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Bank profitability, leverage constraints, and risk-taking

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Non-technical summary

Research Question

The 2008 crisis revealed a surprising amount of risk-taking in very profitable financial institutions. This seems to contradict the traditional predictions of corporate finance models where more profitable firms take less risk, because their shareholders stand to lose more if downside risks realize (Keeley, 1990). Understanding why some of the most profitable financial institutions chose to become exposed to risky market-based instruments on a large scale seems critical for both bank risk management and systemic risk regulation.

Contribution

In traditional corporate finance models, banks choose the risk of a portfolio of a fixed size. We show that bank risk-taking may take a different form when balance sheet size is a bank's choice. In our model, banks are endowed with a stable 'core business' and choose the size and risk of the market-based investments undertaken alongside their core business. Higher profitability of their core business enables banks to borrow more and engage in risky side activities on a larger scale.

Results

The model offers predictions that are consistent with some notable patterns of banks' pre-crisis risk-taking. One pattern is that many banks affected by the crisis were in advanced, rather than developing economies. Our model suggests that risk-taking in profitable banks is more likely when bank leverage constraints are looser, consistent with buoyant credit markets and a pre-crisis weakening of bank capital regulation in many advanced economies. Another pattern is the role of senior funding (e.g. repo) in supporting banks' pre-crisis expansion. Consistent with this evidence, our model confirms that banks' risk-taking in side investments is more severe when these can be financed with senior debt.

Nichttechnische Zusammenfassung

Fragestellung

Die Krise im Jahr 2008 förderte eine überraschend hohe Risikobereitschaft bei sehr profitablen Finanzinstituten zutage. Dies scheint den üblichen Unternehmensfinanzierungsmodellen zu widersprechen, denen zufolge profitablere Unternehmen weniger risikofreudig sind, da deren Anteilseigner mehr zu verlieren haben, sollten sich Abwärtsrisiken realisieren (Keeley, 1990). Sowohl für das Risikomanagement der Banken als auch für die Regulierung des Systemrisikos ist es wichtig zu verstehen, warum einige der ertragsstärksten Finanzinstitute im großen Stil Engagements im Bereich risikobehafteter marktbasierter Instrumente eingegangen sind.

Beitrag

In den üblichen Unternehmensfinanzierungsmodellen wählen die Banken das Risiko eines Portfolios mit fester Größe. In der Studie wird gezeigt, dass sich die Risikoaufnahme der Banken möglicherweise anders gestaltet, wenn Banken stattdessen die Bilanzgröße wählen. Im vorliegenden Modell verfügen Banken über ein stabiles „Kerngeschäft" und bestimmen Umfang und Risiko der neben ihrem Kerngeschäft getätigten marktbasierten Investitionen. Eine höhere Rentabilität ihres Kerngeschäfts versetzt die Banken in die Lage, mehr Mittel aufzunehmen und sich in größerem Umfang risikobehafteten Nebenaktivitäten zu widmen.

Ergebnisse

Das Modell liefert Prognosen, die mit einigen auffälligen Mustern der in der Zeit vor der Krise beobachteten Risikobereitschaft der Banken vereinbar sind. Erkennbar war zum einen, dass sich zahlreiche von der Krise betroffene Banken nicht in Entwicklungs-, sondern in Industrieländern befanden. Das Modell legt den Schluss nahe, dass profitable Banken mit größerer Wahrscheinlichkeit Risiken eingehen, wenn der Verschuldungsgrad der Banken weniger strengen Beschränkungen unterliegt. Genau diese Situation lag bei wachsenden Kreditmärkten und einer in der Zeit vor der Krise zurückgefahrenen Eigenkapitalregulierung in vielen Industrieländern vor. Augenfällig war auch die Bedeutung von vorrangiger Finanzierung (z.B. über Repos), die zur Expansion der Banken in der Zeit vor der Krise beigetragen hat. Im Einklang mit dieser Evidenz wird durch das vorliegende Modell bestätigt, dass die Risikobereitschaft der Banken in Bezug auf nicht zum Hauptgeschäft zählende Anlagen ausgeprägter ist, sobald diese über vorrangige Verbindlichkeiten finanziert werden können.

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Bank Profitability, Leverage Constraints, and Risk-Taking

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Abstract

Traditional theory suggests that higher bank profitability (or franchise value) dissuades bank risk-taking. We highlight an opposite effect: higher profitability loosens bank borrowing constraints. This enables profitable banks to take risk on a larger scale, inducing risk-taking. This effect is more pronounced when bank leverage constraints are looser, or when new investments can be financed with senior funding (such as repos). The model's predictions are consistent with some notable cross-sectional patterns of bank risk-taking in the run-up to the 2008 crisis.

Keywords: Banks; Risk-Taking; Leverage; Funding Structure; Crises.

JEL Classifications: G21, G24, G28.

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1 Introduction

The 2008 crisis revealed a surprising amount of risk-taking in otherwise very profitable financial institutions. For example, UBS in Switzerland had prior to the crisis a unique wealth management franchise with a stable return on allocated capital in excess of 30% (UBS, 2007). It rapidly, over just two years, accumulated a large portfolio of credit default swaps (CDS), lost over $50 billion in 2008, and had to be rescued. Washington Mutual, once called "The Walmart of Banking," had profitable consumer and small business operations. Yet it became one of the most aggressive mortgage lenders, lost $22 billion on subprime exposures, and was liquidated. The insurance company AIG was one of only three AAA-rated companies in the U.S. It started selling CDS protection on senior tranches of asset-backed securities in 2005 and lost over $100 billion – 10% of assets – in 2008 (AIG, 2008), wiping out shareholder equity and inducing a bailout.

Significant risk-taking in profitable financial institutions seems to contradict the traditional predictions of corporate finance models. Shareholders are protected by limited liability and have incentives to take risk to maximize their option-like payoff (Jensen and Meckling, 1976). But the risk-taking incentives should be lower in more profitable firms, because their shareholders stand to lose more if downside risks realize (Keeley, 1990). Understanding why some of the world's most profitable financial institutions chose to become exposed on a large scale to risky market-based instruments seems critical for both bank risk management and systemic risk regulation.

This paper offers a model of bank risk-taking that is consistent with the decision of profitable banks to take large-scale risks. Our starting observation is that in Jensen and Meckling-type models, firms choose the risk of a portfolio of an exogenous (fixed) size. We show that bank risk-taking may take a different form when bank balance sheet size is endogenous. In our model, banks are endowed with a stable 'core business'. They use the implicit equity associated with the profitability of the core business to fund 'side activities' undertaken alongside the core business. Higher core profitability enables banks to borrow more and engage in side activities on a larger scale. Larger scale makes risk-taking in side activities more attractive. When higher core profitability enables an exceeding scale of side activities, even very profitable banks may choose to take risk.[1]

[1] Examples of a core business include a wealth management franchise for UBS, or retail relationships for Washington

The model builds on three key ingredients. The first ingredient is a stylized separation of a bank's overall operation into a core business and side activities. The second ingredient, and a key friction in the model, is that bank makes the decision on the scale and riskiness of its side activities after the funding for the core business has already been attracted. This friction is ubiquitous in risk-shifting models, including Jensen and Meckling (1976). This seems natural in our framework, given the relatively long-term nature of the bank's core business (Allen and Gale, 1997; Boot, 2000), which implies that some of the bank's core business' funding may predate its involvement in side activities. This would most notably be the case when the bank's core business is funded by passive deposits (Huang and Ratnovski, 2011; Damar et al., 2013). Note that when bank's investment decision regarding the side activities is taken after the initial debt is financed, the funding for the core business can still be priced correctly on expectation. The final ingredient of the model is the endogenous bank balance sheet size, driven by the Holmstrom and Tirole (1998) leverage constraint.

Our model's structure and predictions are consistent with some notable patterns of banks' pre-crisis risk-taking.[2] One pattern is that many crisis-stricken banks incurred losses as a result of rapid non-core assets growth (Fahlenbrach et al., 2017; Baron and Xiong, 2017). This observation is directly reflected in the structure of our model, which focuses on bank risk-taking in its scalable non-core activities. Another pattern is that many banks affected by the crisis were in advanced, rather

Mutual. Examples of risky side activities are accumulating senior tranches of asset-backed securities (Gorton, 2010), selling protection on senior tranches of asset-backed securities through CDS contracts (Acharya and Richardson, 2009), undiversified exposures to housing (Shin, 2009), etc. An interesting illustration to the scale-related effect in bank risk-taking is that prior to the crisis UBS ran the largest trading floor by physical size in the world (103,000 sq.ft., the size of two football fields; "Empty Floors Fray Traders' Nerves", *Wall Street Journal*, July 14, 2014). We posit that a bank with a weaker core business would have been unable to support such a massive non-core operation.

[2]In motivating our theoretical analysis, we interpret the pre-crisis risk-taking by profitable financial institutions in the most direct manner: that those institutions chose to take risk. Besides this direct interpretation, there are three alternative explanations for risk-taking by profitable financial institutions. First, some institutions might be profitable because they take risk, i.e. there is reverse causality. However, this interpretation is inconsistent with our motivating examples: the high profits of UBS and AIG came from the relatively safe parts of their business (wealth management or traditional insurance), and not from the risky market-based investments. Second, banks might have faced uncertainty: insufficient information about the risk they were taking. Still, to the extent that the institutions understood that there was some risk, the traditional theory predicts that profitable banks should be less tolerant of uncertainty. In fact, when agents respond to uncertainty with min-max preferences (so-called 'Knightian uncertainty' – based on worst case beliefs, cf. Pritzker, 2013), the unknown risk is over-emphasized and profitable institutions should be particularly averse to it. Third, the risk could have been fully unanticipated (Gennaioli et al., 2012). But this interpretation contradicts the evidence that mortgage originators knew of the risks (Demyanyuk and van Hemert, 2011) and that those risks were – at least in part – priced (Demiroglu and James, 2012). For these reasons, we believe that our initial interpretation is credible enough to motivate deeper theoretical analysis of why profitable financial institutions might have incentives to take risk.

than emerging markets and developing economies (Laeven and Valencia, 2013). Our model suggests that risk-taking in profitable banks is likelier when bank leverage constraints are looser, consistent with buoyant credit markets and a pre-crisis weakening of bank capital regulation in many advanced economies (Krishnamurthy and Muir, 2017; Lopes-Salido et al., 2017). Yet another pattern is the role of senior, often repo, funding in supporting banks' pre-crisis leverage expansion (Adrian and Shin, 2010; Gorton and Metrick, 2012; Acharya and Öncü, 2013). Consistent with this evidence, our model confirms that the risk-shifting problem in banks' side investments is more severe when these can be financed with senior debt. Thus, our paper speaks to the literature on the negative externalities of strategic debt dilution through shorter new debt maturities (Brunnermeier and Oehmke, 2013), accumulating derivatives that are privileged in bankruptcy (Bolton and Oehmke, 2015), or pledging collateral to the detriment of unsecured creditors (Donaldson et al., 2017).

The model offers itself to a number of extensions. In one extension, we allow banks to exert effort to increase the profitability (or, equivalently, reduce the risk) of the core business. We show that a bank may then strategically combine high effort in the core business with increased risk in its side activities. The reason is that a more profitable core business loosens a bank's leverage constraint and lets it carry out the side activities on a larger scale, making risky side activities more attractive. Whereas the literature has often associated the seeming inconsistency of combining a prudent core business with risky side activities with a "clash of cultures" between conservative bankers and risk-loving traders (Froot and Stein, 1998), we explain it based purely on shareholder value maximization in the presence of risk-shifting frictions under endogenous bank balance sheet size.

In another extension, we consider the effects of monetary policy on bank risk-taking, and show that these effects are different for the bank's core business and side activities. Loose monetary policy reduces a bank's cost of funding, increasing its profitability. Higher profitability discourages risk-taking in the core business, because shareholders internalize more of the downside risk realizations. However, higher profitability induces risk-taking in side activities, by enabling the bank to carry them out at a larger scale. The finding of the heterogenous effects of monetary policy on bank risk-taking in various bank activities offers a new angle to the debate on the impact of monetary policy on bank risk-taking (see e.g., Dell'Ariccia et al., 2014, 2017; and Jiménez et al., 2014).

This paper adds to the large literature on the link between bank profitability and risk-taking. Profitability is a static concept, its dynamic counterparts are bank franchise value or charter value. The accepted first-order effect is that higher profitability reduces bank risk-taking incentives (Keeley, 1990; Demsetz et al., 1996; Repullo, 2004; among others). But some papers caution that the relationship is more complex. First, banks may take risk in order to generate profits (e.g., to satisfy higher capital requirements, see Blum, 1999; Hellmann et al., 2000; Matutes and Vives, 2000). Second, profitable banks can build up capital, which makes capital requirements less binding, enabling banks to absorb occasional losses and thus permitting risk-taking (Calem and Rob, 1999; Perotti et al., 2011). Our model proposes a novel effect and shows that a more profitable core business may induce banks to boost their leverage and take risk in side activities.

The framework of our model, which views a bank as a combination of businesses with different risk properties, is common in the literature on financial conglomerates (Santos, 1998; Freixas et al., 2007; Boot and Ratnovski, 2016; and more recently Chami et al., 2017; and Goel et al., 2017). The focus of our analysis is different from other papers in this literature. Santos (1998) focuses on the banks' expansion into the securities business, and contrasts the benefits of the economies of scope with the costs of the conflicts of interest. Freixas et al. (2007) analyze how the organizational structure of financial conglomerates affects their risk-taking incentives, arguing that capital arbitrage within a decentralized structure may strengthen market discipline and increase welfare. In Chami et al. (2017), a bank trades off the benefits of trading, such as more efficient interest rate risk management, with its risks, such as rogue trading. Goel et al. (2017) examine how the allocation of bank capital across business units responds to changes in financing conditions.

Our modelling approach, which builds on the interaction between a bank's core business and its side activities in the presence of credit constraints, is similar to that of Boot and Ratnovski (2016). However, our papers differ on two important dimensions. First, the models analyze different frictions. Boot and Ratnovski (2016) study the time inconsistency of capital allocation between the bank's relationship lending and trading activities. In their model, banks offer discretionary funding commitments (credit lines) to their relationships customers in return for ex ante fees. Such discretionary commitments lose their credibility when banks can divert their capital to trading. Risk-shifting per se plays no role in this capital misallocation. In contrast, our paper squarely focuses on risk-shifting. We study a classical time inconsistency problem of the investment risk decision (risk chosen after some debt has already been attracted), but enhance it to contrast banks' core and side activities. Second, we have a fundamentally different approach to modelling the scale constraints on bank non-core activities. In Boot and Ratnovski (2016), the scale of non-core activities (trading) is fixed. Consequently, when relationship lending is more profitable, banks have lower incentives to divert resources to trading, alleviating the time inconsistency problem in bank capital allocation. We consider a more general environment where the scale of trading is endogenous. This generates the key novel effect of our model: a more profitable core business expands the feasible scale of trading, inducing risk-taking in the trading activities. To put it differently, whereas in Boot and Ratnovski (2016), a more profitable core business reduces time inconsistency problems in trading when its scale is exogenous, we highlight that a more profitable core business can increase the time inconsistency (risk-shifting) problems in trading when its scale is endogenous.

The paper is structured as follows. Section 2 sets up the model. Section 3 solves the baseline model. Section 4 endogenizes the cost of bank funding. Section 5 offers extensions. Section 6 discusses empirical and policy implications. Section 7 concludes. The proofs are in the Appendix.

2 The model

Consider a bank that operates in a risk-neutral economy with three dates $(0, 1, 2)$ and no discounting. The bank is owner-managed, has no initial capital, and maximizes its expected profit.

Projects The bank has four investment opportunities:

1. The bank is endowed with a relationships-based "core" project. Thanks to an endowment of private information about the bank's existing customers, the core project is profitable (due to information rents; Petersen and Rajan, 1995) but not scalable (due to adverse selection in the

market for new customers, Dell'Ariccia and Marquez, 2006; or the difficulty in processing large amounts of soft information, Stein, 2002). For 1 unit invested at date 0, the core project produces $R > 1$ at date 2. Since the size of the core project is normalized to 1, R is also the profitability of the core project: a ratio of profit to project size. (For simplicity we abstract from risk in the core project in the main model. This does not affect the results. A risky core project is analyzed in Section 5.1.)

2-3. The bank may in addition engage in "side activities" – market-based investments. Market-based investments are scalable but less profitable (due to smaller if any information rents). There are two available side investments. A *safe* market-based investment (such as treasury securities) for each unit invested at date 1 produces $1+\varepsilon$ ($\varepsilon > 0$) at date 2. A *risky* market-based investment (such as asset-backed securities that lose value in a crisis) for each unit invested at date 1 produces at date 2: $1 + \alpha$ ($\alpha > \varepsilon$) with probability p, $p < 1$, but 0 with probability $1 - p$. We denote the endogenous scale of the market-based investment X. The fact that the scale of the market-based investment is endogenous allows us to generate predictions that are different from traditional risk-shifting models.

Note that the side investments are initiated after the core project: at date 1 rather than at date 0. This reflects the fact that they are undertaken alongside the bank's pre-existing, long-term business. The bank's project choice is not verifiable, so the bank cannot pre-commit at date 0 to the scale or the type of its future side activities. The sequential investment timing assumption, coupled with the bank's inability to commit to the nature of the later investment, helps justify why bank debt attracted at date 0 may be priced correctly only on expectation. As in other risk-shifting models, the fact that the pricing of some debt does not reflect the actual ex post risk choice drives bank risk-taking incentives – more on this below.

4. Finally, the banker can 'abscond'. Immediately after date 1, the owner-manager may divert the bank's assets to generate private benefits that are proportional to the bank's size. This opportunistic action reduces cash-flows, leaving nothing to creditors. The manager runs the bank normally when:

$$\Pi \geq b(1 + X), \tag{1}$$

where Π is the bank's profit when its assets are employed for normal use, and $b(1+X)$ is the absconding payoff: the initial value of assets $1+X$ multiplied by the conversion factor b $(0<b<1)$ of assets into private benefits. While the returns obtained in the normal course of bank business are fully pledgeable to outside investors, the private benefits received during absconding are not.[3] In our model, the absconding event is out-of-equilibrium: the creditors do not provide funding if they expect the bank to abscond. The expression (1) therefore defines the bank's leverage constraint: its maximal balance sheet size as a $1/b$ multiplier of equity (Holmstrom and Tirole, 1998). The restriction that firms can borrow only up to a multiple of their net worth because of their inability to pledge all the returns to outside creditors is standard in corporate finance models (Holmstrom and Tirole, 2011). For banks, this restriction can be thought of as an economic capital requirement (Adrian and Shin, 2010; Allen et al., 2011). The parameter b thus affects how easy it is for the bank to lever up, with a lower b corresponding to looser leverage constraints. The value of b may be lower, for example, in better institutional environments, consistent with more leveraged banks in advanced economies.

Parametrization We set the parameter space so as to analyze the bank's incentives to opportunistically choose a risky rather than a safe market-based investment. We assume that the risky market-based investment has a lower NPV than the safe market-based investment:

$$p(1+\alpha)-1<\varepsilon, \tag{2}$$

[3]The absconding payoff can capture the proceeds of looting or cash diversion (Calomiris and Kahn, 1991; Akerlof and Romer, 1993; Hart, 1995; Burkart and Ellingsen, 2004; Martin and Parigi, 2013; Boyd and Hakenes, 2014). It can alternatively represent the payoff to shirking, when the manager saves on the cost of effort but ruins the bank (Laffont and Martimort, 2002). More generally, the distortions associated with the private benefits stem from the limited pledgeability of revenues of some investment strategies (Holmstrom and Tirole, 1998).

but once the cost of funding is sunk the expected return to the banker from the risky investment is higher than that from the safe investment, creating risk-shifting incentives:[4]

$$p\alpha > \varepsilon. \tag{3}$$

We also assume that:

$$R - 1 \geq b, \tag{4}$$

so that the leverage constraint (1) is not binding when the bank engages only in the core project, but:

$$\varepsilon < p\alpha < b, \tag{5}$$

so that the leverage constraint becomes tighter as the bank expands market-based investments. The conditions (4) and (5) can be interpreted as that the core project gives the bank spare borrowing capacity, which the bank then uses for side investments.[5]

Funding The bank funds itself with debt. It attracts 1 unit of funds for the core project at date 0 against the interest rate r_0, and X units of funds for market-based investments at date 1 against the interest rate r_1. We call the two groups of creditors "date 0" and "date 1" creditors, respectively.

The creditors are repaid in full at date 2 if the bank is solvent: the payoff from projects exceeds the total amount owed. If the bank is insolvent, which may happen when the risky market-based investment returns 0, the bank goes bankrupt and the value of its assets – the core project's payoff

[4]Of course, some risky market-based investments may be more profitable than safe investments or indeed traditional bank lending (think of successful hedge fund strategies). But in our setup we focus on bank incentives to opportunistically undertake unproductive risky investments. The setup with a binary risky return resembles 'carry trade' strategies that were common in the run-up to the crisis. They generated a small positive return most of the time, but catastrophic losses with a small probability (Acharya et al., 2009).

[5]Banks' traditional lending is indeed usually more profitable than their marked-based investments. In 2000-2014, the average bank net interest margin was 3% (NY Fed, 2015) and the average cost of bank funding 2.5% (according to the Federal Home Loan Bank of San Francisco Cost of Funds Index), making the gross return on lending 5.5%. In the same period, the average gross return on banks' trading assets and securities was substantially lower: less than 2%. A b higher than that in (4), $b > R-1$, would make the bank unable to raise funds even for its core activity (creditors' participation constraint would never be satisfied). A b lower than that in (5), $b < p\alpha$, would enable the bank to undertake market-based investments on an infinite scale (a bank's leverage constraint would never be binding).

R – is distributed to the two groups of creditors according to their relative seniority. The relative seniority of date 1 creditors is given by a parameter θ: the share of their initial investment that they reclaim in bankruptcy. That is, in bankruptcy, date 1 creditors are repaid θX and date 0 creditors $R - \theta X$, where:

$$0 < \theta < \min\{R/X, 1\}. \tag{6}$$

A higher θ implies more senior date 1 creditors. We treat θ as an exogenous parameter in the baseline model. We show separately (in Sections 3.4 and 4.4) that if the bank was able to choose θ after date 0 debt was attracted, and θ was non-contractible at date 0, the bank would always select the highest possible θ at date 1, so as to reduce the cost of date 1 debt. (For example, the bank always has incentives to attract as much date 1 debt as possible through senior repos.) Therefore one can interpret the exogenous θ as the maximum seniority of date 1 debt that is feasible under contractual arrangements available to the bank and its date 1 creditors.[6]

To generate risk-shifting, we need the interest rate on date 0 debt to be set before bank risk choice at date 1 and not to reflect the date 1 risk choice. We consider two alternative formulations for this. In Section 3, we present a simplified model with an exogenous interest rate charged by date 0 creditors: $r_0 = 0$. The setup with an exogenous $r_0 = 0$ can be rationalized with not risk-sensitive deposit insurance[7], or by assuming that date 0 debt is likely bailed out in a bank's failure (Farhi and Tirole, 2012). (This setup allows us to obtain a simple closed-form solution, and demonstrate the economics behind our results most directly.)

In Section 4, we solve the model with a fully endogenous cost of funding. There, the friction is that the bank cannot commit to date 0 creditors about the type of the market-based investment that it will undertake at date 1. This friction seems justified based on the sequential investment timing where market-based investments are undertaken alongside the pre-existing core project. The fact that the price of the pre-existing bank debt does not reflect the risk undertaken, even though

[6] The modeling of debt seniority through the share of the principal repaid to a given class of creditors appears to be a convenient and sufficiently flexible modeling simplification. *Prima facie*, this specification is consistent, for example, with θX of date 1 funding being provided through repos (i.e. senior debt, with protected principal investment), and the remainder as junior debt. Choosing other ways to introduce the seniority parameter into the model would not qualitatively affect our results. The equal seniority of date 0 and date 1 investors corresponds to $\theta = X/(1 + X)$.

[7] See Laeven (2002) for a discussion of bank deposit insurance modalities. Boyd et al. (1998) and Freixas et al. (2007) analyze mispriced safety net as a source of bank risk-shifting.

this debt might be priced correctly on expectation, is accepted in the literature (e.g., Brunnermeier and Oehmke, 2013).

In either case – when the price of date 0 debt is exogenous and risk-insensitive or when date 0 debt is priced correctly on expectation but prior to date 1 – its cost is fixed at date 1 when the bank makes the decision on the risk of the market-based investment. This creates incentives for the bank to take risk in its date 1 market-based investments, at the cost to date 0 creditors. This effect is stronger when date 1 debt is more senior, as that shifts more risk to date 0 debt.[8]

The timeline is summarized in Figure 1.

3 Solution with an exogenous r_0

This section solves the model with an exogenous interest rate charged by date 0 creditors: $r_0 = 0$. This enables us to present a simple closed-form solution, and does not affect the economics of the model. The assumption of a risk-inelastic r_0 corresponds to the cases when date 0 creditors are protected by deposit insurance with risk-insensitive premia or expect to be bailed out if bank fails. Section 4 considers a model with a fully endogenous r_0 and confirms that our results hold.

We solve the model backwards. First, we derive bank profits conditional on a bank's strategy. Next, we establish the profit-maximizing bank strategy. Finally, we show how the bank's strategy (i.e., the decision on whether to engage in risk-shifting) depends on its profitability, and on debt seniority arrangements.

3.1 Bank payoffs

Safe market-based investment When, alongside the core project, the bank makes a safe market-based investment, its profit is:

$$\Pi_{Safe} = (R - 1) + \varepsilon X, \tag{7}$$

[8] For simplicity, we assume that date 1 creditors are able to observe the riskiness of the contemporaneous market-based investment. If date 1 creditors were not able to observe this risk and price date 1 debt accordingly, our results would be stronger.

where $R-1$ is the return on the core project, and εX is the return on the safe investment, both net of repayment to creditors. Here $r_0 = 0$ by assumption, and $r_1 = 0$ because the bank with a safe market-based investment never fails. (Note that $\Pi_{Safe} > R-1$, so making a safe market-based investment always dominates making no market-based investment at all.)

Risky market-based investment Now consider the bank's profit when it makes a risky market-based investment. Recall that the risky market-based investment has a lower NPV than the safe market-based investment. Accordingly, the bank will only make the risky investment when it can shift the downside risk realizations to its creditors. For a small-scale market-based investment, $X < R-1$, that is impossible: even when the risky investment returns 0, the bank's return on the core project R exceeds the total amount owed to creditors $1+X$, so the shareholders internalize the downside. A bank's expected profit from a small-scale risky investment is:

$$\Pi_{Risky}^{X<R-1} = R - 1 + p(1+\alpha)X - X, \tag{8}$$

where R is the return on the core project, 1 is the amount borrowed for the core project, $p(1+\alpha)X$ is the expected return on the risky project, and X is the amount borrowed for the risky investment. From (2), $\Pi_{Risky}^{X<R-1} < \Pi_{Safe}$: a bank's profit from a small-scale risky market-based investment is always lower than that from a safe investment.

Therefore, the bank would only make a risky market-based investment when the investment's scale is large enough, $X > R-1$. Then, the bank's expected profit is:

$$\Pi_{Risky} = p\left(R - 1 + (\alpha - r_1)\, X\right), \tag{9}$$

where p is the probability of success of the risky investment, $R-1$ is the return on the core project, and $(\alpha - r_1)\, X$ is the return to the bank on the risky investment, both net of repayment to creditors. With additional probability $1-p$ the risky investment fails, the bank cannot repay the creditors in full, and its profit is zero. The interest rate r_1 is obtained from the break-even condition of date 1 creditors:

$$p(1+r_1)X + (1-p)\theta X = X, \tag{10}$$

where $(1+r_1)X$ is the repayment to date 1 creditors when the risky investment succeeds and the bank is solvent (with probability p), θX is the repayment when the risky investment fails and the bank goes bankrupt (with probability $1-p$), and X is the date 1 creditors' investment into the bank. From (10):

$$r_1 = \frac{(1-p)(1-\theta)}{p}, \tag{11}$$

making the bank's profit (9):

$$\Pi_{Risky} = p\left(R - 1 + \left(\alpha - \frac{(1-p)(1-\theta)}{p}\right)X\right). \tag{12}$$

Combining a safe and a risky investments is never optimal Hypothetically, the bank could make a combination of a safe and a risky market-based investments. However, this is never optimal, because a single investment type always dominates a combination of the two. To see this, assume that the bank divides the total market-based investment X into a combination of a safe X^S and a risky X^R investments, so that $X^S + X^R = X$. Similar to (7), (8), and (9), the bank's profit is:

$$\Pi_{Safe+Risky} = \begin{cases} R - 1 + \varepsilon X^S + (p(1+\alpha)-1)X^R, & \text{if } R - 1 + \varepsilon X^S - X^R \geq 0 \\ p(R - 1 + (\varepsilon - r_1)X^S + (\alpha - r_1)X^R), & \text{otherwise} \end{cases}. \tag{13}$$

The top line in (13) represents the case when the bank is able to repay all creditors in full even when the risky investment fails (therefore, $r_1 = 0$). Then, choosing a safe investment dominates the combination of a safe and a risky ones (from (2)):

$$\Pi_{Safe+Risky} = R - 1 + \varepsilon X^S + (p(1+\alpha)-1)X^R < R - 1 + \varepsilon X = \Pi_{Safe}. \tag{14}$$

The bottom line in (13) represents the case when the bank cannot repay its creditors in full when the risky investment fails (therefore, $r_1 = (1-p)(1-\theta)/p$). Then, choosing a risky market-based investment dominates the combination of a safe and a risky ones (from (3)):

$$\Pi_{Safe+Risky} = p(R - 1 + (\varepsilon - r_1)X^S + (\alpha - r_1)X^R) < p(R - 1 + (\alpha - r_1)X) = \Pi_{Risky}. \quad (15)$$

Therefore, the combination of a safe and a risky market-based investments is never optimal.

3.2 Bank strategy

We showed above that the bank chooses between making a safe market-based investment that obtains profit Π_{Safe} (7) and a risky market-based investment that obtains profit Π_{Risky} (12). We decompose the analysis of this decision into two parts. One part is the bank's *incentives* to take risk. The other part is the bank's *ability* to take enough leverage to make a risky investment worthwhile. We discuss these in turn.

Incentives to make a risky investment The bank has incentives to choose the risky market-based investment over a safe one for $\Pi_{Safe} < \Pi_{Risky}$, corresponding to (use (7) and (12)):

$$X > X_{\min} = \frac{(1-p)(R-1)}{p\alpha - \varepsilon - (1-p)(1-\theta)}, \quad (16)$$

when:

$$\theta > \theta_{\min} = 1 - \frac{p\alpha - \varepsilon}{1-p}, \quad (17)$$

and never for a lower θ.[9] (For $\theta \leq \theta_{\min}$, the date 1 funding is sufficiently price-sensitive and consequently, the risky market-based investment is always less profitable for the bank than the safe market-based investment.)[10]

The expression (16) suggests that the risky investment can only dominate the safe investment when it is undertaken at sufficient scale. The intuition is that the benefit to the bank of choosing a

[9] One can verify that $X_{\min} > R - 1$, where $R - 1$ is the minimum scale of the risky investment such the the bank becomes insolvent upon a 0 realization of the risky investment ($X > R - 1$ is the constraint that underlies equation (12)). To see this, use (2): $(1-p) - p\alpha > -\varepsilon > -\varepsilon - (1-p)(1-\theta)$, implying for the denominator in (16) that: $p\alpha - \varepsilon - (1-p)(1-\theta) < 1 - p$, and consequently: $X_{\min} > R - 1$.

[10] Condition $\theta > \theta_{\min}$ ensures that the bank repays date 1 creditors in full when the risky investment is successful: $r_1 < \alpha$.

risky investment – the extra return to shareholders, $X(p\alpha - \varepsilon)$ – is proportional to the scale of the investment. The cost of making a risky investment – a possible loss of the core project's profits in bank bankruptcy, $(1-p)(R-1)$ – is invariant to the scale of the risky investment. Thus the scale of the investment X has to be high enough to make the benefit of the risky investment outweigh its cost. Note that consistent with traditional risk-shifting models, the bank's incentives to make a risky investment decline in the profitability of the core project. Thus, a higher profitability R requires a higher scale of the risky investment to make it worthwhile.

Ability to obtain leverage for the risky investment The notion of the minimal scale of the risky investment leads us to analyze whether the bank can lever up sufficiently to achieve the scale at which the risky investment dominates the safe investment. The leverage constraint (1) of a bank that makes a risky investment at scale X is:

$$p\left(R-1+\left(\alpha-\frac{(1-p)(1-\theta)}{p}\right)X\right)\geq b(1+X), \tag{18}$$

where the left hand side $p(\cdot)$ is the bank's profit (same as Π_{Risky} in (12)) and the right hand side $b(1+X)$ is the absconding payoff. From (18), the bank has the ability to make a risky investment for scale X with:

$$X\leq X_{\max}=\frac{p\,(R-1)-b}{b-p\alpha+(1-p)(1-\theta)}. \tag{19}$$

Further, it must hold that $\theta X < R$, because the promised repayment in bankruptcy to date 1 creditors θX cannot exceed the resources available in bankruptcy R (see (6)). Combining (6) and (19) yields an additional restriction:

$$\theta<\theta_{\max}=\frac{R(b-p\alpha+1-p)}{R-b-p}. \tag{20}$$

Expressions (16) and (19) help contrast the effects of profitability on bank risk-taking in traditional risk-shifting models and in our model. Whereas from (16) the bank incentives to take risk decline in the profitability of the core project, from (19) the bank's ability to lever up and take risk increases in its profitability. A higher R enables the bank to achieve larger scale of the risky investment $X_{\max}$. Then the question is whether a higher R (which increases the right-hand side in (16)) might allow for an exceedingly higher endogenous X (which increases the left-hand side in (16)), thus offsetting the traditional effect where a higher R diminishes bank risk-taking incentives for a fixed X.

Figure 2 illustrates the bank's payoffs from different strategies. The bank chooses between a safe and a risky market-based investments, shown with the dashed and the solid lines, respectively. For high enough scale of side investments, $X > X_{\min}$, the risky investment dominates. However the bank is constrained by the leverage constraint, shown with the dotted line, restricting the scale of side investments to $X < X_{\max}$. A change in R affects both $X_{\min}$ and $X_{\max}$.

Bank risk choice To derive the bank's strategy, we restrict the parameter space to $\theta_{\min} < \theta < \theta_{\max}$ (see (17) and (20)). We can summarize the bank's strategy as follows:

Lemma 1 *The bank chooses a risky market-based investment when $X_{\min} < X_{\max}$, corresponding to:*

$$b < b^{*} = \frac{\left(p(\alpha - \varepsilon) - (1-p)(1-\theta)\right)(R-1)}{(1-p)(R-1) + p\alpha - \varepsilon - (1-p)(1-\theta)}. \tag{21}$$

There exist parameter values such that the intersection between conditions $\theta_{\min} < \theta < \min\{\theta_{\max}, 1\}$ and $b < b^{*}$ is non-empty. When the bank chooses a risky market-based investment, it does so at its maximum scale $X_{\max}$. For $b \geq b^{*}$, the bank chooses the safe market-based investment.

Proof. See Appendix. ■

The intuition for Lemma 1 is that an opportunistic risky investment is only attractive at high scale, and thus a bank is more likely to choose it when its leverage constraint is sufficiently loose: b is low (i.e., $b < b^{*}$). A loose leverage constraint may be a result of a better institutional environment, with more protection of creditor rights. This interpretation is consistent with the fact that banks in advanced economies were on average more leveraged and more exposed to risky financial instruments compared to banks in emerging and developing economies prior to the recent crisis (Claessens et al., 2010). Expression (21) summarizes bank risk-taking strategy as a relation between b and other parameters of the model. We can use it to assess how changes in those other parameters – specifically R and θ – affect bank risk-taking by considering how they impact the threshold value b^*. A higher b^* indicates a wider range of parameter values for which a bank chooses the risky investment, which we interpret as higher risk-taking incentives.

3.3 Determinants of bank risk-taking

For the effect of core profitability R on bank risk-taking, we can demonstrate the following:

Proposition 1 (Bank Profitability and Risk-Taking) *Higher profitability of the bank's core business R expands the range of parameter values for which the bank chooses the risky market-based investment: $\partial b^*/\partial R > 0$, and increases the scale of the risky investment: $\partial X_{\max}/\partial R > 0$.*

Proof. See Appendix. ■

Proposition 1 is our key result. It shows that, in our framework, more profitable banks have higher risk-taking incentives. The reason is that a higher profitability R enables the bank to make the risky investment on a larger scale (higher $X_{\max}$), making risk-taking more attractive. This indirect scale-related effect offsets the traditional direct effect where higher profitability reduces a bank's incentives to take risk of fixed size (higher $X_{\min}$). Proposition 1 sheds light on a possible reason why some profitable banks invested so much in risky financial instruments before the recent crisis. High profitability allowed those banks to take risky side exposures on an exceedingly large scale. The additional profit from risky investments taken at high scale compensated the banks for the risk of a loss of a profitable core business franchise. Figure 3 panel A illustrates the impact of the bank's core profitability R on its risk-taking in side activities.

We now examine the effect of the seniority of date 1 bank funding θ on bank risk-taking. We can demonstrate the following:

Proposition 2 (Debt Seniority and Bank Risk-Taking) *Higher seniority of date 1 debt θ expands the range of parameter values for which the bank chooses the risky market-based investment: $\partial b^*/\partial \theta > 0$, and increases the scale of the risky investment: $\partial X_{\max}/\partial \theta > 0$. Moreover, the effects* *of core profitability and debt seniority on bank risk-taking are mutually reinforcing:* $\partial^2 b^*/\partial R\partial\theta > 0$ *and* $\partial^2 X_{\max}/\partial R\partial\theta > 0$*.*

Proof. See Appendix. ■

Proposition 2 highlights the role of bank funding arrangements in inducing risk-shifting through side activities. When the side activities (market-based investments) are financed with senior funding, this shifts the risk from new, date 1 creditors to the incumbent, date 0 creditors. The interest rate required by the new creditors in case of a risky investment declines in their seniority: $\partial r_1/\partial\theta = -(1-p)/p < 0$, making side investments more attractive (lower $X_{\min}$) and enabling the bank to take risk on a larger scale (higher $X_{\max}$). Since the feasible scale of side investments $X_{\max}$ (on which the seniority-related interest rate subsidy is accrued) increases in R, the effects of higher core profitability and new debt seniority on bank risk-taking are mutually reinforcing. Figure 3 panel B illustrates the impact of new bank debt seniority θ on bank risk-taking.

Our model's predictions for the role of bank funding arrangements in bank risk-taking are reminiscent of the discussion on the negative externalities of strategic debt dilution through the accumulation of derivatives (Bolton and Oehmke, 2015). When the costs of highly collateralized derivatives - which benefit from a special treatment in bankruptcy - are imposed on junior creditors, firms may have strong incentives to use derivatives on a larger scale than justified by hedging needs. And that, in turn, would impair the firm's continuation value through collateral calls.

3.4 Endogenous seniority of date 1 debt

Recall that our baseline model treats θ as an exogenous parameter. Consider now the effect of allowing for an endogenous θ. Assume that the bank chooses θ after date 0 debt is attracted, and that θ is not contractible *ex ante*. Assume also that the bank chooses θ from a range $[\theta_1, \theta_2]$ of contractually feasible bank debt seniority arrangements, such that $[\theta_1, \theta_2] \subset (\theta_{\min}, \min\{\theta_{\max}, 1\})$, where $\theta_{\min}$ and $\theta_{\max}$ are defined in (17) and (20), respectively. From (12), bank profits increase in θ:

$$\frac{\partial \Pi_{Risky}}{\partial\theta} = (p\alpha - (1-p)(1-\theta)) \cdot \frac{\partial X_{\max}}{\partial\theta} + (1-p)X_{\max} > 0.$$

This is because a higher θ shifts more risk from date 1 to date 0 creditors, which reduces the cost of date 1 debt and increases bank's expected profits. Consequently, the bank always chooses the highest possible θ: $\theta = \theta_2$.[11] Therefore one can interpret the exogenous θ in our basic model as the maximum feasible seniority of date 1 debt.

4 Solution with an endogenous r_0

The previous section assumed an exogenous interest rate on date 0 debt, $r_0 = 0$, to obtain a closed-form solution and demonstrate the economics of our model most directly. This section allows for an endogenous r_0 set from the date 0 creditors' break-even condition, and verifies that the results of Propositions 1 and 2 hold.[12]

With an endogenous r_0 the solution becomes more complex, due to the interaction between the interest rate charged by date 0 creditors and bank risk-taking. The bank's anticipated risk-taking implies a positive interest rate r_0. In a typical risk-shifting model, a higher r_0 increases the borrowers' risk-taking incentives (Stiglitz and Weiss, 1981): it induces borrowers to undertake projects with a lower probability of success but a higher payoff in case of success, because higher debt repayments absorb much of the return on moderately profitable projects. However, in our model a higher r_0 has an opposite effect: it may reduce bank risk-taking incentives. The reason is that a higher r_0, in effect, reduces the profitability of the bank's core project and thus constrains the bank's borrowing capacity (similar to an effect of a lower R on $X_{\max}$ in (19)), and risk-taking is less attractive at a lower scale. Therefore, in our model there always exists a high enough r_0 that constrains the bank's borrowing capacity sufficiently to prevent risk-taking. But with a high

[11] This analysis relies on an implicit assumption that a bank cannot commit ex ante to a future seniority arrangement θ. This appears plausible. Brunnermeier and Oehmke (2013) argue that a bank cannot commit to the future funding strategy because of "frequent funding needs, opaque balance sheets, and continuous activity in the commercial paper market." Bolton and Oehmke (2015) similarly argue that it is difficult to counteract the future use of derivatives that are privileged in bankruptcy when banks cannot anticipate their hedging needs. Donaldson et al. (2017) highlight that although date 0 creditors' stake can be protected by giving them collateral, that entails asset encumbrance which limits the bank's future access to liquidity – a potentially risky strategy. Also, to the extent that the bank uses date 0 debt to finance relationships-based projects with high asymmetric information, the resulting bank assets may not offer much collateral value. If the bank were nevertheless able to commit to a low θ, that would reduce (as per Proposition 2) or eliminate (e.g., for $\theta < \theta_{\min}$) bank risk-taking incentives identified in our model.

[12] The assumption that the market-based investments are initiated after the core project becomes critical once r_0 is endogenized. This assumption justifies why the bank cannot commit to date 0 creditors to the type of the market-based investment that it will undertake at date 1.

r_0 and no bank risk-taking, date 0 bank creditors would obtain positive rents, which is inconsistent with them being competitive. To meaningfully characterize the equilibrium, we assume that date 0 bank creditors set the minimal interest rate so as to at least break even under correctly anticipated bank risk choices.

4.1 Interest rates and bank risk-taking

We consider the mutually consistent combinations of date 0 interest rates and bank risk choices. There are three such possible combinations in our model.

1. A bank makes a safe investment. Assume that date 0 creditors anticipate that the bank will make a safe market-based investment and set $r_0 = 0$. Then, as in the baseline model, the bank makes a safe investment for $b \geq b^*$ (see (21)). For $b < b^*$, the bank makes a risky investment, so creditors have to set a positive interest rate.

2. A bank makes a risky investment. Assume that $b < b^*$, so date 0 creditors anticipate that the bank will make a risky market-based investment in response to $r_0 = 0$. Then, the creditors can set the interest rate r_0 based on their break-even condition that internalizes bank risk-taking:

$$p(1 + r_0) + (1 - p)(R - \theta X) = 1. \tag{22}$$

This condition is similar to that for date 1 creditors (10), except that the repayment in case of bank bankruptcy is $(R - \theta X)$ rather than θX, reflecting a different relative seniority of date 0 creditors.

Further, the creditors know that the bank's profit (similar to (12)):

$$\Pi^{r_0}_{Risky} = p\left(R - 1 - r_0 + \left(\alpha - \frac{(1-p)(1-\theta)}{p}\right)X\right) \tag{23}$$

is increasing in X: $\partial\Pi^{r_0}_{Risky}/\partial X > 0$. Accordingly, the bank will make the risky investment on the scale (similar to (19)):

$$X = X_{\max}(r_0) = \frac{p(R - (1 + r_0)) - b}{b - p\alpha + (1-p)(1-\theta)}. \tag{24}$$

Substituting X from (24) into (22) and solving for r_0 obtains the interest rate r_0^{Risky} that reflects the bank's anticipated risk-taking:

$$r_0^{Risky} = \frac{1-p}{p} \cdot \frac{(R-1)(\theta - (b - p\alpha + 1 - p)) - b\theta}{b - p\alpha + 1 - p}. \tag{25}$$

It is easy to verify that $\partial r_0^{Risky}/\partial b < 0$. When b declines, the bank's leverage constraint becomes looser, so the bank makes the risky investment on a larger scale. This dilutes the date 0 creditors' claim in a possible bankruptcy, as they have to share the core project's payoff R with a higher mass of date 1 creditors.

3. A sufficiently high r_0 prevents bank risk-taking An alternative response of date 0 creditors to the possibility of bank risk-taking for $b < b^*$ is to set r_0 high enough, so as to reduce the bank's profit and tighten its borrowing capacity sufficiently to make the risky side investment unattractive for the bank. Indeed, from (24), an increase in r_0 decreases the bank's borrowing capacity: $\partial X_{\max}(r_0)/\partial r_0 < 0$. Consequently, for any $b < b^*$ there exists $r_0^{Prevent}(b) > 0$ such that in response to that interest rate the bank does not take risk. To derive $r_0^{Prevent}$, consider the minimal scale at which the bank starts preferring a risky market-based investment to a safe one, $X_{\min}(r_0)$ (similar to (16)):

$$X_{\min}(r_0) = \frac{(1-p)(R-(1+r_0))}{p\alpha - \varepsilon - (1-p)(1-\theta)}, \tag{26}$$

and set $X_{\min}(r_0) = X_{\max}(r_0)$ (use (24)) to obtain:

$$r_0^{Prevent} = (R-1) - \frac{b(p\alpha - \varepsilon - (1-p)(1-\theta))}{p(\alpha - \varepsilon) - (1-p)(1-\theta) - (1-p)b}. \tag{27}$$

Note that $r_0^{Prevent} \to 0$ for $b \to b^*$, whereas $r_0^{Risky} > 0$ for $b \to b^*$. Therefore, $r_0^{Prevent} < r_0^{Risky}$ for $b \to b^*$: when b is below but close to b^*, date 0 creditors can prevent bank risk-taking with a relatively small increase in r_0, and the interest rate that prevents bank risk-taking is lower than the one that prices it in. At the same time, $\partial r_0^{Prevent}/\partial b < 0$: as b declines, the bank's leverage constraint becomes looser, so the interest rate that prevents bank risk-taking increases.

4.2 Equilibrium strategy

Recall that date 0 creditors choose the minimal interest rate consistent with at least breaking even under correctly anticipated bank risk-taking strategy. For $b \geq b^*$, that is $r_0 = 0$. For $b < b^*$, that is either $r_0^{Prevent}$ or r_0^{Risky}, whichever is lower. To derive the equilibrium strategy we solve for: $r_0^{Prevent} = r_0^{Risky}$. We can demonstrate the following:

Proposition 3 (Equilibrium Strategy under Endogenous r_0) *Equilibrium date 0 interest rate and bank risk-taking are characterized by two thresholds: b^* from (21) and $b^{**} < b^*$ given by:*

$$b^{**} = \frac{(R-1)(p(\alpha - \varepsilon) - (1-p)(1-\theta))}{(1-p)(R-1) + p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta} \tag{28}$$

- *For $b \geq b^*$, date 0 creditors set $r_0 = 0$, and the bank makes the safe market-based investment.*
- *For $b^{**} \leq b < b^*$, date 0 creditors set $r_0 = r_0^{Prevent} > 0$ (given by (27)), and the bank makes the safe market-based investment. Date 0 creditors earn positive rents, but a lower interest rate would induce the bank to take risk, which would violate the date 0 creditors' break-even condition.*
- *For $b < b^{**}$, date 0 creditors set $r_0 = r_0^{Risky}$ (given by (25)), and the bank makes the risky market-based investment.*

Proof. See Appendix. ■

Figure 4 illustrates the evolution of r_0^{Risky} (from(25)) and $r_0^{Prevent}$ (from(27)) in the intensity of the leverage constraint b. If $b < b^*$, the bank chooses a risky investment in response to $r_0 = 0$. Yet a small increase in the date 0 interest rate from $r_0 = 0$ to $r_0 = r_0^{Prevent}$ tightens the bank's leverage constraint and prevents bank risk-taking (recall that $r_0^{Prevent} \to 0$ for $b \to b^*$). Interestingly, for $b^{**} \leq b < b^*$, date 0 creditors earn positive rents. Yet $r_0^{Prevent}$ is the smallest interest rate consistent with them at least breaking even: reducing it would induce bank risk-taking and make date 0 creditors lose money on expectation. As b declines, the interest rate that is necessary to prevent bank risk-taking increases. At $b = b^{**}$ the two interest rate functions: $r_0^{Prevent}$ and r_0^{Risky}

intersect, and for lower values of b, $b < b^{**}$, it is cheaper to price in bank risk-taking in the cost of debt than to prevent it. Thus, for $b < b^{**}$, date 0 creditors charge r_0^{Risky} and break even, and the bank makes the risky investment. Therefore, b^{**} is the bank's risk-taking threshold in this model. The key implication of Proposition 3 is that the threshold value b^{**} is lower than b^*, as it was under $r_0 = 0$ in Section 3. Allowing for a positive r_0 reduces the bank's core profitability and hence its risk-taking incentives. Accordingly, the bank makes a risky market-based investment for a narrower range of parameter values.

4.3 Determinants of bank risk-taking

We can now characterize how the threshold b^{**} responds to changes in R and θ:

Proposition 4 (Determinants of Risk-Taking under Endogenous r_0) *The threshold b^{**} increases in R and θ, and is convex in the combination of R and θ.*

Proof. *See Appendix.* ■

The evolution of b^{**} in response to R and θ is therefore similar to the evolution of b^* in Propositions 1 and 2. This confirms that our results hold also under an endogenous date 0 interest rate r_0. Banks take risk for a wider range of parameter values when their core profitability R or the feasible seniority of debt used to finance market-based investments θ, are higher.

4.4 Endogenous seniority of date 1 debt

Similar to Section 3.4, we can verify that when θ is not exogenous, the bank would choose the highest possible θ. As in Section 3.4, consider a range $[\theta_1,\theta_2]$ of contractually feasible bank debt seniority arrangements, where $[\theta_1, \theta_2] \subset (\theta_{\min}, \min\{\theta_{\max}, 1\})$, with $\theta_{\min}$ and $\theta_{\max}$ defined in (17) and (20). From (23) and (24), bank profits increase in θ:

$$\frac{\partial \Pi^{r_0}_{Risky}}{\partial \theta} = \frac{(1-p)b(p(R-1-r_0)-b)}{(b-p\alpha+(1-p)(1-\theta))^2} > 0. \tag{29}$$

Thus, the bank chooses the highest possible θ: $\theta = \theta_2$. This result is consistent with our initial interpretation of an exogenous θ as the maximal feasible seniority of date 1 funding.

5 Extensions

This section offers two extensions to the main model. First, we consider a non-deterministic core project and let the bank exert effort to improve its performance. Second, we consider the effects of monetary policy on bank risk-taking in its core business versus that in its market-based investments.

To derive tractable closed form solutions, we go back to the assumption of an exogenous date 0 interest rate: $r_0 = 0$. (Endogenizing r_0 would not affect the results.)

5.1 Effort in the core project

Assume that the return on the core project is no longer deterministic. Instead, the bank needs to exert effort to improve the performance (increase the probability of success) of the core project. We analyze how access to an opportunistic market-based investment affects bank effort in the core project.

Assume that the return on the core project is R with probability e and 0 otherwise (as opposed to a certain R in the main model). The probability e corresponds to the bank's effort, which carries a private cost $ce^2/2$. We focus on c high enough such that the model admits an interior solution in effort. The bank exerts effort after date 0 funding is attracted. The outcome of the effort becomes public knowledge immediately afterwards. One interpretation of effort is ex-ante screening, which helps the bank to identify safer or more profitable projects. The screening is performed observable through activities such as putting in place more robust lending procedures, or hiring more experienced personnel. since the bank's ex ante screening is observable, date 1 creditors can make correct inference about the bank's core performance. If it becomes known that the core project returns 0, the bank cannot make a market-based investment because the leverage constraint (1) is not satisfied for standalone market-based investments (see (5)). If it becomes known that the core project returns R, the rest of the game is similar to the model in Section 3. The timeline is summarized in Figure 5.

We are interested in two questions. First, how does the bank's effort in the core project e depend on the feasible scale of its market-based side investments, as captured by b (a lower b implying a looser leverage constraint and larger side investments). Second, how effort e is affected by the bank's access to an opportunistic, risky market-based investment, compared to a hypothetical case when the bank can only make a safe market-based investment.

When the core project returns 0, the payoff to the banker is 0. When the core project returns R, the bank makes a safe market-based investment for $b \geq b^*$ and a risky one for $b < b^*$ (with b^* given in (21)). When the bank makes the safe investment, its profit is:

$$\Pi^e_{Safe} = e(R - 1 + \varepsilon X) - \frac{ce^2}{2}, \tag{30}$$

where $(R - 1 + \varepsilon X)$ is the bank's profit conditional on successful effort (similar to (7)), e is the probability of the core project's success, and $-ce^2/2$ is the cost of effort. The scale of the market-based investment X is obtained by setting to equality the leverage constraint (1) that takes form:

$$R - 1 + \varepsilon X \geq b(1 + X), \tag{31}$$

giving: $X = (R - 1 - b) \,/\, (b - \varepsilon)$. Substituting X from (31) into Π^e_{Safe} (30) and maximizing with respect to e gives:

$$e^*_{Safe} = \frac{b}{c} \cdot \frac{R - 1 - \varepsilon}{b - \varepsilon}. \tag{32}$$

When the bank chooses the risky investment, its profit is:

$$\Pi^e_{Risky} = ep\left(R - 1 + \left(\alpha - \frac{(1-p)(1-\theta)}{p}\right) X\right) - \frac{ce^2}{2}, \tag{33}$$

where $p(\cdot)$ is the bank's profit conditional on successful effort (similar to (12)). Establishing X and maximizing Π^e_{Risky} in a manner similar to (32) gives:

$$e^*_{Risky} = \frac{b}{c} \cdot \frac{p\,(R - 1 - \alpha) + (1 - p)\,(1 - \theta)}{b - p\alpha + (1 - p)(1 - \theta)}. \tag{34}$$

It is easy to obtain by differentiating (32) and (34) that the bank's effort increases in the profitability of the core project: $\partial e^*_{Safe}/\partial R > 0$ and $\partial e^*_{Risky}/\partial R > 0$. This is natural: a higher upside induces the bank to exert more effort to make the core project succeed. More interesting, the bank's effort in the core project also increases with the feasible scale of the bank's market-based investments: $\partial e^*_{Safe}/\partial b < 0$ and $\partial e^*_{Risky}/\partial b < 0$, with a lower b capturing a higher feasible scale of market-based investments. The reason is that higher feasible scale makes market-based investments more valuable and a successful core business ensures the bank's ability to undertake them.

We can now proceed with the following exercise. Consider $b < b^*$, so that the bank normally chooses the risky market-based investment when the core project succeeds. Compare this with the case when the bank is restricted to the safe market-based investment only, even for these low values of b. We can demonstrate the following:

Proposition 5 (Safe Core Business, Risky Side Investments) *For $b < b^*$, a bank's effort in the core business when the risky market-based investment is available is higher than that when the bank is restricted to the safe market-based investment only: $e^*_{Risky}|_{b<b^*} > e^*_{Safe}|_{b<b^*}$.*

Proof. See Appendix. ■

Proposition 5 shows that a bank's access to a risky market-based investment may increase its effort in the core project. The reason is that, although the risky market-based investment has a lower NPV, it is still more profitable for bank shareholders than the safe market-based investment for $b < b^*$ (by the nature of risk-shifting). Accordingly, for $b < b^*$, when the bank gains access to a privately-profitable risky market-based investment, it increases its effort in the core business to ensure its ability to undertake this side investment. The bank then strategically combines high effort in the core project (higher than that if the bank was restricted to safe side investments) with risky side activities. While the literature has explained the seeming inconsistency of combining a prudent core business with risky side activities through a "clash of cultures" between conservative bankers and risk-loving traders (Froot and Stein, 1998), our model explains it based on shareholder value maximization under the possibility of privately-profitable risk-shifting in the bank's side activities.

5.2 Monetary policy and bank risk-taking

Now consider how bank risk-taking can be affected by monetary policy in our framework. Assume that monetary policy affects the bank's cost of funding and its return on investment. Assume that while monetary policy has a full pass-through into the bank's cost of funding, its pass-through into the return on bank investments is only partial (Berlin and Mester, 1999). This is indeed likely, for two reasons. First, bank assets are longer-term than bank liabilities, so their return is less responsive to the variations in the short-term monetary policy rate. Second, the variation in the return on bank assets is also affected (and may be cushioned over the cycle) by the effects of interbank competition in lending markets (Dell'Ariccia et al., 2014).

Formally, for the monetary policy rate i, the reference interest rate for bank funding is i, with $r_0 = i$, and the cost of date 1 debt determined by a break-even condition with the reservation return i. The returns on bank investments are $R+\gamma i$, $1+\varepsilon+\gamma i$, and $p(1+\alpha+\gamma i/p)$, with $\gamma \leq 1$ to reflect a possibly incomplete monetary policy pass-through.[13] ($\gamma = 1$ implies a full pass-through.) We allow i to vary, $i \lesseqgtr 0$, as opposed to $i = 0$ in the baseline model.

When the bank makes the safe market-based investment, $r_1 = 1+i$. When the bank makes the risky market-based investment, the break-even condition for date 1 creditors is (similar to (10)):

$$p(1+r_1)X + (1-p)\theta X = (1+i)X, \tag{35}$$

which gives (similar to (11)):

$$r_1 = \frac{i + (1-p)(1-\theta)}{p}. \tag{36}$$

With the cost of funding i, the threshold for the bank's choice of safe versus risky market-based

[13]For the effect on the risky investment, we include the term $/p$ to make the total effect of monetary policy on its profitability, γi, consistent with that for other assets.

investment, b_i^*, becomes (similar to (21)):

$$b_i^* = \frac{(R-1-i(1-\gamma))\,(p(\alpha-\varepsilon)-(1-p)(1-\theta+i(1-\gamma)))}{(1-p)(R-1-i(1-\gamma))+p\alpha-\varepsilon-(1-p)(1-\theta)}. \tag{37}$$

Now consider the bank's effort in the core project, as in the previous section. The equilibrium effort is (similar to (32) and (34)):

$$e^*_{Safe,i} = \frac{b}{c}\cdot\frac{R-1-\varepsilon}{b-\varepsilon+i(1-\gamma)} \text{ for } b \geq b_i^*, \tag{38}$$

$$e^*_{Risky,i} = \frac{b}{c}\cdot\frac{p(R-1-i(1-\gamma))-p\alpha+(1-p)(1-\theta)+i(1-\gamma)}{b-p\alpha+(1-p)(1-\theta)+i(1-\gamma)} \text{ for } b < b_i^*. \tag{39}$$

Differentiation of (37)-(39) obtains the following result:

Proposition 6 (Monetary Policy and Bank Risk-Taking) *For $\gamma < 1$ a decrease in the bank's cost of funding increases the bank's effort in the core project, making it safer: $\partial e^*_{Safe,i}/\partial i < 0$ and $\partial e^*_{Risky,i}/\partial i < 0$, but at the same time increases the bank's risk-taking incentives in its market-based investments: $\partial b_i^*/\partial i < 0$.*

Proof. See Appendix. ■

Proposition 6 suggests a novel heterogeneity in the possible impact of monetary policy on bank risk-taking. A lower cost of funding (corresponding to more accommodative monetary policy) increases bank margins. For fixed scale bank activities, such as the core relationships-based business, higher margins induce higher effort. But for scalable bank activities, such as market-based investments, higher margins imply less binding bank's leverage constraint, which increases the bank's incentives to use such investments for risk-shifting. In the case of a full interest rate pass-through into the return on bank investments, $\gamma = 1$, monetary policy does not affect bank margins and therefore it has no effect on bank risk-taking.

The fact that accommodative monetary policy may differentially affect bank risk-taking in the core business versus that in side investments also suggests that the impact of monetary policy on bank risk-taking may depend on a bank's mix of activities. For example, accommodative monetary policy may have a small effect on bank risk-taking in "local" banks involved in relationship lending, but a large effect on that in large banks active in financial markets (cf. Borio and Zhu, 2012).

6 Discussion

6.1 Empirical illustration

The key empirical prediction of our model is that a bank's higher core profitability increases its ability to borrow, and through this may induce bank risk-taking in side activities. While a formal econometric examination of this relationship is beyond the scope of this paper, we can offer some illustrative evidence that such a channel of risk-taking is consistent with the cross-sectional patterns of risk-taking for large banks in the run up to the 2008 crisis. (That is, our premise that banks with high profitability took risk before the crisis is not limited to the three cases of UBS, Washington Mutual, and AIG discussed in the Introduction.) We use the banks' net income to total assets ratio over 1995-2000 (the time when banks' market-based activities were still relatively limited; Boot, 2014) as a proxy for banks' core profitability. We use the banks' equity losses during the crisis (end 2007 to end 2009) to proxy for banks' pre-crisis risk-taking (as in Beltratti and Stulz, 2012). We focus on U.S. and European banks with assets over 50 billion dollars in 2006. All data are from Bankscope.

Figure 6 (Panel A) shows the resulting scatterplot. The dotted fitted line shows a negative relationship between bank profitability in the late 1990s and bank equity returns in 2007-2009, indicating that historically more profitable banks were more affected by the crisis, indicating higher pre-crisis risk-taking.[14] In addition, the solid fitted line shows this relationship for the banks with above-median assets growth during 2001-2006 (such banks are indicated with solid dots). The relationship between core profitability and equity returns during the crisis is steeper for these banks, indicating that risk-taking in profitable banks was related to the expansion of their side

[14] A long lag in measuring bank profitability, as well as the fact that many bank business models have changed since 1990s with the deepening of financial markets, make the reverse causality interpretation where banks were profitable because they took risk, unlikely.

activities, in line with the predictions of our model.[15]

As an additional check, we now also consider the subsample of banks that during 1995-2000 period had the interest income-to-total income ratio above the median, for an average of about 85%. Those banks thus derived a vast majority of their income from the lending activities. Since they were most focused on their core lending business, for these banks the historic profitability is a more precise proxy of their core profitability. Figure 6 (panel B) shows that the negative relationship between bank profitability in the late 1990s and bank equity returns in 2007-2009 is even more pronounced in this subsample of banks.

While the scatter plot is not a formal econometric test, we feel that it provides a useful illustration for the link between bank profitability, asset growth, and bank risk, which is in line with the predictions of our analysis.

6.2 Bank capital

The key implication of our analysis is that, contrary to the traditional Keeley (1990) intuition, high bank profitability may be ineffective in limiting bank risk-taking, and may in fact induce it. This intuition can be extended to bank capital. Similar to bank profitability, bank capital also captures the exposure of bank shareholders to downside risk realizations, and affects the bank's capacity to take additional leverage. Indeed, in our model, explicit bank equity is a perfect substitute for its implicit equity derived from the NPV of the bank's core project. To see this, assume that at date 0 the bank's owner-manager is endowed with wealth $k < 1$ that she invests into the bank as equity, and finances the rest ($1 - k$ for the core project and X for the market-based investment) with debt. Note that k can also be interpreted as a bank's *ex ante* capital ratio, similar to how R was interpreted as its core profitability. With explicit equity k, one can rewrite thresholds $X_{\min}$

[15] We can obtain similar figures with alternative measures, such as net interest margins instead of net income over total assets to capture banks' core profitability; non-loan assets growth or securities and trading assets growth instead of total assets growth to capture the banks' investment in side activities; as well as for U.S. and European banks separately (although in this case the sample sizes shrink).

and $X_{\max}$ of the benchmark model ((16) and (19)) as:

$$X_{\min}^{k} = \frac{(1-p)(R-1+k)}{p\alpha-(1-p)(1-\theta)-\varepsilon}, \text{ and} \tag{40}$$

$$X_{\max}^{k} = \frac{p(R-1+k)-b(1-k)}{b-p\alpha+(1-p)(1-\theta)}, \tag{41}$$

and the threshold b^* (21) as:

$$b_k^* = \frac{(p(\alpha-\varepsilon)-(1-p)(1-\theta))(R-1+k)}{(1-p)(R-1+k)+(1-k)(p\alpha-\varepsilon-(1-p)(1-\theta))}. \tag{42}$$

Note that bank capital k and profitability $R-1$ only enter the above expressions as a sum, implying that they have identical and substitutable impacts on the model's outcomes. This is not surprising: both represent the shareholder's value at stake. Accordingly, one can show that $\partial b_k^*/\partial k > 0$ (similar to the effect for R in Proposition 1): an increase in explicit bank capital expands the range of parameter values for which the bank chooses the risky market-based investment, because it increases the bank's ability to borrow. All other effects of the model also persist with explicit bank capital.

The idea that bank capital may be ineffective in preventing bank risk-taking is consistent with empirical evidence that the link between pre-crisis bank capital and bank performance during the 2008 crisis was tenuous or positive (i.e., better-capitalized banks appeared to take as much or more risk). In Beltratti and Stulz (2012), higher pre-crisis capital improves bank performance during the 2008 crisis, but only in a sample that includes banks from emerging market economies and even then not in all specifications. In Berger and Bouwman (2013), higher capital improves U.S. banks' performance during multiple banking crises, but not specifically during the 2008 crisis. Coccorese and Girardone (2017) find in a sample of banks from 77 developed and developing economies that over 2000-2013 better capitalized banks were more profitable yet also riskier: they invested more in non-traditional assets. In contrast, studies that focus exclusively on banks in advanced economies during the 2008 crisis suggest a weak or negative link between pre-crisis bank capital and performance. Huang and Ratnovski (2009) on a sample of large OECD banks find no relationship between banks' pre-crisis capital and performance during the 2008 crisis. Camara et al. (2013)

show that better-capitalized European banks took more risk before the 2008 crisis. IMF's Global Financial Stability Report (2009) finds that major global banks that were intervened in during the crisis had statistically higher capital metrics before the crisis than the non-intervened banks.[16] Our model explains how this positive relationship between pre-crisis bank capital and bank risk during the 2008 crisis may have arisen.[17]

While in our model bank capital *per se* cannot reduce bank risk-taking incentives, effective minimum bank capital requirements can. The easiest way to see this is to consider leverage requirements – limits on bank balance sheet size as a multiple of bank equity. For example, restricting the banks' ability to lever up to a multiple $1/b^{reg} \leq 1/b_k^*$ of equity would prevent bank risk-taking in market-based investments. The reason is that the scale of side investments would be limited to at most $X_{\min}$, and the bank never finds it optimal to take risk in side investments at such restricted scale. This leverage requirement is most efficient when applied to total (implicit and explicit) bank equity. When implicit bank equity from the core project is not easily verifiable, it can also be applied to explicit equity k only. In this case, however, the requirement would be suboptimally tight.[18] It would prevent risk-taking in market-based investments, but it would also limit excessively the scale of safe market-based investments to below $X_{\min}$, with some resulting welfare cost.

6.3 Limiting or segregating banks' market-based activities

Another way to limit profitable banks' risk-taking in market-based activities might be to directly restrict their permitted scale. In our model, market-based activities would never be risky if their

[16] Also, on pre-crisis data, Barth et al. (2006) find no relationship between bank capital ratios and stability, and Bichsel and Blum (2004), Lindquist (2004), Jokipii and Milne (2008), and Angora et al. (2011) find no or negative relationship between bank capital and performance.

[17] The link between bank profitability and risk-taking also offers insights into the relationship between interbank competition and financial stability. A common argument is that low competition increases banks' profitability and reduces bank risk-taking incentives. But there are also counter-arguments, based on general equilibrium effects (Boyd and De Nicoló, 2005), or the fact that in the absence of competition banks become less efficient and as a result unstable (Carlson and Mitchener, 2006; Calomiris and Haber, 2013; Akins et al., 2016). Our paper suggests another reason why restricted competition may make banks riskier. A lack of interbank competition may increase bank profits in the core business (cf. Boot and Thakor, 2000), enabling the bank to expand its side activities and use them for opportunistic risk-taking.

[18] The leverage multiple derived disregarding implicit equity from the core project is higher than the true leverage multiple: $1/b_{k-explicit}^* = \frac{(1-p)k+(1-k)(p\alpha-\varepsilon-(1-p)(1-\theta))}{(p(\alpha-\varepsilon)-(1-p)(1-\theta))k} > 1/b_k^*$. Whereas restricting the multiple to $1/b_k^*$ implements the scale of market-based investments equal to $X_{\min}$, restricting the multiple to $1/b_{k-explicit}^*$ therefore implements the scale below $X_{\min}$ – i.e. inefficiently foregoing some safe market-based investment opportunities.

scale was limited to $X < X_{\min}$. Yet there may be practical impediments to implementing quantitative limits on banks' market-based activities. For example, banks may shift to risk-taking through derivatives exposures, the scale of which might be hard to measure. An alternative policy might be to ban banks' market-based activities outright. To an extent that banks engage in welfare-destroying risky market–based activities, this may be desirable. But there is a cost: welfare-enhancing safe market-based investments (at a scale up to $X_{\min}$) would be banned too. Furthermore, some market-based activities, such as hedging, might be essential for the bank's core business, raising concerns about the practicality of an outright ban.

In our model, the two bank activities, core business and side investments, are housed within a single legal entity. Some recent policy debate has highlighted a possibility of the segregation of bank activities into different legal entities, with firewalls between them to ensure their individual stability. Recall that, by assumption (5), standalone market-based activities are impossible in our model: we consider those market-based activities that rely on the implicit equity from the bank's core business. This means that placing market-based activities in a subsidiary without any capital recourse to the core business is equivalent to banning them outright. A better solution in the context of our model might be to allow limited capital recourse, so as to implement $X < X_{\min}$.[19] This arrangement might be especially useful when the size of the capital recourse is more easily verifiable than the scale of market-based exposures.

7 Conclusions

This paper studies risk-taking incentives in banks. Traditional Jensen and Meckling (1976) intuition suggests that more profitable banks should have lower risk-taking incentives. But in the run up to the recent crisis many profitable financial institutions became exposed to risky financial instruments, realizing significant losses. Understanding this contradiction appears important for both bank risk management and systemic risk regulation.

[19]The capital of the segregated entity that enables the maximum scale of safe market-based investment but does not give rise to risk-shifting, E, can be derived by reformulating the threshold equation (21) as $b = \frac{(p(\alpha-\varepsilon)-(1-p)(1-\theta))E}{(1-p)E+p\alpha-\varepsilon-(1-p)(1-\theta)}$, giving: $E = \frac{b(p\alpha-\varepsilon-(1-p)(1-\theta))}{p(\alpha-\varepsilon)-(1-p)(1-\theta+b)}$.

Our model highlights that many banks are organized around a stable core business and take risk through scalable market-based side investments. In the presence of leverage constraints, more profitable banks can borrow more and make side investments on a larger scale. Larger scale makes risk-taking more attractive. Thus, when banks choose not only the risk of their assets, but also the size of the risky exposure, the indirect scale-related effect can offset the traditional effect where more profitable banks have lower incentives to take risk of fixed size. As a consequence, more profitable banks may have higher, rather than lower, risk-taking incentives. We also show that banks have higher risk-taking incentives when their risky investments can be financed with senior funding (e.g., repos), and when bank leverage constraints are looser (consistent with a weakening of bank capital regulation pre-crisis in many advanced economies). Banks may strategically combine high effort in the core business with opportunistic risk-taking in side activities. Accommodative monetary policy makes banks' core activities safer, but their side activities riskier. Overall, the description of bank risk-taking as occurring in non-core bank activities, as well as the cross-sectional patterns of bank risk-taking predicted by the model, appear to match some patterns of bank risk-taking in the run-up to the recent crisis.

The key implication of this analysis is that higher bank profitability (or, similarly, higher bank capital or franchise value) is not by itself panacea against risk-taking. Profitable banks have superior capacity to borrow and therefore can rapidly accumulate risks. Bank risk-taking should be understood as a dynamic concept. Regulators need to consider not only contemporaneous bank risks, but also the banks' ability to increase risk going forward. Such dynamic effects are particularly relevant when banks have easy access to scalable market-based investments. The financial markets have become much deeper since late 1990s. Accordingly, the banks' ability to quickly accumulate large-scale exposures has likely increased - as was evidenced during the crisis (Morrison and Wilhelm, 2007; Boot, 2014). Therefore, the concerns about banks' risky non-core activities that are highlighted by our study are likely to remain pertinent going forward.

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A Proofs

A.1 Proof of Lemma 1

First, we show that the bank's strategy for choosing a risky over a safe market-based investment reduces itself to a condition on b (i.e., $b < b^*$). Second, we show that there exist parameter values such that the intersection between conditions $b < b^*$ and $\theta_{\min} < \theta < \theta_{\max}$ is non-empty.

Recall from (16) and (17) that the bank makes a risky investment only when the investment can be undertaken at a sufficient scale $X > X_{\min}$, conditional on that a minimal seniority $\theta > \theta_{\min}$ is offered to date 1 creditors. Also, recall from (19) and (20) that the bank has the ability to make a risky investment at a maximal scale $X_{\max}$, conditional on promising to date 1 creditors a repayment in case of bankruptcy that does not exceed the bank's available resources, $\theta < \theta_{\max}$. Substituting from (16) and (19) and rearranging terms gives that $X_{\min} < X_{\max}$ for:

$$b < b^* = \frac{(p(\alpha - \varepsilon) - (1-p)(1-\theta))(R-1)}{(1-p)(R-1) + p\alpha - \varepsilon - (1-p)(1-\theta)}.$$

The bank's profit from the risky investment is increasing in X: by differentiation (use (12)) $\partial \Pi_{Risky}(X)/\partial X = p\alpha - (1-p)(1-\theta)$ which is positive for $\theta > \theta_{\min}$. Thus, the bank chooses the maximal scale $X = X_{\max}$ whenever $X_{\min} < X_{\max}$. Note that $X_{\max} > 0$ if $b < p(R-1)$. For $\theta > \theta_{\min}$, $X_{\max} > 0$ if $b < b^*$, since then $X_{\max} > X_{\min} > 0$, implying that $b^* < p(R-1)$.

For $b > b^*$, the bank makes the safe market-based investment. Its scale is given by $\Pi_{Safe} = b(1 + X)$ (from (1) and (7)), which gives: $X_{\max}^{Safe} = (R - 1 - b)/(b - \varepsilon)$.

Next we show that there exist parameter values such that the intersection between conditions $b < b^*$ (corresponding to $X_{\min} < X_{\max}$) and $\theta_{\min} < \theta < \theta_{\max}$ is non-empty. Namely, we derive the conditions under which risk-taking equilibrium exists. For this, we rewrite the condition $X_{\min} < X_{\max}$ as a restriction on the space of θ rather than b as we have it in (21).

Rearranging the terms in (21) gives that $X_{\min} < X_{\max}$ holds for

$$\theta > \theta^e = 1 - \frac{p\alpha}{1-p} + \frac{(1-p)(R-1)b + \varepsilon(p(R-1)-b)}{(1-p)(R-1-b)}, \tag{43}$$

corresponding to $b < b^*$.

Recall that for $\theta < \theta_{\max}$, we have that $X_{\max} < \frac{R}{\theta}$, and that for $\theta > \theta^e$, we have just proved that $X_{\min} < X_{\max}$. To find whether the risk-taking takes place under $\theta_{\min} < \theta < \theta_{\max}$, we need to show that $(\exists)$ $\theta_{\max} > \theta^e$, such that $X_{\max} < \frac{R}{\theta}$ $(\forall)$ $\theta \in [\theta^e, \theta_{\max})$.

To proceed, we first establish the correspondence and the intersection points of $X_{\min}$, $X_{\max}$ and $\frac{R}{\theta}$ as functions of θ. By differentiation (use (16) and (19)):

$$\frac{\partial X_{\min}}{\partial \theta} = -\frac{(1-p)^2(R-1)}{(p\alpha - \varepsilon - (1-p)(1-\theta))^2} < 0, \tag{44}$$

$$\frac{\partial X_{\max}}{\partial \theta} = \frac{(1-p)(p(R-1)-b)}{(b - p\alpha + (1-p)(1-\theta))^2} > 0. \tag{45}$$

Note also that $\partial(\frac{R}{\theta})/\partial\theta = -\frac{R}{\theta^2} < 0$. Using (16) we obtain that $X_{\min} < \frac{R}{\theta}$ for

$$\theta > \hat{\theta} = R\left(1 - \frac{p\alpha - \varepsilon}{1-p}\right), \tag{46}$$

where $\hat{\theta} > \theta_{\min}$ from (17).

This implies that the decreasing function $X_{\min}(\theta)$ intersects from above with the decreasing function $(\frac{R}{\theta})$ at $\theta = \hat{\theta}$, and intersects from above with the increasing function $X_{\max}(\theta)$ at $\theta = \theta^e$. Also, the increasing function $X_{\max}(\theta)$ intersects from below with $\frac{R}{\theta}$ at $\theta = \theta_{\max}$.

Given that, $\theta_{\max} > \theta^e$ (risk-taking region exists) if and only if $(\forall)$ $\theta \geq \theta^e$, both functions $X_{\max}(\theta)$ and $\frac{R}{\theta}$ lie above $X_{\min}(\theta)$. This is equivalent to showing that $(\exists)$ $\theta^e \in (\hat{\theta}, 1)$, such that $X_{\min} < X_{\max}$ $(\forall)$ $\theta > \theta^e$. We proceed with finding the conditions for $\hat{\theta} < \theta^e < 1$.

Using (43) and (46) and rearranging terms we can rewrite these restrictions with respect to b.

We can show that $\hat{\theta} < \theta^e$ holds for

$$b > b_{low} = \frac{(R-1)(1-p-p\alpha+\varepsilon)+\varepsilon(1-p)}{2-2p-p\alpha-\varepsilon},$$

and that $\theta^e < 1$ holds for

$$b < b_{high} = \frac{p(R-1)(\alpha-\varepsilon)}{(1-p)(R-1)+p\alpha-\varepsilon}.$$

implying that $\hat{\theta} < \theta^e < 1$ is equivalent to the condition $b_{low} < b < b_{high}$.

From (4) and (5) it must be that both b_{low} and b_{high} are in the range $(p\alpha, R-1)$. Note that $b_{high} < R-1 \Leftrightarrow \frac{p(\alpha-\varepsilon)}{(1-p)(R-1)+p\alpha-\varepsilon} < 1 \Leftrightarrow \varepsilon < R-1$ (which holds from (4) and (5)). To complete the proof we distinguish between two possible configurations of parameters. For the first configuration, $p\alpha < b_{low} < b_{high}$, which holds for:

$$\begin{cases} 1+p\alpha+\frac{(1-p)(p\alpha-\varepsilon)}{1-p-p\alpha+\varepsilon} < R < \frac{1-p}{1-p-p\alpha+\varepsilon} \\ \varepsilon < \frac{\alpha(2p+p\alpha-1)}{1+\alpha} \\ p(1+\alpha)-1 > -p \end{cases}.$$

For the second configuration, $b_{low} < p\alpha < b_{high}$, which holds for:

$$\begin{cases} 1+\alpha < R < 1+p\alpha+\frac{(1-p)(p\alpha-\varepsilon)}{1-p-p\alpha+\varepsilon} \\ \varepsilon < \frac{\alpha(2p+p\alpha-1)}{1+\alpha} \\ p(1+\alpha)-1 > -p \end{cases}.$$

Thus, there $(\exists)$ $\theta^e \in (\hat{\theta}, 1)$, such that $X_{\min} < X_{\max}$ $(\forall)$ $\theta > \theta^e$, whenever

$$\begin{cases} 1+\alpha < R < \frac{1-p}{1-p-p\alpha+\varepsilon} \\ \varepsilon < \frac{\alpha(2p+p\alpha-1)}{1+\alpha} \\ p(1+\alpha)-1 > -p \end{cases}. \tag{47}$$

QED

A.2 Proof of Proposition 1

By differentiation (use (21)):

$$\frac{\partial b^*}{\partial R} = \frac{(p(\alpha - \varepsilon) - (1-p)(1-\theta))(p\alpha - \varepsilon - (1-p)(1-\theta))}{((1-p)(R-1) + p\alpha - \varepsilon - (1-p)(1-\theta))^2}. \tag{48}$$

Recall that from (17):

$$\theta > \theta_{\min} = 1 - \frac{p\alpha - \varepsilon}{1-p}. \tag{49}$$

Substituting into the numerator of (48) obtains:

$$\frac{\partial b^*}{\partial R} = \frac{(1-p)^2(\theta - \theta_{\min} + \varepsilon)(\theta - \theta_{\min})}{((1-p)(R-1) + p\alpha - \varepsilon - (1-p)(1-\theta))^2}.$$

The numerator of (48) is positive for $\theta > \theta_{\min}$. The denominator is a square, and thus it is always positive. Therefore, $\partial b^*/\partial R > 0$. *QED*

A.3 Proof of Proposition 2

By differentiation (use (21)):

$$\frac{\partial b^*}{\partial \theta} = \frac{(1-p)^2(R-1)(R-1-\varepsilon)}{((1-p)(R-1) + p\alpha - \varepsilon - (1-p)(1-\theta))^2}. \tag{50}$$

Recall that $R - 1 > \varepsilon$ from (4) and (5), therefore, the numerator is positive. The denominator is a square, and thus it is always positive. Hence $\partial b^*/\partial \theta > 0$.

Also, by differentiation (use (19)):

$$\frac{\partial X_{\max}}{\partial \theta} = (1-p)\,\frac{p\,(R-1) - b}{(b - p\alpha + (1-p)(1-\theta))^2}. \tag{51}$$

From (5) and (19), the numerator is positive. The denominator is a square, and thus it is always positive. Therefore, $\partial X_{\max}/\partial \theta > 0$.

Further, by differentiation (use (21)):

$$\frac{\partial^2 b^*}{\partial R \partial \theta} = \frac{(1-p)^2\left((R-1)\varepsilon(1-p) + (2(R-1)-\varepsilon)\left(p\alpha - \varepsilon - (1-\theta)(1-p)\right)\right)}{\left((1-p)(R-1) + p\alpha - \varepsilon - (1-p)(1-\theta)\right)^3}. \tag{52}$$

The numerator and denominator are both positive because $p\alpha - \varepsilon - (1-p)(1-\theta) > 0$ for $\theta > \theta_{\min}$. Finally:

$$\frac{\partial^2 X_{\max}}{\partial R \partial \theta} = \frac{p(1-p)}{\left(b - p\alpha + (1-p)(1-\theta)\right)^2} > 0. \tag{53}$$

QED

A.4 Proof of Proposition 3

First, we find the threshold b^{**}. The threshold b^{**} is defined by $r_0^{Risky} = r_0^{Prevent}$. Since $r_0^{Prevent}$ is defined by $X_{\min}(r_0) = X_{\max}(r_0)$, equation $r_0^{Risky} = r_0^{Prevent}$ is equivalent to $X_{\min}(r_0^{Risky}) = X_{\max}(r_0^{Risky})$. Substituting r_0^{Risky} from (25) into (26) and (24) and rearranging terms obtains:

$$X_{\min}(r_0^{Risky}) = \frac{(1-p)((R-1)(b - p\alpha + (1-p)(1-\theta)) + b\theta(1-p))}{p(b - p\alpha + 1 - p)(p\alpha - \varepsilon - (1-p)(1-\theta))}, \text{ and} \tag{54}$$

$$X_{\max}(r_0^{Risky}) = \frac{R-1-b}{b - p\alpha + 1 - p}. \tag{55}$$

From here, $X_{\min}(r_0^{Risky}) < X_{\max}(r_0^{Risky})$ whenever $b < b^{**}$, where

$$b^{**} = \frac{(R-1)(p\alpha - p\varepsilon - (1-p)(1-\theta))}{(1-p)(R-1) + p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta}. \tag{56}$$

Next, we show the existence of risk-taking region $b < b^{**}$ within the feasible range of model's parameter values. Risk-taking exists if the intersection between $X_{\min}(r_0^{Risky}) < X_{\max}(r_0^{Risky})$ and $\theta X_{\max}(r_0^{Risky}) < R$ is non-empty for the following $b : b > p\alpha$ (see (5)) and $\theta : \theta_{min} < \theta \leq 1$ (see (17)).

Note that $X_{\min}(r_0^{Risky}) < X_{\max}(r_0^{Risky})$ holds for $b < b^{**}$ and $\theta X_{\max}(r_0^{Risky}) < R$ holds for:

$$b > \frac{\theta(R-1) - R(1-p-p\alpha)}{R+\theta} \equiv \underline{b}. \tag{57}$$

From (5) and (57) we obtain that the bank takes risk when $b^{**} > \max[p\alpha, \underline{b}]$. Next, we consider two cases: (1) when $p\alpha > \underline{b}$, and risk-taking occurs when $b^{**} > p\alpha$, and (2) when $p\alpha \leq \underline{b}$, and risk-taking occurs when $b^{**} > \underline{b}$.

Case $p\alpha > \underline{b}$. Using (57) we obtain that $p\alpha > \underline{b}$ whenever:

$$\theta < \frac{R(1-p)}{R-1-p\alpha} \equiv \bar{\theta}. \tag{58}$$

When $\theta < \bar{\theta}$, the constraint $b^{**} > p\alpha$ is binding. Note that $b^{**} > p\alpha$ for:

$$\theta > \theta^a_{\min} = \frac{p^2\alpha(p\alpha - \varepsilon - 1 + p) - (R-1)(p^2\alpha - p\varepsilon - 1 + p)}{(1-p)(R-1-p\alpha)}, \tag{59}$$

and from (17): $\theta_{\min} < \theta^a_{\min}$. From (58), it must be that $\theta^a_{\min} < \min[1, \bar{\theta}]$. Note that $\bar{\theta} < 1$ holds for:

$$R > \frac{1+p\alpha}{p} \equiv \underline{R}, \tag{60}$$

implying that for $R > \underline{R}$, the relevant restriction is $\theta^a_{\min} < \bar{\theta}$ which holds for:

$$R < R_{high} \equiv 1 + p\alpha + \frac{(1-p)^2}{p(\varepsilon - p\alpha + 1 - p)}. \tag{61}$$

Conversely, for $R < \underline{R}$, the relevant restriction is $\theta^a_{\min} < 1$ which holds for:

$$R > R_{low} \equiv 1 + p\alpha + \frac{(1-p)^2\alpha}{p\alpha - \varepsilon}. \tag{62}$$

Thus, risk-taking takes place if either (a) $R_{low} < R < \underline{R}$ or (b) $\underline{R} \leq R < R_{high}$. Alternatively, bank takes risk whenever (R_{low}, R_{high}) is non-empty. Using (61) and (62) we can show that $R_{low} <$

R_{high} holds for

$$\varepsilon < \frac{p^2\alpha(1+\alpha)}{1+p\alpha} \equiv \tilde{\varepsilon}. \tag{63}$$

Note that $\tilde{\varepsilon} > p(1+\alpha) - 1$, i.e. higher than the lower bound of ε (see (2)). Therefore, the bank takes risk for the following set of parameters:

$$\varepsilon < \tilde{\varepsilon} \tag{64}$$

$$R_{low} < R < R_{high} \tag{65}$$

$$\theta^a_{\min} < \theta < \min[1, \bar{\theta}] \tag{66}$$

$$p\alpha < b < b^{**} \tag{67}$$

Case $p\alpha < \underline{b}$. When $\theta \geq \bar{\theta}$ the constraint $b^{**} > \underline{b}$ is binding. From (28) and (57) we obtain that $b^{**} > \underline{b}$ for:

$$\theta > \frac{pR(\varepsilon - p\alpha + 1 - p)}{1-p} \equiv \theta^b_{\min}, \tag{68}$$

and from (17) $\theta^b_{\min} > \theta_{\min}$ for $pR > 1$.

From (68), it must be that $\theta \geq \max[\bar{\theta}, \theta^b_{\min}]$. Note that $\bar{\theta} \leq \theta^b_{\min}$ for $R \geq R_{high}$, and $\theta^b_{\min} < 1$ holds for:

$$R < \frac{1-p}{p(\varepsilon - p\alpha + 1 - p)} \equiv \bar{R}. \tag{69}$$

Thus, risk-taking takes place for $R_{high} \leq R < \bar{R}$. Otherwise, bank takes risk if $\bar{\theta} > \theta^b_{\min}$, i.e. when $\underline{R} < R < R_{high}$. To sum up, bank takes risk whenever $\underline{R} < R < \bar{R}$.[20] This interval is non-empty for $\varepsilon < \tilde{\varepsilon}$. Therefore, the bank takes risk for the following set of parameters:

$$\varepsilon < \tilde{\varepsilon} \tag{70}$$

$$\underline{R} < R < \bar{R} \tag{71}$$

$$\min[\bar{\theta}, \theta^b_{\min}] < \theta < 1 \tag{72}$$

$$\underline{b} < b < b^{**} \tag{73}$$

[20]Note that all thresholds of R are higher than $1 + p\alpha$.

Summing up two cases, the bank takes risk for the following set of parameters:

$$\varepsilon < \tilde{\varepsilon} \tag{74}$$

$$R_{low} < R < \bar{R} \tag{75}$$

$$\theta^a_{\min} < \theta < 1 \tag{76}$$

$$\max[p\alpha, \underline{b}] < b < b^{**} \tag{77}$$

QED

A.5 Proof of Proposition 4

Differentiating b^{**} from (28) with respect to R gives:

$$\frac{\partial b^{**}}{\partial R} = \frac{(p\alpha - p\varepsilon - (1-p)(1-\theta)) \cdot (p(p\alpha - \varepsilon - (1-p)(1-\theta))(1-p)^2\theta)}{((1-p)(R-1) + p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta)^2}. \tag{78}$$

From (17), the term $p\alpha - p\varepsilon - (1-p)(1-\theta)$ in the numerator of (78) is positive, therefore the numerator is positive. The denominator is a square, and thus is also positive. Therefore, $\partial b^{**}/\partial R > 0$. Likewise, by differentiating (28) with respect to θ we obtain:

$$\frac{\partial b^{**}}{\partial \theta} = \frac{(1-p)^2(R-1)(R-p-p\alpha)}{((1-p)(R-1) + p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta)^2}. \tag{79}$$

From (4) and (5), the numerator is positive. The denominator is a square, and thus is also positive. Therefore, $\partial b^{**}/\partial \theta > 0$.

Furthermore, the cross-derivative of b^{**} with respect to R and θ is:

$$\begin{aligned} \frac{\partial^2 b^{**}}{\partial \theta \partial R} &= \frac{(1-p)^2(R-p-p\alpha)(p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta)}{((1-p)(R-1) + p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta)^3} + \\ &+ \frac{(1-p)^2(R-1)(p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta + (1-p)(p(1+\alpha)-1))}{((1-p)(R-1) + p(p\alpha - \varepsilon - (1-p)(1-\theta)) + (1-p)^2\theta)^3}. \end{aligned} \tag{80}$$

The numerator of the first item is positive for $\theta > \theta_{\min}$ and $R > p(1+\alpha)$, which is true from (4) and (5). The numerator of the second item is also positive for any $\theta > \theta_{\min}$. Thus, $\frac{\partial^2 b^{**}}{\partial\theta\partial R} > 0$.

QED

A.6 Proof of Proposition 5

Substitute (38) and (39) into $e^*_{Risky} > e^*_{Safe}$ to obtain:

$$\frac{b}{c} \cdot \frac{p\,(R-1-\alpha) + (1-p)\,(1-\theta)}{b - p\alpha + (1-p)(1-\theta)} > \frac{b}{c} \cdot \frac{R-1-\varepsilon}{b-\varepsilon},$$

Rearranging the terms gives:

$$p(R-1-\alpha)(b-\varepsilon) - (1-p)(1-\theta)(R-1-b) - (R-1-\varepsilon)(b-p\alpha) > 0.$$

Rearranging the terms again gives:

$$b < \frac{(p(\alpha-\varepsilon) - (1-p)(1-\theta))(R-1)}{(1-p)(R-1) + p\alpha - \varepsilon - (1-p)(1-\theta)}.$$

The expression on the right-hand side is equal to b^* from (21), implying that $e^*_{Risky} > e^*_{Safe}$ for $b < b^*$. Recall from Section 3 that the banker makes risky market-based investment if $b < b^*$, with b^* from (21). *QED*

A.7 Proof of Proposition 6

When the policy rate i affects the bank's cost of funding, $X_{\min}$ (similar to (16)) changes to:

$$X > X^i_{\min} = \frac{(1-p)(R-1-i(1-\gamma))}{p\alpha - \varepsilon - (1-p)(1-\theta)}, \tag{81}$$

and $X_{\max}$ (similar to (19)) changes to:

$$X \leq X^i_{\max} = \frac{p(R-1-i(1-\gamma)) - b}{b - p\alpha + (1-p)(1-\theta) + i(1-\gamma)}. \tag{82}$$

The bank makes a risky investment if $X^i_{\min} < X^i_{\max}$, or when $b < b^*_i$. From (37):

$$b^*_i = \frac{(R - 1 - i(1-\gamma))\,(p(\alpha - \varepsilon) - (1-p)(1 - \theta + i(1-\gamma)))}{(1-p)(R - 1 - i(1-\gamma)) + p\alpha - \varepsilon - (1-p)(1-\theta)}. \tag{83}$$

Note that $b^*_i > 0$ if $\theta > 1 + i(1-\gamma) - \frac{p(\alpha-\varepsilon)}{1-p}$. From (17): $\theta_{\min} > 1 + i(1-\gamma) - \frac{p(\alpha-\varepsilon)}{1-p}$ for $i(1-\gamma) < \varepsilon$ (which typically is true in reality, since ε is the return on treasury securities). Thus, for any $\theta > \theta_{\min}$, we obtain $b^*_i > 0$.

Next, by differentiation (use (37)):

$$\frac{\partial b^*_i}{\partial i} = -(1-\gamma)\cdot\left[\frac{(p(\alpha-\varepsilon) - (1-p)(1-\theta+i(1-\gamma)))\cdot(p\alpha - \varepsilon - (1-p)(1-\theta))}{((1-p)(R-1-i(1-\gamma)) + p\alpha - \varepsilon - (1-p)(1-\theta))^2}\right.$$
$$\left.+\frac{(1-p)(R-1-i(1-\gamma))\cdot((1-p)(R-1-i(1-\gamma)) + p\alpha - \varepsilon - (1-p)(1-\theta))}{((1-p)(R-1-i(1-\gamma)) + p\alpha - \varepsilon - (1-p)(1-\theta))^2}\right]. \tag{84}$$

For any $\theta > \theta_{\min}$, both terms of the expression are negative, implying that $\frac{\partial b^*_i}{\partial i} < 0$. Similarly, by differentiation (use (38) and (39)):

$$\frac{\partial e^*_{Safe,i}}{\partial i} = -\frac{b}{c}\cdot\frac{(1-\gamma)(R-1-\varepsilon)}{(b-\varepsilon+i(1-\gamma))^2}, \tag{85}$$

$$\frac{\partial e^*_{Risky,i}}{\partial i} = -\frac{b}{c}\cdot\frac{(1-\gamma)(p(b-p\alpha+(1-p)(1-\theta)-i(1-\gamma))+p(R-1)-b)}{(b-p\alpha+(1-p)(1-\theta)+i(1-\gamma))^2}. \tag{86}$$

Recall that $R - 1 \geq b > p\alpha > \varepsilon$ from (4) and (5). Therefore, the numerator is positive in both expressions. The denominator is a square in both expressions, and thus it is always positive. Hence $\partial e^*_{Safe,i}/\partial i < 0$ and $\partial e^*_{Risky,i}/\partial i < 0$. *QED*

Figure 1. The timeline.

Date 0

- A bank attracts 1 unit of funds at the interest rate r_0 to invest in the core project.

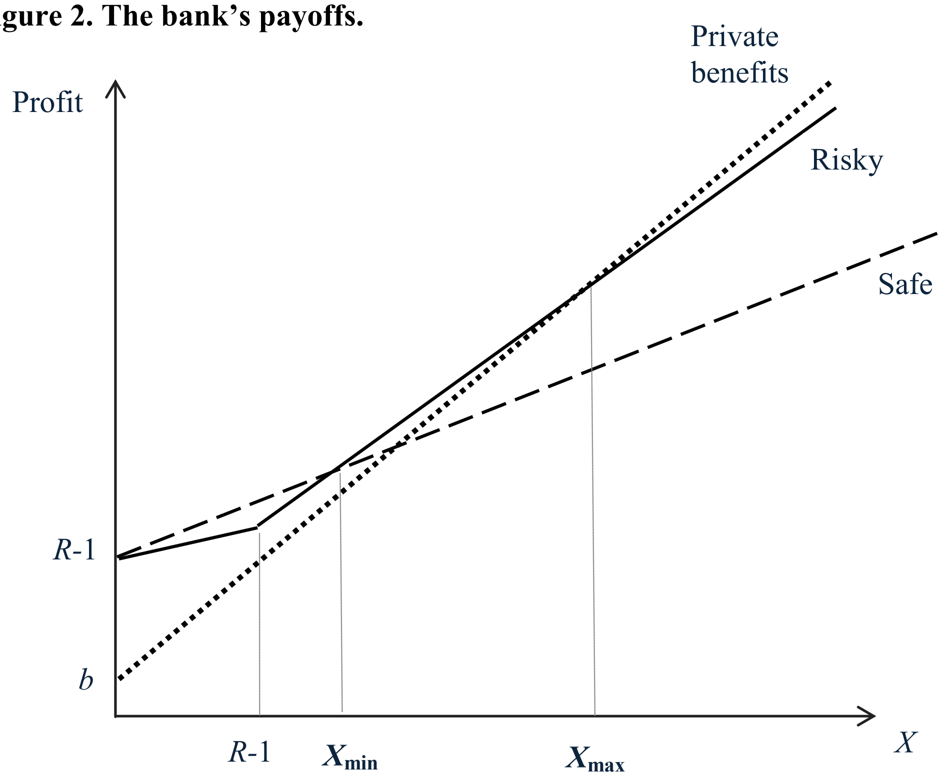
Date 1

- A bank attracts X units of funds at the interest rate r_1 to undertake a market-based investment
- A bank can convert its assets into private benefits $b(1+X)$.

Date 2

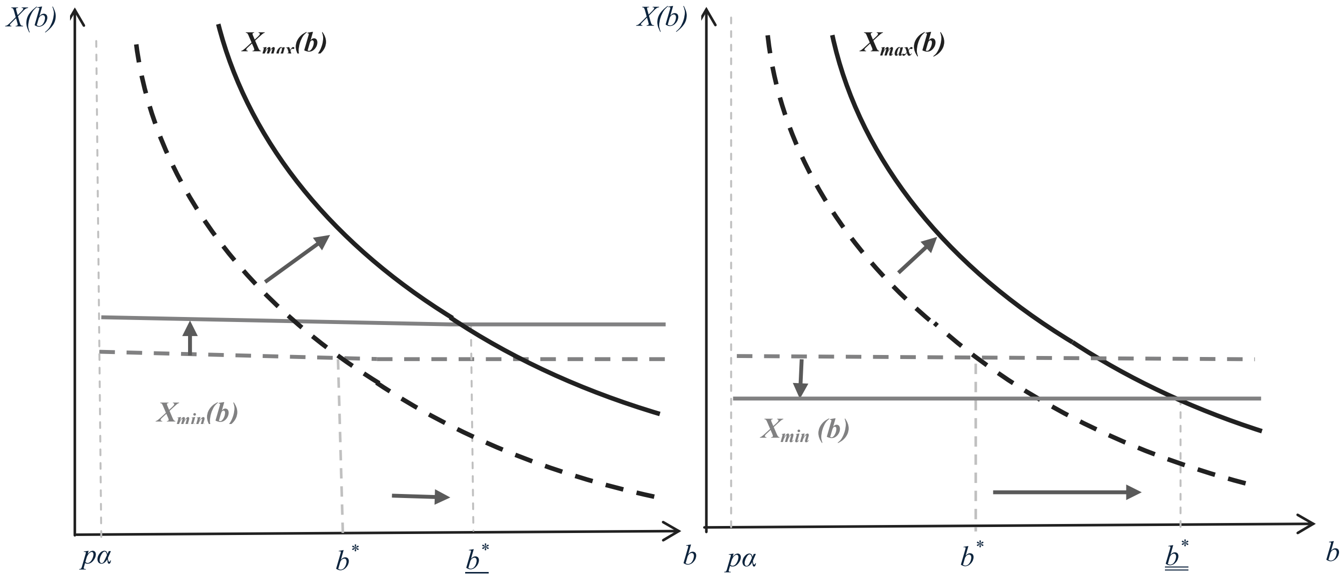
- Projects returns are realized and distributed.

Figure 2. The bank's payoffs.



The figure plots the bank's expected returns from three alternative projects undertaken alongside the core project. The solid line shows the bank's return from the risky market-based investment. For X=0, the bank's return is the core return R-1. For $X \leq R$-1, the bank internalizes the downside risk realizations; the slope of the line is thus gradual (positive if the NPV of the risky project is positive, or negative otherwise). For $X>R$-1, the bank does not internalize the downside risk realizations, and the slope of the line steepens. The dashed line represents the bank's return from the safe market-based investment. The return to the risky investment dominates the return to the safe investment for high enough scale X, $X>X_{min}$. The dotted line is the banker's private benefits $b(1+X)$, which are equal to b if no side investment is made. The bank does not abscond as long as its profit in normal operations exceeds its return from absconding. Therefore, the maximum scale of the risky investment is limited to $X \leq X_{max}$. As a result, the bank can make a risky investment for $X_{min}<X \leq X_{max}$.

Figure 3. The impact of the bank's core profitability and new debt seniority on bank risk-taking.

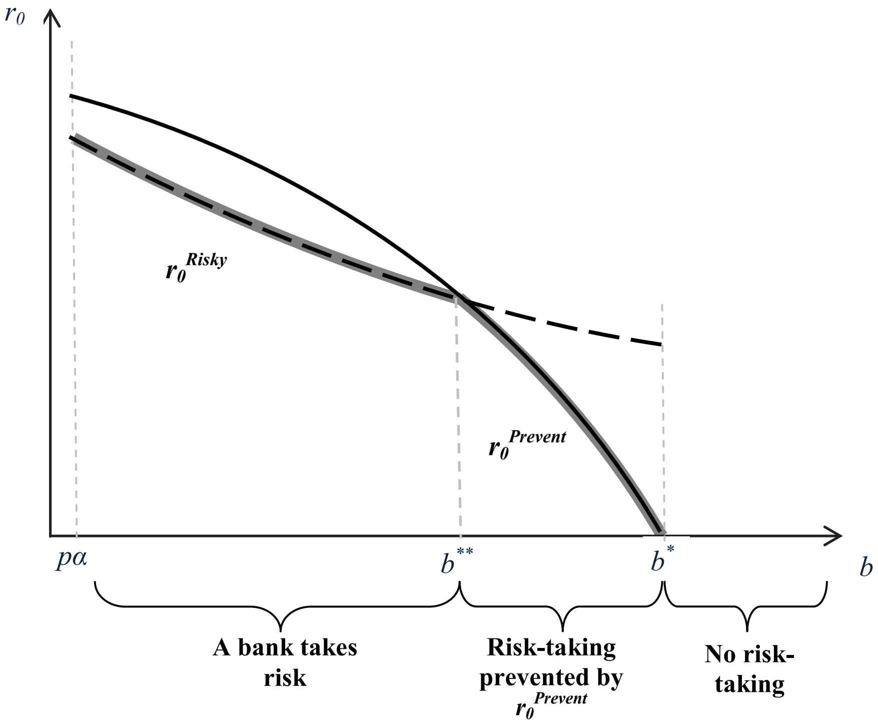


Panel A **Panel B**

Panel A shows the impact of an increase in the bank's core profitability, R, on its risk-taking strategy. A higher R increases the minimal scale at which the bank finds it profitable to make the risky market-based investment, X_{min}, as well as the maximum feasible scale of the risky investment, X_{max}. On net, the effect on X_{max} dominates, so that a higher R leads to a higher intersect b^*, indicating a wider range of parameter values for which the bank undertakes the risky investment.

Panel B shows the impact of an increase in the feasible date 1 debt seniority, θ, on the bank's risk-taking strategy. A higher θ reduces the minimal scale at which the banks finds it profitable to make the risky market-based investment X_{min}, and increases the maximum feasible scale of the risky investment X_{max}. As a result, higher θ leads to a higher intersect b^*, indicating a wider range of parameter values for which a bank undertakes the risky investment.

Figure 4. Equilibrium under endogenous r_0.



The figure shows the evolution of the interest rates required by date 0 creditors, r_0^{Risky} that prices bank risk-taking (dashed line) and $r_0^{Prevent}$ that prevents bank risk-taking (solid black line), depending on b. A looser leverage constraint (a lower b) increases both r_0^{Risky} and $r_0^{Prevent}$. Date 0 creditors choose the minimal interest rate consistent with at least breaking even under correctly anticipated bank risk-taking strategy. For $b>b^*$, $r_0=0$, and the bank makes safe market-based investment. For $b^{**}<b\leq b^*$, $r_0^{Prevent} < r_0^{Risky}$; so date 0 creditors set $r_0=r_0^{Prevent}$ and the bank makes the safe market-based investment. For $b<b^{**}$, $r_0^{Risky}< r_0^{Prevent}$; so date 0 creditors set $r_0=r_0^{Risky}$ and the bank makes the risky market-based investment. Highlighted in grey is the resulting equilibrium interest rate r_0, which is the minimum of the two possible equilibrium rates $r_0^{Prevent}$ and r_0^{Risky}. Note that b is restricted to $b>p\alpha$ under the model parametrization (equation (5)).

Figure 5. Effort in the core project: The timeline.

Date 0

- A bank attracts 1 unit of funds at the interest rate r_0 to invest in the core project
- A bank can exert effort to increase the probability of success of the core project

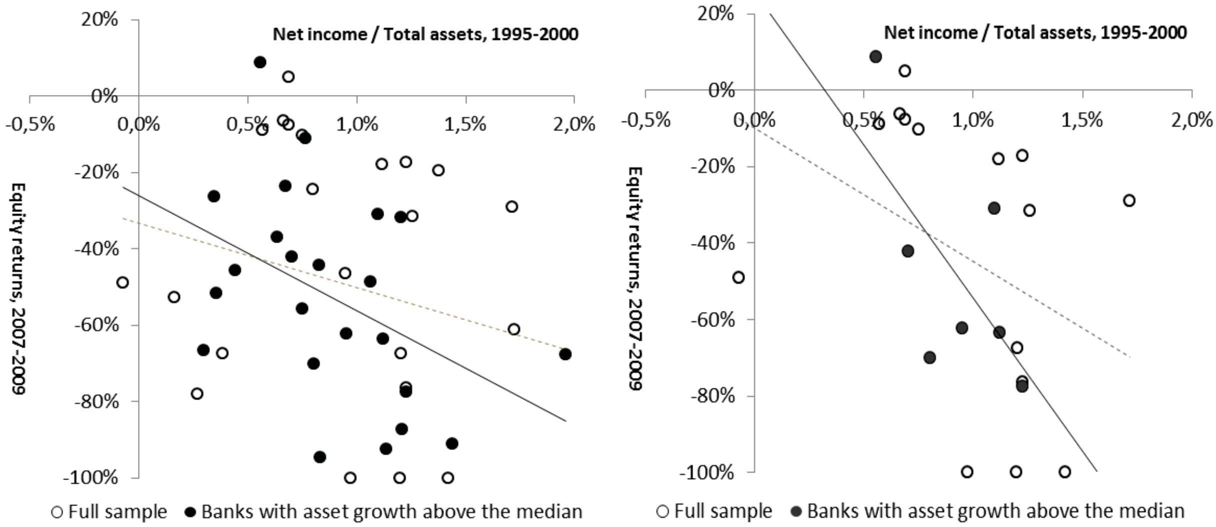
Date 1

- A bank attracts X units of funds at the interest rate r_1 to undertake a market-based investment
- A bank can convert its assets into private benefits $b(1+X)$.

Date 2

- Projects returns are realized and distributed.

Figure 6. The relationship between banks' core profitability and losses during the 2008 crisis.



Panel A **Panel B**

The scatterplots show the relationship between the banks' average net income to total assets over 1995-2000 (a proxy for a bank's core profitability) and bank equity losses during the crisis (end 2007 – end 2009, a proxy for bank risk-taking), for North American and European banks with assets over USD 50 billion at end-2006. The dashed line is the trend conditional on all observations, showing that banks that were more profitable in late 1990s experienced larger losses during the crisis, suggesting higher risk-taking. Solid dots highlight banks with above-median non-loan assets growth in 2001-2006. The solid line is the trend conditional on those observations. It is steeper than the unconditional trend, indicating that risk-taking by profitable banks was related to the expansion of their side activities. Panel A includes all banks independent on their 1995-2000 interest income to total income ratio, whereas Panel B is restricted to the sample of banks with the 1995-2000 interest income to total income ratio above the median of the full sample. Comparison of two panels highlights more striking relationship between core profitability and risk-taking for banks with higher share of relationship lending in 1995-2000 (proxied as interest income to total income ratio in 1995-2000).