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## Dynamic regulation revisited: Signal dampening, experimentation and the ratchet effect

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# Dynamic Regulation Revisited: Signal Dampening, Experimentation and the Ratchet Effect

Thomas D. Jeitschko\*      John A. Withers†

July 2019

## Abstract

Regulators and the firms they regulate interact repeatedly. Over the course of these interactions, the regulator collects data that contains information about the firm's idiosyncratic private characteristics. This paper studies the case in which the regulator uses information gleaned from past cost observations when designing the current period's contract. Cost observations are obscured in stochastic settings and so perfect inferences about underlying private information are not possible. However, the design of the regulatory contract affects how much information is gleaned. When learning more about the firm's type, the regulator increases expected second period welfare by reducing distortions tied to asymmetric information. In contrast, by learning less about the firm's type, the regulator reduces incentive payments in first period. The trade-off between the desire to be more informed and to reduce incentive payments leads to a contracting dynamic that aligns with anecdotal, experimental and empirical evidence of the ratchet effect.

**Keywords:** Dynamic Contracts, Dynamic Agency, Ratchet Effect, Experimentation, Signal Dampening, Regulation

**JEL Classification:** D8, C73, L5

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# 1 Introduction

In regulated industries, firms and regulators have long-term relationships with one another. The rules and procedures that govern these relationships are revised over time. When the regulator cannot commit at the outset of the relationship to how these rules and procedures will be updated in the future, the ratchet effect arises.

In repeated agency interactions, the ratchet effect describes the agent's response to the principal's inability to commit to long term contracts. The principal learns about the agent's ability, or the economic environment, by observing his performance. The principal then adjusts the agent's compensation in the future based on what she learns from this observation. The more the principal learns about the agent, the more rent she is able to extract. To obscure the principal's learning process, the agent restricts his performance, or reduces his effort. This allows the agent to avoid more stringent incentives in the future.

Take, for example, a regulated monopoly that provides electricity to consumers. Periodically, the regulator will undertake a rate case to evaluate whether current electricity prices offer the utility a fair return on capital. During the rate case, the regulator observes the utility's operating expenses, along with other measures such as the firm's rate base (capital), taxes and depreciation expenses. Based on these measures, the regulator determines the revenue that the firm needs to earn to recoup operating expenses and make a fair return for their investors. This revenue target in turn determines the prices that the utility can charge consumers.

During this process, the regulator learns about the firm's efficiency by observing the firm's operating expenses. The regulator expects that a firm with high operating expenses in the current rate cycle will have high operating expenses again in the next rate cycle, and is thus more willing to give a generous reimbursement. Therefore, the firm has little incentive reduce operating costs, since a better performance today implies a less generous revenue requirement in the next rate cycle.

Some of the earliest anecdotal evidence of the ratchet effect comes from studies of piece

rate factory workers (see Matthewson (1931), Roy (1952), Montgomery (1979) and Clawson (1980)). Matthewson (1931) documented that piece-rate workers understood that a good performance today ultimately made them worse off in the long run. To see this, suppose a worker produces more units of output in the current period than in the previous pay period. Since the worker is paid per unit, the worker earns more in the current period than in the previous period. Workers learned, however, that the factory manager’s response to this improved performance was an increase in his performance expectations. In response to this behavior by factory managers, Matthewson documented that workers “never worked at anything like full capacity.” Berliner (1957) documented that factory managers in the Soviet Union responded similarly to incentive systems based on output targets.

The anecdotal evidence discussed above suggests that agents restrict their performance (i.e., reduce effort) when the principal bases future compensation on information that she gathers about them. Recent empirical evidence supports this notion. Macartney (2016) adapts the theoretical model of Weitzman (1980) to examine if teacher value-added schemes induce dynamic effort distortions among teachers in North Carolina. Teachers in a given school receive a bonus in the current year if the school-wide average on a standardized test is above a pre-specified target. The key feature of these schemes is that the target score is a function of the school’s average standardized test score in the previous year. Clearly, the higher is the school’s average test score this year, the more difficult it will be for teachers to exceed next year’s target and receive a bonus. Macartney exploits differences in grade composition across schools to show that teachers respond to the value-added schemes by reducing their effort on improving their students standardized test scores.

In the kind of repeated interactions described by Matthewson (1931) and Macartney (2016), agents with high ability have the strongest incentive to reduce effort in the present to maintain information rents in the future. Charness, Kuhn, and Villeval (2011) use an experimental design to study the effects of labor market competition on the ratchet effect. As a baseline case, they examine a two-period relationship between one firm and one worker.

In this baseline case, roughly 60 percent of the experimental subjects who are designated as having high ability reduce their effort in the first period so that they can maintain a second period information rent. In a related experimental paper, Cardella and Depew (2018) study the impact of evaluating performance at the individual versus group level on the ratchet effect. The authors find that workers suppress effort when evaluated individually.

In contrast to the anecdotal, empirical, and experimental evidence discussed above, in most theoretical models of the ratchet effect, the good agent’s effort does not evolve as one would expect. For example, Laffont and Tirole (1987) examine a two-period interaction between a regulator and a regulated firm in which the firm completes a project for the regulator. The observable outcome is the project’s cost. The project cost depends on the firm’s intrinsic cost level, which is the firm’s private information. The regulator cannot commit, in the first period, to the second period incentive scheme.

In this setting, if a separating contract can be supported, then the low-cost firm exerts the first best level of effort in the first and second period—that is, there is no change in the equilibrium effort being exerted. One reason the low-cost firm’s effort in Laffont and Tirole (1987) does not evolve in a manner that fits with received evidence is because the firm is assumed to have perfect control over the observable outcome. That is, the only way for the low cost agent to hide his private information is to mimic (pool with) the high cost firm.

Contrast this with the case in which the agent does not have perfect control over the observable outcome (i.e., the relationship between the agent’s actions and the project’s outcome is stochastic). While Laffont and Tirole (1986) and Laffont and Tirole (1993) show that an additive, zero-mean noise term has no impact on incentives in a static setting (assuming that both the firm and the regulator are risk neutral), in a dynamic setting noise impedes the principal’s learning process. Jeitschko, Mirman, and Salgueiro (2002) and Jeitschko and Mirman (2002) demonstrate this in two-period interactions in which an agent produces output for a principal. They show that as a result of impeded learning two opposing incentives determine the first period output targets. First, the principal can design the first period

contract to increase what she learns about the agent’s private information. By doing so, she increases her expected second period payoff. Second, the principal can design the first period contract to decrease what she learns about the agent’s private information. By doing so, she decreases the first period transfer to the high productivity agent.

We revisit the equilibrium dynamics in a regulatory context when the regulated firm does not have complete control over the outcomes tied to their actions. Borrowing from the framework of Laffont and Tirole (1987), a firm completes a project for a regulator. The observable outcome is the project’s cost. In contrast to Laffont and Tirole (1987), however, the project’s cost is stochastic. The principal uses the cost observation to update her beliefs about the firm’s type. We show that when the noisy component of the project’s cost follows a general distribution, the low-cost agent has his effort increased over time. Therefore, we present a theoretical model whose predictions match with anecdotal, empirical and experimental evidence of the ratchet effect.

This paper is related to two strands of dynamic principal-agent literature. First, this paper is related to theoretical models of the ratchet effect. The ratchet effect has most famously been studied in the context of regulation and procurement (see, e.g., Freixas et al. (1985), and Laffont and Tirole (1988) in addition to the aforementioned papers). It has also been studied in settings such as piece-rate incentive contracts (Gibbons (1987)), optimal income taxation (Dillen and Lundholm (1996)), and government corruption (Choi and Thum (2003)). These papers differ from the current paper in that the agent is assumed to have perfect control over the observable outcome.

This paper is also related to a growing dynamic mechanism design literature. Athey and Segal (2013) and Pavan et al. (2014) derive efficient and revenue maximizing dynamic mechanisms, respectively, when the principal can commit to future mechanisms and the agent’s private information changes over time (for a survey of dynamic mechanism design when the principal can commit to future incentive schemes, see Bergemann and Valimaki (2017)). Because the principal is assumed to commit to future mechanisms, the ratchet effect



problem does not arise.

The dynamic mechanism design literature most closely related to this paper studies dynamic mechanisms in which the principal has limited commitment power. First, Skreta (2015) studies a two period model in which a seller cannot commit not to re-sell an indivisible good if the first period mechanism fails to allocate the good to one of several buyers. Deb and Said (2015) study a sequential screening problem that builds off of Courty and Li (2000). The seller can commit in the first period to the terms of consumption of a good in the second period, but cannot commit to the selling mechanism offered in the second period. The principal in both Skreta (2015) and Deb and Said (2015) is concerned with maximizing revenue, while the principal in our paper maximizes welfare. Additionally, consumption only occurs once in each paper; in either the first or second period in Skreta (2015), and at the end of the second period in Deb and Said (2015). In our paper, the agent completes a task for the principal in each period. The principal gathers information about the agent from the outcome of the first period project, and uses this information to increase the efficiency of the second period interaction.

Finally, Gerardi and Maestri (2017) study an infinitely repeated principal-agent interaction. The principal is uninformed about the agent's private cost characteristic, which may be high or low. The agent produces a good of observable and verifiable quality for the principal. Depending on the principal's prior beliefs and the discount factor, the principal learns the agent's type immediately, over time, or never at all. Because Gerardi and Maestri (2017) study a pure adverse selection setting, there are no direct comparisons between our paper and theirs about how the low cost agent's effort evolves over time.

## 2 Model

Consider a two period interaction between a welfare-maximizing regulator (she) and a regulated firm (he). In each period, the regulator offers the firm a contract to complete a project

that has gross-benefit  $S$ . In return for completing the project each period, the regulator reimburses the firm for the project's cost,  $c_t$ , and pays the firm an additional transfer,  $t_t(c_t)$ . The additional transfer is a function of the project's realized cost in each period, and incentivizes cost-reducing effort. The project's cost in each period depends on the firm's intrinsic cost parameter,  $\beta$ , its unobservable effort,  $e_t$ , and a homoskedastic, zero mean noise term,  $\varepsilon_t$ :

$$c_t = \beta - e_t + \varepsilon_t, \quad t = 1, 2. \quad (1)$$

The random variable  $\varepsilon_t$  is assumed to be distributed over the entire real line according to the distribution function  $G(\varepsilon)$  with associated density  $g(\varepsilon)$ . The density satisfies the monotone likelihood ratio property. While the full support assumption is analytically convenient, it raises two issues that bear mention.

The first issue is that the low cost firm's effort from mimicking the high cost type may be negative in the second period. This occurs when the first period cost realization is sufficiently low. A common assumption in static models is that the regulator's prior belief that the firm has low costs is small enough that this situation does not arise. However, in this dynamic-stochastic setting, the regulator's second period beliefs are endogenous, and depend on the first period cost realization. Thus, the analysis allows for negative effort levels. Second, the full support assumption implies that negative cost realizations are possible. While unrealistic, the possibility of negative costs does not affect the results of this paper.

It is important to note that  $\varepsilon_t$  is unobservable both ex-ante and ex-post. Thus, while the regulator is able to observe total cost  $c_t$  in each period, she cannot determine the individual impacts of the firm's type, its effort, and noise. This captures the intuition that the firm does not have perfect control over the project's cost. The firm affects the distribution of costs by exerting effort, but the project's cost depend on factors outside of the firm's control. Another interpretation of noise is that of an "accounting error." Given the complexity of accounting rules, and constraints on her time, the regulator may not be able to perfectly discern which costs should and shouldn't be reimbursed after observing the firm's income statement

or other supporting documents.

The firm's type can be either  $\underline{\beta}$  or  $\bar{\beta}$ , with  $0 < \underline{\beta} < \bar{\beta}$ , and remains constant over the course of the interaction. Throughout, type  $\underline{\beta}$  is referred to as the “low cost type” or “low cost firm,” and type  $\bar{\beta}$  as the “high cost type” or “high cost firm.” The firm's type is its private information; the regulator's prior belief that the firm is the low cost type is given by  $\rho$ . The firm experiences a disutility of effort that can be expressed in monetary terms by

$$\psi(e_t) = \begin{cases} \frac{\gamma}{2}e_t^2, & e_t > 0, \\ 0, & e_t \leq 0, \end{cases} \quad (2)$$

where  $\gamma > 0$ . Thus, the firm's per period utility is given by

$$U_t = t_t(c_t) - \psi(e_t). \quad (3)$$

Although project costs are stochastic, the firm's effort is not; in each period, the firm chooses his effort before the realization of  $\varepsilon_t$ .

The regulator's objective in each period is to maximize expected welfare, which is the sum of taxpayer surplus and the firm's utility. In each period, welfare is given by

$$W_t = S - (1 + \lambda)(c_t + t_t(c_t)) + U_t. \quad (4)$$

Taxpayers enjoy benefit  $S$  from the project, compensate the firm for its costs  $c_t$ , and pay out the incentive fee  $t_t(c_t)$ . Since the cost reimbursement and incentive transfer are raised via distortionary taxation, one dollar paid to the firm costs taxpayers  $\$(1 + \lambda)$ , where  $\lambda > 0$  denotes the shadow cost of public funds.

The solution concept used is that of a perfect Bayesian equilibrium. In each period, the regulator designs an incentive scheme to maximize expected welfare. The incentive scheme depends on the regulator's beliefs about the firm's type. In the first period, the regulator

considers the impacts of the first period contract on expected second period welfare.

At the beginning of the second period, the regulator observes the first period project cost, and updates her beliefs about the firm's type using Bayes' rule. Contracts are short term; thus, when designing the second period contract, the regulator cannot commit to ignore any information she learns about the firm's type from observing the realized first period project cost.

The firm chooses whether to participate or not in each period. If the firm chooses to participate, he chooses his effort to maximize his expected utility given the transfer designed by the regulator. In the first period, he considers the impact that his actions have on the regulator's second period beliefs, and thus his expected second period payoffs.

In the analysis to follow, the regulator's problem in each period is to maximize expected welfare by choosing a cost target for each type of firm. These targets serve two purposes. First, whatever cost the firm decides to target determines the firm's effort. To see this, recall that effort is chosen before the realization of  $\varepsilon_t$ . Thus, the firm simply chooses its effort such that its expected cost,  $E[c_t] = \beta - e_t$ , is equal to its chosen cost target.

Second, for a given type of firm, the cost target serves as the mean of the distribution of project costs in each period. Since the incentive transfer is a function of project costs, the expected transfer in each period depends on the cost target. Thus, at the beginning of each period the regulator chooses cost targets that, in expectation, form an incentive feasible menu.

Framing the regulator's problem as a choice of cost target for each type of firm is without loss of generality as long as there exists an incentive transfer, based solely on realized costs, that satisfies the three following properties in expectation. First, the high cost firm's expected utility from targeting  $\bar{c}_t$  must be equal to his outside option of zero. Second, the low cost firm's expected utility from targeting  $\underline{c}_t$  must be equal to his expected utility from targeting  $\bar{c}_t$ . Third, the firm's expected utility from targeting  $c_t \notin \{\underline{c}_t, \bar{c}_t\}$  is lower than his expected utility from targeting either  $\underline{c}_t$  or  $\bar{c}_t$ .

When these three properties are satisfied, the high cost firm's participation constraint and the low cost firm's incentive constraint are satisfied in expectation in each period. Further, neither firm has an incentive to target a cost level other than the cost target designed for him by the regulator. The paper proceeds by assuming that there exists a transfer based on observed costs,  $t_t(c_t)$ , such that the expected transfer,  $E[t_t(c_t)]$ , satisfies the three aforementioned properties.

Caillaud, Guesnerie, and Rey (1992), Picard (1987) and Melumad and Reichelstein (1989) study the existence of such reward schedules when the agent's type space is continuous. When the agent's type may only take on two values, there are fewer constraints placed on the reward schedule. However, the lower envelope of the high and low cost agent's indifference curves is kinked, which implies that it may not be possible to implement the high cost firm's exact cost target. However, one can implement a cost target that is arbitrarily close (see Jeitschko and Mirman (2002)).

Throughout the paper, the focus is on deriving an equilibrium that is "separating in actions." Because cost observations are noisy, and this uncertainty is not resolved ex-post, the regulator is not able to determine with certainty the firm's type by only observing the cost realization. That is, even when the first period contract is designed in a way that the low cost firm and high cost firm target distinct cost levels, the regulator does not have full information about the firm's type in the second period. Thus, the equilibrium is separating in actions when the regulator designs distinct targets for each type of firm, and each type of firm targets the expected cost designed for for him by the regulator. This means in period  $t = 1, 2$ , the low cost firm targets  $\underline{c}_t$ , and the high cost firm targets  $\bar{c}_t$ .

### 3 Second period

Sine the model is solved using backward induction, the analysis begins with the second period. Suppose that the first period contract is separating in actions. At the beginning of

the second period, the regulator observes the first period cost realization and updates her beliefs about the firm's type using Bayes' rule. Therefore, her second period belief that the firm is the low cost type is given by

$$\rho_2 := \frac{\rho g(c_1 - \underline{c}_1)}{\rho g(c_1 - \underline{c}_1) + (1 - \rho)g(c_1 - \bar{c}_1)}. \quad (5)$$

Consider the numerator of (5). The regulator's prior belief that the firm has low costs is given by  $\rho$ . In the first period, the low cost firm targets  $\underline{c}_1$ ; when the firm targets  $\underline{c}_1$ , the first period cost realization is  $c_1 = \underline{c}_1 + \varepsilon_1$ . Since  $g(\varepsilon_1)$  represents the density of noise in the first period,  $g(c_1 - \underline{c}_1)$  is the probability density of first period costs when the agent targets  $\underline{c}_1$ . Thus,  $g(c_1 - \underline{c}_1)$  gives the value of the probability density function when the cost realization is  $c_1$  and the agent targets  $\underline{c}_1$ .

Similarly, the probability density of costs when the agent targets  $\bar{c}_1$  is given by  $g(c_1 - \bar{c}_1)$ . Since noise has full support on the real line, both  $g(c_1 - \bar{c}_1)$  and  $g(c_1 - \underline{c}_1)$  are strictly positive on the entire real line. Thus, the principal never believes to be fully informed about the agent's type in the second period. That is, because of the full support assumption,  $\rho_2 \in (0, 1)$ .

With beliefs given in (5), the regulator's problem is to choose expected costs  $\underline{c}_2$  and  $\bar{c}_2$  to maximize expected welfare, subject to incentive and participation constraints (which are derived below):

$$\begin{aligned} \max_{\underline{c}_2, \bar{c}_2} \quad & \rho_2 \int_{\mathbb{R}} \left[ S - (1 + \lambda)(c_2 + t_2(c_2)) + t_2(c_2) - \frac{\gamma}{2}(\underline{\beta} - \underline{c}_2)^2 \right] g(c_2 - \underline{c}_2) dc_2 \\ & + (1 - \rho_2) \int_{\mathbb{R}} \left[ S - (1 + \lambda)(c_2 + t_2(c_2)) + t_2(c_2) - \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 \right] g(c_2 - \bar{c}_2) dc_2 \end{aligned} \quad (6)$$

Because the second period game is static, and both the regulator and the firm are risk neutral, zero-mean noise has no impact on incentives. Thus, the binding constraints on the regulator's problem are the low cost type's incentive compatibility constraint and the high

cost firm's participation constraint.<sup>1</sup>

First, consider the low cost type's incentive compatibility constraint. The optimal second period cost targets make the low cost firm's expected utility from targeting  $\underline{c}_2$  equal to his expected utility from targeting  $\bar{c}_2$ . When the low cost firm targets  $\underline{c}_2$ , he chooses his effort in the second period such that  $\underline{e}_2 = \underline{\beta} - \underline{c}_2$ , and thus his private cost of effort is equal to  $\frac{\gamma}{2} (\underline{\beta} - \underline{c}_2)^2$ .

When the low cost firm chooses his effort in this manner, it is easy to see that

$$E[c_2] = E[\underline{\beta} - \underline{\beta} + \underline{c}_2 + \varepsilon_2] = \underline{c}_2. \quad (7)$$

Therefore, the second period project cost can be written as  $c_2 = \underline{c}_2 + \varepsilon_2$ , which implies that the density of second period costs is given by  $g(c_2 - \underline{c}_2)$ . And, the low cost firm's expected second period utility from targeting  $\underline{c}_2$  is given by

$$E[U_2 | \underline{c}_2] := \int_{\mathbb{R}} \left[ t_2(c_2) - \frac{\gamma}{2} (\underline{\beta} - \underline{c}_2)^2 \right] g(c_2 - \underline{c}_2) dc_2 = \underline{t}_2 - \frac{\gamma}{2} (\underline{\beta} - \underline{c}_2)^2, \quad (8)$$

where  $\underline{t}_2 := \int_{\mathbb{R}} t_2(c_2) \cdot g(c_2 - \underline{c}_2) dc_2$ .

Similarly, when the low cost type targets  $\bar{c}_2$ , his effort is given by  $\bar{e}_2 - \Delta\beta = \underline{\beta} - \bar{c}_2$ , and the density of second period costs is given by  $g(c_2 - \bar{c}_2)$ . Thus, his expected utility from targeting  $\bar{c}_2$  is

$$E[U_2 | \bar{c}_2] := \int_{\mathbb{R}} \left[ t_2(c_2) - \frac{\gamma}{2} (\underline{\beta} - \bar{c}_2)^2 \right] g(c_2 - \bar{c}_2) dc_2 = \bar{t}_2 - \frac{\gamma}{2} (\underline{\beta} - \bar{c}_2)^2, \quad (9)$$

where  $\bar{t}_2 := \int_{\mathbb{R}} t_2(c_2) \cdot g(c_2 - \bar{c}_2) dc_2$ . The low cost firm's incentive compatibility constraint makes him indifferent, in expectation, between targeting  $\underline{c}_2$  and  $\bar{c}_2$ :

$$E[U_2 | \underline{c}_2] = E[U_2 | \bar{c}_2] \implies \underline{t}_2 - \frac{\gamma}{2} (\underline{\beta} - \underline{c}_2)^2 = \bar{t}_2 - \frac{\gamma}{2} (\underline{\beta} - \bar{c}_2)^2. \quad (10)$$

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<sup>1</sup>The low cost firm's incentive constraint depends on whether the low cost type's effort from mimicking the high cost type is positive or negative. This issue is addressed shortly.

The second period game is designed to extract all expected rent form the high cost type. When the high cost type targets  $\bar{c}_2$ , his cost of effort is  $\bar{e}_2 = \bar{\beta} - \bar{c}_2$ , and the density of expected costs is given by  $g(c_2 - \bar{c}_2)$ . Thus, the high cost type's expected second period rent is given by

$$E [\bar{U}_2 | \bar{c}_2] := \int_{\mathbb{R}} \left[ t_2(c_2) - \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 \right] g(c_2 - \bar{c}_2) dc_2 = \bar{t}_2 - \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2. \quad (11)$$

Therefore, the high cost type's participation constraint is given by

$$E [\bar{U}_2 | \bar{c}_2] = 0 \implies \bar{t}_2 - \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2 = 0. \quad (12)$$

Simplifying the objective function in (6) and using (10) and (12) to substitute for the expected transfers leaves the following unconstrained problem:

$$\begin{aligned} \max_{\underline{c}_2, \bar{c}_2} \quad & S - \rho_2 \left[ (1 + \lambda) \left( \underline{c}_2 + \frac{\gamma}{2}(\underline{\beta} - \underline{c}_2)^2 \right) + \lambda \left( \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_2)^2 \right) \right] \\ & - (1 - \rho_2)(1 + \lambda) \left( \bar{c}_2 + \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 \right), \end{aligned} \quad (13)$$

where  $\frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_2)^2$  is the low cost firm's expected information rent.

The first order conditions of this problem imply the following equilibrium efforts and cost targets:

$$\underline{e}_2 = \underline{\beta} - \underline{c}_2 = \frac{1}{\gamma}, \quad (14)$$

and

$$\bar{e}_2 = \bar{\beta} - \bar{c}_2 = \frac{1}{\gamma} - \frac{\rho_2}{1 - \rho_2} \frac{\lambda}{1 + \lambda} \Delta\beta. \quad (15)$$

Thus, the low cost type exerts the first best effort in the second period, and the high cost type's effort is distorted away from the first best according to the principal's second period beliefs. Notice that the effort levels given in (14) and (15) correspond to the standard static



game in which beliefs are given by  $\rho_2$ . This illustrates that in a static setting, additive noise has no impact on incentives when the regulator and firm are risk neutral.

One concern in this model is that the low cost firm's effort from mimicking the high cost firm,

$$\bar{e}_2 - \Delta\beta = \underline{\beta} - \bar{c}_2 = \frac{1}{\gamma} - \frac{1 + \lambda - \rho_2}{(1 - \rho_2)(1 + \lambda)} \Delta\beta, \quad (16)$$

can be less than zero for values of  $\rho_2$  close to one. “Negative effort” captures any measures taken to increase rather than decrease the project cost. To understand why the low cost type might have to increase project costs in order to mimic the high cost type, recall that the expected cost for the high cost type are equal to its type minus its cost-reducing effort. When the first period cost observation is low, this leads the regulator to believe that she is very likely to be contracting with the low cost type in the second period. In response, she reduces the effort of the high cost type in order to extract rent from the low cost type. When this effort is small enough (i.e. when second period beliefs are close to one),  $\bar{c}_2 = \bar{\beta} - \bar{e}_2 > \underline{\beta}$ .

This possibility is usually assumed away in static models. However, as  $\varepsilon$  has full support on the real line, it must be considered in this setting. Since  $g$  satisfies the monotone likelihood ratio property, the principal's posterior belief that the firm has low costs is monotone decreasing in first period cost realizations. Therefore, there exists a unique value of  $\rho_2$ , defined

$$\rho_2^0 := \rho_2(c_1^0) = \frac{(1 + \lambda)(1 - \gamma\Delta\beta)}{1 + \lambda - \gamma\Delta\beta} < 1, \quad (17)$$

such that for every  $c_1 \leq c_1^0$ , the low cost type's effort from mimicking the high cost type is negative.

Since the firm cannot experience a dis-utility from negative effort (that is,  $\psi(e_t) = 0$  when  $e_t \leq 0$ ), the low cost type's second period incentive compatibility constraint is written

$$t_2 - \frac{\gamma}{2}(\underline{\beta} - c_2)^2 = \bar{t}_2. \quad (18)$$

The high cost firm's participation constraint remains unchanged. Together, this implies that

the regulator's unconstrained problem when  $c_1 \leq c_1^0$  is given by

$$\begin{aligned} \max_{\underline{c}_2, \bar{c}_2} \quad & S - \rho_2 \left[ (1 + \lambda) \left( \underline{c}_2 + \frac{\gamma}{2} (\underline{\beta} - \underline{c}_2)^2 \right) + \lambda \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2 \right] \\ & - (1 - \rho_2)(1 + \lambda) \left( \bar{c}_2 + \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2 \right), \end{aligned} \quad (19)$$

where the low cost firm's expected information rent is now given by  $\frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2$ .

The first order condition for this problem with respect to  $\bar{c}_2$  implies the following equilibrium effort for the high cost type (the low cost type still exerts the first best effort):

$$\bar{e}_2^0 = \bar{\beta} - \bar{c}_2 = \frac{1}{\gamma} \frac{(1 - \rho_2)(1 + \lambda)}{1 + \lambda - \rho_2}. \quad (20)$$

The following proposition summarizes the second period game:

**Proposition 1.** *When  $c_1 > c_1^0$ , the regulator's problem is given by (13), while for  $c_1 \leq c_1^0$ , the regulator's problem is given by (19). The first order conditions of (13) and (19) with respect to  $\underline{c}_2$  and  $\bar{c}_2$  imply that the low cost firm's equilibrium expected rent is given by*

$$\underline{U}_2(\rho_2) = \begin{cases} \frac{\gamma}{2} (\bar{e}_2)^2 - \frac{\gamma}{2} (\bar{e}_2 - \Delta\beta)^2 =: \underline{u}_2, & \text{if } c_1 > c_1^0, \\ \frac{\gamma}{2} (\bar{e}_2^0)^2 =: \underline{u}_2^0, & \text{if } c_1 \leq c_1^0, \end{cases} \quad (21)$$

where  $\bar{e}_2$  is given in (15),  $\bar{e}_2 - \Delta\beta$  in (16), and  $\bar{e}_2^0$  in (20). Similarly, equilibrium expected second period welfare is given by

$$W_2(\rho_2) = \begin{cases} S - \rho_2 \left[ (1 + \lambda) \left( \underline{\beta} - \frac{1}{2\gamma} \right) + \lambda \underline{u}_2 \right] - (1 - \rho_2)(1 + \lambda) \left( \bar{\beta} - \bar{e}_2 + \frac{\gamma}{2} (\bar{e}_2)^2 \right) =: w_2, \\ S - \rho_2 \left[ (1 + \lambda) \left( \underline{\beta} - \frac{1}{2\gamma} \right) + \lambda \underline{u}_2^0 \right] - (1 - \rho_2)(1 + \lambda) \left( \bar{\beta} - \bar{e}_2^0 + \frac{\gamma}{2} (\bar{e}_2^0)^2 \right) =: w_2^0, \end{cases} \quad (22)$$

when  $c_1$  is greater than  $c_1^0$  and less than  $c_1^0$ , respectively.

Regardless of the size of  $c_1$ , the second period game exhibits the classic rent extrac-

tion/efficiency trade-off present in static adverse selection models:

$$\frac{dU_2(\rho_2)}{d\rho_2} = \begin{cases} \frac{du_2}{d\bar{e}_2} \frac{d\bar{e}_2}{d\rho_2} = \frac{-1}{(1-\rho_2)^2} \frac{\lambda}{1+\lambda} \gamma \Delta \beta^2 < 0, & \text{if } c_1 > c_1^0, \\ \frac{du_2^0}{d\bar{e}_2^0} \frac{d\bar{e}_2^0}{d\rho_2} = \frac{-\lambda(1+\lambda)^2}{\gamma} \frac{1-\rho_2}{(1+\lambda-\rho_2)^3} < 0, & \text{if } c_1 \leq c_1^0. \end{cases} \quad (23)$$

This is an important consideration for the regulator in the first period, since  $\rho_2$  is a function of  $\underline{c}_1$  and  $\bar{c}_1$ .

To see how second period beliefs, and thus second period welfare, depend on the first period contract, consider  $\tilde{c}_1 = \underline{c}_1 + x$ , for some fixed value  $x$ . From (5), the closer together are  $\underline{c}_1$  and  $\bar{c}_1$ , the closer together are the values of  $\underline{g}(\tilde{c}_1)$  and  $\bar{g}(\tilde{c}_1)$ . The closer together are  $\underline{g}(\tilde{c}_1)$  and  $\bar{g}(\tilde{c}_1)$ , the closer  $\rho_2$  is to the prior,  $\rho$ ; indeed, if  $\underline{c}_1 = \bar{c}_1$ , then  $\underline{g}(\tilde{c}_1) = \bar{g}(\tilde{c}_1)$  for all  $x$ , and the posterior is equal to the prior. Conversely, the further apart are  $\underline{c}_1$  and  $\bar{c}_1$ , the smaller is  $\bar{g}(\tilde{c}_1)$  relative to  $\underline{g}(\tilde{c}_1)$ , and the closer the posterior is to one.

Thus, the distance between first period cost targets directly influences how much the regulator updates her prior, given a first period cost realization. The further apart are the first period cost targets, the more accurate are the regulator's second period beliefs; the more accurate are the regulator's second period beliefs, the closer second period welfare is to the first-best. However, this information comes at a cost. Since the low cost firm's second period rent is decreasing in  $\rho_2$ , spreading the cost targets apart decreases (in expectation) the low cost firm's rent from targeting  $\underline{c}_1$ , and increases his rent from targeting  $\bar{c}_1$  in the first period. This increases the low cost type's first period transfer. Thus, the regulator faces a tradeoff between increasing the expected second period welfare *or* preserving the low cost firm's expected second period rent.

## 4 First period

The second period beliefs,  $\rho_2$ , serve as the link between the first and second period contracts. When choosing the first period cost targets, the regulator considers not only the impact that

they have on first period welfare, but what impact they have on expected second period welfare as well. The regulator's first period problem is to maximize the expectation of first and (discounted) second period welfare, subject to incentive compatibility and participation constraints, which are derived below:

$$\begin{aligned} \max_{\underline{c}_1, \bar{c}_1} \quad & S - \rho \int_{\mathbb{R}} \left[ (1 + \lambda) (c_1 + t_1(c_1)) + t_1(c_1) - \frac{\gamma}{2} (\underline{\beta} - \underline{c}_1)^2 \right] g(c_1 - \underline{c}_1) dc_1 \\ & - (1 - \rho) \int_{\mathbb{R}} \left[ (1 + \lambda) (c_1 + t_1(c_1)) + t_1(c_1) - \frac{\gamma}{2} (\bar{\beta} - \bar{c}_1)^2 \right] g(c_1 - \bar{c}_1) \\ & + \delta E[W_2(\rho_2)], \end{aligned} \quad (24)$$

where  $W_2(\rho_2)$  is given in (22), and

$$E[W_2(\rho_2)] = \int_{\mathbb{R}} W_2(\rho_2) [\rho g(c_1 - \underline{c}_1) + (1 - \rho)g(c_1 - \bar{c}_1)] dc_1. \quad (25)$$

A well known issue in dynamic games is that the first period payment to the low cost firm may be so large that the high cost type's incentive compatibility constraint binds (the so-called “take the money and run” strategy). For now, consider the low cost firm's incentive compatibility constraint and the high cost firm's participation constraint.<sup>2</sup> The low cost firm's incentive constraint requires that his expected utility from targeting  $\underline{c}_1$  equal his expected utility from targeting  $\bar{c}_1$ . That is,

$$\begin{aligned} E[\underline{U}_1 | \underline{c}_1] &:= \int_{\mathbb{R}} \left[ t_1(c_1) - \frac{\gamma}{2} (\underline{\beta} - \underline{c}_1)^2 + \delta \underline{U}_2(\rho_2) \right] \underline{g} dc_1 \\ &= \int_{\mathbb{R}} \left[ t_1(c_1) - \frac{\gamma}{2} (\underline{\beta} - \bar{c}_1)^2 + \delta \underline{U}_2(\rho_2) \right] \bar{g} dc_1 =: E[\underline{U}_2 | \bar{c}_2], \end{aligned} \quad (26)$$

where  $\underline{g} := g(c_1 - \underline{c}_1)$  and  $\bar{g} := g(c_1 - \bar{c}_1)$ . The left hand side of (26) is the low cost firm's expected utility when he targets  $\underline{c}_1$  in the first period. He exerts effort  $\underline{e}_1 = \underline{\beta} - \underline{c}_1$ , and receives an expected first period transfer and expected second period rent, where expectations

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<sup>2</sup>In the Appendix it is shown that in sufficiently noisy environments, the high cost firm's incentive constraint is slack.

are taken over the real line according to the density  $\underline{g}$ . If the low cost firm instead chooses to target  $\bar{c}_1$ , he experiences a disutility from effort  $\bar{e}_1 - \Delta\beta = \underline{\beta} - \bar{c}_1$ , and receives an expected first period transfer and expected second period rent. These expectations are taken according to the density  $\bar{g}$ .

From the perspective of the high cost firm, the first period game is essentially static since the second period game extracts all the rent from the high cost type. Therefore, the high cost firm's participation constraint requires that his expected first period utility from targeting  $\bar{c}_1$  be equal to his outside option of zero:

$$E[\bar{U}_1 | \bar{c}_1] := \int_{\mathbb{R}} \left[ t_1(c_1) - \frac{\gamma}{2} (\bar{\beta} - \bar{c}_1)^2 \right] \bar{g} dc_1 = 0. \quad (27)$$

By defining  $\underline{t}_1$  and  $\bar{t}_1$  analogously to  $\underline{t}_2$  and  $\bar{t}_2$ , one can simplify (26) and (27) and solve for the low cost firm's expected first period transfer:

$$\underline{t}_1 = \frac{\gamma}{2} (\underline{\beta} - \underline{c}_1)^2 + \frac{\gamma}{2} (\bar{\beta} - \bar{c}_1)^2 - \frac{\gamma}{2} (\underline{\beta} - \bar{c}_1)^2 + \delta \int_{\mathbb{R}} \underline{U}_2(\rho_2) (\bar{g} - \underline{g}) dc_1. \quad (28)$$

The first three terms on the right hand side of (28) comprise the familiar static transfer: the low cost firm must be compensated for the cost of its effort, and also for the ability to “hide behind” the high cost firm.

In dynamic games, there is an additional component of the low cost firm's first period transfer. Because the density of noise,  $g$ , satisfies the monotone likelihood ratio property, the distribution of costs induced by targeting  $\bar{c}_1$  first order stochastically dominates the distribution induced by targeting  $\underline{c}_1$ . Therefore, the low cost firm enjoys a higher expected second period rent when he targets  $\bar{c}_1$  than he does when he targets  $\underline{c}_1$ .<sup>3</sup> The first period transfer must compensate him for this opportunity cost to induce him to target  $\underline{c}_1$ .

In a deterministic setting, unless the the firm cares little about the future (i.e., the firm heavily discounts future payoffs), this additional component of the low cost firm's first period

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<sup>3</sup>That is, because  $g$  satisfies the monotone likelihood ratio property,  $\int_{\mathbb{R}} \underline{U}_2(\rho_2) (\bar{g} - \underline{g}) dc_1 > 0$ .

transfer can make it impossible to induce a separating equilibrium. To see this, recall that in a deterministic setting, the firm has perfect control over the project's cost. Suppose the regulator's contract specifies that the high and low cost firms complete the project at different cost levels. If the firm accepts such a contract, his actions perfectly reveal his type to the regulator; information revelation in a deterministic separating equilibrium is an "all-or-nothing" proposition.

Thus, when the low cost firm follows the equilibrium in the first period, the regulator believes with probability one that she is contracting with the low cost type in the second period, and he is held to his reservation utility. Further, when the low cost firm takes out-of-equilibrium actions in the first period and mimics the high cost firm, at the beginning of the second period the regulator believes the firm to be the high cost type. In this case the low cost firm enjoys his highest possible second period information rent,  $\underline{U}_2(0)$ . To induce him to target  $\underline{c}_1$ , the principal must increase the low cost firm's first period transfer by  $\delta \underline{U}_2(0)$ .

This rationale changes in a stochastic setting. First, simply by following the equilibrium and targeting  $\underline{c}_1$  in the first period, the low cost firm enjoys expected second period rent

$$\int_{\mathbb{R}} \underline{U}_2(\rho_2) \underline{g} dc_1 > 0. \quad (29)$$

Second, the low cost firm's gains from mimicking the high cost firm are diminished. Suppose the low cost firm deviates and targets  $\bar{c}_1$  in the second period. The corresponding density of first period costs is  $\bar{g}$ , so that the low cost firm's expected second period rent from targeting  $\bar{c}_1$  is

$$\int_{\mathbb{R}} \underline{U}_2(\rho_2) \bar{g} dc_1 < \int_{\mathbb{R}} \underline{U}_2(0) \bar{g} dc_1 = \underline{U}_2(0). \quad (30)$$

Therefore, the additional component of the low cost firm's first period transfer is smaller in a stochastic setting than it is in a deterministic environment.

To proceed with the principal's first period problem, consider the following assumption:

**Assumption 1.** *The single crossing property holds in the first period. That is,*

$$\begin{aligned}\gamma(\bar{\beta} - c) &\geq \gamma(\underline{\beta} - c) + \delta \int_{\mathbb{R}} \frac{d\underline{U}_2}{d\rho_2} \frac{d\rho_2}{dc_1} g(c_1 - c) dc_1 \\ \implies \gamma\Delta\beta &\geq \delta \int_{\mathbb{R}} \frac{d\underline{U}_2}{d\rho_2} \frac{d\rho_2}{dc_1} g(c_1 - c) dc_1.\end{aligned}\tag{31}$$

The single crossing assumption guarantees a regular first period problem by ensuring that the high cost type's marginal cost of decreasing the cost target  $c$  is higher than the low cost type's marginal cost of decreasing the cost target for every  $c$ . From (31), this condition is satisfied when  $\frac{d\rho_2}{dc_1}$  is small, i.e. when the posterior beliefs are not too sensitive to changes in first period cost. Since the magnitude of  $\frac{d\rho_2}{dc_1}$  depends on the slope of the density, and the slope of the density goes to zero when the variance is large, this condition is satisfied in sufficiently noisy environments. The single crossing condition is also more likely to be satisfied when the difference between the low and high cost firm's intrinsic cost levels,  $\Delta\beta$ , is large.

**Proposition 2.** *The regulator's full first period problem is given by*

$$\begin{aligned}\max_{\underline{c}_1, \bar{c}_1} \quad & S - \rho \left[ (1 + \lambda) \left( \underline{c}_1 + \frac{\gamma}{2}(\underline{\beta} - \underline{c}_1)^2 \right) + \lambda \left( \frac{\gamma}{2}(\bar{\beta} - \bar{c}_1)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_1)^2 + \delta \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1 \right) \right] \\ & - (1 - \rho)(1 + \lambda) \left( \bar{c}_1 + \frac{\gamma}{2}(\bar{\beta} - \bar{c}_1)^2 \right) + \delta E[W_2(\rho_2)],\end{aligned}\tag{32}$$

where  $E[W_2(\rho_2)]$  is given in (25). The first order conditions imply the following first period efforts (and cost targets):

$$\underline{c}_1 = \underline{\beta} - \underline{c}_1 = \frac{1}{\gamma} + \frac{\delta}{\gamma\rho(1 + \lambda)} \frac{d}{d\underline{c}_1} \left[ \rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1 - E[W_2] \right],\tag{33}$$

and

$$\begin{aligned}\bar{e}_1 = \bar{\beta} - \bar{c}_1 = & \frac{1}{\gamma} - \frac{\rho\lambda}{(1-\rho)(1+\lambda)}\Delta\beta \\ & + \frac{\delta}{\gamma(1-\rho)(1+\lambda)}\frac{d}{d\bar{c}_1}\left[\rho\lambda\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 - E[W_2]\right].\end{aligned}\quad (34)$$

If the regulator were able to commit to the first and second period cost targets at the outset of her relationship with the firm, she would implement the same contract in each period. In Periods 1 and 2, the low cost agent exerts the first best level of effort,

$$\underline{e}^c = \underline{e}^* = \frac{1}{\gamma}.\quad (35)$$

The high cost firm's effort distortion remains the same in Periods 1 and 2:

$$\bar{e}^c = \frac{1}{\gamma} - \frac{\rho\lambda}{(1-\rho)(1+\lambda)}\Delta\beta.\quad (36)$$

Comparing (35) to (33) and (36) to (34), one can see that each type of firm's effort is distorted away from the commitment optimum. Whether the low cost firm exerts more or less effort than in the commitment optimum depends on how the additional component of the low cost firm's first period transfer and expected second period welfare change with the low cost firm's first period cost target.

In particular, if

$$\frac{d}{d\bar{c}_1}\left[\rho\lambda\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 - E[W_2]\right] < 0,\quad (37)$$

the low cost firm exerts less effort in the first period than he does in the second period. To see this, recall that the second period game is static. In a static game, the low cost firm exerts the first best effort. The low cost firm also exerts the first best effort in every period when the principal can commit. Therefore, if the low cost firm's first period effort, given in (33), is less than the commitment effort given in (35), then his first period effort is lower



than his effort in the second period.

This case is of particular interest in light of the discussion of the ratchet effect in the introduction. If  $\underline{e}_1 < \underline{e}_2$ , then the theoretical predictions of this paper match with anecdotal, experimental and empirical evidence which shows that high ability agents decrease their effort at the beginning of their relationship with a principal to maintain information rents in the future.

In order to explore this, we consider the competing incentives discussed at the outset: the desire to reduce up-front payments to the low-cost firm that are required to induce the equilibrium, and principal's desire to learn in order to reduce distortions and rents in the second period. We consider each of these in turn.

## 4.1 Signal dampening

Recall that the low cost firm's expected second period rent is higher when he targets  $\bar{c}_1$  than it is when he targets  $\underline{c}_1$ . The additional component of the low cost type's first period transfer,

$$\delta \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1, \quad (38)$$

compensates him for this difference in expected second period rents. Without this additional component, the principal cannot induce the low cost firm to target  $\underline{c}_1$ . Clearly, the larger is (38), the larger is the low cost firm's first period transfer, given in (28). This subsection demonstrates that the principal can decrease (38), and thus decrease the low cost firm's expected first period transfer, by reducing the distance between the first period cost targets.

The intuition for this argument is simple. Because the density of noise satisfies the monotone likelihood ratio property, the principal's belief that the firm is the low cost type is monotone decreasing in the first period cost realization. That is, the higher is the first period cost, the lower is the principal's second period belief that the firm is the low cost type.

The lower is the principal's belief that the firm is the low cost type, the more effort the high cost firm exerts in the second period. The more effort that the high cost firm exerts, the higher is the low cost firm's information rent. Thus, the less the principal's second period beliefs change depending on which cost level the firm targets, the lower is the low cost firm's incentive to mimic the high cost firm. To see this, consider Figure 1.

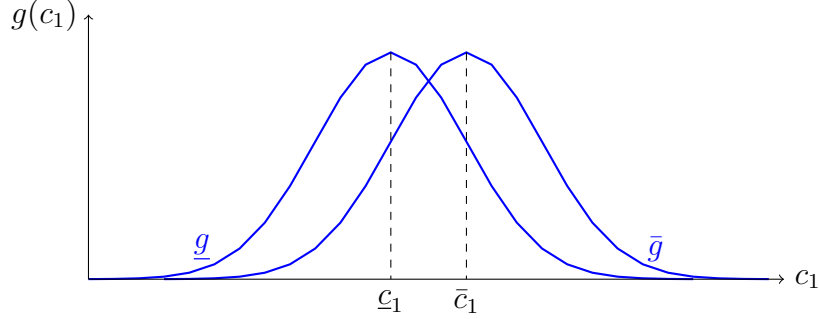


Figure 1: The probability density of costs depends on the agent's effort choice

When the firm targets  $\underline{c}_1$ , the density of first period costs is given by  $\underline{g}$  in Figure 1. Similarly, when the firm targets  $\bar{c}_1$ , the density of first period costs is  $\bar{g}$ . The closer together are  $\underline{c}_1$  and  $\bar{c}_1$ , the closer together are the values of  $\underline{g}$  and  $\bar{g}$  for any given first period cost realization. The closer together are the values of  $\underline{g}$  and  $\bar{g}$ , the closer second period beliefs (given in (5)) are to the prior,  $\rho$ .

The less the regulator updates her beliefs for any given first period cost realization, the closer is the low cost firm's expected second period rent from targeting  $\underline{c}_1$  compared to when he deviates and targets  $\bar{c}_1$ . This decreases the low cost type's incentives to mimic the high cost type in the first period, which reduces the low cost type's first period transfer, and thus alleviates the first period incentive problem.

The following proposition formalizes this logic by, for the time being, abstracting from the impacts of the first period contract on expected second period welfare. The proof makes use of the connection between effort and cost targets; an increased cost target implies a decrease in effort, and vice-versa. The proof formalizes the intuition that the regulator can decrease the low cost firm's first period transfer by decreasing the distance between  $\bar{c}_1$  and

$\underline{c}_1$ . To do this, the proof shows that the first period transfer is decreasing in  $\underline{c}_1$  and increasing in  $\bar{c}_1$ . This equilibrium transfer effect decreases (increases) the low cost (high cost) type's equilibrium first period effort.

**Proposition 3.** *The effect of the dynamic portion of the low cost firm's first period transfer is to decrease (increase) the low cost (high cost) firm's first period effort. That is,*

$$\frac{d}{d\underline{c}_1} \left[ \rho \lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) d c_1 \right] < 0, \quad (39)$$

and

$$\frac{d}{d\bar{c}_1} \left[ \rho \lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) d c_1 \right] > 0. \quad (40)$$

The proof of Proposition 3 (found in the Appendix) establishes that even though the regulator cannot commit to ignore information she learns about the firm when designing the second period contract, in a stochastic environment the regulator can commit to learn less via her choice of first period cost targets. Doing so preserves the low cost firm's equilibrium expected second period rent and decreases his gains from deviation, which in turn decreases his first period transfer, alleviating the dynamic incentive problem.

Tying cost targets to efforts also allows a discussion of how the ratchet effect behaves in a stochastic setting versus a deterministic one. In a deterministic separating equilibrium, the high cost type has his effort decreased over time, while the low cost type always exerts the first best effort. As Proposition 3 shows, and as the above intuition argues, in a stochastic setting the regulator distorts the efforts of both types of firm in the first period, as opposed to just the high cost firm. In particular, to decrease the low cost firm's first period transfer, the principal decreases the low cost type's effort, and increases the high cost type's effort, relative to the commitment optimum.

## 4.2 Experimentation

Proposition 3 establishes that the regulator has an incentive to restrict how much information she gathers about the firm. However, an opposing incentive exists as well. The more the regulator learns about the firm's type by observing the first period project cost, the better she can tailor the second period contract to the firm's type. The stronger is the regulator's belief that the firm is the low cost type (i.e., the closer  $\rho_2$  is to one), the lower is the high cost agent's effort. This extracts rent from the low cost firm in the second period. The stronger is the regulator's belief that the firm is the high cost type (i.e., the closer  $\rho_2$  is to zero), the higher is the high cost type's cost-reducing effort.

Thus, the better is the principal's information in the second period, the less-distortionary is the high cost firm's effort in the second period. This improves expected second period welfare by either inducing more cost-reducing effort from the high cost firm or extracting more rent from the low cost firm. The following lemma establishes that information about the firm's type is valuable to the regulator in the second period.<sup>4</sup>

**Lemma 1.** *Information is valuable. That is, expected second period welfare is convex in second period beliefs:*

$$\frac{d^2 W_2(\rho_2)}{d\rho_2^2} > 0. \quad (41)$$

The proof of Lemma 1 is a straightforward envelope theorem argument, which is given in the Appendix. Given that information is valuable, one can show that the regulator increases expected second period welfare,  $E[W_2(\rho_2)]$ , by increasing the distance between first period cost targets.

To see the intuition for this result, return attention to Figure 1. As the distance between first period cost targets grows, so does the difference between the value of  $\underline{g}$  and  $\bar{g}$  for any given first period cost realization. The further apart are the values of  $\underline{g}$  and  $\bar{g}$ , the more the regulator updates her prior beliefs for any given first period cost realization.

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<sup>4</sup>Information is valuable in the sense of Blackwell (1951).

Thus, the information asymmetry between the regulator and the firm in the second period diminishes with the distance between first period cost targets. Since welfare distortions in the second period arise because of asymmetric information, an increase in the distance between  $\underline{c}_1$  and  $\bar{c}_1$  increases expected second period welfare.

This incentive to manipulate first period cost targets to increase how much the principal learns about the agent's type can be interpreted in terms of equilibrium first period efforts. As the following proposition shows, the principal increases expected second period welfare by increasing the low cost firm's effort, and decreasing the high cost firm's effort.

**Proposition 4.** *The effect of expected second period welfare is to increase (decrease) the low cost (high cost) firm's first period effort. That is,*

$$\frac{dE[W_2(\rho_2)]}{d\underline{c}_1} < 0, \quad (42)$$

and

$$\frac{dE[W_2(\rho_2)]}{d\bar{c}_1} > 0 \quad (43)$$

The proof of Proposition 4 (also in the Appendix) establishes that the principal increases expected second period welfare by increasing the distance between the first period cost targets. Since the game ends after the second period interaction, the only welfare distortions in the second period arise because of the presence of asymmetric information (i.e., there are no dynamic considerations as there are in the first period). Thus, any measures the regulator can take to decrease the information asymmetry in the first period, increase expected second period welfare.

## 5 Equilibrium ratchet effect

The analysis has shown that two opposing incentives determine the optimal first period contract. To decrease the low cost firm's first period transfer, the regulator must decrease

the distance between the first period cost targets, and restrict how much she learns about the firm's type. To increase expected second period welfare, the regulator must increase the distance between first period cost targets, and increase how much she learns about the firm's type.

To determine the combined effect of these competing incentives on the first period cost targets, consider the following re-formulation of the regulator's first period problem:

$$\begin{aligned}
\max_{\underline{c}_1, \bar{c}_1} \quad & S - \rho \left[ (1 + \lambda) \left( \underline{c}_1 + \frac{\gamma}{2} (\underline{\beta} - \underline{c}_1)^2 \right) + \lambda \left( \frac{\gamma}{2} (\bar{\beta} - \bar{c}_1)^2 - \frac{\gamma}{2} (\underline{\beta} - \bar{c}_1)^2 \right) \right] \\
& - (1 - \rho)(1 + \lambda) \left( \bar{c}_1 + \frac{\gamma}{2} (\bar{\beta} - \bar{c}_1)^2 \right) + \delta \left[ \rho \underline{w}^{FB} + (1 - \rho) \left( \bar{w}^{FB} - \frac{1 + \lambda}{2\gamma} \right) \right] \\
& + \delta \int_{-\infty}^{\underline{c}_1^0} \left\{ (1 - \rho)(1 + \lambda) \bar{e}_2^0 - (1 + \lambda - \rho) \frac{\gamma}{2} (\bar{e}_2^0)^2 \right\} \bar{g} dc_1 \\
& + \delta \int_{\underline{c}_1^0}^{\infty} \left\{ (1 - \rho)(1 + \lambda) \bar{e}_2 - (1 + \lambda - \rho) \frac{\gamma}{2} \bar{e}_2^2 + \rho \lambda \frac{\gamma}{2} (\bar{e}_2 - \Delta \beta)^2 \right\} \bar{g} dc_1. \quad (44)
\end{aligned}$$

Note that  $\underline{w}^{FB} = S - (1 + \lambda) \left( \underline{\beta} - \frac{1}{2\gamma} \right)$  and  $\bar{w}^{FB} = S - (1 + \lambda) \left( \bar{\beta} - \frac{1}{2\gamma} \right)$  are the first best welfare for the low and high cost firm, respectively.

In (44), the expected transfers have already been substituted using the low cost firm's incentive constraint and the high cost firm's participation constraint. The second period welfare distortions (how much rent to leave the low cost firm and how much effort to induce in the high cost firm) are captured by the two integrals. Recall that the high cost firm's second period effort determines how much rent is left to the low cost firm. Now, define

$$A := (1 - \rho)(1 + \lambda) \bar{e}_2^0 - (1 + \lambda - \rho) \frac{\gamma}{2} (\bar{e}_2^0)^2, \quad (45)$$

and

$$B := (1 - \rho)(1 + \lambda) \bar{e}_2 - (1 + \lambda - \rho) \frac{\gamma}{2} \bar{e}_2^2 + \rho \lambda \frac{\gamma}{2} (\bar{e}_2 - \Delta \beta)^2. \quad (46)$$

The first order conditions of this problem imply the following effort levels for the low and

high cost firm:

$$\underline{e}_1 = \underline{\beta} - \underline{e}_1 = \frac{1}{\gamma} - \frac{\delta}{\rho(1+\lambda)\gamma} \frac{d}{d\underline{e}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right], \quad (47)$$

$$\begin{aligned} \bar{e}_1 = \bar{\beta} - \bar{e}_1 = & \frac{1}{\gamma} - \frac{\rho\lambda}{(1-\rho)(1+\lambda)} \Delta\beta \\ & - \frac{\delta}{(1-\rho)(1+\lambda)\gamma} \frac{d}{d\bar{e}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right]. \end{aligned} \quad (48)$$

Again, the equilibrium efforts in (47) and (48) are distorted relative to the commitment optimum targets in (35) and (36). The overall effect of the first period contract is to restrict how much the regulator learns about the firm's type if  $\bar{e}^c < \bar{e}_1 < \underline{e}_1 < \underline{e}^c$ , and to increase learning if  $\bar{e}_1 < \bar{e}^c < \underline{e}^c < \underline{e}_1$ .

Here we show that the net effect of the two competing incentives is to restrict learning; that is, the low cost firm has his effort increased over the course of his interaction with the regulator. This result that an agent with favorable private information increases his effort over time is in contrast with the deterministic theoretical analysis, but comports with anecdotal, experimental, and empirical evidence of the ratchet effect. Anecdotal evidence of piece-rate factory workers documented that skilled workers learned to restrict their output in order to avoid either an increase in their output quotas or a decrease in their piece rates (see, e.g., Matthewson (1931), Clawson (1980) Montgomery (1979) and Roy (1952)). In experimental settings that study two-period principal agent interactions, high ability workers restrict their output (reduce their effort) in the first period to maintain a second period information rent (see Charness et al. (2011) and Cardella and Depew (2018)). Empirical studies of the ratchet effect show that teachers reduce their effort on improving student's standardized test scores when their compensation in the future depends on their student's scores today (Macartney (2016)).

With this discussion on the relevance of the ratchet effect in mind, we find:

**Proposition 5. *The Ratchet Effect:*** *If the low cost firm's second period effort from mimicking the high cost firm is positive for all first period cost realizations  $c_1 \geq \underline{c}_1$ , then the low cost firm has his effort increased over the course of the relationship with the regulator. That is,*

$$\frac{d}{d\underline{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right] > 0, \quad (49)$$

$$\frac{d}{d\bar{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right] < 0. \quad (50)$$

The important implication of Proposition 5 is that the low cost firm's first period effort, given in (47), is less than his effort when the regulator can commit, (35). Since the low cost firm exerts the first best effort in the first period when the principal can commit, and he exerts the first best effort in the second period regardless of the principal's commitment powers, this implies that the low cost firm's effort increases over time. Put differently, compared to the second period, the low-cost agent decreases his effort in the first period.

Since the low cost firm's first period effort is less than in the commitment optimum and the high cost firm's effort is greater than in the commitment optimum, the first period cost targets are closer together than the commitment optimum targets. Therefore, the optimal first period contract favors reducing the first period transfer to the low cost firm at the expense of having worse information about the firm's type in the second period.

Proposition 5 requires that the low cost firm's effort from mimicking the high cost firm in the second period be positive for all first period cost realizations greater than the low cost firm's first period cost target. Recall from the discussion of the second period game that there exists a unique first period cost realization,  $c_1^0$ , such that for all  $c_1 \leq c_1^0$ , the low cost firm's effort from mimicking the high cost firm in the second period is negative, and for all  $c_1 > c_1^0$  the low cost firm exerts positive effort to mimic the high cost firm. Therefore, Proposition 5 requires that  $c_1^0$  be less than or equal to the low cost firm's first period cost



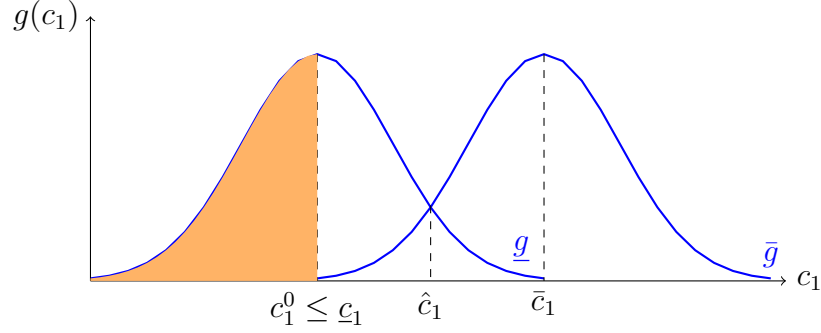


Figure 2: If  $c_1^0$  lies in the shaded region, the low cost firm has his effort increased over time target.

Figure 2 illustrates the restriction that Proposition 5 places on  $c_1^0$ , which we consider to be natural. Suppose that  $c_1^0 > \underline{c}_1$ . This implies that for some cost realizations greater than the low cost firm's cost target,  $\rho_2$  is close enough to one that the high cost firm's second period effort is close to zero. When the high cost firm's effort is close to zero, the low cost firm has to increase costs above its intrinsic cost level,  $\underline{\beta}$ , to mimic the high cost firm.

Under the conditions outlined in Proposition 5, the value of learning is decreased in a repeated relationship; not only is the regulator content to have imperfect information in the second period, but she chooses to learn less than she could by implementing the commitment optimum. This is because the benefit of better information in the second period does not outweigh the concomitant increase in the low cost type's expected first period transfer.

## 6 Conclusion

In this two-period model of regulation, the regulator and the firm contract over the completion of a socially valuable project. The firm has private information about its intrinsic cost level, which can be high or low, and has imperfect control over the project's final cost (costs are stochastic). Due to the noise in the environment learning is impeded and the regulator determines how much information she gleans about the firm's type via her choice of first period cost targets.

The regulator can gather more information about the firm by increasing the distance between first period cost targets. The better the regulator's information is about the firm's type in the second period, the higher is expected second period welfare. Conversely, the regulator gathers less information about the firm by decreasing the distance between first period cost targets. The less the regulator learns about the firm's type, the higher is the low cost firm's equilibrium expected second period rent, and the lower is its benefit from mimicking the high cost firm. Thus, by decreasing the distance between first period cost targets, the regulator decreases the low cost firm's first period transfer.

Given a natural restriction on the regulator's second period beliefs, the net effect of the first period contract is to decrease the distance between the first period cost targets. Thus, the regulator's desire to reduce the first period transfer is stronger than her desire to improve expected second period welfare.

This implies that the low cost type exerts less than the first-best effort in the first period, and has his effort ratcheted up over the course of his interaction with the regulator. Anecdotal, experimental and empirical evidence of the ratchet effect suggests that agents with favorable private information preserve their future information rents by taking actions to keep this information private. Thus, the prediction that the low cost firm increases his effort over time aligns closely with observed repeated principal-agent interactions.

## Appendix

### High cost type's first period incentive constraint

Given the expression for the low and high cost firm's equilibrium efforts, one can verify that the high cost firm's incentive constraint is satisfied in sufficiently noisy environments. Since the high cost type's participation constraint binds in expectation, it is sufficient to check that

$$t_1 - \frac{\gamma}{2} (\bar{\beta} - \underline{c}_1)^2 \leq 0. \quad (51)$$

Substituting for  $\underline{t}_1$  from (28) and simplifying, this requires

$$\frac{\delta}{\gamma\Delta\beta} \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 \leq \bar{c}_1 - \underline{c}_1. \quad (52)$$

Now, from (33) and (34),<sup>5</sup>

$$\begin{aligned} \bar{c}_1 - \underline{c}_1 &= \frac{1 + \lambda - \rho}{(1 - \rho)(1 + \lambda)} \Delta\beta \\ &\quad + \frac{\delta}{\gamma\rho(1 - \rho)(1 + \lambda)} \frac{d}{d\underline{c}_1} \left[ \rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 - E[W_2] \right]. \end{aligned} \quad (54)$$

Thus, the high cost firm's incentive constraint is satisfied when

$$\begin{aligned} \frac{1 + \lambda - \rho}{(1 - \rho)(1 + \lambda)} \Delta\beta &\geq \frac{\delta}{\gamma\rho(1 - \rho)(1 + \lambda)} \frac{d}{d\underline{c}_1} E[W_2] \\ &\quad - \frac{\delta\lambda}{\gamma(1 - \rho)(1 + \lambda)} \frac{d}{d\underline{c}_1} \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 \\ &\quad + \delta \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1. \end{aligned} \quad (55)$$

From Proposition 4,  $\frac{d}{d\underline{c}_1} E[W_2] < 0$ . Therefore, it must be checked that when the variance is sufficiently large,

$$- \frac{\delta\lambda}{\gamma(1 - \rho)(1 + \lambda)} \frac{d}{d\underline{c}_1} \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 + \delta \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 \approx 0. \quad (56)$$

From Proposition 3,

$$\frac{d}{d\underline{c}_1} \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 = \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2} k [\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'] d\underline{c}_1 + \int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2} k [\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'] d\underline{c}_1. \quad (57)$$

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<sup>5</sup>And using the fact that

$$\frac{d}{d\underline{c}_1} \left[ \rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 - E[W_2] \right] = - \frac{d}{d\underline{c}_1} \left[ \rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 - E[W_2] \right] \quad (53)$$

As the variance of first period cost increases, the slope of the density goes to zero. As the slope of the density goes to zero, so too does  $\frac{d}{dc_1} \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1$ .

Turning attention to  $\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1$ , integration by parts yields

$$\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 = - \left[ \int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2} \frac{d\rho_2}{dc_1} [\bar{G} - \underline{G}] dc_1 + \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2} \frac{d\rho_2}{dc_1} [\bar{G} - \underline{G}] dc_1 \right]. \quad (58)$$

Since  $\frac{d\rho_2}{dc_1} = \frac{\rho(1-\rho)[\underline{g}'\bar{g} - \underline{g}\bar{g}']}{D^2}$  goes to zero as the slope of the density goes to zero, this term is close to zero when the variance is large. Thus, the high cost type's incentive constraint is satisfied in noisy enough environments.

### Proof of Proposition 3

**Proposition.** *The effect of the dynamic portion of the low cost firm's first period transfer is to decrease (increase) the low cost (high cost) firm's first period effort. That is,*

$$\frac{d}{dc_1} \left[ \rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 \right] < 0, \quad (59)$$

and

$$\frac{d}{dc_1} \left[ \rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 \right] > 0. \quad (60)$$

*Proof.* Consider the expression for the low cost type's first period effort given by (33). Abstracting from the effect of the first period contract on expected second period welfare, the low cost type's equilibrium first period effort is less than in a deterministic separating equilibrium (that is, less than  $\frac{1}{\gamma}$ , the first best) when (59) is true. To show that (59) holds, consider

$$\begin{aligned} \frac{d}{dc_1} \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 &= \frac{d}{dc_1} \left[ \int_{-\infty}^{c_1^0} \underline{u}_2^0(\bar{g} - \underline{g})dc_1 + \int_{c_1^0}^{\infty} \underline{u}_2(\bar{g} - \underline{g})dc_1 \right] \\ &= \int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2} \frac{d\rho_2}{dc_1} (\bar{g} - \underline{g}) + \underline{u}_2^0 \underline{g}' dc_1 + \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2} \frac{d\rho_2}{dc_1} (\bar{g} - \underline{g}) + \underline{u}_2 \underline{g}' dc_1. \end{aligned} \quad (61)$$

Integrate the second term under each integral on the right hand side of (61) by parts. Doing so yields

$$\begin{aligned} \int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2} \left[ \frac{d\rho_2}{d\underline{c}_1} \bar{g} - \left( \frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1} \right) \underline{g} \right] dc_1 + \underline{u}_2^0 \underline{g} \Big|_{-\infty}^{c_1^0} \\ + \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2} \left[ \frac{d\rho_2}{d\underline{c}_1} \bar{g} - \left( \frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1} \right) \underline{g} \right] dc_1 + \underline{u}_2 \underline{g} \Big|_{c_1^0}^{\infty}. \end{aligned} \quad (62)$$

Now,

$$\frac{d\rho_2}{d\underline{c}_1} = \frac{-\rho(1-\rho)\underline{g}'\bar{g}}{D^2}, \quad (63)$$

and

$$\frac{d\rho_2}{dc_1} = \frac{\rho(1-\rho)[\underline{g}'\bar{g} - \underline{g}\bar{g}']}{D^2}, \quad (64)$$

where  $D = \rho\underline{g} + (1-\rho)\bar{g}$ . Thus,

$$\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1} = \frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2}. \quad (65)$$

Further,

$$\underline{u}_2^0 \underline{g} \Big|_{-\infty}^{c_1^0} + \underline{u}_2 \underline{g} \Big|_{c_1^0}^{\infty} = \underline{g}(c_1^0) [\underline{u}_2^0(\rho_2^0) - \underline{u}_2(\rho_2^0)]. \quad (66)$$

When  $\rho_2 = \rho_2^0$ , it is easily verified that

$$\underline{u}_2^0(\rho_2^0) = \frac{\gamma}{2} \Delta\beta^2 = \underline{u}_2(\rho_2^0). \quad (67)$$

After substituting for the relevant terms and simplifying, (62) becomes

$$\int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2} k [\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'] dc_1 + \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2} k [\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'] dc_1, \quad (68)$$

where  $k = \frac{-\rho(1-\rho)}{D^2}$ .

Because  $\frac{du_2}{d\rho_2} < 0$  and  $\frac{du_2^0}{d\rho_2} < 0$ , to show that

$$\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}' < 0, \quad \forall c_1, \quad (69)$$

it is sufficient to show that the above integrals are negative over their respective limits of integration. This follows from the monotone likelihood ratio property (see, e.g., the proof of Theorem 2 in Jeitschko and Mirman (2002)). Thus,

$$\begin{aligned} \frac{d}{dc_1} \rho \lambda \int_{\mathbb{R}} U_2(\rho_2)(\bar{g} - \underline{g}) dc_1 \\ = \rho \lambda \left[ \int_{-\infty}^{c_1^0} \frac{du_2^0}{d\rho_2} k [\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'] dc_1 + \int_{c_1^0}^{\infty} \frac{du_2}{d\rho_2} k [\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'] dc_1 \right] < 0, \end{aligned} \quad (70)$$

and the low cost firm's first period effort is decreased. A similar proof shows that

$$\frac{d}{d\bar{c}_1} \left[ \int_{\mathbb{R}} U_2(\rho_2)(\bar{g} - \underline{g}) dc_1 \right] = -\frac{d}{dc_1} \left[ \int_{\mathbb{R}} U_2(\rho_2)(\bar{g} - \underline{g}) dc_1 \right] > 0. \quad (71)$$

Thus, the effect of the dynamic portion of the low cost firm's first period transfer is to decrease the distance between cost targets, and reduce how much the regulator updates her prior for any given cost realization.  $\square$

## Proof of Proposition 4

We first prove Lemma 1.

**Lemma.** *Information is valuable. That is, expected second period welfare is convex in second period beliefs:*

$$\frac{d^2 W_2(\rho_2)}{d\rho_2^2} > 0. \quad (72)$$

*Proof.* From the perspective of the second period, expected second period welfare is given

by (22). When  $c_1 > c_1^0$ , welfare can be expressed as

$$w_2 = \operatorname{argmax}_{\underline{e}_2, \bar{e}_2} S - \rho_2 \left( (1 + \lambda) \left( \underline{\beta} - \underline{e}_2 + \frac{\gamma}{2} (\underline{e}_2)^2 \right) + \lambda \underline{u}_2 \right) - (1 - \rho_2)(1 + \lambda) \left( \bar{\beta} - \bar{e}_2 + \frac{\gamma}{2} (\bar{e}_2)^2 \right). \quad (73)$$

By the envelope theorem,

$$\begin{aligned} \frac{dw_2}{d\rho_2} &= -(1 + \lambda) \left( \underline{\beta} - \underline{e}_2(\rho_2) + \frac{\gamma}{2} (\underline{e}_2(\rho_2))^2 \right) - \lambda \underline{u}_2(\bar{e}_2(\rho_2)) + (1 + \lambda) \left( \bar{\beta} - \bar{e}_2(\rho_2) + \frac{\gamma}{2} (\bar{e}_2(\rho_2))^2 \right) \\ &= (1 + \lambda) \left( \Delta\beta + \frac{1}{2\gamma} \right) - \lambda \underline{u}_2(\bar{e}_2(\rho_2)) - (1 + \lambda) \left( \bar{e}_2(\rho_2) - \frac{\gamma}{2} (\bar{e}_2(\rho_2))^2 \right). \end{aligned} \quad (74)$$

Thus,

$$\frac{d^2 w_2}{d\rho_2^2} = -\lambda \frac{d\underline{u}_2}{d\bar{e}_2} \frac{d\bar{e}_2}{d\rho_2} - (1 + \lambda)(1 - \gamma\bar{e}_2(\rho_2)) \frac{d\bar{e}_2}{d\rho_2} > 0, \quad (75)$$

since  $\frac{d\underline{u}_2}{d\bar{e}_2} > 0$  and  $\frac{d\bar{e}_2}{d\rho_2} < 0$ , and the high cost type's effort is less than the first best, which implies  $(1 - \gamma\bar{e}_2(\rho_2)) > 0$ . Because  $(1 - \gamma\bar{e}_2^0) > 0$  and  $\frac{d\underline{u}_2^0}{d\bar{e}_2^0} > 0$  and  $\frac{d\bar{e}_2^0}{d\rho_2} < 0$  as well, the proof is identical for  $w_2^0$ . Thus, information is valuable.  $\square$

**Proposition.** *The effect of expected second period welfare is to increase (decrease) the low cost (high cost) firm's first period effort. That is,*

$$\frac{dE[W_2(\rho_2)]}{dc_1} < 0, \quad (76)$$

and

$$\frac{dE[W_2(\rho_2)]}{d\bar{c}_1} > 0 \quad (77)$$

*Proof.* From the perspective of the first period,

$$E[W_2(\rho_2)] = \int_{-\infty}^{c_1^0} w_2^0 [\rho \underline{g} + (1 - \rho) \bar{g}] dc_1 + \int_{c_1^0}^{\infty} w_2 [\rho \underline{g} + (1 - \rho) \bar{g}] dc_1. \quad (78)$$

First, consider

$$\begin{aligned} \frac{dE[W_2(\rho_2)]}{d\underline{c}_1} &= \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} \frac{d\rho_2}{d\underline{c}_1} [\rho \underline{g} + (1 - \rho) \bar{g}] - w_2^0 \rho \underline{g}' d\underline{c}_1 \\ &\quad + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} \frac{d\rho_2}{d\underline{c}_1} [\rho \underline{g} + (1 - \rho) \bar{g}] - w_2 \rho \underline{g}' d\underline{c}_1. \end{aligned} \quad (79)$$

Integrate the second term under each integral by parts. Doing so yields

$$\begin{aligned} \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} \left[ \left( \frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{d\underline{c}_1} \right) \rho \underline{g} + \frac{d\rho_2}{d\underline{c}_1} (1 - \rho) \bar{g} \right] d\underline{c}_1 - w_2^0 \rho \underline{g} \Big|_{-\infty}^{c_1^0} \\ + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} \left[ \left( \frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{d\underline{c}_1} \right) \rho \underline{g} + \frac{d\rho_2}{d\underline{c}_1} (1 - \rho) \bar{g} \right] d\underline{c}_1 - w_2 \rho \underline{g} \Big|_{c_1^0}^{\infty}. \end{aligned} \quad (80)$$

Now,

$$- w_2^0 \rho \underline{g} \Big|_{-\infty}^{c_1^0} - w_2 \rho \underline{g} \Big|_{c_1^0}^{\infty} = - w_2^0 \rho \underline{g} \Big|_{c_1^0} + w_2 \rho \underline{g} \Big|_{c_1^0} = 0. \quad (81)$$

From the proof of Proposition 3,  $\frac{d\rho_2}{d\underline{c}_1} = \frac{-\rho(1-\rho)\underline{g}'\bar{g}}{D^2}$ ,  $\frac{d\rho_2}{d\underline{c}_1} = \frac{\rho(1-\rho)[\underline{g}'\bar{g}-\underline{g}\bar{g}']}{D^2}$ , and  $\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{d\underline{c}_1} = \frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2}$ .

Substituting the above into (80) yields

$$\begin{aligned} \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} \left[ \frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2} \rho \underline{g} - \frac{\rho(1-\rho)\underline{g}'\bar{g}}{D^2} (1-\rho) \bar{g} \right] d\underline{c}_1 \\ + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} \left[ \frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2} \rho \underline{g} - \frac{\rho(1-\rho)\underline{g}'\bar{g}}{D^2} (1-\rho) \bar{g} \right] d\underline{c}_1 \end{aligned} \quad (82)$$

$$\begin{aligned} = - \left[ \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} \rho_2^2 (1-\rho) \bar{g}' d\underline{c}_1 + \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} (1-\rho_2)^2 \rho \underline{g}' d\underline{c}_1 \right] \\ - \left[ \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} \rho_2^2 (1-\rho) \bar{g}' d\underline{c}_1 + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} (1-\rho_2)^2 \rho \underline{g}' d\underline{c}_1 \right]. \end{aligned} \quad (83)$$



Using the fact that  $(1 - \rho_2)^2 = 1 - \rho_2 - \rho_2(1 - \rho_2)$ , re-write (83) as

$$\begin{aligned} & - \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} \rho_2 [\rho_2(1 - \rho)\bar{g}' - \rho(1 - \rho_2)\underline{g}'] dc_1 - \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1 \\ & - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} \rho_2 [\rho_2(1 - \rho)\bar{g}' - \rho(1 - \rho_2)\underline{g}'] dc_1 - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1. \end{aligned} \quad (84)$$

Since  $\rho_2 = \frac{\rho g}{D}$  and  $1 - \rho_2 = \frac{(1 - \rho)\bar{g}}{D}$ ,

$$\rho_2(1 - \rho)\bar{g} - \rho(1 - \rho_2)\underline{g} = \frac{\rho(1 - \rho)}{D} [\bar{g}'\underline{g} - \bar{g}\underline{g}'] = -\frac{d\rho_2}{dc_1} D. \quad (85)$$

Thus, (84) becomes

$$\begin{aligned} & \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} \frac{d\rho_2}{dc_1} \rho_2 D dc_1 - \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1 \\ & + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} \frac{d\rho_2}{dc_1} \rho_2 D dc_1 - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1. \end{aligned} \quad (86)$$

Once again, use the fact that  $D\rho_2 = \rho g$ , and (86) becomes

$$\begin{aligned} & \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 - \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1 \\ & + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1. \end{aligned} \quad (87)$$

Integrate the second and fourth integrals in (87) by parts:

$$\begin{aligned} & \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1 \\ & = \frac{dw_2^0}{d\rho_2} (1 - \rho_2) \rho \underline{g} \Big|_{-\infty}^{c_1^0} - \int_{-\infty}^{c_1^0} \left( \frac{d^2 w_2^0}{d\rho_2^2} \frac{d\rho_2}{dc_1} (1 - \rho_2) - \frac{dw_2^0}{d\rho_2} \frac{d\rho_2}{dc_1} \right) \rho \underline{g} dc_1, \end{aligned} \quad (88)$$

and

$$\begin{aligned} \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2} (1 - \rho_2) \rho \underline{g}' dc_1 \\ = \frac{dw_2}{d\rho_2} (1 - \rho_2) \rho \underline{g} \Big|_{c_1^0}^{\infty} - \int_{c_1^0}^{\infty} \left( \frac{d^2 w_2}{d\rho_2^2} \frac{d\rho_2}{dc_1} (1 - \rho_2) - \frac{dw_2}{d\rho_2} \frac{d\rho_2}{dc_1} \right) \rho \underline{g} dc_1. \end{aligned} \quad (89)$$

Substituting back in to (87) yields

$$\begin{aligned} \frac{dE[W_2(\rho_2)]}{dc_1} = \int_{-\infty}^{c_1^0} \frac{d^2 w_2^0}{d\rho_2^2} (1 - \rho_2) \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 + \int_{c_1^0}^{\infty} \frac{d^2 w_2}{d\rho_2^2} (1 - \rho_2) \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 \\ + (1 - \rho_2) \rho \underline{g} \left( \frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2} \right) \Big|_{c_1^0}. \end{aligned} \quad (90)$$

Since  $\frac{d\rho_2}{dc_1} < 0$  by the monotone likelihood ratio property, by Lemma 1 the integrals are negative for all  $c_1$ . It is left to show that, when evaluated at  $c_1^0$ ,

$$\frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2} = 0. \quad (91)$$

Lemma 1 gives the expression for  $\frac{dw_2}{d\rho_2}$ , and a similar argument yields

$$\frac{dw_2^0}{d\rho_2} = (1 + \lambda) \left( \Delta\beta + \frac{1}{2\gamma} \right) - \lambda \underline{u}_2^0(\rho_2) - (1 + \lambda) \left( \bar{e}_2^0(\rho_2) - \frac{\gamma}{2} (\bar{e}_2^0(\rho_2))^2 \right). \quad (92)$$

Thus, when evaluated at  $c_1^0$ ,

$$\begin{aligned} \frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2} = \lambda [\underline{u}_2^0(\rho_2^0) - \underline{u}_2(\rho_2^0)] \\ + (1 + \lambda) \left[ \bar{e}_2^0(\rho_2^0) - \bar{e}_2(\rho_2^0) + \frac{\gamma}{2} (\bar{e}_2(\rho_2^0))^2 - \frac{\gamma}{2} (\bar{e}_2^0(\rho_2^0))^2 \right]. \end{aligned} \quad (93)$$

From Proposition 1,  $\underline{u}_2^0(\rho_2^0) - \underline{u}_2(\rho_2^0) = 0$ . Further,

$$\bar{e}_2^0(\rho_2^0) = \Delta\beta = \bar{e}_2(\rho_2^0). \quad (94)$$

Thus,

$$\frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2} = 0, \quad (95)$$

and

$$\frac{dE[W_2(\rho_2)]}{d\underline{c}_1} = \int_{-\infty}^{c_1^0} \frac{d^2 w_2^0}{d\rho_2^2} (1 - \rho_2) \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 + \int_{c_1^0}^{\infty} \frac{d^2 w_2}{d\rho_2^2} (1 - \rho_2) \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 < 0. \quad (96)$$

A similar proof shows that

$$\frac{dE[W_2(\rho_2)]}{d\bar{c}_1} = - \left[ \int_{-\infty}^{c_1^0} \frac{d^2 w_2^0}{d\rho_2^2} (1 - \rho_2) \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 + \int_{c_1^0}^{\infty} \frac{d^2 w_2}{d\rho_2^2} (1 - \rho_2) \frac{d\rho_2}{dc_1} \rho \underline{g} dc_1 \right] > 0. \quad (97)$$

Thus, the effect of expected second period welfare is to increase the distance between the first period cost targets.  $\square$

## Proof of Proposition 5

**Proposition. *The Ratchet Effect:*** *If the low cost firm's effort from mimicking the high cost firm is positive for all  $c_1 > \underline{c}_1$ , then the low cost firm has his effort increased over the course of the relationship with the regulator. That is,*

$$\frac{d}{d\underline{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right] > 0, \quad (98)$$

$$\frac{d}{d\bar{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right] < 0. \quad (99)$$

*Proof.* To prove Proposition 5, consider

$$\frac{d}{d\underline{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right] = \int_{-\infty}^{c_1^0} A' \frac{d\bar{e}_2^0}{d\rho_2} \frac{d\rho_2}{d\underline{c}_1} \bar{g} dc_1 + \int_{c_1^0}^{\infty} B' \frac{d\bar{e}_2}{d\rho_2} \frac{d\rho_2}{d\underline{c}_1} \bar{g} dc_1, \quad (100)$$

where

$$A' = (1 - \rho)(1 + \lambda) - (1 + \lambda - \rho)\gamma \bar{e}_2^0, \quad (101)$$

and

$$B' = (1 - \rho)(1 + \lambda)(1 - \gamma\bar{e}_2) - \rho\lambda\gamma\Delta\beta. \quad (102)$$

First, focus on

$$\int_{-\infty}^{c_1^0} A' \frac{d\bar{e}_2^0}{d\rho_2} \frac{d\rho_2}{d\underline{c}_1} \bar{g} dc_1. \quad (103)$$

Since

$$\gamma\bar{e}_2^0 = \frac{(1 - \rho_2)(1 + \lambda)}{1 + \lambda - \rho_2}, \quad (104)$$

it follows that

$$A' = \frac{1 + \lambda}{1 + \lambda - \rho_2} [(1 - \rho)(1 + \lambda - \rho_2) - (1 - \rho_2)(1 + \lambda - \rho)] = \frac{\lambda(1 + \lambda)(\rho_2 - \rho)}{1 + \lambda - \rho_2}. \quad (105)$$

Since  $c_1 \leq c_1^0 < \hat{c}_1$ , where  $\hat{c}_1$  is implicitly defined by  $\bar{g}(\hat{c}_1) \equiv \underline{g}(\hat{c}_1)$  (see Figure 2),  $\rho_2 > \rho$ .

Thus,  $A' > 0$ .

Further, for every  $c_1 \leq \underline{c}_1$ ,  $\underline{g}$  is increasing, so  $\underline{g}' \geq 0$  (see Figure 2); thus,

$$\frac{d\rho_2}{d\underline{c}_1} = \frac{-\rho(1 - \rho)\underline{g}'\bar{g}}{D^2} < 0. \quad (106)$$

Therefore, since  $\frac{d\bar{e}_2^0}{d\rho_2} < 0$  for all  $c_1$ , and since  $c_1^0 \leq \underline{c}_1$ ,

$$\int_{-\infty}^{c_1^0} A' \frac{d\bar{e}_2^0}{d\rho_2} \frac{d\rho_2}{d\underline{c}_1} \bar{g} dc_1 > 0 \quad (107)$$

for  $c_1 \in (-\infty, c_1^0]$ .

Now, return attention to

$$\int_{c_1^0}^{\infty} B' \frac{d\bar{e}_2}{d\rho_2} \frac{d\rho_2}{d\underline{c}_1} \bar{g} dc_1. \quad (108)$$

Using the definition of  $\frac{d\rho_2}{d\underline{c}_1}$ , (108) can be re-written

$$\frac{-\rho}{1 - \rho} \int_{c_1^0}^{\infty} B' \frac{d\bar{e}_2}{d\rho_2} (1 - \rho_2)^2 \underline{g}' dc_1. \quad (109)$$

Integrating by parts yields

$$\begin{aligned}
& B' \frac{d\bar{e}_2}{d\rho_2} (1 - \rho_2)^2 \underline{g} \Big|_{c_1^0}^{\infty} \\
& - \int_{c_1^0}^{\infty} \left[ -(1 - \rho)(1 + \lambda) \gamma \left( \frac{d\bar{e}_2}{d\rho_2} \right)^2 (1 - \rho_2) + B' \left[ \frac{d^2 \bar{e}_2}{d\rho_2^2} (1 - \rho_2) - 2 \frac{d\bar{e}_2}{d\rho_2} \right] \right] (1 - \rho_2) \frac{d\rho_2}{dc_1} \underline{g} dc_1.
\end{aligned} \tag{110}$$

Notice that

$$\frac{d^2 \bar{e}_2}{d\rho_2^2} (1 - \rho_2) - 2 \frac{d\bar{e}_2}{d\rho_2} = \frac{-2\lambda\Delta\beta(1 - \rho_2)}{(1 - \rho_2)^3(1 + \lambda)} - \frac{-2\lambda\Delta\beta}{(1 - \rho_2)^2(1 + \lambda)} = 0. \tag{111}$$

Thus, (109) becomes

$$\frac{-\rho}{1 - \rho} \left[ -B' \frac{d\bar{e}_2}{d\rho_2} (1 - \rho_2)^2 \underline{g} \Big|_{c_1^0} + \int_{c_1^0}^{\infty} (1 - \rho)(1 + \lambda) \gamma \left( \frac{d\bar{e}_2}{d\rho_2} \right)^2 (1 - \rho_2)^2 \frac{d\rho_2}{dc_1} \underline{g} dc_1 \right]. \tag{112}$$

First, notice that

$$\frac{-\rho}{1 - \rho} \int_{c_1^0}^{\infty} (1 - \rho)(1 + \lambda) \gamma \left( \frac{d\bar{e}_2}{d\rho_2} \right)^2 (1 - \rho_2)^2 \frac{d\rho_2}{dc_1} \underline{g} dc_1 > 0, \tag{113}$$

since  $\frac{d\rho_2}{dc_1} < 0$  for all  $c_1$ , and every other term under the integral in (113) is positive. Now, consider

$$\frac{\rho}{1 - \rho} B' \frac{d\bar{e}_2}{d\rho_2} (1 - \rho_2)^2 \underline{g} \Big|_{c_1^0}. \tag{114}$$

Since

$$\frac{d\bar{e}_2}{d\rho_2} = \frac{-\lambda\Delta\beta}{(1 - \rho_2)^2(1 + \lambda)}, \tag{115}$$

(114) can be simplified to

$$\frac{-\rho\lambda\Delta\beta}{(1 - \rho)(1 + \lambda)} B'(c_1^0) \underline{g}(c_1^0). \tag{116}$$

When evaluated at  $c_1 = c_1^0$ ,  $\bar{e}_2 = \Delta\beta$ . Thus,

$$B'(c_1^0) = (1 - \rho)(1 + \lambda) \left[ 1 - \frac{1 + \lambda - \rho}{(1 - \rho)(1 + \lambda)} \gamma \Delta\beta \right], \quad (117)$$

and (114) further simplifies to

$$- \rho \lambda \Delta\beta \left[ 1 - \frac{1 + \lambda - \rho}{(1 - \rho)(1 + \lambda)} \gamma \Delta\beta \right] \underline{g}(c_1^0). \quad (118)$$

Clearly, the term in brackets in (118) is less than one. It is also equal to  $\gamma(\bar{e}_2(\rho) - \Delta\beta)$ , where  $\bar{e}_2(\rho) - \Delta\beta$  is the low cost firm's effort from mimicking the high cost firm in a static game in which the regulator's beliefs are given by  $\rho$ . This is assumed to be positive; thus, the expression given in (118) is negative. However, the terms multiplying  $\underline{g}(c_1^0)$  are small, and if  $\underline{g}(c_1^0) \approx 0$ , the term in (118) can be ignored in signing the first order condition.

Thus,

$$\begin{aligned} \frac{d}{d\bar{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right] &\approx \int_{-\infty}^{c_1^0} A' \frac{d\bar{e}_2^0}{d\rho_2} \frac{d\rho_2}{d\bar{c}_1} \bar{g} dc_1 \\ &- \frac{\rho}{1 - \rho} \int_{c_1^0}^{\infty} (1 - \rho)(1 + \lambda) \gamma \left( \frac{d\bar{e}_2}{d\rho_2} \right)^2 (1 - \rho_2)^2 \frac{d\rho_2}{d\bar{c}_1} \bar{g} dc_1 > 0, \end{aligned} \quad (119)$$

and the desired result is obtained. A similar proof shows that

$$\frac{d}{d\bar{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right] = - \frac{d}{d\bar{c}_1} \left[ \int_{-\infty}^{c_1^0} A \bar{g} dc_1 + \int_{c_1^0}^{\infty} B \bar{g} dc_1 \right]. \quad (120)$$

Thus, the low cost (high cost) firm's effort in the first period is below (above) the commitment optimum, and his effort is increased (decreased) over the course of the interaction with the regulator.  $\square$

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