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Discounting in the Presence of Scarce Ecosystem Services

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Discounting in the Presence of Scarce Ecosystem Services

Abstract

Discounting has to take account of ecosystem services in consumption and production. Previous literature focuses on the first aspect and shows the importance of the relative price effect, for given growth rates of consumption and ecosystem services. This paper focuses on intermediate ecosystem services in production and shows that for limited substitutability and a low growth rate of these ecosystem services, the growth rate of consumption, and thus the discount rate, declines towards a low value. Using a Ramsey optimal-growth framework, the paper distinguishes three cases. If ecosystem services can be easily substituted, then the discount rate converges to the usual value in the long term. Secondly, if ecosystem services can be easily substituted in production but not in consumption, the relative price effect is important. Finally, and most interestingly, if ecosystem services cannot be easily substituted in production, the discount rate declines towards a low value and the relative price effect is less important. Another part of the previous literature has shown that a declining discount rate is the result of introducing several forms of uncertainty, but this paper reaches that conclusion from an endogenous effect on the growth rate of the economy.

JEL-Codes: C610, D900, E430, O440, Q570.

Keywords: discount rate, ecosystem services, consumption value, production value, growth rate, Ramsey optimal balanced growth.

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1. Introduction

Discounting is the way in which we relate the future and the current costs and benefits in cost-benefit analyses for public policy and project evaluation. Since the pioneering work of Fisher and Krutilla (1975) and later Weitzman (1994), the literature has given a lot of attention recently to the role and the relative scarcity of the natural environment in the provision of ecosystem services (e.g., Guesnerie, 2004; Weikard and Zhu, 2005; Hoel and Sterner, 2007; Gollier, 2010; Traeger, 2011). An important argument is that the growth rates of the economy and the ecosystem services differ. This implies that the usual assumption of constant relative prices does not hold, since the valuation of environmental benefits relative to produced consumption goods changes over time. Cost-benefit analyses should either take the expected change in relative prices into account or use different discount rates for consumption and for ecosystem services. The elasticity of substitution plays, of course, an important role. This literature derives the discount rate from the Ramsey rule[5] with a reduced form for welfare depending on the flow of consumption and on the quality of the environment. An interesting result is that the change in relative prices, or the gap between the discount rates, is proportional to the difference between these growth rates, and inversely related to the elasticity of substitution. Empirical work (Baumgärtner et al, 2015; Koetse et al, 2016; Drupp, 2018) shows that this gap is in the order of 1% and varies with the type and the role of the ecosystem service that is considered.

A reduced form for welfare, with exogenous growth rates for consumption and for the quality of the environment, does not take the different roles of ecosystem services into account. Ecosystem services can have a direct amenity value, but more often ecosystem services have an intermediate role as an input factor into production. It follows that the growth rate of consumption depends on the availability of ecosystem services and on the possibilities of substitution. More specifically, technological progress may drive the economic growth, but growth may be restricted if ecosystem services in production do not grow and cannot be easily substituted in production. This has an important effect on the discount rate. The literature on changing relative prices assumes that the growth rates of consumption and ecosystem services are exogenous and given. This paper adds the endogenous effect of limited availability and substitutability of ecosystem services on the growth rate of the economy, and therefore on the discount rate.

[5] The Ramsey rule yields the discount rate as the rate of pure time preference plus the product of the elasticity of marginal utility and the growth rate.

In order to analyse the roles of ecosystem services both in utility and in production, we use a full Ramsey optimal-growth framework. Intermediate ecosystem services are a factor of production and final ecosystem services are an argument in welfare. In order to allow for substitution, we use both a *CES* utility function and a *CES* production function. We derive the steady-state conditions for balanced growth. If the elasticities of substitution are high, we get the standard growth rate and discount rate in the long run. However, if the elasticity of substitution in production is low, we get a low growth rate in the long run, with a low discount rate given by the Ramsey rule. If the elasticity of substitution is high in production but low in welfare, we get a high growth rate in the long run, but also the relative price effect that was considered in the extant literature. Moreover, the full Ramsey framework allows considering the paths of the growth rate and the discount rate towards the balanced-growth steady state. In this way, we can show the effects of the limited availability and substitutability of ecosystems services on the term structure of the discount rate.

An important conclusion is that when the restrictions of the natural environment start to kick in, the path of the discount rate is declining towards a very low level in the long run. The term structure of the discount rate will inherit the negative slope from the term structure of the growth rate (Gollier and Hammitt, 2014). A declining discount rate also results from various effects of uncertainty (Gollier, 2002, 2010, 2013; Newell and Pizer, 2003; Weitzman, 2007, 2010). Several countries have implemented this in their official policy (Groom and Hepburn, 2017). The main idea is that increasing uncertainty about the growth rate of the economy increases the effects of risk aversion and of prudence, assuming concavity of utility and convexity of marginal utility. We leave uncertainty out in this paper, but it is clear that all arguments point in the same direction: a lower discount rate in the long term. This is very important for cost-benefit analyses with a very long time horizon, such as the costs and benefits of climate change mitigation and adaptation. A flat discount rate effectively means that the benefits in the long run hardly count, but a declining discount rate changes the picture.

This paper provides a framework for analysing the effects on the discount rate that are caused by a possible low growth rate and limited substitutability of ecosystem services. It is, of course, an empirical question to determine the sectors for which this is relevant, and to quantify the effects by determining growth rates and elasticities of substitution.

Section 2 analyses the role of ecosystem services in production, using the full Ramsey optimal-growth framework. Section 3 adds the role of ecosystem services in utility, in order to get the full picture with the three different cases. Section 4 provides numerical simulations to show the paths of the growth rate and the discount rate for the different cases. Section 5 concludes.

2. Ecosystem services in production

Ecosystems are natural capital. Services from ecosystems provide essential inputs (e.g. pollination, water quantity and water quality) into production processes and thus have production value. Ecosystems follow the dynamics of biophysical processes. These are influenced by human activities, but we abstract from this and take an exogenous growth rate g_E (positive or negative) for the ecosystems. Ecosystem services are either a stock or a flow variable. For example, pollination that is important for agricultural production processes can be measured by the number of bees in a neighbouring wild area at some point in time, which is a stock variable. A change in the wild area changes the number of bees and therefore the pollination capacity. However, extractions of water and other resources to be used in production are flow variables. We simply assume here that the availability or the quality of the stock *E* affects the production. We want to investigate how the constant rate g_E of growth (or de-growth) of this stock *E* affects the growth rate of the economy.

We use the Ramsey optimal-growth framework, with ecosystem services as an input in the production function. This allows us to consider the effect of intermediate ecosystem services on the growth rate of the economy and on the discount rate.

In the Ramsey growth model the optimal allocation of investment and consumption is determined by maximizing the integral of discounted welfare *U* of consumption *C* over time, subject to the accumulation of capital *K*:

$$\max \int_0^\infty e^{-\rho t} U(C(t))dt,$$
$$\dot{K}(t) = F(K(t), E(t), H(t)) - C(t), K(0) = K_0, \tag{1}$$

where F denotes the (net) production function, E ecosystem services, and ρ the pure rate of time preference. Effective labour input H is labour input scaled by human capital and labour-augmenting technology. It grows at exogenous constant rate g_H. Ignoring population growth and human capital, we may simply refer to the growth rate g_H as the rate of technical change. In order to capture the increasing scarcity of the ecosystem services, we assume throughout the paper that $g_E < g_H$.

The (consumption) discount factor D is the marginal contribution to welfare of future consumption relative to the marginal contribution to welfare of current consumption, or the marginal rate of substitution:

$$D(t) = e^{-\rho t} U'(C(t)) / U'(C(0)). \tag{2}$$

The (consumption) discount rate r is therefore the rate at which the discount factor falls, $r(t) = -\dot{D}(t) / D(t)$, which implies:

$$r(t) = \rho - \dot{U}'(C(t)) / U'(C(t)). \tag{3}$$

The discount rate reflects the additional minimum amount of consumption the society requires at time t, in exchange for giving up one unit of consumption at time $t - dt$ (with dt arbitrarily small), without suffering a decline in welfare.[6]
In problem (1), with the current-value Hamiltonian function G,

$$G(C, K, \lambda) = U(C) + \lambda(F(K, E, H) - C), \tag{4}$$

the optimal allocation requires

$$\begin{aligned} &U'(C) = \lambda, \\ &\dot{\lambda}(t) - \rho\lambda(t) = -F_K(K(t), E(t), H(t))\lambda(t), \end{aligned} \tag{5}$$

[6] For the more general welfare function *U(C,E)*, cf. below, we can write the consumption discount rate as $r^C(t) = \rho - \dot{U}_C(C(t), E(t)) / U_C(C(t), E(t))$, and we can similarly define an environmental discount rate as $r^E(t) = \rho - \dot{U}_E(C(t), E(t)) / U_E(C(t), E(t))$.

where λ denotes the shadow value of capital and $F_K(K(t),E(t),H(t))$ the (net) marginal product of capital. By comparing (3) and (5), it is easy to see that optimality requires that the discount rate, *r*, is exactly equal to the (net) marginal product of capital, F_K.

With a constant relative risk aversion (*CRRA*) utility function $U(C) = C^{1-\gamma}/(1-\gamma)$, where γ denotes the inverse of the elasticity of intertemporal substitution, substitution of the first part of (5) into the second part of (5) leads to the Keynes-Ramsey rule for the optimal consumption path

$$\dot{C}(t) = \gamma^{-1}(F_K(K(t),E(t),H(t)) - \rho)C(t), \tag{6}$$

where a transversality condition has to hold. In order to find the optimal path of all the variables, we have to solve the system:

$$\begin{aligned} &\dot{K}(t) = F(K(t),E(t),H(t)) - C(t), K(0) = K_0, \\ &\dot{C}(t) = \gamma^{-1}(F_K(K(t),E(t),H(t)) - \rho)C(t), \\ &\dot{H}(t) = g_H H(t), H(0) = H_0, \\ &\dot{E}(t) = g_E E(t), E(0) = E_0. \end{aligned} \tag{7}$$

The solution of (7) allows us to identify the time path of the discount rate $r = F_K$.

2.1 Cobb-Douglas production system with ecosystem services

For a Cobb-Douglas production function, it is easy to show that the economy converges to a steady state with balanced growth in which the growth rate *g* depends on the growth rates of technological change and ecosystem services.

Suppose that the production function is given by

$$F^{CD}(K,E,H) = AK^{\alpha}E^{\beta}H^{1-\alpha-\beta}, \tag{8}$$

where *A* denotes total factor productivity and α, β and 1 - α - β the respective shares of capital, ecosystem services and labour-augmenting technological change in production.

Defining the composite input $X \equiv E^{\beta/(1-\alpha)}H^{1-\beta/(1-\alpha)}$, we can write production function (8) as a Cobb-Douglas function with *K* and *X* as inputs:

$$F^{CD}(K,X) = AK^{\alpha}X^{1-\alpha}, \tag{9}$$

where *X* grows at a constant exogenous rate *g*, the weighted average of the growth rates of ecosystem services and effective labour input:

$$g = \left(1-\frac{\beta}{1-\alpha}\right)g_H + \frac{\beta}{1-\alpha}g_E. \tag{10}$$

It follows immediately that on a balanced growth path, output F^{CD} and capital *K* grow at rate *g*. Brock and Taylor (2010) have a comparable approach but they consider the growth rate of the abatement technology in a problem with environmental pollution. Moreover, since the problem is now isomorphic to the standard Ramsey growth model with a Cobb-Douglas production function and an exogenous constant rate of technical change (where our composite input *X* replaces the effective labour input in the standard model, e.g. Acemoglu (2009)), we can state the following proposition:

Proposition 1. For the Cobb-Douglas production function (8), the economy converges to a balanced growth path along which output, consumption and capital grow at growth rate *g*, given by (10), and the discount rate is constant.
If $K_0 < (>) X_0(\alpha A/(\rho+\gamma g))^{1/(1-\alpha)}$, the discount rate de(in)creases monotonically over time along the transition towards balanced growth.

Proof. It is convenient to define the variables $c = C/X$ and $k = K/X$ and to rewrite problem (1), using $X(t) = X_0 e^{gt}$, as follows:

$$\begin{aligned} &\max \int_0^{\infty} e^{-(\rho+(\gamma-1)g)t} X_0^{1-\gamma} U(c(t))dt, U(c) = \frac{c^{1-\gamma}}{1-\gamma}, \\ &\dot{k}(t) = f(k(t)) - gk(t) - c(t),\ f(k) = Ak^{\alpha}, k(0) = K_0/X_0. \end{aligned} \tag{11}$$

The Keynes-Ramsey rule becomes

(12) $$\dot{c}(t) = \gamma^{-1}(f'(k(t) - \rho - \gamma g)c(t).$$

In the steady state *(k*, c*)*, consumption *C*, capital *K* and output F^{CD} grow at the same rate *g*, and the discount rate is constant and given by the Ramsey rule

(13) $$f'(k^*) = \rho + \gamma g.$$

The specification of the production function *f* in (11) implies that $f'(k^*) = \alpha A(k^*)^{\alpha-1}$. It follows from (13) that $K(t)/X(t) = k^* = (\alpha A/(\rho + \gamma g))^{1/(1-\alpha)}$ along the balanced growth path. The transitional dynamics is monotonic. If $k(0) = K_0/X_0 < (>) k^*$, then $k(t)$ converges to k^* from below (above). It follows that the discount rate de(in)creases monotonically over time on the transition path. Q.E.D.

Equation (10) for the growth rate *g* has an easy interpretation. If the ecosystem services grow at the same rate as the technology g_H, the economy grows at that rate as well, but if the ecosystem services grow at a lower or even negative rate g_E, the economy grows at a lower rate *g*. This downsizing effect depends on the share β of ecosystem services in the production. We calibrate the model such that it has a conventional value of 0.3 for the share of capital and generates a value slightly below 2% for the balanced growth rate (cf. Jones, 2016). We normalize the values of *A*, K_0 and X_0, choose conventional values for the utility parameters ρ and γ (cf. Nordhaus, 2008; Stern, 2006), and choose a growth rate of the ecosystem services close to 0. With the following set of parameter values:

(14) $$\gamma = 1.45, g_H = 0.02, \rho = g_E = 0.001,$$
$$\alpha = 0.3, \beta = 0.2, A = 0.1, K_0 = X_0 = 1,$$

the discount rate $f'(k) = \alpha A k^{\alpha-1}$ is initially equal to 0.03, and it converges to 0.022, according to (10) and (13). The stable manifold of the system consisting of (12) and the second part of (11) yields the path for the discount rate $r = f'(k)$ depicted in Figure 1.

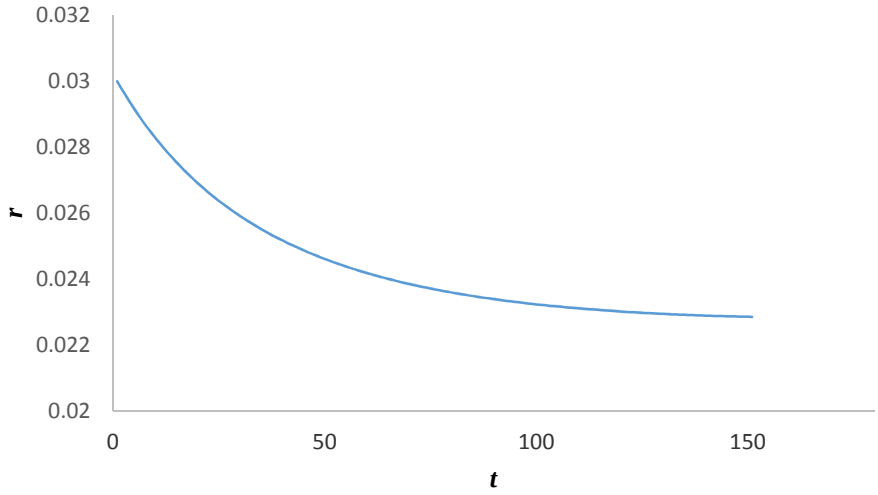

Figure 1 Time path for discount rate: the Cobb-Douglas case

2.2 CES production

The Cobb-Douglas production function of the previous sub-section implies that the production elasticities of each of the inputs are constants, i.e. no matter how scarce or abundant an input is, a 1% increase has always the same proportionate effect on output, because the elasticity of substitution is one. In the current sub-section we generalize the production function to a *CES* specification in which production elasticities rise or fall with abundance, depending on whether the elasticity of substitution is above or below unity. We specify the *CES* production function as

$$F(K,E,H) = A\left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta E^{\frac{\sigma-1}{\sigma}} + (1-\alpha-\beta)H^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{15}$$

where A denotes total factor productivity, α, β and 1 - α - β the respective shares of capital, ecosystem services and labour-augmenting technological change in production, and σ the elasticity of substitution. It is convenient again to aggregate the exogenously growing inputs into a composite input, which we again denote by X. In this way, we can write the production function as a *CES* function of the endogenously evolving state variable K and the exogenously evolving input variable X:

$$
\begin{aligned}
&F(K,X) = A\left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)X^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \\
&X(E,H) = \left(\frac{X_1 + X_2}{1-\alpha}\right)^{\frac{\sigma}{\sigma-1}}, X_1 = \beta E^{\frac{\sigma-1}{\sigma}}, X_2 = (1-\alpha-\beta)H^{\frac{\sigma-1}{\sigma}}.
\end{aligned} \tag{16}
$$

It is easy to show that

$$
\dot{X}(t) = g_X(t)X(t), g_X = \frac{X_1 g_E + X_2 g_H}{X_1 + X_2}, g_E < g_X < g_H, \tag{17}
$$

where g_X is the growth rate of the composite production input *X* of ecosystem services and labour-augmenting technological change. This growth rate g_X is the solution of a simple differential equation, because it is straightforward to show from (16) that

$$
\begin{aligned}
&\dot{g}_X(t) = \frac{\sigma-1}{\sigma}\left[\frac{X_1(t)g_E^2 + X_2(t)g_H^2}{X_1(t) + X_2(t)} - \frac{(X_1(t)g_E + X_2(t)g_H)^2}{(X_1(t) + X_2(t))^2}\right] \Rightarrow \\
&\dot{g}_X(t) = \frac{1-\sigma}{\sigma}(g_X(t) - g_E)(g_X(t) - g_H).
\end{aligned} \tag{18}
$$

The solution of the differential equation (18) is

$$
g_X(t) = \frac{g_H - g_E}{1 + he^{\frac{1-\sigma}{\sigma}(g_H - g_E)t}} + g_E, h = \frac{\beta E_0^{\frac{\sigma-1}{\sigma}}}{(1-\alpha-\beta)H_0^{\frac{\sigma-1}{\sigma}}}, \tag{19}
$$

which, of course, also follows directly from (17), (16) and (7).

We have assumed that ecosystem services become relatively scarcer over time, so that $g_E < g_H$. Equation (19) shows that the growth rate of the composite production input *X* of ecosystem services and labour-augmenting technological change, g_X, converges either to the growth rate of the ecosystem services g_E (if $\sigma < 1$) or to the growth rate of the technological change g_H (if $\sigma > 1$). If man-made inputs can easily substitute for

ecosystem services, the importance of the ecosystem services in production declines and technological change drives the economy in the end. However, if man-made inputs cannot easily substitute for ecosystem services, the low growth rate of the ecosystem services restricts the growth possibilities of the economy. The path of the growth rate g_X follows an (inverse) logistic growth curve. It starts between g_E and g_H, and moves down towards g_E or up towards g_H, depending on the elasticity of substitution σ.

From (19) it follows that the initial condition $g_X(0)$ is determined by E_0 and H_0. If $\sigma<1$, so that the growth rate g_X converges to g_E in the long run, this initial condition is close to g_H, if the variable *h* in (19) is close to 0 or if $E_0 \gg H_0$. This means that if ecosystem services are initially abundant, the growth rate g_X starts close to g_H and it only comes down in order to converge to g_E when ecosystem services become scarce. If $\sigma>1$, so that the growth rate g_X converges to g_H in the long run, ecosystem services do not become scarce because of substitution.

In the long run the economy will converge to a balanced-growth path with the growth rate of the economy equal to g_E or to g_H. In order to identify the transient path of the economy towards this long-run balanced-growth path, and to find the time path of the discount rate, we have to solve the system (7).

We have the following proposition:

Proposition 2. With *CES* production function (15), the economy converges to the steady state in which output, consumption and capital grow at a common growth rate *g** and the discount rate converges to the steady-state value that is given by the Ramsey rule $r=\rho+\gamma g^*$. If the elasticity of substitution $\sigma<1$, the steady-state growth rate of the economy, *g**, equals the growth rate of the ecosystem services g_E. If the elasticity of substitution $\sigma>1$, the steady-state growth rate of the economy, *g**, equals the growth rate of the technological change g_H.

Proof. It is convenient to define the variables $u=F/K$ and $v=C/K$, and to rewrite the system (7). From the capital accumulation in (7), it follows that

(20) $$\frac{\dot{K}(t)}{K(t)} = u(t) - v(t).$$

The discount rate F_K becomes

(21) $$F_K = \alpha A^{\frac{\sigma-1}{\sigma}} \left(\frac{F}{K}\right)^{\frac{1}{\sigma}} = \alpha A^{\frac{\sigma-1}{\sigma}} u^{\frac{1}{\sigma}}.$$

From the first part of (16), it is easy to show that

(22) $$\frac{\dot{F}(t)}{F(t)} = \varepsilon_K(t)\frac{\dot{K}(t)}{K(t)} + (1-\varepsilon_K(t))\frac{\dot{X}(t)}{X(t)} = \varepsilon_K(t)\frac{\dot{K}(t)}{K(t)} + (1-\varepsilon_K(t))g_X(t),$$

where the production elasticity ε_K of capital *K* is equal to $\varepsilon_K = KF_K / F = F_K / u$. Using (20), (22) and (18), it follows that the system (7) becomes

(23) $$\begin{aligned} \dot{u}(t) &= \left[(1 - F_K(u(t))/u(t))(g_X(t) + v(t) - u(t))\right]u(t), \\ \dot{v}(t) &= \left[\gamma^{-1}(F_K(u(t)) - \rho) + v(t) - u(t)\right]v(t), \\ \dot{g}_X(t) &= (\sigma^{-1} - 1)(g_X(t) - g_E)(g_X(t) - g_H), \end{aligned}$$

where (19) gives the explicit solution of the third differential equation. Accordingly, in the long run, the growth rate g_X converges to:

(24) $$\lim_{t\to\infty} g_X(t) = g^* = \begin{cases} g_E & \text{if } \sigma < 1 \\ g & \text{if } \sigma = 1 \\ g_H & \text{if } \sigma > 1 \end{cases}.$$

The initial condition $u(0)$ is equal to $F(K_0, E_0, H_0)/K_0$, the initial condition of $g_X(0)$ is given by (19), E_0 and H_0, but the initial condition of $v(0)$ is not predetermined. The $\dot{u} = 0$, $\dot{v} = 0$ and $\dot{g}_X = 0$ isoclines are given by

(25) $$v = u - g_X, v = u - \gamma^{-1}(F_K(u) - \rho), g_X = g^*,$$

respectively. It follows that the steady state of the system (23) becomes (u^*, v^*, g^*) where

(26) $$F_K(u^*) = \rho + \gamma g^*, v^* = u^* - g^*.$$

This means that the steady-state discount rate is given by the Ramsey rule. In the steady state (u^*, v^*, g^*), output, consumption and capital grow at the same rate g^*, which is given by (24). Q.E.D.

The path of the growth rate g_X is fully characterized by (19), so that the system (23) is essentially a two-dimensional two-point boundary value problem with an exogenous time-dependent input. The phase diagram in the (u, v)-plane is depicted in Figure 2.

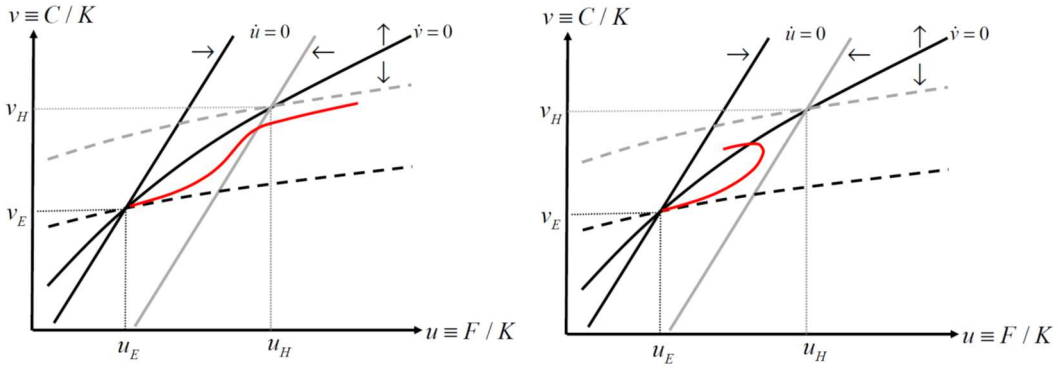

Figure 2 Phase diagrams for system (23)

The $\dot{u} = 0$ isocline is either the line $v = u - g_E$ or $v = u - g_H$. The $\dot{v} = 0$ isocline cuts the line $v = u - g_E$ in (u_E, v_E), with $u_E = F_K^{-1}(\rho + \gamma g_E)$, and the line $v = u - g_H$ in (u_H, v_H), with $u_H = F_K^{-1}(\rho + \gamma g_H)$. In case the growth rate was fixed at either g_E or g_H, we would find standard stable manifolds through (u_E, v_E) and (u_H, v_H), respectively (see Figure 2). It is helpful to make the following thought experiment. Suppose that the growth rate is first equal to g_H but suddenly and unexpectedly drops to g_E (an unexpected tipping point). It follows that the economy first jumps to the stable manifold that is approaching

(u_H, v_H) but when tipping is observed, the economy jumps to the stable manifold that is approaching (u_E, v_E). In our case, we do not have an unexpected tipping point so that the economy will prepare for a decline in the growth rate. As we have seen above in (19): if the elasticity of substitution $\sigma < 1$ and ecosystem services are initially abundant, i.e. $E_0 \gg H_0$, the growth rate g_X starts close to g_H and at some point comes down and converges to g_E. In Figure 2, this implies that the stable manifold of the system (23) starts close to the upper stable manifold in the figure and at some point, it comes down and converges to the lower stable manifold in the figure. In any case, the stable manifold of (23) lies between those upper and the lower stable manifolds.

In order to derive results about the time pattern of the discount rate, we take a closer look at the phase diagram in Figure 2. We consider the case with $\sigma < 1$, in which the economy converges in the long run to the balanced-growth steady state (u_E, v_E). For any point in time *t*, we can draw the $\dot{v} = 0$ isocline in the phase diagram in Figure 2 as the line $v = u - g_X(t)$. This line is parallel to, and in between, the two steepest lines in Figure 2, which represent $v = u - g_E$ and $v = u - g_H$. We also know that over time this lien is moving to the left, because g_X declines over time. If the stable manifold of the system (23) starts to the right of this line, it cannot cross this line, because *u* has to move down. This happens for $u(0) \geq u_H$, but also for $u(0) < u_H$ in case the line $v = u - g_X(t)$ has already moved sufficiently to the left. However, for $u(0) < u_H$, in case the growth rate $g_X(t)$ is still close to g_H, so that the line $v = u - g_X(t)$ is only very slowly moving to the left, the stable manifold of (23) jumps up close to the upper stable manifold, and the economy starts moving up and to the right. Because the economy ultimately has to converge to the balanced-growth steady state (u_E, v_E), it has to cross the $\dot{v} = 0$ isocline, so that *v* starts moving down, and then it has to meet and cross the line $v = u - g_X(t)$, so that *u* starts moving to the left.

We can derive the slope of the stable manifold from the system (23):

$$\frac{dv}{du} = \frac{\dot{v}}{\dot{u}} = \frac{\left[\gamma^{-1}(F_K(u) - \rho) + v - u\right]v}{\left[(1 - F_K(u)/u)(g_X + v - u)\right]u}. \tag{27}$$

It is immediately clear that when the stable manifold crosses the $\dot{v}=0$ isocline, the slope is 0, and when it crosses the line $v=u-g_X(t)$, the slope is infinite. In the last point, *u*, and thus the discount rate $F_K(u)$, changes direction and starts decreasing after having increased first (Figure 2, right-hand side panel). If $\sigma>1$, so that the economy converges in the long run to the balanced-growth steady state (u_H, v_H), the analysis is basically the same, and a mirror of the analysis above.

We can derive an important qualitative conclusion from the phase diagram in Figure 2. If the economy starts with sufficiently abundant inputs *X* relative to capital *K*, so that $u(0)\geq u_H$, and if substitution of ecosystem services is poor, i.e. $\sigma<1$, then the discount rate declines monotonically over time, i.e. the term structure is declining.

In order to find the precise time path of the discount rate $F_K(u)$, we have to solve the system (23), in order to find the time path of *u*. It is not possible to solve the system (23) analytically, so that we have to resort to numerical methods. The time paths of the growth rate g_X and the exogenous inputs *H* and *E* are given by (19) and (7), with the initial conditions E_0 and H_0. Our algorithm fixes and, if necessary, adjusts a time *T*, where the growth rate g_X has converged close to *g**. Then the algorithm calculates $H(T)$ and $E(T)$, and uses the steady-state values u^* and v^* from (26) and (21) to calculate the approximations of $K(T)$, $C(T)$ and $F(T)$, with (15) and the definitions $u=F/K$ and $v=C/K$. With these final values, a standard algorithm for the Ramsey optimal-growth model yields the time paths for *K* and *F*, and thus for *u*, and thus for the discount rate, given by (21). We could also apply an algorithm directly to the two-point boundary value problem, but that would amount to the same thing.

For $\sigma<1$, *g** is equal to g_E. This is the case of limited substitutability of ecosystem services in production. This is the most important case, since estimates of substitution between different factors of production indicate that natural resources and man-made inputs are usually poor substitutes, with substitution elasticities between 0.17 and 0.65 (van der Werf, 2008). We take the same parameter values as in (14). In order to calibrate the model to the stylized fact that the average (structural) growth rate has been trendless over the past decades (e.g., Jones, 2016), we take initial values for *E* and *H* such that

$X_0 = 1$, according to (16), and $g_X(0) = 0.99 g_H$, according to (19). Then it follows that $u(0) = F(K_0, X_0)/K_0 = A = 0.1$, so that the discount rate $F_K(u)$ is again initially equal to 0.03 (for all σ) according to (21), and converges to 0.00245, according to (26). In the phase diagram in Figure 2, this means that we start in $u(0) = u_H$ and that we jump to the stable manifold close to the point (u_H, v_H), because the initial growth rate $g_X(0)$ is close to g_H. As long as the growth rate g_X stays close to g_H, the optimal path hardly moves but when the growth rate g_X starts to decline, the line $v = u - g_X(t)$ moves to the left and the optimal path moves to the left and comes down, following the stable manifold. The growth rate g_X of the composite input *X* converges from the initial 0.0198 to the steady-state value, which is equal to g_E (i.e., 0.001). Consequently, the discount rate follows the same pattern, starting at 0.03, staying just below 0.03 for some time, and converging to 0.0245. Figure 3 and Figure 4 show the graphs for the growth rate g_X and the discount rate $F_K(u)$, for different values of the elasticity of substitution $\sigma < 1$. For comparison, we also show the growth rate and the discount rate for a value of $\sigma > 1$. For this value, the growth rate g_X of the composite input *X* converges to g_H (i.e., 0.02), according to (19), and the discount rate converges to 0.03, according to (26). Since we start with $g_X(0)$ close to g_H, the growth rate remains high, and the discount rate is almost flat. If man-made inputs can easily substitute for ecosystem services, the standard flat discount rate is the right choice in a deterministic setting, but if man-made inputs cannot easily substitute for ecosystem services, it is more appropriate to choose a declining discount rate.

The patterns in Figure 3 and Figure 4 are interesting and intuitively clear. If the growth rate and the discount rate decrease towards their low steady-state value, they decrease faster in case the elasticity of substitution σ is small. In that case, the effect of the low growth rate of the ecosystem services is stronger and drives down the growth rate of the economy and the discount rate faster. For policy, this implies that if the substitution possibilities are very limited, one can start with a discount rate of 3% but this number decreases relatively fast in the long term. If the substitution possibilities are better, but with the elasticity of substitution σ still smaller than 1, the decrease of the discount rate in the long term is slower.

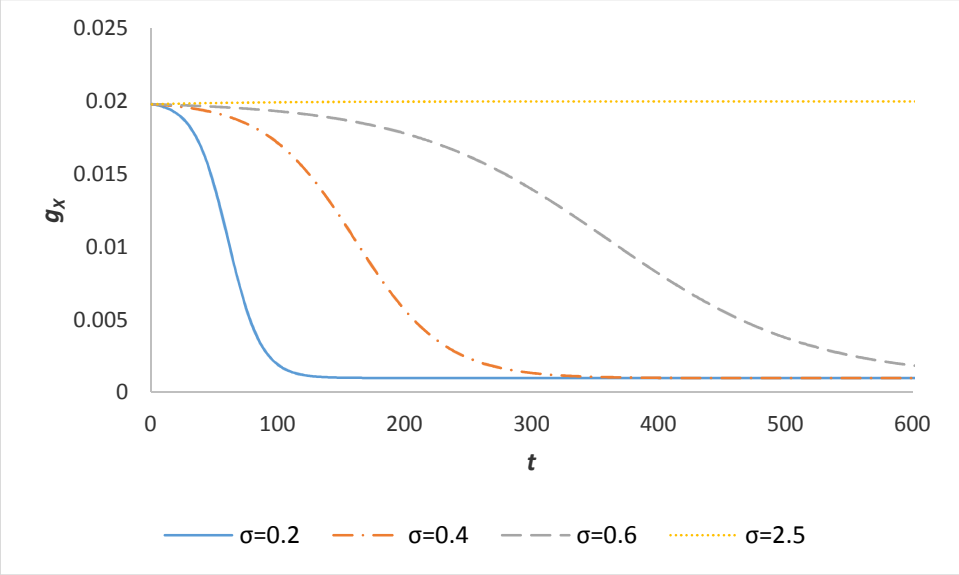

Figure 3 Time paths for growth rate: the CES case

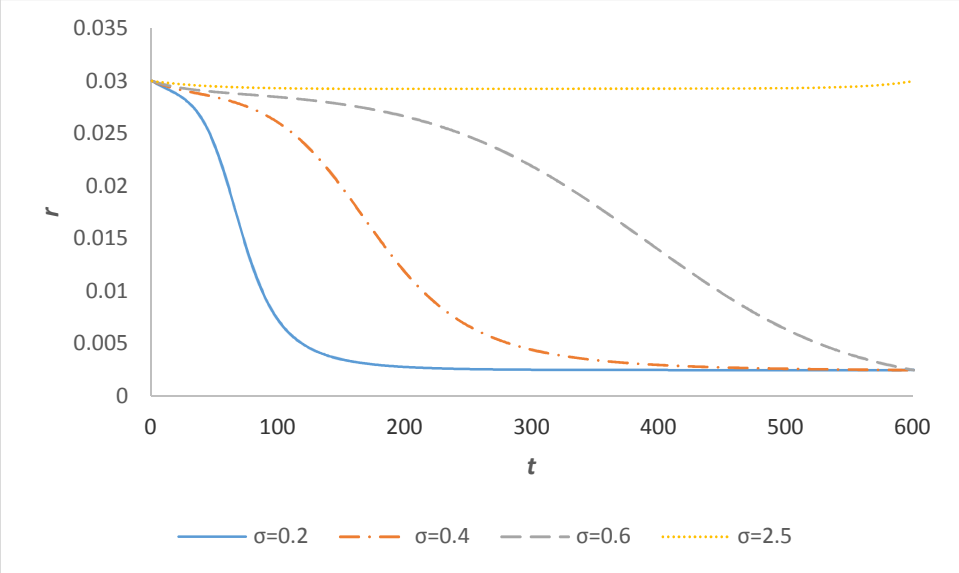

Figure 4 Time paths for discount rate: the CES case

An important conclusion is that we can get a declining discount rate. Uncertainty is the usual argument for the declining discount rate (Gollier, 2002, 2010, 2013; Newell and Pizer, 2003; Weitzman, 2007, 2010), but in this analysis it is the effect of the low growth rate of ecosystem services that are used in production and cannot easily be substituted.

3. Ecosystem services in utility and production

We extend the model in the previous section in order to consider ecosystem services in both the utility function and the production function. We assume that the ecosystem

services providing an amenity value in the utility function grow at the same exogenous rate g_E. Therefore, we can denote both types of ecosystem services by *E*. The Ramsey optimal-growth model (1) changes into:

$$\max_C \int_0^\infty e^{-\rho t} U(C(t), E(t))dt, \qquad \dot{K}(t) = F(K(t), E(t), H(t)) - C(t), K(0) = K_0, \tag{28}$$

and the optimal allocation requirements (5) become

$$U_C(C, E) = \lambda, \qquad \dot{\lambda}(t) - \rho\lambda(t) = -F_K(K(t), E(t), H(t))\lambda(t), \tag{29}$$

so that the Keynes-Ramsey rule for the optimal consumption path (6) becomes

$$\dot{C}(t) = \gamma_{CC}^{-1}(F_K(K(t), E(t), H(t)) - \gamma_{CE} g_E - \rho)C(t), \tag{30}$$

where $\gamma_{CC} = -CU_{CC}/U_C$ and $\gamma_{CE} = -EU_{CE}/U_C$ denote the elasticities of the marginal utility of consumption. Again, a transversality condition has to hold.
Note that (30) implies that the discount rate can be written as

$$F_K = \rho + \gamma_{CC} g_C + \gamma_{CE} g_E, \tag{31}$$

where g_C denotes the growth rate of consumption (see Weikard and Zhu, 2005; Hoel and Sterner, 2007).
We want to allow for substitution between consumption and the amenity of ecosystem services in consumer utility, and therefore we follow Hoel and Sterner (2007) and use the *CES* utility function:

$$U(C, E) = \frac{1}{1-\gamma}\left[(1-\pi)C^{\frac{\zeta-1}{\zeta}} + \pi E^{\frac{\zeta-1}{\zeta}}\right]^{\frac{(1-\gamma)\zeta}{\zeta-1}}, \tag{32}$$

where γ is again the inverse of the elasticity of intertemporal substitution, π denotes the relative shares, and ζ is elasticity of substitution between consumption and ecosystem services in utility. It is straightforward to derive the two elasticities γ_{CC} and γ_{CE}. The result is:

$$\gamma_{CC} = \gamma - \gamma_{CE}, \gamma_{CE} = \delta(\gamma - \zeta^{-1}), \tag{33}$$

where

$$\delta = \frac{\pi E^{\frac{\zeta-1}{\zeta}}}{(1-\pi)C^{\frac{\zeta-1}{\zeta}} + \pi E^{\frac{\zeta-1}{\zeta}}}. \tag{34}$$

This δ can be interpreted as the value share of the ecosystem services in the consumer expenditure. It is generally not constant over time when there is substitutability between consumption and ecosystem services in utility. In Hoel and Sterner (2007) and Traeger (2011) the growth rates of *C* and *E* are fixed, so that δ converges to 0 or 1, depending on the elasticity of substitution ζ. However, if the growth rate of *C* converges to the growth rate of *E*, then δ converges to a number between 0 and 1.

In order to understand the results below, we first focus on the adjusted Ramsey rule for this problem, given by (31). Using (33), we can write this as:

$$F_K(t) = \rho + \gamma\left[(1-\delta(t))g_C(t) + \delta(t)g_E\right] + \zeta^{-1}\delta(t)\left[g_C(t) - g_E\right], \tag{35}$$

where δ is given by (34).

Hoel and Sterner (2007) have essentially the same result, but we regroup the terms in order to facilitate a clear interpretation. Equation (35) clearly shows the determinants of the discount rate F_K (see also Traeger, 2011). The three terms at the right-hand side represent "impatience", "(full) consumption smoothing", and "relative price effects", respectively. Note that the welfare at a point in time *t* not only depends on the produced consumption *C* but on "full consumption", i.e. *C* and ecosystem services *E*, weighted by $(1 - \delta)$ and δ, respectively. The second term shows that if the full consumption grows

fast, the discount rate becomes high, because the marginal value of full consumption falls. This is the standard story. The strength of this effect is governed by the elasticity of intertemporal substitution γ^{-1} (the intergenerational inequality aversion γ). The third term shows that if consumption *C* becomes more abundant than ecosystem services *E*, the relative price of consumption *C* gets lower, provided it is an imperfect substitute for ecosystem services *E*. The (consumption) discount rate becomes higher. Therefore, the growing relative scarcity of ecosystem services provides an additional reason to value the future consumption lower and to use a higher (consumption) discount rate. The strength of this effect is governed by the elasticity of intra-temporal substitution ζ.
For a given growth rate of consumption g_C, a decline in the growth rate of ecosystem services g_E has two opposing effects. On the one hand, it slows down full consumption growth, and thus lowers the discount rate. On the other hand, it makes consumption *C* relatively more abundant, and therefore increases the (consumption) discount rate. The second effect dominates, if the elasticity of intra-temporal substitution ζ is smaller than the elasticity of intertemporal substitution γ^{-1}.

As in the previous section, we assume that the economy is driven by labour augmenting technological change *H*, with growth rate g_H, but that the ecosystem services, which provide intermediate inputs in production, grow at a lower rate g_E. Instead of (7), we have to solve the system

$$
\begin{aligned}
&\dot{K}(t) = F(K(t), E(t), H(t)) - C(t), K(0) = K_0, \\
&\dot{C}(t) = (\gamma - \gamma_{CE}(t))^{-1}(F_K(K(t), E(t), H(t)) - \gamma_{CE}(t) g_E - \rho) C(t) \\
&\dot{H}(t) = g_H H(t), H(0) = H_0, \\
&\dot{E}(t) = g_E E(t), E(0) = E_0,
\end{aligned}
\tag{36}
$$

where γ_{CE} is given by (33) and (34). Note that γ_{CE} is function of $C(t)$ and $E(t)$.
In this system, some ecosystem services *E* provide an amenity value to consumers, and therefore affect the elasticities of the marginal utility of consumption. Other ecosystem services *E* are a production factor, and thus affect the growth rate of consumption. It follows that the ecosystem services *E* affect the discount rate in two ways.

In order to investigate the long-run properties of the system, we can follow the analysis of the previous section. In the right-hand side of the second equation of system (23), however, the growth rate of consumption in equation (6) changes into the growth rate of consumption in equation (36). Furthermore, γ_{CE} in equation (36) can be replaced by $\delta(\gamma-\zeta^{-1})$, according to (33). This leads to the more complicated system

$$
\begin{aligned}
\dot{u}(t) &= \left[(1-F_K(u(t))/u(t))(g_X(t)+v(t)-u(t))\right]u(t),\\
\dot{v}(t) &= \begin{bmatrix}(\gamma-\delta(t)(\gamma-\zeta^{-1}))^{-1}(F_K(u(t))-\delta(t)(\gamma-\zeta^{-1})g_E-\rho)\\ +v(t)-u(t)\end{bmatrix}v(t),\\
\dot{g}_X(t) &= (\sigma_2^{-1}-1)(g_X(t)-g_E)(g_X(t)-g_H),
\end{aligned}
\tag{37}
$$

where δ is given by (34), as a function of *C/E*. It is a tedious, but straightforward, to express the variable *C/E* in the variables *u*, *v* and g_X (see the Appendix), so that (37) is a well-defined system.

As in the previous section, the $\dot{u}=0$ isocline is the line $v=u-g_X(t)$, which is located in between the lines $v=u-g_E$ and $v=u-g_H$ in the (*u*, *v*)-plane. We can characterize the $\dot{v}=0$ isocline by two extreme positions as well. For $\delta=0$, the $\dot{v}=0$ isocline is the same as in the previous section, and for $\delta=1$, the $\dot{v}=0$ isocline reaches the other extreme (because $0\le\delta\le1$):

$$
\begin{aligned}
&v=u-\gamma^{-1}(F_K(u)-\rho), \quad \delta=0,\\
&v=u-\zeta(F_K(u)-(\gamma-\zeta^{-1})g_E-\rho), \quad \delta=1.
\end{aligned}
\tag{38}
$$

Note that the two curves in (38) coincide for $\gamma=\zeta^{-1}$, and that both curves in (38) cut the line $v=u-g_E$ in (u_E,v_E), with $u_E=F_K^{-1}(\rho+\gamma g_E)$. For $\gamma>\zeta^{-1}$, the second curve in (38) cuts the line $v=u-g_H$ in a point with $u<u_H$, and for $\gamma<\zeta^{-1}$, the second curve in (38) cuts the line $v=u-g_H$ in a point with $u>u_H$, where $u_H=F_K^{-1}(\rho+\gamma g_H)$. Since it is reasonable to assume that $\gamma>\zeta^{-1}$ (Drupp, 2018), we can depict the basic elements of the phase diagram in the (*u*, *v*)-plane as in Figure 5.

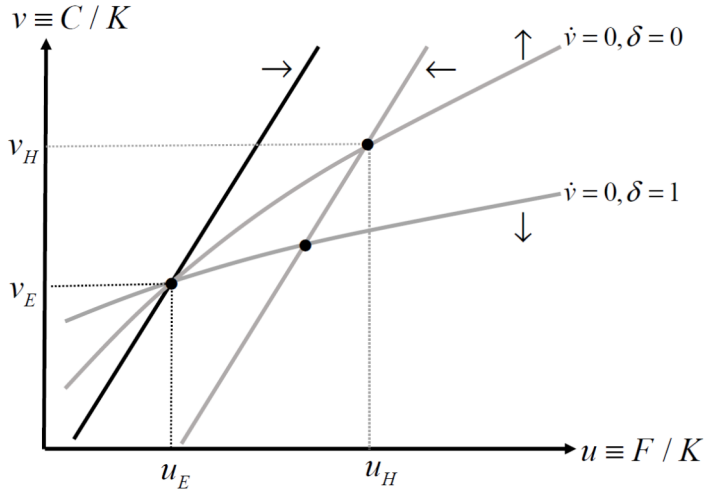

Figure 5 Steady states for system (37) with $\gamma > \zeta^{-1}$

The steady-state version of the extended Ramsey rule becomes:

(39) $$F_K(u^*) = \rho + \delta^*(\gamma - \zeta^{-1})g_E + (\gamma - \delta^*(\gamma - \zeta^{-1}))g^*,$$

where δ^* denotes the steady-state value of δ.

Depending on the elasticities of substitution ζ and σ, we can distinguish three cases that correspond to the three possible steady states in Figure 5.

Case 1. If $\zeta > 1$ and $\sigma > 1$, g_X converges to g_H and δ converges to 0. The steady state is the intersection of the first curve in (38) and the line $v = u - g_H$, and the discount rate becomes

(40) $$F_K(u^*) = \rho + \gamma g_H,$$

which is the case in which it is simply assumed that the economy can substitute away from ecosystem services.

Case 2. If $\zeta < 1$ and $\sigma > 1$, g_X converges to g_H and δ converges to 1. The steady state is the intersection of the second curve in (38) and the line $v = u - g_H$, and the discount rate becomes

(41) $$F_K(u^*) = \rho + (\gamma - \zeta^{-1})g_E + \zeta^{-1}g_H = \rho + \gamma g_H - (\gamma - \zeta^{-1})(g_H - g_E),$$

which is the case in which consumers cannot easily substitute produced consumption goods for the amenity value of ecosystem services, although the producers can easily substitute away from the intermediate use of ecosystem services in the production. As we have shown in Figure 5, for $\gamma > \zeta^{-1}$, the second curve in (38) cuts the line $v = u - g_H$ in a point with $u^* < u_H$, where $u_H = F_K^{-1}(\rho + \gamma g_H)$, so that the steady-state discount rate is lower than in case 1. For $\gamma < \zeta^{-1}$, the second curve in (38) cuts the line $v = u - g_H$ in a point with $u^* > u_H$, so that the steady-state discount rate is higher than in case 1 (see also Hoel and Sterner, 2007).

Case 3. If $\sigma < 1$, g_X converges to g_E and δ converges to a number δ^* between 0 and 1. The number δ^* is determined by ζ and σ and by the steady-state values u^* and v^* (see the Appendix). The steady state is the intersection of the line $v = u - g_E$ and a $\dot{v} = 0$ isocline in between the two curves in (38). The steady state discount rate becomes

$$F_K(u^*) = \rho + \gamma g_E, \tag{42}$$

which is the case where the limited substitutability of ecosystem services in production restricts the economic growth. At the end, the substitutability in utility does not matter, because the growth rate of the economy in the long run converges to g_E.

In order to study the development of the discount rate over time, we consider the phase diagram in Figure 5 more closely. For fixed values of δ between 0 and 1, we consider the $\dot{v} = 0$ isoclines given by

$$v = u - (\gamma - \delta(\gamma - \zeta^{-1}))^{-1}(F_K(u) - \delta(\gamma - \zeta^{-1})g_E - \rho), \tag{43}$$

which are positioned between the extremes for $\delta = 0$ and $\delta = 1$ in equation (38). In Figure 6, we have depicted the $\dot{v} = 0$ isoclines for a fixed $\delta(0)$ and two different values of ζ. It is easy to show that the $\dot{v} = 0$ isocline in equation (43) rotates downwards for an increasing ζ.

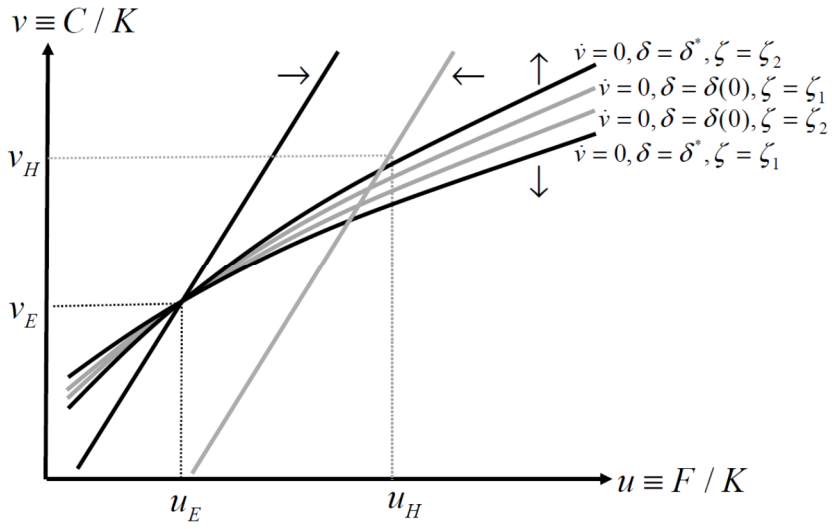

Figure 6 Dynamics of $\dot{v}=0$ isoclines for $\gamma^{-1}<\zeta_1<1$ and $\zeta_2>1$

Note that $\delta(0)$ in (34) is not predetermined, because $C(0)$ is not predetermined. It is reasonable to assume that initially the value share of ecosystem services in consumer expenditure is small, so that $\delta(0)$ is close to 0, and both corresponding $\dot{v}=0$ isoclines in Figure 6 are close to the one for $\delta=0$. It is clear from (34) that δ is decreasing over time if the elasticity of substitution in utility $\zeta>1$, and that δ is increasing over time if $\zeta<1$. It implies that in the dynamical process, not only the line $v=u-g_X(t)$ is moving to the right or to the left but also the $\dot{v}=0$ isocline is rotating upwards (in case $\zeta>1$) or downwards (in case $\zeta<1$). In *cases 1* and *2*, the line $v=u-g_X(t)$ starts close to the line $v=u-g_H$, and moves towards this line. If we start in $u(0)=u_H=0.1$, with the discount rate equal to 0.03, we cannot immediately jump close to the point (u_H,v_H), as in the previous section, because we have to stay below the $\dot{v}=0$ isoclines. This implies that the optimal path moves a bit to the left until it is blocked by the line $v=u-g_X(t)$, so that the discount rate initially decreases somewhat. If the elasticity of substitution in utility $\zeta>1$, δ converges to 0 and the $\dot{v}=0$ isocline rotates upwards, so that the optimal path changes direction, and the discount rate converges to 0.03. Easy substitution of ecosystem services in utility implies that the path of the discount rate is close to the one we found in the previous section. However, if the elasticity of substitution in utility $\zeta<1$, δ converges to 1 and the $\dot{v}=0$ isocline rotates downwards, so that the optimal path moves down and to the left, and the discount rate declines in the long run to the steady-state value given by (41). This shows the relative price effect as was introduced

in the previous literature (Weikard and Zhu, 2005; Hoel and Sterner, 2007). In *case 3*, the line $v = u - g_X(t)$ starts close to the line $v = u - g_H$ and moves to the left. The $\dot{v} = 0$ isocline rotates upwards (in case $\zeta > 1$) or downwards (in case $\zeta < 1$), but not all the way to the extreme positions in (38). The value share of ecosystem services in consumer expenditure δ converges to a long-run value δ^* in between $\delta(0)$ and 0 (in case $\zeta > 1$), or to a δ^* in between $\delta(0)$ and 1 (in case $\zeta < 1$). These values of δ^* can be calculated with the expressions in the Appendix. The main difference with *cases 1* and *2* is that the line $v = u - g_X(t)$ moves to the left, so that the optimal path will come down at some point and converge to the steady state (u_E, v_E). This implies that the relative price effect almost disappears in the long run, because the growth rate of consumption converges to the growth rate of ecosystem services. In the short run, however, the growth rate of consumption is higher than the growth rate of ecosystem services, and a similar effect occurs as in *cases 1* and *2*. If we start in $u(0) = u_H = 0.1$, with the discount rate equal to 0.03, we have to stay below the $\dot{v} = 0$ isoclines. This implies that the optimal path starts to move a bit to the left, until it is temporarily blocked by the line $v = u - g_X(t)$, so that the discount rate initially decreases a bit. This effect is small, but it is a little bit larger in case $\zeta > 1$ than in case $\zeta < 1$, because the initial position of the $\dot{v} = 0$ isocline is a bit lower in the first case. Otherwise, the optimal paths are not much different from the one described in the previous section. The production effect leading to a declining discount rate dominates the relative price effect in utility. In the next section, we will present an example.

We can formulate the following proposition:

Proposition 3. For the *CES* production function (15) and *CES* utility function (32), the economy converges to the steady state in which output, consumption and capital grow at a common growth rate g^*. If the elasticity of substitution in the production $\sigma < 1$, the steady-state growth rate of the economy, g^*, equals the growth rate of the ecosystem services g_E, and the discount rate converges to the steady-state value that is given by the standard Ramsey rule $r = \rho + \gamma g_E$. If the elasticity of substitution in the production $\sigma > 1$, the steady-state growth rate of the economy, g^*, equals the growth rate of the technological change g_H. The discount rate converges to the steady-state value that is

given by the standard Ramsey rule $r = \rho + \gamma g_H$, if the elasticity of substitution in utility $\zeta > 1$, and converges to the steady-state value that is given by an adjusted Ramsey rule $r = \rho + \gamma g_H - (\gamma - \zeta^{-1})(g_H - g_E)$, if the elasticity of substitution in utility $\zeta < 1$.
If the economy starts with sufficiently abundant inputs *X* relative to capital *K*, so that $u(0) \geq u_H$, and if $\sigma < 1$ and $\gamma > \zeta^{-1}$, then the discount rate declines monotonically over time, i.e. the term structure is declining.

In order to present the precise time path of the discount rate in all these cases, we have to resort to numerical methods again. We use the same algorithm as in section 2. In the next section, we give an example that characterizes the different possible paths for the discount rate.

4. Numerical example

In this section, we present a numerical example for the paths of the discount rate in the three cases that we identified in the previous section. For each case, we take the same set of parameter values and initial values as in section 2, that is

$$\begin{aligned} &\gamma = 1.45, g_H = 0.02, \rho = g_E = 0.001, \alpha = 0.3, \beta = 0.2, \\ &K_0 = X_0 = 1, u(0) = A = 0.1, g_X(0) = 0.99 g_H. \end{aligned} \tag{44}$$

We choose the share π in the utility function (32) such that the initial value share of the ecosystem services in the consumer expenditure $\delta(0) = 0.2$. Note that this requires an iterative process, because $\delta(0)$ is not predetermined. The discount rate is initially equal to 0.03, according to (21). We take different values for ζ and σ in order to distinguish the three cases of the previous section. For σ, we return to the central values of Section 2.2: $\sigma = 2.5 > 1$ and $\sigma = 0.4 < 1$. For ζ, we take the mean value and the lowest value from the empirical literature (Drupp, 2018): $\zeta = 2.31 > 1$ and $\zeta = 0.86 < 1$, respectively. Combining $\zeta = 2.31$ and $\sigma = 2.5$ yields *case 1*. In this case, the ecosystem services are perceived to be substitutable in production and consumption. Combining $\zeta = 0.86$ and $\sigma = 2.5$ yields *case 2*, and combining $\sigma = 0.4$ with $\zeta = 0.86$ and $\zeta = 2.31$ yields *cases 3a* and *3b*, respectively. The iterative process in order to get the initial value $\delta(0) = 0.2$ yields $\pi = 0.563$ in *case 1*, $\pi = 0.133$ in *case 2*, $\pi = 0.351$ in *case 3a*, and $\pi = 0.0166$ in

case 3b. Using the Appendix, it is easy to show that with the last two values of π that yield the initial value $\delta(0)=0.2$, the steady-state values δ^* become 0.304 for $\zeta=0.86$ and 0.034 for $\zeta=2.31$. In the phase diagram in Figure 6, the $\dot{v}=0$ isocline rotates downwards for $\zeta=0.86$ and upwards for $\zeta=2.31$. It is interesting to note that δ does not become larger than 0.304 in *case 3ab*, so that the relative price effect is small. In the long run, the relative price effect almost disappears, because the growth rate of consumption converges to the growth rate of ecosystem services.

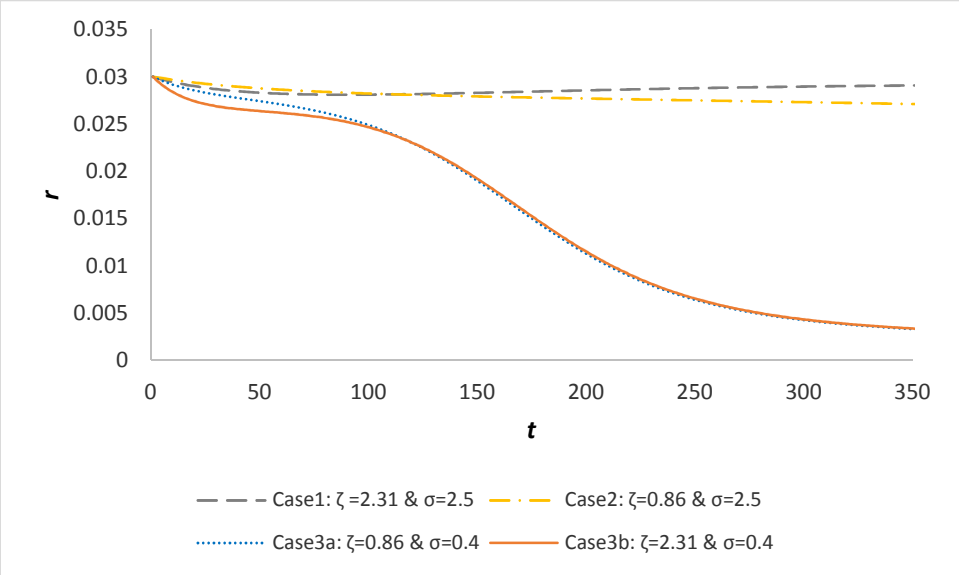

Figure 7 Time paths for the discount rate for different elasticities of substitution

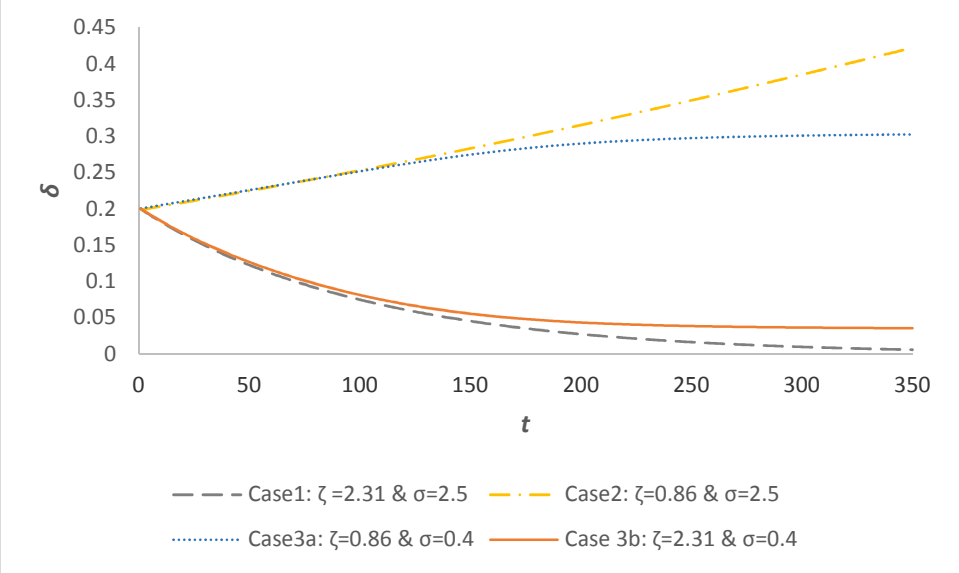

Figure 8 Time paths for the value share of ecosystem services

Figure 7 shows the time paths of the discount rate. Figures 8 shows the time paths of the value share of ecosystem services in consumer expenditure δ in *cases 3a* and *3b*.

The discount rate starts at 0.03 in all cases. In *case 1*, the discount rate remains close to this level, and the time path is almost flat, because the growth rate is high and ecosystem services can be easily substituted in production and in utility. As we have seen in our discussion of the phase diagram in Figure 6, initially the discount rate moves down a bit, but then it moves up again and converges to the steady-state value 0.03. This is the case in which the ecosystem services are perceived to be substitutable in production and consumption.

In *case 2*, the growth rate also remains high, because ecosystem services can be easily substituted in production. In utility, however, the ecosystem services become relatively scarce and since we assume on the basis of empirical observations that the elasticity of intra-temporal substitution ζ is larger than the elasticity of intertemporal substitution γ^{-1}, the discount rate decreases further. It is interesting to note that it takes a long time before the decrease becomes substantial. We have not depicted the full convergence to the steady state in this case, which is $F_K(u^*) = 0.025$ according to (41). However, it is clear that the relative price effect is small, and kicks in only after a long time.

In *case 3*, the discount rate declines because the growth rate declines. The *cases 3a* and *3b* hardly differ. As we have seen in our discussion of the phase diagram in Figure 6, initially the discount rate moves down a bit, and a bit more in *case 3b* than in *case 3a* because of the different starting positions. Then the relative price effect causes the paths to move closer to each other. Figure 8 shows how the value share of ecosystem services in consumer expenditure δ develops in the two cases, with different steady-state values δ^* (i.e., $\delta^* = 0.304$ in *case 3a* and $\delta^* = 0.034$ in *case 3b*). When the paths for the two discount rates meet in these cases, the effect of the scarcity of ecosystem services in production dominates the relative price effect in utility, and the paths come down and stay close together. In the long run, the discount rate converges to the steady-state value 0.00245, according to (42).

5. Conclusion

This paper considers the discount rate in case ecosystem services are important for the production but cannot grow at the same rate as the usual drivers of economic growth, such as technological change. The literature on the discount rate mostly assumes that

the growth rate is given but in case ecosystem services are important for production and cannot be easily substituted, growth of the economy will be restricted and the discount rate will decline.

This paper considers the ecosystem services in production and directly in utility, with in both cases high or low elasticities of substitution. If substitution is easy, the discount rate in the long term follows the standard Ramsey rule with a high growth rate. On the balanced-growth path, discounting over any time horizon occurs at a constant rate. If substitution in production is easy but substitution in utility is not easy, the relative price effect from the previous literature shows up. Growing scarcity of ecosystem services requires an adjusted Ramsey rule for the discount rate. Most importantly, however, if substitution in production is not easy, the discount rate declines towards a low value given by the standard Ramsey rule with a low growth rate. Moreover, in this context the relative price effect is small and almost disappears in the long run.

In order to analyse these issues, this paper uses a Ramsey optimal-growth framework, with both a *CES* utility function and a *CES* production function. The balanced-growth steady states are derived, the system dynamics are characterized in phase diagrams, and a numerical procedure is used to calculate the paths of the growth rate and the discount rate in a calibrated model. In this way, the paper compares, across different scenarios, not only the steady-state values of the discount rate but also the paths towards these steady-state values.

The main result of the paper is that the appropriate discounting rule crucially depends on the role of ecosystem services in production. If the ecosystem services can be easily substituted, growing scarcity will not slow down the growth from technological change, and the discount rate can be based on the current balanced-growth rate of consumption. However, if the ecosystem services cannot be easily substituted, and if the growth rate of ecosystem services is low, future growth will slow down and the discount rate will decline towards a low value. This main result implies that the role of ecosystem services in utility is not very strong. In the case in which the economy can achieve a high growth rate in the long term, the previous literature has already pointed out the relative price effect that occurs if the growth rate of the ecosystem services and the elasticity of substitution in utility are low. However, we show that this effect is not large and only occurs after a long term. In the case in which the economy cannot achieve a high growth rate in the long term, we show that this relative price effect hardly plays a role. These are theoretical results. In practice, it is necessary to identify production sectors where

ecosystem services are essential, and to determine the growth rates and the elasticities of substitution. This is a topic for further research.

The previous literature has shown extensively that a declining discount rate may also result from the introduction of several forms of uncertainty. This paper has left out this uncertainty, but it is clear that many arguments point in the same direction, and thus support the idea of a declining discount rate. This paper has specifically focused on the role of a limited availability and substitutability of ecosystem services in production. A declining discount rate is very important for the cost-benefit analyses with a long time horizon, because a flat discount rate would make the costs and benefits in the far future hardly count.

This paper assumes an exogenous (low) growth rate for ecosystem services, but ignores a possible feedback effect of production and consumption (and thus emissions into the natural environment) on the availability of ecosystem services. This is a topic for further research as well. Moreover, this paper contains interesting simulations to extend and illustrate the theoretical results, but calibrations with real data are needed in order to quantify the policy advice. This is also a direction for future research.

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Appendix: Specification of δ as a function of the variables of system (37)

According to (34), δ is a function of C/E:

(A.1) $$\delta = \frac{\pi}{(1-\pi)(C/E)^{\frac{\zeta-1}{\zeta}} + \pi}.$$

Since $C/E = (C/K)(K/F)(F/E) = (v/u)(F/E)$, we have to show that F/E is a function of the system variables. From (15), it follows that:

(A.2) $$F_E = \beta A^{\frac{\sigma-1}{\sigma}} \left(\frac{F}{E}\right)^{\frac{1}{\sigma}} \Rightarrow \frac{F}{E} = A\left(\frac{1}{\beta}\frac{EF_E}{F}\right)^{\frac{\sigma}{1-\sigma}},$$

and from (16) and (15), it follows that:

(A.3)
$$\frac{\dot{F}}{F} = \frac{KF_K}{F}\frac{\dot{K}}{K} + \left(1 - \frac{KF_K}{F}\right)\frac{\dot{X}}{X} = \frac{KF_K}{F}\frac{\dot{K}}{K} + \frac{EF_E}{F}\frac{\dot{E}}{E} + \left(1 - \frac{KF_K}{F} - \frac{EF_E}{F}\right)\frac{\dot{H}}{H}.$$

With (21) and the definitions of the growth rates, this implies that:

(A.4)
$$\left(1 - \frac{F_K(u)}{u}\right)g_X = \frac{EF_E}{F}g_E + \left(1 - \frac{F_K(u)}{u} - \frac{EF_E}{F}\right)g_H.$$

Combining (A.2) and (A.4) yields:

(A.5)
$$\frac{C}{E} = \frac{v}{u}\frac{F}{E} = \frac{Av}{u}\left(\frac{1}{\beta}\left(\frac{g_H - g_X}{g_H - g_E}\right)\left(1 - \frac{F_K(u)}{u}\right)\right)^{\frac{\sigma}{1-\sigma}}.$$

Q.E.D.