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On interest rate policy and asset bubbles

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On Interest Rate Policy and Asset Bubbles*

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Abstract

In a provocative paper, Galí (2014) showed that a policymaker who raises interest rates to rein in a potential bubble will only make a bubble bigger if one exists. This poses a challenge to advocates of lean-against-the-wind policies that call for raising interest rates to mitigate potential bubbles. In this paper, we argue there are situations in which the lean-against-the wind view is justified. First, we argue Galí's framework abstracts from the possibility that a policymaker who raises rates will crowd out resources that would have otherwise been spent on the bubble. Once we modify Galí's model to allow for this possibility, policymakers *can* intervene in ways that raise interest rates and dampen bubbles. However, there is no reason policymakers *should* intervene to dampen the bubble in this case, since the bubble that arises in Galí's setup is not one that society would be better off without. We then further modify Galí's model to generate the type of credit-driven bubbles that alarm policymakers, and argue there may be justification for intervention in that case.

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Introduction

In a provocative paper, Galí (2014) argued that a policymaker who raises interest rates out of concern about the possibility of an asset bubble – a policy known as "leaning against the wind" – will paradoxically only make a bubble larger if one is present. This result stands in sharp contrast to conventional wisdom on how interest rate policy can be used as a tool for financial stability. Although central banks have long been reluctant to use interest rates to combat asset bubbles, this was not out of concern that raising rates would exacerbate bubbles. To the contrary, policymakers viewed interest rates as an effective but blunt tool that would not only mitigate bubbles but also affect economic activity and inflation in ways that may not be desirable, a view formalized in the work of Bernanke and Gertler (1999). Galí's result represents a more fundamental challenge to the lean-against-the-wind view, since it suggests that even if it were desirable to act against bubbles, raising interest rates would be counterproductive for achieving this goal.

In this paper, we argue that these results notwithstanding, there are circumstances in which the lean-against-the wind view might be justified. We begin by showing that Galí's framework precludes certain channels that allow raising interest rates to dampen bubbles. Galí was certainly aware of this possibility, noting in the conclusion to his paper that while he assumes agents are rational, policymakers in practice may be concerned about bubbles driven by "irrational exuberance" against which raising rates might be effective.[1] We argue Galí's model precludes a more elementary reason for why raising rates can mitigate bubbles that applies even with rational agents. In particular, we show that his framework rules out the possibility that raising interest rates crowds out resources that would otherwise go to the bubble. Essentially, Galí considers an economy in which a higher interest rate is associated with higher equilibrium savings by households. Since Galí assumes the bubble is the only asset agents can use to save, and since this asset is available in fixed supply, then the price of the asset must rise when agents attempt to save more. A higher interest rate draws more resources towards the bubble asset rather than crowd them out.

We demonstrate that with modest modifications to an economy that is qualitatively similar to Galí's, interventions that raise interest rates can crowd out resources from the bubble asset. For example, if we allow agents to save using either the bubble asset or government bonds, as opposed to the bubble alone, we can construct policy interventions that raise interest rates and dampen the bubble. In particular, if the government issues more debt in a way that leaves households with sufficiently fewer resources to buy the bubble asset, interest rates will rise and the price of the bubble asset will fall. As another example, if higher interest rates induce a temporary fall in output, as is true of many models that feature nominal price rigidity, agents will reduce their savings as interest rates rise. In that case, agents would spend less on the bubble asset as their income temporarily contracts, and the bubble in the asset market would dampen.

One of the contributions of this paper, then, is to identify when leaning-against-the-wind policies can be effective against bubbles. This not only offers a contrast to Galí's result that raising interest rates is

[1] Indeed, the phrase was notably coined by a central banker in a speech on policy; see Greenspan (1996).

counterproductive, but provides microfoundations for the view common among policymakers that raising rates helps mitigate bubbles.[2] Developing such a framework offers several insights on the use of interest rate policy to combat bubbles. For example, the channel we identify suggests policymakers seeking to use interest rate policy to combat bubbles should not simply be satisfied if their intervention raises real interest rates, but should also seek to gauge whether the rise in interest rates is accompanied by lower savings or a change in the portfolio of assets held by the public that is consistent with crowding out. Our results also suggest that it may be difficult to dampen asset bubbles without inducing a recession, since a reduction in the resources agents earn and can use to buy assets may be essential for dampening the bubble. In one of our examples, the only reason the bubble is mitigated is because economic activity falls.

The case for leaning-against-the-wind policies rests not only on the argument that higher interest rates can dampen asset bubbles, but that dampening bubbles is beneficial in the first place. However, the bubble that arises in our adaptation of Galí's model is not one that society is better off avoiding, and its existence does not necessitate intervention. Nor does that setup speak to the type of credit-driven bubbles that seem to most alarm policymakers, i.e., bubbles fuelled by borrowing that can trigger defaults and financial crises if and when they burst. In the second part of the paper, we further modify the model to allow for both credit and the possibility of bursting bubbles. These modifications give rise to a different type of bubble that more closely resembles the credit-driven bubbles policymakers express concern about. We confirm that for this type of bubble, there may be interventions that raise interest rates and crowd out resources that would have otherwise been spent on the bubble. If the bubble asset is available in fixed supply, such interventions still fail to produce a Pareto improvement. But when the supply of the bubble asset is variable, depressing bubbles may discourage the creation of more bubble assets and reduce the amount of aggregate risk in the economy. Thus, policymakers should aim not to correct asset prices *per se* but to curb excessive creation of bubble assets. This logic would suggest that the case for intervention is weaker for bubbles on assets whose stock cannot be easily augmented, like land or the paintings of a deceased artist, than for bubbles on assets like housing or commercial real estate whose supply can respond to changes in prices.

The paper is organized as follows. In Section 1, we lay out our model and use it to demonstrate Galí's result that when agents can only save using the bubble asset, a higher real interest rate will be associated with a larger bubble. In Section 2, we allow agents to save using either a bubble asset or government debt, and show that there exist policy interventions that drive up interest rates and reduce the bubble. In Section 3 we allow agents to hold the bubble asset, bonds, and money, and show that there exist monetary interventions that raise interest rates and reduce the bubble. Since neither of these settings involve borrowing against bubble assets or bursting bubbles, we modify the model in Section 4 to allow for credit-driven bubbles that can burst and trigger default. We argue that in this case, interventions that raise rates can still serve to dampen bubbles, and that these models more closely correspond to the type of bubbles policymakers are worried about. We then conclude in Section 5.

[2] In independent work, Dong, Miao, and Wang (2017) offer a different model in which a move to raise rates can dampen bubbles. They study a model with credit frictions in which agents hold bubbles as collateral to relax the constraints they face. Monetary policy impacts credit market frictions, which in turn affects bubble assets. See also related work by Ikeda (2017).

1 Reproducing Galí's Result

We begin by replicating Galí's result. The environment we present differs along some dimensions from the one Galí described, although both are variants of Samuelson (1958). Our framework is analytically more convenient, and we will argue below that our framework captures the key features behind Galí's results.

Time is discrete and indexed $t = 0, 1, 2, ...$ There is a single consumption good available at each date. This good can be stored at no cost, allowing agents to convert a unit of consumption goods at date t into a unit of consumption goods at date $t + 1$.

We assume overlapping generations of agents with two-period lives. Agents are risk neutral. They derive utility only from the goods they consume in the second period of life. Specifically, the utility of agents born at date t from consumption c_t and c_{t+1} at dates t and $t + 1$, respectively, is given by

$$u(c_t, c_{t+1}) = c_{t+1} \tag{1}$$

The cohort born at date t is endowed with $e_t > 0$ units of consumption goods when young and none when old. The amount of resources successive cohorts are endowed with grows over time according to

$$e_t = (1 + g)^t e_0 \tag{2}$$

The only problem agents face is how to convert the goods they are endowed with when young into goods they can consume when old. One option is to store their endowment. However, they might be able to do even better by exchanging their endowment for assets when young and then back into goods when old.

Galí essentially assumes there is only one asset that agents can trade.[3] We begin with this case as well, although we eventually relax this assumption. The asset is available in fixed supply, normalized to 1, and yields a constant dividend flow of $d \geq 0$ consumption goods per period. Galí assumed $d = 0$, rendering the asset intrinsically worthless. For now, we also assume $d = 0$, although we will eventually allow $d > 0$.

The asset is initially endowed to the old at date 0. Let p_t denote the price of one unit of the asset at date t, measured in consumption goods. Given our normalization, p_t is also the asset's total market value.

Asset market clearing in our overlapping generations economy requires that at each date t, the old agree to sell all of their asset holdings to the young. This ensures all assets will be owned by someone each period. Hence, an equilibrium is a path of prices $\{p_t\}_{t=0}^{\infty}$ such that for each date t, the old are willing to sell their asset holdings for a price p_t and the young are willing to buy them at this price. In principle, $\{p_t\}_{t=0}^{\infty}$ can be a sequence of random variables. However, we restrict attention to deterministic price paths.

[3] More precisely, Gali allows agents to borrow and lend, so in principle agents can also hold privately issued debt. But since only the young would ever borrow or lend and young agents are identical, there is no borrowing or lending between agents in equilibrium. In Section 4 below we introduce within-cohort heterogeneity to allow private debt to circulate in equilibrium.

Let r_t denote the rate of return that those who buy the asset at date t anticipate to earn from it in equilibrium. Since we are assuming the asset yields no dividends, the return to buying the asset at date t is just the rate at which the price of the asset grows between dates t and $t+1$, i.e.,

$$1 + r_t = \frac{p_{t+1}}{p_t} \tag{3}$$

We refer to r_t as the real interest rate. The presence of storage implies $r_t \geq 0$ in equilibrium. Otherwise, the asset market would fail to clear as the young at date t would prefer storage to buying the asset.

To characterize the equilibrium paths $\{p_t\}_{t=0}^{\infty}$, consider first the possibility that the real interest rate r_t at date t implied by a candidate equilibrium path of asset prices is positive. In this case, the cohort born at date t would strictly prefer buying the asset to storage. Since we normalized the supply of the asset to 1, and since agents will trade all of their endowment for the asset, it follows that

$$p_t = e_t$$

Hence, in any period t in which the interest rate $r_t > 0$, the price of the asset is uniquely determined.

If the real interest rate were instead equal to zero at date t, i.e., $r_t = 0$, the young would be indifferent between storing their endowment and using it to buy the asset. While the equilibrium price p_t would no longer be unique, there is still something we can say about the path of prices in this case. Specifically, if $r_t = 0$, then (3) implies $p_{t+1} = p_t$. Since $p_t \leq e_t$, then the fact that $r_t = 0$ implies the price in the next period p_{t+1} will necessarily be less than the endowment of agents, since

$$p_{t+1} = p_t \leq e_t < e_{t+1}$$

But this implies $r_{t+1} = 0$, since if agents at date $t+1$ do not use all their endowment to buy the asset, storage must not be dominated. Hence, a zero real interest rate is absorbing: Once the real interest rate falls to 0, it will remain there indefinitely. Moreover, since the price of the asset grows at the rate of interest, it follows that once $r_t = 0$, from then on the price of the asset will remain equal to its value at date t.

The above observations allow us to characterize all deterministic equilibrium price paths for our economy. Any such equilibrium can be described in terms of a cutoff date t^* such that before t^* we have $r_t > 0$ and $p_t = e_t$, and after t^* we have $r_t = 0$ and p_t stops growing. At t^*, the price of the asset can assume any value between e_{t^*-1} and e_{t^*} if we adopt the convention that $e_{-1} \equiv 0$. Formally,

Proposition 1: Suppose $d = 0$. A deterministic path $\{p_t\}_{t=0}^{\infty}$ is an equilibrium iff there exists a cutoff date t^* with $0 \leq t^* \leq \infty$ such that p_{t^*} can assume any value in $[e_{t^*-1}, e_{t^*}]$, and for $t \neq t^*$, we have

$$p_t = \begin{cases} e_t & \text{if } t < t^* \\ p_{t^*} & \text{if } t > t^* \end{cases} \tag{4}$$

Figure 1 illustrates some typical price paths implied by Proposition 1. The equilibria of this economy can be indexed by the asset price p_{t^*} at the cutoff date t^*, which is also the asymptotic price of the asset

$\lim_{t\to\infty} p_t$. That is, for any $\rho \geq 0$, there exists a unique equilibrium for which $\lim_{t\to\infty} p_t = \rho$. When $\rho \leq e_0$, the threshold date $t^* = 0$. For $\rho > e_0$, the threshold t^* is the value of t^* for which $e_{t^*-1} < \rho \leq e_{t^*}$.

In order to determine whether the asset in this economy should be viewed as a bubble, we first need to define a fundamental value for the asset to which we can compare its price. We follow the common practice of defining the fundamental value of the asset f_t as the value of the dividends the asset is expected to generate, properly discounted to date t. When $d = 0$, what discount rate we use is irrelevant as f_t would equal 0 for any discount rate. We will refer to an asset as a bubble whenever $p_t \neq f_t$, i.e., when the asset trades at a price that deviates from its fundamental value, and we define the size of the bubble as

$$\Delta_t \equiv p_t - f_t \tag{5}$$

Since the fundamental value f_t of an intrinsically worthless asset is 0, for such an asset $\Delta_t = p_t$. That is, the bubble and the price of the asset are the same when $d = 0$. Proposition 1 establishes that equilibria with $p_t > 0$ can occur in our economy, meaning we can use the model to study bubbles.

The specific question we are interested in is how changing the path of interest rates $\{r_t\}_{t=0}^{\infty}$ would affect the size of the bubble Δ_t. At this point, it isn't obvious whether this is even a well-posed question. First, interest rates are endogenous in our setup: The path of interest rates is directly implied by the path of asset prices $\{p_t\}_{t=0}^{\infty}$ from (3), and the latter are equilibrium objects. How can we talk about a policymaker changing interest rates? And even if we could settle this question, recall that our model exhibits multiple equilibria. Can we make any clear predictions about the size of the bubble for a given interest rate path?

Galí faced these same questions in his paper. He began his discussion with a partial equilibrium model in which he took the interest-rate path $\{r_t\}_{t=0}^{\infty}$ as exogenous. Condition (3) provides a first-order difference equation that the path of asset prices $\{p_t\}_{t=0}^{\infty}$ must satisfy. Given a path for interest rates, we can use this equation to determine the path of equilibrium p_t up to an initial price p_0. But since the model exhibits multiple equilibria, the initial price p_0 is indeterminate; it can assume any value between 0 and e_0. To compare asset prices under two interest rate paths, we need a way to assign an initial price p_0 to each path. Galí initially suggests requiring p_0 to be the same for the two paths. He argued this rules out the case where policy affects the bubble indirectly through some "indeterminacy" channel, i.e., where asset prices change because we jump from one equilibrium to the other as we switch from one exogenous interest rate path to another. If we restrict p_0 to be the same for both interest rate paths, then the fact that p_0 starts at the same point and p_t grows at the rate of interest implies p_t will be higher after date 0 when interest rates are higher. Higher interest rates are associated with larger asset bubbles.

The above argument might give the impression that Galí's result rests on an arbitrary (even if plausible) assumption about initial conditions for a given interest rate path, and that alternative assumptions would lead to different conclusions. However, Galí offers his partial equilibrium analysis only as motivation, and ultimately turns to a general equilibrium model to explore how interest rates and bubbles are related. As we now argue, a general equilibrium model provides additional structure that governs p_0. This structure

renders assumptions about p_0 redundant and potentially in conflict with the model. Notwithstanding the intuition that emerges from Galí's partial equilibrium analysis, we show that the full equilibrium model yields unambiguous predictions on the relationship between interest rates and bubbles.

We develop the argument using our model and then relate it to Galí's setup. Recall that in Proposition 1 we showed that the model exhibits multiple equilibrium price paths. Each equilibrium path for asset prices is associated with a corresponding equilibrium path for interest rates. That is, we can interpret Proposition 1 to mean that there exists a set of deterministic equilibrium interest rate paths $\{r_t\}_{t=0}^{\infty}$ that can arise in our model. These interest rate paths can be similarly characterized by the cutoff date t^*, since an interest rate path $\{r_t\}_{t=0}^{\infty}$ is an equilibrium in our economy iff

$$r_t = \begin{cases} g & \text{if } t < t^* - 1 \\ 0 & \text{if } t > t^* - 1 \end{cases} \tag{6}$$

for some date $t^* = 1, 2, ...$, and the interest rate at date $t^* - 1$ assumes any value between 0 and g. This structure implies that the set of equilibrium interest rate paths can be ordered: Given any two interest-rate paths, one will be weakly higher than the other at each and every date. A policymaker can set interest rates by selecting a particular interest-rate path from the set of all equilibrium paths in (6). Indeed, this is essentially what Galí assumes when he analyzes a general equilibrium model later in his paper.[4] The effect of policy thus reduces to how asset prices and interest rates covary across the equilibria in Proposition 1: If a policymaker selects an equilibrium with higher interest rates, what asset prices would she face?

As we noted above, the deterministic equilibrium interest rate paths in our model can be ordered. Given we are considering interest rate paths that diverge at some point, we might as well allow the interest rates to differ from date 0; before the interest rate paths diverge, both would have to equal g to allow interest rates to diverge later, and so whatever happens before rates diverge is inconsequential and can be ignored. Figure 2 shows how the initial asset price p_0 and the initial interest rate r_0 vary across equilibria. As evident from the figure, any equilibrium in which the initial asset price p_0 is below e_0 will be associated with an interest rate of $r_0 = 0$. Likewise, any equilibrium in which the interest rate r_0 is positive will be associated with an equilibrium asset price of $p_0 = e_0$. Note that in this case, the initial price p_0 is uniquely determined, and so it will be inappropriate to impose restrictions on p_0 as Galí does in his partial equilibrium analysis. Figure 2 shows that if the policymaker selects an interest rate path with a higher initial interest rate r_0, the associated equilibrium price p_0 must be the same or higher. Since a higher initial interest rate r_0 implies all subsequent interest rates will be weakly higher as well, choosing a higher initial rate will result in a bubble that is at least as large at date 0, grows more between dates 0 and 1, and then grows at least as fast thereafter. Raising rates will force the policymaker to tolerate a strictly larger bubble from date 1 on.

[4] Gali assumes the central bank can select the nominal interest rate. Since he assumes prices are sticky, this means it also selects the real interest rate. To be sure, Gali's model is much richer, including stochastic shocks that are only immediately observable to the central bank. This allows him to deal with the fact that price setters might anticipate central bank policy. But this shouldn't obscure from the fact that Gali effectively assumes the central bank selects the interest rate without intervening in asset markets, presumably by promising to take certain actions if the nominal rate it wants does not prevail. One can similarly assume a policymaker in our model promises to intervene if the real interest rate differs from its desired level.

Galí's insight can thus be understood as the finding that in certain models that admit multiple equilibria, the equilibria with higher interest rates also feature larger bubbles.[5] The intuition for this result can be understood as follows. In our economy, a higher interest rate leads agents to shift from storage to buying assets. Thus, a higher interest rate will be associated with weakly higher demand for assets. Galí's model features no storage, but agents value consumption both when young and old. Agents in his model therefore choose between consumption and saving rather than storage and saving. Given his assumptions, a higher equilibrium interest rate will induce young agents to allocate more resources to buying assets. Since there is only one asset agents can buy, which is available in fixed supply, this asset necessarily becomes more valuable as demand for assets rises. In Galí's setup, in fact, the analog of Figure 2 is a strictly upward-sloping curve, so a higher interest rate path in his model will be associated with a strictly larger bubble starting at date 0, contrary to the restriction he imposes in his partial equilibrium analysis.[6]

Our model can thus elucidate why Galí's model implies a higher real interest rate is associated with a larger bubble. His model, like ours, features multiple equilibria, and those equilibria with higher interest rates also feature larger bubbles. If policymakers intervene by selecting those equilibria with higher interest rates, they will end up with larger bubbles. The fact that policymakers select equilibria rather than intervene directly in markets plays an important role in this result, as does the fact that there is only one asset agents can use to save. In the next section, we modify the model so that it admits a unique equilibrium. Policymakers will only be able to affect interest rates by intervening in financial markets, e.g. by changing the amount of government debt the public must hold. In this case, we will show that policymakers can intervene in ways that both raise interest rates and dampen asset bubbles.

2 Overturning Galí's Result

Our first modification to the model in Section 1 is to replace $d = 0$ with $d > 0$. As we show below, this seemingly trivial change reduces the set of equilibria to a singleton. Moreover, the unique equilibrium features a bubble, i.e., the equilibrium asset price necessarily exceeds the fundamental value of the asset.[7]

[5] This result was already shown in work on OLG models of money for a particular class of preferences, although it was interpreted quite differently. If we view the intrinsically worthless asset as money, then the fact that equilibria where the bubble grows faster are those where the bubble is larger can be interpreted to mean that with a fixed money supply, higher inflation is associated with a lower value of money and lower real balances. See Blanchard and Fischer (1989, p158-9).

[6] While agents in Gali's model and ours can only buy a bubble asset, Diamond (1965) and Tirole (1985) let agents buy a bubble asset or invest in physical capital. These models also exhibit multiple equilibria, and still imply that equilibria with higher interest rates feature larger bubbles. Intuitively, if a policymaker selected an equilibrium with a high interest rate without intervening in markets, she would have to choose an equilibrium with less capital to raise the marginal product of capital. Agents who hold less capital but want to save more given higher interest rates must spend more on the bubble asset.

[7] These results do not rely on constant dividends. Suppose we allowed time-varying nonnegative dividends d_t. Our proof of Proposition 2 below shows the equilibrium is unique iff $\sum_{t=0}^{\infty} d_t = \infty$. This equilibrium will be a bubble if $\lim_{t\to\infty} d_t/e_t = 0$, i.e., if d_t grows at a rate less than g. Tirole (1985) and Rhee (1991) show bubbles cannot arise if $\lim_{t\to\infty} d_t/e_t > 0$.

To show that $d > 0$ implies a unique equilibrium in the asset market, we now argue that the return to holding the asset r_t cannot equal 0 in equilibrium. For suppose r_t did equal 0. Then (3) would imply

$$p_{t+1} = p_t - d < p_t$$

Since $p_t \leq e_t$, it follows that $p_{t+1} < e_t < e_{t+1}$, which implies that $r_{t+1} = 0$. By this logic, the price would continue to decline by d at dates $t+1$, $t+2$, and so on, until the price would eventually turn negative. But this cannot be an equilibrium, since at a negative price the cohort that owns them would prefer not to sell them. The only candidate equilibrium price path is one with strictly positive interest rates at all dates. But in that case, buying the asset dominates storage and young agents will exchange all of their endowment for the asset. The price of the asset will thus be uniquely determined in every period. We confirm this in the next proposition. The proof of this and other propositions are contained in an appendix.

Proposition 2: Suppose $d > 0$. Then the unique equilibrium path $\{p_t\}_{t=0}^{\infty}$ is given by

$$p_t = e_t \tag{7}$$

and the unique equilibrium interest rate is given by

$$r_t = \frac{d}{e_t} + g \tag{8}$$

Note that while Proposition 1 allowed for the existence of additional stochastic equilibria in which $\{p_t\}_{t=0}^{\infty}$ are random variables, in the case where $d > 0$ the equilibrium is unique, meaning there are no stochastic equilibria. This result is noteworthy, since previous work on bubbles with intrinsically worthless assets has at times explored the possibility of stochastically bursting bubbles, i.e., equilibria in which the bubble can collapse to zero at a random date. Such phenomena are not possible when $d > 0$ without modifying the model: The bubble in our model persists indefinitely. We return to this point in Section 4.

We now argue that the equilibrium in Proposition 2 constitutes a bubble. We first compute the fundamental value of the asset f_t. We opt to discount dividends using the market interest rate r_t, i.e.,

$$f_t = \sum_{j=1}^{\infty} \left(\prod_{i=0}^{j-1} \frac{1}{1 + r_{t+i}} \right) d \tag{9}$$

The reason for using the market interest rate is that if we take resources from a young agent at date t, he would demand $1 + r_t$ units at date $t+1$ to remain equally well off. The interest rate thus faithfully captures the way society trades off resources between adjacent dates.

From Proposition 2, $r_t > g > 0$ for all t. Hence, the expressions in (9) are all bounded above, specifically

$$f_t \leq \sum_{j=1}^{\infty} \left(\frac{1}{1+g} \right)^j d = \frac{d}{g}$$

Furthermore, $\lim_{t\to\infty} f_t = d/g < \infty$. The fundamental value of the asset is thus bounded. At the same time, the asset price grows without bound, since $\lim_{t\to\infty} p_t = \lim_{t\to\infty} (1+g)^t e_0 = \infty$. The asset price must

therefore eventually exceed its fundamental value, so the asset is a bubble, at least asymptotically. But we can show that the equilibrium price of the asset exceeds the fundamental value at all dates rather than just asymptotically. To see this, note that the equilibrium interest rate r_t in Proposition 2 implies that

$$p_t = \frac{d + p_{t+1}}{1 + r_t}$$

At the same time, the fundamental value f_t in (9) satisfies

$$f_t = \frac{d + f_{t+1}}{1 + r_t}$$

Subtracting the latter expression from the former reveals that the difference $\Delta_t \equiv p_t - f_t$ must satisfy

$$\Delta_t = \frac{\Delta_{t+1}}{1 + r_t}$$

Since $\Delta_T > 0$ as $T \to \infty$, it follows that $\Delta_0 > 0$. Hence, the price of the asset exceeds its fundamental value at all dates, and the asset is necessarily a bubble.[8] If we take the limit as $d \to 0$, we can think about allowing for an arbitrarily small but positive d to effectively select the largest among the many possible bubbles in an economy where $d = 0$, i.e., the bubble in which agents spend all available resources on the asset. We will return to the observation below.

Given that our model admits a unique equilibrium, we can no longer model policy interventions as selecting equilibria. Instead, we need to allow policy to affect the unique equilibrium interest rate directly. Towards this end, we introduce government debt into the model. Not only is debt a natural channel through which policy can affect interest rates, but it also allows agents to save using more than one asset, and we previously argued that the restriction to one asset played an important role behind Galí's result. To the extent that leaning against the wind falls under the responsibility of central banks, the most natural intervention to study would be open-market operations in which the monetary authority sells bonds if it wants to raise rates. But modelling this would require us to introduce money as well as bonds. For simplicity, we start by ignoring money and focusing on the consequences of interventions that involve issuing more debt. While these would more naturally be interpreted as fiscal interventions, they still help demonstrate why interventions that raise interest rates can dampen bubbles. We introduce money and monetary policy in the next section.[9]

Consider a government that issues real one-period debt. In a slight abuse of notation, we refer to the promised return on this debt as r_t, even though we previously used r_t to refer to the return on the asset. That is, the government promises $1 + r_t$ units of consumption at date $t + 1$ for each unit it collects at date t.

[8] In a closely related setting, Tirole (1985) showed that if dividends are positive and the limiting interest rate without a bubble is nonpositive, the equilibrium is unique and features a bubble. The fact that the return to storage is zero in our model plays a similarly essential role. Intuitively, since the fundamental value of the asset would be infinite if the bubble vanished, equilibrium must feature an asymptotic bubble, and there is only one bubble path that does not vanish asymptotically.

[9] Although policy interventions against bubbles are often framed in terms of monetary policy, issuing debt has been proposed as a policy in economies that are vulnerable to bubbles. For example, Diamond (1965) interprets his model to mean that issuing public debt can potentially restore dynamic efficiency to an economy that would otherwise be vulnerable to bubbles.

In equilibrium, r_t will correspond to the return on any risk-free asset. In addition to issuing debt, we allow the government to collect lump-sum taxes from the young. In principle, the government could also buy assets or store goods, but allowing these options would yield little new insight. Let b_t denote the resources the government collects from issuing debt at date t and let τ_t denote lump-sum taxes collected at date t, where $\tau_t \geq 0$ for all t. The government must collect resources at date $t+1$ to cover its promises from date t, i.e., it must set $b_{t+1} + \tau_{t+1} = (1 + r_t)\, b_t$. Rearranging yields the flow government budget constraint

$$b_{t+1} = (1 + r_t)\, b_t - \tau_{t+1} \tag{10}$$

A fiscal policy for the government reduces to two objects: an initial obligation by the government to the old at date 0, which we denote $(1 + r_{-1})\, b_{-1}$, and a path of lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$. Given a path for $\{r_t\}_{t=0}^{\infty}$, we can use (10) to derive an implied path for debt $\{b_t\}_{t=0}^{\infty}$. Feasibility requires $(1 + r_{t-1})\, b_{t-1} \leq e_t$ for all $t = 0, 1, 2, ...$, i.e., the government cannot promise to pay more resources than it could ever collect.

We now describe the equilibrium of our economy given a path for fiscal policy. Market clearing requires that at each date t, the young are willing to buy all of the debt b_t issued by the government and all shares of the asset held by the old. The old must be willing to sell all of their shares of the asset. This requires that the interest rate on government bonds and the asset be equal, i.e.,

$$1 + r_t = \frac{d + p_{t+1}}{p_t} \tag{11}$$

Note that (11) is no longer a definition but an equilibrium condition that requires two different financial instruments to deliver the same rate of return. If $r_t > 0$, young agents will spend all of their available resources, which now correspond to $e_t - \tau_t - b_t$, to purchase the asset. If $r_t = 0$, they will be indifferent between storage and buying the asset. Hence, the equilibrium price is given by

$$p_t = \begin{cases} \text{any } \rho \in [0, e_t - b_t - \tau_t] & \text{if } r_t = 0 \\ e_t - \tau_t - b_t & \text{if } r_t > 0 \end{cases} \tag{12}$$

An equilibrium for a given fiscal policy corresponds to a path $\{p_t, r_t, b_t\}_{t=0}^{\infty}$ that satisfies (10), (11), and (12) at all dates. Our next lemma shows that adding fiscal policy continues to yield a unique equilibrium.

Lemma 1: Suppose $d > 0$. Given an initial obligation $(1 + r_{-1})\, b_{-1} \leq e_0$ and a path $\{\tau_t\}_{t=0}^{\infty}$ where $\tau_t \geq 0$, there is a unique $\{p_t, r_t, b_t\}_{t=0}^{\infty}$ that satisfies (10), (11), and (12). Along this path, $r_t > 0$ for all t.

In what follows, we restrict attention to fiscal policy paths in which the government sector doesn't eventually crowd out all economic activity. We do this to avoid the case where young agents ultimately hand over all of their endowment to the government, either as tax payments or as purchases of government bonds. Formally, we restrict attention to paths for fiscal policy for which

$$\lim_{t \to \infty} \frac{\tau_t + b_t}{e_t} = \chi < 1 \tag{13}$$

where b_t is the debt issuance implied by the path of fiscal policy that is summarized in Lemma 1. Condition (13) ensures the unique equilibrium price p_t must grow arbitrarily large over time.

Lemma 2: Suppose $d > 0$. If (13) holds, then the equilibrium price p_t satisfies $\lim_{t\to\infty} p_t = \infty$.

To study the effect of policy interventions, it will be convenient to further restrict attention to policies that can be summarized with a single parameter. Consider fiscal paths that imply a constant level of debt b over time. That is, suppose the path of fiscal policy satisfies

$$\begin{aligned} (1+r_{-1})\, b_{-1} &= (1+r_{-1})\, b \\ b_t &= b \text{ for } t = 0, 1, 2, ... \end{aligned} \tag{14}$$

Under this formulation, a higher b corresponds to an increase in government liabilities at all dates. To ensure debt remains equal to b, the fiscal authority must set taxes τ_t to equal the interest obligation $r_{t-1}b$, leaving the principal balance unchanged.

An important implication of (14) is that the present discounted value of debt, evaluated at the unique equilibrium path of interest rates described in Lemma 1, always vanishes asymptotically regardless of b:

$$\lim_{T\to\infty} \left(\prod_{s=0}^{T} (1+r_s) \right)^{-1} b_{T+1} = 0 \tag{15}$$

Equation (15) requires that a higher level of obligations b be accompanied by higher taxes to ensure the debt doesn't grow asymptotically. In representative agent economies, this is a constraint fiscal policy has to satisfy; otherwise, the agent holding bonds would violate his transversality condition. But as Diamond (1965) showed, in a dynamically inefficient economy such as ours, the government can roll over its debt indefinitely. Restricting policy to (14) thus focuses on policies in which the government pays down its debt. We return to this point below. Our next result characterizes how equilibria in our economy vary with b.

Proposition 3: Suppose fiscal policy is given by (14) where $r_{-1} > 0$. Then the equilibrium asset price p_t is decreasing in b for all t and the equilibrium interest rate r_t is increasing in b for all t.

Intuitively, recall that we are considering policies in which a higher b is financed by higher taxes τ_t.[10] If agents pay more taxes and hold more debt, they will have fewer available resources with which to buy the asset. As a result, the price of the asset will fall. Issuing more bonds crowds out spending on the bubble.

Although it might seem equally intuitive that issuing more debt should drive interest rates up, the argument is in fact a bit subtle. In our model higher rates do not induce agents to hold the additional bonds the government issues, since agents want to save all of their endowment regardless of the interest rate.[11] The reason a higher b leads to higher interest rates, then, is not that rates must rise to induce

[10] Our assumption that $r_{-1} > 0$ ensures τ_0 and r_0 both increase in b. If we instead assumed $r_{-1} = 0$, increasing b would have only increased taxes and interest rates from date 1 on.

[11] In principle, we could introduce such a feature even with our preferences by assuming agents value government bonds more than other assets, as in Krishnamurthy and Vissing-Jorgensen (2012). In that case, a change in the supply of government bonds would affect the equilibrium return on those bonds. We do something like this in the next section, where we introduce money and assume agents value it differently from other assets.

households to buy them, but that under (14) a higher b leads the government to collect more taxes. To see why this matters for interest rates, note that in equilibrium agents born at date t use all of their disposable income $e_t - \tau_t$ to buy assets and bonds, and use the returns on these investments to buy up all available $e_{t+1} + d$ units of the consumption good at date $t+1$. The return on private savings for those born at date t can thus be expressed as a function of endowments and taxes:

$$1 + r_t = \frac{e_{t+1} + d}{e_t - \tau_t} \tag{16}$$

As is clear from (16), higher taxes imply higher interest rates. In essence, any intervention that makes agents temporarily poor, as a higher lump sum tax on the young does, leaves agents with fewer resources to save but does not diminish what they earn on their savings. When we incorporate monetary policy in the next section and allow money to affect real activity, we show that a tight monetary policy can play a similar role in temporarily impoverishing agents. By contrast, if the government increased its promise to the initial old at date 0 but then financed it by perpetually rolling over its debt without ever raising taxes, setting $b_{t+1} = (1 + r_t)\, b_t$ and $\tau_t = 0$ for all t, interest rates would remain unchanged. In that case, the government effectively replaces one bubble with another, since anything agents do not spend on the bubble asset they spend on bonds. The intervention we consider goes beyond exchanging one bubble for another by raising taxes and thereby crowding out the resources agents can save. This crowding out acts to both raise the return to savings and further drive down the amount agents spend on the bubble asset.

Proposition 3 establishes that there exists a policy intervention that increases interest rates and drives down the price of the asset. However, with $d > 0$, a lower price need not correspond to a smaller bubble. Since a higher b implies higher interest rates, the fundamental value f_t as defined in (9) will fall, and the fundamental could fall by the same amount or even more than the price. Our next result confirms that under assumption (13), an increase in b will also reduce the size of the bubble $\Delta_t = p_t - f_t$ at all dates.

Proposition 4: If fiscal policy is given by (14), and the path of fiscal policy satisfies (13), then $\Delta_t \equiv p_t - f_t$ is decreasing in b for all t.

We thus have an example of an intervention that, contrary to Galí's result, leads to higher interest rates and a smaller bubble. To gain some insight into why this intervention reduces the bubble, consider the case with no government intervention, i.e., where $b = 0$, and let the government issue a little debt. As we noted earlier, the unique equilibrium in our economy corresponds to the maximal possible bubble, the one in which agents spend all available resources to buy the asset. If government intervention were successful in raising rates, then, it must either lead to a smaller bubble or render the bubble unsustainable. By reducing the amount agents can save, the intervention we consider succeeds in raising rates and reducing the bubble.

While our example suggests raising rates can help mitigate bubbles, it is also the case that in this example there is no reason for a policymaker to act against the bubble. In fact, the interventions we consider have no effect on allocations or welfare in our model: Regardless of the fiscal policy the government implements, each cohort consumes the full endowment of the next cohort, e_{t+1}, as well as the dividend d. Issuing more

debt merely changes the way in which resources are transferred between generations. If the government issues no debt, the young transfer their resources to the old directly by paying them for the asset. If the government issues debt, the young will transfer resources to the government when they pay taxes and buy bonds, which the government then transfers to the old when it repays its outstanding debt. Intervention may affect market interest rates and prices, but it does not affect what any cohort consumes.

Our irrelevance result may give the impression that our intervention merely exchanges government debt for the bubble asset. But as we argued earlier, this is only true when the government opts to roll over its debt without raising taxes. The intervention we consider involves higher taxes at all dates, but it has no effect on welfare because the preferences we assume ensure old agents consume all available resources each period regardless of policy. If we instead assumed agents have the same preferences over consumption that Galí assumed, the intervention we analyze would affect allocations; a higher interest rate would induce agents to maintain a steeper consumption profile. In that case, the intervention we consider would matter for welfare. But it would still not make government intervention desirable. The bubble in our model survives asymptotically, in the sense that $\lim_{t\to\infty} \Delta_t/e_t > 0$. As Tirole (1985) and others have argued, such bubbles are Pareto efficient in environments like ours. Hence, any intervention which changes allocations must make at least some agents worse off. Acting to dampen a bubble cannot make society as a whole better off.

This is not to say that it is impossible to modify such models to allow for welfare-improving interventions. For example, if agents could create additional assets at a cost, a bubble could lead to excessive creation of assets: Society would be better off having the government issue debt than wasting resources to create assets that achieve the same thing. While this offers a reason for intervening against bubbles, it is not the one policymakers typically invoke to argue for leaning against potential bubbles. They tend to worry about bubbles collapsing and the consequences thereof. Recall that in our economy where $d > 0$, the bubble persists indefinitely and does not burst, so introducing variable asset supply would not touch on these concerns even if it provided a reason for intervention. We return to these considerations in Section 4. For now, we simply note that while we have shown that policymakers can successfully fight bubbles by raising interest rates, we have yet to offer a compelling reason why they should.

3 Monetary Policy

In the previous section, we showed that a fiscal intervention involving more debt issuance led to higher interest rates and smaller bubbles. It might be tempting to conjecture that contractionary monetary policy should work similarly. After all, one way to contract the money supply is through open market operations that induce the public to hold more bonds. However, open market operations replace one asset held by the public with another, i.e., money with bonds. It isn't obvious, then, that such a policy would still crowd out spending on the bubble asset. To understand the effects of monetary interventions, we need to explicitly add fiat money as another asset agents can hold. In this section, we sketch out how to add money to our model and the consequences of doing so. The details of the analysis are described in Appendix B. To make

the analysis transparent, we continue to use the model in Section 2 in which there is no welfare gain from deflating a bubble. We turn to the question of why intervention might be beneficial in the next section.

To anticipate our main findings, we show that when goods prices are flexible and respond instantly to changes in the stock of money, a monetary intervention can result in higher interest rates and a smaller bubble, but only if it forces a fiscal intervention of the type discussed in Section 2. In particular, a monetary intervention that lowers the price of goods will increase the real value of the government's outstanding nominal obligations. For the real interest rate to rise, the monetary intervention must induce the government to collect more taxes; if the government finances its obligations using offsetting monetary policy at a later date, or by rolling over its debt, real rates will be unchanged. With flexible prices, then, monetary policy shapes interest rates only by forcing changes in fiscal policy. We then show that if prices are sticky, a monetary intervention can raise rates and dampen bubbles even when taxes are held fixed. This is because sticky prices allow a monetary contraction to temporarily reduce economy activity. This lowers the income of the agents who buy the bubble, leaving them with fewer resources to spend on the asset. The reduction in income plays the same role as a tax increase in our example based on fiscal interventions.[12]

To establish these results, we need to modify the model from Section 2 in two key ways. First, we introduce money as an additional financial asset agents can hold. If money is a perfect substitute for other assets, however, it will have to carry the same return as other assets, and money injections or withdrawals will not affect interest rates. So we assume instead that money provides liquidity services to agents, which we model by the expedient of putting money in the utility function. Second, we assume agents are endowed with labor rather than with a fixed amount of goods. This allows monetary policy to affect the quantity of goods produced, and thus the income of young agents in the model who buy the bubble assets.

We begin by introducing money. Young agents can still store their endowment or exchange it for the dividend-bearing asset and government debt. But now they can also exchange it for money, i.e., a non-interest bearing asset issued by the government. Let M_t denote the amount of money circulating at date t, so that $M_t - M_{t-1}$ corresponds to the amount of money issued or removed from circulation at date t. We model these injections and withdrawals as lump sum taxes on the old and transfers to the old, respectively. Let P_t denote the price of one unit of consumption goods in terms of money. Conversely, $1/P_t$ represents the real price of one unit of money, and $m_t = M_t/P_t$ denotes real money balances. Let Π_t denote the gross inflation rate between dates t and $t+1$, i.e., $\Pi_t = P_{t+1}/P_t$. It will be convenient to define

$$x_{t+1} = \frac{M_{t+1} - M_t}{P_{t+1}}$$

as the injection of money between dates t and $t+1$ measured in terms of how much this amount could buy at date $t+1$. If young individuals opt to set aside m_t of their original wealth to exchange for money, their

[12] The reason this effect does not arise in Gali's model is that he only considered contractionary monetary policy in response to a shock that increases the wealth of young agents. In that case, even if the income of young agents does decline, the amount they can spend on bubble assets can still rise. Dong, Miao, and Wang (2017) make a similar point.

real money balances at date $t+1$ will equal

$$m_{t+1} = \Pi_t^{-1} m_t + x_{t+1}$$

where agents view x_{t+1} as fixed and unaffected by their actions. We continue to denote the real rate of return on government debt by $1 + r_t$, and we use $1 + i_t$ to denote the nominal interest rate $(1 + r_t)\,\Pi_t$.

As we already noted, if money were just another asset that can be used as a store of value, there would be limited scope for monetary policy to influence inflation or interest rates. Thus, we need agents to value money beyond its use as a store of value. We follow Galí's approach (in Appendix 3 of his paper) of assuming that real balances enter the utility function directly.[13] This specification is meant to be a stand-in for the liquidity value of holding money, since it makes agents willing to hold money even when it offers a lower return. In particular, we assume agents derive log utility from the real balances they set aside while young, $m_t = M_t/P_t$. As we discuss below, the log utility specification is convenient because it implies money is superneutral, i.e., changing the growth rate of money will have no effect on economic activity in equilibrium.

The second way we modify our model is to assume young agents are endowed not with goods but with labor. Agents born at date t are endowed with one unit of labor. Production is linear, so if an agent devotes $n_t \in [0, 1]$ units of labor he will produce $A_t n_t$ units of output, where productivity A_t evolves as

$$A_t = (1 + g)^t A_0$$

Galí also assumed young agents are endowed with labor rather than goods, but he assumed they supply their labor inelastically. We instead follow Adam (2003) in assuming agents value time spent not working, and so might vary n_t in response to policy. In particular, we assume that the cohort of agents born at date t incur disutility $v_t(n_t)$ from supplying n_t units of time, where $v_t(\cdot)$ is increasing and convex. To simplify the exposition, we will proceed here as if agents are yeoman farmers who operate their own technology. In Appendix B, we assume some young agents are workers and others are entrepreneurs who know how to deploy labor, and allow workers to supply their labor on a market at a market-clearing wage.

The modifications above mean we need to replace the preferences in (1) with

$$u(c_t, c_{t+1}, m_t, n_t) = c_{t+1} + \theta \ln(m_t) - v_t(n_t) \tag{17}$$

where $\theta > 0$. It will be convenient to assume that $v_t(n_t) = A_t v(n)$ for some common function $v(\cdot)$. Agents find it more costly to put in a given amount of effort as they become more productive, so employment can remain stable over time. We assume $\lim_{n \to 0} v'(n) = 0$ and $\lim_{n \to 1} v'(n) = \infty$ to ensure an interior solution.

Agents maximizing the utility in (17) are subject to the budget constraint

$$c_{t+1} = (1 + r_t)(A_t n_t - m_t - \tau_t) + \Pi_t^{-1} m_t + x_{t+1} \tag{18}$$

[13] Waldo (1985) offers another example of a monetary OLG economy where agents value real balances directly.

In equilibrium, both the government bond and the asset yield the same return r_t, and so the agent's portfolio decision boils down to choosing between interest-bearing instruments and money. The first order conditions of the problem above with respect to real balances and labor effort are given by

$$m_t = \frac{\theta \Pi_t}{(1+r_t)\Pi_t - 1} = \frac{\theta \Pi_t}{i_t} \tag{19}$$

$$v'(n_t) = 1 + r_t \tag{20}$$

Finally, we need to modify the intertemporal government budget constraint in (10) to take into account money injections and withdrawals. This constraint is now given by

$$\begin{aligned} b_t &= (1+r_{t-1})\, b_{t-1} - \tau_t - \frac{M_t - M_{t-1}}{P_t} \\ &= (1+r_{t-1})\, b_{t-1} - \tau_t - m_t + \frac{m_{t-1}}{\Pi_{t-1}} \end{aligned} \tag{21}$$

An equilibrium in this economy is a path $\{1/P_t, p_t, r_t\}_{t=0}^{\infty}$ for the price of money and the asset relative to goods, respectively, as well as the real interest rate, together with a path $\{n_t\}_{t=0}^{\infty}$ for hours, that satisfy the first order conditions (19) and (20), the intertemporal government budget constraint (21), and the equilibrium condition that the return to buying the asset $(d + p_{t+1})/p_t$ must be the same as the real interest rate $1 + r_t$.[14] Since there is nothing to impede the price of goods P_t from changing when the amount of money in circulation $\{M_t\}_{t=0}^{\infty}$ changes, we will refer to this setup as a flexible price economy.

As anticipated above, in Appendix B we show that log preferences ensure money is superneutral, in the sense that changes in $\{M_t\}_{t=0}^{\infty}$ have no effect on the real interest rate r_t or the employment decision n_t. In particular, given the demand for money in (19), the equilibrium real interest rate r_t will be given by

$$1 + r_t = \frac{A_{t+1} n_{t+1} + \theta + d}{A_t n_t - \tau_t} \tag{22}$$

Note the similarity between (22) and (16). Although n_t is endogenous, in Appendix B we show that it is still the case that the real interest rate will only change if the path of taxes τ_t does. A change in the path of money $\{M_t\}_{t=0}^{\infty}$ on its own will have no effect on the real interest rate given our specification.

While the path of $\{M_t\}_{t=0}^{\infty}$ will not affect the real interest rate, employment, or output, it will in general affect the price of goods P_t, the level of real balances $m_t = M_t/P_t$ agents hold, and government revenues. It will also affect the real price of the asset p_t. To see this, consider the effect of a change in $\{M_t\}_{t=0}^{\infty}$ that leads to a reduction in the initial price level P_0. Since storage is still dominated, the equilibrium price of the asset p_t will continue to equal the income agents have available, i.e., the amount $A_t n_t$ agents earn from production, net of taxes τ_t, government debt b_t, and real balances M_t/P_t. Applied to date 0, this implies

$$p_0 = A_0 n_0 - \tau_0 - b_0 - \frac{M_0}{P_0} \tag{23}$$

[14] The nominal rate is not included as an equilibrium object because this rate is defined as $1 + i_t = (1 + r_t)\, P_{t+1}/P_t$.

From the intertemporal government budget constraint in (21), we know that

$$b_0 + \tau_0 + \frac{M_0}{P_0} = (1 + r_{-1})\, b_{-1} + \frac{M_{-1}}{P_0}$$

Substituting this into (23) yields

$$p_0 = A_0 n_0 - (1 + r_{-1})\, b_{-1} - \frac{M_{-1}}{P_0}. \tag{24}$$

Both $(1 + r_{-1})\, b_{-1}$ and M_{-1} are predetermined, and above we argued that n_0 is independent of $\{M_t\}_{t=0}^{\infty}$. Hence, if changing $\{M_t\}_{t=0}^{\infty}$ reduces P_0, it will also decrease p_0. Intuitively, a higher price level P_0 reduces the seniorage revenue of the government, forcing it to either issue more debt or raise taxes. Either would leave agents with fewer resources to buy the asset, so its price will fall. The same intuition would apply if the initial debt obligation $(1 + r_{-1})\, b_{-1}$ were issued as a nominal obligation.

Hence, monetary interventions can affect the real price of the asset p_t when goods prices are flexible. However, since we argued that n_t will not change as we vary $\{M_t\}_{t=0}^{\infty}$, monetary policy affects the price of the asset without affecting r_0, the real interest rate at date 0, as evident from (22). Only if monetary policy forces a change in τ_0 will the real rate r_0 change. Recall this is also what we found in our analysis of fiscal policy, i.e., the interest rate r_t only changes when τ_t does. A monetary intervention will only affect the real interest rate r_0 if it is not undone by subsequent monetary policy and if it leads the government to increase taxes. In that case, a monetary intervention can once again result in both a higher real interest rate and a smaller bubble, but only because it forces a fiscal intervention along the lines in Section 2. Monetary policy operates on real interest rates not directly but through fiscal policy.

The fact that monetary policy will not influence interest rates except through changes in fiscal policy is a consequence of assuming goods prices are flexible. However, much of the literature on monetary policy assumes prices are sticky. For this reason, we also consider in Appendix B a version of our model that allows for price-setting which builds on Galí (2014) and Adam (2012). In that version, agents produce not a final good but different intermediate goods that can be combined into a final good. Each intermediate good producer is a monopolist supplier and can set the price of her respective good. This formulation allows us to analyze what happens if producers set their prices before the monetary authority moves. We show in Appendix B that if the monetary authority unexpectedly withdraws money at date 0 but then reverts to a fully predictable policy thereafter, there exists an intervention that raises the real interest rate r_0 and lowers equilibrium employment n_0. Since young agents earn less, they will have less to spend on the asset. As evident from (24), since P_0 is fixed and both $(1 + r_{-1})\, b_{-1}$ and M_{-1} are predetermined, a fall in n_0 would cause the price of the asset p_0 to fall. Note the analogy to the way an increase in taxes τ_t raised the interest rate and dampened asset prices in Section 2: A tight monetary policy with rigid prices reduces the resources agents can use to save just as a lump sum tax on agents would, and this raises interest rates and lowers asset prices. With a bit more work, we show that the bubble term $\Delta_0 = p_0 - f_0$ also falls. Thus, with nominal price rigidity, there exist purely monetary interventions that raise the real interest rate and mitigate the bubble, even when the path of taxes remains fixed.

Interestingly, the expression for p_0 in (24) implies that if P_0 cannot respond to changes in the stock of money, the only way a surprise monetary contraction at date 0 would affect p_0 is if n_0 fell. In other words, the only way to dampen bubbles is to generate a recession. Intuitively, dampening the bubble requires the crowding out of resources that agents spend to buy assets. In our sticky-price economy, the only way to achieve this is if agents earn less income. This suggests it might be difficult for a policymaker intent on dampening an asset bubble to deflate the bubble without also curtailing economic activity; the logic of crowding out dictates that income must fall for spending on the asset to fall. More generally, our analysis suggests that a policymaker seeking to deflate a bubble should not simply be content with raising interest rates, but should look to see if the increase in rates was associated with crowding out, i.e., whether it was associated with a decline in saving by agents or a shift in the composition of the portfolio of assets held by the public. In other words, since our theory emphasizes a particular channel through which higher rates depress bubbles, policymakers can verify whether there is any evidence that this channel is in fact operative.

Finally, we remark briefly on welfare. Consider the case of flexible prices. Recall that we argued that a monetary intervention that lowers P_0 would reduce the bubble even if it has no impact on the real interest rate. As before, there is nothing inherently desirable about reducing the bubble *per se*. Such a policy will have no effect on output, and agents will consume the same amount even if we reduce the bubble. The new wrinkle is that even if there is no consequence for reducing the bubble, monetary interventions will matter for welfare, since they affect the real balances m_t that agents care about. But the effect is independent of the price of the asset p_t or the extent to which the asset is overvalued. Even if some paths for the stock of money are better than others, it is not because of the way they affect the bubble on the asset.

4 Credit-Driven Bubbles

At this point, our analysis offers mixed justification for the lean-against-the-wind view. On the one hand, we show that there can be interventions that raise the real interest rate and depress bubbles, in line with the view that the way to dampen bubbles is to raise interest rates. At the same time, our examples rely on models in which there is no reason for a policymaker to intervene to dampen bubbles. Of course, advocates of leaning against the wind would be quick to point out that the types of bubbles we have discussed so far do not coincide with the scenarios they worry about. Policymakers do not seem especially alarmed when a strong desire to save leads agents to bid up the price of assets above their fundamental value, especially if, as is true in our model, such a bubble will persist indefinitely. What worries policymakers is the prospect of a bubble collapsing, especially when its purchase was financed with debt. For example, Mishkin (2011) singles out what he calls credit-driven bubbles as a particularly acute concern for policymakers:

> [N]ot all asset price bubbles are alike. Financial history and the financial crisis of 2007-2009 indicates that one type of bubble, which is best referred to as a credit-driven bubble, can be highly dangerous. With this type of bubble, there is the following typical chain of events: Because of either exuberant expectations about economic prospects or structural changes in

financial markets, a credit boom begins, increasing the demand for some assets and thereby raising their prices... At some point, however, the bubble bursts. The collapse in asset prices then leads to a reversal of the feedback loop in which loans go sour, lenders cut back on credit supply, the demand for the assets declines further, and prices drop even more.

In this section, we show how our model can give rise to credit-driven bubbles along these lines. To do this requires both the possibility of a bubble bursting as well as private credit. We discuss each in turn.

4.1 Adding Shocks

To keep things simple, we ignore money and return to our setup in Section 2. We begin by modifying that model to accommodate a collapse of the bubble. Recall that when we assume the asset pays a positive dividend $d > 0$, the unique equilibrium features a bubble that persists indefinitely. To get the bubble to burst requires a shock that can trigger its collapse. As we mentioned earlier, this stands in contrast to what happens when $d = 0$. In that case, the set of all equilibria include stochastic price paths in which the asset is initially overvalued but the price of the asset can collapse to 0 at some stochastic date.[15]

One way to try to get a bubble to collapse is to introduce a shock that would render a bubble non-viable if the shock materialized. In our economy, a bubble requires the endowment agents use to buy the asset to grow as fast as the interest rate. Suppose, then, that there is a possibility that the endowment stops growing from some random date T on. Formally, we replace the endowment process in (2) with

$$e_t = \begin{cases} (1+g)^t e_0 & \text{if } t < T \\ (1+g)^{T-1} e_0 & \text{if } t \geq T \end{cases} \tag{25}$$

where T is a random variable. We can interpret this specification as a risk of secular stagnation.[16] Since the return on buying the asset is positive given $d > 0$, a bubble should not be possible beyond date T. We now verify that indeed no bubble can exist from date T on once the endowment stops growing.

For expositional ease, let us temporarily assume that the government issues no debt and agents can only hold the dividend-bearing asset, i.e., let us set $(1 + r_{-1}) b_{-1} = 0$ and $\tau_t = 0$ for all t. The argument for the non-existence of a bubble beyond date T goes through when we allow the government to issue debt. The

[15] A necessary condition of these equilibria is that even if the bubble can burst, it must still potentially survive for arbitrarily long periods. Blanchard and Watson (1982) were among the first to discuss such stochastically bursting bubbles, referring to them as "rational" bubbles to highlight that even rational agents would be willing to hold them. Later work by Weil (1987) showed that such price paths can arise as equilibria in dynamically inefficient economies with an intrinsically worthless asset.

[16] To the extent that the bubble collapses when growth stops, a period of secular stagnation would further coincide with the collapse of a bubble and the onset of low real interest rates. These features seem to characterize the empirical pattern Summers (2016) associated with the notion of secular stagnation when he revived the term originally coined by Alvin Hansen.

reason for ignoring government debt in this subsection is to simplify the derivation of asset prices; without public debt or taxes, the price of the asset is equal to the endowment of the cohort that buys it.

We use a hat to denote the value of a variable once the endowment stops growing. That is, $\widehat{p}_t$ denotes the price of the asset for $t \geq T$ and $\widehat{f}_t$ denotes its fundamental value for $t \geq T$. Since storage is dominated in equilibrium and young agents trade all of their endowment for the asset, we have

$$\widehat{p}_t = \widehat{e}_t = e_{T-1}$$

The real interest rate for any date $t \geq T$ is given by

$$1 + \widehat{r}_t = \frac{d + \widehat{p}_{t+1}}{\widehat{p}_t} = 1 + \frac{d}{e_{T-1}}$$

We can use this interest rate to compute the fundamental value of the asset for any date $t \geq T$:

$$\widehat{f}_t = \sum_{j=1}^{\infty} \left(\prod_{i=0}^{j-1} \frac{1}{1 + \widehat{r}_{t+i}} \right) d = \sum_{j=1}^{\infty} \left(\prod_{i=0}^{j-1} \frac{1}{1 + d/e_{T-1}} \right) d = e_{T-1}$$

From date T on, then, both $\widehat{p}_t$ and $\widehat{f}_t$ are equal to e_{T-1}. Hence, there can be no bubble beyond date T. Note that the fundamental value $\widehat{f}_t$ depends on T. This is because even though dividends are constant, the equilibrium interest rate $\widehat{r}_t$ depends on T.

We have confirmed that even if a bubble did exist before date T, it could not remain once the endowment stopped growing. To determine the price p_t for $t < T$, or while the endowment is growing, we need to specify a distribution for T. We assume T is geometrically distributed, meaning that, at any date $t < T$, the probability that $T = t + 1$ is a constant $\pi > 0$. Formally,

$$\Pr(T = k) = (1 - \pi)^{k-1} \pi \text{ for } k = 1, 2, 3, ...$$

We now show that when T is distributed geometrically, bubbles can be ruled out before date T as well as after date T. That is, in our attempt to accommodate a bursting bubble, we have actually ruled out bubbles altogether. We hasten to add that once we modify the model so that private credit does circulate, it will be possible for a bubble to arise that will survive until date T. Such a bubble is truly credit-driven in the sense that it relies on the existence of private debt. This is consistent with Mishkin's comment above that credit-driven bubbles are fundamentally different types of bubbles.

To show that a bubble cannot arise before date T when private debt does not circulate, we must first define the fundamental value of the asset when the endowment process is stochastic. The natural extension of the notion of fundamental value to this case is the *expected* present discounted value of dividends, i.e.,

$$f_t = E\left[\sum_{j=1}^{\infty} \left(\prod_{i=0}^{j-1} \frac{1}{1 + r_{t+i}} \right) d \right] \tag{26}$$

where $(1 + r_t)^{-1}$ denotes the value as of date t of one unit of resources to be delivered with certainty at date $t + 1$. For dates $t > T$, we already argued that $\widehat{r}_t = d/e_{T-1}$. For $t \leq T$, the fact that agents are risk

neutral implies they are indifferent between a unit of resources at date $t+1$ with certainty and a random amount of resources at date $t+1$ that equals 1 in expectation. The return on a risk-free instrument is thus the same as the expected return on the asset, i.e.

$$1 + r_t = \frac{d + E\left[p_{t+1}\right]}{p_t}$$

Since $p_{t+1} = e_t$ with probability π and $(1+g)\, e_t$ with probability $1-\pi$, the risk-free rate r_t if $t \leq T$ equals

$$r_t = (1-\pi)\, g + \frac{d}{e_t}$$

With this definition for the fundamentals, we can establish the following result:

Proposition 5: Suppose $d > 0$. If e_t is given by (25) and agents are homogeneous, a bubble will not arise, i.e., $p_t = f_t$ for $t < T$ and $\widehat{p}_t = \widehat{f}_t$ for $t \geq T$.

Note that the mere absence of a bubble beyond date T does not automatically mean bubbles cannot occur before date T. When $d = 0$, we know from Weil (1987) that there exist equilibria that feature a bubble through date T. Only when $d > 0$ can we rule out a bubble while the endowment is still growing.[17]

4.2 Adding Credit

Now that we have a shock that can trigger the collapse of a bubble, we move on to modify our economy to allow for private credit. After we describe the features that allow private debt to circulate, we show how credit markets allow for a bubble to persist for as long as the economy is growing.

As a first step, we return to allowing the government to issue positive debt. This will allow us to discuss the effect of policy interventions, which we do in Section 4.3. As before, we need to impose some restrictions. To ensure the government will never be forced to default, we restrict the path of $\{\tau_t\}_{t=0}^{\infty}$ so that

$$(1 + r_t)\, b_t < e_t \text{ for all } t$$

We assume taxes τ_t are not contingent on the realization of T but on calendar time. To ensure that the government never crowds out economic activity under any circumstances, we assume

$$\lim_{t \to \infty} b_t + \tau_t < e_0$$

As we noted in an earlier footnote, although we can allow for private debt in our economy, such debt will not circulate when all young agents are the same given only the young are able to borrow or willing to

[17] Technically, the set of equilibrium prices is upper hemicontinuous in d, since $p_t = e_t$ is the unique equilibrium price when $d > 0$ and continues to be an equilibrium – just not the unique one – when $d = 0$. By contrast, the fundamental value of the asset is discontinuous in d at 0, since $f_t = e_t > 0$ for $d > 0$, but is equal to 0 when $d = 0$. Essentially, as d tends to 0, the interest rate $\widehat{r}_t$ tends to 0. Thus, for small values of d, we have an infinite stream of vanishingly small dividends discounted at a vanishingly small rate. This allows the fundamental value to remain bounded strictly away from 0 for arbitrarily small d.

lend. For private debt to circulate, we need some young agents who want to save and others who want to borrow. Suppose, then, that in addition to the young agents born at date t with endowment e_t who wish to save for period $t+1$, there is a mass κ of entrepreneurs each period who are endowed with no resources but have the technical expertise to convert a unit of goods at date t into $1+y$ units of good at date $t+1$. We assume y is large, in the sense that the return to production is higher than the asset can deliver:

$$1 + y > \sup_t \left\{ \frac{p_{t+1} + d}{p_t} \right\} \tag{27}$$

Although (27) concerns an endogenous object, p_t, the price is driven by e_t and fiscal policy, and so (27) amounts to a restriction on y relative to paths for fiscal policy. Suppose each entrepreneur faces a capacity constraint and can convert at most one unit of goods into $1+y$ units of goods one period later, and that the mass of entrepreneurs κ is strictly below e_0. For now, we assume agents trade only through debt contracts, i.e., young entrepreneurs receive goods at date t and promise to pay a fixed amount at date $t+1$. We discuss whether agents would use such contracts later. We denote the interest rate on private debt issued at date t by R_t to distinguish it from the return on government debt r_t should the two differ.

Without credit market frictions, savers lend entrepreneurs the κ resources they need and buy assets and government debt with the remaining $e_t - \kappa$. The return on loans R_t will equal the return r_t on government debt. This economy is essentially equivalent to an economy without entrepreneurs where savers are endowed with $e_t - \kappa$ instead of e_t and can only buy assets and public debt. But we already know from Proposition 5 that no bubble is possible in such an economy. For a bubble to exist requires credit market frictions.

4.3 Equilibrium Credit-Driven Bubbles

One candidate credit-market friction that can allow bubbles is a constraint that limits the amount entrepreneurs can borrow. Suppose some friction prevented entrepreneurs from borrowing all κ units of resources they can deploy. Several papers have now demonstrated that such frictions allow bubbles to arise. Intuitively, trading bubble assets can substitute for credit as a way to transfer resources to entrepreneurs: Agents buy intrinsically worthless assets with the intent of selling them when they need to produce; in turn, agents looking to save buy these assets. This channel is explored in Farhi and Tirole (2012), Martin and Ventura (2012), and Hirano and Yanagawa (2017). Other papers have emphasized that purchasing intrinsically worthless assets that trade at a positive price can help agents relax the borrowing constraints they face; see, for example, Kocherlakota (2009), Miao and Wang (2015), and Martin and Ventura (2016).[18]

We study a different friction. Unlike the aforementioned papers, we assume entrepreneurs can secure all κ units they need. However, as in Allen and Gorton (1993), Allen and Gale (2000), and Barlevy

[18] Kocherlakota (1992), Santos and Woodford (1997), and Kocherlakota (2008) also argued that the existence of borrowing constraints can give rise to bubbles, although they considered endowment economies with constraints on consumers. Rocheteau and Wright (2013) argue that, at least in some cases, constraints on consumers and firms are isomorphic.

(2014), we introduce an informational friction that allows agents who are not entrepreneurs to blend in with entrepreneurs and borrow from savers. Non-entrepreneurs use these funds to speculate on risky assets. Thus, we consider frictions that induce non-entrepreneurs to borrow for non-productive purposes rather than frictions that force entrepreneurs to borrow less than they need. The two frictions can certainly coexist. We focus on informational frictions that lead to too much borrowing because they speak to policymaker concerns about credit-driven bubbles. Bubbles that arise because entrepreneurs are borrowing-constrained tend to be socially beneficial, precisely because they facilitate trade. As such, it is often not optimal to lean against these bubbles; if anything, policymakers should intervene to prevent their collapse. By contrast, bubbles that arise because agents can exploit an informational friction to speculate provide no social benefit.

To capture this informational friction requires a third type of agent. Consider a new group of agents, whom we shall call speculators, that have neither endowments nor technical expertise. Like all other agents, they have linear utility in the amount they consume when old. Savers cannot tell entrepreneurs and speculators apart nor monitor their actions. Even though speculators bring neither resources nor skills to the table, they can still affect the trade between savers and entrepreneurs by borrowing from the former to buy assets.[19] We allow for infinitely many speculators each period, effectively allowing for free entry into speculation.

Why would speculators borrow to buy assets? Buying riskless government bonds is pointless: Savers can buy bonds themselves, and would require at least as much in compensation for lending as the return r_t on government bonds. There would similarly be no point in buying the dividend-bearing asset after date T: The return to buying the asset is deterministically equal to $(\widehat{p}_{t+1} + d)/\widehat{p}_t$, which must equal the interest rate on government bonds $\widehat{r}_t$. But they might benefit from buying the asset before date T. At this point, the return to buying the asset is stochastic: It will equal $(p_{t+1} + d)/p_t$ if the endowment continues to grow and $(\widehat{p}_{t+1} + d)/p_t$ if the endowment stops growing. If the interest rate on loans, $1 + R_t$, falls between these two values, speculators will earn profits as long as the endowment grows. If the endowment stops growing, they will default and be left with nothing, no worse than what they start with. Of course, free entry of speculators will ensure that the profits from speculation equal zero in equilibrium, i.e., $p_t = d + p_{t+1}$.

As we noted above, we restrict agents to trade via debt contracts. That is, each saver can offer a menu of debt contracts at date t, each of which stipulates an amount w_t to be transferred from the saver at date t and an amount $(1 + R_t)\, w_t$ he must be repaid at date $t + 1$. Entrepreneurs and speculators then select which contract if any to enter. In Appendix C, we show that even when agents can offer other contracts, it will be an equilibrium for savers to only offer debt contracts. We further show this equilibrium is stable.

Before date T, while there is still uncertainty, creditors must set the interest rate in each contract they

[19] The assumption that lenders cannot distinguish entrepreneurs and speculators seems plausible for new, hard-to-evaluate technologies where it can be difficult to distinguish those who can profitably deploy assets specific to the new technology from those who buy assets in the hope their price appreciates. The assumption seems less plausible for housing, where lenders scrutinize borrowers and the assets they purchase. But in that case, the difference between speculators and good borrowers is not the assets they buy but their willingness to default if prices fall. See Barlevy and Fisher (2014) for an example of a model of risk-shifting bubbles where lenders can't discern the preferences of borrowers rather than the assets they buy.

offer high enough to leave speculators with zero profits. Otherwise, given free entry into speculation, demand for resources would exceed the finite amount e_t agents can lend out. Hence,

$$1 + R_t \geq \max\left\{\frac{d + p_{t+1}}{p_t}, \frac{d + \widehat{p}_{t+1}}{p_t}\right\} = \frac{d + p_{t+1}}{p_t} \tag{28}$$

Under (28), speculators remit all their earnings to lenders. If entrepreneurs and speculators were charged different interest rates that both satisfy (28), neither speculators nor lenders would be affected by replacing the interest rate offered to speculators with that offered to entrepreneurs. Hence, without loss of generality, we can focus on equilibria where lenders charge a single interest rate on all loans. There is also no loss in assuming lenders extend entrepreneurs loans of size 1, as there is no gain lending entrepreneurs either more or less than they need. Finally, below we argue that in equilibrium speculators must always borrow enough to buy the full supply of the asset. Hence, offering smaller loans to speculators will not affect equilibrium aggregate borrowing by speculators. This means that without loss of generality we can focus on a pooling equilibrium with a single debt contract of size 1 and an interest rate R_t that satisfies (28).

If lenders issue a single debt contract, we can characterize an equilibrium for our economy as a set of paths $\{p_t, r_t, R_t\}_{t=0}^{\infty}$ and $\{\widehat{p}_t, \widehat{r}_t, \widehat{R}_t\}_{t=0}^{\infty}$ for prices and interest rates that ensure asset and credit markets clear. The equilibrium price of the asset is once again easy to pin down. Agents will spend their entire collective endowment e_t to pay their tax obligations, buy the bonds issued by government, finance entrepreneurs and speculators, and buy the asset. The equilibrium asset price before and after T are thus given by

$$\begin{aligned} p_t &= e_t - b_t - \tau_t - \kappa \\ \widehat{p}_t &= \widehat{e}_t - b_t - \tau_t - \kappa \end{aligned} \tag{29}$$

From date T on, there is no uncertainty and hence no way for speculators to profit. In that case,

$$\widehat{R}_t = \widehat{r}_t = \frac{\widehat{p}_{t+1} + d}{\widehat{p}_t} \tag{30}$$

Substituting in for $\widehat{p}_t$ from (29), we can express $\widehat{R}_t$ and $\widehat{r}_t$ in terms of the primitives of the model.

Finally, we turn to R_t and r_t, the returns on loans and government debt before date T while the economy is growing. Recall from (28) that R_t must be set to deny speculators any profits. We focus on the equilibrium where (28) holds with equality, i.e., where creditors charge no more than necessary to keep potential speculators at bay. In that case, R_t is equal to $(d + p_{t+1})/p_t$, which we can express in terms of primitives by substituting in for p_t from (29). There are additional equilibria where (28) holds as a strict inequality, but these will still give rise to bubbles just as we argue below will occur when (28) holds with equality.[20]

[20] Consider an equilibrium in which (28) is satisfied with equality. If both speculators and entrepreneurs were charged a slightly higher interest rate, speculators would be unaffected and would simply default for both values of e_{t+1}. Entrepreneurs would earn slightly lower but still positive profits. So entrepreneurs would still be fully funded and speculators would earn zero profits. The interest rate on government bonds would have to rise to match the return on loans, but the equilibrium would be qualitatively similar to one in which (28) holds with equality.

To solve for the risk-free rate r_t on government debt, note that in equilibrium creditors must be indifferent between making loans and buying government debt. Lenders expect entrepreneurs to pay them back in full given our restriction on y in (27). Creditors also expect that speculators will pay them back in full if the endowment keeps growing and default and pay at a rate of $(d + \widehat{p}_{t+1})/p_t$ if the endowment stops growing. The expected return to lending thus depends on the share of borrowing by entrepreneurs and speculators. Since we are assuming (28) holds with equality, entrepreneurs earn strictly positive profits, and so all κ entrepreneurs strictly prefer to borrow. Hence, the amount entrepreneurs borrow is κ. As for speculators, we can show that in equilibrium they will purchase all assets. Intuitively, speculators value assets more than agents who purchase them with their own funds, since the former have an option to default if the endowment stops growing. Speculators will therefore borrow p_t, the value of the asset. Savers thus lend a total of $\kappa + p_t$, and the expected return on loans will equal r_t if and only if

$$1 + r_t = (1 + R_t)\left[\frac{\kappa}{\kappa + p_t} + (1 - \pi)\frac{p_t}{\kappa + p_t}\right] + \pi\left(\frac{d + \widehat{p}_{t+1}}{\kappa + p_t}\right)$$

Rearranging this equation and substituting in (29), we obtain

$$R_t - r_t = \frac{\pi\left(p_{t+1} - \widehat{p}_{t+1}\right)}{\kappa + p_t} = \frac{\pi g e_t}{e_t - b_t - \tau_t} > 0 \tag{31}$$

Thus, there will be a spread between the rate agents charge on loans R_t and the risk-free rate paid on government bonds r_t. This spread compensates lenders for losses they would incur on their loans to speculators if the endowment failed to grow between dates t and $t+1$. We can rearrange (31) to arrive at an expression for r_t in terms of primitives. This completes our characterization of the equilibrium in this economy.

To recap, in our economy savers want to trade with entrepreneurs. In the equilibrium we study, savers buy government debt and lend to both entrepreneurs and speculators while the economy is growing. Speculators use the funds they borrow to buy up the entire supply of the asset. When the endowment stops growing, the speculators who borrowed in the previous period default. From that point on, speculators cannot profit from buying assets as there is no risk to shift. Thus, our economy begins with a period of speculative growth during which agents borrow to speculate on assets, until eventually secular stagnation starts and speculation stops.[21] We now argue that the speculative growth regime features a bubble, i.e., the demand for the asset by speculators will cause the price of the asset to exceed fundamentals until the bubble bursts. Formally, we show that $\Delta_t = p_t - f_t > 0$ for $t < T$ and $\Delta_t = 0$ for $t \geq T$.

To see this, we combine (31), the fact that $1 + R_t = (d + p_{t+1})/p_t$, and some additional algebra to express the risk-free rate r_t before T as follows:

$$1 + r_t = \frac{(1 - \pi)\, p_{t+1} + \pi \widehat{p}_{t+1} + d}{p_t} + \frac{\pi \kappa g e_t}{p_t\left(p_t + \kappa\right)} \tag{32}$$

[21] Previous work on speculative growth has focused on the idea that bubbles encourage capital accumulation and hence growth. For example, Caballero, Farhi, and Hammour (2006) argue bubbles can lead to more savings that translate into capital accumulation, while Martin and Ventura (2012) and Hirano and Yanagawa (2016) argue bubbles improve the allocation of resources in economies with borrowing constraints. Our analysis highlights the opposite direction: growth can foster speculation, since it feeds into growth in asset prices that make it profitable to speculate on how long growth will continue.

The first term in (32) is the expected return on the asset, $(E_t[p_{t+1}] + d)/p_t$. If $\kappa > 0$, the risk-free rate r_t will exceed the return to the asset. This supports our earlier claim that lenders will not buy the asset, since holding the asset will be dominated by holding either private or public debt. Only speculators buy the asset in equilibrium. Let us define $1 + r_t^A$ as the expected return to buying the asset at date t, i.e.,

$$1 + r_t^A \equiv \frac{E_t[p_{t+1}] + d}{p_t}$$

In Proposition 5, we showed that if we discount dividends at the rate $1 + r_t^A$ above, the expected present discounted value of dividends must equal the price, i.e.

$$p_t = E\left[\sum_{t=1}^{\infty}\left(\prod_{s=0}^{t-1}\frac{1}{1+r_s^A}\right)d\right]$$

By contrast, the equilibrium fundamental value of the asset f_t is defined as the expected present value of dividends discounted at the riskless rate $1 + r_t$, i.e.,

$$f_t = E\left[\sum_{t=1}^{\infty}\left(\prod_{s=0}^{t-1}\frac{1}{1+r_s}\right)d\right]c$$

Since (32) implies $r_t > r_t^A$ for $t < T$, it follows that the price of the asset will strictly exceed the fundamental value for as long as $t < T$. This fact is summarized in the next proposition.

Proposition 6: If $\kappa > 0$, then $\Delta_t > 0$ for $t < T$ and $\Delta_t = 0$ for $t \geq T$. That is, the economy will exhibit a stochastically bursting bubble.

The reason that a bubble emerges before date T is that entrepreneurs cross-subsidize speculators and lower the spread over the risk-free rate that creditors charge speculators. Interestingly, the bubble in our model does not arise because speculators drive up the price of the asset. Recall from (29) that p_t is pinned down by the endowment and fiscal policy. Rather, the bubble emerges because speculators raise the risk-free rate, driving down the fundamental value of the asset. This is due to our assumption that agents only consume when old, and so inelastically supply their entire (after-tax) endowment to savings. More generally, speculation can lead to a higher asset price, as shown in Allen and Gale (2000) and Barlevy (2014).

4.4 Policy Interventions

Proposition 6 above establishes that our model gives rise to a credit-driven bubble that eventually bursts once economic growth stops. We now examine whether government intervention can still drive up the interest rate and dampen the bubble for this type of bubble. Our first result shows that, if we restrict fiscal policies to paths in which the size of debt obligations can be parameterized by a single parameter b as in (14), increasing b will once again increase the risk-free rate and lower the price of the asset.

Proposition 7: Suppose fiscal policy is given by (14), where $r_{-1} > 0$. Then the equilibrium interest rate r_t and loan rate R_t are increasing in b and the equilibrium asset price p_t is decreasing in b for all t.

Recall that a lower asset price p_t need not correspond to a smaller bubble, since a higher b will generally imply both a lower price p_t and a lower fundamental f_t. In fact, we will argue below that for credit-driven bubbles, a higher b no longer necessarily implies a smaller bubble. This highlights an important difference between credit-driven bubbles and the bubbles in our original framework inspired by Galí's model. For credit-driven bubbles, both the price of the asset and the fundamental value of the asset can be viewed as the expected present value of the same stream of dividends, but evaluated at different discount rates: p_t corresponds to discounting dividends at the rate $1+r_t^A$, the expected return on the asset, while f_t corresponds to discounting dividends at the rate $1 + r_t$, the return on risk-free assets. The effect of intervention on the bubble depends on how the intervention affects these two rates, and in principle can make the bubble smaller or larger. By contrast, for the bubble we analyzed earlier, the return on the asset r_t^A and the risk-free rate r_t were equal. The reason a bubble emerged in that setting is because the asset produced a shadow dividend reflecting the value of the bubble as a superior savings instrument, and the price was equal to the present discounted value of the shadow dividend which exceeded d. In that case, there was no ambiguity about the effect an intervention on the bubble: Raising rates lowered the discounted value of both dividends and shadow dividends. Since the latter is larger, the higher rates have a larger effect on the price than on fundamentals. The effect of interventions on credit-driven bubbles can be different.

Calculating the fundamental value f_t when T is distributed geometrically turns out to be unwieldy. However, we can obtain analytical insights if we assume T has a two point-support, i.e.,

$$T = \begin{cases} 1 & \text{with probability } \pi \\ 2 & \text{with probability } 1-\pi \end{cases}$$

In this case, the endowment will either grow for one period or not at all. Speculators at date 0 can gamble on whether the endowment will grow between dates 0 and 1, but there is no scope for gambling beyond date 0. This change still admits a bubble. We can then establish the following result:

Proposition 8: Suppose fiscal policy is given by (14), where $r_{-1} > 0$. There exists a $\kappa^* \in (0, e_0)$ such that the initial bubble Δ_0 is increasing in b if $0 < \kappa < \kappa^*$ and decreasing in b if $\kappa^* < \kappa < e_0$.

In words, as long as there are enough safe borrowers, an intervention to issue more government debt will increase rates, drive down the price of the asset, and dampen the bubble. Intuitively, a larger κ implies entrepreneurs use more resources, leaving less to be used for purchasing the asset. As a result, the price of the asset will be low. Since the fundamental value cannot exceed the price, it too must be small. When the fundamental value is low, a higher interest rate has a small effect on the fundamental value. By contrast, the price will always fall by at least b. In this case, issuing more debt will depress the bubble.

4.5 Welfare

The credit-driven bubble above is very much in line with the bubble Mishkin identifies as a particular source of concern for policymakers: Credit increases demand for the asset, which results in the asset being

overvalued, and agents default if and when the bubble eventually bursts. We showed that, at least under certain conditions, intervention to raise rates while the bubble exists can dampen the bubble. Thus, we have a setup that seemingly captures the scenario that would most justify leaning against the wind. And yet, even in this case, the model still does not imply intervention can make society as a whole better off. This is because when the supply of the asset is fixed, as we have assumed so far, any intervention would simply redistribute resources from entrepreneurs to lenders. Some agents benefit from intervention, but others lose. To see this, recall that speculators earn zero profits in equilibrium, by virtue of (28), and are therefore unaffected by the intervention. Next, observe that savers always spend $e_t - \tau_t$ to buy bonds and make loans, regardless of how much government debt circulates. Lenders earn $(1 + r_t)\, b_t$ on the government bonds they hold, $(1 + R_t)\, \kappa$ from their loans to entrepreneurs, and expect to earn $\left(1 + r_t^A\right) p_t$ from lending to speculators to buy the asset. Their expected return is thus

$$\frac{(1 + r_t)\, b_t + (1 + R_t)\, \kappa + \left(1 + r_t^A\right) p_t}{e_t - \tau_t}$$

Using the expressions for p_{t+1} and $\widehat{p}_{t+1}$ from (29), the expected return to savers above is equal to

$$\frac{(1 + (1 - \pi)\, g)\, e_t + d + R_t \kappa}{e_t - \tau_t}$$

From Proposition 7, we know that intervening to raise the risk-free rate will increase the interest rate R_t on loans. But an increase in R_t helps savers at the expense of entrepreneurs, who must hand over more of the $1 + y$ units of output they produce. Beyond this redistribution, raising r_t and driving down p_t has no effect on welfare. In particular, the risk from the uncertain growth rate for the economy is always fully borne by young agents who wish to save, regardless of b. Reducing the bubble cannot eliminate this risk which is inherent in the endowment of agents. Dampening the bubble is not Pareto improving.

To break this logic, we need to allow the supply of assets to be variable. In that case, society might be worse off if a larger bubble encouraged the creation of too many bubble assets. Developing a full-fledged example of this is beyond the scope of this paper. However, in Appendix D we illustrate the potential for a model with variable asset supply to admit a role for policy intervention. In that section, we consider a different setup in which the uncertainty that speculators gamble on comes from the asset's dividend rather than the aggregate endowment. The key distinction is that the risk due to an uncertain aggregate endowment process is exogenously fixed, whereas the risk due to the assets themselves depends on the *endogenous size* of the stock of assets. Our example in Appendix D shows that the type of credit-driven bubbles we have analyzed here can emerge in an environment where the stock of assets is variable and the amount of aggregate risk is endogenous.

We have not shown that an intervention is desirable in this example if and when a bubble arises; but our example suggests how one might go about establishing this. First, if we assumed agents were risk-averse rather than risk-neutral, any effects on the endogenous amount of aggregate risk would certainly matter for welfare. Since the agents who create new assets are responding to its price rather than its fundamental, there is no reason to think a laissez-faire equilibrium would be associated with the socially optimal amount of risky assets. But adding risk-aversion significantly complicates solving for an equilibrium.

As a separate argument, more aggregate risk magnifies the losses lenders would realize if and when the bubble bursts. Lenders would take this into account and set higher rates on loans. But the realized losses can still be greater the larger the bubble. Developing this line of reasoning would require us to model the consequences of losses to financial intermediaries, which again is beyond the scope of our paper. Hence, our analysis only suggests how intervention *might* be desirable. But it identifies the key components that would be needed to justify a policy of leaning against the wind. This in itself is informative. As we noted in the introduction, our results suggest there is probably less scope for intervening against bubbles on assets available in fixed supply, e.g., land, than on assets that can be accumulated. And it also suggests that the reason to move against an incipient bubble is not so much to align its price with fundamentals or to curb speculation, but to mitigate the excessive creation of risk that bubbles might foster.

5 Conclusion

In this paper, we argued that moving to raise interest rates can be an effective tool against asset bubbles. This result stands in contrast to recent work by Galí (2014) which showed that moving to raise interest rates only serves to make a bubble larger if one were in fact present. We began by replicating Galí's result in a slightly different setting. We argued that the intervention Galí analyzed amounted to selecting equilibria, and that in his model equilibria that feature higher interest rates also feature larger bubbles. We then showed that introducing a seemingly inconsequential modification to our model – assuming the bubble asset is not intrinsically worthless but yields actual dividends – can eliminate the multiplicity of equilibria, forcing us to think of policy interventions not in terms of selecting an equilibrium but in terms of a direct intervention in financial markets such as issuing or selling government bonds. We construct examples of interventions that simultaneously raise the interest rate and reduce the extent to which the bubble asset is overvalued. Thus, the notion that a policymaker intent on mitigating a bubble should intervene to raise interest rates can be plausible. Once we show that raising rates can dampen bubbles, the next question is whether dampening bubbles is indeed desirable. We argued that the model Galí used, a variant of the Samuelson (1958) model in which bubbles arise when agents are eager to save, does not on its own suggest such interventions are desirable. We then showed how we can modify our model to give rise to credit-driven bubbles. That setup seems to better capture the reasons policymakers are concerned about bubbles. We also argued that in such an environment there may be a role for policy in curbing aggregate risk.

We conclude our analysis with a few observations. First, one of the points Galí emphasizes in his paper is that theoretically, higher interest rates ought to be associated with more rapid asset price appreciation. In subsequent work, Galí and Gambetti (2015) provide empirical evidence in support of this proposition. Our model, which also features rational agents, is consistent with this implication. However, whether a policymaker who intervenes to raise rates succeeds in depressing a bubble depends not on how interest rates affect the growth rate of asset prices, but how they affect asset prices as compared to fundamentals. Unfortunately, since this requires measuring the fundamental value of assets, the latter is difficult if not impossible to resolve empirically. But our analysis nevertheless shows that the case in which higher rates

affect asset prices more than they do fundamentals is a plausible theoretical possibility.

Even if empirical evidence on how interest rates impact prices and fundamentals is hard to come by, our analysis does suggest empirical measures that policymakers can use to assess whether contractionary policies are likely to have their intended effect on bubbles, at least in line with the channels we emphasize here. The reason contractionary policy mitigates bubbles in our model is that it crowds out resources from overvalued assets. This suggests that for an interest rate hike to depress bubbles through the channel we describe in this paper, it should be accompanied by a decline in savings or in the share of the bubble asset in the aggregate wealth portfolio of investors. Absent these, our model would suggest an interest rate hike would not be successful in reining in bubbles.

Of course, there may be other channels through which a higher interest rate might depress bubbles that our model fails to capture. For example, some policymakers argue that low interest rates trigger a search for yield that induces agents to take on more risks. Our model does not admit such a channel. We also ignore any role for interest rate policy to affect bubbles by affecting credit markets. Although we do describe a model in Section 4 where agents borrow to speculate on assets, the amount speculators borrow in our model in unaffected by the interest rate. In principle, a higher interest rate might make speculation unprofitable, even if a higher rate increases the expected rate of appreciation. The notion that higher rates would directly discourage speculation suggests another channel by which higher rates might crowd out spending on bubbles, and thus is also missing in the original framework analyzed in Galí (2014). However, formalizing this reasoning would presumably require a richer model of credit spreads than we develop here.

The last point we wish to make is that while our analysis here can be viewed as a first step towards a model that can be used to justify leaning against the wind policies, further work is needed to determine whether such policies represent the best way to combat asset bubbles. Even if one can show that dampening credit-driven bubbles is welfare improving by reducing macroeconomic volatility, there is more than one way to attempt to reduce a bubble. For example, a central bank could impose regulatory restrictions on credit markets that discourage or mitigate credit-driven bubbles. In the model we develop, for example, a restriction on the amount agents can borrow against an asset would have implications on the size of the bubble. Such restrictions would also discourage socially valuable trade between creditors and entrepreneurs. Determining what is the most cost effective way to dampen bubbles remains a challenge for future work.

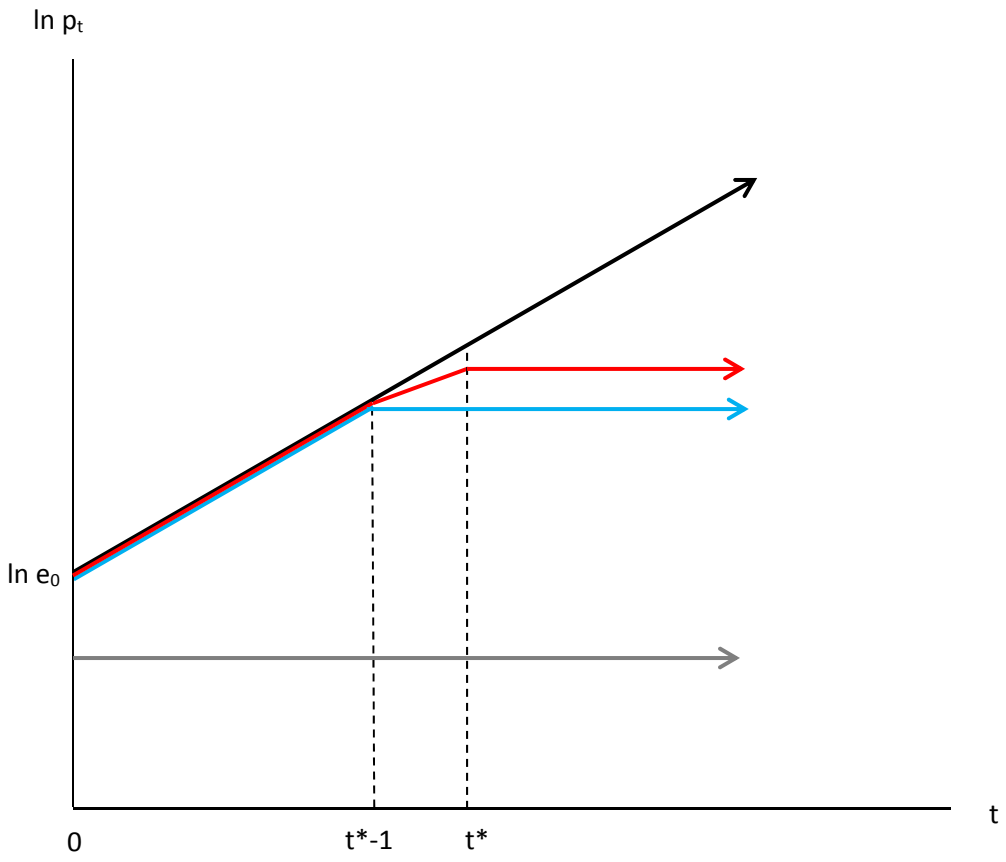

Figure 1: Sample equilibrium price paths for the case where d = 0

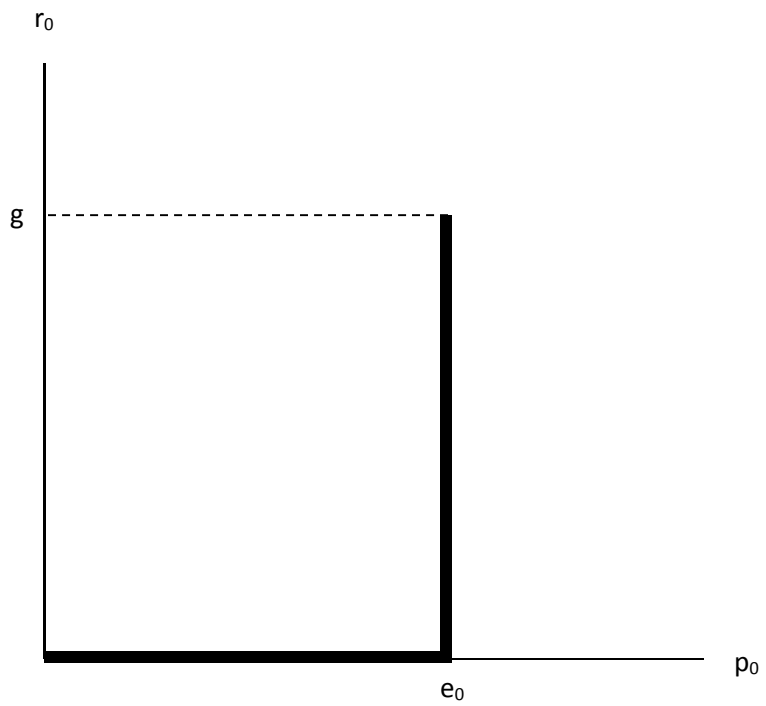

Figure 2: Set of possible equilibrium values (p_0,r_0) at date 0

References

[1] Adam, Klaus, 2003. "Learning and Equilibrium Selection in a Monetary Overlapping Generations Model with Sticky Prices" *Review of Economic Studies*, 70(4), October, p887-907.

[2] Allen, Franklin and Douglas Gale, 2000. "Bubbles and Crises" *Economic Journal*, 110(460), January, p236-255.

[3] Allen, Franklin and Gary Gorton, 1993. "Churning Bubbles" *Review of Economic Studies*, 60(4), October, p813-836.

[4] Barlevy, Gadi, 2014. "A Leverage-based Model of Speculative Bubbles" *Journal of Economic Theory*, 153, p459-505.

[5] Barlevy, Gadi and Jonas Fisher, 2014. "Mortgage Choice and House Price Speculation" Federal Reserve Bank of Chicago working paper.

[6] Bernanke, Ben and Mark Gertler, 1999. "Monetary Policy and Asset Price Volatility" *Federal Reserve Bank of Kansas City Economic Review*, 4th Quarter, p17-51.

[7] Blanchard, Olivier and Stanley Fischer, 1989. *Lectures on Macroeconomics*. Cambridge, Massachusetts: MIT Press.

[8] Blanchard, Olivier and Mark Watson, 1982. "Bubbles, Rational Expectations, and Financial markets" *Crisis in the Economic and Financial Structure: Bubbles, Bursts, and Shocks*, ed. P. Wachtel. Lexington, Massachusetts: Lexington Books, p295–315.

[9] Caballero, Ricardo, Emmanuel Farhi, and Mohamad Hammour, 2006. "Speculative Growth: Hints from the U.S. Economy" *American Economic Review*, 96(4), September, p1159-92.

[10] Diamond, Peter, 1965. "National Debt in a Neoclassical Growth Model" *American Economic Review*, 55(5), Part 1, December, p1126-50.

[11] Dong, Feng, Jianjun Miao, and Pengfei Weng, 2017. "Asset Bubbles and Monetary Policy" Boston University working paper.

[12] Farhi, Emmanuel and Jean Tirole, 2012. "Bubbly Liquidity" *Review of Economic Studies*, 79, p678-706.

[13] Galí, Jordi, 2014. "Monetary Policy and Rational Asset Price Bubbles" *American Economic Review*, 104(3), March, p721-52.

[14] Galí, Jordi and Luca Gambetti, 2015. "The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence" *American Economic Journal: Macroeconomics*, 7(1), p233-57.

[15] Greenspan, Alan, 1996. "The Challenge of Central Banking in a Democratic Society" Speech presented at the American Enterprise Institute in Washington DC on December 5. Available at http://www.federalreserve.gov/boarddocs/speeches/1996/19961205.htm.

[16] Hirano, Tomohiro and Noriyuki Yanagawa, 2017. "Asset Bubbles, Endogenous Growth, and Financial Frictions" *Review of Economic Studies*, 84(1), p406-43.

[17] Ikeda, Daisuke, 2017. "Monetary Policy, Inflation and Rational Asset Price Bubbles" Bank of England Working Paper.

[18] Kocherlakota, Narayana, 1992. "Bubbles and Constraints on Debt Accumulation" *Journal of Economic Theory*, 57 (1), p245—56.

[19] Kocherlakota, Narayana, 2008. "Injecting Rational Bubbles" Journal of Economic Theory, 142(1), September, p218-32.

[20] Kocherlakota, Narayana, 2009. "Bursting Bubbles: Consequences and Cures" FRB Minneapolis working paper.

[21] Kirshnamurthy, Arvind and Anette Vissing-Jorgensen, 2012. "The Aggregate Demand for Treasury Debt" *Journal of Political Economy*, 120(2), April, p233-67.

[22] Martin, Alberto and Jaume Ventura, 2012. "Economic Growth with Bubbles" *American Economic Review*, 102(6), p3033-58.

[23] Martin, Alberto and Jaume Ventura, 2016. "Managing Credit Bubbles" *Journal of the European Economic Association*, 14(3), June, p753-89.

[24] Miao, Jianjun and Pengfei Wang, 2015 "Bubbles and Credit Constraints" Boston University working paper.

[25] Mishkin, Frederic, 2011. "Monetary Policy Strategy: Lessons from the Crisis', in M Jarociński, F Smets and C Thimann (eds), *Approaches to Monetary Policy Revisited – Lessons from the Crisis*, European Central Bank, Frankfurt, p67–118.

[26] Samuelson, Paul, 1958. "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money" *Journal of Political Economy*, 66, p467-82.

[27] Santos, Manuel and Michael Woodford, 1997. "Rational Asset Pricing Bubbles" *Econometrica*, 65(1), January, p19-57.

[28] Summers, Lawrence, 2016. "The Age of Secular Stagnation" *Foreign Affairs*, 95, February 15.

[29] Tirole, Jean, 1985. "Asset Bubbles and Overlapping Generations" *Econometrica*, 53(6), November, p1499-1528.

[30] Rhee, Changyong, 1991. "Dynamic Inefficiency in an Economy with Land" *Review of Economic Studies*, 58(4), June, p791-7.

[31] Rocheteau, Guillaume and Randall Wright, 2013. "Liquidity and Asset Market Dynamics" *Journal of Monetary Economics*, 60, p275-94.

[32] Waldo, Douglas, 1985. "Open Market Operations in an Overlapping Generations Model" *Journal of Political Economy*, 93(6), p1242-57.

[33] Weil, Philippe, 1987. "Confidence and the Real Value of Money in an Overlapping Generations Economy" *Quarterly Journal of Economics*, 102(1), February, p1-22.

Appendix A: Proofs of Propositions

Proof of Proposition 1: In the text, we argued that $r_t = 0$ is an absorbing state. That is, there exists a t^* where $0 \leq t^* \leq \infty$ such that $r_t > 0$ for $t < t^*$ and $r_t = 0$ for $t \geq t^*$. For $t < t^*$, we know that $r_t > 0$ implies storage is dominated, and so $p_t = e_t$ for $t < t^*$. Since $r_t = 0$ for $t \geq t^*$ we can use (3) to conclude that $p_{t+1} = p_t$ for $t \geq t^*$, and by induction we can infer $p_t = p_{t^*}$ for all $t \geq t^*$.

The last step is to show that at date t^*, any $\rho \in [e_{t^*-1}, e_{t^*}]$ can be an equilibrium, and only these values can be an equilibrium. Since $r_t \geq 0$, $p_{t^*} \geq p_{t^*-1} = e_{t^*-1}$. Since $p_t \leq e_t$ at all dates t, this is also true for date t^*. For any $\rho \in (e_{t^*-1}, e_{t^*}]$, the rate of return on the asset to those purchasing the asset at date $t^* - 1$ will be positive, so they would buy the asset at this price at date $t^* - 1$ and sell all their holdings at date t^*. Since $r_{t^*} = 0$, the young at date t^* are indifferent between storage and buying the asset, so they would be willing to buy any amount the old sell. Hence, we can construct an equilibrium where $p_{t^*} = \rho$ for any $\rho \in (e_{t^*-1}, e_{t^*}]$. ■

Proof of Proposition 2: We prove a more general result which implies the Proposition. In particular, we show that for a sequence of nonnegative dividends $d_t \geq 0$, there is a unique equilibrium iff $\sum_{t=0}^{\infty} d_t = \infty$. Since the case where $d_t = d > 0$ satisfies this condition, the claim will follow.

Suppose $d_t \geq 0$ for all t and $\sum_{t=0}^{\infty} d_t = \infty$. We argue that the interest rate r_t must be positive at all dates. For suppose $r_t = 0$ for some date t. Then $p_{t+1} = p_t - d_t$ and since $p_t \leq e_t$, it follows that $p_{t+1} < e_t < e_{t+1}$, which implies that $r_{t+1} = 0$. By this logic, the price declines by d_{t+i} at each date $t + i$, for $i = 1, 2, ...$ Since $\sum_{t=0}^{\infty} d_t = \infty$ implies $\lim_{k\to\infty} \sum_{i=1}^{k} d_{t+i} = \infty$, there must be a date $t + k$ such that $p_{t+k} = p_t - \sum_{i=1}^{k} d_{t+i} < 0$, i.e., there must be some date $t+k$ at which the price of the asset turns negative. But this is incompatible with equilibrium. Hence, $r_t > 0$ at all dates. This implies storage is dominated, and so the unique equilibrium price is $p_t = e_t$ for all t. The corresponding return on holding the asset is then

$$1 + r_t = \frac{d_t + p_{t+1}}{p_t} = (1 + g) + \frac{d_t}{e_t}$$

We can further show that the condition $\sum_{t=0}^{\infty} d_t = \infty$ is a necessary condition for uniqueness. The proof is constructive. Suppose $\sum_{t=0}^{\infty} d_t < \infty$. Then there must be some date t^* such that $e_{t^*} - \sum_{t=t^*}^{\infty} d_t > 0$. We now propose an equilibrium price sequence as follows. For $t < t^*$, set $p_t = e_t$; for $t = t^*$, set p_t to be any

value such that $\max\left\{e_{t^*-1} - d_{t^*}, \sum_{s\geq t^*} d\right\} < p_t \leq e_{t^*}$; and for $t > t^*$, let $p_t = p_{t*} - d_{t^*+1} - \ldots - d_t$. The price sequence is positive for every date t because $p_t = e_t > 0$ for $t < t^*$ and $p_t \geq p_{t^*} - \sum_{s\geq t^*} d_s > 0$ for $t > t^*$. The price sequence p_t corresponds to an equilibrium because, for $t < t^*$, the interest rate is positive and the entire endowment is invested in the asset and, for $t \geq t^*$, the interest rate is zero. Thus, there are multiple equilibria in this case. ■

Proof of Lemma 1: We first argue that $r_t > 0$ for all t. For suppose $r_t = 0$ at some date. From (10), we would then have

$$b_{t+1} + \tau_{t+1} = b_t$$

Moreover, from (12), we have

$$\begin{aligned} p_t &\leq e_t - \tau_t - b_t \\ &= e_t - \tau_t - (b_{t+1} + \tau_{t+1}) \\ &< e_{t+1} - (b_{t+1} + \tau_{t+1}) \end{aligned}$$

where the last step uses the fact that $e_{t+1} > e_t$ and $\tau_t \geq 0$ for all t. Hence, just as before, $r_t = 0$ is an absorbing interest rate. From (11), this implies

$$p_{t+1} = p_t - d$$

which means the price would eventually turn negative. So $r_t > 0$ for all t. It then follows from (12) that $p_t = e_t - b_t - \tau_t$.

We now proceed by induction. Suppose we are given histories $\{b_{-1}, ..., b_{t-1}\}$ and $\{r_{-1}, ..., r_{t-1}\}$ up to but not including date t. We want to show that these histories imply a unique candidate pair b_t and r_t that satisfy both the government budget constraint and the equilibrium interest rate condition. From the government's flow budget constraint, we have

$$b_t = (1 + r_{t-1})\, b_{t-1} - \tau_t$$

Since τ_t, b_{t-1}, and r_{t-1} are all given to us, b_t is uniquely determined. Next, consider the equilibrium condition for interest rates (11). We already argued that when $d > 0$, the equilibrium interest rate r_t must be positive or else the price of the asset will eventually turn negative. Hence, storage will be dominated, and agents will exchange all of their endowment net of taxes for bonds and the asset, i.e.,

$$p_t = e_t - \tau_t - b_t$$

Substituting in for p_t and p_{t+1} into (11), we get

$$(1 + r_{t-1})\, b_{t-1} - b_t = (1 + r_{t-1})\,(e_{t-1} - \tau_{t-1}) - (e_t - \tau_t) - d$$

But from (10), we know that

$$(1 + r_{t-1})\, b_{t-1} - b_t = \tau_t$$

and so we have

$$\tau_t = (1 + r_{t-1})(e_{t-1} - \tau_{t-1}) - (e_t - \tau_t) - d$$

which reduces to

$$d = (1 + r_t)(e_t - \tau_t) - e_{t+1} \tag{33}$$

Since e_t, e_{t+1}, and τ_t are given, r_t is uniquely determined. The claim then follows by induction. ■

Proof of Lemma 2: Condition (13) implies that

$$\lim_{t\to\infty} \frac{\tau_t + b_t}{e_t} = \chi$$

for some $\chi < 1$. Hence,

$$\begin{aligned} \lim_{t\to\infty} p_t &= \lim_{t\to\infty} e_t - \tau_t - b_t \\ &= \lim_{t\to\infty} e_t - \chi e_t \\ &= (1-\chi) \lim_{t\to\infty} e_t = \infty \end{aligned}$$

where the last step uses the fact that $\chi < 1$. ■

Proof of Proposition 3: At $t = 0$, we have $\tau_0 = r_{-1}b$ which is increasing in b. From the intertemporal budget constraint (10) and the equilibrium condition (11), we know that

$$d = (1 + r_t)(e_t - \tau_t) - (1 + g)\, e_t$$

Since τ_0 is increasing in b, it follows that r_0 is increasing in b as well. Next, suppose we know $r_0, ..., r_{t-1}$ are increasing in b. Then we have $\tau_t = r_{t-1}b$ and by the same logic as above, we can conclude that τ_t and r_t are increasing in b. To finish the claim, we need to prove that p_t is decreasing in b. But since

$$p_t = e_t - b - \tau_t$$

the result follows. ■

Before proving Proposition 4, we establish the following lemma:

Lemma A1: Suppose fiscal policy is given by (14) and satisfies (13). Then $\lim_{t\to\infty} r_t = g$.

Proof of Lemma A1: Recall from (33) that

$$d = (1 + r_t)(e_t - \tau_t) - e_{t+1}$$

Using the fact that $e_{t+1} = (1 + g)\, e_t$ and the fact that when $b_t = b$ for all t,

$$\begin{aligned} \tau_t &= (1 + r_{t-1})\, b_{t-1} - b_t \\ &= (1 + r_{t-1})\, b - b = r_{t-1}b \end{aligned}$$

we can rewrite (33) as

$$d = (r_t - g)\, e_t - (1 + r_t)\, r_{t-1} b$$

which, upon rearranging, yields a difference equation that relates r_t to r_{t-1}:

$$r_t = \frac{d + ge_t + r_{t-1} b}{e_t - r_{t-1} b} \equiv h_t\left(r_{t-1}\right) \tag{34}$$

Since $r_{t-1} b = b_t + \tau_t$, we can rewrite this as

$$r_t = \frac{d + ge_t + b_t + \tau_t}{e_t - b_t - \tau_t}$$

From (13), we know that

$$\lim_{t\to\infty} b_t + \tau_t = \lim_{t\to\infty} \chi e_t$$

for some $\chi < 1$. Hence, taking limits as t tends to infinity, we get

$$\begin{aligned}
\lim_{t\to\infty} r_t &= \lim_{t\to\infty} \frac{d + ge_t + b_t + \tau_t}{e_t - b_t - \tau_t} \\
&= \lim_{t\to\infty} \frac{d + (g + \chi)\, e_t}{(1 - \chi)\, e_t} \\
&= \frac{g + \chi}{1 - \chi}
\end{aligned}$$

where the last expression is finite. But if $\lim_{t\to\infty} r_t$ is finite, then

$$\begin{aligned}
\lim_{t\to\infty} r_t &= \lim_{t\to\infty} \frac{d + ge_t + r_{t-1} b}{e_t - r_{t-1} b} \\
&= \lim_{t\to\infty} \frac{g + \frac{d + r_{t-1} b}{e_t}}{1 - \frac{r_{t-1} b}{e_t}} \\
&= g
\end{aligned}$$

where the last equality follows from the fact that if $\lim_{t\to\infty} r_t$ is finite, so is $\lim_{t\to\infty} r_t b$. Hence, (13) together with (14) imply that $\lim_{t\to\infty} \frac{b_t + \tau_t}{e_t} = \chi = 0$ and $\lim_{t\to\infty} r_t = g$, as claimed. ■

Proof of Proposition 4: By repeated substitution, we can relate Δ_t to any Δ_{t+h} for any horizon h:

$$\Delta_t = \left(\prod_{j=0}^{h-1} \frac{1}{1 + r_{t+j}} \right) \Delta_{t+h}$$

From Lemma A1, we know that $\lim_{t\to\infty} r_t = g$. Hence, for any value of b, we have

$$f_t \equiv \lim_{t\to\infty} \sum_{j=0}^{\infty} \left(\prod_{i=1}^{j} \frac{1}{1 + r_{t+j}} \right) d \to \frac{d}{g}$$

By contrast, $p_t = e_t - r_{t-1} b$, which tends to $e_t - gb$. This implies that the bubble term $\Delta_t = p_t - f_t$ tends to $e_t - gb - \dfrac{d}{g}$, which is decreasing in b. In other words, there exists a T such that for $t \geq T$, Δ_t is decreasing in

b. Since r_{t+j} is increasing in b by Proposition 3, it follows that $\Delta_t = \left(\prod_{j=0}^{h-1} \frac{1}{1+r_{t+j}}\right) \Delta_{t+h}$ will be decreasing in b. ■

Proof of Proposition 5: We show the statement holds for the fundamentals at date 0. The argument for other dates is similar. The fundamental value of the asset at date 0 if growth stops until a random date T_n is given by

$$f_0^n = \sum_{T=1}^{n} \frac{(1-\pi)^{T-1}\pi}{1-(1-\pi)^n} \left(\sum_{s=1}^{T-1} \left(\prod_{i=0}^{s} \frac{1}{1+r_i} \right) d + \left(\prod_{i=0}^{T-1} \frac{1}{1+r_i} \right) e_T \right)$$

Here we use the fact that $f_t^n = e_t$ for $t \geq n$. The above summation can be written as two sums:

$$f_0^n = \sum_{T=1}^{n} \frac{(1-\pi)^{T-1}\pi}{1-(1-\pi)^n} \sum_{t=1}^{T-1} \left(\prod_{i=0}^{s} \frac{1}{1+r_i} \right) d + \sum_{T=1}^{n} \frac{(1-\pi)^{T-1}\pi}{1-(1-\pi)^n} \left(\prod_{i=0}^{T-1} \frac{1}{1+r_i} \right) e_T$$

Since for $t < n$, we have

$$r_t = (1-\pi)\, g + \frac{d}{e_t} > (1-\pi)\, g > 0$$

we can easily establish that the first term converges. As for the second term, once again using the expression for r_t when $t < n$, we have

$$\begin{aligned}
\sum_{T=1}^{n} \frac{(1-\pi)^{T-1}\pi}{1-(1-\pi)^n} \left(\prod_{i=0}^{T-1} \frac{1}{1+r_i} \right) e_T &= \sum_{T=1}^{n} \frac{(1-\pi)^{T-1}\pi}{1-(1-\pi)^n} \left(\prod_{i=0}^{T-1} \frac{1+g}{1+(1-\pi)\, g + d/e_i} \right) e_0 \\
&= \sum_{T=1}^{n} \frac{\pi}{1-(1-\pi)^n} \left(\prod_{i=0}^{T-1} \frac{(1-\pi)(1+g)}{1+(1-\pi)\, g + d/e_i} \right) \frac{e_0}{1-\pi} \\
&< \sum_{T=1}^{n} \frac{\pi}{1-(1-\pi)^n} \left(\prod_{i=0}^{T-1} \frac{(1-\pi)(1+g)}{1+(1-\pi)\, g} \right) \frac{e_0}{1-\pi}
\end{aligned}$$

Since the last term converges whenever $\pi > 0$, we know that $\lim_{n\to\infty} f_0^n$ exists. Moreover, we know that $f_0^n = e_0$. This can be computed directly, but it also follows from a simple unravelling argument for bubbles with finite horizons. Hence, $f_0 = \lim_{n\to\infty} f_0^n = e_0$. From this, we can conclude that $f_0 = e_0$. A similar argument can be applied at any date. ■

Proof of Proposition 7: The implications of higher b for r_t and p_t can be established in the same way as in Proposition 3. From (31), we have that

$$\begin{aligned}
R_t &= r_t + \frac{\pi g e_t}{e_t - b_t - \tau_t} \\
&= r_t + \frac{\pi g e_t}{e_t - (1+r_{t-1})\, b}
\end{aligned}$$

Since both r_t and $(1+r_{t-1})\, b$ are increasing in b, we can conclude that R_t is increasing in b as well. ■

Proof of Proposition 8: Observe that the price of the asset at date 0 is given by

$$\begin{aligned}
p_0 &= e_0 - b_0 - \tau_0 - \kappa \\
&= e_0 - (1+r_{-1})\, b - \kappa
\end{aligned}$$

Next, from (32), we have

$$(1+r_0)(e_0-b_0-\tau_0-\kappa)=(1-\pi)ge_0+e_0-b_1-\tau_1-\kappa+d+s_0\kappa$$

where

$$s_0=\frac{\pi g e_0}{e_0-b_0-\tau_0}$$

Using the parameterization (14), we have

$$\tau_t=r_{t-1}b$$

and so

$$(1+r_0)(e_0-b-r_{-1}b-\kappa)=(1-\pi)ge_0+e_0-(1+r_0)b-\kappa+d+s_0\kappa$$

Rearranging, we have an expression for r_0 in terms of b and other primitives.

$$1+r_0=\frac{(1-\pi)ge_0+r_{-1}b+d+s_0\kappa}{e_0-r_{-1}b-\kappa}$$

The fundamental value at date 0 is given by

$$f_0=\frac{d+(1+(1-\pi)g)e_0-(1+r_0)b-\kappa}{1+r_0}$$

Substituting in for r_0 and evaluating $\Delta_0=p_0-f_0$ yields

$$\Delta_0=\frac{\pi g e_0\kappa(r_{-1}b-e_0+\kappa)}{(1+r_{-1})b(d+(1+(1-\pi)g)e_0-\kappa)-e_0(d+e_0(1+(1-\pi)g)-\kappa(1-\pi g))}$$

Differentiating Δ_0 with respect to b and simplifying (with Mathematica) yields

$$\frac{d\Delta_0}{db}=\frac{(1+(1-\pi)g)e_0^2-(2+(1+g)r_{-1}+g(1-\pi))e_0\kappa+(1+r_{-1})\kappa^2+(e_0-(1+r_{-1})\kappa)d}{[(1+r_{-1})b(d+(1+(1-\pi)g)e_0-\kappa)-e_0(d+e_0(1+(1-\pi)g)-\kappa(1-\pi g))]^2}\pi g e_0\kappa$$

The sign of this derivative corresponds to the sign of the numerator in the fraction above, which is independent of b, and represents a quadratic in κ. This quadratic is convex, since the coefficient on κ^2 is $1+r_{-1}>0$. Evaluating the quadratic at $\kappa=0$ yields

$$(1+(1-\pi)g)e_0^2+e_0d$$

which is positive, while evaluating the quadratic at $\kappa=e_0$ yields

$$-e_0(d+ge_0)r_{-1}$$

which is negative. This tells us that the roots of the quadratic are both real, with one between 0 and e_0 and the other above e_0. It follows that we can find a $\kappa^*\in(0,e_0)$ such that $\frac{d\Delta_0}{db}>0$ if $\kappa\in(0,\kappa^*)$ and $\frac{d\Delta_0}{db}<0$ if $\kappa\in(\kappa^*,e_0)$. ■

Appendix B: Monetary Policy

In this Appendix, we describe a monetary OLG economy, building on the framework we sketch out in Section 3. The key difference between the model here and the one sketched out in Section 3 is that rather than assuming workers are yeoman farmers who operate their own technology, we now introduce producers who price their goods and hire labor, allowing us to incorporate the possibility of price rigidity. Our framework borrows elements from both Adam (2003) and Appendix 3 in Galí (2014). As in Galí, we assume agents hold money because they derive utility directly from money holdings without modelling why. However, we follow Adam in allowing for variable labor supply and in assuming that the entrepreneurs who hire workers are young rather than old. The implication of this is that all output generated within the period will be used to buy assets and not just a fraction of the output that accrues to the young.

Following Adam (2003), suppose each cohort consists of a unit mass of workers and a unit mass of entrepreneurs. This is in contrast to the model described in Section 3, where we assume there is only a single unit mass of individuals in each cohort. When agents are young, those who are workers supply their labor services to entrepreneurs who know how to deploy labor to produce goods. When they turn old, neither type is productive any more and each must rely on previous earnings to consume. We assume that the two cohorts equally bear the burden of lump sum taxes, so that when the government collects τ_t from the young, it collects $\frac{1}{2}\tau_t$ from workers and entrepreneurs.

As in the text, we use M_t to denote the amount of money circulating at date t, Π_t to denote the gross inflation rate P_{t+1}/P_t, and

$$x_{t+1} = \frac{M_{t+1} - M_t}{P_{t+1}}$$

as the injection of money between dates t and $t+1$ measured in terms of how much this amount could buy at date $t+1$. We assume the injection is split equally between old workers and old entrepreneurs, so each expects to receive $\frac{1}{2}x_{t+1}$ when old.

We start with workers. Each worker is endowed with one unit of time and must decide how to allocate it. The cost of providing effort n_t for the cohort born at date t is given by a convex function $v_t(n_t)$. As in the text, we assume $v_t(n_t) = A_t v(n_t)$ where $\lim_{n\to 0} v'(n) = 0$ and $\lim_{n\to 1} v'(n) = \infty$. In addition to caring about consumption when old and leisure when young, workers derive utility from their money holdings as in Galí (2014). Specifically, we replace (1) with

$$u\left(c_t^w, c_{t+1}^w, m_t^w, n_t\right) = c_{t+1}^w + \frac{\theta}{2}\ln\left(m_t^w\right) - v_t\left(n_t\right) \tag{35}$$

The budget constraint of a worker is given by

$$c_{t+1}^w = (1+r_t)\left(\frac{W_t}{P_t}n_t - m_t^w - \frac{1}{2}\tau_t\right) + \Pi_t^{-1}m_t^w + \frac{1}{2}x_{t+1} \tag{36}$$

where W_t denotes the nominal wage per unit labor. Since government bonds and the asset are perfect substitutes, agents will be indifferent between them. Hence, workers face only two non-trivial choices: How

hard to work and how much of their wealth to hold as money. The optimal effort level n_t will satisfy

$$A_t v'(n_t) = (1 + r_t) \frac{W_t}{P_t}$$

and the optimal level of money demand will satisfy

$$m_t^w = \frac{\theta}{2\left((1 + r_t) - \Pi_t^{-1}\right)} = \frac{\theta \Pi_t}{2 i_t}$$

Next, we turn to entrepreneurs. They are endowed not with labor but with the knowledge of how to deploy labor to produce goods. Each entrepreneur $i \in [0, 1]$ can produce a different intermediate good i. These goods are sold to competitive final goods producers who produce the goods old agents consume. Entrepreneurs derive utility from consuming when old and from holding money, i.e.

$$u\left(c_t^e, c_{t+1}^e, m_t^e\right) = c_{t+1}^e + \frac{\theta}{2} \ln\left(m_t^e\right) \tag{37}$$

The budget constraint of an entrepreneur is given by

$$c_{t+1}^e = (1 + r_t)\left(\rho_{it} - m_t^e - \frac{1}{2}\tau_t\right) + \Pi_t^{-1} m_t^e + \frac{1}{2} x_{t+1} \tag{38}$$

where ρ_{it} denote the profits of entrepreneur i after paying the workers he hires. As will become clear below, entrepreneurs also face two non-trivial choices: How much of their earnings to hold as money, just like workers, and what price to charge for the particular good they produce. Their demand for money is the same as workers, i.e.

$$m_t^e = \frac{\theta \Pi_t}{2 i_t}$$

As for what price to set, we first need to specify the production technology for both intermediate and final goods. Suppose that if entrepreneur $i \in [0, 1]$ hires n_{it} units of labor to work at date t, he will produce

$$y_{it} = A_t n_{it}$$

units of good i, where $A_t = A_0 (1 + g)^t$. These goods can be combined to produce final goods according to a Dixit-Stiglitz production function, i.e., y_{it} of each good $i \in [0, 1]$ combine to yield Y_t of final goods, where

$$Y_t = \left(\int_0^1 y_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$$

The production of final goods is competitive. Final goods producers will therefore choose intermediate goods y_{it} to produce the Y_t final goods at the lowest possible cost. That is, they solve

$$\max P_t \left(\int_0^1 y_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} y_{it} di$$

The first-order condition with respect to y_{it} for final goods producers yields the demand for each intermediate good as follows:

$$y_{it} = Y_t \left(\frac{P_t}{P_{it}}\right)^{\frac{1}{\sigma}} \tag{39}$$

where recall Y_t denotes the output of final goods in equilibrium. Substituting in for y_{it}, we can compute the cost of producing a unit of final goods. Since the market for final goods is competitive, the price of final goods P_t must equal this cost. Equating the two yields the familiar Dixit-Stiglitz price index:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

Each entrepreneur chooses the price P_{it} of his good to maximize profits ρ_{it} given demand (39) for his good,

$$\left(P_{it} - \frac{W_t}{A_t} \right) Y_t \left(\frac{P_t}{P_{it}} \right)^{\frac{1}{\sigma}}$$

From the first-order condition for this problem, we have

$$P_{it} = \frac{W_t}{(1-\sigma) A_t} \tag{40}$$

Since the price P_{it} will determine demand, the choice of price will fully determine how much labor effort entrepreneur i will hire. The equilibrium nominal wage W_t will equate this demand with the amount workers are willing to supply.

Finally, we turn to the government sector. The government budget constraint still corresponds to (21). By repeated substitution, we get

$$\begin{aligned} (1+r_{t-1}) b_{t-1} &= \sum_{s=0}^{\infty} \left(\prod_{i=0}^{s-1} \frac{1}{1+r_{t+i}} \right) \left[\tau_{t+s} + \left(\frac{M_{t+s} - M_{t+s-1}}{P_{t+s}} \right) \right] \\ &= \sum_{s=0}^{\infty} \left(\prod_{i=0}^{s-1} \frac{1}{1+r_{t+i}} \right) \left[\tau_{t+s} + \frac{1}{P_t} \frac{M_{t+s} - M_{t+s-1}}{\prod_{i=0}^{s-1} \Pi_{t+i}} \right] \end{aligned} \tag{41}$$

Equation (41) states that the outstanding government liability $(1+r_{t-1}) b_{t-1}$ at date t must equal the present discounted value of taxes and seniorage revenue the government is set to collect. We define a monetary intervention as a change in the path $\{M_t\}_{t=0}^{\infty}$ holding the path of lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$ fixed.

We can now examine the effects of monetary policy. Let p_t denote the real price of the asset relative to the final good. An equilibrium is a path for prices $\{p_t, P_t, W_t, r_t\}_{t=0}^{\infty}$ and quantities $\{n_t, c_t^w, c_t^e\}_{t=0}^{\infty}$ such that agents optimize and markets clear. We first consider the case where prices are flexible, i.e., where entrepreneurs set prices P_{it} knowing the full path of $\{M_t\}_{t=0}^{\infty}$ and the implied equilibrium nominal wage W_t associated with this path. We then contrast this with the case where entrepreneurs set their prices before the monetary authority and thus before seeing the realized nominal wage W_t.

Case I: Flexible Prices

We begin with the case where prices are flexible. Since intermediate goods producers all choose the same price according to (40), the price of final goods will be given by

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} = \frac{W_t}{(1-\sigma) A_t}$$

The real wage will then equal $W_t/P_t = (1-\sigma) A_t$, and so the equilibrium amount of labor will solve

$$A_t v'(n_t) = (1+r_t)(1-\sigma) A_t$$

Since entrepreneurs each set the same price and face the same technology, they will hire the same amount of labor in equilibrium. Since there is a unit mass of entrepreneurs, this implies each entrepreneur will hire n_t units of labor where n_t denotes aggregate employment. The total output of final goods will be given by

$$Y_t = A_t \left(\int_0^1 n_t^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = A_t n_t$$

We now argue that when prices are flexible, the equilibrium real interest rate r_t, employment n_t, output of final goods $Y_t = A_t n_t$, and total consumption $C_t = c_t^e + c_t^w$ are all independent of monetary policy, i.e., they will not depend on M_{-1} or $\{M_t\}_{t=0}^{\infty}$. This feature follows from our assumption that agents have log preferences over real money balances. These preferences ensure that even though changes in $\{M_t\}_{t=0}^{\infty}$ affect equilibrium real money balances, changes in real balances do not influence consumption and employment decisions.

Formally, the equilibrium real return on the asset must equal the return on government debt $1+r_t$, i.e.

$$(1+r_t) p_t = d + p_{t+1}$$

Since agents will not use storage in equilibrium, we can substitute in for the price of the asset p_t, i.e.

$$(1+r_t)(A_t n_t - \tau_t - b_t - m_t) = d + (A_{t+1} n_{t+1} - \tau_{t+1} - b_{t+1} - m_{t+1})$$

where aggregate real money balances under log utility are given by

$$m_t = m_t^e + m_t^w = \frac{\theta \Pi_t}{i_t}$$

Substituting the government budget constraint (21) into the equilibrium condition above implies

$$\begin{aligned}
(1+r_t)(A_t n_t - \tau_t) &= d + \left(A_{t+1} n_{t+1} + \left(1 + r_t - \Pi_t^{-1}\right) m_t \right) \\
&= d + \left(A_{t+1} n_{t+1} + i_t \frac{m_t}{\Pi_t} \right) \\
&= d + (A_{t+1} n_{t+1} + \theta)
\end{aligned}$$

and so

$$1 + r_t = \frac{d + A_{t+1} n_{t+1} + \theta}{A_t n_t - \tau_t} \tag{42}$$

From the worker's first order condition, we know that they will choose n_t so that

$$v'(n_t) = (1+r_t)(1-\sigma) \tag{43}$$

Conditions (42) and (43) together yields a difference equation in n_t:

$$\frac{d + A_{t+1} n_{t+1} + \theta}{A_t n_t - \tau_t} = \frac{v'(n_t)}{1-\sigma}$$

In the limit as $t \to \infty$, this converges to the condition

$$n_{t+1} = \frac{v'(n_t)\, n_t}{(1+g)(1-\sigma)}$$

This condition has a fixed point at $n_* = 0$ and one other fixed point at $n^* = v'^{-1}[(1+g)(1-\sigma)]$, and the latter is an unstable fixed point. Hence, the limiting condition $\lim_{t\to\infty} n_t = n^*$ provides a boundary condition for the difference equation in n_t. It follows that n_t is independent of $\{M_t\}_{t=0}^{\infty}$, and from (43) is as well.

Next, we turn to the variables that do respond to monetary policy. Consider first the price of goods relative to money, P_t. From the demand for money balances, we have

$$\frac{M_t}{P_t} = \frac{\theta \Pi_t}{i_t} = \frac{\theta \Pi_t}{(1+r_t)\,\Pi_t - 1}$$

Since $\Pi_t = P_{t+1}/P_t$ for all t, we can rearrange this condition into a difference equation where P_{t+1} can be expressed as a function of P_t:

$$P_{t+1} = \frac{1}{(1+r_t)/P_t - \theta/M_t}$$

The boundary condition for this difference equation comes from the government budget constraint (21) evaluated at date 0. That is, given a path for inflation $\{\Pi_t\}_{t=0}^{\infty}$, the initial price P_0 must be such that the amount of seniorage revenue just offsets the difference between the initial obligation $(1+r_{-1})\, b_{-1}$ and the present discounted of lump-sum taxes the government collects. This allows us to solve for the path of prices $\{P_t\}_{t=0}^{\infty}$. We then can easily back out nominal wages and the real price of the asset:

$$\begin{aligned} W_t &= (1-\sigma)\, A_t P_t \\ p_t &= A_t n_t - \tau_t - b_t - \frac{M_t}{P_t} \end{aligned}$$

In short, when prices are flexible, monetary policy has no effect on the real interest rate, employment, or output, but it can affect the real price of the asset p_t. Consider the price p_t at date $t = 0$. From the intertemporal government budget constraint in (21), we have

$$b_0 + \tau_0 + \frac{M_0}{P_0} = (1+r_{-1})\, b_{-1} + \frac{M_{-1}}{P_0}$$

Substituting this into the expression for p_t, the real price of the asset at date 0 is given by

$$p_0 = A_0 n_0 - (1+r_{-1})\, b_{-1} - \frac{M_{-1}}{P_0}$$

Since n_t is independent of monetary policy, while $(1+r_{-1})\, b_{-1}$ and M_{-1} are fixed, the effect of monetary policy on the price of the asset at date 0 works entirely through the initial price level P_0. In particular, a policy that lowers the nominal price of goods, or alternatively that increases the real value of money, will lower the real price of the asset. This is also what we argue in the text.

Note that a monetary policy intervention that drives down the price level P_0 will also depress any bubble in the asset. This is because the real interest rate r_t is independent of monetary policy. Hence, the

fundamental value of the asset, i.e., the present discount value of the dividends it yields evaluated at the market real interest rate, will be unchanged, and so the gap between the price and the fundamentals will be lower.

Reducing the price level arguably represents a contractionary monetary policy, since it makes money more valuable. However, this policy will not result in a higher real interest rate, since recall the real interest rate $1 + r_t$ is independent of $\{M_t\}_{t=0}^{\infty}$ when prices are flexible. It has ambiguous implications for the nominal rate: (41) suggests that the price level generally depends on the entire future path of money, so a lower initial price level P_0 could in principle be associated with either a higher or lower nominal interest rate $1 + i_0$. To generate an example of contractionary monetary policy that both raises the real interest rate and depresses the bubble, we now turn to the case where goods prices are set in advance of monetary policy.

Case II: Sticky Prices

Suppose entrepreneurs must set their prices P_{it} before they get to observe monetary policy, based only on their expectation of the nominal wage W_t. We want to study what would happen if at date t the monetary authority unexpectedly announced a different path for money from what entrepreneurs expected. For ease of exposition, we will refer to any variables after the intervention with a hat whenever these variables might differ from what would have happened without the change in monetary policy. Hence, the new path for monetary policy will be denoted by $\{\widehat{M}_t\}_{t=0}^{\infty}$. The change in path is unanticipated as of date t, but is perfectly anticipated from date t on. Hence, any changes in the path of money beyond date t are anticipated and incorporated into entrepreneurs' pricing decisions.

We want to show that there exists an unanticipated monetary policy intervention starting from date t that both raises the real interest rate and reduces the bubble. In particular, suppose the government shrinks the amount of money by a small amount Δ at date t, i.e.,

$$\widehat{M}_t = M_t - \Delta$$

From the government budget constraint (21), we know that

$$\widehat{b}_t = (1 + r_{t-1})\, b_{t-1} - \tau_t - \frac{M_t - \Delta - M_{t-1}}{P_t}$$

Since $(1 + r_{t-1})\, b_{t-1}$, τ_t, M_{t-1}, and P_t are all preset variables, it follows that as a result of this intervention, the government at date t must issue an additional Δ/P_t units of real debt, i.e.,

$$\widehat{b}_t = b_t + \frac{\Delta}{P_t} \tag{44}$$

We then assume that the remaining path $\{\widehat{M}_{t+1}, \widehat{M}_{t+2}, ...\}$ will be set to ensure that the real price of the asset at date $t + 1$ is unchanged, i.e., they will be set to ensure

$$\widehat{p}_{t+1} = p_{t+1}$$

To appreciate what this condition implies for the path of money balances, recall that the equilibrium price of the asset is given by

$$p_t = An_t - \tau_t - b_t - m_t$$

Since $\{\widehat{M}_{t+1}, \widehat{M}_{t+2}, ...\}$ is anticipated, our analysis of the case where prices are flexible applies. This implies $\widehat{n}_{t+1} = n_{t+1}$, i.e., employment at date $t+1$ would be the same as if there was no surprise change in money. The lump-sum tax τ_{t+1} is unaffected by monetary policy by assumption. Hence, $\widehat{p}_{t+1} = p_{t+1}$ implies

$$b_{t+1} + m_{t+1} = \widehat{b}_{t+1} + \widehat{m}_{t+1}$$

Substituting in from (44), we can rewrite this as

$$\widehat{m}_{t+1} - m_{t+1} = (r_t - \widehat{r}_t)\, b_t - (1 + \widehat{r}_t)\, \frac{\Delta}{P_t}$$

We will argue below that $\widehat{r}_t > r_t$, so this implies that real money balances at date $t+1$ must be lower than they would have been absent the shock.

Given a path $\{\widehat{M}_t, \widehat{M}_{t+1}, ...\}$ that satisfies these conditions, we can now characterize the effect on certain equilibrium prices at date t. Since the old exchange all of their resources for goods, they will want to consume

$$\frac{M_t - \Delta}{P_t} + (1 + r_{t-1})\, b_{t-1} + (\widehat{p}_t + d)$$

Since the old are the only ones who consume, the amount of final goods Y_t produced at date t must equal the above amount net of the d units of consumption available as a dividend. That is,

$$A_t \widehat{n}_t = \frac{M_{t-1} - \Delta}{P_t} + (1 + r_{t-1})\, b_{t-1} + \widehat{p}_t \tag{45}$$

Note that most of the variables on the right-hand side of (45) are pre-determined, including the price level P_t. The only endogenous variable on the right-hand side is the price of the asset $\widehat{p}_t$. But we can rewrite this as

$$\begin{aligned} A_t \widehat{n}_t &= \frac{M_{t-1} - \Delta}{P_t} + (1 + r_{t-1})\, b_{t-1} + \frac{d + \widehat{p}_{t+1}}{1 + \widehat{r}_t} \\ &= \frac{M_{t-1} - \Delta}{P_t} + (1 + r_{t-1})\, b_{t-1} + (d + \widehat{p}_{t+1}) \left(\frac{A_t n_t - \tau_t}{A_{t+1} n_{t+1} + d + \theta} \right) \end{aligned}$$

where the second equation uses the fact that $1 + \widehat{r}_t$ must satisfy (42) whether prices are flexible or not. Since young agents at date $t+1$ must be able to afford the asset, we know $\widehat{p}_{t+1} \leq A_{t+1} n_{t+1}$. Hence,

$$\frac{d + \widehat{p}_{t+1}}{A_{t+1} n_{t+1} + d + \theta} < 1$$

Since we considered a path that ensures $\widehat{p}_{t+1} = p_{t+1}$, then $\widehat{p}_{t+1}$ is by construction independent of Δ. This implies we have

$$\left[1 - \frac{d + \widehat{p}_{t+1}}{A_{t+1} n_{t+1} + d + \theta} \right] A_t \widehat{n}_t = \frac{M_t - \Delta}{P_t} + (1 + r_{t-1})\, b_{t-1} - \frac{\tau_t\, (d + \widehat{p}_{t+1})}{A_{t+1} n_{t+1} + d + \theta} \tag{46}$$

where all terms above are independent of Δ. From this, it follows that $\widehat{n}_t$ is decreasing in Δ, since

$$\frac{d\widehat{n}_t}{d\Delta} = -\left[1 - \frac{d + \widehat{p}_{t+1}}{A_{t+1}n_{t+1} + d + \theta}\right]^{-1} \frac{P_t}{A_t} < 0$$

This shows that an unexpected reduction in money, followed by an adjustment in money balances that sterilizes the affect on asset prices at date $t+1$, would reduce employment and thus output. Intuitively, the shock leaves old agents with fewer resources to consume, and so less will be produced to meet their demand.

From (42), we can further deduce that the real interest rate $1 + \widehat{r}_t$ at date t will be higher than $1 + r_t$. This is because all date $t+1$ variables will be unchanged, while $\widehat{n}_t < n_t$. As for the price of the asset, (46) reveals that output $A_t\widehat{n}_t$ falls more than one for one with Δ/P_t, implying the real price of the asset $\widehat{p}_t < p_t$. Appealing to a similar argument as in the case of fiscal policy, we can use the fact that any bubble would not change asymptotically as a result of this policy, and yet the interest rate $1 + \widehat{r}_t$ is higher than $1 + r_t$, to argue that the bubble at date t must fall. Hence, an unexpected monetary contraction would not only temporarily raise the real interest rate, but it would temporarily depress the bubble. This is precisely what we wanted to demonstrate.

Finally, we note that from the equation for labor supply, we have

$$v'\left(\widehat{n}_t\right) = \left(1 + \widehat{r}_t\right) \frac{\widehat{W}_t}{\widehat{P}_t}$$

Since $\widehat{n}_t < n_t$ while $1 + \widehat{r}_t > 1 + r_t$, we can conclude that the real wage $\widehat{W}_t/\widehat{P}_t < W_t/P_t$. That is, since employment must fall when goods prices are sticky, the real wage must fall to induce workers to put in less effort. Since $\widehat{P}_t = P_t$ when prices are set in advance, it follows that the nominal wage $\widehat{W}_t < W_t$, i.e., the monetary policy we consider will cause the nominal wage to fall. But this means that if some producers could respond to monetary policy, they would want to lower their prices. This suggests that if some producers had flexible prices while others were not, then the policy we consider would lead the price level to fall. Thus, when some prices are flexible, we would expect both a fall in output, meaning young agents have fewer resources with which to buy the asset, and an increase in the amount of debt issued to rise as the value of previous nominal government obligations increased. Thus, in a version where some but not all prices were flexible, contractionary monetary policy would depress the bubble both because there are fewer resources agents can use to buy the asset and because there is more public debt that agents must hold instead of the asset.

Appendix C: General Contracting

In Section 4, we restricted savers to only offer debt contracts. In this appendix we consider the case where savers can offer more general financial contracts and show that equilibria in which a single debt contract is offered to all agents is indeed an equilibrium. We assume each contract is exchanged on a different submarket. Because the interest rate is specified in the contract, there is no "price" that can be adjusted to clear the market. Instead, rationing provides the signal for agents, entrepreneurs, and speculators to move among submarkets. We represent this process as a matching market, in which savers, entrepreneurs, and speculators take as given the probability of exchanging each contract with a counterparty and then choose the submarket that offers the highest expected payoff.

If demand and supply are unbalanced, we assume that rationing is efficient in the sense that everyone on short side of the market is matched with someone on the long side, but the long side there will be rationing. The matching probability affects payoffs in two ways: it represents the probability of exchanging a contract and for, savers, it represents the probability of being matched with an entrepreneur or a speculator, respectively.

In what follows, we describe how markets clear at a single date t, taken as given the current asset price p_t and the expected future asset prices, p_{t+1} and $\hat{p}_{t+1}$. In Section 4, we showed that when the government issues no debt, the equilibrium prices are determined independently of the contract chosen at date t since $p_t = e_t - \kappa$. In this case, we can analyze the choice of the contract taking the payoffs to speculators who purchase the asset as given.

Parameters There is a unit measure of savers with $\omega > 0$ units of the good to invest, a measure $\kappa > 0$ of entrepreneurs with no endowment, and an unbounded measure of speculators, also with no endowment. Entrepreneurs can invest one unit and earn a return $1 + y > 0$ with probability 1. Speculators can invest in the asset and earn a return r with probability π and $\hat{r}$ with probability $1 - \pi$ for each unit of the good invested. The parameters r and $\hat{r}$ represent the equilibrium returns

$$r = \frac{d + p_{t+1}}{p_t} \text{ and } \hat{r} = \frac{d + \hat{p}_{t+1}}{p_t},$$

for some fixed but arbitrary date t. As in the text, we assume that $1 + y \gg r > \hat{r} > 0$.

Contracts A contract $c = (x, \hat{x})$ specifies a transfer of one unit to the borrower at time t, a repayment at date $t + 1$ of x (resp. $\hat{x}$) if r (resp. $\hat{r}$) is the return on the asset. The set of feasible contracts is denoted by $C = X \times \hat{X}$, where

$$X = \{x_1, ..., x_m\} \text{ and } \hat{X} = \{\hat{x}_1, ..., \hat{x}_n\} \tag{47}$$

$$x_1 < x_2 < \ \ldots \ < x_m \text{ and } \hat{x}_1 < \hat{x}_2 < \ \ldots \ < \hat{x}_n, \tag{48}$$

$$x_1 = r, \ \hat{x}_1 = \hat{r}, \ x_m = \hat{x}_n = 1 + y \text{ and } r \in \hat{X}. \tag{49}$$

Condition (47) says that X and $\hat{X}$ each consist of a finite number of payment levels and condition (48) strictly orders them without loss of generality. Condition (49) restricts X and $\hat{X}$ so that

$$\min X \geq r, \ \ \min \hat{X} \geq \hat{r}, \ \text{and} \ \max X, \ \ \max \hat{X} \leq 1 + y.$$

The requirement that $\min X \geq r$ eliminates contracts that offer speculators a positive expected payoff. Since speculators must receive a zero payoff in equilibrium, this does not affect the equilibrium set materially. For convenience, condition (49) also assumes that the parameters $\{r, 1+y\}$ and $\{r, \hat{r}, 1+y\}$ belong to the sets X and $\hat{X}$, respectively. We denote the generic element of C by $c_{ij} = (x_i, \hat{x}_j)$, for $i = 1, ..., m$ and $j = 1, ..., n$.

Allocations An allocation is an ordered triple $(\mathbf{a}, \mathbf{e}, \mathbf{s}) \in \mathbf{R}_+^{mn} \times \mathbf{R}_+^{mn} \times \mathbf{R}_+^{mn}$. We write $\mathbf{a} = (a_{ij})$, $\mathbf{e} = (e_{ij})$ and $\mathbf{s} = (s_{ij})$, where a_{ij} is the measure of savers who supply contract c_{ij}, e_{ij} is the measure of entrepreneurs who demand contract c_{ij}, and s_{ij} is the measure of speculators who demand contract c_{ij}. An allocation $(\mathbf{a}, \mathbf{e}, \mathbf{s})$ is attainable if

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} = \omega,$$

and

$$\sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} = \kappa.$$

Here we are restricting our attention to equilibria in which all savers want to lend and all entrepreneurs want to borrow.

Rationing A matching probability is a vector $\boldsymbol{\lambda} \in \mathbf{R}_+^{mn} \times \mathbf{R}_+^{mn} \times \mathbf{R}_+^{mn}$, where $\boldsymbol{\lambda}_{ij} = \left(\lambda_{ij}^a, \lambda_{ij}^e, \lambda_{ij}^s\right)$, for any $i = 1, ..., m$ and $j = 1, ..., n$. We interpret λ_{ij}^a as the probability that an entrepreneur or speculator demanding contract c_{ij} is matched with an agent and the contract c_{ij} is exchanged. Similarly, λ_{ij}^e and λ_{ij}^s are the probabilities that an agent offering contract c_{ij} is matched with an entrepreneur and speculator, respectively, and the contract c_{ij} is exchanged. A matching probability $\boldsymbol{\lambda}$ is consistent with an attainable allocation $(\mathbf{a}, \mathbf{e}, \mathbf{s})$ if

$$\lambda_{ij}^a = \min\left\{\frac{e_{ij} + s_{ij}}{a_{ij}}, 1\right\}, \text{ for any } c_{ij} \text{ such that } a_{ij} > 0, \tag{50}$$

$$\lambda_{ij}^e = \frac{e_{ij}}{e_{ij} + s_{ij}} \min\left\{\frac{a_{ij}}{e_{ij} + s_{ij}}, 1\right\}, \text{ for any } c_{ij} \text{ such that } e_{ij} + s_{ij} > 0, \tag{51}$$

$$\lambda_{ij}^s = \frac{s_{ij}}{e_{ij} + s_{ij}} \min\left\{\frac{a_{ij}}{e_{ij} + s_{ij}}, 1\right\} \text{ for any } c_{ij} \text{ such that } e_{ij} + s_{ij} > 0. \tag{52}$$

Consistency ensures that trade is efficient in the sense that, for any given allocation, it maximizes the number of contracts exchanged. More precisely,

$$a_{ij} > e_{ij} + s_{ij} > 0 \text{ implies } \lambda_{ij}^a = 1,$$

so that entrepreneurs and speculators exchange the contract with probability one and savers are rationed, and

$$e_{ij} + s_{ij} > a_{ij} > 0 \text{ implies } \lambda^e_{ij} + \lambda^s_{ij} = 1,$$

so savers exchange the contract with probability one and entrepreneurs and speculators are rationed. In other words, only the long side of the market is rationed and the short side always clears, for any contract c_{ij} such that $a_{ij} > 0$ and $e_{ij} + s_{ij} > 0$.

Payoffs Let V^e_{ij} denote the payoff from contract c_{ij} if the agent is matched with an entrepreneur and let V^s_{ij} denote the agent's payoff from contract c_{ij} if the agent is matched with a speculator. Then

$$V^e_{ij} = (1-\pi)\min\{x_i, 1+y\} + \pi \min\{\hat{x}_j, 1+y\},$$

and

$$V^s_{ij} = (1-\pi)\min\{x_i, r\} + \pi \min\{\hat{x}_j, \hat{r}\},$$

for any $i = 1, ..., m$ and $j = 1, ..., n$.

An entrepreneur's payoff from a contract c_{ij} is defined as

$$U^e_{ij} = (1-\pi)\max\{1+y-x_i, 0\} + \pi \max\{1+y-\hat{x}_j, 0\},$$

and a speculator's payoff from a contract c_{ij} is defined as

$$U^s_{ij} = (1-\pi)\max\{r-x_i, 0\} + \pi \max\{\hat{r}-\hat{x}_j, 0\},$$

for any $i = 1, ..., m$ and $j = 1, ..., n$.

Equilibrium An equilibrium consists of an attainable allocation $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*)$ and a matching probability $\boldsymbol{\lambda}^*$, such that $\boldsymbol{\lambda}^*$ is consistent with $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*)$, $\mathbf{a}^*$ solves the problem:

$$\max_{\mathbf{a}\geq\mathbf{0}} \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\left(\lambda^{e*}_{ij}V^e_{ij} + \lambda^{s*}_{ij}V^s_{ij}\right)$$
$$\text{s.t. } \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} = \omega;$$

$\mathbf{e}^*$ solves the problem

$$\max_{\mathbf{e}\geq\mathbf{0}} \sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}\lambda^{a*}_{ij}U^e_{ij}$$
$$\text{s.t. } \sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij} = \kappa;$$

and $\mathbf{s}^*$ solves the problem

$$\max_{\mathbf{s}\geq\mathbf{0}} \sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij}\lambda^{a*}_{ij}U^s_{ij}.$$

The equilibrium conditions require that the distributions of agents, entrepreneurs and speculators are concentrated on the most profitable contracts, respectively, taking the matching probabilities as given. The fact that the number of speculators is not bounded implies that $U^s_{ij} = 0$ for any contract c_{ij} such that $\lambda^a_{ij} > 0$.

Stable outcomes The equilibrium concept requires the matching probabilities to be consistent with the allocation, but this only constrains the probabilities for the contracts that are actively traded. For example, suppose that $\boldsymbol{\lambda}_{ij} = \mathbf{0}$ for some contract c_{ij}. Since the probability of exchanging c_{ij} is zero, it is optimal for agents, entrepreneurs, and speculators to avoid this contract. Then the equilibrium allocation satisfies $a_{ij} = e_{ij} = s_{ij} = 0$ and the belief that $\boldsymbol{\lambda}_{ij} = \mathbf{0}$ becomes self-fulfilling. In this way, it is possible to ensure that an arbitrary set of contracts will not be traded in equilibrium. Equilibrium becomes indeterminate for rather trivial reasons as a result.

One way to avoid this kind of indeterminacy is to perturb the equilibrium allocation slightly, so that each contract has a positive measure of agents, entrepreneurs, and speculators in the market for each contract. Then the matching probabilities will be well defined and reflect the optimal choices of the agents, entrepreneurs, and speculators in every market.

A perturbation is a vector $\boldsymbol{\varepsilon} \in \mathbf{R}^{mn}_+ \times \mathbf{R}^{mn}_+ \times \mathbf{R}^{mn}_+$ such that $\boldsymbol{\varepsilon}_{ij} \gg \mathbf{0}$, for every contract c_{ij}. We write $\boldsymbol{\varepsilon}_{ij} = \left(\varepsilon^a_{ij}, \varepsilon^e_{ij}, \varepsilon^s_{ij}\right)$ and interpret ε^a_{ij}, ε^e_{ij}, and ε^s_{ij} as the measure of savers, entrepreneurs, and speculators assigned to contract c_{ij}. We say that an attainable allocation $(\mathbf{a}, \mathbf{e}, \mathbf{s})$ is consistent with the perturbation $\boldsymbol{\varepsilon}$ if $(\mathbf{a}, \mathbf{e}, \mathbf{s}) \geq \boldsymbol{\varepsilon}$. Let $(\mathbf{a}, \mathbf{e}, \mathbf{s})$ be an attainable allocation that is consistent with a perturbation $\boldsymbol{\varepsilon} \gg \mathbf{0}$. Because the perturbation ensures a positive measure of agents, entrepreneurs, and speculators are assigned to the market for each contract c_{ij}, the matching probabilities $\boldsymbol{\lambda}_{ij} = \left(\lambda^a_{ij}, \lambda^e_{ij}, \lambda^s_{ij}\right)$ will be defined by the consistency conditions (50) - (52).

For any perturbation $\boldsymbol{\varepsilon} \gg \mathbf{0}$, an $\boldsymbol{\varepsilon}$-perturbed equilibrium is an attainable allocation $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*)$ consistent with $\boldsymbol{\varepsilon}$ and a matching probability $\boldsymbol{\lambda}^*$ satisfying (50) - (52), such that $\mathbf{a}^*$ solves the problem:

$$\max_{\mathbf{a}} \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \left(\lambda^{a*}_{ij} \frac{e^*_{ij} V^e_{ij} + s^*_{ij} V^s_{ij}}{e^*_{ij} + s^*_{ij}} \right)$$
$$\text{s.t. } \boldsymbol{\varepsilon}^a \leq \mathbf{a} \text{ and } \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} = \omega;$$

$\mathbf{e}^*$ solves the problem

$$\max_{\mathbf{e}} \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} \left(\lambda^{*es}_{ij} U^e_{ij} \right)$$
$$\text{s.t. } \boldsymbol{\varepsilon}^e \leq \mathbf{e} \text{ and } \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} = \kappa;$$

and $\mathbf{s}^*$ solves the problem

$$\max_{\mathbf{s}} \sum_{i=1}^{m} \sum_{j=1}^{n} s_{ij} \left(\lambda_{ij}^{*es} U_{ij}^{s} \right)$$
$$\text{s.t. } \boldsymbol{\varepsilon}^s \leq \mathbf{s}.$$

An equilibrium $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ is *strategically stable* if, for any sequence of perturbations $\{\boldsymbol{\varepsilon}^n\}$ converging to $\mathbf{0}$, there exists an $\boldsymbol{\varepsilon}^n$-perturbed equilibria, $(\mathbf{a}^n, \mathbf{e}^n, \mathbf{s}^n, \boldsymbol{\lambda}^n)$, for each n and the sequence of perturbed equilibria $\{(\mathbf{a}^n, \mathbf{e}^n, \mathbf{s}^n, \boldsymbol{\lambda}^n)\}$ converges to $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$.

Proposition C1: Let $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ be an equilibrium in which there is a unique contract c^* such that $a_{ij}^* = e_{ij}^* = 0$ for any $c_{ij} \neq c^*$ and

$$a_{ij}^* = e_{ij}^* + s_{ij}^*$$

for $c_{ij} = c^*$. Assume also that the equilibrium payoffs of wealthy agents and entrepreneurs are both strictly positive. Then, for any $\boldsymbol{\varepsilon} \gg \mathbf{0}$ sufficiently close to $\mathbf{0}$, there exists an $\boldsymbol{\varepsilon}$-perturbed equilibrium $(\mathbf{a}^\varepsilon, \mathbf{e}^\varepsilon, \mathbf{s}^\varepsilon)$ such that $a_{ij}^\varepsilon = \varepsilon_{ij}^a$ and $e_{ij}^\varepsilon = \varepsilon_{ij}^e$ for any contract $c_{ij} \neq c^*$.

Proof. Let $(\mathbf{a}^\varepsilon, \mathbf{e}^\varepsilon)$ be defined by putting $a_{ij}^\varepsilon = \varepsilon_{ij}^a$ and $e_{ij}^\varepsilon = \varepsilon_{ij}^e$, for any contract $c_{ij} \neq c^*$, and put

$$a_{c^*}^\varepsilon = \omega - \sum_{c_{ij} \neq c^*} \varepsilon_{ij}^a$$

and

$$e_{c^*}^\varepsilon = \kappa - \sum_{c_{ij} \neq c^*} \varepsilon_{ij}^e$$

for $c_{ij} = c^*$. It is clear that the speculators' payoff is zero by construction. So we can allocate speculators to contracts in any way we want. For any contract $c_{ij} \neq c^*$, we can ensure that the payoffs of entrepreneurs are arbitrarly close to zero by choosing s_{ij}^ε sufficiently large. This follows directly from the fact that $\left(\lambda_{ij}^a\right)^\varepsilon$ converges to zero as s_{ij}^ε diverges to infinity. In the case of wealthy agents, the payoff from exchanging a contract with speculators is independent of the contract and lower than the contract c^*, so increasing the proportion of speculators at the contract $c_{ij} \neq c^*$ eventually reduces the agents' payoff to the payoff from exchanging a contract with a speculator. We therefore define the allocation at c^* by putting $s_{c^*}^\varepsilon = a_{c^*}^\varepsilon - e_{c^*}^\varepsilon$. Then the payoffs for the equilibrium contract c^* will be close to the payoffs in the limit equilibrium and converge to the equilibrium payoffs as $\boldsymbol{\varepsilon} \to \mathbf{0}$. This establishes that $(\mathbf{a}^\varepsilon, \mathbf{e}^\varepsilon, \mathbf{s}^\varepsilon)$ is an $\boldsymbol{\varepsilon}$-perturbed equilibrium for every $\boldsymbol{\varepsilon}$ sufficiently small. It is also clear that $(\mathbf{a}^\varepsilon, \mathbf{e}^\varepsilon) \to (\mathbf{a}^*, \mathbf{e}^*)$ as $\boldsymbol{\varepsilon} \to \mathbf{0}$. We can also show that s_{ij}^ε can be chosen so that $s_{ij}^\varepsilon \to 0$ for any $c_{ij} \neq c^*$, because the payoffs to agents and entrepreneurs depend only on the *ratio*, $e_{ij}^\varepsilon / s_{ij}^\varepsilon$, and $e_{ij}^\varepsilon \to 0$. Finally, in the case of the equilibrium contract c^*, we have defined $s_{c^*}^\varepsilon$ so that $s_{c^*}^\varepsilon \to s_{c^*}^*$. ■

The intuition for this result is quite simple. Because speculators receive zero payoffs in equilibrium, we can allocate large numbers of them to each contract other than (x_i, x_i). This ensures that the agents' payoff from these contracts is smaller than their equilibrium payoff and so only agents who are perturbed will be in those markets. This in turn makes the probability of exchanging a contract very small on the other side of the market, so only entrepreneurs who are perturbed will be in those markets.

Non-contingent contracts In what follows we assume $X \subset \hat{X}$. A non-contingent contract $(x_i, \hat{x}_j)$ satisfies $x_i = \hat{x}_j$. Obviously, we can identify the set of non-contingent contracts with X. Although the contractual payments are non-contingent, the actual payments will be contingent on r and $\hat{r}$ (for the speculators, at least) because $\min\{x_i, r\} = r$ and $\min\{x_i, \hat{r}\} = \hat{r}$, for any $x_i \in X$. We can interpret non-contingent contracts as debt contracts that demand a fixed repayment x_i and allow the borrower to default if the payment is infeasible. These are the types of contracts we restricted agents to use in the text. We now argue that such a contract can in fact be an equilibrium. First, let us call an equilibrium non-contingent if the only contracts exchanged are non-contingent contingent.

Proposition C2: For any $x \in X$, there exists a non-contingent equilibrium in which the only contract exchanged has the form $c = (x, x)$.

Proof. Let $x_i \in X$ and define the allocation $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ by

$$\left(a^*_{kj}, e^*_{kj}, s^*_{kj}\right) = \begin{cases} (\omega, \kappa, \omega - \kappa) & \text{if } (k,j) = (i,i) \\ (0,0,0) & \text{if } (k,j) \neq (i,i) \end{cases},$$

and

$$\left(\lambda^{a*}_{kj}, \lambda^{e*}_{kj}, \lambda^{s*}_{kj}\right) = \begin{cases} \left(1, \frac{\kappa}{\omega}, \frac{1-\kappa}{\omega}\right) & \text{if } (k,j) = (i,i) \\ (0,0,1) & \text{if } (k,j) \neq (i,i) \end{cases}.$$

It is clear from inspection that the allocation $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*)$ is attainable and the matching probability $\boldsymbol{\lambda}^*$ is consistent with the allocation. As we have already noted, speculators receive zero profits from every contract. Given the matching probability $\boldsymbol{\lambda}^*$, neither agents nor entrepreneurs have an incentive to deviate from the contract (x_i, x_i). Thus, $(\mathbf{a}^*, \mathbf{e}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ is an equilibrium. ■

This equilibrium will be stable, of course, as indicated by Proposition C1, so for any small perturbation there will be a perturbed equilibrium very close to this one. In that sense, these equilibria are not supported by "unreasonable" beliefs.

Appendix D: Credit-Driven Bubbles and Variable Supply of Assets

In this Appendix, we provide an example of credit-driven bubbles in which an intervention that raises the interest rate and mitigates a bubble can affect the amount of aggregate risk in the economy. This example requires two departures from the model we analyzed in Section 4. First, we need to modify the model so that additional units of the asset can be created. This allows an intervention that mitigates the bubble to affect the quantity of bubble assets created. Second, for a change in the number of assets created to influence aggregate risk, we need the source of risk in the economy to come from the asset rather than the aggregate endowment. If the source of risk in the economy concerns the aggregate endowment as in the model we analyze in Section 4, the amount of assets created would have no effect on the risk the economy is exposed to.

Our formulation follows the setup in Section 4 in which young agents who want to save can buy government bonds, buy the asset, or lend to a combination of entrepreneurs or speculators. We consider two modifications. First, we assume $g = 0$, i.e., $e_t = e_0$ for all t and there is no uncertainty about next period's endowment. Second, rather than assume that the initial old are endowed with a fixed stock of assets that yields a fixed dividend d, we assume that the initial old can create assets by converting output into assets, and that the dividend on these assets is stochastic. The production technology for assets features increasing marginal cost. In particular, we assume that creating the q-th unit of the asset requires $c(q)$ units of output. For simplicity, suppose $c(q) = q^a$ for some constant $a > 0$. As before, we use p_0 to denote the real price of the asset at date 0. Since producers earn p_0 on each unit they produce, they will create assets up to the point where the cost of the last asset is equal to the price at which they can sell the asset, i.e.,

$$p_0 = c(q) = q^a$$

We continue to assume the asset yields a constant dividend. However, the value of this dividend is only revealed at date $t = 1$, when those who purchased the asset at date 0 are old. Thus, only the initial cohort who buys the asset is uncertain about its payoff. To keep things simple, suppose the dividend d has a binomial distribution and will equal $\underline{d} > 0$ with probability $1 - \pi$ and $\overline{d} > \underline{d}$ with probability π.

Since $\underline{d} > 0$, agents who are young at date 0 will not rely on storage. Since we assume the return y on production is large, all entrepreneurs will wish to borrow. They will therefore receive an amount κ of the resources available at date 0. This implies

$$e_0 - b_0 - \tau_0 - \kappa = p_0 q$$

That is, total spending on the asset at date 0 is equal to the endowment of agents at date date net of the amount they use to pay taxes, buy bonds, and finance entrepreneurs. Substituting in $q^a = p_0$ yields an expression for the initial price of the asset as well as the quantity produced:

$$\begin{aligned} p_0 &= (e_0 - b_0 - \tau_0 - \kappa)^{\frac{a}{a+1}} \\ q &= (e_0 - b_0 - \tau_0 - \kappa)^{\frac{1}{a+1}} \end{aligned}$$

The higher is a, the more an increase in $p_0 q$ will be reflected in a higher price as opposed to a higher quantity. When $a = 1$, the two effects are equal.

To show that the model still gives rise to a bubble, let us begin at date 1 when the value of the dividend d is revealed. The total amount spent on the asset at this date will equal

$$p_1 q = e_0 - b_1 - \tau_1 - \kappa$$

This amount will be the same whether $d = \overline{d}$ or $d = \underline{d}$. However, the realization of dividends will instead affect the equilibrium interest rate. In particular, for $t \geq 1$, we have

$$1 + r_t = \frac{d + p_{t+1}}{p_t} \tag{53}$$

where $d \in \left\{\underline{d}, \overline{d}\right\}$. Thus, if dividends are low, the interest rate will be low as well. Indeed, the interest rate will fall by exactly the amount needed to push up the present discounted value of dividends to be the same as when dividends are high. In other words, if we denote $1 + \underline{r}_t$ as the interest rate at date t if $d = \underline{d}$ and $1 + \overline{r}_t$ if $d = \overline{d}$, then

$$\sum_{t=1}^{\infty} \left(\prod_{s=1}^{t} \frac{1}{1 + \underline{r}_s} \right) \underline{d} = \sum_{t=1}^{\infty} \left(\prod_{s=1}^{t} \frac{1}{1 + \overline{r}_s} \right) \overline{d} = p_1 = \frac{e_0 - b_1 - \tau_1 - \kappa}{q}$$

To prove this result formally, we use (53) to obtain

$$\begin{aligned}
f_1 &= \sum_{t=1}^{\infty} \left(\prod_{s=1}^{t} \frac{1}{1 + r_s} \right) d \\
&= d \left[\frac{p_1}{p_2 + d} + \frac{p_2}{p_3 + d} \frac{p_1}{p_2 + d} + \frac{p_3}{p_4 + d} \frac{p_2}{p_3 + d} \frac{p_1}{p_2 + d} + \cdots \right] \\
&= d \times \frac{p_1}{d} \times \left[\frac{d}{p_2 + d} + \frac{p_2}{p_3 + d} \frac{d}{p_2 + d} + \frac{p_3}{p_4 + d} \frac{p_2}{p_3 + d} \frac{d}{p_2 + d} + \cdots \right] \\
&= d \times \frac{p_1}{d} \times \left[\frac{d}{p_2 + d} + \frac{d}{p_3 + d} \frac{p_2}{p_2 + d} + \frac{d}{p_4 + d} \frac{p_3}{p_3 + d} \frac{p_2}{p_2 + d} + \cdots \right]
\end{aligned}$$

We now argue that the last expression in brackets is equal to 1. Consider a random variable that describes the date of an arrival, where the arrival can occur at any date $t \geq 2$, and where the probability of arrival at date t conditional on no arrival up to date t is equal to $\dfrac{d}{p_t + d} \in (0, 1)$. Then the expression in brackets is the probability of an arrival at any date, which is 1. Hence, $f_1 = p_1$, regardless of the realization of d.

At date 0, the fundamental value of the asset is equal to

$$f_0 = \frac{E\left[d + f_1\right]}{1 + r_0} = \frac{E\left[d\right] + p_1}{1 + r_0}$$

We now want to show that the price of the asset at date 0 exceeds f_0. To see this, note that because there is free entry into speculation, the interest rate on loans R_0 must ensure speculators earn zero profits from borrowing if the high state is realized, i.e.,

$$1 + R_0 = \frac{\overline{d} + p_1}{p_0}$$

Since creditors could have purchased riskless government bonds and earned $1 + r_0$ on the resources they lend out, and since lenders are risk-neutral, the expected profits from lending out resources must yield the same return as the risk-free rate, i.e.

$$(1 + r_0)(\kappa + p_0 q) = (1 + R_0)(\kappa + (1 - \pi) p_0 q) + \pi (\underline{d} + p_1) q \tag{54}$$

By adding and subtracting $\pi \left(\overline{d} + p_1\right) q$ to the RHS implies

$$\begin{aligned}(1 + r_0)(\kappa + p_0 q) &= (1 + R_0)(\kappa + (1 - \pi) p_0 q) + \pi \left(\overline{d} + p_1\right) q - \pi \left(\overline{d} + p_1\right) q + \pi (\underline{d} + p_1) q \\ &= (1 + R_0)(\kappa + p_0 q) - \pi \left(\overline{d} - \underline{d}\right) q\end{aligned}$$

where the second step uses the fact that $1 + R_0 = \left(\overline{d} + p_0\right)/p_0$. Rearranging, we can derive an expression for the risk free interest rate on government debt r_0 in terms of the loan rate R_0:

$$r_0 = R_0 - \frac{\pi \left(\overline{d} - \underline{d}\right) q}{\kappa + p_0 q}$$

This implies $R_0 > r_0$. Next, observe that from (54), we have

$$\begin{aligned}(1 + r_0)(\kappa + p_0 q) &= \frac{\overline{d} + p_1}{p_0} (\kappa + (1 - \pi) p_0 q) + \pi (\underline{d} + p_1) q \\ &= \frac{\overline{d} + p_1}{p_0} \kappa + (1 - \pi) \left(\overline{d} + p_1\right) q + \pi (\underline{d} + p_1) q \\ &= (1 + R_0) \kappa + [E[d] + p_1] q\end{aligned}$$

Dividing both sides by $1 + r_0$ and rearranging the above equation yields

$$(p_0 - f_0) q = \frac{R_0 - r_0}{1 + r_0} \kappa$$

Since we argued above that $R_0 > r_0$, this implies that if $\kappa > 0$, there will be a bubble in the asset market.

Finally, we want to study the effects of intervention. As before, we parameterize policy so that $b_t = b$ for $t = -1, 0, 1, ...$ From the government budget constraint, we have

$$b_0 + \tau_0 = (1 + r_{-1}) b_{-1} = (1 + r_{-1}) b$$

Substituting this into the expression for the price of the asset at date 0 yields an expression in terms of b and other primitives:

$$p_0 = (e_0 - (1 + r_{-1}) b - \kappa)^{\frac{a}{a+1}} \tag{55}$$

Next, we turn to the real interest rate. Using the expression for the spread between R_0 and r_0 above, as well as the expression for R_0 above, we can solve for r_0 as the value which solves

$$\begin{aligned}1 + r_0 &= \frac{\overline{d} q + p_1 q}{p_0 q} - \frac{\pi \left(\overline{d} - \underline{d}\right) q}{\kappa + p_0 q} \\ &= \frac{d_1 q + (e_0 - (1 + r_0) b - \kappa)}{e_0 - (1 + r_{-1}) b - \kappa} - \frac{\pi (d_1 - d_0) q}{e_0 - b - r_{-1} b} \\ &= \frac{d_1 (e_0 - (1 + r_{-1}) b - \kappa)^{\frac{1}{a+1}} + (e_0 - (1 + r_0) b - \kappa)}{e_0 - (1 + r_{-1}) b - \kappa} \\ &\quad - \frac{\pi (d_1 - d_0) (e_0 - (1 + r_{-1}) b - \kappa)^{\frac{1}{a+1}}}{e_0 - b - r_{-1} b}\end{aligned}$$

The analytical expression for r_0 in terms of primitives is messy, and differentiating it with respect to b does not offer much insight. However, we can easily produce numerical examples in which an increase in b will raise the risk-free interest rate r_0, reduce the initial price of the asset p_0, lower the bubble $\Delta_0 = p_0 - f_0$, and reduce the quantity of the asset supplied q. For example, suppose we set the initial endowment $e_0 = 1$. For the asset, we assume $\overline{d} = 0.1$ and $\underline{d} = 0.05$, each equally likely so $\pi = 0.5$. We set $\kappa = 0.99$, so most lending finances entrepreneurial activity rather than speculation. This is consistent with our finding in Proposition 7 that a high κ is more compatible with bond issuance reducing the size of the bubble. We set the initial interest rate at $r_{-1} = 0.15$. Finally, we set $a = 9$ so the effect of bonds on total spending on the asset primarily affects the price of the asset p_0 rather than the quantity of the asset created q. In this case, we can compute numerically that for all values of b that ensure the asset price p_0 and p_1 will be positive, a higher b will in fact raise the real interest rate, reduce the price and quantity of the asset produced at date 0, and reduce the bubble component at date 0.

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