

Campbell, Jeffrey R.; Weber, Jacob P.

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Discretion Rather than Rules: Equilibrium Uniqueness and Forward Guidance with Inconsistent Optimal Plans

Jeffrey R. Campbell* Jacob P. Weber†

September 7, 2018

Abstract

New Keynesian economies with active interest rate rules gain equilibrium determinacy from the central bank's incredible off-equilibrium-path promises (Cochrane, 2011). We suppose instead that the central bank sets interest rate *paths* and occasionally has the discretion to change them. Private agents taking future central bank actions and their own best responses to them as given reduces the scope for self-fulfilling prophecies. With empirically-reasonable frequencies of central-bank reoptimization, the monetary-policy game has a unique Markov-perfect equilibrium wherein forward guidance influences current outcomes without displaying a forward-guidance puzzle.

*Federal Reserve Bank of Chicago and CentER, Tilburg University. Jeff.Campbell@chi.frb.org

†Department of Economics, University of California at Berkeley. jacob_weber@berkeley.edu

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1 Introduction

This paper replaces the interest rate rule in the three-equation New Keynesian (NK) model with benevolent central bankers who can commit to interest rate *paths* for random terms in office. If central bankers turn over frequently enough, then the economy has a unique symmetric Markov Perfect Equilibrium (MPE) even though each one of them commits to a completely passive interest rate rule. Simple intuition explains this result: central bankers optimize in the future, private agents optimally respond to the resulting interest rates when they are set, and the private sector takes these future outcomes *as given* when making current consumption and pricing decisions. This basic logic of Markov-perfect Nash equilibrium reduces the scope for self-fulfilling prophecies to influence current choices.

Our model builds upon the quasi-commitment framework of [Roberds \(1987\)](#), which [Schaumburg and Tambalotti \(2007\)](#), [Bodenstein, Hebden, and Nunes \(2012\)](#) and [Debor-toli and Nunes \(2014\)](#) have previously applied to the NK model. In it, the central banker commits to a deterministic path for interest rates while facing a constant probability of being replaced by a successor who will herself reoptimize. The future interest rates along this path constitute the central banker’s forward guidance.

No major central bank provides complete and perfectly credible details of its policy actions for the indefinite future. In the discussion of [Campbell, Evans, Fisher, and Justiniano \(2012\)](#), Donald Kohn asserted that “no central bank to date has adopted an Odyssean commitment strategy,” and Laurence Meyer stated that 98 of the 100 market participants he interviewed understood forward-looking statements of the Federal Reserve to be forecasts rather than commitments ([Romer and Wolfers, 2012](#)). Therefore, the benchmark from which an empirically-useful consideration of forward guidance should deviate is closer to complete discretion than to perfect commitment. On the other hand, monetary policy committees adjust their policies slowly ([Blinder, 1998](#)), so their choices and forward guidance embody some commitment. Indeed, slow adjustment and long periods of inaction are natural features of any institutional setting where decisions have to be made by consensus. [Riboni and Ruge-Murcia \(2010\)](#) demonstrate how such “status quo bias” arises naturally when monetary policy committee members have heterogenous goals, and they argue that such models explain the actual behavior of central banks well. [Ruge-Murcia and Riboni \(2017\)](#) empirically validate this prediction with a natural experiment created by changes to the Bank of Israel’s governance structure. Thus committee gridlock effectively endows a central bank with some degree of commitment, even when individual members vote with discretion ([Riboni, 2010](#)).

We interpret our model’s central banker replacement as a metaphor for considerations which could lead committees of central bankers to renege on their prior commitments, and

quasi-commitment represents this commitment “technology” in an analytically tractable way. We expect no real-world central banker to recognize our model’s stochastic reoptimization within the details of her daily life, just as no real-world entrepreneur believes that he stochastically chooses when to adjust prices independently of economic conditions. In particular, our implementation of quasi-commitment abstracts from central bankers’ freedom to respond to non-contractable but relevant variables, as discussed by [Kocherlakota \(2016\)](#). Nevertheless we believe that the quasi-commitment framework represents monetary policy formation more realistically than the standard interest rate rule, which also abstracts from many observable variables used by central bankers in practice. While such rules describe observed central bank behavior conditional upon very limited information sets well, their embodiment within the NK model imposes the incredible off-equilibrium-path promises detailed by [Cochrane \(2011\)](#). *Not only are these not plausible descriptions of real-world central bank operating procedures, the canonical model’s predictions hinge upon them.*

Our work is related to many other examinations of equilibrium multiplicity in the NK model. [Adão, Correia, and Teles \(2011\)](#) gain equilibrium uniqueness with a passive interest rate rule which exactly cancels out forward looking terms in the IS curve. Unlike [Adão, Correia, and Teles](#), we have central bankers choosing deterministic paths for interest rates. [Adão, Correia, and Teles \(2014\)](#) deliver determinacy by introducing long-term debt and having the central bank manipulate bond term premiums. Our framework incorporates no deviation from the expectations theory of the term structure. Both [King and Wolman \(2004\)](#) and [Armenter \(2008\)](#) establish multiplicity of MPEs in nonlinear NK models with an inflation bias, while [Siu \(2008\)](#) shows that these results are sensitive to the specification of firms’ nominal pricing technology. We demonstrate our result within the standard linearized NK model based on the standard Calvo pricing technology, which lacks an inflation bias. Finally, [Blake and Kirsanova \(2012\)](#) showed that linearized NK models with endogenous state variables, such as the stock of government debt, can have multiple MPEs. Our model’s only endogenous state variables are the interest-rate commitments of incumbent central bankers. Empirically-relevant NK models include many endogenous state variables, such as capital and past wage and price inflation. Just as the Taylor principle is only suggestive of equilibrium determinacy in such environments ([Coibion and Gorodnichenko, 2011](#)), our results provide a useful baseline for the examination of DSGE models with quasi-commitment ([Debortoli and Lakdawala, 2016](#)).

An older game-theoretic approach to the time-consistency problem in monetary policy characterizes all subgame perfect equilibria of the game between an infinitely-lived monetary authority and the private sector ([Chari and Kehoe, 1990](#); [Stokey, 1991](#); [Chang, 1998](#); [Phelan and Stacchetti, 2001](#)). [Chari, Christiano, and Eichenbaum \(1998\)](#) label those equilibria with

outcomes inside the policy frontier “expectation traps,” and they interpret regime switches in U.S. macroeconomic performance as movements between “bad” and “good” equilibria. This approach to solving the time-consistency problem conceives of commitment as a characteristic of the solution concept: the best subgame perfect equilibrium. In contrast, we model commitment more heuristically as a technologically-given probability of the central bank keeping its promises. Therefore, we can compare economies with different amounts of central-bank commitment; which seems to us a central precondition for quantitative empirical applications.

To demonstrate our framework’s quantitative potential, we calculate an upper bound on the average duration of central bank commitment consistent with equilibrium uniqueness using the parameter values estimated by Galí and Gertler (1999) and other values employed by Schaumburg and Tambalotti (2007). This upper bound varies between three and six quarters, so the model can have both equilibrium uniqueness and nontrivial implications for forward guidance.¹ Campbell and Weber (2018) note that the standard NK model’s forward guidance employs an unspecified ability to coordinate expectations on one of many equilibria. Because our model has only one equilibrium given any interest rate path, its forward guidance does not depend on such “open mouth operations.”

In the next section, we lay the foundations for our analysis by reviewing the Ramsey planning formulation of the central bank’s problem. We then introduce quasi-commitment by *assuming* that the current central banker’s successor returns inflation and the output gap to zero with some fixed probability in each time period, as in Schaumburg and Tambalotti (2007). In Section 3, we relax that assumption by explicitly modeling the game associated with quasi-commitment. In it, an initial central banker chooses a path of interest rates while aware that she may be succeeded in each period by another central banker who will reoptimize. We show that if the the probability of reoptimization is high enough, then the resulting unique symmetric Markov perfect equilibrium coincides with the Ramsey planning allocation under quasi-commitment. Section 4 demonstrates that the quasi-commitment framework with a unique equilibrium lacks a forward guidance puzzle. Section 5 contains concluding remarks.

¹ As noted above, each central banker in our model has a *stochastic* tenure which can be arbitrarily long. The relevant modification of our model with term-limited central bankers always has a unique Markov-perfect Nash equilibrium. However, we demonstrate in Campbell and Weber (2018) that its monetary policy advice relies excessively on the forward-guidance puzzle when these term limits are the only bounds on commitment.

2 Ramsey Planning with Quasi-Commitment

Our analysis characterizes monetary policy for a standard NK economy.² The economy’s Phillips curve (PC) is

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}] + m_t \quad (1)$$

with PC intercept $m_0 \neq 0$ and $m_t = 0$ for all $t > 0$.³ Its intertemporal substitution (IS) curve is

$$y_t = -\frac{1}{\sigma} (i_t - \mathbb{E}_t [\pi_{t+1}] - i^{\natural}) + \mathbb{E}_t [y_{t+1}]. \quad (2)$$

Here i_t and i^{\natural} denote the nominal interest rate and the natural interest rate, while π_t and y_t denote inflation and the output gap. The parameters satisfy $\sigma, \kappa \in (0, \infty)$ and $\beta \in (0, 1)$. The expectation operators in (1) and (2) allow for the possibility of stochastic monetary policy.

In addition to the PCs and IS curves, we require that the output gap always lies between finite lower and upper bounds, which we denote with $\hat{y} < 0$ and $\check{y} > 0$. We motivate the upper bound on y_t with resource constraints: there is only so much that can be produced at one time from finite resources. The lower bound comes from [Campbell \(2016\)](#), who shows that price aggregation based upon CES preferences for differentiated goods imposes a lower bound on output. This equals output in a flexible-price economy with all firms currently unable to adjust their prices (and their goods) removed. Note that these upper and lower bounds can never be reached: households consume *some* leisure, and sticky-price firms continue producing. Therefore, we express them with *strict* inequalities, $\hat{y} < y_t < \check{y}$ for all t . Henceforth, we refer to these requirements on y_t as the *output feasibility constraints*. In our analysis, these bounds eliminate from consideration otherwise-possible outcomes with either exploding or collapsing output.⁴

As in [Schaumburg and Tambalotti \(2007\)](#), a sequence of central bankers sets monetary policy. The initial central banker, named 0, faces a constant hazard of replacement each period, α . If her term ends in period $t > 0$, then central banker t replaces her. She and all other central bankers face the same replacement hazard. A central banker at the beginning of her term chooses an infinite sequence of interest rates. For central banker t , we label these with i_t^t, i_{t+1}^t, \dots . If central banker t survives until period $t + j$, then the interest rate equals i_{t+j}^t .⁵ Central bankers make no other choices, and they seek to minimize the same quadratic

²See [Galí \(2008\)](#) for a derivation of these now-standard equations.

³In a stochastic environment, m_t is sometimes called the “cost-push shock.”

⁴The bounds on y_t do *not* generate equilibrium determinacy by themselves, as can be demonstrated with a basic analysis of equilibrium determinacy using (1) with $m_0 = 0$, (2), and a constant interest rate.

⁵The relevant extension of our model to a stochastic environment would have each central banker choosing state-contingent sequences of interest rates, $i_{t+j}^t(s^{t+j})$, where s^{t+j} gives the history of *exogenous* shocks up

social loss function:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \pi_t^2 + \frac{\lambda}{2} y_t^2 \right) \right].$$

Denote inflation in period $t+j$ conditional on central banker t being in power as π_{t+j}^t , so that π_t^t is the inflation that will prevail in the first period of central banker t 's term. Since $m_t = 0$ for all $t \geq 1$, the social bliss point with $\pi_{t+j}^t = y_{t+j}^t = 0$ for all $j \geq 0$ is consistent with the PCs and IS curves. [Blanchard and Galí \(2007\)](#) call this the ‘‘divine coincidence,’’ so we refer to this outcome hereafter as the divine coincidence allocation. We simplify this section’s analysis by assuming that all central bankers except the first one successfully implement this allocation. In this case, the initial central banker’s expected loss given her choices π_{0+j}^0 and y_{0+j}^0 for all $j \geq 0$ is

$$\sum_{j=0}^{\infty} (1-\alpha)^j \beta^j \left(\frac{1}{2} (\pi_j^0)^2 + \frac{\lambda}{2} (y_j^0)^2 \right). \quad (3)$$

The PCs and IS curves yield the following constraints on the initial central banker after taking expectations:

$$\pi_j^0 = \kappa y_j^0 + \beta(1-\alpha)\pi_{j+1}^0 + m_j, \quad (4)$$

$$y_j^0 = -\frac{1}{\sigma} (i_j^0 - (1-\alpha)\pi_{j+1}^0 - i^{\natural}) + (1-\alpha)y_{j+1}^0. \quad (5)$$

Furthermore, the output feasibility constraints require

$$y_j^0 \in (\hat{y}, \check{y}) \quad \forall j \geq 0. \quad (6)$$

The standard computational approach to similar Ramsey problems solves for optimal paths of inflation and the output gap constrained only by the PCs, and then backs out the required interest rates using the IS curves. This procedure applies here as well. The loss function and PCs only differ from their standard counterparts by a multiplicative discount factor adjustment, so one might (prematurely) conclude that quasi-commitment has no qualitative impact on the Ramsey planning problem. After all, the quasi-commitment solutions to this problem are just those of the perfect commitment problem with less patience. However, this is incorrect, because the same *cannot* be said of the IS curve (5). The change in the IS curve can eliminate the indeterminacy which plagues the standard model with perfect commitment.

To see how this is so, we analyze this problem in terms of instruments instead of allocations. Plug (4) into (5) to obtain a second order difference equation in terms of π_j^0 with the

to and including $t+j$.

forcing function $x_j^0 \equiv -\frac{\kappa}{\sigma}(i_j^0 - i^{\natural}) + m_j - (1 - \alpha)m_{j+1}$,

$$\pi_j^0 - (1 - \alpha) \left(1 + \beta + \frac{\kappa}{\sigma}\right) \pi_{j+1}^0 + \beta(1 - \alpha)^2 \pi_{j+2}^0 = x_j^0. \quad (7)$$

Trivially, we can create a system with a unique solution by setting α to one. This would leave the central banker with no ability to commit to future policy actions, as in [Blake and Kirsanova \(2012\)](#). On the other hand, if we set α to zero, (7) becomes the standard difference equation governing inflation with commitment to a completely passive monetary policy rule. This has a continuum of solutions. Does an intermediate case exist where the initial central banker can commit to future policy with some credibility, but the system still has a unique feasible solution given any non-explosive $\{x_j^0\}_{j=0}^{\infty}$?

To answer this in the affirmative, first note that the characteristic polynomial for (7) has roots equal to

$$\psi, \varphi = \frac{(1 + \beta + \frac{\kappa}{\sigma}) \pm \sqrt{(1 + \beta + \frac{\kappa}{\sigma})^2 - 4\beta}}{2\beta(1 - \alpha)}. \quad (8)$$

Label the roots so that $\psi > \varphi$. When $\alpha = 0$, then $\psi > 1/\beta$ and $\varphi < 1$. Obviously, both roots are increasing in α . If

$$\alpha > \alpha^* \equiv \frac{-(1 - \beta + \frac{\kappa}{\sigma}) + \sqrt{(1 + \beta + \frac{\kappa}{\sigma})^2 - 4\beta}}{2\beta} \quad (9)$$

then $\varphi > 1$. In this case, we can express the set of solutions to (7) as

$$\pi_j^0 = a_0 \varphi^j + b_0 \psi^j + \sum_{l=0}^{\infty} \varphi^{-l} \sum_{n=0}^{\infty} \psi^{-n} x_{j+n+l}^0. \quad (10)$$

Since extreme choices for $i_t^0 - i^{\natural}$ can obviously require y_t^0 to violate the output feasibility constraints, we impose an exogenous bound on these choices by bounding x_t^0 .

$$|x_t^0| < \max\{-\hat{y}, \check{y}\} \frac{\kappa(1 - \psi^{-1})(1 - \varphi^{-1})}{1 - \beta(1 - \alpha)} \quad (11)$$

This bound guarantees that the solution to (10) with $a_0 = b_0 = 0$ satisfies the output feasibility constraints. All other solutions for π_j^0 diverge. There is nothing *per se* inadmissible about such paths for inflation, but when combined with the PCs they imply that the output gap's path *conditional on the initial central banker remaining in office* violates the output feasibility constraints.⁶ Thus, we have

⁶Note that the explosive solutions with $b_0 = 0$ and $a_0 \neq 0$ do *not* display exploding paths for the *expected*

Proposition 1. *If $\alpha > \alpha^*$ and, $\{x_t^0\}_{t=0}^\infty$ satisfies (11), then there exists exactly one pair of sequences for π_j^0 and y_j^0 which satisfy (4), (5) and (6). In this pair, π_j^0 is that from (10) with $a_0 = b_0 = 0$.*

Proposition 1 matters because it implies that the initial central banker can minimize her loss by solving her Ramsey problem, calculating the interest rates consistent with its solution and the IS curves, and announcing them as a deterministic path.⁷

Qualitatively, replacing the model’s interest rate rule with an optimizing central banker subject to quasi-commitment can eliminate the model’s equilibrium multiplicity and thereby provide clearer guidance to real-world monetary policy makers. To assess its quantitative implications for the equilibrium set, consider the quarterly calibration of [Schaumburg and Tambalotti \(2007\)](#). Given their values of $\kappa = 0.1, \beta = 0.99$ and $\sigma = 2/3$; $\alpha^* \approx 0.32$. Since α^{-1} equals a central banker’s expected tenure, her commitment to her forward guidance is expected to last about three quarters if $\alpha = \alpha^*$. With any lower expected duration, we can apply Proposition 1 to show that the initial central banker can guarantee a unique outcome merely by choosing a *path* for interest rates.

[Schaumburg and Tambalotti \(2007\)](#) use a relatively high estimate of the PC’s slope. [Gali and Gertler \(1999\)](#) estimate $\kappa = 0.023$. Using this estimate for the slope, but holding the other parameters fixed yields $\alpha^* \approx 0.17$, with a corresponding maximum expected duration of about six quarters.⁸ Figure 1 provides values of α^* for each ratio $\kappa/\sigma \in [0, 1/2]$, which covers most of the plausible ratios of empirical interest. It is easy to verify that α^* always increases with κ/σ , as in the figure.⁹ By construction, $\alpha^* = 0$ if $\kappa = 0$. Thus, Proposition 1 applies even when credibility is nearly perfect if the PC is very flat. Even if we use a high estimate for κ such as the 0.1 from [Schaumburg and Tambalotti \(2007\)](#), and a low σ of 0.2 (which corresponds to an intertemporal elasticity of substitution of 5), this would yield an $\alpha^* \approx 1/2$. [Bodenstein, Hebden, and Nunes \(2012\)](#) estimate that for the U.S. in the recent period following the financial crisis, α was most likely above one half, with $\alpha \approx 0.6$ as a tentative point estimate. We conclude that quasi-commitment has quantitative promise for resolving the positive and normative problems that arise from the NK model’s equilibrium indeterminacy.

output gap, because the explosion that occurs if the current central banker remains in office indefinitely is tempered by a very low probability of doing so.

⁷Since the initial central banker’s optimal choices for $i_t^0 - i_t^d$ are proportional to m_0 , an appropriate bound on m_0 can guarantee (11).

⁸[Gali and Gertler \(1999\)](#) have an accompanying quarterly value of $\beta = 0.942$. (See the estimated equation at the top of their page 207.) Lowering β from 0.99 to this lower number makes practically no difference to our results.

⁹To develop intuition for this result, note that if $\kappa = 0$, then (4) requires any bounded solution to satisfy $\pi_t^0 = \pi_{t+1}^0 = 0$. Similarly, if $\sigma = \infty$, then (5) requires $y_t^0 = y_{t+1}^0 = 0$ from bounded solutions. Equilibrium multiplicity only becomes possible by raising κ or by lowering σ .

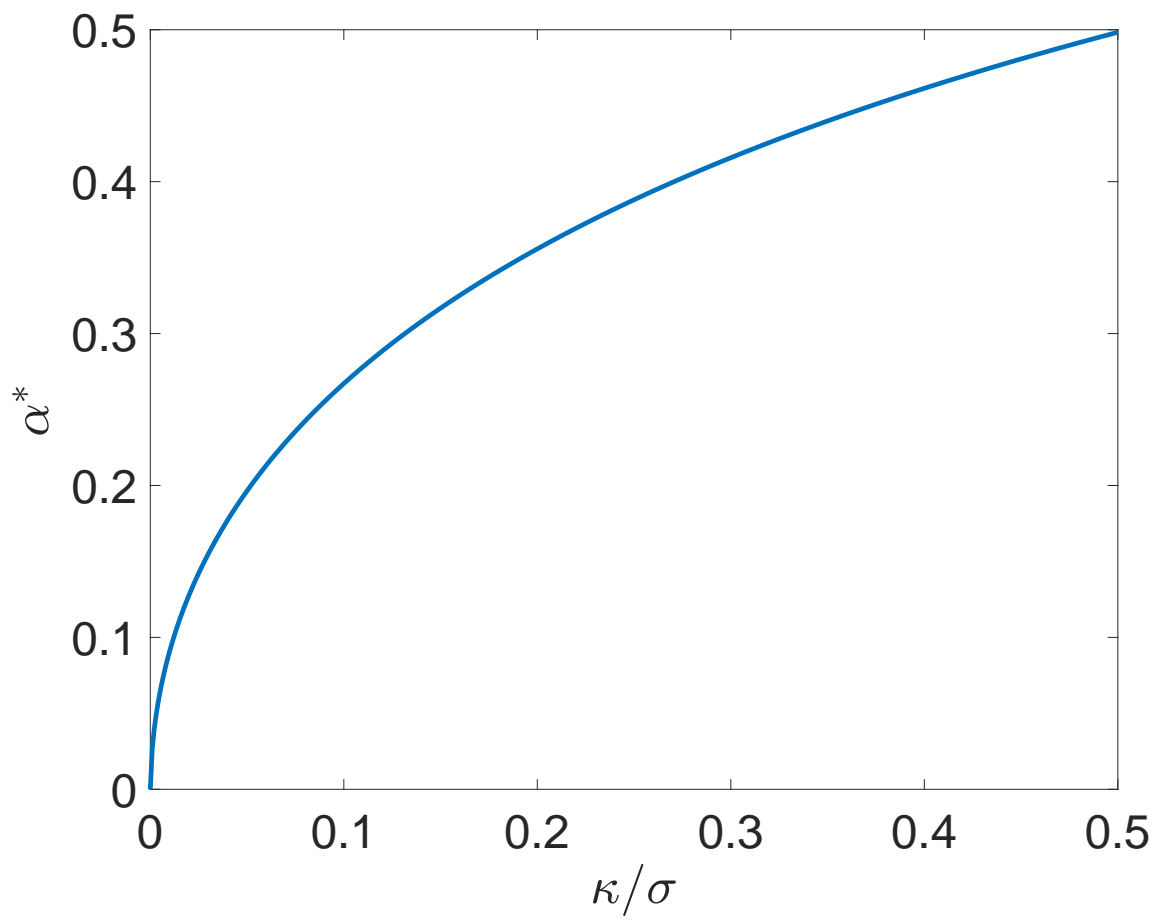


Figure 1: α^* versus κ/σ for $\beta = 0.99$

3 Quasi-Commitment Games

In this section we specify two monetary policy games which endogenize future central bankers' choices. In both games, central bankers make one-time choices during their initial periods in office and seek to minimize a common social loss function. In the first game, each central banker directly chooses the inflation and output sequences that will occur during her tenure constrained only by the sequence of PCs. We call this the *allocation* game, and it is the foundation of [Debortoli and Lakdawala's \(2016\)](#) empirical DSGE analysis. When m_0 is small, it has a unique symmetric Markov-perfect equilibrium (MPE) *regardless of the frequency of central bankers' replacement*.¹⁰ We view this game as a theoretically illuminating baseline for the second game, which we call the *interest rate* game. In it, each central banker chooses a path of interest rates that will be set during her tenure, and the private sector chooses values for inflation and output each period. In this game, any private sector strategy that depends solely on the current and future interest rates promised by the current central banker and the remaining Phillips-curve intercepts can in principle be part of an MPE. We prove that this game has a unique symmetric MPE when the hazard of central banker replacement is high enough. Therefore, the equilibrium uniqueness results of Section 2 do not depend on the simplification of fixing economic outcomes after the initial central banker's tenure.

3.1 The Allocation Game

The initial central banker begins play by choosing sequences of inflation and output. So long as she is in office, these chosen sequences govern the realizations of actual inflation and output. She faces a constant probability of her term ending each period. If this occurs in period t , then central banker t replaces her and makes her own choices of inflation and output. Her choices must satisfy the output feasibility constraints and the PCs taking her possible successors' choices as given:

$$\pi_{t+j}^t = \kappa y_{t+j}^t + \beta(1 - \alpha)\pi_{t+j+1}^t + \beta\alpha\pi_{t+j+1}^{t+j+1} + m_{t+j}. \quad (12)$$

Finally, all central bankers seek to minimize the common social-loss function given above. Since this game reduces to the Ramsey problem with perfect commitment when $\alpha = 0$, we henceforth require $\alpha > 0$.

In an MPE to this game, each central banker's choices depend only on current and future

¹⁰[Blake and Kirsanova \(2012\)](#) demonstrate that there exists a unique stationary Markov-perfect equilibrium in the special case of this game with $\alpha = 1$. Our analysis builds on theirs by considering smaller values of α .

values of m_t ; and in a *symmetric* MPE, two central bankers facing exactly the same future values of m_t make the same choices. Since all central bankers but the initial one face the same constant sequence $m_t = 0$, a symmetric MPE consists of two pairs of sequences: one for the initial central banker's choices of inflation and the output gap and one for all of her possible successors' choices.

Begin the allocation game's analysis by calculating successor central banker t 's desired initial inflation rate, π_t^t , given the inflation rates that will prevail under all of her possible successors, $\pi_{t'}^{t'}$. Since $t \geq 1$, straightforward calculation yields the best-response curve

$$\pi_t^t = \frac{\alpha\beta\theta}{\alpha\beta\theta + 1 - \beta\theta} \pi_{t'}^{t'}, \quad (13)$$

where $\theta \in (0, 1)$ solves

$$1 - \left(1 + \frac{\kappa^2}{\lambda} + \beta(1 - \alpha)\right) \theta + \beta(1 - \alpha)\theta^2 = 0.$$

This best-response curve's only fixed point is $\pi_t^t = 0$, so each successor central banker can and will achieve the divine coincidence allocation. Given this necessary outcome, the initial central banker minimizes the loss function in (3) subject to the sequence of PCs in (4). Since this loss function is strictly convex and this constraint set is convex, the initial central banker's optimal choices are unique. It is not hard to show that these choices are proportional to m_0 , so we can bound m_0 to ensure that the chosen output gaps satisfy the output feasibility constraints. The divine coincidence allocation implemented by all other central bankers satisfies these constraints automatically, so we have

Proposition 2. *There exists an $\tilde{m} \in \mathbb{R}^+$ such that if $|m_0| < \tilde{m}$, then the allocation game has one and only one MPE.*

3.2 The Interest Rate Game

When $\alpha = 1$, the current central banker and private agents take *all* future interest rates and macroeconomic outcomes as given. Therefore, the current central banker's single interest rate choice uniquely implements her chosen outcome. The connection between interest rates and macroeconomic outcomes can be looser when $\alpha < 1$, so we examine next a game in which central bankers only control interest rates. For this, we add continua of infinitely-lived households and firms, and we refer to these new players collectively as the "private sector." They inhabit the standard NK economy examined above, and their actions determine inflation and the output gap after any interest rate choices made by a central banker.

Households derive utility from consumption and leisure, and firms seek to maximize their stock market values. Instead of providing detailed descriptions of these private objectives, we follow [Atkeson, Chari, and Kehoe \(2010\)](#) by requiring that the stochastic processes for π_t and y_t satisfy the sequences of PCs and IS curves.

All central bankers seek to minimize the same expected loss as in the allocation game. The initial central banker begins play by choosing and publicly announcing a sequence of interest rates that will prevail for as long as she is in office. We denote her choice for time t with i_t^0 . After setting this path, she makes no further choices. If her term ends at date t , central banker t replaces her and makes her own once-and-for-all public choice of current and future interest rates, $i_t^t, i_{t+1}^t, i_{t+2}^t, \dots$

A symmetric MPE for the interest-rate game collects four objects: an interest rate path for the initial central banker, another for all successor central bankers, and mappings from the continuation of the current central banker's interest rate path and the continuation of the sequence of PC intercepts into outcomes for the current inflation rate and output gap. These must jointly satisfy the PCs, the IS curves, and the output feasibility constraints for any possible interest rate path. Furthermore, the two chosen interest rate paths must minimize the expected loss for the initial and successor central bankers given the private-sector's mappings and the successor central bankers' common path.

Before proceeding with our analysis, we need to highlight a technical difficulty. First, if $\alpha \leq \alpha^*$, then *all* of the sequences for π_{t+j}^t and y_{t+j}^t which satisfy the PCs and IS curves *are backward looking*. That is, π_{t+j}^t and y_{t+j}^t depend nontrivially on past interest rates, past PC intercepts and past expectations of inflation. This is because self-fulfilling prophecies require agents to believe that their own future behavior will depend on payoff-irrelevant variables. In this case, it is impossible to find mappings from the continuations of interest rates and PC intercepts into the current inflation rate and output gap. Therefore, we ensure that the set of private sector Markov strategies satisfying the PCs and IS curves is non-empty by henceforth requiring $\alpha > \alpha^*$.¹¹

With this assumption in place, we may proceed with the interest rate game's equilibrium analysis. First, construct a symmetric MPE. For this, first set the initial central banker's interest rate path to that which satisfies the sequence of IS curves evaluated at her chosen inflation rates and output gaps in that equilibrium. Second, set the interest rate path for all successor central bankers to i^\natural always. Third, set

$$\pi_{t+j}^t = \sum_{l=0}^{\infty} \varphi^{-l} \sum_{n=0}^{\infty} \psi^{-n} (m_{t+j+n+l} - \frac{\kappa}{\sigma} (i_{t+j+n+l}^t - i^\natural)). \quad (14)$$

¹¹See Footnote 1 for a brief discussion of the model with exogenous term limits on central bankers, which *always* has a symmetric MPE.

This is the appropriate modification of (10) which selects the unique non-explosive solution to the PCs and IS curves given that all central bankers with names exceeding t successfully implement the divine-coincidence allocation. Fourth and finally, use the PCs to set

$$y_{t+j}^t = (\pi_{t+j}^t - \beta(1 - \alpha)\pi_{t+j+1}^t)/\kappa.$$

Given these mappings from central bank choices into private-sector outcomes, the successor central bankers' paths always achieve the divine-coincidence allocation. Therefore, all successor central bankers' choices minimize their expected losses. Furthermore, the initial central banker achieves her minimum loss. Finally, the private sector mappings satisfy the PCs and IS curves by construction. Therefore, this collection forms one symmetric MPE.

To see that this is the *only* symmetric MPE, note that the PCs, the IS curves, and the output feasibility constraints together require

$$\begin{aligned} \pi_{t+j}^t &= \sum_{l=0}^{\infty} \varphi^{-l} \sum_{n=0}^{\infty} \psi^{-n} (m_{t+j+n+l} - \frac{\kappa}{\sigma} (i_{t+j+n+l}^t - i^t)) \\ &+ \frac{(\alpha^2 \beta + \alpha \kappa / \sigma) \pi_t^{t'} + \alpha \kappa y_t^{t'}}{1 - (1 - \alpha)(1 + \beta + \kappa / \sigma) + \beta(1 - \alpha)^2}, \end{aligned} \quad (15)$$

where $\pi_t^{t'}$ and $y_t^{t'}$ equal all successor central bankers' common initial inflation rate and initial output gap.¹² Equation (15) implies that there is one and only one inflation response from the private sector to any given interest rate path chosen by the current central banker. Therefore, each successor central banker can *guarantee* that the private sector implements her best response to $\pi_t^{t'}$ by choosing the interest rates consistent with her Ramsey planning problem's solution. In turn, this implies that the best-response relationship (13) from the allocation game is a *necessary* condition for a symmetric MPE of the interest-rate game. This requires that $\pi_t^t = y_t^t = 0$ for all $t \geq 1$. Finally, (15) also implies that the initial central banker can guarantee that the private sector implements her optimal allocation by setting the interest rates consistent with it and the IS curves. Therefore, we have

Proposition 3. *There exists an $\check{m} \in \mathbb{R}^+$ such that if $|m_0| < \check{m}$ and $\alpha > \alpha^*$, then the interest rate game has one and only one MPE. Its outcomes equal those in the allocation game's unique MPE.*

Proposition 3 demonstrates that equilibrium uniqueness is not simply an artifact of the simplifying assumptions of Section 2.

¹²Recall that $m_t = 0$ for $t \geq 1$ by assumption. If instead future PC intercepts were nonzero, then the expression in (15) would require modification.

4 The Forward-Guidance Puzzle

The interest rate game produces a unique prediction given values of its primitives. Moreover, that prediction conforms to the conventional, old-Keynesian wisdom that higher interest rates today *lower* contemporaneous inflation as seen in (15), arguably unlike in the canonical NK model (Cochrane, 2017).

Additionally, the model makes intuitive predictions about changes in long-dated forward guidance. Similar changes to forward guidance have challenged empirical DSGE models used at central banks (Del Negro, Giannoni, and Patterson, 2015). In those frameworks, a monetary authority promising a very small change to the policy rate delayed for a very long time can substantially influence current output and inflation. Furthermore, postponing the promise’s implementation *raises* its effects on current outcomes. This is the forward-guidance puzzle of Carlstrom, Fuerst, and Paustian (2015). Studies of actual central bank forward guidance provide no evidence supporting this puzzling prediction (Campbell, Fisher, Justiniano, and Melosi, 2016). Moreover, the forward-guidance puzzle impedes applying the NK model to calculate appropriate future interest rate policy and to forecast economic outcomes conditional upon that policy’s adoption, as FOMC participants do for the Survey of Economic Projections.¹³

In this section, we demonstrate that the quasi-commitment model with $\alpha > \alpha^*$ lacks a forward-guidance puzzle. To see this, first consider the canonical model’s forward-guidance puzzle. With an active interest rate rule and $m_t = 0 \forall t \geq 0$, the unique non-explosive equilibrium achieves the divine-coincidence allocation: $\pi_t = y_t = 0$. Suppose that the central banker follows such an active interest rate rule on and after some date $\tau > 0$, but the interest rate is *pegged* to an exogenous path $i_0, i_1, \dots, i_{\tau-1}$ before then. Define *the standard forward guidance experiment* as a reduction of $i_{\tau-1}$ announced in period 0. Figure 2 reports the results of such an experiment using parameter values from Schaumburg and Tambalotti (2007). Here, $\tau = 8$ and the interest rate is lowered five basis points in period 7. This has very modest contemporaneous effects on output and inflation, but the impacts on outcomes when the forward guidance is announced in period 0 exceed these by an order of magnitude. It is not hard to show analytically that the impacts of a given change in $i_{\tau-1}$ on y_0 and π_0 *explode* as $\tau \rightarrow \infty$.

The same forward-guidance experiment conducted under quasi-commitment comes out quite differently. Figure 3 plots the results for an experiment with the same parameter values used to construct Figure 2 but with $\alpha = 0.4$. In this experiment, the initial central banker announces an arbitrary reduction of i_7^0 by $5/(1 - \alpha)^7$ basis points. By construction, this has

¹³See Campbell and Weber (2018) for a brief but thorough exploration of this point.

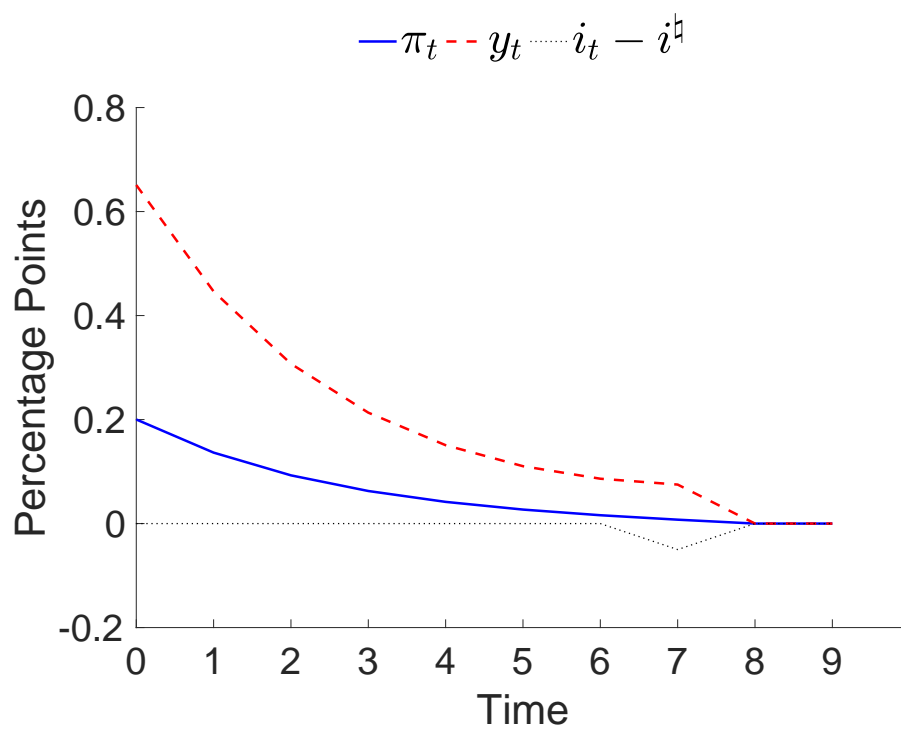


Figure 2: The Forward Guidance Puzzle

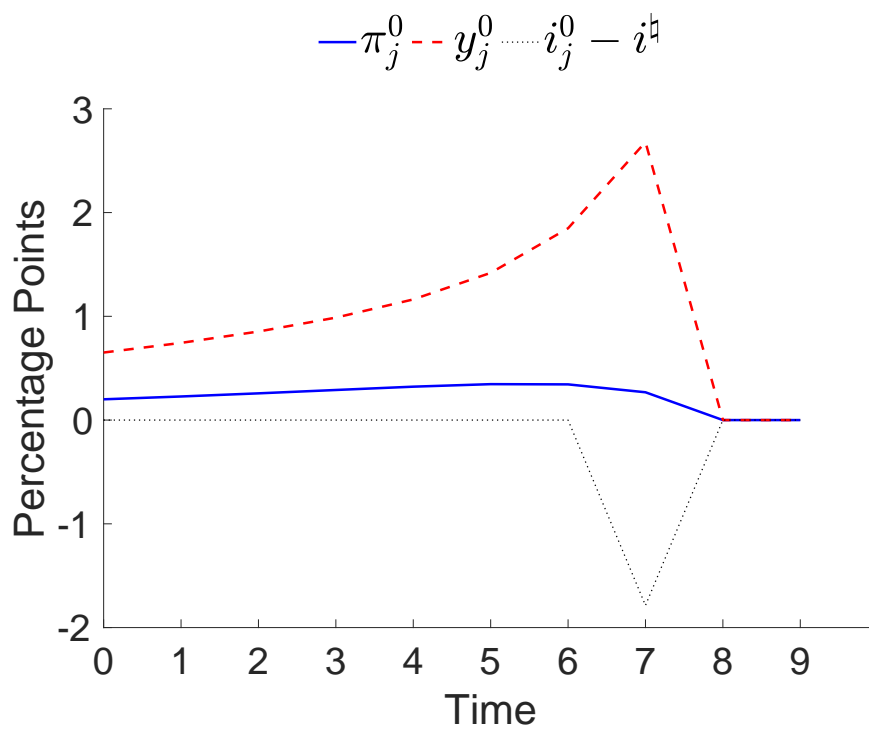


Figure 3: Forward Guidance with Quasi Commitment

the same impact on $\mathbb{E}[i_7]$ as a certain five basis point reduction. Here, the conditional expectations of inflation and output both *increase* as the date of forward-guidance implementation becomes closer. We can show analytically that the effects of a given announced change to $i_{\tau-1}^0$ on y_0 and π_0 go to zero as $\tau \rightarrow \infty$. Thus, the NK model with quasi-commitment can provide a framework for formulating optimal forward guidance *without a forward-guidance puzzle*.¹⁴

5 Conclusion

The standard NK model provides no unique prediction for macroeconomic outcomes given a particular *path* of interest rates. In this paper, we have demonstrated that the quasi-commitment NK model produces a unique prediction from a promised (but imperfectly credible) path for interest rates if the probability of monetary policy reoptimization is high enough. Thus, the model can be used for forecasting and for policy formation without assuming that central bankers can make incredible promises to follow active interest rate rules. Of course, there exist other well-known options for either eliminating or mitigating the forward guidance puzzle. [Gabaix \(2016\)](#) shows that introducing rationally inattentive households introduces a discounting of future outcomes in the IS curve very similar to that in (5). [Angeletos and Lian \(2016\)](#) and [Barrdear \(2017\)](#) mitigate the forward guidance puzzle by relaxing the assumption of common knowledge across agents, and [Del Negro, Giannoni, and Patterson \(2015\)](#) do so by introduce discounting of future outcomes in the IS curve with a perpetual youth lifecycle model. Our analysis complements these approaches by realistically relaxing the assumption of perfect monetary policy commitment. This requires no change to the standard model’s market structure or information-processing technology.

Central bankers interest in forward guidance rises substantially when the effective lower bound (ELB) constrains interest rates. For the special case with $\alpha = 1$, [Armenter \(2016\)](#) demonstrates that our results do *not* immediately carry over to such an environment. Specifically, the ELB creates a second equilibrium reminiscent of the liquidity trap found by [Benhabib, Schmitt-Grohé, and Uribe \(2001\)](#) in the NK model with an active interest rate rule. A more thorough exploration of the quasi-commitment interest rate game with an ELB is on our research agenda.

¹⁴In a particular sense, this result arises directly from the initial central banker’s limited commitment. If we allow the promised interest rate change to grow as $\tau \rightarrow \infty$ so that the change in *expected* future interest rates is held constant, the forward-guidance puzzle reappears. Nevertheless, the inability of current central bankers to have arbitrarily large impacts on current macroeconomic outcomes without arbitrarily large promised changes to future interest rates makes the model with quasi-commitment more useful for the calculation of appropriate interest rate policy and its consequences.

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