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Reformatory Policies and Factor Prices in a Developing Economy with Informal Sector

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Abstract

Effects of different reformatory policies have always been a pulsating concern for the researchers and policy makers. Considering this concern, this paper attempts to check various effects of reformatory policies such as labor market reform, tariff cut, change in subsidy, bureaucratic reform in a typical small open economy comprising of both formal and informal sectors. It has been found that the implications of labor market reform and tariff liberalization for factor prices and wage disparity are distinctly opposite. However, skilled labor of the economy benefits from both labor market reform and export subsidy. Next we extend the basic model to bring in related corruption in the informal sector for its illegal nature. This calls for the existence of a sector which helps hassle free informal production. There we find that unskilled workers lose owing to both bureaucratic reform and labor market reform. Nevertheless, though traditionally labor market reform is supposed to harm workers, wage disparity gets ameliorated whereas tariff reform leads to worsening of it.

Keywords: International Trade, Wages, General Equilibrium, Economic Policy, Informal Sector, Extortion.

JEL classification: F1, J31, D5, F11, F68, D73.

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1. Introduction

The debate over restriction versus reform has been a pulsating area of research for quite a long time. This concern mainly hovers around if reformatory policies may end-up with undesired outcomes. Protectionism started with the initiation of Corn Laws in Great Britain during the first half of nineteenth century, these laws were the best examples of British mercantilism and thereafter the abolishment of these laws marked a significant stride forward towards economic reform. Proponents of protectionist policies claim that control over imports saves jobs, help flourishing domestic industries and conductive to trim down trade deficits. On the other hand protectionism comes with some costs in the form of increased commodity prices, less money to buy goods, more domestic taxes etc. Thus it is believed that protectionism more often than not hurts the protecting country (Bhagwati, 1988). However, there might be a few cases where protectionism may raise country's welfare (Helpman and Krugman, 1989). Thus, one significant feature of recent economic environment has been the extent of trade liberalization in developing countries. The reason is the conviction: liberalization is conducive for growth (Dixon, 1998).

In line of this backdrop many developing countries have abandoned the policy of government intervention in order to embrace free trade; India is no exception. It has initiated the policy of economic reform way back in 1991. Conventionally, liberalization policies consist of structural reform, product market reform, trade reform, bureaucratic reform, labour market reform and tariff reduction in the import competing sector, foreign capital inflow, removal of export subsidies etc. In addition to massive implementation of these policies in developing countries, the phenomenon of informal sector has also been growing rapidly. The size of informal labour market has changed from approximately 4-6% in developed countries to over

50% in developing countries. ¹ Related literature comprises of Tokman (2001), Agenor & Montiel (1996), Schneider (2007), Charmes (2000) etc. The informal sector is often characterized by small scale unskilled, less skilled workers who generally earn lower wages than formal sectors. In this backdrop, many economists and policy makers have examined the effects of some reformatory policies on the interconnectedness between formal and informal sectors, wages, unemployment etc. Empirical studies by Marjit and Kar (2005) and Marjit and Maiti (2006) have argued that the liberal trade policies have contracted the size of import competing manufacturing sector and informal wage goes up in the presence of capital mobility between formal and informal sectors. Goldberg and Pavcnik (2003), Paz (2012), on the other hand, have shown the effects of trade policy on industry level outcomes and average wages of formal and informal workers.

In line with the above mentioned empirical works, many theoretical works have also been done dealing with formal informal interactions and wage rates. Carruth and Oswald (1981), Agenor and Monteil (1996), Marjit and Kar (2011, 2015) also have discussed the impact of economic reform on informal wage rate. Fiess, Fugazza and Maloney (2002, 2008) have developed a theoretical model where the relationship between relative formal-informal earnings and the relative size of the labour force is analyzed. In another interesting contribution Marjit, Kar and Maiti (2008) have also shown the impact of both labour market and tariff reform on informal wage and poverty.

¹ Specifically 82 per cent in South Asia, 66 per cent in Sub-Saharan Africa, 65 per cent in East and Southeast Asia, 45 per cent in the Middle East and North Africa and 51 per cent in Latin America of total employment are considered as informal labour force (Vanek et al. 2014). According to ILO India Labour Market Update (2016) and NSSO data (2011-12), more than 90 percent of the employment in the agricultural sector and close to 70 percent in the non-agricultural sector falls under the informal category.

Taking clue from this literature, our paper looks at the impact of different reformatory policies in terms of labour reform, tariff reform, on the factor prices and output of both formal and informal sectors in a four sector trade theoretic framework. In this paper we also highlight the effect of fiscal reform in the form of a decrease in export subsidy. Export subsidies are distinct from production/consumption subsidies and it only lowers the commodity price in the international market, make them more competitive against foreign competition. In spite of the fact the subsidies are one of the major instruments of government expenditure policy; the effect of export subsidy reduction is less focused in contemporary research. Bagwell and Staiger (2004) have analyzed international rules regarding subsidies to domestic production. Schwartz and Clements (1999) have shown the use of government subsidies as a fiscal policy and examined their reformatory effects. Salunkhe and Deshmukh (2012) have focused on the overview of agriculture subsidies in India with the help of provisions of funds for agriculture in five years plans and annual budget and have also studied the types of agricultural subsidies and distribution criteria in India. Informality, however, was not a prime concern in these papers. Chaudhuri (2000) made an attempt to look at the effects of wage or price subsidy to the rural economy in the presence of informal sector. In this line we have tried to show the effect of reduction of export subsidy on factor prices and sectoral composition.

Our main motivation, behind the present study generates from the literature related to informal sector. Hence, the present study is built up with a theoretical model having both formal and informal sectors, and where labour market and product market reforms have simultaneously implemented and the effect of change in subsidy is also experimented which is something new in the context of existing literature. Here tariff reform and labour market reform have generated conflicting results for the informal counterpart of the economy. We also try to examine the

possible consequences of reduction in export subsidies. Then we extend our model introducing corruption in the informal sector. As informal sector is usually not registered, not recorded, non-taxpaying and government rules and regulations are not binding, sometimes they are beset with extortion related corruption.² Anderson and Marcouiller (2002), Marjit, Mukherjee and Kolmar (2006), Marjit and Kar (2011), Dutta, Kar and Roy (2013), Marjit and Mandal (2012) are some evidence for the theory of corruption and informality. In most of the cases informal units have to pay an extra cost as fee for corruption or bribery or extortion fee to comply with government rules and regulations to sustain their production (Mandal et al. 2018). Follwing Mandal et al. (2018), this paper also considers extortion as a facilitating activity for organizing production in the informal sector. So, one important aspect of this paper over the existing papers is that, given this set up we want to show the effect of various reformatory policies including bureaucratic reform on factor prices and outputs which is not done in the existing stuffs. .

Remaining paper is arranged as follows. Section 2 explains the model. Section 3 consists of three sub sections explaining the effect of three different reformatory policies; one of them discusses the impact of labour market reform, another one shows the impact of tariff cut and the third one describes the effect of a change in export subsidies on factor prices and output. Section 4 talks about the effect of bureaucratic reform in terms of reduction in the cost of corruption. Finally, the concluding remarks are placed in section 5. Mathematical details are, however, relegated to the Appendix.

2. The Basic Model and Solutions

² Extortion is sometimes referred as payment for protection from unspecified parties. Sometimes negotiation is done by politically supported intermediaries, the "extortionists".

We have a small open economy with four sectors A, I, M and Y. Here, A produces agricultural good which is exportable in nature and uses unskilled labour (L) and land (T) as inputs. We further assume that A is land intensive and enjoys an export subsidy (s) per unit. Export subsidy is an important export promotion instrument and considered to be very helpful for farming group as it acts as a minimum price for A that helps more production, employment and income. I is non - traded labour intensive urban informal sector which uses unskilled labour (L) and capital (K). These unskilled workers neither have any technical knowledge nor they are fortunate enough to get employed in formal sectors. When workers do not find job in formal sector, they have to get employed somewhere for survival. This phenomenon leads to surfacing of informal sector (I) in our model. Notice that L is perfectly mobile among A and I and hence the workers of A and I earn identical market determined competitive wage rate, W, which is lower than formal wage rate (say \overline{W}). M is a formal import competing manufacturing sector which also uses unskilled labour (L) and capital (K) and this sector is assumed to be capital intensive in nature. Since M belongs to formal activities and workers are paid a unionized fixed wage rate \overline{W} . This wage rate may be determined through collective bargaining or set a-priori by the government. Therefore, lowering of \overline{W} is synonymous with more flexible labour market conditions as it becomes more market oriented. We also assume that import competing sector is protected by ad-valorem tariff (t). Another formal sector is Y which is considered here as exportable service sector. Government subsidy (s) also applies to this sector. However, the workers in Y is skilled³ or trained in a specific field and earn skilled wage rate W_s which is market determined. Y is skill intensive. Thus, arranging the different wages of different

³ Special training includes higher education, technical training, computer literacy, software knowledge etc.

categories of labour, we may write $W_s > \overline{W} > W$. We have assumed that K is perfectly mobile among M, Y and I. All the markets are competitive in nature. Each sector operates under standard neo-classical technology of constant returns to scale and diminishing marginal productivity.

The following notations are used to describe the equational structure of the model.

 X_i = product produced by the ith sector, i = A, I, M, Y.

 P_A (1+s) = subsidy inclusive domestic price of good A

 P_I = price of commodity I

 P_M^* = world price of good M

 $P_M = P_M^*$ (1+t) = tariff inclusive domestic price of good M

 P_Y (1+s) = subsidy inclusive domestic price of good Y

W = competitive wage rate of unskilled labour

 \overline{W} = fixed wage rate of formal unskilled labour in the manufacturing sector

 W_S = Wage of skilled labour in service sector.

r = common rate of return to capital.

R = return to land.

K = economy's aggregate capital stock.

T = total amount of land.

L = economy's total supply of unskilled labour.

S = endowment of skilled labour in the economy.

 a_{ij} = input coefficients.

t = ad-valorem rate of tariff on the import of commodity M.

s = ad - valorem rate of subsidy on the export of commodity A and Y.

 λ_{ji} =employment share of jth factor/input in the production of ith commodity; j = L, K, S, T and i = A, I, M, Y.

 \wedge = proportional change

 θ_{ii} = distributive share of the jth input in the ith industry.

The competitive equilibrium conditions in the product market for the four sectors give us the following equations.

$$Wa_{LA} + Ra_{TA} = P_A(1+s) \tag{1}$$

$$Wa_{II} + ra_{KI} = P_I \tag{2}$$

$$\overline{W}a_{LM} + ra_{KM} = P_M^* (1+t) \tag{3}$$

$$W_{S}a_{SY} + ra_{KY} = P_{Y}(1+s) \tag{4}$$

The unskilled labour and specific skilled labour endowment equations are as follows

$$a_{LA}X_A + a_{LI}X_I + a_{LM}X_M = L (5)$$

$$a_{SY}X_Y = S \tag{6}$$

Full employment conditions for K and T can be expressed as follows

$$a_{KI}X_I + a_{KM}X_M + a_{KY}X_Y = K (7)$$

$$a_{TA}X_A = T \tag{8}$$

So, there are eight unknown variables in the system such as W, r, R, W_S , X_A , X_I , X_M and X_Y with eight independent equations. Thus the system can be solved. From equations (3) we can determine the value of r. Plugging the value of r into equations (1), (2) and (4), W, R and W_S are obtained respectively. Input coefficients $a_{ij}s$ denote the labour and capital requirement per unit of output. Once factor prices are determined we can solve for all $a_{ij}s$. Thus, X_A , X_I , X_M and X_Y are simultaneously solved from equations (5), (6), (7) and (8).

3. Effects of Reformatory Policies

3.1. Effects of Labour Market Reform

Labour market liberalization is one of the most important structural reforms. This reform naturally deregulates the labour laws or rigidity. Hence, wage flexibility may create jobs for workers in the formal sector and workers can freely move between informal and formal segments. In reality, freely mobile capital between the formal and informal sectors in a flexible labour market conditions increases the return to capital in the formal sector. Thus capital is drawn into the formal sector from the informal sector and informal wage rate gets reduced due to supply side response. So, in this model we want to show the effect of labour market reform in the form of flexible labour market condition by lowering \overline{W} . To do so, we assume unchanged t and s.

A decline in formal wage rate \overline{W} would increase the return to capital r. $\hat{r} = -\frac{\theta_{LM}}{\theta_{KM}} \hat{\overline{W}} > 0$ as

 $\hat{\overline{W}}$ < 0. Increase in r leads to a deterioration in the informal wage (W) and skilled wage (W_s) because of higher capital cost and fixed commodity prices due to small country assumption.

These are as follows
$$\hat{W_S} = \frac{\theta_{LM} \, \theta_{KY}}{\theta_{KM} \, \theta_{SY}} \hat{\overline{W}} < 0$$
 and $\hat{W} = \frac{\theta_{LM} \, \theta_{KI}}{\theta_{KM} \, \theta_{LI}} \hat{\overline{W}} < 0$ as $\hat{\overline{W}} < 0$.

As agricultural workers and informal workers earn same wage W, the reduction in W leads to an increase in R which is the return to the specific factor T in A. $\hat{R} = -\frac{\theta_{LM}\theta_{KI}\theta_{LA}}{\theta_{KM}\theta_{LI}\theta_{TA}}\hat{\overline{W}} > 0$ as

 $\hat{\overline{W}}$ < 0. Therefore, on the output front, depending on the factor intensity comparison and elasticity of substitution we have experienced various possibilities. Based on these outcomes we have the following proposition.

<u>Proposition 1</u>: Labour market reform leads to: (i) a decline in W, W_s and an increase in r and R; (ii) contraction of I and Y but expansion of A and M.

Explanation

A labour market reform or a fall in the negotiated wage in the formal sector would cause a lower cost of production in the formal manufacturing sector for a given cost of capital and P_M. As M is capital intensive than I and Y, this sector may appropriate this cost advantage by investing more on capital and producing more output. Thus, M expands and the demand for capital increases which helps to increase the return to capital r⁴. So, capital will be redirected from other sectors to M as capital is mobile among M, I and Y. Subsequently, this lowers the return to the unskilled workers (W) and skilled workers (W_s) as well. Following changes in factor prices the output effects of different sectors depend solely on factor substitutability. Change in Y is shown in equation (A.7) of the Appendix A.1. Labour market reform increases r and decreases W_s which helps producers to economize on the usage of K and substitutes K by S and as S is fixed in supply, Y contracts. On the other hand, when W decreases, agricultural sector may take advantage of this cost reduction on investing more on land and the return on land increases and results in an expansion of A⁵. Increase in r enhances the cost of using capital in production of informal good. Consequently I may substitute K by L. But expansion of A requires more L to fulfill the demand. This creates shortage of labor supply for I. And hence I contracts.

⁴ See Eq. (A.11) in Appendix A.1

⁵ This produces a sort of Rybczynski effect. As A is land intensive. See Eq. (A.9) in the Appendix A.1.

<u>Proposition 2</u>: Labour market reform decreases the wage gap between the skilled and unskilled labour in the economy.

Explanation

As mentioned before, this model nicely explains why both both skilled and unskilled wage declines due to labour market reform. This result has nothing to do with the factor intensity assumption of goods. However, skilled-unskilled wage disparity crucially depends on the factor intensity assumption and thus the rates of change in W_S and W. The expression for the wage gap is given by $(\hat{W}_S - \hat{W}) = \frac{\theta_{LM} \left(\theta_{KY} \theta_{LI} - \theta_{KI} \theta_{SY}\right)}{\theta_{LI} \theta_{KM} \theta_{SY}} \hat{W}$. So, if Y is more capital intensive than I i.e. $\theta_{KY} > \theta_{KI}$, then wage gap decreases owing to labour market reform, $(\hat{W}_S - \hat{W}) < 0$. This is owing to the fact that the factor intensity comparison leads to a higher increase in return to capital in Y than I. This implies that the decline in W_S is larger than the decrease in W.

3.2. Effect of Tariff Reduction

Tariff liberalization is another important reformatory policy for developing nations. So, we attempt to derive the effects of tariff liberalization on factor prices and output. Consequent upon liberalized trade policy in the form of declining tariff, r goes down since formal manufacturing labour is unionized and gets a pre-determined wage \overline{W} . The exact effect is shown as $\hat{r} = \frac{\alpha}{\theta_{KM}} \hat{t} < 0$ as $\hat{t} < 0$. This leads to an improvement in the wages earned by the informal labour and skilled labour of the economy as prices are given. The mathematical results are as follows $\hat{W}_S = -\frac{\alpha \theta_{KY}}{\theta_{KM} \theta_{SY}} \hat{t} > 0$ and $\hat{W} = -\frac{\alpha \theta_{KI}}{\theta_{KM} \theta_{LI}} \hat{t} > 0$ as $\hat{t} < 0$. Again when W goes up R must

fall. This is apparent from equation (1) as P_A is given. Therefore, $\hat{R} = \frac{\alpha \theta_{KI} \theta_{LA}}{\theta_{KM} \theta_{LI} \theta_{TA}} \hat{t} < 0$ as $\hat{t} < 0$.

In what follows output effects depend on elasticity of substitution between factors used in production. These results can be summarized in the following proposition.

Proposition 3: Tariff Liberalization leads to:

- (i) an increase in W, W_s and a decrease in r and R;
- (ii) expansion of I and Y and contraction of both A and M sector;
- (iii) increase in skilled-unskilled wage gap if $\theta_{\rm KY} > \theta_{\rm KI}$.

Explanation

When t decreases, due to less protectionist effect M, manufacturing sector, which is protected by tariff is contracted. The resources are redirected from M to other sectors. Since M is capital intensive compared to other sectors (I and Y), K released per unit of L by M is higher than the K demanded per unit of L by I and Y. This leads to an excess supply of capital in the economy and it causes a reduction in the return to capital (r). When r decreases, the cost of using capital in production process becomes lower and this cost reduction can be appropriated by appointing more labour in the informal sector through factor substitution and profit maximizing behavior of the producers. This helps to increase informal wage rate W, and hence I sector expands. Changes in I is shown in Eq. A.19 of Appendix A.2. Again, reduction in r makes capital constraint more binding and an increase in W makes the labour constraint less binding. Subsequently, the output effects of A and Y depend on factor substitutability. As A uses

⁶ See Eq. A.20 in Appendix A.2.

⁷ The exact effect is shown in Eq. (A.17) and (A.18) of the Appendix A.2.

unskilled labour and land, increase in W creates a cost enhancement in agricultural sector which depresses the return to the other factor T, and thus A must contract. This happens because T is specific to A. Again when r decreases, W_s goes up for unchanged commodity price of Y. S being the specific factor, Y inflates owing to the fact W_s is higher now leading to a fall in a_{sy} .

Skilled and unskilled labour force both are benefitted owing to tariff reduction but the skilled-unskilled wage gap $(\hat{W_S} - \hat{W})$ depends on the magnitude of increase of W_S and W. The wage gap is given by $(\hat{W_S} - \hat{W}) = \frac{\alpha(\theta_{KI}\theta_{SY} - \theta_{KY}\theta_{LI})}{\theta_{LI}\theta_{KM}\theta_{SY}}\hat{t}$. Higher capital intensity in Y compared to I, leads to higher return to capital in Y than I. This causes W_S to rise more than W. So, tariff liberalization worsens the skilled-unskilled wage gap, if $\theta_{KY} > \theta_{KI}$.

3.3. Effect of Reduction in Export Subsidy

A subsidy is generally described as the converse of a tax. It signifies the difference between domestic market prices and world prices and the objective of subsidy is also to create a wedge between consumer prices and producer costs. Export subsidies are thought of as internal price supports to create more production in the home country in support of actively promoting export interests. Thus, subsidies may be implemented in the form of reduced tax-liability, low interest government loans or government equity participation. Export subsidies can also be spent for wage hikes demanded by workers. Therefore, exports of subsidized products affect domestic production of the same product in the importing country. Because of this, the WTO has taken an agreement to prohibit export subsidies to control trade-distorting effect. So, export subsidy reduction is an important trade policy reform in liberalized regime. In line with this view, in this model we want to show the effect of subsidy (s) reduction on agricultural and service sectors.

In our model, we have introduced export subsidy (s) in two exportable sectors A and Y. So, change in s will be directly appropriated by R and W_s . Note that, there will be no change in W and r since these are already determined from equations (2) and (3) for given P_I and P_M. Through constant returns to scale assumption output of I and M would not change. So, from Eq. (1) and (4) it is proven that return to skilled labour (W_s) and land (R) would decrease as subsidy decreases. The expressions are as follows $\hat{W_S} = \frac{\beta}{\theta_{SV}} \hat{s} < 0$ and $\hat{R} = \frac{\beta}{\theta_{TA}} \hat{s} < 0$ as $\hat{s} < 0$. Subsequently, using the concept of elasticity of substitution we can derive the following expression of the output effect of A and Y. These are given by $\hat{X}_A = \beta \sigma_A \left(\frac{\theta_{LA}}{\theta_{TA}} \right) \hat{s}$ and

 $\hat{X}_{Y} = \beta \sigma_{Y} \left(\frac{\theta_{KY}}{\theta_{CY}} \right) \hat{s}$, respectively. Based on these outcomes we have the following proposition

Proposition 4 Y and A contract along with decrease in W_s and R owing to export subsidy reduction. Subsidy reform also lowers skilled-unskilled wage gap.

Explanation

When per unit subsidy goes down, the effective price of A and Y decreases. Producers intend to produce less and this lowers the demand for factors. We have already mentioned that W and r remain unchanged due to non-changing commodity prices and tariff rate. So, A wants to substitute labour by land in production. As, land is specifically used in A, fixed supply of this factor results in a contraction of this sector. For similar reason, sector Y also shrinks due to subsidy reduction.

On wage disparity front, the skilled-unskilled wage difference $(W_S - W)$ depends on the rates of change of W_S and W. Though, subsidy reform decreases the skilled wage rate $\hat{W_S}$, \hat{W} remains unchanged. We have mentioned earlier the reasons underlying such effects. We also know about the ranking of wages of different categories, thus we know $W_S > W$. So, wage gap between skilled and unskilled workers gets lowered due to subsidy reduction.

4. Extended Model with Corruption in Informal Sector

Informal sector plays a crucial role in employment generation in developing economy. This argument quite well accepted by now, and does not require much clarification. The problems of informality, nevertheless, arise because of its unrecorded, non taxpaying, illegal characteristics. In such cases, this sector has to pay an extra cost in the form of bribe or extortion fee to local political leaders or govt. officials to protect their production process from legal hassles. Government bureaucracy could also be a part of this negotiation process. Bureaucracy is an administrative institution managed by government officials which works according to the government policies, rules and regulations. Bureaucrat gives services to be used for policy formulating, coordinating, monitoring or effort to limit corruption. Informal sector does not abide by these government rules and regulations because of its characteristics; this sector is used to pay such extra cost of corruption to continue their output process. As the activities of informal sector is lubricated by the process of intermediation of extortionists or by so called lobbying by government bureaucrats for protection, in this model we assume the existence of such workers who takes care of the problems related to informal production. We call them unproductive

workers⁸ (L_c) in informal production activities as without them the amount of output would not change. However, the survival of informal sector crucially depends on the existence of such sector where extortionists are paid the fee of extortion or cost of corruption which is defined as 'u'. This is the fraction of the value of informal output that is lost due to such activities. Now, we modify the structure of the basic model by introducing a rate of cost of corruption 'u' in the informal sector. We also assume that, these workers get identical wage as informal workers. In a competitive set up total expenditure on L_c has to be equal to the lost⁹ value of output (uP_IX_I) which is actually paid to the extortionists. Moreover, we have the standard neo-classical assumptions of constant returns (CRS) to scale and diminishing return to factors.

Therefore, Eq. (2) and Eq. (5) of the original model are now modified as

$$Wa_{II} + ra_{KI} = P_I(1-u) \tag{2A}$$

$$a_{LA}X_A + a_{LI}X_I + a_{LM}X_M = L - L_c (5A)$$

We have mentioned earlier that L_c people are also paid at the rate W which is same as the amount earned by informal productive workers and the total value of output lost in I is uP_IX_I . In a competitive set up this must be identical to the payment made for such people engaged in intermediation related extortion activities. Thus, the value cost equality of corruption is

$$uP_IX_I = WL_c \tag{9}$$

⁸ Intermediaries are unproductive implies that their marginal productivities in terms of the volume of output are zero though they get positive return for their work. In our model we have used this sort of intermediations as directly unproductive profit-seeking activities (Bhagwati, 1982) in the concept of corruption.

⁹ By the word 'lost' we only indicate loss from the value of production that does not come back to the factors of production. This 'lost' value is, however, very much needed for production in the informal sector. Notice that some workers are also paid by the 'lost' value. So, apparently this is not 'lost' from the economy (Mandal 2018).

The other equations and assumptions of the model remain same as in the basic model. Now, There are nine unknown variables in the system such as W, r, R, W_s , X_A , X_I , X_Y , X_M and L_c with nine independent equations. From Eq. (3) for given tariff rate, formal wage \overline{W} andr remains unchanged. When r is unchanged, from Eq. (4) W_s is also constant for given s. Then W can be examined in terms of \hat{u} from Eq. (2), and R would be solved from (1). Thus, all $a_{ij}s$ are determined through CRS assumption. Then using endowment equations we calculate the output effects. Hence equations (6) and (8) give us the value of X_Y and X_A as endowment of S and T are constants. Thus, X_M and X_I are simultaneously solved from equation (5) and (7). Consequently, L_c gets solved from equation (9). Detailed mathematical results are provided in Appendix B which helps us to show the effect of decrease in cost of corruption on factor prices and outputs of different sectors along with wage gap between skilled and unskilled workers.

4.1. Effect of Bureaucratic Reform on Informal Wage and Output

In order to consider the effect of bureaucratic reform we assume that owing to some reasons cost of corruption or extortion fee (u) falls. This extortion fee could be lowered because of qualitative improvement of administrative officials, effective delivery of services and betterment in local governance. A decrease in u leads to an increase in effective price of I. From equation (3) it is evident that r is unchanged. Hence, wage rate of unskilled workers W would increase as u decreases. $\hat{W} = -\frac{\delta}{\theta_{II}}\hat{u} > 0$ as $\hat{u} < 0$. When W increases, from Eq. (1) R decreases as P_A is given. Subsequently, using the concept of elasticity of substitution we can solve the value of

 X_A^{10} . The magnitude of effect on I and M can be expressed as $\hat{X}_I = \frac{\lambda_{KM}}{|\lambda|} D_2 \hat{u} > 0$ and

$$\hat{X}_M = -\frac{\lambda_{KI}}{|\lambda|} D_2 \hat{u} < 0 \text{ as } \hat{u} < 0.$$

Where,
$$D_1 = \sigma_A \left(\frac{\theta_{TA} + \theta_{LA}}{\theta_{TA} \theta_{LI}} \right) \theta_{LA} > 0$$
, $D_2 = \left(\lambda_u + \frac{\delta}{\theta_{LI}} + \lambda_{LA} D_1 \right) > 0$,

$$\left|\lambda\right| = \left[\lambda_{LM} \lambda_{KI} - \left(\lambda_{LI} + \lambda_{u}\right) \lambda_{KM}\right] < 0$$

Proposition 5 Due to bureaucratic reform

- (i) W increases,
- (ii) Wage gap ($\hat{W_s}$ \hat{W}) falls,
- (iii) Informal output expands.

Explanation

When cost of corruption decreases, informal producers have to pay less amount of money to extortionists to sustain their production. Then, this sector takes advantage of this cost reduction and invests on other factors, and can try for output expansion. Thus, W increases. Interestingly, though, we incorporate the cost of corruption only in informal sector in this model, this also induces changes in Ws and R. Ws remains constant and R falls. T being the specific factor in A, a fall in R implies an increase in a_{TA}. Hence, A shrinks [from equation (8)] thus it releases L that moves to I and I expands.

¹⁰ See Eq. (B.3) in Appendix B.

Bureaucratic reform increases \hat{W} but $\hat{W_S}$ is unchanged. Therefore, it is apparent that the wage gap between skilled and unskilled labour $(\hat{W_S} - \hat{W})$ is also depressed.

4.1. Effect on Number of Extortionists

In this extended part of the model extortion or corruption is an important issue.. Extortionists or unproductive workers (L_c) are involved in informal production to combat legal hassles. Here we have already mentioned that these workers get wage (W) equal to the unskilled wage. Equation $uP_1X_1 = WL_c$ has described the payment that made for people who are engaged in corruption. Reduction in cost of corruption increases the informal wage, W and leads to an expansion of I. From factor price effects it is understandable that both A and M contracts and releases some L which can either be employed in I or in extortion activity. So, it is not guaranteed that L_c would rise. This is also corroborated by equation (9). Detailed mathematical calculations are provided in Appendix B (see equation (B.4) and (B.7) for further clarification). One interesting point to note that though I expands L_c may not rise. This is because extortionists are now more efficient which is reflected in higher W.

Therefore, the effect on L_c is uncertain. The desired mathematical expression is as follows $\hat{L}_c = (\frac{\lambda_u \lambda_{KM}}{|\lambda|} + \lambda_u + \frac{\delta}{\theta_{II}})\hat{u}$. Thus we have the following proposition:

Proposition 6 Bureaucratic reform leads to an ambiguous effect on number of extortionists.

5. Conclusion

Reformatory policies are quite common and supposed to have desirable results in developing economy. In order to examine such intention this paper starts with a four sector general

equilibrium model where both formal and informal sectors are embedded. The economy has a formal sector that uses skilled labour as specific factor whereas unskilled labour is assumed to be perfectly mobile among other three sectors. Capital is also mobile among two formal sectors and the informal one. In this backdrop it has been found that labour market reform and tariff reform produces distinctly opposite outcomes on the factor returns and output. Labour market reform leads to the contraction of both informal and service sector with decreasing unskilled and skilled wage rate, but it expands manufacturing and agricultural output with increasing return to capital. A tariff reduction policy on the other hand comes up with totally opposite results. While, a reduction in subsidy yields effect similar with labour market reform in respect to skilled labour and similar with tariff cut in capital front. However, skilled-unskilled wage disparity is reduced owing to labour market reform and subsidy reduction but tariff liberalization policy worsens it. Later on the basic model is extended to incorporate corruption in informal sector and it is found that a decrease in the cost of corruption helps increasing informal wage along with expanding informal output but the effect on number of extortionists is ambiguous.

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APPENDIX

Appendix A

Appendix A.1. Labor market reform or a reduction in \overline{W}

Differentiating equation (3) and using envelope condition, we get,

$$\hat{\overline{W}}\theta_{LM} + \hat{r}\theta_{KM} = 0 \tag{A.1}$$

From equations (A.1)

$$\hat{r} = -\frac{\theta_{LM}}{\theta_{KM}} \hat{\overline{W}} > 0 \text{ as } \hat{\overline{W}} < 0 \tag{A.2}$$

Differentiating equations (2) and (4) and substituting the expression for \hat{r} we get

$$\hat{W}_{S} = \frac{\theta_{LM} \theta_{KY}}{\theta_{KM} \theta_{SY}} \hat{\overline{W}} < 0 \text{ as } \hat{\overline{W}} < 0$$
(A.3)

$$\hat{W} = \frac{\theta_{LM} \theta_{KI}}{\theta_{KM} \theta_{LI}} \hat{\overline{W}} < 0 \text{ as } \hat{\overline{W}} < 0$$
(A.4)

The wage gap between skilled and unskilled wages are derived from (A.3) and (A.4)

$$\left(\hat{W}_{S} - \hat{W}\right) = \frac{\theta_{LM} \left(\theta_{KY} \theta_{LI} - \theta_{KI} \theta_{SY}\right)}{\theta_{LI} \theta_{KM} \theta_{SY}} \hat{\overline{W}} < 0 \text{ as } \hat{\overline{W}} < 0$$

If,
$$(\theta_{KY} > \theta_{KI})^{11}$$

Again, differentiating equation (1) and substituting \hat{W} we obtain

$$\hat{R} = -\frac{\theta_{LM}\theta_{KI}\theta_{LA}}{\theta_{KM}\theta_{LI}\theta_{TA}}\hat{\overline{W}} > 0 \text{ as } \hat{\overline{W}} < 0$$
(A.5)

Equation (7) yields

$$\hat{X}_{Y} = -\hat{a}_{SY} \tag{A.6}$$

By definition, the elasticity of substitution between S and K in Y is given by

$$\sigma_{\scriptscriptstyle Y} = rac{\hat{a}_{\scriptscriptstyle KY} - \hat{a}_{\scriptscriptstyle SY}}{\hat{W}_{\scriptscriptstyle S} - \hat{r}}$$

¹¹ Y uses capital more intensively than I.

Using expression for elasticity of substitution and Envelope theorem between S and K in Y we have

$$\hat{a}_{SY} = -\sigma_{Y} (\hat{W}_{S} - \hat{r}) \theta_{KY}$$

Substituting the values from equations (A.3) and (A.2) in the above equation, we obtain

$$\hat{a}_{SY} = -\sigma_{Y} \left(\frac{\theta_{LM} \, \theta_{KY} + \theta_{LM} \, \theta_{SY}}{\theta_{KM} \, \theta_{SY}} \right) \theta_{KY} \hat{\overline{W}}$$

Using equation (A.6) and simplifying this becomes

$$\hat{X}_{Y} = A_{l}\hat{\overline{W}} < 0 \text{ as } \hat{\overline{W}} < 0 \tag{A.7}$$

Where,
$$A_{l} = \sigma_{Y} \left(\frac{\theta_{LM} \theta_{KY} + \theta_{LM} \theta_{SY}}{\theta_{KM} \theta_{SY}} \right) \theta_{KY} > 0$$

Again from equation (8) we get

$$\hat{X}_A = -\hat{a}_{TA} \tag{A.8}$$

The elasticity of substitution between L and T in A is expressed as $\sigma_A = \frac{\hat{a}_{TA} - \hat{a}_{LA}}{\hat{W} - \hat{R}}$. Application of Envelope theorem and zero profit condition ensures $\hat{a}_{TA} = \sigma_A (\hat{W} - \hat{R}) \theta_{LA}$.

Substituting the values in the above equation, we obtain

$$\hat{a}_{TA} = \sigma_{A} \left(\frac{\theta_{LM} \theta_{KI} \theta_{TA} + \theta_{LM} \theta_{LA} \theta_{KI}}{\theta_{KM} \theta_{TA} \theta_{LI}} \right) \theta_{LA} \hat{\overline{W}}$$

Again equation (A.8) can be rewritten as

$$\hat{X}_A = -A_2 \hat{\overline{W}} > 0 \text{ as } \hat{\overline{W}} < 0 \tag{A.9}$$

Where,
$$A_2 = \sigma_A \left(\frac{\theta_{LM} \theta_{KI} \theta_{TA} + \theta_{LM} \theta_{LA} \theta_{KI}}{\theta_{KM} \theta_{TA} \theta_{LI}} \right) \theta_{LA} > 0$$

Differentiating equation (5) and (6) and substituting the values of equations (A.7) and (A.9)

$$\lambda_{II}\hat{X}_I + \lambda_{IM}\hat{X}_M = \lambda_{IA}A_2\hat{\overline{W}}$$

$$\lambda_{KI}\hat{X}_{I} + \lambda_{KM}\hat{X}_{M} = -\lambda_{KY}A_{I}\hat{\overline{W}}$$

Rearranging the above equations in a matrix form we solve for

$$\hat{X}_I = \frac{1}{|\lambda|} A_3 \hat{\overline{W}} \tag{A.10}$$

$$\hat{X}_{M} = -\frac{1}{|\lambda|} A_{4} \hat{\overline{W}} \tag{A.11}$$

Where,

$$A_3 = (\lambda_{LA}\lambda_{KM}A_2 + \lambda_{LM}\lambda_{KY}A_1) > 0 \quad A_4 = (\lambda_{LI}\lambda_{KY}A_1 + \lambda_{LA}\lambda_{KI}A_2) > 0 \quad |\lambda| = (\lambda_{LI}\lambda_{KM} - \lambda_{LM}\lambda_{KI}) > 0$$

If, I is more labour-intensive than M in comparison with capital.

Appendix A.2 Tariff reform or a reduction in t

From equation (3) we get,

$$\hat{r}\theta_{KM} = \alpha \hat{t} \tag{A.12}$$

Where,
$$\alpha = \frac{t}{1+t} > 0$$

Equation (A.12) can be rewritten as

$$\hat{r} = \frac{\alpha}{\theta_{\text{\tiny EM}}} \hat{t} < 0 \text{ as } \hat{t} < 0 \tag{A.13}$$

From equation (2) and (4) values of change in Ws and W can be expressed as

$$\hat{W}_{S} = -\frac{\alpha \theta_{KY}}{\theta_{KM} \theta_{SY}} \hat{t} > 0 \text{ as } \hat{t} < 0$$
(A.14)

$$\hat{W} = -\frac{\alpha \theta_{KI}}{\theta_{KM} \theta_{IJ}} \hat{t} > 0 \text{ as } \hat{t} < 0$$
(A.15)

So, the expression for the wage gap between the skilled and unskilled labour is

$$(\hat{W}_S - \hat{W}) = \frac{\alpha(\theta_{KI}\theta_{SY} - \theta_{KY}\theta_{LI})}{\theta_{LI}\theta_{KM}\theta_{SY}} \hat{t} > 0 \text{ as } \hat{t} < 0$$

If,
$$\theta_{KY} > \theta_{KI}$$

Again, from equation (1)

$$\hat{R} = \frac{\alpha \theta_{KI} \theta_{LA}}{\theta_{KM} \theta_{IJ} \theta_{TA}} \hat{t} < 0 \text{ as } \hat{t} < 0$$
(A.16)

Differentiating equation (7) and using expression for elasticity of substitution by substituting the values from equations (A.13) and (A.14), we obtain

$$\hat{X}_Y = -\alpha B_1 \hat{t} > 0 \text{ as } \hat{t} < 0 \tag{A.17}$$

Where,
$$B_1 = \sigma_Y \left(\frac{\theta_{KY} + \theta_{SY}}{\theta_{KM} \theta_{SY}} \right) \theta_{KY} > 0$$

Similarly, Differentiating equation (8)

$$\hat{X}_A = -\alpha B_2 \hat{t} > 0 \text{ as } \hat{t} < 0 \tag{A.18}$$

Where,
$$B_2 = \sigma_A \left(\frac{\theta_{KI} \theta_{TA} + \theta_{LA} \theta_{KI}}{\theta_{KM} \theta_{TA} \theta_{LI}} \right) \theta_{LA} > 0$$

Differentiating equation (5) and (6) and then solving them in matrix form to yield the following sets of equations

$$\hat{X}_I = -\frac{\alpha}{|\lambda|} B_3 \hat{t} > 0 \tag{A.19}$$

$$\hat{X}_M = \frac{\alpha}{|\lambda|} B_4 \hat{t} < 0 \tag{A.20}$$

Where,

$$B_3 = (\lambda_{LA}\lambda_{KM}B_2 + \lambda_{LM}\lambda_{KY}B_1) > 0 \quad B_4 = (\lambda_{LI}\lambda_{KY}B_1 + \lambda_{LA}\lambda_{KI}B_2) > 0 \quad |\lambda| = (\lambda_{LI}\lambda_{KM} - \lambda_{LM}\lambda_{KI}) > 0$$

Appendix A.3 Removal of export subsidy or a reduction in s

Differentiating equations (2) and (3) and simplifying we obtain no change in r and W.

$$\hat{W} = \hat{r} = 0$$

Note that, nothing would happen to I and M as $\hat{W} = \hat{r} = \overline{W} = 0$. Factor substitution is not permitted due to non-changing factor prices and in addition to this, unchanged factor supply confirms constancy of I and M.

Differentiating equation (1) and substituting \hat{W} we get

$$\hat{R} = \frac{\beta}{\theta_{TA}} \hat{s} < 0 \text{ as } \hat{s} < 0 \tag{A.21}$$

Where,
$$\beta = \frac{s}{1+s} > 0$$

Again differentiating equation (4) and substituting \hat{r} we get

$$\hat{W}_{S} = \frac{\beta}{\theta_{SY}} \hat{s} < 0 \text{ as } \hat{s} < 0 \tag{A.22}$$

Differentiating equation (7) and using expression for elasticity of substitution by substituting the values from equations (A.22), we obtain

$$\hat{X}_{Y} = \beta C_1 \hat{s} < 0 \text{ as } \hat{s} < 0$$
 (A.23)

Where,
$$C_1 = \sigma_Y \left(\frac{\theta_{KY}}{\theta_{SY}} \right) > 0$$

Similarly equation (8) yields

$$\hat{X}_A = \beta C_2 \hat{s} < 0 \text{ as } \hat{s} < 0$$
 (A.24)

Where,
$$C_2 = \sigma_A \left(\frac{\theta_{LA}}{\theta_{TA}} \right) > 0$$

Appendix B

Effects of bureaucratic reform

Bureaucratic reform is considered in the form of decrease in the cost of corruption. Corruption is incorporated in the informal sector only.

Differentiating equations (3) and (4) and simplifying we obtain

$$\hat{W_S} = \hat{r} = 0$$

Differentiating equation (2) and substituting \hat{r} we get

$$\hat{W} = -\frac{\delta}{\theta_{IJ}}\hat{u} > 0 \text{ as } \hat{u} < 0 \tag{B.1}$$

Where,
$$\delta = \frac{u}{1-u} > 0$$

Again differentiating equation (1) and substituting Eq. (B.1)

$$\hat{R} = \frac{\delta \theta_{LA}}{\theta_{LI} \theta_{TA}} \hat{u} < 0 \text{ as } \hat{u} < 0$$
(B.2)

Differentiating equation (8) and using expression for elasticity of substitution, we obtain

$$\hat{X}_A = \delta D_1 \hat{u} < 0 \text{ as } \hat{u} < 0 \tag{B.3}$$

Where,
$$D_1 = \sigma_A \left(\frac{\theta_{TA} + \theta_{LA}}{\theta_{TA} \theta_{LI}} \right) \theta_{LA} > 0$$

Differentiating Eq. (9)

$$\hat{L}_c = \lambda_u \hat{X}_I + (\lambda_u + \frac{\delta}{\theta_{II}})\hat{u}$$
(B.4)

Differentiating equation (5) and (6)

$$(\lambda_{IJ} + \lambda_u)\hat{X}_I + \lambda_{IM}\hat{X}_M = -D_2\hat{u}$$

$$\lambda_{KI} \hat{X}_I + \lambda_{KM} \hat{X}_M = 0$$

Rearranging the above equations in a matrix form and manipulating them we derive the expressions for

$$\hat{X}_I = \frac{\lambda_{KM}}{|\lambda|} D_2 \hat{u} > 0 \tag{B.5}$$

$$\hat{X}_{M} = -\frac{\lambda_{KI}}{|\lambda|} D_{2} \hat{u} < 0 \tag{B.6}$$

Where,

$$D_{2} = \left(\lambda_{u} + \frac{\delta}{\theta_{LI}} + \lambda_{LA}D_{1}\right) > 0, \ \left|\lambda\right| = \left[\lambda_{LM}\lambda_{KI} - \left(\lambda_{LI} + \lambda_{u}\right)\lambda_{KM}\right] < 0.$$

Substituting the value of \hat{X}_I in Eq. (B.4), it yields

$$\hat{L}_{c} = (\frac{\lambda_{u}\lambda_{KM}}{|\lambda|} + \lambda_{u} + \frac{\delta}{\theta_{LI}})\hat{u}$$
(B.7)