i.e., offering a scholarship to some high ability students. But, again, the caveat referred to in footnote (138) applies here.

\textsuperscript{161} Putting aside the saturation solution as a remote possibility, this suggests that there will be a positive correlation between $y$ and $a$ in the lower end of $a$, and then a positive correlation in the upper end, which may be testable with empirical data.
even among those students who have successfully secured the school’s admission by demonstrating a high enough academic achievement, i.e., the amount of $e_2$ purchased. The high performance may be the result of 1) student’s intrinsic ability, 2) family’s high income, and information about $y$ is not that helpful. For an effective price discrimination, schools may have to require the students to submit additional information\(^{162}\).

### 5-4. Effects of School Equalization Measure

In the early 1970s, the Korean government adopted a rather radical reformatory measures in secondary education system by banning all forms of school entrance examinations which had been in use ever since the adoption of the modern education system after WW II. The major goal of this measure was to reduce private tutoring, which was viewed “unreasonably excessive.” Before this measure was taken, each student had to pass an entrance examination which was administered by individual schools. There had been no zoning restrictions either. In that sense, there was a national level competition to enter a better school, and there had been a well-established hierarchical order amongst secondary schools. The authority perceived that the excessive demand for private tutoring had originated in large measure from this school-order, and that by eliminating this rank-order system, one could dampen or root out the passion for private tutoring. Under the new equalization measure, students were randomly allocated, basically by lottery, to schools within the newly created school districts in the neighborhood of the students’ residence.

In a comparative sense, therefore, we can see that there exists a close similarity between the proposals aired these days in the US to improve its school system and what the Korean government did three decades ago to reduce the excessive competition, but in an opposite way. The so-called “choice” policies or proposals include inter- and intra-district open enrollment, formation of magnet and charter schools, and vouchers for private schools. Thus, the goals that the choice proposals aim to achieve can be restated as increased school choice through an elimination of territorial restrictions and a mitigation of the familial financial constraints, which are in a large extent diametrically opposite to what the Korean government had implemented: a new creation of school districts, an introduction of mandatory neighborhood schooling, and prohibition of competition among schools and students.

In order to analyze the effect of the school equalization measure, I assume in this section a more concrete form of school quality choice function. Specifically, I assume,

\(^{162}\) Of course, schools can be assured of the lowest ability, because $q^*$ is increasing in $a$. 

so that $\psi' = k$. In this formulation, $k$ stands for the marginal return of private tutoring in enhancing the quality of school. The equalization measure can then be interpreted as setting the value of $k$ at zero, or reducing it to a small value, rendering the private tutoring less potent or impotent at all in its power to raise the school quality. I take the equalization measure as marginally reducing the value of $k$, and perform the following comparative statics$^{163)}$.

A differentiation of the first order condition of the first interior solution with respect to $k$ gives$^{164)}$,

$$
\frac{\partial \xi}{\partial k} = - \frac{\xi e_2 [U_x(q+1)-pU_{xx}]}{|H_{11}|} - \frac{U_{xx} \left( x \xi' + \zeta \right)}{|H_{11}|}
$$

where $\varepsilon_{kq}$ is the elasticity of $\xi$ with respect to $q$ and $\varepsilon_{\eta k}$ the elasticity of $\eta$ with respect to $k$.

In the range of the first interior solution, $\xi$ is positive. The first term, which gives the direct effect of the $k$ increase is negative. For a given $e_2$, an increase in $k$ brings about a higher $q$, via $q = ke_2$, which increases $e_1$. This reduces the demand for private tutoring. The second term is indeterminate because the elasticity of $\eta$ with respect to $k$ can be positive or negative. If it is negative, then the overall effect which combines both the first term and the second, is

163) There are some reasons for not setting $k=0$. Obviously, it makes the analysis easier. In addition to this, the school equalization measure was in some sense incomplete. In the case of junior high school, only those schools located within the major metropolitan cities were subjected to this new ordinance, and schools in local provinces were left free to take either form of the admission methods: free competition or compulsory randomization. Furthermore, in tandem with this equalization measure, a handful of special schools were newly launched to accommodate the high ability students such as special schools for science and special schools for foreign languages. These schools were allowed to have entrance examination to select students, and later became very competitive. Still another reason is that the colleges and universities have never been equalized.

164) As already explained above, the analysis for the second interior solution is almost similar to the first case, except that the two effects are now separated.
negative, implying that private tutoring will decrease as $k$ increases. If it is positive, the overall effect is ambiguous, and will depend on which one is bigger in absolute terms.

School equalization measure lowers the value of $k$, and does not increases it. Therefore, we have to reverse all the signs in the above results. Under this reversal, the first term is positive, implying we will have more private tutoring as a result of the equalization measure. This is opposite to what is intended. The second term is either positive or negative. If it is also positive, that will be a fatal blow to the equalization measure because the overall effect is unabashedly opposite to what was aimed for. If it is negative, the overall effect is indeterminate. In short, the equalization measure may have been altogether detrimental to the reduction of private tutoring, the result so desperately hoped for, or may have been neutral or ineffective because of the countervailing forces at work. It may have been effective if, and only if, the indirect elasticity effect is so large that it completely cancels out the first direct effect and works more to the negative direction, which is a rather stringent requirement to satisfy.

The intuitive reason for this intriguing conclusion is not difficult to figure out. Let’s take the direct effect first, the demand for private tutoring to supplement school education. The channel through which students can attain a larger education within the school system by attending a high-quality school is now damaged by the equalization measure. As a result, students with a given $e_2$ now have a smaller $e_1$ to consume than before. Consequently, they have to resort to alternative means to fill up their unsatisfied educational demand, which is buying the same good in the private market. In short, people buy more private tutoring because they can not get what they want within their schools.

The second effect, i.e., the demand for private tutoring for the purpose of going to better schools, is a bit involved. A change in $k$ causes $\eta$ to vary, the marginal productivity of private tutoring in increasing the school education. Note that $\eta=\xi k$, and when $k$ decreases, $\xi$ moves to the opposite direction because $\xi$ is decreasing in $q$). Thus the overall effect of a reduction in $k$ on $\eta$ depends on how much $\xi$ increases in response to the $k$ reduction. If $\xi$ increases more than proportionately, that is, if $\xi$ is elastic with respect to $k$, $\eta$ increases. This induces students to take even more private tutoring in response to the $k$ reduction. In this case, lowering $k$ will do more harm than good, as far as the volume of private tutoring is concerned. On the other hand, if $\xi$ increases less than proportionately, i.e., if $\xi$ is inelastic, a reduction in $k$ decreases $\eta$, and thus discourages private tutoring. In this case, the direct effect and indirect effect work in the conflicting manner, one to increase and the other to decrease the private tutoring, and the overall effect becomes uncertain.

---

165) This is the second order condition we have seen earlier. If it doesn’t hold, every student will try to go to the highest possible quality school, destroying the equilibrium.
Recall that $\xi$ stands for the marginal productivity of quality in increasing the amount of education a student can get within the school. In other words, it is an indicator about what significant difference it can make to attend a school whose quality is higher by one unit. We know that this is diminishing. The question is then how fast it decreases: More than proportionately as $q$ increases or less than proportionately? This is the elasticity question we have had above.

The elasticity of $\xi$ function depends on the shape of $e_1^1(q)$ function and the values of $q$ and other parameters. More specifically, it can be shown that in order for $e_2^q$ to be inelastic, one must have

$$-\frac{\hat{e}_1 q}{\hat{e}_1} < 1 - \frac{4\beta q}{\alpha \hat{e}_1}.$$ \hspace{1cm} (27)

This implies that for a given $e_1(q)$ function, the smaller (larger) the $\beta (\alpha)$ is, the value of the right hand side of the above inequality will become larger, rendering it easier for the above condition to be satisfied\(^{166}\).

There are at least two more things that we have to consider with regards to the equalization measure adopted by the Korean government. With the abolition of all forms of entrance examinations, the authority had to provide some means of allocating students amongst the schools. As mentioned earlier, the measure adopted was to randomize the allocation through a lottery. Many new school districts have been defined based on, among others, regional proximity, in such a way that each district contains multiple schools, and within each school district, students were randomly distributed among schools\(^{167}\).

The net effect was a huge increase in the variance of student ability distribution within each school. Before the equalization measure, students self-selected themselves according to their ability and income, making it possible for each school to hold a comparatively homogeneous ability group. Under the newly introduced equalization measure, they were forced to attend a school whose student ability distribution is not known, or, worse than that, where the distribution was manipulated to be almost the same as the distribution of the student population within a school district in every school. Within the current model, it means that the area of free disposability has been turned into no-disposability, thus increasing the range of the ability distribution of every school.

\(^{166}\) For a given set of parametric values, a logarithmic function of the type $e_1 = \log(1+q)$ yields the left hand side of the inequality to lie between 0 and 1, and a power function of the type $e_1 = 1/(q+1)$ between 0 and 2. Therefore, it is more likely that the elasticity condition holds under the latter function. For example, in the case of logarithmic function, if the value of $\beta$ goes to zero, $e_2^q$ will be always inelastic, thus making the sign of the second term positive.

\(^{167}\) In reality, it was not purely random, though, because the authority consciously tried to achieve an equalization of school even in student ability distribution among schools, probably to perfect the spirit of equalization in every aspect of the matter or to ward off possible complaints from parents. It somehow ranked each student's ability within a given school district, and then assigned the students to different schools in a descending order such that the ex post mean and variance of ability distribution in each school should be almost comparable across schools.
If we interpret this as an increase in $\beta$ in our school education function, it may have increased private tutoring further, at least among those whose ability is relatively high.

Lastly, the equalization measure has effectively destroyed the distribution of school quality itself and caused the distribution to degenerate. First, the measure literally ordained the shutdown of several top-quality schools. Second, among those that survived, little incentives were there to continue to try to provide top-quality education to students. Third, even if they tried, it was difficult to achieve the desired outcome because the range of the distribution of student ability within a school was practically too wide to have effective teaching and learning\textsuperscript{168}). A degeneration of school quality distribution in our model implies that a large number of students must have dropped out of school. As mentioned earlier, this does not necessarily mean that all the students in this range literally have stayed out of school. It might have been difficult for those students to actually quit coming to school under social and family pressure. In this regard, a testimony on the Chinese experiment which is similar to what Korea had, is suggestive about what would have happened: "What only became clear to me later,... was the collapse of real learning in the primary and secondary schools; in Unger’s words, ‘most of China’s urban students simply stopped paying attention in class’\textsuperscript{169}). In the context of our model, it means that a large number of students located near the end points of the ability spectrum must have been driven to go directly to the private tutoring market to satisfy their demand for education.

All in all, it is impossible to draw any definite conclusion about the effects that the equalization measure may have created on the amount of private tutoring. Other than the last one, which is definitely positive, the signs of the first two effects are mixed. It is clear, though, that if we confine ourselves to the direct effects only, the measure must have been detrimental to what is desired for, because all those terms are decidedly positive. Furthermore, some of the available circumstantial evidences are also pointing to that overall direction\textsuperscript{170}).

\textsuperscript{168}) There exists abundant testimonies to that effect.
\textsuperscript{169}) Dore (1997), p 196.
\textsuperscript{170}) See Kim and Lee (2001) which shows that the private tutoring has been larger in the metropolitan cities than in medium and small cities, even after the effects of other variables are controlled.
Whether we like it or not, the way to approach educational problems has to be founded on facts and reality. One of the undeniable facts about education is that it is a good, just like any other ordinary goods. People want to have it, and more of it is preferred than less. It is, after all, demanded by people. It is produced by people through combination of various real resources which otherwise might be used for the production of other goods. More of education means, therefore, less of other goods from the social standpoint. So we have supply of and demand for education. This is an inevitable fact about education from which no society can escape.

Once we acknowledge that education is a good that is produced and consumed, we are inexorably confronted with the choice problem of how much, at what prices, for whom, etc. If a society makes judicious choices concerning all these inherently economic issues, it will prosper, the more so because education is considered as one of the most important factors affecting the pace and the quality of economic progress. If instead, a society makes inept decisions, inadvertently or deliberately, the society will suffer unnecessary wastes, inefficiency, social tensions and encumbered progress.

Fortunately enough, we know how to make the best decisions on all of these, at least theoretically, best in the sense that nobody can improve upon it further no matter how smart he/she is. Something demanded by people must be produced exactly as much as the marginal benefit from it is matched by the marginal cost of producing it. The beneficiaries have to be asked to pay its market clearing price in order to prevent over- or under-consumption from happening. Production cost must be borne fully by producers in order to avoid over- or under-production. In short, for an optimal allocation of scarce resources of a society, the value, the price and the cost of a good must always be equated. So simple is the fact, few policy-makers or practitioners would blunder if tested in a formal exam over these questions.

The real tragedy, however, is that many forget this simple truth when they are dealing with real world problems. Not a few unwittingly put aside textbook solutions as high-sounding and then lapse into more down-to-earth solutions, believing that they are digging out better, more fitting answers to the problems at hand. The consequences are that things are often messed up, sometimes irrevocably, and a series of patching-up remedies follow, prepared
again offhandedly in an anxious frenzy until things get into a hopeless state. This might be what we have today with regard to education policies, especially those concerned with the private tutoring problem.

Somehow, tuitions and enrollment quotas of colleges have long been regulated, thus creating an artificial scarcity in the higher education market. The artificial scarcity gives rise to a deadweight loss. It also inevitably causes a divergence between value and price, and price and cost. The gap between the value and the prices makes the good in question a privilege, something that bestows a windfall gain to its lucky owners. Everybody covets it. But the quantity is limited. People begin to compete, and make an investment in whatever they believe would increase their chances to get an access to it.

College education has long been one of those coveted items. Until quite recently, about 900 thousand students wanted to have it. But only about 300 thousand students were allowed to get it, with 600 thousand left unsatisfied at the on-going prices. Students started to make investments in private tutoring, sometimes excessively to the eyes of policy makers, to mark a good score in entrance exams and to enhance their odds of admittance. Public authorities also started to chase them to discourage their “excessive, wasteful and irrational” zeal for private tutoring when the social outcry against it started mounting up.

It must be, however, emphasized that one can never lower or heighten a price, the full price the demanders are willing and have to pay to get a good. He/she can only change the composition of the forms by which the payment is made. If a monetary price is suppressed, a non-monetary form of payment will be tried. People sometimes stand long in line to buy something whose price is regulated. They pay less in money, but more in time instead, until the time value sacrificed is equated to the gap between the willing price and the regulated one. So the full price is the same as before.

College tuitions have been regulated. Thereby students were induced to pay to their schools less in money, but more in private tutoring outside school. Their tutoring expenses reached 20 trillion won in 1997. As social tensions was about to burst, schools and authorities instituted additional reforms in entrance exam systems, other non-exam selection methods, and even a complete ban of private tutoring, and so on, as if they could thereby successfully lower the burden the students are bearing, or equivalently the full price students were paying. The consequence? No avail, as a natural course of the matter. Economics has the law of constancy of full price like the law of constancy of mass in physics!

Why regulation, then? We have examined several possible arguments in depth for educational subsidies, both general and special, and refuted that none of them are tenable. Unlike the basic education at the elementary level, college education, most of which are vocational, cannot be claimed to be a public good, because most of the benefits from education solely accrues to the prac-
tioner. An educational subsidy for the purpose of improving income distribution doesn’t make sense, either, because the subsidy is most likely to aggravate income distribution, and so on. In particular, it has been clearly demonstrated that the all-too-often-touted argument, — that college education is one of the most effective and surest means to improve one’s future income and social status, — either cannot be true at all in a free open market, or, if it is true nonetheless, there must be something terribly wrong with the workings of the college education market. A balloon cannot stay underneath water. If it does somehow, you would rightfully suspect that there be something artificial that hinders the natural buoyancy. This is exactly what the above proposition about college education implies. College education remains a sure means to improve one’s social status only because the access to it is regulated. Otherwise, purchasing a college education would not be an effective means to improve one’s future income any more than purchasing a pencil or a pair of shoes would make you rich in the future.

Strangely enough, college enrollment quotas have also been regulated in tandem with tuition. The reason being a prevention of over-supply of college graduates, this measure, however, diametrically contradicts the earlier arguments for educational subsidy. The latter says education would be underconsumed if not unassisted, whereas the former voices education will be overly indulged if unchecked. Obviously, both cannot be true at the same time. If one is right, the other must be wrong by logical necessity. No matter how contradictory they may be, we have successfully kept both safe and sound for long. The result was a cumulative worsening of private tutoring problem, and probably an aggravation of income distribution and an exclusive inter-generational reproduction of social classes. Generation after generation, the rich and talented march through the narrowed gate of college education, while the poor and powerless are being pushed away. The big wage gap between college graduates and non-graduates evinces a conclusive evidence for this.

Powerful evidences — the ever worsening tutoring problem and others, however, have not frightened the authorities away from their ungainly policies. Many still have a faith in a regulated and managed education system, clinging to sheer myths and unbaked half-truths. As a matter of fact, one cannot but get surprised to find how many fabulous myths are hovering in our minds when it comes to matters related to education, as some of them have been uncovered in our earlier discussions.

Do all these really mean then that the colleges and universities should admit all students who want a college education regardless of their qualifications as long as they are willing to pay? The answer is yes and no. It is yes from the standpoint of college education industry as a whole in principle. No one should be denied based on his/her qualification, as no one should be denied based on their ages, gender, race, etc.. This is the economic principle to be established to secure an efficient allocation of scarce resources: what matters
and should matter in economic transactions is the customer’s marginal value, not the color of their skin, ages and other things, or even the readiness to learn.

From the standpoint of individual colleges and universities, however, the answer is no. Each college must carefully assess the qualification and readiness of each student, in addition to checking their willingness to pay. The qualification or ability of students matters to an individual school because the range of student ability distribution that a particular school can effectively serve is limited. Students have to be sorted into various ability groups, and each school should concentrate in educating a particular ability group. This is a cost-effective way of maximizing the educational outputs.

Education, or more correctly schooling, is a good, but a special good in the sense that in the production of it, both seller and buyer must jointly participate. It is not produced first by a seller and then sold to a buyer. Both sellers and buyers must work together to get anything out of it. And due to the particular cost conditions, oftentimes multiple students participate in a single educational process. This joint and collective nature of schooling dictates a sorting of productive inputs as inevitable steps to be taken. For this purpose of sorting, entrance exams are administered by schools.

Exams for sorting, however, must be distinguished from exams for screening. Earlier, we have seen the exams used as a rationing device, to select a limited number of students from a large applicant pool. The major function of an exam there was denial. But an exam for sorting is different. This is not for denial, but for efficient provision of educational services. As explained earlier, the difference between the two is analogous to the difference between licenses and certificates. Licenses are for denial: a non-licensed doctor is strictly prohibited from practicing medical operations. On the other hand, certificates are for dissemination of information. You can sell both KS marked and unmarked products in the market. The former is efficiency-reducing, and the latter is efficiency-enhancing. The same is true with exams: an exam for screening is efficiency-reducing whereas an exam for sorting is efficiency-enhancing.

With all these suggested reforms faithfully implemented, which would certainly render the educational good to be something like traded in the auction markets, wouldn’t the specter of the “first-class-school fervor” be revived among parents and the dismaying “cliquism”, especially among the graduates from those so-called top-notch schools, pervade down the street again? As has been discussed more fully in the text, and summarized once more later, a large part of the excessive zeal for top-class school comes from tuition control and the resultant imperfect pricing of school qualities. With price regulation completely resolved, a substantial portion of the top-class frenzy will be also gone. If there still remains an excessive fervor, nonetheless, it has to be dealt with a more extensive and thorough deregulation in other parts of economy. In this way, we would be able to deplete the grains to grind in the mill. Regulating and suppressing school education merely diverts the way through which the
ground flour is distributed with no possible effects on the operation of the mill. If one distribution route is blocked, other alternative routes will surely be tried and further developed.

In the preceding two chapters 4 and 5, we have shifted our attention from more or less normative to positive analysis of the private tutoring problem, with chapter 4 concentrating on entrance exam system and chapter 5 on the secondary school equalization measure. In chapter 4, we have examined the effects of various reforms on entrance exams, suggested or actually implemented to suppress private tutoring. The major theoretical conclusions reached are summarized below. On behalf of the readers who may have skipped the chapters because of the technicalities, I will elaborate here somewhat in detail, even taking the risk of some duplication. To put the overall conclusion first, most of the suggestions for entrance exam reform appear to be sensible and, therefore, effective on first impression in generating the desired results. Yet most of them do not survive scrutiny. Our theoretical examination predicts that some measures would turn out to be totally ineffective or even harmful, others uncertain in their likely effects, and still others effective only within a very limited range or magnitude, sometimes only under rather stringent conditions.

One suggestion we most frequently hear in relation to private tutoring is that school authorities should make the test easier to lighten the students’ burden to prepare for the exam. An easier test will relieve students of the strain, the advocates contend, and will consequently make students take less tutoring. Appealing it may sound, this suggestion, however, has little value of significance when we consider what is really important for students is not their absolute scores but their relative ranks. Only those students who mark higher scores over others can succeed in grabbing the tickets for college. Making the test easier across the board makes every student’s score improve, leaving the relative rank altogether intact. Knowing this, students will continue to strive to get higher marks and thus continue to take private tutoring.

In response, some people may argue that the suggestion for an easier test in fact means that school authorities should increase the admission quota. They say that an easier test with an unchanged admission quota would generate little impact, but an easier test accompanied with an increase in quota will surely bring down the demand for private tutoring. It sounds again quite persuasive. Unfortunately, however, the layman’s common sense fails once more in this front. Our theoretical analysis suggests that an easier entry to college would not necessarily bring about a reduced demand for private tutoring, either. Surprising as it may be, a closer scrutiny discloses the intuitive reasons why it must be so.

Lowering the cut-off score to accommodate more students will indeed cause the students with a high expected score to choose less private tutoring. They no longer need study as hard as they used to. For those students whose expected score is lower than the cut-off point, however, it is a totally different
story. With the lowered cut-off score, a little bit more effort on their part will bring out a substantial increase in their odds of being admitted. This induces them to take more tutoring. Even those students who would otherwise have given up the college education altogether enter the game with a renewed hope. Therefore, the aggregate effect crucially hinges on the relative size of these two student groups, one who are favorably affected by the change in the cut-off score, and the other who are adversely affected.

If the competition to enter college is very fierce and severe at the point of policy change, fierce in the sense that only a small number of students succeed to enter colleges with the remaining large chunk of population failing, an easier test will surely make private tutoring more prevalent. Only when a large portion of students can enter college with relatively small effort, would an easier test result in a reduction in the aggregate tutoring. In short, one has to make a hard test even harder if he/she really wants to discourage tutoring, or an easy test easier. If, instead, one makes an easier test harder or a hard test easier, it will only instigate more tutoring. Since the college competition rate in Korea have been quite high, often standing in the range of 3:1 to 4:1, this implies that an attempt to make entrance exam easier would have caused a tremendous increase in private tutoring.

Another proposal advanced to cut down tutoring demand is to put more basic and fundamental questions on exam for which private tutoring would have a relatively little additional value in terms of improving the test scores. This may be similar to the previous proposal to make exam easier in many respects, but is different because it is specifically targeted toward private tutoring, and therefore, generates different effects. Making private tutoring less potent in improving test score causes the demand for it to shrink for most of students, but with one exception. For a large number of students, private tutoring is less worth purchasing because of its lessened usefulness. However, those students who have bought already a large amount of tutoring, thus expecting high scores, purchase even more tutoring when tutoring is made less effective. This is because as their expected score gets lowered, their odds of being admitted is threatened. In other words, more is at stake now in terms of their expected benefit and they are driven to take a protective action against the threat.

A third policy proposal we have examined is the one that recommends raising the cost of private tutoring, including an outright ban of all tutoring activities outside school. Making tutoring more costly to take will make all the students buy less of it for certain. But we must be careful yet before rushing to the conclusion that we finally have one instance in which both our common sense and theory have reached a common conclusion. To the surprise of many, the above statement is true only in real terms, the physical magnitude of tutoring, not in nominal terms, which is far more significant for us. Even with a smaller quantity, the nominal volume may still be larger because the price is higher now. The final answer, therefore, crucially depends on the elasticity of demand
with respect to price, which we do not know. Consequently, the effect of raising the cost of private tutoring is again uncertain.

Finally, we have examined the effect of putting more essay type questions than multiple choice questions on exam, which is also frequently recommend-ed by many with an eye to suppress private tutoring. Here again, the offhand common sense fails. First of all, it is not clear, on second thought, why people recommend it, sometimes so enthusiastically, as a possible solution to the private tutoring problem. Conceivably, some may have believed that putting more essay type questions would somehow provide some magical ways of ascertaining the true qualification of students, which is in itself rather doubtful. Even if one accepts this highly hypothetical proposition on faith, it doesn’t necessarily follow that private tutoring will decrease as a result. A sure way of checking the quality of students has nothing to do with private tutoring because the quality of students may still improve through private tutoring. If the quality cannot be improved by tutoring, then the issue becomes the same as the one we have already investigated, designing tests in such a way that private tutoring is rendered relatively impotent.

Another possible reason might be that putting more essay type questions would make private tutoring a less reliable means to improve test scores. Essay type questions are likely to reduce the predictability of final outcomes. If a tool produces unpredictable, thus unreliable outputs, people would opt to use less of it, some might conjecture. This proposition has been examined in the chapter with some rigor. The theoretical conclusion says that the conjecture is partly wrong. As the variance of the test outcome increases, some students, especially those in the middle range of expected scores, buy less of private tutoring, but those students in the outer tails of the distribution buy more of it. The intuitive reason is that as the variance increases, the density function becomes flattened in such a way that the middle range is squeezed down and the tail ranges are thickened. Therefore, the aggregate effect on private tutoring is again uncertain. Worse than that, an increased dispersion of the distribution is likely to cause a deterioration of the overall quality of student body. If we divide the student distribution into two subsections along the expected scores, the students in the lower half distribution enjoy either increased or indeterminate odds while the students in the upper half suffer lowered or indeterminate odds as a result of the variance change, implying that the student body will be increasingly composed of ill-prepared students.

In Chapter 5, the focus shifted to the issue of competition to enter a better school, with school quality allowed to vary. Once we allow variation in school quality, it is natural for us to allow student ability to also vary amongst themselves. In general, the amount of education a student can get from a school increases as the school quality increases. As a result, students aspiring to get into a higher quality school, take private tutoring to upgrade the quality level of their attendance. However, this demand for school quality is bounded
because a school with too high a quality can be detrimental to the educational attainment of a student with a given ability. That is, if the gap between school quality and student ability is widened too much, the student would get very little education from the school. This implies there exists a variation in the amount of education that each student can get. Some students would get a relatively large amount of education while others get relatively little depending on what ability he/she is endowed with. This variation creates another important motive for private tutoring, i.e., supplementing school education. Those students who receive a relatively small amount of education from school would try to make up for the deficiency by purchasing additional education outside school. Thus we have two motives for tutoring, one for advancing to a better school, the other to supplement school education.

This is the framework we have set up in the chapter. One general conclusion that follows from this framework is that the competition for better schools arises because of the imperfect pricing of school quality. If all schools are uniformly priced despite the quality differential, as is true with most of the secondary schools, and more or less so with colleges, students compete to get into a better school because a better school is certainly a better deal. In other words, a uniformity in nominal price necessarily results in a divergence in real effective, quality-adjusted prices. After all, if a Grandeur is sold for the same price as a Tico, every consumer would rush to buy Grandeurs. Who else wouldn’t? This implies that if diversified pricing is instituted, which would be instantaneously achieved if the current regulation on school tuition is dismantled, a substantial portion of the “better school fervor” and the resultant demand for private tutoring would disappear in no time.

Another conclusion of significance is that a hierarchy in school quality among schools will naturally arise in equilibrium in the sense that a higher quality school attracts a group of higher ability students. In other words, the mean ability of student body of a school increases as the quality index of the school increases. The lower and the upper limit of student ability distribution also increase as the index gets higher. This implies that the so-called “quality rank among schools” is a natural equilibrium phenomenon which cannot be easily demolished at will as some critics of the rank system contend.

Comparative static analysis with respect to some parameters in the model has also been performed. Major findings of practical significance are: first, demand for private tutoring will increase as the parent’s income increases, and/or, the price of private tutoring is lowered, as long as both education and other non-educational goods are normal. Second, the higher a student’s individual ability, the greater the school-quality-upgrading demand for private tutoring. This is because a more abled student can extract a greater benefit from a quality school. The school-education-supplementing demand for tutoring is a bit different. It depends crucially on the gap between the school quality and individual ability in such a way that the greater the gap, the larger the
demand for tutoring, implying that those top- or bottom-ranked students would buy relatively larger amounts of tutoring, and the middle-range students less. Combining these two, we can easily conjecture that demand for private tutoring would be more pervasive among the better performers in school. Those students who are falling behind in a given school would certainly want to have some tutoring to catch up, but would never want to move up to a still higher quality school.

Third, under fairly general and plausible assumptions, it can be shown that a poor family would buy private tutoring for their kids only when their children demonstrate an exceedingly high ability, whereas a rich family would buy lavishly even when their kids exhibit a mediocre ability. Fourth, as the efficiency of school education system improves, the demand for private tutoring will diminish because students now can get more education within school with no additional increase in school tuition. This suggests that if some schools offer a better education in return for increased tuition, some students may prefer to go to the private market to meet their educational demand. A general taxation for improving school quality is, of course, different from this, because in this case the increase in school quality is not directly linked to the cost that a student has to individually bear.

Fifth, an increase in the marginal educational gain a student can get by attending a higher quality school will make the quality upgrading demand for private tutoring increase for an obvious reason. The school quality matters more now by the assumed hike in returns. The demand for private tutoring for the purpose of supplementing the school education within a given quality school, however, behaves differently. It will decline in response to the hike. Students can get more and better education within the school, so they don’t need go outside the school for more education, which would certainly entail an additional cost. When these two conflicting forces are combined, its overall effect on private tutoring is indeterminate.

Sixth, an increase in the penalty a student has to pay for the mismatch between the school quality and his/her individual ability will make the supplementary demand for tutoring rise for every student except those who have a perfect match. For those students whose ability fall far away from the school quality, the educational attainment is now smaller because of the increased penalty, and they will want to make up the reduction through an increased private tutoring.

In response to the penalty increase, the aspiration to seek a better school, and thus the quality upgrading demand for private tutoring, is heightened only among the higher ability students. They want to move up to a higher quality school to avoid the penalty that they are now bearing at the lower-than-his/her-ability school. Surely, moving up to a higher quality school is costly because it implies that students have to buy additional tutoring. But this is now worthwhile because otherwise they would have to continue to pay the
increased penalty at the lower quality school. Those students who are already attending a higher quality school than their individual ability, however, would not want to further upgrade their school quality. As a result, their quality upgrading demand for private tutoring will decrease.

As is well known, the secondary school equalization measures taken during the late 1960s and early 1970s has substantially increased the variance of the student ability distribution within each school. This has forced a large number of students to suffer from the increased penalty for the mismatch. By an analogy to the above result, it suggests that the supplementary demand for tutoring must have increased as a result of that equalization measure. Of course, it may have brought along some reduction in the quality upgrading demand by making it impossible or harder for students to choose their own best matching schools. So the question would be which effect has been bigger, to which I will now return.

In analyzing the effects the equalization measures might have created, it is important to recognize that it was in the late 1970s to 1990s, that is, in the wake of the measures and not before, that much of the social tension concerning the private tutoring problem has blown out in a tremendous proportion. So the timing in itself attests that the measure had not succeeded in bringing out the desired outcomes. If the measures had been effective in extinguishing the tutoring fervor, no further social outburst surrounding the tutoring problem would have continued to show up, not to mention in such a bloated fashion. One might well say that, in terms of its effect on reducing private tutoring, it was a complete fiasco. The analytical results obtained in the chapter also confirm this conclusion, since most of the theoretical conclusions predict a likely failure.

The model predicts that the measure must have caused a substantial surge in the school-education-supplementing demand for private tutoring through its negative effect on school choice and education within schools. Firstly, the equalization measure has caused a complete degeneration of the distribution of school quality, thus shrinking the range of student ability that the whole secondary school system can effectively serve. It must have caused a large increase in school drop-outs, who would have naturally headed for private market for their education. Secondly, among the existing school distribution, it has demolished the self-selection mechanism of students by the compulsory random allocation of students among schools. It has, therefore, caused a tremendous increase in the variance of student ability distribution in every school, thus prompting more tutoring to make up for the insufficient school education. The increased variance has also made effective teaching within classes almost impossible.

Moreover, the effect of the measure even on the “top-quality- school hunger” of students, thus on the quality upgrading demand for tutoring, turns out to be also doubtful. Apparently, it has quelled the quest for better schools
because there remained not many better schools any longer after the measure. But parents could still migrate to better school districts, causing the so-called “Eighth District Syndrome”. More than anything else, however, the colleges and universities, the ultimate goal of ever hungry searches for better schools, have never been equalized. If one can accept that better colleges and universities are indeed the ultimate destination of all those chases, the measure has simply rendered one of the routes of the chase, school education, to be unrewarding, and thus caused the quest for better universities to stampede to another alternative route, private education.

Third, making private tutoring less effective in upgrading the school quality, which can be regarded as one of the policy objectives, may have backfired if the marginal return from school quality is diminishing and is elastic with respect to quality. As it is, the issue appears to be more of a theoretical curiosity, but may still have some practical relevance. This works as follows. Private tutoring is desired for its effect of increasing the school quality, and this increased quality produces a larger education. In this two-step channel, if the marginal productivity of tutoring is lowered, it necessarily reduces the quality level that a student can attain. A reduced quality then raises the marginal productivity of quality in augmenting the amount of education, because the marginal productivity of quality is diminishing as assumed. With one being reduced and the other increased, the final change in the product of the two depends on the latter’s elasticity with respect to the former. If it is elastic, the product will increase as a result of the reduction in the former. If it is inelastic, the product will decline. This suggests that a reduction in effectiveness of tutoring in terms of its quality upgrading power may backfire, and thus cause an increase in demand for tutoring, to the dismay of naive policy-makers.

Putting all these together, it would be really surprising if the secondary school equalization measure had achieved its objective of reducing private tutoring since most of the theoretical results are pointing to the other direction.

Where should we go from here to resolve the private tutoring problem, or to be more specific, the education problem in general? The analysis and discussion above clearly suggest that we should now go back to the fundamentals and to the basics. Without going directly to the causes of the problem, trying to revamp the structure with mere allopathic treatments such as additional reforms in entrance exams would only elongate or aggravate the current deplorable situation. Our theoretical analysis clearly proves that they are all doomed to fail. Using the metaphor once more, they all amount to trying to push down a balloon underneath the water, probably with one finger. If one finger doesn’t work, two fingers may be tried, and later three fingers or the whole palm. Thereby, you may be able to hold it temporarily, but surely not for long. It is not, after all, a matter of the number of fingers, but a matter of a physical law; the fundamentals.

The regulation on tuition and enrollment quotas must be dismantled com-
pletely, the sooner, the better. This is the first fundamental we have to restore in the educational market. It will remove the disparity between the value and the price. It will also make possible the differentiated pricing of various school qualities. Once the proper and free pricing is restored, almost all problems associated with private tutoring will be gone, because much of private tutoring itself will wither away and cease to exist. Everybody who wants to have higher education will get it by paying its cost. On a societal level, we will have just the optimum amount of educational service, no less, no more. Moreover, the seemingly excessive zeal for the first-class schools will also disappear as the appropriately differentiated pricing is instituted. Somebody who wants a higher quality product will pay more, while others who want modest quality will pay less because they have other better things to have.

Authorities do not need endeavor to develop and come up with a new reform in student selection methods or school equalization measures. Colleges and universities will develop their own best methods and procedures, to attract their own best students by ability and concentration, and thereby specialize themselves in their best niches. Rather than trying to patch up the current school equalization measure, the authority has to revoke it completely. It does more harm than good in regards to private tutoring. Tutoring aside, it defies the fundamental organizing principle of educational institutions. Grouping together the students with comparable abilities is the cost-efficient method of educating students. If one does not mix different graders in one classroom, public authorities should not do that either by the equalization measure, which is the same thing.

Finally, we should always resist the temptation to select a person or a group of persons for differential economic favors and privileges, because it will certainly lead to waste, inefficiency, cliquism, corruption, degradation of human mind, and thereby impeded social progress. Nature never breeds and nurtures privileges. Only man does, thus defiling the world, unwittingly or deliberately.

All in all, we have fortunately good solutions at our hands to turn the long-deserted ruins into well-watered gardens, in which students, unburdened from unnecessary distortions, once again joyously study and prepare themselves for their own promising futures. Let their dreams and visions all come true in this new promised-land!
References


Steiner, P.O., “Peak Loads and Efficient Pricing,” *QJE*, March 1964, pp.54-76.


