

Pierpaolo, Benigno; Karantounias, Anastasios G.

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Overconfidence, Subjective Perception, and Pricing Behavior

Pierpaolo Benigno and Anastasios G. Karantounias

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Abstract: We study the implications of overconfidence for price setting in a monopolistic competition setup with incomplete information. Our price-setters overestimate their abilities to infer aggregate shocks from private signals. The fraction of uninformed firms is endogenous; firms can obtain information by paying a fixed cost. We find two results: (1) overconfident firms are less inclined to acquire information, and (2) prices might exhibit excess volatility driven by nonfundamental noise. We explore the empirical predictions of our model for idiosyncratic price volatility.

JEL classification: D4;D8;E3

Key words: overconfidence, imperfect common knowledge, information acquisition, inflation volatility

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Please address questions regarding content to Pierpaolo Benigno, LUISS Guido Carli and Einaudi Institute of Economics and Finance, Viale Romania 32, 00197 Roma, Italy, pbenigno@luiss.it, or Anastasios Karantounias, Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree Street NE, Atlanta, GA 30309, anastasios.karantounias@atl.frb.org.

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1 Introduction

This paper studies the implications of the well-documented psychological bias of *overconfidence* on the price-setting behavior of firms in a monopolistically competitive market with incomplete information. In our model, firms receive a private signal about an aggregate shock that influences marginal costs. They can acquire better information by paying a fixed cost. Our price-setters are overconfident in their signals, i.e. they overestimate the precision of their private information.

There are two important conclusions that we draw from this framework: first, overconfidence implies that a large fraction of firms is going to optimally stay uninformed; second, overconfident firms set prices that may be excessively volatile, since they are driven by underestimated noise. This volatility at the individual level is coupled with an aggregate inflation rate that is smooth and persistent. So we provide a unified “noise” explanation of prices at both the idiosyncratic and the aggregate level.

There is a vast psychological literature on overconfidence. The term has borne different interpretations depending on the particular study but here we focus on the concept of *overprecision*.¹ The concept of overconfidence is rather new in macro models but has been influential in the finance literature. For example, [Barber and Odean \(2001\)](#) analyze the implications of overconfident male investors for excessive stock trading whereas [Daniel et al. \(1998\)](#) and [Daniel et al. \(2001\)](#) focus on the effects of investor’s overconfidence on market overreaction and asset pricing.

Overconfidence has not been found to be important only in experimental studies or in setups where monetary loss is inconsequential: [Oberlechner and Osler \(2009\)](#) and [Oberlechner and Osler \(2012\)](#) provide empirical evidence that overconfidence, in the sense of underestimation of uncertainty, is ubiquitous in currency markets and that overconfident traders actually survive in the long-run. In a similar vein, [Burnside et al. \(2011\)](#) associate investor’s overconfidence to the forward premium puzzle.

The prevalence of overconfidence has led to work that generates endogenously optimistic biases in beliefs, by taking into account the utility benefits of good outcomes. A prominent example is the theory of optimal expectations of [Brunnermeier and Parker \(2005\)](#). Another example is the work of [Ortoleva and Snowberg \(2015\)](#), who generate overconfidence as a consequence of correlational neglect and explore the implications for political ideology. For our own purposes, we take overconfidence as *given* but explore how it interacts with fundamental and *strategic* uncertainty in our monopolistic competition setup. In the baseline version of our economy, overconfidence can deter agents from obtaining information, since

¹See the related literature section for details and for an overview of the use of overconfidence in various fields of economics.

they rely more on their subjective perception of the world, but can also increase the incentives to obtain information by increasing the implicit cost of staying uninformed through the mechanism of higher-order beliefs.

In a dynamic extension of our economy, the presence of strategic complementarities and the interaction with higher-order beliefs generates persistent responses of output to nominal shocks, as in the imperfect common knowledge setup of [Woodford \(2002\)](#), where all agents are uninformed. The persistence at the aggregate level, together with the excess volatility of overconfident uninformed price-setters at the idiosyncratic level, leads to interesting empirical predictions. To flesh them out, we explore how much idiosyncratic price volatility can be generated by overconfidence.

Using micro data on consumer prices for the U.S. economy, [Klenow and Kryvtsov \(2008\)](#) have documented that prices change frequently with an average absolute size of 8.5% on a monthly basis and that these frequent movements are equally likely to be positive or negative in sign. In contrast, aggregate inflation averages just at 0.6% over an horizon of 3 months. We find that with rational price setters, our model implies that the average absolute size of price changes is of the order of only 2%. For a reasonable degree of overconfidence documented in experimental studies, we find that the average absolute size of price changes is of the order of 4 – 5%. This is quite large for a model that does not rely at all on idiosyncratic fundamental shocks like idiosyncratic productivity or demand shocks.

2 Related literature

Our setup is based on the imperfect common knowledge framework of [Woodford \(2002\)](#). [Lorenzoni \(2009\)](#) provides a theory of demand shocks in setups with incomplete information, [Melosi \(2014\)](#) devises methods for the structural estimation of these models and [Adam \(2007\)](#) considers optimal monetary policy.

More generally, the imperfect common knowledge setup shares similarities with the rational inattention theory of [Sims \(2003\)](#) in its emphasis on the difficulty of processing information. [Moscarini \(2004\)](#) analyzes in a continuous time model the optimal sampling frequency of noisy information under information-processing constraints and shows that it can also generate inertia. [Morris and Shin \(2006\)](#) have emphasized that in models with forward-looking expectations even the existence of a small fraction of uninformed agents about the future path of fundamentals can generate persistence in the price behavior.

The option to obtain information resembles [Reis \(2006\)](#), who builds a model where producers decide when to acquire information but do not receive any private signals. The contribution of [Hellwig and Veldkamp \(2009\)](#) focuses on the strategic complementarities in

information acquisition and their implications for multiple equilibria in setups without overconfidence.

The literature in psychology is nicely summarized by [Moore and Healy \(2008\)](#). They provide an extensive survey of the studies that document overconfidence and differentiate between *overestimation* (of one’s abilities), *overplacement* (relative to others), and *overprecision*, which is the type of overconfidence we focus on. An example of overestimation is [Clayson \(2005\)](#), who shows that students overestimate their performance in exams. A prominent example of overplacement is the study of [Svenson \(1981\)](#): in a simultaneous study of US and Swedish drivers, 88% of the US group and 77% of the Swedish group asked believed that they are safer drivers than the median. Another example is [Guthrie et al. \(2001\)](#): 90% of 168 federal magistrate judges thought that they are above average as far as their reversal rate on appeal is concerned. Regarding overprecision, studies like [Soll and Klayman \(2004\)](#) have shown that subjective confidence intervals are systematically too narrow given the accuracy of one’s information. Another classic reference is [Lichtenstein et al. \(1982\)](#).

In finance, other influential studies beyond the ones mentioned in the introduction are [Malmendier and Tate \(2005\)](#), who devise methods of measuring CEO overconfidence based on corporate investment and [Scheinkman and Xiong \(2003\)](#), who explore the potential of overconfidence to generate speculative bubbles in a dynamic setup. [Daniel et al. \(2002\)](#) consider the policy implications of overconfidence. The survey of [Daniel and Hirshleifer \(2015\)](#) provides additional references.

Various studies in experimental economics have explored the interactions of overconfidence and excess entry, see for example [Camerer and Lovo \(1999\)](#). Furthermore, in micro theory there are models on the effects of overconfidence on performance as in [Compte and Postlewaite \(2004\)](#) and on the optimal menu of wage contracts as in [Fan and Moscarini \(2005\)](#). In macroeconomics, [Caliendo and Huang \(2008\)](#) analyze the implications of overconfidence for consumption and savings problems.

3 Economy

In this section, we present a partial-equilibrium model of price-setting behavior in which firms have full information on the structure, parameters and variables of interest.² We consider a continuum of firms indexed by i on the unit interval $[0, 1]$. Each firm produces a good that is differentiated in the preferences of consumers. We do not explicitly model neither consumer preferences nor their optimization problem. We just assume what is needed to characterize

²The model is similar to the one used in [Ball and Romer \(1989\)](#), [Ball and Romer \(1991\)](#) and [Blanchard and Kiyotaki \(1987\)](#).

the price-setting problem of firms. Firms are profit maximizers and set their prices in a monopolistic-competitive market. The problem of a generic firm j is to choose the price of its product $P(j)$ to maximize real profits given by

$$\frac{P(j)}{P}Y(j) - \frac{W}{P}L(j) \quad (1)$$

where $Y(j)$ is the demand of good j given by

$$Y(j) = \left(\frac{P(j)}{P} \right)^{-\varepsilon} Y \quad (2)$$

that depends on the relative price of the good j with respect to the general price index P given by

$$P = \left[\int_0^1 P(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

and on aggregate production Y . The parameter ε (with $\varepsilon > 1$) denotes the elasticity of substitution across differentiated goods in consumer preferences. Firms use labor $L(j)$ to produce goods through the production function $Y(j) = AL(j)$, where A is a productivity shock common to all firms; W is the nominal wage paid for one unit of labor in the labor market.

In this market (not modeled here) we assume that the labor-supply schedule implies the following relation between real wage and aggregate production

$$\frac{W}{P} = Y^\eta \quad (4)$$

with $\eta > 0$.³ We assume the existence of a monetary authority that has a perfect control on the level of nominal spending in the economy. It follows that

$$M = PY \quad (5)$$

where M , which may be labeled as money supply, is indeed controlled by the monetary authority.

³This labor-supply schedule can be derived from the optimizing-behavior of households in a general-equilibrium model. In particular, η would be a combination of the risk-aversion coefficient in consumer preferences and of the Frisch elasticity of substitution of labor supply, or in case of local labor market of ε as well. Assuming a more general labor-supply schedule does not change the subsequent analysis.

We can substitute (2), (4) and (5) into (1) to define a profit function of firm j as

$$\pi(P(j), P, \theta) \equiv \left[\frac{P(j)}{P} - \frac{1}{A} \left(\frac{M}{P} \right)^\eta \right] \left(\frac{P(j)}{P} \right)^{-\varepsilon} \left(\frac{M}{P} \right). \quad (6)$$

Profits of firm j are a function of the action (in game-theoretic sense) of firm j , $P(j)$, of the actions of all other firms synthesized by the index P and the vector $\theta \equiv (A, M)$. Firm j is of measure zero with respect to the aggregate, so its pricing decision does not affect the general price index P .

3.1 Full information

In the full information economy firm j chooses price $P(j)$ to maximize profits (6). Let $P^\dagger(j)$ denote the profit-maximizing price, given by

$$P^\dagger(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{W}{A} = \frac{\varepsilon}{\varepsilon - 1} \frac{P}{A} \left(\frac{M}{P} \right)^\eta \quad (7)$$

which is just the familiar markup rule over nominal marginal cost. Since the right-hand side of (7) is independent of j , all firms set the same price, $P^\dagger(j) = P$. This implies a full information output equal to $Y^* = \left(\frac{A}{\varepsilon/(\varepsilon-1)} \right)^\frac{1}{\eta}$. The full information price level is a function of the exogenous money supply and equilibrium output,

$$P^* = \frac{M}{Y^*} = \left(\frac{\varepsilon}{\varepsilon - 1} \frac{1}{A} \right)^\frac{1}{\eta} M.$$

P^* is function solely of the aggregate shock θ and will be a useful statistic in the next section. For later use, we can use the definition of P^* to write (7) as

$$P^\dagger(j) = P^{1-\eta} P^{*\eta}. \quad (8)$$

This rewriting shows that the parameter η determines whether price-setting decisions are strategic complements (the case $0 < \eta < 1$) or strategic substitutes ($\eta > 1$). In the strategic-complements case, there is a positive elasticity between the individual optimal price and the aggregate price level. The elasticity is negative when pricing decisions are strategic substitutes.

3.2 Incomplete information

Our incomplete information setup is based on two assumptions. First, firms do not know the realization of the aggregate shock θ . Second, following the lead of [Woodford \(2002\)](#) and the imperfect common knowledge literature, we assume that the general price level P is not in the information set of price-setters.

In particular, each firm receives a private signal about θ , that captures its “subjective perception” about the aggregate state. The firm uses its private signal to make inferences on productivity and money supply shocks that affect marginal costs and aggregate demand. Furthermore, the signal is used to infer the beliefs, and therefore the pricing decisions, of other firms. An entire hierarchy of beliefs about the price level is formed and will be the topic of a later section. Thus, price-setters face both fundamental and strategic uncertainty. Overconfidence enters the inference problem by making agents put too much weight on their own subjective perception of the aggregate shock. Moreover, we allow the fraction of uninformed firms to be *endogenous*: each firm j can pay a real fixed cost \tilde{c}^j and obtain full information about the shock θ and the pricing decisions of the rest of the firms.

The timing is as follows. The firm receives a private signal. Given the signal, the firm decides to obtain or not information. If the firm becomes “informed”, it sets price $P^\dagger(j)$. If the firm decides to stay “uninformed,” it sets price $\tilde{P}(j)$. We can characterize this problem by working backwards.

Informed firm. The optimal price when the firm has complete information is just the typical markup over nominal marginal cost, which can be written as in (8), $P^\dagger(j) = P^{1-\eta} P^{*\eta}$. The price level P is obviously not the same as in an environment with full information.

Uninformed firm. If the firm does not obtain information, it sets a price that maximize its *subjective* expectation of real profits. Let E^j denote the subjective expectation operator of firm j , given its private information set. $\tilde{P}(j)$ maximizes $E^j \pi(P(j), P, \theta)$ which implies that

$$\tilde{P}(j) = \frac{E^j \{P^\dagger(j)Z\}}{E^j \{Z\}}, \quad (9)$$

where Z is defined as $Z \equiv MP^{\varepsilon-2}$ and $P^\dagger(j)$ is given by (8). According to (9), when there is incomplete information, a generic firm j sets its price as a subjective expectation (appropriately weighted) of nominal marginal cost, i.e. as an average of the price that would be set under complete information.

Obtaining information. A generic firm j chooses to acquire complete information when the expected increase in profits in doing this is higher than the cost \tilde{c}^j

$$E^j\{\pi(P^\dagger(j), P, \theta) - \pi(\tilde{P}(j), P, \theta)\} \geq \tilde{c}^j. \quad (10)$$

Having observed the realization of its own signal, a firm j evaluates the left-hand side (LHS) of (10). The firm acquires information and sets price $P^\dagger(j)$, if (10) holds, otherwise it chooses $\tilde{P}(j)$.

3.3 Approximation to the incomplete-information model

We have not specified yet the signal structure and the filtering problem that is hiding behind the expectation operator E^j . Before we do that, we will simplify the problem with an approximation around a deterministic steady state where $\theta = \bar{\theta} = (\bar{A}, \bar{M})$.

Proposition 1. *As second-order approximation to the LHS of (10) leads to a criterion of the form*

$$\text{var}^j\{p^\dagger(j)\} \geq c^j, \quad (11)$$

where $\text{var}^j\{\cdot\}$ denotes the variance operator conditional on the subjective information set, while $p^\dagger(j)$ is the log of the price that firm j would set with complete information and c^j is a reparametrization of the cost \tilde{c}^j .

Proof. See Appendix. □

The decision of acquiring or not information depends on whether the subjective variance of the price that a firm sets under complete information is higher than the cost c^j . An important implication of proposition 1 is that (11) can be evaluated using just a log-linear approximation to the equilibrium conditions. In this log-linear approximation, equation (9) implies that the log of the price under incomplete information is the expected value of the log of the price under complete information

$$\tilde{p}(j) = E^j p^\dagger(j), \quad (12)$$

where lower-case letters denote log of the respective variables.⁴ Moreover $p^\dagger(j)$ is independent of j and from (8) it can be seen that it satisfies an exact log-linear relationship given by

$$p^\dagger = (1 - \eta)p + \eta p^*, \quad (13)$$

⁴In the steady-state all firms set the same constant price.

where p^* is the log of the full-information equilibrium price level and p is given by

$$p = \int_0^1 p(i) di$$

as a result of a first-order approximation of (3).

Let us denote with μ the fraction of firms that in equilibrium decide to keep the subjective information set and assuming without loss of generality that agents $j \in [0, \mu]$ are the ones who remain uninformed, we can write the above equation as

$$p = \mu \tilde{p} + (1 - \mu) p^\dagger, \quad (14)$$

where we have defined with \tilde{p} the average price of the subjectively-informed firms as

$$\tilde{p} \equiv \frac{1}{\mu} \int_0^\mu \tilde{p}(i) di. \quad (15)$$

We can then plug (14) into (13) to obtain

$$p^\dagger = \delta p^* + (1 - \delta) \tilde{p}, \quad (16)$$

where

$$\delta \equiv \frac{\eta}{\eta + (1 - \eta)\mu}.$$

The weight on the full information price is equal to the degree of complementarity, $\delta = \eta$, if every firm is uninformed ($\mu = 1$) as in [Woodford \(2002\)](#). The weight δ is a decreasing function of μ and $\eta \leq \delta \leq 1$ in case of strategic complements ($\eta < 1$) and δ an increasing function of μ with $1 \leq \delta \leq \eta$ in case of strategic substitutes ($\eta > 1$).

The set of equations (12), one for each firm that remains uninformed, together with (15) and (16) determine the equilibrium prices of informed and uninformed firms in a first-order approximation to the equilibrium conditions.

3.4 Signal extraction and overconfidence

We describe now the information structure in detail. We note that uncertainty about the aggregate shock θ has collapsed to uncertainty about the full-information price p^* , which is a linear combination of the logarithms of the technology and the money supply shock, $p^* = \text{const.} + \ln M - \frac{1}{\eta} \ln A$. Without loss of generality for our purpose of exploring the effects of overconfidence on pricing, we are going to assume that firms receive signals about

this particular linear combination p^* . Let p^* be a random variable of the form

$$p^* = \bar{p}^* + u,$$

where \bar{p}^* is a constant and u is a Gaussian white-noise process with variance σ_u^2 . This distribution is common knowledge and corresponds to the objective probability distribution of p^* .

Overconfidence. Each firm receives a *private* signal s^j that is linearly related to p^* as

$$s^j = p^* + \xi^j,$$

where ξ^j is an idiosyncratic Gaussian noise with mean zero. Let $\bar{\sigma}_\xi^2$ and σ_ξ^2 denote the *perceived* and *true* variance of ξ^j . We introduce overconfidence as in the influential contributions of [Daniel et al. \(1998\)](#) and [Daniel et al. \(2001\)](#) in the behavioral finance literature: we assume that $\bar{\sigma}_\xi^2 < \sigma_\xi^2$. So price-setters overestimate their ability to infer the hidden state from their private signals. This corresponds to the behavioral bias of *overprecision*.⁵ Moreover, ξ^j is statistically independent of u as well as of ξ^i for each $i \neq j$. All this information is common knowledge.

Given this information structure, each firm can form its own expectation of the full-information price p^* as in a standard signal-extraction problem

$$E^j p^* = (1 - \bar{r}^j) \bar{p}^* + \bar{r}^j s^j, \tag{17}$$

where the weight \bar{r}^j is defined as

$$\bar{r}^j = \bar{r} \equiv \frac{1}{1 + \bar{\lambda}} \quad \text{with} \quad \bar{\lambda} = \frac{\bar{\sigma}_\xi^2}{\sigma_u^2}.$$

Since $\bar{\lambda}$ is common and common knowledge across the different firms, then \bar{r}^j is independent of j and equal to a common \bar{r} . In particular, $\bar{\lambda}$ represents the noise-to-fundamental variance ratio and can be interpreted as an index of confidence in how a firm's private signal is a good representation of the full-information price. Lower values of $\bar{\lambda}$ implies a higher weight to the signal when firms form expectations of the full-information price and then a high degree of confidence on the subjective information set. We can define a 'true' degree of confidence λ as $\lambda \equiv \frac{\sigma_\xi^2}{\sigma_u^2}$ with a respective weight in the filtering problem, $r = 1/(1 + \lambda)$.

⁵See the survey of [Moore and Healy \(2008\)](#) for a succinct taxonomy of overconfidence studies in terms of overestimation, overplacement and overprecision.

Under overconfidence, we have $\bar{\lambda} < \lambda$ and therefore $\bar{r} > r$, so there is an excessive weight on the private signal.

3.5 Higher-order beliefs and optimal pricing

Each firm forms its own expectation of the signals of others as

$$E^j s^i = E^j p^* = (1 - \bar{r})\bar{p}^* + \bar{r}s^j$$

which is then a first-order expectation belief.⁶ Furthermore, each firm can form its subjective expectation of others' first-order expectation belief as

$$E^j E^i s^k = E^j [(1 - \bar{r})\bar{p}^* + \bar{r}s^i] = (1 - \bar{r})\bar{p}^* + \bar{r}E^j p^*$$

and so on.

To solve for the equilibrium prices of informed and uninformed agents and for the equilibrium fraction of firms that acquire or not information, we first guess that μ is known to each firm j . We then verify that this is indeed the case. Given this guess, we can substitute (16) into (12) to get

$$\tilde{p}(j) = \delta E^j p^* + (1 - \delta)E^j \tilde{p} \quad (18)$$

which can be averaged across all uninformed price setters to obtain

$$\tilde{p} = \delta \bar{E} p^* + (1 - \delta)\bar{E} \tilde{p},$$

where we have defined the operator $\bar{E}(\cdot) \equiv \frac{1}{\mu} \int_0^\mu \bar{E}(\cdot) di$. This operator represents the average expectation among the uninformed firms. We can iterate the above expression to obtain

$$\tilde{p} = \delta \sum_{k=0}^{\infty} (1 - \delta)^k \bar{E}^{(k+1)} p^*, \quad (19)$$

where the $(k+1)$ -order average expectation operator is defined as $\bar{E}^{(k+1)}(\cdot) \equiv \bar{E}(\bar{E}^{(k)}(\cdot))$ for each $k \geq 1$.⁷ It follows that the average price of uninformed firms is a linear combination of their higher-order average expectations of the full-information price. We can plug (19) into

⁶See [Allen et al. \(2006\)](#), [Amato and Shin \(2003\)](#) and [Amato and Shin \(2006\)](#) for examples of problems with iterated expectations.

⁷Under the restriction that $\eta(2\mu - 1) < 2\mu$, δ is such that $|1 - \delta| < 1$.

(18) to obtain

$$\tilde{p}(j) = \delta E^j p^* + \delta(1 - \delta) \sum_{k=0}^{\infty} (1 - \delta)^k E^j \bar{E}^{(k+1)} p^*.$$

The price set by an uninformed firm depends on its own expectation of the full-information price and its own expectation of the average expectation (of uninformed firms) of the full-information prices as well as on all higher-order expectations. Since (17) holds for all uninformed firms, we can obtain that

$$\bar{E} p^* = (1 - \bar{r}) \bar{p}^* + \bar{r} p^*, \quad (20)$$

where we have implicitly assumed that the law of large number holds on a positive measure⁸

$$\frac{1}{\mu} \int_0^{\mu} \xi^i di = 0. \quad (21)$$

It follows that the firm i 's expectation of the average expectation of the full-information price is given by

$$\begin{aligned} E^i \bar{E} p^* &= (1 - \bar{r}) \bar{p}^* + r E^i p^* \\ &= (1 - \bar{r}) \bar{p}^* + \bar{r} (1 - \bar{r}) \bar{p} + \bar{r}^2 s^i \end{aligned}$$

from which it follows that the second-order average estimate is given

$$\bar{E}^{(2)} p^* = (1 - \bar{r})(1 + \bar{r}) \bar{p}^* + \bar{r}^2 p^*.$$

By re-iterating the above arguments, we get that the k -fold average expectation of the full-information price is

$$\bar{E}^k p^* = (1 - \bar{r}^k) \bar{p}^* + \bar{r}^k p^*. \quad (22)$$

We can substitute (22) into (19) to finally obtain

$$\begin{aligned} \tilde{p} &= \frac{1 - \bar{r}}{1 - (1 - \delta) \bar{r}} \bar{p}^* + \frac{\bar{r} \delta}{1 - (1 - \delta) \bar{r}} p^* \\ &= \frac{\eta(1 - \bar{r}) + (1 - \eta)(1 - \bar{r}) \mu}{\eta + (1 - \eta)(1 - \bar{r}) \mu} \bar{p}^* + \frac{\bar{r} \eta}{\eta + (1 - \eta)(1 - \bar{r}) \mu} p^*. \end{aligned} \quad (23)$$

⁸See Uhlig (1996) for the conditions under which this holds.

Substituting (23) into (16) we obtain that the price set by any informed firm is

$$p^\dagger = \frac{(1 - \bar{r})(1 - \eta)\mu}{\eta + (1 - \eta)(1 - \bar{r})\mu} \bar{p}^* + \frac{\eta}{\eta + (1 - \eta)(1 - \bar{r})\mu} p^*. \quad (24)$$

3.6 Information acquisition

Given (24), it follows that a generic firm j decides to acquire information if the following inequality holds

$$\left[\frac{\eta}{\eta + (1 - \eta)(1 - \bar{r})\mu} \right]^2 \text{var}^j\{p^*\} \geq c^j,$$

where $\text{var}^j\{p^*\}$ is the variance of the full-information price level conditional on the subjective information set of firm j . This is given by

$$\text{var}^j\{p^*\} = \sigma_u^2(1 - \bar{r}^j).$$

We can then write the above inequality as

$$\left[\frac{\eta}{\eta + (1 - \eta)(1 - \bar{r})\mu} \right]^2 \sigma_u^2(1 - \bar{r}^j) \geq c^j, \quad (25)$$

where we have kept the distinction –since it matters for the discussion that follows– between the own degree of confidence \bar{r}^j and the others’ degree of confidence \bar{r} – although we have assumed that they are the same.⁹

According to (25), several parameters of the model drive the incentives for firm j to acquire information. The higher is the prior on the variance of the full-information price, σ_u^2 , the higher are the incentives to acquire information. Obviously, the lower the cost c^j , the higher those incentives. In the case of strategic complementarity in the pricing decision, $0 < \eta < 1$, the higher is the fraction of firms that are acquiring information (i.e. the lower the μ) the higher are the incentives for the individual firm to acquire information. This result is of the same nature as the one found by Ball and Romer (1989) in a similar model but with only imperfect information, in which firms’s decisions are on whether to change or not prices.

Each firm’s decision is also influenced by the degree of confidence in the informativeness of the signal. If \bar{r}^j is high ($\bar{\lambda}^j$ is low), then the firm will not have incentives to acquire finer information. A high degree of confidence implies that firms are going to be stuck with their perceptions of the world when setting their prices.

Interestingly, if the confidence of others increases ($\bar{\lambda}$ decreases and \bar{r} increases) then the

⁹Indeed, (24) could have been derived even if we didn’t have the same \bar{r}^i by defining $\bar{r} \equiv (\int_0^\mu \bar{r}^i di) / \mu$ and by assuming a law of large numbers to hold for $\int_0^\mu \xi^i \bar{r}^i di = 0$.

price under complete information has higher subjective variance since the average price of uninformed firms is getting close to the full information price, as shown in (20). Then, each individual firm has higher incentives to acquire information and imitate other firms –when pricing decisions are strategic complements.

We move to characterize the equilibrium value of μ , under the assumption $\bar{r}^j = \bar{r}$ for each j . We define

$$c^* \equiv \left[\frac{\eta(1 - \bar{r})^{\frac{1}{2}}}{\eta + (1 - \eta)(1 - \bar{r})\mu} \right]^2 \sigma_u^2$$

and note that (25) implies that all firms with c^j less than c^* acquire information. Let the cumulative distribution function of costs be F with respective density f on support $[\underline{c}, \bar{c}]$. The fraction μ of uninformed firms satisfies

$$1 - F(c^*) = \mu. \quad (26)$$

This solution confirms our initial guess that μ is a function of known parameters and then known to each firm j . The properties of $F(\cdot)$ determine the existence and the characteristics of the equilibrium. Indeed, when $c^j = c$ for each j , multiple equilibria are possible for the same reasons as they occur in the imperfect-information model of Ball and Romer (1989). For other $F(\cdot)$ multiple equilibria might disappear. Since this is not the focus of this work, we assume that $F(\cdot)$ and $f(\cdot)$ are such that there exists a unique equilibrium. We get the following proposition.

Proposition 2. (*“Overconfidence and uninformed firms”*)

Assume that we are at a stable equilibrium where $1 + f(c^) \frac{\partial c^*}{\partial \mu} > 0$.*

$$\text{If } (1 - \eta)(1 - \bar{r})\mu < \eta \quad \text{then} \quad \frac{d\mu}{d\bar{r}} > 0. \quad (27)$$

Thus, the fraction of uninformed firms increases when the weight on the private signal increases.

Proof. See Appendix. □

The proposition implies that overconfidence can lead to a larger mass of uninformed firms relative to a rational signal-extraction benchmark.

Discussion. Proposition 2 is *not* a trivial consequence of the overconfidence of the agents. It involves the effects of firm’s own confidence and the opposing effects of the confidence

of other firms through the mechanism of higher-order expectations. An increase in one's confidence decreases the incentives to acquire information. But an increase in the confidence of others makes the price of informed firms more volatile, since there is larger reliance on private signals in equilibrium, *amplifying* the effect of higher-order beliefs. To see that, consider the extreme case where private signals are completely *uninformative*, $\bar{r} = 0$. In that case, the entire mechanism of higher-order beliefs is mute: uninformed firms set a price equal to their prior, $\tilde{p} = \bar{p}^*$, and the price of informed firms becomes $p^\dagger = (1 - \delta)\bar{p}^* + \delta p^*$. So the weight on the full information price p^* reaches its minimum, leading to small volatility and reduced incentives to acquire information.

The inequality condition in (27) requires that the effect of the firm's own confidence, which leads to a larger equilibrium fraction μ , is *stronger* than the higher-order beliefs effect, which reduces μ . As expected, the condition always holds in the case of strategic substitutes, $\eta > 1$. In the case of strategic complements, the condition holds if we effectively limit the effect of higher-order beliefs. This would happen in an equilibrium where μ is small (so the complementarities are not strong enough), $\mu < \frac{\eta}{(1-\eta)(1-\bar{r})}$. From another angle, the higher-order beliefs effect would be contained if we bounded η away from a lower bound, by writing the condition as $\eta > \frac{(1-\bar{r})\mu}{1+(1-\bar{r})\mu}$. The lower bound for η is always smaller than 1/2 and decreases to zero when \bar{r} increases to unity. The conclusion is that if complementarities are not too "large," overconfidence increases the equilibrium fraction of uninformed firms.

4 Price implications of incomplete information and overconfidence

In the previous section, we have shown how to determine the fraction of firms that in equilibrium decide to remain uninformed. A high degree of confidence under standard regularity conditions implies a higher fraction of uninformed firms. In this section we study the price implications of the model and in particular the relation between excess volatility of prices and overconfidence.

A first important implication is that the model displays two levels of heterogeneity: at a first stage, there are differences in prices between informed and uninformed firms. At a second stage, within uninformed firms, prices are related to the realization of subjective signals. We can rewrite equation (24) and show that the price of the informed firms is

$$p^\dagger = \bar{p}^* + (1 + \bar{\lambda})\hat{k}u, \quad (28)$$

where

$$\hat{k} = \frac{\bar{r}\eta}{\eta + (1 - \eta)(1 - \bar{r})\mu}.$$

The prices of the informed firms react only to the fundamental shocks of the model. On the opposite the uninformed firms set their prices as a subjective expectation of p^\dagger , based on their signals which include also non-fundamental noises. We obtain that a generic uninformed firm j sets

$$\tilde{p}(j) = \bar{p}^* + \hat{k}u + \hat{k}\xi^j, \quad (29)$$

while the average price of uninformed firms is given by

$$\tilde{p} = \bar{p}^* + \hat{k}u.$$

We first discuss how prices react to fundamental shocks. Following (28) prices of informed firms react less than proportionally to fundamental shocks when pricing decisions are strategic complements, since $(1 + \bar{\lambda})\hat{k} < 1$, but more than proportionally in the strategic-substitute case. As shown in (29), the response of uninformed firms is always smaller than that of informed firms, since $\bar{\lambda} > 0$. The discrepancy is coming from the fact that the informed firms do not have to filter the hidden shock.

However, prices of uninformed firms react also to non-fundamental shocks, ξ^j , in the same proportion as they do to fundamental shocks.

Overconfidence can affect the volatility of prices. Using equation (29), we obtain that the “true” variance of prices for a generic uninformed agent j is

$$\text{var}\{\tilde{p}(j)\} = (1 + \lambda)\hat{k}^2\sigma_u^2.$$

Equation (28) implies that the variance of the prices of informed firms is given by

$$\text{var}\{p^\dagger\} = (1 + \bar{\lambda})^2\hat{k}^2\sigma_u^2.$$

It follows that the ratio of the volatilities of prices of uninformed and informed firms is given by

$$\frac{\text{var}\{\tilde{p}(j)\}}{\text{var}\{p^\dagger\}} = \left[\frac{(1 + \lambda)^{\frac{1}{2}}}{(1 + \bar{\lambda})} \right]^2.$$

When the signal-extraction problem is rational (i.e. $\lambda = \bar{\lambda}$), prices of uninformed firms are always less volatile than informed firms. With overconfident firms, it is instead possible for the reverse to happen. It is sufficient that $(1 + \bar{\lambda}) < (1 + \lambda)^{\frac{1}{2}}$, which requires that the true volatility of the idiosyncratic noise σ_ξ^2 is large enough relative to the perceived $\bar{\sigma}_\xi^2$.

A second important implication of overconfidence is that it is even possible to have excess volatility of the price of an individual uninformed firm with respect to the full-information (fundamental) price. Indeed, we obtain that

$$\frac{\text{var}\{\tilde{p}(j)\}}{\text{var}\{p^*\}} = \left[\frac{\zeta(1+\lambda)^{\frac{1}{2}}}{(1+\bar{\lambda})} \right]^2$$

where ζ is a positive parameter given by $\zeta = \hat{k}/\bar{r}$ such that $\zeta < 1$ ($\zeta > 1$) when pricing decisions are strategic complements (substitutes). To have excess volatility of the prices of uninformed firms with respect to fundamentals, it is required that $(1+\bar{\lambda}) < \zeta(1+\lambda)^{\frac{1}{2}}$ which is then a more (less) stringent condition than before when pricing decisions are strategic complements (substitutes).¹⁰

Overconfidence has two important roles in this model. On one side, it implies that a higher fraction of firms is going to decide optimally not to acquire information and just pay attention to their own private signals. On the other side, the prices of uninformed firms can be more volatile than fundamental disturbances and this volatility is driven by the noise in the perception of the full-information price. In the dynamic extension of the above model, the fact that overconfident price setters are less prone to acquire information implies that there can be a high proportion of this kind of subjectively-driven price setters. [Woodford \(2002\)](#) has shown that higher-order expectations matter for determining persistent effects of output and prices following exactly those shocks agents are subjectively informed about. In our model, the existence of subjectively-informed firms with overconfident beliefs can produce an excess *volatility* of prices with respect to fundamentals.

5 Infinite-horizon model

In this section, we consider an extension of the previous model to an infinite horizon. We assume that each firm does not know the realization of the sequence $\{\theta_t\}_{t=t_0}^{\infty}$. However, each firm has a prior distribution on the sequence $\{\theta_t\}_{t=t_0}^{\infty}$ that coincides with the correct distribution and which is common knowledge. In each period and contingency, each firm can observe a private signal s_t^j . In particular the sequence of signals $\{s_t^j\}_{t=t_0}^{\infty}$, one for each j , is related to the sequence $\{\theta_t\}_{t=t_0}^{\infty}$ through a likelihood function which is known and common knowledge but, as before, does not necessarily coincide with the correct likelihood function. As in the previous model, incomplete information is modelled by assuming that each firm

¹⁰Note that with no overconfidence ($\lambda = \bar{\lambda}$) the ratio is always smaller than unity even in the case of strategic substitutes ($\zeta > 1$). This is clear if we note that $\hat{k} = \delta/(\delta + \lambda)$ and that the ratio is less than unity when $Q(\delta) = \lambda\delta^2 - 2\lambda\delta - \lambda^2 < 0$, which holds for the permissible δ , i.e. such that $|1 - \delta| < 1$.

knows only its own realization of the signals and not those of the others, as well as not the price index and the individual prices. Each firm has the option to acquire information by paying a cost \tilde{c}^j . Once the cost is paid, the firm remains in the ‘informed’ state forever.

We assume that firms choose prices to maximize the expected discounted value of profits given by

$$E_{t_0}^j \sum_{t=t_0}^{\infty} \beta^{t-t_0} \pi(P_t(j), P_t, \theta_t),$$

where β is such that $0 < \beta < 1$.¹¹ $E_{t_0}^j$ is the appropriate expectation operator conditional on the information at time t_0 . Prices are set freely in each period. An ‘informed’ firm sets its price as

$$P_t^\dagger(j) = P_t^{1-\eta} P_t^{*\eta}$$

for each period t after having paid the information cost. An ‘uninformed’ firm instead sets its price as

$$\tilde{P}_t(j) = \frac{E_t^j \{P_t^\dagger(j) Z_t\}}{E_t^j \{Z_t\}},$$

where Z_t has the same definition as before and the expectation operator is conditional on the information set of firm j at t .

To characterize the decision for a generic firm j to acquire or not information, we guess an equilibrium and then verify that prices and information decisions are consistent with that equilibrium. The analysis is simplified by noting that the fraction of firms that remain uninformed each period cannot increase over time, i.e. $\{\mu_t\}_{t=t_0}^{+\infty}$ is a non-increasing sequence. Of the many equilibria that can exist, we are interested in ones in which $\mu_t = \mu$ for each $t \geq t_0$. In particular, in these stationary equilibria, whichever firm decides to be informed does it in the first period. For this to be optimal, the strategy of getting information in the first period should give higher expected discounted profits than the strategy of waiting until a generic time T , given the equilibrium strategies of all other firms. In particular at time t_0 the expected profits to acquire immediately information and pay the cost should be higher than the strategy of remaining with the subjective information until a generic period T and pay the cost in that period. For a generic firm j to become informed at time t_0 , the following inequality should hold for each $T > t_0$

$$E_{t_0}^j \sum_{t=t_0}^{T-1} \beta^{t-t_0} \pi(\tilde{P}_t(j), P_t, \theta_t) - \beta^T \tilde{c}^j \leq E_{t_0}^j \sum_{t=t_0}^{T-1} \beta^{t-t_0} \pi(P_t^\dagger(j), P_t, \theta_t) - \tilde{c}^j,$$

¹¹We can generalize the analysis that follows by assuming a stochastic discount factor to evaluate real profits across contingencies and time.

which can be rewritten

$$E_{t_0}^j \sum_{t=t_0}^{T-1} \beta^{t-t_0} \left\{ E_t^j [\pi(P_t^\dagger(j), P_t, \theta_t) - \pi(\tilde{P}_t(j), P_t, \theta_t)] \right\} \geq \tilde{c}^j (1 - \beta^T). \quad (30)$$

We take a second-order approximation of the above problem around a stationary point with unitary relative prices to obtain

$$E_{t_0}^j \sum_{t=t_0}^{T-1} \beta^{t-t_0} \text{var}_t^j \{p_t^\dagger\} \geq c^j (1 - \beta^T).$$

We guess, and verify later, that in the equilibrium $\text{var}_t^j \{p_t^\dagger\}$ is a constant that does not depend on j and is also independent of t in a stationary filtering problem. The above condition then simplifies to

$$\text{var}_t^j \{p_t^\dagger\} \geq c^j (1 - \beta), \quad (31)$$

which is also independent of T .

We verify now that $\text{var}_t^j \{p_t^\dagger\}$ is constant and independent of j , and that μ is also a constant and known within the information set of each type of firm at time t_0 . As before, we just need to characterize the equilibrium values of prices in a log-linear approximation to the equilibrium. It is still true that the set of equations (12), one for each firm that remains uninformed, together with (15) and (16) determine the equilibrium prices of informed and uninformed firms in a first-order approximation to the equilibrium conditions. We continue to assume that each firm receives a private signal s_t^j that is related linearly to p_t^* as

$$s_t^j = p_t^* + \xi_t^j, \quad (32)$$

where ξ_t^j is an idiosyncratic Gaussian noise with mean zero, perceived variance $\bar{\sigma}_\xi^2$ and true variance σ_ξ^2 for each j , with $\bar{\sigma}_\xi^2 < \sigma_\xi^2$. We assume that ξ_t^j , for each j , is statistically independent of the sequence $\{p_t^*\}$ as well as of the sequence $\{\xi_t^i\}$ for each $i \neq j$. All this information is common knowledge, but the realization of the private signal s_t^j is private information. We allow now $\{p_t^*\}$ to be a first-order autoregressive stochastic process of the form

$$p_t^* = \bar{p}^* + \rho p_{t-1}^* + u_t \quad (33)$$

with $|\rho| \leq 1$ where u_t is Gaussian noise with mean zero and variance σ_u^2 . The assumption of persistence of the unobservable shock can in principle be a source of complication in the solution of the model, for an infinite dimensional state might be necessary to keep track of

the higher-order beliefs of other firms. [Woodford \(2002\)](#) has shown that the dimension of the hidden-state space is finite in the same model as the one presented here but with all firms assumed to be uninformed and without any endogenous decision of acquiring information.

Proposition 3. (*“Optimal prices in the dynamic model”*)

- The general price index evolves according to

$$p_t = \bar{p}^* + \rho(1 - \hat{k})p_{t-1} + \rho\hat{k}p_{t-1}^* + [\delta(1 - \mu)(1 - \hat{k}) + \hat{k}]u_t,$$

where ¹²

$$\hat{k} \equiv \frac{1}{2\rho^2} \left\{ \rho^2 - 1 - \frac{\delta}{\bar{\lambda}} + \sqrt{\left[(1 - \rho^2) + \frac{\delta}{\bar{\lambda}} \right]^2 + 4\rho^2 \frac{\delta}{\bar{\lambda}}} \right\}.$$

- The price of informed firms follows

$$p_t^\dagger = \bar{p}^* + \rho(1 - \hat{k})p_{t-1}^\dagger + \rho\hat{k}p_{t-1}^* + [\delta(1 - \hat{k}) + \hat{k}]u_t. \quad (34)$$

- The price of uninformed firms follows

$$\tilde{p}_t(j) = \bar{p}^* + \rho(1 - \hat{k})\tilde{p}_{t-1}(j) + \rho\hat{k}p_{t-1}^* + \hat{k}(u_t + \xi_t^j). \quad (35)$$

- The contemporaneous variance of p_t^\dagger in a stationary solution is given by

$$\text{var}_t^j\{p_t^\dagger\} = \frac{1 + \bar{\lambda}[1 - \rho^2(1 - \hat{k})]^2}{[1 - \rho^2(1 - \hat{k})^2]} \bar{\lambda}\hat{k}^2\sigma_u^2. \quad (36)$$

Proof. See the Appendix for the details of the calculations. Our approach is a variant of [Woodford \(2002\)](#), properly modified to account for the fact that there is a fraction of firms that is uninformed and acquires endogenously information. \square

Note that the static model is nested under the assumption that $\rho = 0$. We can now evaluate (31). It follows that equilibrium fraction of uninformed firms is implicitly defined

¹²The parameter \hat{k} in the dynamic model is a different function of other (exogenous) parameters than the \hat{k} in the static model and represents a linear combination of the vector of Kalman gains. We use the same notation, since when $\rho = 0$ the two expressions coincide.

by the same condition as (26) where now instead c^* is given by

$$c^* \equiv \frac{1 + \bar{\lambda}[1 - \rho^2(1 - \hat{k})]^2}{[1 - \rho^2(1 - \hat{k})^2]} \frac{\bar{\lambda}\hat{k}^2\sigma_u^2}{(1 - \beta)}. \quad (37)$$

The main qualitative results of the static model hold in this extension with some qualifications. Indeed it is still the case that overconfidence is needed for the volatility of prices of uninformed to be higher than that of informed. The ratio of the unconditional variances between informed and uninformed firms is higher than the unitary value when the following criterion holds:

Proposition 4. (*“Excess price volatility”*)

$$\frac{\text{var}\{\tilde{p}_t(i)\}}{\text{var}\{p_t^\dagger\}} > 1$$

if and only if

$$\lambda > 2\bar{\lambda} + \bar{\lambda}^2[1 - \rho^2(1 - \hat{k})^2]. \quad (38)$$

Proof. See Appendix. □

Note that the criterion nests the static case result for $\rho = 0$. In this dynamic model, it does not only matter the difference between the ‘true’ and the subjective degree of confidence, but also other parameters. Indeed since $\hat{k} < 1$, the discrepancy between λ and $\bar{\lambda}$ that is needed in order to have excessive volatility of the uninformed prices is smaller than in the static case. The reason is that the persistence of the shock process makes past estimates useful to forecast the future evolution of the state. But this leads to a larger reliance on their subjective perception (signals) and therefore, comparatively to the static case, agents are driven more by their perceptions. As a consequence, the amount of overconfidence needed to have excess volatility is less. This is also the case if the mass of uninformed agents (μ) increases since \hat{k} becomes smaller and if the degree of strategic complementarity increases, i.e. η becomes smaller.

6 An application

In this section we explore the empirical implications of our model for individual price volatility and aggregate inflation persistence. There are two empirical facts of interest. First, [Klenow and Kryvtsov \(2008\)](#) have shown that the average absolute size of price changes for individual goods composing the CPI index amounts to 8.5% on a monthly basis. Second, VAR analyses

on the response of output to monetary disturbances have shown a hump-shaped behavior with a peak that ranges from 4 till 6 quarters like in [Christiano et al. \(2005\)](#).

There are many possible sources for price volatility which we have completely abstracted from in our analysis, like idioyncratic productivity or demand shocks. Our objective here is to show that the mechanism presented in this paper can in some ways complement other explanations. To this end, we ask to what extent, under a reasonable calibration of the model, degrees of overconfidence found in experimental studies can explain the variability of prices found in the data. With reasonable calibration we mean that our model should be also consistent with some other moments of the data reported in [Klenow and Kryvtsov \(2008\)](#) and at the same time require information costs which are in line with other empirical studies. Moreover, we want to show that our explanation can be consistent with a model that allows for a persistent response of output to a nominal spending disturbance.¹³

To start the exercise, we note that [Bils and Klenow \(2004\)](#) found that prices are sticky with a median duration of 4.3 months. But in our model firms change their price in each period, even if they have incomplete information. For that reason, we decide to measure the time period of our model in quarters, during which we might reasonably assume that all the firms had the time to adjust their prices.

The parameters of the model are $(\rho, \bar{p}^*, \sigma_u^2, \bar{\lambda}, \frac{\lambda}{\bar{\lambda}}, \eta, \varepsilon, F)$, where F indicates the parameters that characterize the distribution of the costs. We calibrate $\rho = 1$ in a way that our model can be consistent with a unit root in the general price index, which in general is not rejected by the data.¹⁴ Under this assumption equilibrium output in deviation from the steady state (i.e. $y_t \equiv \ln Y_t / \bar{Y}$) follows the process

$$y_t = (1 - \hat{k})y_{t-1} + (1 - c)u_t,$$

where $c = (\delta\mu + 1 - \delta)\hat{k} + \delta(1 - \mu)$ and $\delta = \eta/(\eta + (1 - \eta)\mu)$. So \hat{k} is a parameter that summarizes the output persistence to a monetary shock like in [Woodford \(2002\)](#). From this process we can deduce that the half-life of the response of output to a nominal spending shock is given by $\tau = -\ln 2 / \ln(1 - \hat{k})$. Since we want our model to display a persistent response of output and at the same time be consistent with all the other facts, we assume that τ is equal to 4 quarters as it is suggested by the VAR literature and which is a reasonable value according to the discussion of [Woodford \(2003, ch. 3\)](#). This assumption implies that

¹³For calibration purposes we are going to treat the shock in the full information price as a nominal spending shock.

¹⁴We have experimented with data on the non-shelter CPI for the sample period of KK (1988:1-2003:4) and found that a null of a unit root in the price level cannot be rejected.

$\hat{k} = 0.159$. Under the assumption of $\rho = 1$, the process of the inflation rate is given by¹⁵

$$\pi_t = \hat{k}\bar{p}^* + (1 - \hat{k})\pi_{t-1} + cu_t - (c - \hat{k})u_{t-1}. \quad (39)$$

In their sample, KK find an average monthly percentage price change (excluding sales) among firms that change prices of 1.11 % with a standard deviation of 1.14%. This corresponds to a calibration of $\bar{p}^* = 1.11$ for the mean price change among all firms in our model. Furthermore, their data on the variance of the inflation provide an additional moment condition to match for the variance of the model inflation rate, σ_π^2 , which according to (39) is just a function of the form

$$\sigma_\pi^2 = f(\mu, \hat{k}, \eta, \sigma_u^2) \quad (40)$$

pinning down a relation between η , σ_u^2 and μ for given \hat{k} , σ_π^2 .

Studies on the costs of price adjustment like Zbaracki et al. (2004) have shown that managerial and customer costs of price adjustment constitute a large fraction of firms profits even if actual menu costs are quite small. In fact, managerial costs (which refer to information gathering, decision-making and communicating-to-sales-team costs) are 4.61% of the profits, whereas customer costs (communication and negotiation costs) can reach even 15.01% of profits. The concept of managerial costs is the closest to the notion of information-processing costs in our model and therefore we use the figure of 4.61% of profits as the most relevant as an average measure of cost. More specifically, we assume a uniform distribution of costs per period as a fraction of steady state profits with a minimum cost of zero and a mean cost of 4.61%.¹⁶ Furthermore, we assume an elasticity of substitution between the differentiated products $\varepsilon = 6$, which corresponds to a markup under full information of 20%. Given these assumptions, and noting that $\bar{\lambda} = \delta(1 - \hat{k})/\hat{k}^2$ equations (26) and (37) imply the following relation

$$g(\mu, \hat{k}, \eta, \sigma_u^2) = 0 \quad (41)$$

which is then another implicit relation between η , σ_u^2 and μ for given \hat{k} .¹⁷

The parameter η is critical for determining whether pricing decisions are strategic complements or substitutes and plays a crucial role in determining the persistence of the response of output to a monetary shock, as discussed in Woodford (2003). Indeed, Woodford (2003, ch. 3) has shown that when a sufficient degree of strategic complementarity is assumed, i.e. a low value of η around 0.15, then a sticky-price model can account for a prolonged response of

¹⁵Note that since the idiosyncratic noise washes out in the aggregate, the amount of overconfidence doesn't affect the aggregate dynamics of inflation and the relevant mean and standard deviation.

¹⁶Note that the maximum cost in this parametrization (9.22%) is still smaller than the measure of customer costs of Zbaracki et al. (2004).

¹⁷See details in the Appendix.

output. On the opposite case, [Chari et al. \(2000\)](#) assume a value of η equal to 2.25, leading to pricing decisions that are strategic complements and then have argued that a sticky-price model is not able to generate enough persistence. In this work, we take an agnostic view on η and experiment how our results may differ for a range of values for this parameter. In particular, we report results for a range of η that goes from the low number assumed in [Woodford \(2003, ch. 3\)](#) to the high number used in [Chari et al. \(2000\)](#). Having fixed η , then equations (40) and (41) determine μ and σ_u^2 . We can determine δ and, bearing in mind that $\bar{\lambda} = \delta(1 - \hat{k})/\hat{k}^2$, we can also determine the degree of confidence compatible with our calibration strategy.

Table 1 presents the results of the calibration and in particular how the parameters and the equilibrium fraction μ are influenced by the chosen value of η . We observe that lower values of η require higher variance of the fundamental shock σ_u^2 to match the variance of price changes in the data and at the same time have the model consistent with a half-life of 4 quarters and reasonable information costs. However, the degree of confidence on the signals, which is $\bar{\lambda}$, that measures how good the signals are as a proxy of the fundamental shock, is low for low values of η . This perhaps indicates that the model is more reasonable when η is in this range and a sufficient degree of price complementarity is assumed. Using $\eta = 0.15$ we obtain that 83% of firms maintain a subjective information set when setting their prices. This value decreases as η increases and reaches 57% when $\eta = 2.25$.

Table 1: Calibration and Equilibrium Fraction μ

	$\eta=0.15$	$\eta=0.5$	$\eta=0.9$	$\eta=1.1$	$\eta=1.5$	$\eta=1.9$	$\eta=2.25$
\bar{p}^*	1.11	1.11	1.11	1.11	1.11	1.11	1.11
σ_u	4.234	2.97	2.310	2.131	1.903	1.764	1.678
$\bar{\lambda}$	5.81	20.6	31.20	35.05	41.07	45.53	48.62
μ	0.831	0.611	0.580	0.577	0.573	0.571	0.570

We then proceed to simulate the model in order to characterize the pricing behavior of 20000 firms over the sample 1988:1-2003:4 as in KK. Over this sample, we compute the same statistics that they report and in particular we focus on the average over the sample and across firms of the absolute of the changes in the individual prices for two subsequent observations. This statistic corresponds to the statistic $|dp|$ for regular prices of all items in their Table 1. By construction, our simulations are in line with the statistics that they report

on the mean and variance of the price changes.¹⁸

Our results on the variable $|dp|$ depend on the degree of overconfidence assumed. We have defined overconfidence as the ratio $\lambda/\bar{\lambda}$ which is equivalent to $\sigma_\xi^2/\bar{\sigma}_\xi^2$, the ratio of the true variance of the noise with respect to the perceived variance. We choose as an index of overconfidence the parameter $\gamma = \sigma_\xi/\bar{\sigma}_\xi$ which then gives a measure on how much the true standard deviation of the noise exceeds the believed one. We repeat our simulations for the chosen values of η by letting γ varies from 1 to 8. In particular $\gamma = 1$ corresponds to the rational signal extraction problem. The results are presented in Table 2.

Table 2: Average absolute value of price changes (%) and overconfidence

	$\eta=0.15$	$\eta=0.5$	$\eta=0.9$	$\eta=1.1$	$\eta=1.5$	$\eta=1.9$	$\eta=2.25$
$\gamma = 1$	1.83	2.012	1.99	1.97	1.95	1.94	1.93
$\gamma = 2$	2.75	3.02	2.89	2.86	2.8	2.76	2.73
$\gamma = 3$	3.80	4.05	3.86	3.79	3.69	3.63	3.59
$\gamma = 4$	4.88	5.13	4.83	4.73	4.6	4.51	4.46
$\gamma = 5$	5.98	6.21	5.81	5.68	5.52	5.40	5.33
$\gamma = 6$	7.07	7.28	6.79	6.65	6.43	6.30	6.21
$\gamma = 7$	8.18	8.37	7.77	7.60	7.36	7.19	7.08
$\gamma = 8$	9.30	9.45	8.76	8.56	8.28	8.08	7.96

Table 2 shows for each pair γ and η the average absolute value of price changes ($|dp|$) implied by this model. The reference value is the 8.5% of KK. In a rational signal extraction problem ($\gamma = 1$) we obtain a value close to 2%. But to match their reported value we need a degree of overconfidence close to 7 or 8, which can be considered as a large number. Indeed, experimental studies, like [Soll and Klayman \(2004\)](#), have shown that on a series of questions where individuals are asked to form an 80% confidence interval the actual hit rate is around 40%, which can be translated in γ being approximately equal to 2.5.¹⁹ In more complicated tasks, as forecasting the level of the exchange rate with a confidence interval of 90% (see [Oberlechner and Osler \(2009\)](#) and [Oberlechner and Osler \(2012\)](#)), the hit rate ranges from 5% to 70% with an average of 40%, rationalizing values of γ higher than 3. In general values from 2 to 4 can be considered as reasonable.

¹⁸We have repeated the described simulation 1000 times to smooth out any influence of the small sample on the results and we average the statistics across these repeated simulations.

¹⁹This value can be obtained by rough computation on confidence intervals for normal distributions.

If we set our target less ambitiously and ask what is the implied $|dp|$ for a degree of overconfidence close to experimental evidence, we find that this can range from 3% to 5%, doubling the value under a fully rational model and being close to explain more than the 50–60% of the value found in the data.²⁰ We think that these results suggest that our proposed mechanism can have some value in explaining price volatility –although we acknowledge that there can be other important mechanisms from which we have abstracted in this analysis.

7 Concluding remarks

In this paper, we study the behavior of individual and aggregate prices in an economy with monopolistic-competitive firms that is driven by aggregate shocks observed with noise. Each firm receives a private signal about the hidden state. We assume that firms are overconfident in their signals, that is, they overestimate the precision of their own perception of the aggregate state.

This model can rationalize a persistent response of output to the aggregate hidden state and be consistent at the same time with excess volatility of individual prices, providing therefore a unified “noise” interpretation of aggregate and idiosyncratic prices. We see our approach as complementary to setups where idiosyncratic fundamental shocks are the main driver of price volatility.

More generally, we believe that behavioral biases, especially in the processing and analysis of information, are not easy to dismiss. The way forward is to conduct careful studies of actual decision makers in real market conditions. For example, [Bachmann and Elstner \(2015\)](#) use confidential German manufacturing survey data and document substantial instances of systematic positive and negative expectation errors. Further studies that carefully document second-moment biases are necessary in order to clarify the quantitative importance of overconfidence.

²⁰ As Table 1 shows, this result does not depend on the value of η assumed.

A Static model

A.1 Proof of proposition 1

Derivation of condition (11) from (10). We first note that by using condition (7) we can rewrite marginal costs in terms of the complete information price and by using also condition (9) we can express the expected profits as functions of $P^\dagger(j)$ and $\tilde{P}(j)$ and the variable $Z \equiv MP^{\varepsilon-2}$ as follows

$$E^j\{\pi(P^\dagger(j), P, \theta)\} = \frac{1}{\varepsilon} E^j\{(P^\dagger(j))^{1-\varepsilon} Z\}, \quad (\text{A.1})$$

$$E^j\{\pi(\tilde{P}(j), P, \theta)\} = \frac{1}{\varepsilon} \tilde{P}^{1-\varepsilon}(j) E^j Z. \quad (\text{A.2})$$

We take a second order approximation of expected profits around a deterministic steady state where $\theta = \bar{\theta}$ and as a result $P = P^\dagger = \tilde{P} \equiv \bar{P}$. Let lowercase variables denote log-deviations from the steady state and let $\|p\|$ and $\|z\|$ denote a bound on the size of fluctuations for the price of each differentiated good and for the variable Z respectively. We can obtain by approximating equation (A.1) that

$$\begin{aligned} E^j\{\pi(P^\dagger(j), P, \theta)\} &= \frac{1}{\varepsilon} \bar{P}^{1-\varepsilon} \bar{Z} \left[1 + (1-\varepsilon) E^j p^\dagger + \frac{1}{2} (1-\varepsilon)^2 E^j (p^\dagger)^2 + \right. \\ &\quad \left. + E^j z + \frac{1}{2} E^j z^2 + (1-\varepsilon) E^j p^\dagger z \right] + \mathcal{O}(\|p, z\|^3). \end{aligned} \quad (\text{A.3})$$

Similarly by approximating equation (A.2) we obtain that

$$\begin{aligned} E^j\{\pi(\tilde{P}(j), P, \theta)\} &= \frac{1}{\varepsilon} \bar{P}^{1-\varepsilon} \bar{Z} \left[1 + (1-\varepsilon) \tilde{p}(j) + \frac{1}{2} (1-\varepsilon)^2 \tilde{p}(j)^2 \right. \\ &\quad \left. + E^j z + \frac{1}{2} E^j z^2 + (1-\varepsilon) \tilde{p}(j) E^j z \right] + \mathcal{O}(\|p, z\|^3). \end{aligned} \quad (\text{A.4})$$

Note that $\bar{P}^{1-\varepsilon} \bar{Z} = \frac{\bar{M}}{\bar{P}} = \bar{Y}$ and let

$$W(j) \equiv \pi(P^\dagger(j), P, \theta) - \pi(\tilde{P}(j), P, \theta)$$

denote the difference in profits. Then using (A.3) and (A.4) we have

$$\begin{aligned} E^j\{W(j)\} &= \frac{\varepsilon-1}{\varepsilon} \bar{Y} \left\{ \tilde{p}(j) - E^j p^\dagger + \frac{1}{2} (\varepsilon-1) \left[E^j (p^\dagger)^2 - (\tilde{p}(j))^2 \right] \right. \\ &\quad \left. - E^j[(p^\dagger - \tilde{p}(j)) z] \right\} + \mathcal{O}(\|p, z\|^3). \end{aligned} \quad (\text{A.5})$$

Note that the price of the uninformed agents (1) in a first order approximation is

$$\tilde{p}(j) = E^j p^\dagger + \mathcal{O}(\|p, z\|^2). \quad (\text{A.6})$$

Furthermore, if we take a second-order approximation of it we obtain

$$\tilde{p}(j) - E^j p^\dagger = \frac{1}{2} Var^j(p^\dagger) + E^j [p^\dagger - E^j p^\dagger] z + \mathcal{O}(\|p, z\|^3), \quad (\text{A.7})$$

where $Var^j(p^\dagger) \equiv E^j(p^\dagger)^2 - (E^j p^\dagger)^2$. Using (A.6) and (A.7) into (A.5) we observe that the terms involving z cancel out and that $E^j(p^{\dagger 2} - \tilde{p}(j))^2 = Var^j(p^\dagger)$, so ignoring third order terms we obtain

$$\begin{aligned} E^j \{W(j)\} &= \frac{1}{2} \frac{\varepsilon - 1}{\varepsilon} \bar{Y} \{Var^j(p^\dagger) + (\varepsilon - 1) Var^j(p^\dagger)\} \\ &= \frac{\bar{Y}}{2} (\varepsilon - 1) Var^j(p^\dagger). \end{aligned}$$

Thus firms acquire information if and only if

$$Var^j(p^\dagger) \geq c^j$$

where $c^j \equiv \frac{2}{\bar{Y}(\varepsilon-1)} \tilde{c}^j$ which is expression (11) in the text. In the text, we also denote with lower-case letters the logs of the respective variable.

Note that exactly the same calculations apply in the dynamic case, where each variable is indexed with t . In this case the expected difference of profits at time t , in (30), is equal to

$$E_t^j \{\pi(P_t^\dagger(j), P_t, \theta_t) - \pi(\tilde{P}_t(j), P_t, \theta_t)\} = \frac{\bar{Y}}{2} (\varepsilon - 1) Var_t^j(p_t^\dagger)$$

in a second-order approximation, where the expectation operator is conditional on the private history of signals including time t . So $Var_t^j(p_t^\dagger)$ corresponds to the subjective contemporaneous variance of p_t^\dagger .

A.2 Proof of proposition 2

Equation (26) defines implicitly the equilibrium fraction μ as a function of \bar{r} , $\mu(\bar{r})$. We have $c^* = c^*(\bar{r}, \mu(\bar{r}))$. Differentiate implicitly (26) to get

$$\frac{d\mu}{d\bar{r}} = - \frac{f(c^*) \frac{\partial c^*}{\partial \bar{r}}}{1 + f(c^*) \frac{\partial c^*}{\partial \mu}}. \quad (\text{A.8})$$

Note that

$$\frac{\partial c^*}{\partial \mu} = \frac{-2\eta^2(1-\bar{r})^2\sigma_u^2(1-\eta)}{(\eta + (1-\eta)(1-\bar{r})\mu)^3} \quad (\text{A.9})$$

$$\frac{\partial c^*}{\partial \bar{r}} = \frac{\eta^2\sigma_u^2}{(\eta + (1-\eta)(1-\bar{r})\mu)^3} [(1-\eta)(1-\bar{r})\mu - \eta] \quad (\text{A.10})$$

As we noted in the text, when $\eta < 1$, we have strategic complementarities in information

acquisition, $\frac{\partial c^*}{\partial \mu} < 0$. The opposite happens when $\eta > 1$, $\frac{\partial c^*}{\partial \mu} > 0$.

The denominator of (A.8) is always positive when we have strategic substitutes. In the case of strategic complements, we are going to assume that the complementarities are not large enough to make the denominator negative, so we assume that $-f(c^*)\frac{\partial c^*}{\partial \mu} < 1$. This makes the LHS of (26) an increasing function of μ with slope less than unity. This is what we call a “stable” equilibrium.

The above discussion implies that the sign of $dc^*/d\bar{r}$ is determined by the sign of $\partial c^*/\partial \bar{r}$. The result follows from (A.10).

B Infinite-horizon model

B.1 Proof of proposition 3

Derivations of (34) and (35). We proceed using the method developed in [Woodford \(2002\)](#). We claim that the relevant hidden state is

$$X_t = \begin{bmatrix} p_t^* \\ p_t \end{bmatrix}$$

and guess that it evolves according to a linear law of motion

$$X_t = f + MX_{t-1} + mu_t, \quad (\text{B.1})$$

where

$$f \equiv \begin{bmatrix} \bar{p}^* \\ \bar{p} \end{bmatrix}, \quad M \equiv \begin{bmatrix} \rho & 0 \\ a & b \end{bmatrix}, \quad m \equiv \begin{bmatrix} 1 \\ c \end{bmatrix},$$

are vectors and matrices to be determined. Note that our variables of interest are the prices of the informed firms which can be written as $p^\dagger = \bar{\eta}' X_t$ and that of the uninformed which can be written as $\tilde{p}_t(i) = \bar{\eta}' E_t^i X_t$, where $\bar{\eta}' = (\eta, 1 - \eta)$.

Let $e_1 = (1, 0)'$. We can write the following system

$$\begin{aligned} X_t &= f + MX_{t-1} + mu_t \\ s_t^i &= e_1' X_t + \xi_t^j \end{aligned}$$

where the second line corresponds to the observational equation. We proceed assuming a stationary filtering problem. The filtering equation of a generic uninformed firm j is given by

$$E_t^j X_t = E_{t-1}^j X_t + K(s_t^j - E_{t-1}^j p_t^*), \quad (\text{B.2})$$

where K is the vector of Kalman gains pre-multiplied with M^{-1} . Using (B.1) we obtain that $E_{t-1}^j X_t = f + ME_{t-1}^j X_{t-1}$ and $E_{t-1}^j p_t^* = \bar{p}^* + \rho E_{t-1}^j p_{t-1}^*$; we can then write (B.2) as

$$E_t^j X_t = f + ME_{t-1}^j X_{t-1} + K(s_t^j - \bar{p}^* - \rho E_{t-1}^j p_{t-1}^*).$$

Aggregating among all agents j that are uninformed and guessing that in equilibrium μ will

be non-random (as in the static case) we obtain

$$\begin{aligned}\bar{E}_t X_t &= f + M \bar{E}_{t-1} X_{t-1} + K (p_t^* - \bar{p}^* - \rho \bar{E}_{t-1} p_{t-1}^*) , \\ &= f + M \bar{E}_{t-1} X_{t-1} + \rho K (p_{t-1}^* - \bar{E}_{t-1} p_{t-1}^*) + K u_t,\end{aligned}\tag{B.3}$$

which is the law of motion of the average estimate, where we have used the law of large numbers.

Our target is to express the price level p_t in terms of X_{t-1} . The general price index can be expressed as a function of the full information price and average expectations as

$$\begin{aligned}p_t &= \delta [(1 - \mu) p_t^* + \mu \bar{E}_t p_t^*] + (1 - \delta) \bar{E}_t p_t \\ &= (\delta \mu, 1 - \delta) \bar{E}_t X_t + \delta (1 - \mu) p_t^* \\ &= (\delta \mu, 1 - \delta) \bar{E}_t X_t + \delta (1 - \mu) (\bar{p}^* + \rho p_{t-1}^* + u_t) .\end{aligned}$$

Using the law of motion (B.3) to substitute for $\bar{E}_t X_t$ and collecting terms we have

$$\begin{aligned}p_t &= [(\delta \mu, 1 - \delta) f + \delta (1 - \mu) \bar{p}^*] + \rho [\delta (1 - \mu) + \bar{K}] p_{t-1}^* + (\delta \mu, 1 - \delta) M \bar{E}_{t-1} X_{t-1} \\ &\quad - \rho \bar{K} \bar{E}_{t-1} p_{t-1}^* + [\delta (1 - \mu) + \bar{K}] u_t,\end{aligned}$$

where $\bar{K} \equiv (\delta \mu, 1 - \delta) K$. Finally, using the definition of M and $\bar{E}_{t-1} X_{t-1}$ and noting that

$$(\delta \mu, 1 - \delta) f + \delta (1 - \mu) \bar{p}^* = \delta \bar{p}^* + (1 - \delta) \bar{p}$$

we obtain that

$$\begin{aligned}p_t &= \delta \bar{p}^* + (1 - \delta) \bar{p} + \rho [\delta (1 - \mu) + \bar{K}] p_{t-1}^* + [\delta \mu \rho + (1 - \delta) a - \rho \bar{K}] \bar{E}_{t-1} p_{t-1}^* \\ &\quad + (1 - \delta) b \bar{E}_{t-1} p_{t-1} + [\delta (1 - \mu) + \bar{K}] u_t.\end{aligned}\tag{B.4}$$

Since

$$p_{t-1} = \delta \mu \bar{E}_{t-1} p_{t-1}^* + (1 - \delta) \bar{E}_{t-1} p_{t-1} + \delta (1 - \mu) p_{t-1}^*,$$

we can use this expression to substitute for $\bar{E}_{t-1} p_{t-1}$ in (B.4) and arrive at

$$\begin{aligned}p_t &= \delta \bar{p}^* + (1 - \delta) \bar{p} + [\delta (1 - \mu) (\rho - b) + \rho \bar{K}] p_{t-1}^* \\ &\quad + [\delta \mu (\rho - b) + (1 - \delta) a - \rho \bar{K}] \bar{E}_{t-1} p_{t-1}^* \\ &\quad + b p_{t-1} + [\delta (1 - \mu) + \bar{K}] u_t.\end{aligned}\tag{B.5}$$

We note that (B.1) implies

$$p_t = \bar{p} + a p_{t-1}^* + b p_{t-1} + c u_t.\tag{B.6}$$

We can then match the coefficients between (B.5) and (B.6) and obtain

$$\begin{aligned}\delta \bar{p}^* + (1 - \delta) \bar{p} &= \bar{p} \\ \delta (1 - \mu) (\rho - b) + \rho \bar{K} &= a \\ \delta \mu (\rho - b) + (1 - \delta) a - \rho \bar{K} &= 0 \\ \delta (1 - \mu) + \bar{K} &= c.\end{aligned}$$

Solving this system and using the definition of $\bar{K} = (\delta\mu, 1 - \delta) K$, we get

$$\begin{aligned}\bar{p} &= \bar{p}^* \\ a &= \frac{\rho}{\delta\mu + 1 - \delta} \bar{K} = \rho \bar{\eta}' K = \rho \hat{k}\end{aligned}$$

since $\eta = \delta\mu/(\delta\mu + 1 - \delta)$ and $\hat{k} \equiv \bar{\eta}' K$. Moreover

$$b = \rho - a = \rho (1 - \hat{k})$$

and

$$c = (\delta\mu + 1 - \delta) \hat{k} + \delta (1 - \mu).$$

The vector of (pre-multiplied) Kalman gains satisfies the equation

$$K = \Sigma e_1 (e_1' \Sigma e_1 + \bar{\sigma}_\xi^2)^{-1}, \quad (\text{B.7})$$

where Σ is the variance of the one step ahead forecast error which satisfies the following stationary version of the Riccati equation

$$\Sigma = M \Sigma M' + m m' \sigma_u^2 - (e_1' \Sigma e_1 + \bar{\sigma}_\xi^2)^{-1} M \Sigma e_1 e_1' \Sigma M'. \quad (\text{B.8})$$

Thus in our guess-and-verify approach we expressed M and m as a function of \hat{k} which depends on the vector of Kalman gains K which in turn depends on Σ . But Σ depends on M and m by (B.8). So it remains to solve for this fixed point. Let

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}.$$

Solving the upper left block of the Riccati equation (B.8) we find that σ_{11} satisfies the quadratic

$$\sigma_{11}^2 + [(1 - \rho^2) \bar{\sigma}_\xi^2 - \sigma_u^2] \sigma_{11} - \bar{\sigma}_\xi^2 \sigma_u^2 = 0.$$

The positive root (since σ_{11} is a variance) of this quadratic is

$$\sigma_{11} = \frac{1}{2} \sigma_u^2 \left\{ 1 - (1 - \rho^2) \bar{\lambda} + \sqrt{(1 - (1 - \rho^2) \bar{\lambda})^2 + 4 \bar{\lambda}} \right\}$$

where $\bar{\lambda} \equiv \frac{\bar{\sigma}_\xi^2}{\sigma_u^2}$. From the lower left block of the Riccati we derive

$$\sigma_{12} = \frac{\rho^2 \hat{k} \sigma_{11} \bar{\sigma}_\xi^2 + (\sigma_{11} + \bar{\sigma}_\xi^2) [(\delta\mu + 1 - \delta) \hat{k} \sigma_u^2 + \delta (1 - \mu) \sigma_u^2]}{[1 - \rho^2 (1 - \hat{k})] \bar{\sigma}_\xi^2 + \sigma_{11}}$$

and -using (B.7)- we obtain

$$\hat{k} = \bar{\eta}' K = \eta \frac{\sigma_{11}}{\sigma_{11} + \bar{\sigma}_\xi^2} + (1 - \eta) \frac{\sigma_{12}}{\sigma_{11} + \bar{\sigma}_\xi^2}.$$

This is a system of two equations in the two unknowns (σ_{12}, \hat{k}) . Solving the system and using our solution for σ_{11} we finally arrive at the quadratic expression for \hat{k}

$$Q(\hat{k}) = \rho^2 \bar{\lambda} \hat{k}^2 + [\bar{\lambda} (1 - \rho^2) + \delta] \hat{k} - \delta = 0. \quad (\text{B.9})$$

At first, note that the \hat{k} that solves the quadratic for $\rho = 0$ agrees -as expected- with the static case. Proceeding with the case of $\rho \neq 0$, we note that the discriminant of (B.9) is positive, so there are two real roots. Furthermore, since $Q(0) < 0$ and $Q(1) = \bar{\lambda} > 0$, one is negative and the other positive and less than unity. Note that subtracting p_t from p_t^* we get an expression for the output deviation

$$y_t = \rho(1 - \hat{k})y_{t-1} + (1 - c)u_t,$$

since $p_t^* + \ln \bar{Y} = \ln M_t = p_t + \ln Y_t$. In order to have a stationary solution for output we need $|1 - \hat{k}| < |\rho|^{-1}$. It follows that \hat{k} should satisfy the restriction $1 - \rho^{-1} < \hat{k} < 1 + \rho^{-1}$. Since $Q(1 - \rho^{-1}) = -\rho^{-1}((\rho - 1)^2 \lambda + \delta) < 0$, only the positive root of the quadratic satisfies the restriction. Thus

$$\hat{k} = \frac{1}{2\rho^2} \left\{ \rho^2 - 1 - \frac{\delta}{\bar{\lambda}} + \sqrt{\left[(1 - \rho^2) + \frac{\delta}{\bar{\lambda}} \right]^2 + 4\rho^2 \frac{\delta}{\bar{\lambda}}} \right\}.$$

Having solved for the laws of motion of X_t and $E_t^j X_t$, we can derive the laws of motion of the prices of interest $p^\dagger = \bar{\eta}' X_t$ and $\tilde{p}_t(j) = \bar{\eta}' E_t^j X_t$ and obtain expressions (34) and (35) in the main text, respectively.

Derivation of (36). Defining $q_t^j \equiv p_t^\dagger - \tilde{p}_t(j) = p_t^\dagger - E_t^j p_t^\dagger$ we obtain

$$q_t^j = \rho(1 - \hat{k})q_{t-1}^j + \delta(1 - \hat{k})u_t - \hat{k}\xi_t^j. \quad (\text{B.10})$$

At first notice that $E_t^j q_t^j = E_t^j(p_t^\dagger - \tilde{p}_t(j)) = 0$ and $E_{t-1}^j q_t^j = E_{t-1}^j(p_t^\dagger - \tilde{p}_t(j)) = E_{t-1}^j p_t^\dagger - E_{t-1}^j E_t^j p_t^\dagger = 0$. Calculating variances conditional on the private history until last period we get

$$\text{Var}_{t-1}^j(q_t^j) = \rho^2(1 - \hat{k})^2 \text{Var}_{t-1}^j(q_{t-1}^j) + \delta^2(1 - \hat{k})^2 \sigma_u^2 + \hat{k}^2 \bar{\sigma}_\xi^2.$$

Note that $\text{Var}_{t-1}^j(q_{t-1}^j) = E_{t-1}^j(q_{t-1}^j)^2 = \text{Var}_{t-1}^j(p_{t-1}^\dagger)$. Moreover, $\text{Var}_{t-1}^j(q_t^j) = E_{t-1}^j(q_t^j)^2 = E_{t-1}^j\{E_t^j(q_t^j)^2\} = E_{t-1}^j \text{Var}_t^j(p_t^\dagger) = \text{Var}_{t-1}^j(p_t^\dagger)$, where the last step follows from the non-randomness of the variances of the filter. The expression in the text for the contemporaneous variance follows by using the stationarity of the filter and the fact that (B.9) implies $\delta(1 - \hat{k}) = \bar{\lambda}\hat{k}[1 - \rho^2(1 - \hat{k})]$.

B.2 Proof of proposition 4

We will now proceed to derive the condition for excess volatility of the prices of the uninformed firms. Taking unconditional variances in (B.10) we obtain that

$$\text{var}(q_t^j) = \frac{1}{1 - \rho^2(1 - \hat{k})^2} [\delta^2(1 - \hat{k})^2 \sigma_u^2 + \hat{k}^2 \sigma_\xi^2].$$

Furthermore note that, since

$$\text{var}(q_t^j) = \text{var}(p_t^\dagger) + \text{var}(\tilde{p}_t(j)) - 2\text{cov}(p_t^\dagger, \tilde{p}_t(j))$$

and

$$\text{cov}(p_t^\dagger, \tilde{p}_t(j)) = \text{cov}(q_t^j, \tilde{p}_t(j)) + \text{var}(\tilde{p}_t(j)),$$

we have

$$\text{var}(p_t^\dagger) = \text{var}(q_t^j) + 2\text{cov}(q_t^j, \tilde{p}_t(j)) + \text{var}(\tilde{p}_t(j)).$$

Dividing over $\text{var}(p_t^\dagger)$ we obtain

$$\frac{\text{var}(\tilde{p}_t(j))}{\text{var}(p_t^\dagger)} = 1 - \frac{\text{var}(q_t^j) + 2\text{cov}(q_t^j, \tilde{p}_t(j))}{\text{var}(p_t^\dagger)}.$$

So the ratio can exceed unity only if $I \equiv \text{var}(q_t^j) + 2\text{cov}(q_t^j, \tilde{p}_t(j)) < 0$. Note that

$$\text{cov}(q_t^j, \tilde{p}_t(j)) = \frac{1}{1 - \rho^2(1 - \hat{k})^2} [\rho^2(1 - \hat{k})\hat{k} \cdot \text{cov}(p_{t-1}^*, q_{t-1}^j) + \delta(1 - \hat{k})\hat{k}\sigma_u^2 - \hat{k}^2\sigma_\xi^2].$$

Using the law of motion for the full information price and q_t^j we derive that

$$\text{cov}(p_t^*, q_t^j) = \frac{\delta(1 - \hat{k})\sigma_u^2}{1 - \rho^2(1 - \hat{k})}$$

and plugging it in the previous expression we finally obtain

$$\text{cov}(q_t^j, \tilde{p}_t(j)) = \frac{1}{1 - \rho^2(1 - \hat{k})^2} \left[\frac{\delta(1 - \hat{k})\hat{k}}{1 - \rho^2(1 - \hat{k})} \sigma_u^2 - \hat{k}^2 \sigma_\xi^2 \right].$$

Therefore

$$I = \frac{\hat{k}^2}{1 - \rho^2(1 - \hat{k})^2} \left[\frac{\delta(1 - \hat{k})}{\hat{k}} \left(\frac{\delta(1 - \hat{k})}{\hat{k}} + \frac{2}{1 - \rho^2(1 - \hat{k})} \right) - \lambda \right] \sigma_u^2.$$

In order to have $I < 0$, we need

$$\frac{\delta(1 - \hat{k})}{\hat{k}} \left[\frac{\delta(1 - \hat{k})}{\hat{k}} + \frac{2}{1 - \rho^2(1 - \hat{k})} \right] - \lambda < 0.$$

Using as before the fact that $\delta(1 - \hat{k}) = \bar{\lambda}\hat{k}[1 - \rho^2(1 - \hat{k})]$ we derive condition (38).

C Information costs

We convert the lifetime costs to costs per period by multiplying it by the factor $(1 - \beta)$. The steady state real profits are $\bar{\pi} = \bar{Y}/\varepsilon$. Recall that

$$c^j = \frac{2}{(\varepsilon - 1)} \frac{\tilde{c}^j}{\bar{Y}}.$$

Then the equilibrium fraction of uninformed agents is given by

$$\mu = 1 - Pr(c^j \leq c^*).$$

where, Pr refers to the probability measure of costs. In particular

$$Pr(c^j \leq c^*) = Pr(c^j (1 - \beta) \leq c^* (1 - \beta) = Var_t^j(p_t^\dagger))$$

and

$$Pr\left(\frac{\tilde{c}^j (1 - \beta)}{\bar{Y}} \leq \frac{(\varepsilon - 1)}{2} Var_t^j(p_t^\dagger)\right) = Pr\left(\frac{\tilde{c}^j (1 - \beta)}{\bar{\pi}} \leq \frac{\varepsilon (\varepsilon - 1)}{2} Var_t^j(p_t^\dagger)\right).$$

Given our assumption of a uniform distribution for the costs as a fraction of steady state real profits with minimum zero and mean 4.61%, it follows that

$$\mu = 1 - U\left(\frac{\varepsilon (\varepsilon - 1)}{2} Var_t^j(p_t^\dagger)\right),$$

where $U(\cdot)$ is the corresponding c.d.f. This equation gives the implicit equation (41).

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