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A prospect-theory model of voter turnout

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A prospect-theory model of voter turnout

Abstract

We incorporate prospect-theory preferences in a game-theoretic model to study voter turnout. We show that voter turnout is heavily affected by agents having subjective reference points with respect to the vote or abstain decision and their subjective probability weighting in the decision-making process. Using empirically based parameter values, we show that our model has lower prediction error than other game-theoretic models with standard expected-utility preferences. We also find that our model maintains desirable comparative statics effects and leads to higher turnout predictions in larger electorates.

JEL-Codes: D720.

Keywords: voting behavior, Downsian paradox, prospect-theory preferences.

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1 Introduction

Voter turnout is most often high in large-scale democratic elections. However, since the probability of being decisive in such elections is negligible, the rational choice model of Downs (1957) predicts that an individual voter should abstain from voting. The reason is that, in the presence of positive voting costs, the benefits will never outweigh the costs of voting. This is the so-called ‘Downsian paradox’, which is also known as the paradox that ate rational choice theory (Fiorina, 1990). In this paper, we use prospect theory as proposed by Kahnemann and Tversky (1976) to study voter turnout.

Prospect theory is a well-established descriptive theory in behavioral economics and cognitive psychology, and developed by Kahnemann and Tversky (1979) and Kahnemann and Tversky (1992). It has proven to provide a more accurate description of decision-making under uncertainty than the standard model of expected utility maximization in several contexts such as consumption, investment and insurance behavior (Barberis, 2013). Here, we apply it to voting.

Our approach fits in with two strands of research that have emerged to address the Downsian Paradox. The first proposes to depart from standard rational choice theory by widening the range of determinants in the cost-benefit analysis. Including non-instrumental benefits (and costs) in the voters’ utility function, derived from the act of voting, can rationalize voter turnout. These determinants include civic duty, warm glow or peer pressure (Riker and Ordeshook, 1968; Tullock, 1967), altruism (Fowler, 2006), moral behavior (Kirchgässner, 1992), or, more generally, expressive behavior (Brennan and Buchanan, 1984; Brennan and Lomasky, 1984).

The second strand of research focuses on the uncertainty of outcome in elections. Ferejohn and Fiorina (1974) pioneered this strand and impose a minimax regret strategy in a decision-theoretic framework to explain voter turnout. Many scholars criticized their approach as it relies on the assumption that voters act under full uncertainty regarding election outcomes (see e.g. Strøm, 1975; Beck,

1975; Stephens, 1975). Despite this caveat, their notion of uncertainty of outcomes in elections has spurred a series of game-theoretic models analyzing voter turnout.

In particular, Palfrey and Rosenthal (1983) argue that, with a game-theoretic model, the probabilities of being decisive and turnout decisions are simultaneously determined in a noncooperative (Nash) equilibrium (i.e. the probability of being decisive is not exogenous). They show that with homogeneous voting costs, equilibria with significant voter turnout do exist. Palfrey and Rosenthal (1985) make, arguably, assumptions that are more realistic such as heterogeneous voting costs as well as private information about voting costs. These more realistic assumptions unfortunately predict that voter turnout converges to zero when the electorate grows very large, since only voters with negligible voting costs will participate in the elections.

Levine and Palfrey (2007) study a laboratory experiment based on a reconsideration of the Palfrey and Rosenthal (1985) model. They find that the model predicts experimentally observed voter turnout well. They further find empirical evidence for different comparative statics effects that follow from their model. These effects include a size effect (i.e., turnout is negatively related to electorate size), a competition effect (i.e., turnout is positively related to the closeness of the election outcome), and an underdog effect (i.e., turnout is relatively higher in the group that supports the candidate who is the underdog in the election).

In this paper, we combine the aforementioned strands of research and build a prospect-theory model of voter turnout. On the one hand, we rely on the game-theoretic framework as proposed by Levine and Palfrey (2007). On the other hand, we depart from the standard rational choice approach by including prospect-theory preferences that replace the standard expected-utility preferences. Central to prospect theory is that individuals evaluate perceived losses in a different way than perceived gains, and whether outcomes are perceived as gains or losses depend on a subjective reference point. Furthermore, individuals assign subjective (biased) weights to objective probabilities using non-standard weighting functions. Notably, Riker and Ordeshook (1968) hinted on

an approach that takes account of subjectively chosen probability weights and Quattrone and Tversky (1988) have elaborated on a similar approach. However, to the best of our knowledge, we are the first to formalize prospect theory in a game-theoretic model of voting.

We investigate the predictive performance of our prospect-theory model in explaining voter turnout observed in the experiment of Levine and Palfrey (2007), using empirically based values of our model parameters. To preview our main findings, we find that our model leads to smaller prediction errors than the already strong results obtained by Levine and Palfrey (2007) for their model with standard preferences. Our model also provides new insight in the voting decision individuals make. We find that the reference point of an individual has a particularly large impact on the propensity to vote. That is, individuals who have voting as their reference point have a much higher likelihood to vote than those that have abstention as their reference point. We also find that non-standard probability weighting (as opposed to the assumption of voters using objective probabilities) leads turnout to decrease at a slower pace and to higher turnout predictions in larger electorates. This is because individuals tend to overvalue small probabilities and to undervalue large probabilities. Consequently, the chance of being pivotal in large elections is overestimated and turnout is increased. Finally, our model maintains the interesting and intuitively appealing comparative statics effects of the model of Levine and Palfrey (2007).

The paper is organized as follows. We first review the standard game-theoretic model of voting and characterize its equilibrium in Section 2. In Section 3, we introduce prospect-theory preferences into the model and present the corresponding equilibrium conditions. Section 4 compares the properties of the prospect-theory model and standard model. We conclude in Section 5.

2 The standard model

We consider individuals who belong to group A or group B . Group A and group B have, respectively, N_A and N_B members, with $N_B > N_A \geq 1$. Note that A is the minority group and B the majority group. An election between two candidates is held and each individual in either group is given the choice of whether to vote or not. All members of group A prefer candidate A , the members of group B prefer candidate B . Whichever candidate has more votes has won the election, with ties being broken by the flip of a fair coin. We say that a group wins the election if the candidate preferred by this group wins the election. If a group wins, each member of the group receives a reward of (money) value H , while the members of the losing group each receive a reward L , with $H > L \geq 0$. An individual i who votes incurs a voting cost c_i . The value of c_i is drawn by Nature, using the density function $f(c_i)$, which is positive everywhere on its support $[0, \bar{c}]$, with $\bar{c} > 0$. The corresponding distribution function is denoted by $F(\cdot)$. We assume that $H - L > \bar{c}$. Each individual is privately informed about the size of her own voting cost before deciding whether to abstain or vote, but only knows that the voting costs of the other individuals are drawn with density function $f(\cdot)$. The voting costs of the different individuals are drawn independently. This setup is common knowledge.

Let us first consider the standard model of Levine and Palfrey (2007). We denote the number of voters excluding individual i that vote for the candidate of group A or group B by n_A^{-i} and n_B^{-i} , respectively. Tables 1 and 2 describe the expected payoffs of different outcomes for members of the two groups. The payoffs depend both on the action chosen by the individual herself, as well as on the actions of all other individuals. The payoff matrix is unique for each individual, since c_i is drawn separately for every individual.

Following Levine and Palfrey (2007), we focus on quasi-symmetric (Bayesian-Nash) equilibria in which all members of a group employ the same strategy.¹ Denoting equilibrium values with a star (*), in a quasi-symmetric equilibrium

¹Levine and Palfrey (2007, p. 145) discuss that there exists a unique quasi-symmetric equilibrium for the parameter values used in their experiment, which can be shown numerically.

Table 1: Expected payoff matrix for individual i of group A .

	Vote	Abstain
$n_A^{-i} > n_B^{-i} + 1$	$H - c_i$	H
$n_A^{-i} = n_B^{-i} + 1$	$H - c_i$	H
$n_A^{-i} = n_B^{-i}$	$H - c_i$	$\frac{H+L}{2}$
$n_A^{-i} = n_B^{-i} - 1$	$\frac{H+L}{2} - c_i$	L
$n_A^{-i} < n_B^{-i} - 1$	$L - c_i$	L

Table 2: Expected payoff matrix for individual i of group B .

	Vote	Abstain
$n_A^{-i} > n_B^{-i} + 1$	$L - c_i$	L
$n_A^{-i} = n_B^{-i} + 1$	$\frac{H+L}{2}$	L
$n_A^{-i} = n_B^{-i}$	$H - c_i$	$\frac{H+L}{2}$
$n_A^{-i} = n_B^{-i} - 1$	$H - c_i$	H
$n_A^{-i} < n_B^{-i} - 1$	$H - c_i$	H

each individual i of group A uses a cut-point strategy such that she votes if and only if $c_i < c_A^*$, where the equilibrium threshold c_A^* is shared by everyone in her group. If this individual votes, we say that $\tau_A(c_i) = 1$, else $\tau_A(c_i) = 0$. Similarly, and using obvious notation, every individual i of group B votes (so that $\tau_B(c_i) = 1$) if and only if $c_i < c_B^*$. Thus, the equilibrium is described by the pair of thresholds (c_A^*, c_B^*) .²

Given that everybody in a group has the same cut-point strategy, the equilibrium aggregate voting probabilities of the two groups, (p_A^*, p_B^*) , are

$$p_A^* = \int_0^{\bar{c}} \tau_A(c) f(c) dc = \int_0^{c_A^*} f(c) dc = F(c_A^*), \quad (1)$$

$$p_B^* = \int_0^{\bar{c}} \tau_B(c) f(c) dc = \int_0^{c_B^*} f(c) dc = F(c_B^*). \quad (2)$$

Given these voting probabilities for each group, we can calculate the following

²We assume that $c_A^* < \bar{c}$ and $c_B^* < \bar{c}$. This also holds in our numerical analysis below.

probabilities that each voter uses to calculate the benefit of casting her vote:

$$\begin{aligned} P_{A,break}^* &= Prob(\text{voter in group } A \text{ breaks a tie}) \\ &= \sum_{k=0}^{N_A-1} \binom{N_A-1}{k} \binom{N_B}{k} (p_A^*)^k (1-p_A^*)^{N_A-1-k} (p_B^*)^k (1-p_B^*)^{N_B-k}, \end{aligned} \quad (3)$$

$$\begin{aligned} P_{A,create}^* &= Prob(\text{voter in group } A \text{ creates a tie}) \\ &= \sum_{k=0}^{N_A-1} \binom{N_A-1}{k} \binom{N_B}{k+1} (p_A^*)^k (1-p_A^*)^{N_A-1-k} (p_B^*)^{k+1} (1-p_B^*)^{N_B-1-k}, \end{aligned} \quad (4)$$

$$\begin{aligned} P_{B,break}^* &= Prob(\text{voter in group } B \text{ breaks a tie}) \\ &= \sum_{k=0}^{N_A} \binom{N_A}{k} \binom{N_B-1}{k} (p_A^*)^k (1-p_A^*)^{N_A-k} (p_B^*)^k (1-p_B^*)^{N_B-1-k}, \end{aligned} \quad (5)$$

$$\begin{aligned} P_{B,create}^* &= Prob(\text{voter in group } B \text{ creates a tie}) \\ &= \sum_{k=0}^{N_A-1} \binom{N_A}{k+1} \binom{N_B-1}{k} (p_A^*)^{k+1} (1-p_A^*)^{N_A-1-k} (p_B^*)^k (1-p_B^*)^{N_B-1-k}. \end{aligned} \quad (6)$$

In equilibrium, individuals in groups A and B are indifferent between abstaining and voting if and only if the expected benefit of voting equals the cost of voting, i.e. we must have

$$P_{A,break}^* \times \left(H - \frac{H+L}{2} \right) + P_{A,create}^* \times \left(\frac{H+L}{2} - L \right) = c_A^*, \quad (7)$$

$$P_{B,break}^* \times \left(H - \frac{H+L}{2} \right) + P_{B,create}^* \times \left(\frac{H+L}{2} - L \right) = c_B^*. \quad (8)$$

The equilibrium pair of thresholds (c_A^*, c_B^*) simultaneously solves (1) – (8), and in turn yields the equilibrium aggregate voting probabilities for each group.

For later use, we also define

$$P_A^* = P_{A,break}^* + P_{A,create}^*, \quad (9)$$

$$P_B^* = P_{B,break}^* + P_{B,create}^*. \quad (10)$$

Here P_A^* denotes the equilibrium probability that an individual of group A is pivotal, i.e. breaks or makes a tie. In the same way, we can interpret P_B^* .

3 Prospect-theory model

Building on the standard model, we now introduce prospect-theory preferences and determine their impact on aggregate turnout. Hence, agents now evaluate

their actions in terms of gains and losses that result from them, relative to some reference point. In our setup the reference point can be to either abstain or to vote. A priori it is not clear which reference point is used. We therefore look at both cases in our prospect-theory model.

Tables 3 and 4 detail, for individuals of group A , the state-dependent gains and losses, respectively, when deviating from one's reference point. Since $H - L > \bar{c}$, payoffs can be unambiguously determined as either gains or losses. Each individual in the group receives an independent draw from the distribution of voting costs. Hence, the gains and loss matrices are unique for each individual i in their respective group.

Consider individual i of group A , who has the reference point of abstaining (see the second column in Tables 3 and 4). She evaluates the alternative action of voting in terms of the state-dependent gains and losses that result from that action. For example, if group A would lead in the election by more than one vote before taking into account individual i (see the third row in Tables 3 and 4), then her action of voting would not result in any gain received by her group winning the election. Yet, she would incur the individual voting cost c_i , so that a net loss would result. The same calculation is performed for the other four states of the world. Note that $(H - L - c_i, \frac{1}{2}; 0, \frac{1}{2})$ in Table 3 denotes that the gain equals $H - L - c_i$ with probability $\frac{1}{2}$, and 0 with probability $\frac{1}{2}$. The entry $(0, \frac{1}{2}; c_i, \frac{1}{2})$ in the third column of this table, where voting is the reference point, can be interpreted analogously. The entries in Table 4 can be interpreted in a similar way, but then in terms of losses. For brevity, we omit here the gains and loss matrices of individuals of group B .

Let us examine the case with abstaining as the reference point. Take individual i of group A and suppose that she votes. Tables 3 and 4 imply that she will gain $H - L - c_i$ with probability $P_A^*/2$ and incur a loss c_i with probability $1 - P_A^*/2$, with P_A^* given by (9). Following Tversky and Kahneman (1992) (see also Barberis, 2013), individual i then attaches the following value to voting:

$$V_A(c_i) = \pi^+ \left(\frac{P_A^*}{2} \right) \times u(H - L - c_i) + \pi^- \left(1 - \frac{P_A^*}{2} \right) \times u(-c_i). \quad (11)$$

Table 3: Gains matrix for individual i of group A .

Reference point:	Abstain	Vote
Strategy:	Vote	Abstain
$n_A^{-i} > n_B^{-i} + 1$	0	c_i
$n_A^{-i} = n_B^{-i} + 1$	0	c_i
$n_A^{-i} = n_B^{-i}$	$(H - L - c_i, \frac{1}{2}; 0, \frac{1}{2})$	$(0, \frac{1}{2}; c_i, \frac{1}{2})$
$n_A^{-i} = n_B^{-i} - 1$	$(H - L - c_i, \frac{1}{2}; 0, \frac{1}{2})$	$(0, \frac{1}{2}; c_i, \frac{1}{2})$
$n_A^{-i} < n_B^{-i} - 1$	0	c_i

Table 4: Loss matrix for individual i of group A .

Reference point:	Abstain	Vote
Strategy:	Vote	Abstain
$n_A^{-i} > n_B^{-i} + 1$	c_i	0
$n_A^{-i} = n_B^{-i} + 1$	c_i	0
$n_A^{-i} = n_B^{-i}$	$(0, \frac{1}{2}; c_i, \frac{1}{2})$	$(H - L - c_i, \frac{1}{2}; 0, \frac{1}{2})$
$n_A^{-i} = n_B^{-i} - 1$	$(0, \frac{1}{2}; c_i, \frac{1}{2})$	$(H - L - c_i, \frac{1}{2}; 0, \frac{1}{2})$
$n_A^{-i} < n_B^{-i} - 1$	c_i	0

Here the utility function for outcome x is given by

$$u(x) = \begin{cases} x^\alpha & x \geq 0, \\ -\lambda(-x)^\beta & x < 0, \end{cases} \quad (12)$$

with $0 < \alpha < 1$, $0 < \beta < 1$ and $\lambda > 1$. We have a gain if $x > 0$ and a loss if $x < 0$. The probability weighting functions for the objective probability p are given by

$$\pi^+(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{\frac{1}{\gamma}}}, \quad (13)$$

$$\pi^-(p) = \frac{p^\delta}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta}}}, \quad (14)$$

with $0 < \gamma < 1$ and $0 < \delta < 1$.

The utility function $u(\cdot)$ reflects the following three features: (i) reference dependence, i.e. the gains and losses are relative to a given reference point

(here to abstain or to vote), so that the same action may be assigned a different utility value for distinct reference points; (ii) diminishing sensitivity, i.e. the utility function is concave in gains and convex in losses, which implies that voters are risk-averse in gains and risk-loving in losses; (iii) loss aversion, i.e. the increase in utility from gaining a given payoff is smaller than the (absolute value of the) decrease in utility from a loss of the same size. The individual evaluates the utility for each outcome and calculates the corresponding expected value given her subjective probabilities. The subjective probabilities are given by the weighting functions $\pi^+(\cdot)$ and $\pi^-(\cdot)$ (associated with gains and losses, respectively), which depend on the objective probabilities of each state. The weighting functions imply that small probabilities are overestimated, while large probabilities are underestimated. We allow that the weighting functions are different for gains and losses. In the special case with $\gamma = \delta = 1$, the individual calculates the standard expected value based on the objective probabilities.

Next, consider the case where voting is the reference point, and suppose that individual i of group A abstains from voting. Using Tables 3 and 4, we see that her value of voting is then given by

$$W_A(c_i) = \pi^+ \left(1 - \frac{P_A^*}{2} \right) \times u(c_i) + \pi^- \left(\frac{P_A^*}{2} \right) \times u(-(H - L - c_i)). \quad (15)$$

Obviously, we can define the values $V_B(c_i)$ and $W_B(c_i)$ for individual i of group B in the same manner, by replacing P_A^* with P_B^* , defined by (10), in (11) and (15), respectively. Note that we assume that all individuals of both groups have the same utility function (12) and probability weighting functions (13) and (14).

Turning to the quasi-symmetric equilibrium of the model, we use Tables 3 and 4 in order to determine the thresholds of the voting costs in the cut-point strategies used by the members of each group. First, assume that all voters use abstaining as their reference point. Then in order to be indifferent between switching or not switching to voting, the value of voting must equal zero. The equilibrium pair of thresholds of the voting costs, (c_A^*, c_B^*) , simultaneously solves

equations (1)–(6), (9), (10), $V_A(c_A^*) = 0$ and $V_B(c_B^*) = 0$.³ Second, assume that voting is the reference point. The equilibrium pair (c_A^*, c_B^*) then simultaneously solves equations (1)–(6), (9), (10), $W_A(c_A^*) = 0$ and $W_B(c_B^*) = 0$. In the special case with $\alpha = \beta = \lambda = 1$ and $\gamma = \delta = 1$, the equilibrium of the model coincides with the equilibrium of the standard model of Section 2.

Clearly, the assumption that all voters share the same reference point is restrictive, with neither reference point being the more obvious and rational choice. Hence, we now assume that in addition to being randomly assigned a voting cost, each individual is independently and privately assigned (by Nature) either the reference point abstaining or the reference point voting with probabilities q and $1 - q$, respectively. This is also common knowledge. The threshold for the voting cost in an individual's cut-point strategy then varies with both group and reference point type. Let $c_{A,a}^*$ and $c_{A,v}^*$ denote the equilibrium thresholds for a member of group A who has the reference point abstaining or voting, respectively. In the same way, we define $c_{B,a}^*$ and $c_{B,v}^*$ for members of group B . Hence, individual i with the reference point abstaining in group A votes if and only if $c_i < c_{A,a}^*$, and so on. The two types of individuals per group have a distinct threshold cost level, but the equilibrium aggregate voting probability per group is simply their probability weighted average, i.e. we have

$$p_A^* = q \times F(c_{A,a}^*) + (1 - q) \times F(c_{A,v}^*), \quad (16)$$

$$p_B^* = q \times F(c_{B,a}^*) + (1 - q) \times F(c_{B,v}^*). \quad (17)$$

The equilibrium set of thresholds $(c_{A,a}^*, c_{A,v}^*, c_{B,a}^*, c_{B,v}^*)$ simultaneously solves (3)–(6), (9), (10), (16), (17), $V_A(c_{A,a}^*) = 0$, $W_A(c_{A,v}^*) = 0$, $V_B(c_{B,a}^*) = 0$ and $W_B(c_{B,v}^*) = 0$.

We denote the prospect-theory model with the given mixture of reference points as the PT(q)-model. The special cases where all individuals have either abstaining or voting as their reference point are the PT(1)-model and PT(0)-model, respectively.

³For notational simplicity we also denote equilibrium values of the prospect-theory model by means of a star (*). The context makes clear to which model the equilibrium applies.

4 Comparison of models

In this section, we assess the performance of the prospect-theory model compared to the standard model of Section 2. Since there is no closed-form equilibrium solution for these models, our analysis will be done numerically. We examine the turnouts predicted by the prospect-theory model and standard model, as given by the groups' equilibrium aggregate voting probabilities, while using the parameter values used in the experiment of Levine and Palfrey (2007). We investigate how the predictions of the two models match with the turnouts actually observed in the experiment of Levine and Palfrey. In addition, we verify whether the predictions of the prospect-theory model satisfy a number of comparative statics effects discussed by Levine and Palfrey with regard to the standard model.

Following Levine and Palfrey (2007), we look at cases where $N_A = N_B - 1$ (i.e. group A is slightly smaller than group B) or $N_A = N_B/2$ (i.e. group A is much smaller than group B), $H = 105$ and $L = 5$, and the voting cost c_i is uniformly distributed on $[0, 55]$. We set the preference parameter values in the prospect-theory model equal to the median estimates of previous empirical (experimental) studies. Following Abdellaoui (2000), we take $\alpha = 0.89$, $\beta = 0.92$, $\gamma = 0.6$ and $\delta = 0.7$. These values are very close to those of Tversky and Kahneman (1992). Using Kahneman and Tversky (1979), we set $\lambda = 1.69$.

4.1 Predictive performance

We first turn to a comparison of the predictive performance of the two models. As the main criterion, we look at the models' ability to predict the turnouts observed in the experiment of Levine and Palfrey (2007). Table 5 gives the experimental turnouts of group A and group B , denoted as \hat{p}_A and \hat{p}_B , respectively, for the seven different combinations of group sizes considered by Levine and Palfrey (cf. Table 2 of their study). We also present the observed turnout of the total electorate for each combination, i.e. $\hat{p} = (N_A \times \hat{p}_A + N_B \times \hat{p}_B)/N$, where $N = N_A + N_B$ is the size of the total electorate. In the same way, Ta-

ble 5 gives the turnouts of group A and group B predicted by the standard model, and the corresponding prediction for the total electorate, i.e. $p^* = (N_A \times p_A^* + N_B \times p_B^*)/N$.

As noted by Levine and Palfrey (2007, pp. 148-149), the (experimentally) observed turnouts are smaller than the predictions of the standard model for $N = 3$ (except for group A in case $N_A = 1$ and $N_B = 2$), they are approximately equal to each other for $N = 9$, while for $N = 27$ and $N = 51$ the observed turnouts are larger than the predicted ones. As a simple measure of the fit between the observed and predicted turnouts, we calculate the average of the absolute differences between the observed and predicted turnouts of the total electorate across the seven group size combinations of Table 5. We call this the average prediction error of the standard model, and express it as a percentage. It amounts to 6.58%.

Table 5: Turnouts in the experiment of Levine and Palfrey (2007), and predicted turnouts in the standard model and PT(0.7)-model, for groups A and B , and the total electorate, for different group sizes.

		Experiment			Standard model			PT(0.7)-model		
N_A	N_B	\hat{p}_A	\hat{p}_B	\hat{p}	p_A^*	p_B^*	p^*	p_A^*	p_B^*	p^*
1	2	0.539	0.573	0.562	0.537	0.640	0.606	0.566	0.598	0.587
3	6	0.436	0.398	0.411	0.413	0.374	0.387	0.437	0.416	0.423
4	5	0.479	0.451	0.463	0.460	0.452	0.456	0.490	0.488	0.489
9	18	0.377	0.282	0.314	0.270	0.228	0.242	0.311	0.281	0.291
13	14	0.385	0.356	0.370	0.302	0.297	0.299	0.371	0.368	0.369
17	34	0.333	0.266	0.288	0.206	0.171	0.183	0.249	0.220	0.230
25	26	0.390	0.362	0.376	0.238	0.235	0.236	0.314	0.312	0.313

Next, we turn to the PT(q)-model and explore whether this model, which allows for a mixture of reference points regarding prospect-theory preferences, can provide accurate predictions of the observed turnouts. To err on the safe side, we evaluate the model performance for $0 \leq q \leq 1$, while considering the predicted turnouts of group A , group B , and the total electorate, for the seven

group size combinations of Table 5.⁴ Figure 1 shows the average prediction error for different values of q . We find that the value of q yielding the smallest average prediction error for the total electorate is 0.7, whereas all PT(q)-models with $0.5 < q < 0.85$ have smaller prediction errors than the standard model.

Even though the true value of q is unknown, a recent survey among American adults finds that only 34% of the electorate can be classified as consistent voters, who participate regardless of election type, as opposed to voters that vote only in some elections or not at all (Jones et al., 2018). In other words, the point estimate of q is 0.66 with some confidence interval, which is reasonably close to the PT(0.7)-model specification and well within the range of specifications that outperform the standard model. From here, we continue with the PT(0.7)-model, i.e., the model with the smallest prediction error, for further evaluation of the prospect-theory model.

Table 5 gives the predicted turnouts associated with the PT(0.7)-model. The average prediction error of this model is equal to 2.97%, which is less than one half of that of the standard model (6.58%). Table 5 further shows that the observed turnouts are quite close to the predictions of the PT(0.7)-model for $N = 3$, $N = 9$ and $N = 27$, while the observed outcomes are larger than the predicted ones for $N = 51$. Also note that, except for $N = 3$, all predicted turnouts of the PT(0.7)-model are somewhat larger than those of the standard model.

In sum, we conclude that the PT(0.7)-model has better predictive performance than the standard model in terms of explaining observed turnout of the total electorate in the experiment of Levine and Palfrey (2007). For the extreme case where everyone shares the reference point of abstaining ($q = 1$), we find an average prediction error of 12.39%; if all voters have the reference point of

⁴Calculations were performed using the program ‘Mathematica’. Uniqueness of equilibrium in the PT(1)-model was shown by plotting conditions $V_A(c_A^*) = 0$ and $V_B(c_B^*) = 0$, and identifying the unique intersection representing the equilibrium pair (c_A^*, c_B^*) . In a similar way, we have shown uniqueness of equilibrium in the PT(0)-model. Due to the dimensionality of the problem, uniqueness of the equilibrium in the PT(q)-model with $0 < q < 1$ cannot be shown graphically. However, variation in starting values of the root finding algorithm revealed no additional equilibria in all cases considered by us. The code is available upon request.

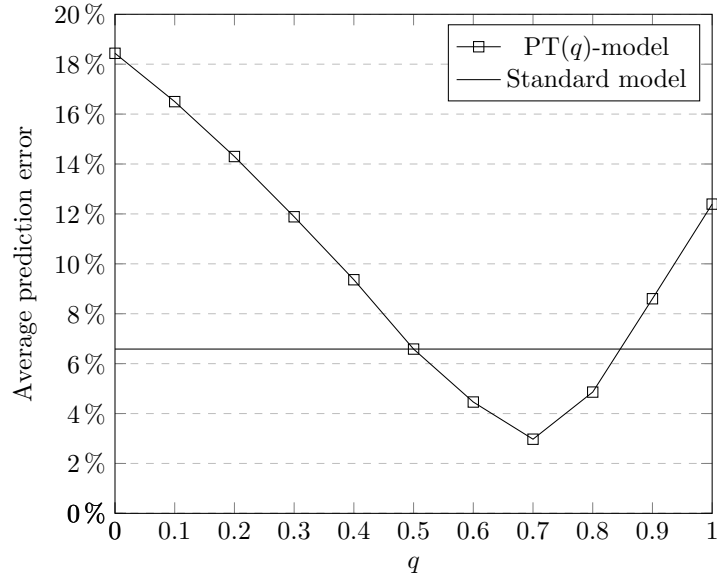


Figure 1: Comparison of the average prediction error of the total electorate of the standard model and PT(q)-model.

voting ($q = 0$), the average prediction error equals 18.44%. Based on this, we conclude that the prospect-theory model in which all individuals have the same reference point performs worse than the standard model.

4.2 Decomposition of effects

We now want to enhance our understanding of the difference between the predictions of the PT(0.7)-model and standard model. Tables 6 and 7 present the predicted turnouts of groups A and B , and the total electorate, for the PT(1)-model and PT(0)-model, in which all individuals share the same reference point of abstaining or voting, respectively. One can verify that (except for group A in case $N_A = 1$ and $N_B = 2$), the turnout predictions of the PT(0.7)-model are reasonably close to the weighted average of the predictions of the PT(1)-model and PT(0)-model, with weights 0.7 and 0.3, respectively.⁵ Hence, in order

⁵The larger the electorate size, the closer are the predictions of the PT(q)-model to the weighted average of the extreme cases. For the case $N_A = 1$ and $N_B = 2$, p_A^* deviates noticeably from the weighted average of the PT(1)-model and PT(0)-model. This is due to

to understand the difference between the predictions of the PT(0.7)-model and standard model, it is useful to compare the PT(1)-model and PT(0)-model with the standard model.

Table 6 also presents the predictions of a variant of the PT(1)-model in which both probability weighting functions are set equal to the identity function (i.e. $\delta = \gamma = 1$), and voters thus use the objective probabilities to calculate expected values. We call this variant the PTOB(1)-model. It can be used to separate the effects of probability weighting from those of reference dependence, diminishing sensitivity and loss aversion in the prospect-theory model. In a similar way, Table 7 presents results for the PTOB(0)-model, which is obtained by using objective probabilities in the PT(0)-model.

Table 6: Predicted turnouts in the PT(1)-model and PTOB(1)-model, for groups A and B , and the total electorate, for different group sizes.

N_A	N_B	PT(1)-model			PTOB(1)-model		
		p_A^*	p_B^*	p^*	p_A^*	p_B^*	p^*
1	2	0.459	0.437	0.445	0.457	0.417	0.430
3	6	0.328	0.294	0.305	0.288	0.238	0.255
4	5	0.337	0.326	0.331	0.283	0.266	0.274
9	18	0.230	0.203	0.212	0.179	0.144	0.156
13	14	0.247	0.244	0.246	0.177	0.172	0.174
17	34	0.186	0.162	0.170	0.136	0.108	0.117
25	26	0.209	0.207	0.208	0.137	0.135	0.136

Let us examine Table 6, where the reference point of all individuals is abstaining. Comparing the PTOB(1)-model with the standard model, we see that using the utility function (12) rather than the linear utility function of the standard model leads to lower predicted turnouts. This is firstly due to the fact that in (12) the voting costs (considered a loss) are multiplied by the loss aversion parameter $\lambda = 1.69 > 1$ such that voting is more costly. In addition, there is a

the fact that for $q = 0$, turnout in group B dominates that of group A , so that P_A^* at first increases strongly with q before gradually decreasing again. As a result, for this case the predictions of the PT(0.7)-model differ noticeably from the weighted average.

Table 7: Predicted turnouts in the PT(0)-model and PTOB(0)-model, for groups A and B , and the total electorate, for different group sizes.

N_A	N_B	PT(0)-model			PTOB(0)-model		
		p_A^*	p_B^*	p^*	p_A^*	p_B^*	p^*
1	2	0.465	0.916	0.766	0.496	0.802	0.700
3	6	0.556	0.608	0.591	0.488	0.478	0.481
4	5	0.651	0.815	0.724	0.592	0.650	0.624
9	18	0.424	0.399	0.407	0.334	0.287	0.303
13	14	0.670	0.696	0.683	0.425	0.421	0.423
17	34	0.338	0.304	0.316	0.254	0.211	0.225
25	26	0.568	0.571	0.569	0.327	0.325	0.326

similar curvature in (12) for gains and losses implied by the very close parameter values $\alpha = 0.89$ and $\beta = 0.92$, which means that diminishing sensitivity makes voting unattractive as well. In order to see this, note from Tables 3 and 4 that for members of group A , the potential gain of voting equals $H - L - c_A^*$, while the potential loss of voting is c_A^* . Since $H - L - c_A^*$ is larger than c_A^* , the decrease in utility associated with the gain of voting is larger than the increase of the (negative) utility of the loss of voting, if we move from linear utility to utility function (12). The same holds for group B .

Going from the PTOB(1)-model to the PT(1)-model, we see that the effect of probability weighting works in the opposite direction. Since the probability of a gain is now overweighted (because $P_A^*/2$ and $P_B^*/2$ are small), while the probability of a loss is underweighted (because $1 - P_A^*/2$ and $1 - P_B^*/2$ are large), probability weighting leads to higher predicted turnout relative to the PTOB(1)-model, where objective probabilities are used. Since the smaller the objective probabilities are, the more they are overweighted, the impact of probability weighting increases with the number of voters. Nevertheless, for the group size combinations considered in Table 6, the predictions of the PT(1)-model are still somewhat smaller than those of the standard model.⁶

⁶Numerical calculations show that for large group sizes (total electorate size 169 and greater), the PT(1)-model shows slightly higher turnouts than the standard model. Hence, while for a small number of voters reference dependence, diminishing sensitivity and loss aversion offset the impact of probability weighting, the latter has a higher impact when the group

Next, take Table 7, where all individuals have the reference point voting. In this situation, the PTOB(0)-model shows that reference dependence, diminishing sensitivity and loss aversion lead (except for group A in case $N_A = 1$ and $N_B = 2$) to *higher* predicted turnout than in the standard model. This is due to the fact that, if members of group A abstain in this situation, then opposite to the previous case, c_A^* is considered as a potential gain and $H - L - c_A^*$ as a potential loss. The same holds for group B . Moving from the PTOB(0)-model to the PT(0)-model, we see that the effect of probability weighting is to further increase predicted turnout. The intuition behind this result is that the probability of receiving a gain from abstaining is now underweighted, while the probability of a loss is overweighted.

In sum, we see that the turnout predictions of the PT(0)-model are substantially larger than those of the standard model, whereas the predictions of the PT(1)-model are somewhat smaller. We have also explained the driving forces behind these results. Combining, and recalling that the predictions of the PT(0.7)-model are quite close to the weighted average of the predictions of the PT(1)-model and PT(0)-model (with weights 0.7 and 0.3, respectively), we have also identified the main determinants of the difference in the predictions of the PT(0.7)-model and standard model. Loosely speaking, the difference in the predictions can be seen as the net outcome of the (weighted) interplay between the two extreme cases in which all voters have either the reference point abstaining or voting.

4.3 Comparative statics hypotheses

Based on the predictions of their model, Levine and Palfrey (2007) stipulate seven comparative statics hypotheses, with the main ones being the ‘size’, ‘competition’ and ‘underdog’ effect.

The size-effect hypothesis states that, holding constant relative group sizes, turnout in each group decreases with electorate size. For the group sizes studied here, the hypothesis implies twenty four pairwise inequalities of the form that size gets larger.

if the total electorate sizes N and M satisfy $N < M$, then $p_j^*(\frac{N-1}{2}, \frac{N+1}{2}) > p_j^*(\frac{M-1}{2}, \frac{M+1}{2})$ and $p_j^*(\frac{N}{3}, \frac{2N}{3}) > p_j^*(\frac{M}{3}, \frac{2M}{3})$ for $j = A, B$ (we attach here the size of groups A and B to p_j^*). In the case where $N_A = N_B - 1$, one example is that $p_A^*(4, 5) > p_A^*(13, 14)$. And for the case where $N_A = N_B/2$, one of the inequalities is $p_B^*(1, 2) > p_B^*(17, 34)$. The intuition behind this effect is that the larger the electorate, the smaller is the probability of being pivotal, and with it turnout.

According to the competition-effect hypothesis, turnout is decreasing with the gap between group sizes. This hypothesis implies six pairwise inequalities: $p_j^*(4, 5) > p_j^*(3, 6)$, $p_j^*(13, 14) > p_j^*(9, 18)$ and $p_j^*(25, 26) > p_j^*(17, 34)$, for $j = A, B$. This hypothesis is again intuitive, with closer group sizes implying a higher chance of individual's being pivotal, and thus higher turnout.

Lastly, the underdog-effect hypothesis posits that, except in the case that $N = 3$, turnout in the smaller group is higher. The six pairwise inequalities in this case are: $p_A^*(4, 5) > p_B^*(4, 5)$, $p_A^*(3, 6) > p_B^*(3, 6)$, $p_A^*(13, 14) > p_B^*(13, 14)$, $p_A^*(9, 18) > p_B^*(9, 18)$, $p_A^*(25, 26) > p_B^*(25, 26)$ and $p_A^*(17, 34) > p_B^*(17, 34)$. Underlying this hypothesis is the fact that the larger the group size, the larger the free-rider problem, so that turnout in the larger group is expected to be lower. For the case where $N = 3$, the relationship is reversed, i.e. $p_A^*(1, 2) < p_B^*(1, 2)$. Consequently, Levine and Palfrey (2007) refer to the hypothesis as the 'counter-example to the underdog effect'.

The experimental data of Levine and Palfrey (2007) is overall in line with the seven hypotheses derived from the standard model, with only three quantitatively small deviations. One of these violations is in regards to the size-effect hypothesis: in the experimental data, $\hat{p}_j(13, 14) < \hat{p}_j(25, 26)$ for $j = A, B$. However, as Levine and Palfrey (2007) argue, some deviations from the hypotheses can be expected due to sampling variation.

Since the empirically supported comparative statics hypotheses are economically meaningful and intuitively appealing, we would like the prospect-theory model to generate them as well. It turns out that the PT- (q) -model satisfies all seven hypothesis for $0.7 < q \leq 0.85$. Hence, based on the criterion that the

model also satisfies the discussed comparative statics effects, we conclude that a range of $0.7 \leq q \leq 0.85$ is best able to explain the observed turnout patterns.⁷

4.4 What happens when N is large?

Finally, we turn to the prospect-theory model's predictions in elections with large electorates. As is well known, an individual voter's probability of being pivotal tends to zero as the electorate grows arbitrarily large. As a consequence, the equilibrium turnout rate in the standard model converges to zero in that case. This is no different for the prospect-theory model since the objective probability p tends to zero and so do the weighted probabilities defined by equations (13) and (14).

However, there is a significant difference between the prospect-theory model and the standard model when the size of the electorate grows large. Namely, the speed of convergence with which voter turnout goes to 0 is in favor of the prospect-theory model. This is due to the presence of probability weighting. In particular, it can be shown that $\lim_{p \rightarrow 0} \frac{\pi^+(p)}{p} = \infty$ and $\lim_{p \rightarrow 0} \frac{\pi^-(p)}{p} = \infty$. That is, arbitrarily small probabilities are infinitely overweighted. Therefore, even in elections with a large electorate, when the chance of being pivotal is very small, the prospect-theory model allows for significant turnout.

Due to computational limitations, we cannot calculate the equilibrium turnout for electorate sizes of several millions.⁸ We can, however, determine with reasonable precision the turnout for electorates with size up to 2000. Figure 2 illustrates voter turnout for the PT(0.7)-model and standard model for different electorate sizes. It is evident that while the gap between the predictions of the two models is fairly constant in absolute terms for different electorate sizes, their ratio increases substantially when the electorate size grows, indicating that

⁷For larger values of q , however, the counter-example to the underdog-effect hypothesis is violated since group A turnout is smaller than that of group B with $N_A = 1$ and $N_B = 2$. Conversely, the underdog-effect hypothesis itself is violated for $q < 0.7$; essentially, the reversion of pattern observed only for $N_A = 1$ and $N_B = 2$ in the standard model now extends to larger group sizes as the proportion of individuals with voting as reference point increases.

⁸As electorate size grows, the terms of equations (3) to (6) become too small to be represented as a normalized machine number, resulting in increasing loss of precision.

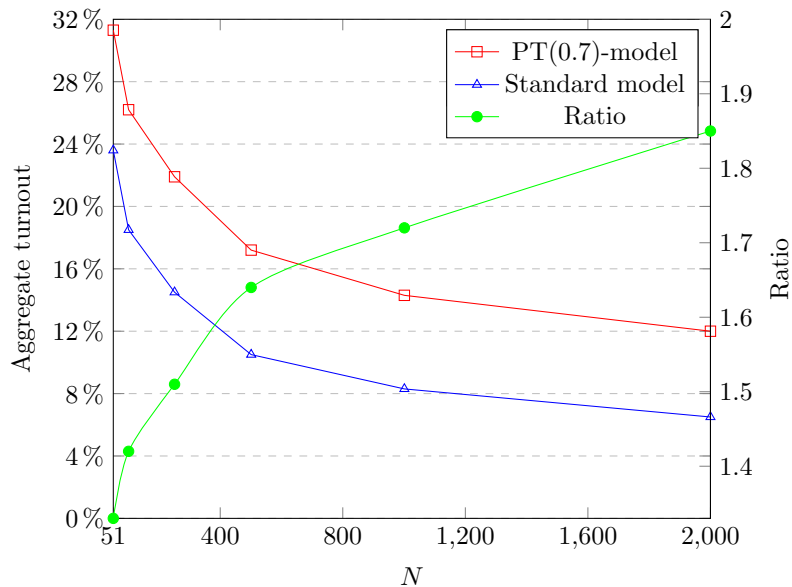


Figure 2: Predicted turnout for the total electorate in the PT(0.7)-model and standard model, for larger electorates, and the ratio of the two models' turnout predictions.

the prospect-theory model keeps performing well in comparison to the standard model for larger electorate sizes.

It should be noted that Levine and Palfrey (2007) dictate that the cost-benefit distribution in the model (and experiment) regarding winning and losing elections should be such that uniqueness of the equilibrium is ensured. We followed their approach. However, despite the possible existence of multiple equilibria, in real elections the benefits of the pivotal voter are, relative to the costs, substantially larger than in the model. This would imply that in very large electorates, voter turnout in absolute terms can be substantial.

5 Conclusion

We have shown that the implementation of prospect-theory preferences in a game-theoretic model of voting yields more accurate predictions of turnout than standard game-theoretic models, and that these predictions satisfy a number

of intuitive comparative statics hypotheses. In analyzing the prospect-theory model of voting, we find that voters' reference point and probability weighting have a large impact on voter turnout. The results also show that instrumental voting plays a more important role in determining real world election turnout than standard (game-theoretic) rational choice models predict.

The prospect-theory model improves upon the existing two strands of research that seek to explain voter turnout, both of which have limitations. First, behavioral approaches based on non-instrumental benefits or costs cannot explain turnout in experimental elections as in Levine and Palfrey (2007), since their experimental design precludes any possibility of such motivations. Second, the game-theoretic approach with standard risk preferences has a hard time explaining the observed heterogeneity in individual strategies. However, the prospect-theory model, on the other hand, has smaller prediction error and is able to explain inter-individual differences based on reference points and risk preference parameters.

It should be noted that there is a striking similarity between the loss matrix of the prospect-theory model and the 'matrix of regrets' from Ferejohn and Fiorina (1974). However, relative to their work, the prospect-theory model also takes into account the impact of potential gains. The model could thus be considered a generalization of the approach of Ferejohn and Fiorina (1974), which (unlike Ferejohn and Fiorina) also takes into account the probability of being decisive.

Levine and Palfrey (2007) apply the Quantal Response Equilibrium (QRE) solution concept to the standard model in order to show that when individuals make unsystematic random errors in choosing their optimal strategy, significant turnout occurs even in the presence of very large electorates. The prospect-theory model we develop in this paper is, in principle, also compatible with this set-up. However, as Haile et al. (2008) show, QRE is in general not falsifiable and can be used to rationalize any observed behavior. This casts doubt on the usefulness of QRE. In contrast, the behavioral assumptions of the prospect-theory model as well as its model predictions can be empirically verified.

The prospect theory of voter turnout opens up a whole new research agenda. For example, the inclusion of endogenous reference point formation would be an extension of the model that requires serious study. Furthermore, empirical investigation of the relation between individual risk preferences, reference points and voting behavior is necessary to test the model and its possible extensions.

References

- Abdellaoui, M. (2000). Parameter-Free Elicitation of Utility and Probability Weighting Functions. *Management Science*, vol. 46, pp. 1497-1512.
- Barberis, N.C. (2013). Thirty Years of Prospect Theory in Economics: A Review and Assessment. *Journal of Economic Perspectives*, vol. 27, pp. 173-196.
- Beck, N. (1975). The Paradox of Minimax Regret, *American Political Science Review*, vol. 69, p. 918.
- Brennan, G. and Lomasky, L. (1984). Inefficient Unanimity. *Journal of Applied Philosophy*, vol. 1, pp. 151-163.
- Brennan, G. and Buchanan, J. (1984). Voter Choice - Evaluating Political Alternatives. *American Behavioral Scientist*, vol. 28, pp. 185-201.
- Downs, A. (1957). *An Economic Theory of Democracy*. New York: Harper & Row.
- Ferejohn, J.A. and Fiorina, M.P. (1974). The Paradox of Not Voting: A Decision Theoretic Analysis. *American Political Science Review*, vol. 68, pp. 525-536.
- Fiorina, M.P. (1990). Information and Rationality in Elections. *Information and Democratic Processes*. Urbana IL: University of Illinois Press, pp. 329-342.
- Fowler, J.H. (2006). Altruism and Turnout. *Journal of Politics*, vol. 68, pp. 674-683.
- Haile, P.A., Hortasu, A. and Kosenok, G. (2008). On the Empirical Content of Quantal Response Equilibrium. *American Economic Review*, vol. 98, pp. 180-200.
- Jones, R.P., Cox, D., Griffin, R., Fisch-Friedman, M. and Vandermaas-Peeler, A. (2018). American Democracy in Crisis: The Challenges of Voter Knowledge, Participation, and Polarization. *PRRI/The Atlantic 2018 Voter Engagement*

Survey.

Kahneman, D. and Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, vol. 47, pp. 263-292.

Kirchgässner, G. (1992). Towards a Theory of Low-Cost Decisions. *European Journal of Political Economy*, vol. 8, pp. 305-320.

Levine, D.K. and Palfrey, T.R. (2007). The Paradox of Voter Participation? A Laboratory Study. *American Political Science Review*, vol. 101, pp. 143-158.

Palfrey, T.R. and Rosenthal, H. (1983). A Strategic Calculus of Voting. *Public Choice*, vol. 41, pp. 7-53.

Palfrey, T.R. and Rosenthal, H. (1985). Voter Participation and Strategic Uncertainty. *American Political Science Review*, vol. 79, pp. 62-78.

Quattrone, G.A. and Tversky, A. (1988). Contrasting Rational and Psychological Analyses of Political Choice. *American Political Science Review*, vol. 82, pp. 719-736.

Riker, W. and Ordeshook, P. (1968). A Calculus of Voting. *American Political Science Review*, vol. 6, pp. 28-42.

Stephens, S. (1975). The Paradox of Not Voting: A comment. *American Political Science Review*, vol. 69, pp. 914-915.

Strøm, G. (1975). On the Apparent Paradox of Participation: A New Proposal. *American Political Science Review*, vol. 69, pp. 908-113.

Tullock, G. (1967). *Towards a Mathematics of Politics*. Ann Arbor: University of Michigan Press.

Tversky, A. and Kahneman, D. (1992). Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty*, vol. 5, pp. 297-323.