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Subsistence – A Bio-economic Foundation of the Malthusian Equilibrium*

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Abstract. This paper develops a bio-economic Malthusian growth model. By integrating recent research on allometric scaling, energy consumption, and ontogenetic growth we provide a model where subsistence consumption is endogenously linked to body mass and fertility. The theory admits a two-dimensional Malthusian equilibrium characterized by population density and body mass (metabolic rate) of the representative adult. As a result, the analysis allows us to examine the link between, in particular, human biology and long run income, body mass and population size. Off the steady-state we investigate the possibility of cyclical behavior of the size of a population and the size of its representative member. We also demonstrate that a take-off into sustained growth should be associated with increasing income, population size, and body mass. The increase in the latter is, however, bounded and can be viewed as convergence to a biologically determined upper limit.

Keywords: Subsistence, Nutrition, Metabolism, Population Growth, Ontogenetic Growth, Malthus.

JEL: O11, I12, J13.

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Subsistence is not located at the edge of a nutritional cliff, beyond which lies demographic disaster. ...[R]ather than one level of subsistence, there are numerous levels at which a population and a food supply can be in equilibrium, in the sense that they can be indefinitely sustained. (Robert W. Fogel, 1994)

1. Introduction

Economic analysis often assume the existence of a level of "subsistence consumption", i.e. a strictly positive lower bound on consumption choice. Importantly, from this assumption flows a number of strong predictions.[1] Perhaps most fundamentally, introducing a minimum consumption constraint leads to the prediction that the savings rate increases with income.[2] From a macroeconomic perspective, an income-dependant savings rate has substantial implications for the impact of income inequality on growth (e.g. Galor and Moav, 2004), and for the convergence process. Indeed, if subsistence consumption is introduced into otherwise standard growth models, multiple steady state equilibria (poverty traps) may arise, which implies that initial conditions determine whether poor countries will be able to catch up or not (e.g. Azariadis, 1996). From a policy perspective, the notion that poor countries may be stuck in a subsistence-induced poverty trap has to a considerable extent formed the logic behind the currently proposed "scaling up" of foreign aid transfers (see Sachs et al., 2004).

A minimum consumption constraint is also a central component of recent theories which aim to provide an understanding of the mechanics of development in the very long run. According to the received perception in this literature, economies were for the bulk of human history "trapped" in a environment best described by Malthusian forces. That is, a situation where the population size ensures an equilibrium involving stagnating living standards around subsistence; a level of income which is required for survival but not much more.[3] A positive shock to average income will induce individuals to increase fertility, but will not instigate capital accumulation (human or otherwise) due to the fact that individuals are near subsistence. As a result of diminishing returns to labor, a rising population will therefore eventually restore the initial level of GDP per capita thus choking off any further expansion of population.

[1]The most common way of introducing subsistence consumption into a growth model involves Stone-Geary preferences, i.e. $U(c) = \frac{(c-\bar{c})^{1-\sigma}}{1-\sigma}$,where $\bar{c} > 0$ is the level of subsistence.
[2]See Dynan et al. (2004) for supportive evidence of this proposition.
[3]See Galor (2006) for a comprehensive survey of the literature which models the Malthusian regime as well as the transition into a modern growth regime where GDP per capita rises exponentially.

In spite of its importance to analytical results, subsistence consumption is always treated as exogenous in the economics literature.[4] However, from a biological standpoint minimum consumption requirements, in the sense of basal metabolism, is inescapably linked to fertility (i.e. pregnancy leads to elevated consumption needs) and body size.[5] For a fuller understanding of the long run historical record, the number *and* size of individuals (and thus their subsistence requirements) should be an equilibrium outcome. Providing such an extension of the Malthusian framework, based on first principles, is the fundamental objective of the present paper. The model comprises three central elements which generate the novel results of the present analysis.

First, consumption during childhood determines body mass as an adult. Accordingly, we emphasize the non-reversible body mass component; "mass" and "height" can therefore be viewed as proportional. The link between child nutrition and body size has long been recognized by economists (see e.g. Fogel, 1994). Here, however, we derive the law of motion for body mass from fundamental biological and physical principles. Biological parameters, reflecting e.g. the energy costs associated with cell maintenance, will therefore turn out to matter for the steady state of the model. In other words, human biology will importantly affect economic outcomes, such as per capita income, in the Malthusian environment.

Second, body mass as an adult and the fertility rate determines subsistence consumption. Again, the link between subsistence, defined as basal metabolism, and fertility-body size is firmly grounded in theory and evidence stemming from the field of biology, as explained below. The fact that child bearing affects energy requirements of an adult (in the model people reproduce asexually) introduces a natural quantity-quality trade-off. Parents derive utility from the number and consumption level of their children; both are conceived to be normal goods. However, increasing fertility requires the parent to extent her own consumption, to cover enhanced subsistence needs, which comes at the cost of less consumption and future body mass for the offspring.

Finally, larger individuals are assumed to be more healthy and therefore more productive. The assumption that "health" matters for labor productivity is not original to this paper (see e.g. Strulik, 2005). However, since it is a well documented fact that larger individuals are

[4]Lagerlöf (2006b) is an exception to the rule. However, our analysis differs from his in an number of dimensions, as explained below.

[5]Metabolism refers to the biochemical processes by which nutrients are transformed into energy, which allows the organs of the body (i.e. ultimately the cells of the body) to function. The basal metabolic rate is defined as the amount of energy expended while at rest.

more healthy and productive, it is important to incorporate this mechanism nevertheless so as to demonstrate the viability of the Malthusian equilibrium in the presence of "health capital accumulation".[6]

The model admits a generalized Malthusian equilibrium, involving constant income per person, a constant population size, and constant body mass of individuals. The first central result of the analysis is that the steady state is unique, which raises some doubts as to whether theories that generate poverty traps through the subsistence channel will be robust to a more rigorous treatment of minimum consumption requirements.

The model holds further predictions which are revealed upon a shock to the system. A permanent one-off increase in productivity will imply that the parent expands fertility, and child consumption. Consequently, the next generation of adults will be bigger and more numerous. The increase in body mass implies greater subsistence requirements for each adult. The expansion of the quantity of individuals will tend to reduce labor productivity, whereas the expansion of quality (i.e. mass) works in the opposite direction. If the former effect dominates, which is required for the existence of a Malthusian equilibrium, labor productivity will fall from one generation to the next. The combination of slightly lower income and higher adult subsistence requirements implies a lower fertility rate and lower child consumption. Accordingly, adults of the following generation will tend to be more numerous but have lower body mass. The process will continue until population growth ceases, and body mass is constant once again. In the new steady state the population is larger, but the size of each person is unaltered. In addition, the adjustment from one steady state to the next may feature oscillations.

Hence, the model preserve the standard implications of Malthusian models of long-run growth as regards the evolution of population size.[7] In addition, the predictions of the model are broadly consistent with historical evidence on the evolution of body size.

Figure 1 speaks to the evolution of body size (as measured by height) during the last two millenia. The data underlying Figure 1 derives from recent work by historians who have started investigating skeleton remains, thereby producing evidence of body size going far back in time (Steckel, 2001; Koepke and Baten, 2005). The overall impression (which is confirmed by formal tests) is that body size was roughly trend less from the first century A.D until the 18th century,

[6]See e.g. Strauss (1986), Fogel (1994, 1997) or Weil (2005) for evidence of the link between health and productivity.
[7]Oscillatory adjustment of population to steady state, while not a standard result in a Malthusian growth model, is not inconsistent with the data; see e.g. the discussion in Lagerlöf (2006a), and Lee and Anderson (2002).

Figure 1: Height Developments in Europe, 1-18th Century

Source: Koepke and Baten (2005).

consistent with the steady state predictions of the present theory (see also Clark, 2006). However, another visually arresting theme of the figure is the fluctuations which height seems to have undergone from one century to the next. According to the model, this pattern could be ascribed to transitional dynamics.

Interestingly, a *permanent* acceleration in productivity growth, as would be associated with a "take-off" into a modern growth regime, will *permanently* affect body mass. Specifically, during a process whereby countries in this way emerge from a Malthusian regime, the model predict rising body mass as well as population and average income. Importantly, however, the gain in body size is bounded, and can be viewed as convergence to a biologically determined upper limit. The prediction that body size (and population growth) rises during the transition to "modern growth" is consistent with evidence for Europe during the 19th century and 20th century (e.g. Steckel, 1995).

The introduced biological elements allow for a series of novel experiments to be conducted. For example, we are able to examine the impact of changes in diet, as reflected in the energy contents of food. As an illustration, whereas as the calorie content of wheat is 3,190 cal. per edible kg., the corresponding number is 2,140 cal. for beef (Livi-Bacci, 1991, Ch. 5). Hence, in practise the composition of food in the diet will therefore determine the mapping from the

physical quantity of consumption into energy intake. Historically, diets have changed as new foods became available. For example, in the context of the Neolithic revolution, or, in Europe following the age of discovery.[8]

We also consider changes in the bio-economic environment related to the impact of increasing work effort. According to existing evidence, work effort increased considerably in the context of the Neolithic revolution; hunter-gatherers were more leisured than (at least the early) agriculturalists (see Weisdorf, 2005). Persistent changes in work intensity matters for metabolism, and the model allows us to trace its consequences for long-run body size, population size and income per capita.

The present paper is related to the literature on growth in the very long run, which models Malthusian stagnation and the transition to modern growth (Galor and Weil , 2000; Lucas, 2002; Hansen and Prescott, 2002; see Galor, 2006 for a survey). These theories focus on the intricate and changing relationship between income and population during long-run development. In contrast to the present paper, however, these studies ignore the evolution of body size, and assume the level of subsistence consumption is exogenous. The present paper does not model the transition to modern growth. Instead we focus on the Malthusian regime, and the process of "take-off".

The closest precursor to our work is Lagerlöf (2006b), who aims to explain why body size seems to have followed a hump-shaped trajectory over the last 1,000,000 years. In contrast to the present paper, Lagerlöf (2006b) assumes body size is exogenous at the level of the individual, thus fully determined by the genetic make-up of individuals. Changes in the composition of the population, brought forth by changing evolutionary pressures, is therefore required to produce changes in average body size.[9] As mentioned above, our analysis do link genetically determined aspects of the human physiology, like certain properties of the cells in the body, to body size. However, given the more limited time horizon for the present analysis (2 millennia as opposed to 1000), human biology is assumed time-invariant. Instead, changes in childhood nutrition is what generates changes in body size at the individual level, going from one generation to the next, in the model developed below. As in our model, Lagerlöf (2006b) allow body size to influence the income of individuals, and their subsistence needs (metabolism). However, fertility does not

[8]During the transition from hunter-gathering to organized farming, diets changed considerably. In particular, meat consumption appears to have dropped significantly (e.g. Fairweather-Tait, 2003). The age of discovery brought new high- calorie foods like beans (3,340 cal. per edible kg; Livi-Bacci, 1991, Ch. 5).

[9]The integration of Darwinian evolution and economic growth theory was pioneered by Galor and Moav (2002).

affect subsistence needs, as in the analysis below. Moreover, our approach to formalizing the link between body size and basal metabolism differ's from Lagerlöf's, in being based on deep microfoundations and biological evidence, as explained in the next section.

The paper proceeds as follows. Section 2 presents recent research from the field of allometric scaling and ontogenetic growth. On this basis, Section 3 develops a bio-economic model of Malthusian stagnation. In this section, subsistence consumption is endogenous. It is worth noting, that our concept of subsistence consumption is not identical with a level of consumption below which individuals are incapable of any economic activity. Section 4 therefore discuss the implications of introducing such a lower bound, and demonstrates the viability of the subsistence equilibrium derived in Section 3. Section 5 analyzes the impact of a "take-off" into sustained growth on income, population size and body size. Finally, Section 6 concludes.

2. Allometric Scaling, Energy Consumption, and Ontogenetic Growth

Allometry is a technique used in the biological sciences to describe how a variable of interest regresses against body mass. The fundamental relationship between energy consumption B and mass of a mammal m is described by Kleiber's Law (Kleiber, 1932):

$$B = B_0 \cdot m^b, \quad \text{with } b = 3/4. \tag{1}$$

Here B is the basal metabolic rate and B_0 is a species-dependent constant. Thus, drawn on log-log paper the energy-body mass relationship is linear with slope of 3/4, see Figure 2. A slope of 3/4 has been verified by Brody (1945) for almost all terrestrial animals yielding the famous mouse-to-elephant curve. Recently, it has also been detected at the level of single cells and mitochondria (West and Brown, 2004). Although there exists still a debate about the *exact* magnitude of b, it seems to be fair to say that the possible variance under discussion is trivialized by the precision of parameter estimates found for so called "laws" in the social sciences.[10] It is also worth observing that the functional association, as captured by (1), also holds for exercising animals. The implication of physical activity is that b takes on a higher value; Darveau et al. (2002) report an exponent of around 0.82.

Biologists have been puzzled by the 3/4 finding for a long time because the most obvious result from theoretical reasoning would be that b equals 2/3. An animal that is x times as big

[10]For example, Darveau et al., 2002, found exponents between 0.76 and 0.79.

Figure 2: Kleiber's Law

Kleiber's original Figure as reproduced in West and Brown (2005).

as another in height has an x^2 as big surface and an x^3 as big volume or mass m. Because heat exchanges through body surface, the metabolic rate should be proportional to x^2 which is itself proportional to $m^{2/3}$. However, recently researchers have argued that the fallacy of this reasoning lies in its application of Euclidian geometry when fractal geometry is appropriate.

The beauty of the new theory, first developed by West, Brown and Enquist (1997), lies in its foundations on first principles. This makes it very general, and in fact it has already been applied to a multitude of biological problems from "genomes to ecosystems" (West and Brown, 2005). Some of the applications and extensions, for example, those on fertility and mortality are in particular relevant for economic analyses.

A living organism needs to feed its cells. For that purpose energy and material is transported through hierarchically branching networks like blood vessels in mammals. The basic fact exploited by West et al. is that the terminal branches of the network (the capillaries) are size-invariant units. From this and the conservation of the fluid as it flows through the system follows that the total number of capillaries is proportional to the metabolic rate. The network

in use, however, is not of arbitrary structure. Given that organisms have evolved through natural selection, it must be one that minimizes energy used for transport i.e. one that minimizes hydrodynamic resistance.

More precisely, consider a network of branching vessels with ν levels of branching and η branches per node. Let $k \in \{1, \ldots, \nu\}$ indicate the level of branching. Nature optimizes through choice of the radii r_k and lengths l_k of the vessels at every level k. West et al. (1997) have solved the optimization problem given the hydrodynamic and elasticity equations for blood flow and a space-filling condition (requiring that all cells are served). They show that the optimal network is a self-similar fractal with two characteristics. (i) It is volume preserving so that $\nu_k l_k^3 \approx \nu_{k+1} l_{k+1}^3$ where ν_k is the number of branches at level k. (ii) It is area preserving so that $\pi r_k^2 = \eta \pi r_{k+1}^2$, i.e. the cross-sectional area of a branch at level k equals the sum of the cross-sectional area of branches at the next (lower) level. Noting that $\nu_{k+1}/\nu_k = \eta$ one gets two invariant scale factors, $\beta \equiv r_{k+1}/r_k = \eta^{-1/2}$ and $\gamma \equiv l_{k+1}/l_k \approx \eta^{-1/3}$. Hence, as blood flows down through a bifurcating hierarchy ($\eta = 2$) the radii of vessels decrease with factor $1/\sqrt{2}$ whereas the length of vessels decreases at rate $1/\sqrt[3]{2}$. Finally, West et al. show that the total volume of blood in an energy minimizing network is proportional to body mass m. This feature combined with the fractal nature of the network implies scaling according to (1). To see this calculate the total volume of blood.

$$V_b = \sum_{k=0}^{\nu} \pi {r_k}^2 l_k \eta^k = \frac{1 - (\eta\gamma\beta^2)^{-(\nu+1)}}{1 - (\eta\gamma\beta^2)^{-1}} \eta^\nu V_c \approx \frac{(\gamma\beta^2)^{-\nu}}{1 - (\eta\gamma\beta^2)^{-1}} V_c$$

with V_c denoting the volume of a capillary, an invariant unit. Conclude that blood volume and thus mass is proportional to $(\gamma\beta^2)^{-\nu}$ implying that $\log(\nu)$ is proportional to $-\log(m)/\log(\gamma\beta^2)$. The metabolic rate is proportional to the number of capillaries n^ν implying that $\log(B)$ is proportional to $\nu \log(n)$ and thus to $-\log(m) \cdot \log(n)/\log(\gamma\beta^2)$. Insert β and γ to find that $\log(B)$ is proportional to $3/4 \log(m)$, which is Kleiber's law.

Kleiber's law states that larger animals are more efficient because each capillary serves a larger number of cells. An economist would probably take the inverse of (1), i.e. $m = B^{4/3}$ and conclude that body tissue is produced with increasing returns to scale. The theory of West, Brown, and Enquist provides a deep foundation of this fact: energy dissipation per cell decreases with body mass. Thus, as an organism gets smaller each single cell is forced to work harder. This basic insight can explain a multitude of biological phenomena like, for example, why a mouse has

a faster heartbeat, sleeps more, and lives shorter than an elephant. It provides also the deep determinant of some phenomena discussed by the economics profession like, for example, why human health and productivity are positively correlated with body size (see Fogel, 1994).

One of the major applications of the theory is ontogenetic growth (West, Brown, and Enquist, 2001). It can be expected that the new theory will soon replace older formal theories of ontogenetic growth used in a special field of biology called life-history (Charnov, 1993, 2001). Consider the energy flow in a human body. This is generally given by

$$B(t) = B_c N(t) + E_c \dot{N}(t) \tag{2}$$

where $N(t)$ is the number of cells at time t, B_c is the metabolic rate of a single cell (inclusive maintenance and replacement) and E_c is the metabolic energy required to create a new cell. If we insert the fact that body mass consists of the mass of a single cell $\bar{m}$ times the number of cells, i.e. $m(t) = \bar{m}N(t)$, and solve for the change of body mass, we obtain:

$$\dot{m}(t) = \left(\frac{\bar{m}}{E_c}\right) B(t) - \left(\frac{B_c}{E_c}\right) m(t).$$

Finally, insert (1) to get a differential equation for body mass:

$$\dot{m}(t) = a \cdot m(t)^b - d \cdot m(t), \tag{3}$$

where $a \equiv B_0 \bar{m}/E_c$ and $d \equiv B_c/E_c$.

Ontogenetic growth according to (3) should look very familiar to economists. In fact, "accumulation" of body mass is structurally equivalent to accumulation of capital in the neoclassical growth model (which would be given by $\dot{k} = sk^\alpha - \delta k$ in standard notation; Solow, 1956). New body tissue is produced with decreasing returns whereas existing tissue depreciates at a constant rate. Thus, there exists a stable steady-state at $m = m_s \equiv (a/d)^{1/(1-b)}$, implying $m_s = (a/d)^4$ using Kleiber's law.

Equation (3) is a simple Bernoulli equation with explicit solution

$$\left(\frac{m(t)}{m_s}\right)^{1/4} = 1 - \left[1 - \left(\frac{m_0}{m_s}\right)^{1/4}\right] \cdot e^{-\frac{1}{4}dt}. \tag{4}$$

If we take a and d as species-specific biological constants, determined by the genetic make-up, how can we then explain the variation in body mass among adult humans? One possibility

would be that reproduction and death occurs before maximum body mass is (asymptotically) reached. While this argument applies certainly to some species like, for example, cod, it is less convincing for humans.

For a more sensible interpretation of (4) in the context of humans, it is important to note that it does not preclude an adult size well below the asymptotic size (Charnov, 2001). Two channels are possible.

First, imagine that maximum adult height (mass) is reached at a genetically predetermined age $t = T$. It is then determined by m_0, the birth-weight or, more specifically, child mass after weaning. Yet m_0 is individual-specific and depends in particular on the biological and economic condition of the mother.

Second, we may observe that equation (4) specifies *unconstrained growth* or, in other words, the *demand side* for energy. If energy, i.e. food, is in limited supply, new body tissue will be accumulated with less speed than the biological maximum, growth will be slower and so will be mass at time T. In other words, while the average Korean born 1980 is taller and heavier than his father (see Steckel, 1995) both have grown according to (4). Only, the new generation started out better initially and consumed more energy (more or better food) in childhood. It is through these channels of initial child mass and child nutrition and its consequences on adult productivity where economics interacts with biology in their joint determination of subsistence consumption.

3. A Bio-Economic Model of Subsistence Consumption

3.1. **Intergenerational Evolution of Body Mass and Subsistence Consumption.** Life is separated into two periods: childhood (after weaning) defined as the period of body growth and dependence on food provided by the parent, and adulthood defined as the period of constant body size and reproduction. Integrating (2) over the period of childhood one gets

$$\epsilon c_t = B_c N_t + E_c(N_{t+1} - N_t). \tag{5}$$

Note the shift from flows to stocks: ϵc_t is now total energy used for ontogenetic growth of a child comprising consumption during childhood and the "energy exchange rate, ϵ, which is measured in kcal. per consumption good (or per dollar). The energy exchange rate is an exogenous

parameter; the parameter B_c denotes the energy required to maintain a cell through childhood, and E_c is energy costs associated with cell generation.

To obtain mass of a grown up child we substitute body mass $m_{c,t} = \bar{m}N_t$:

$$\epsilon c_t = \frac{B_c}{\bar{m}} m_{c,t} + \frac{E_c}{\bar{m}} m_{c,t+1} - \frac{E_c}{\bar{m}} m_{c,t} \quad \Rightarrow \quad m_{c,t+1} = \frac{\bar{m}}{E_c}\epsilon c_t + \left(1 - \frac{B_c}{E_c}\right) m_{c,t}.$$

This gives a relationship between the mass of a child after weaning $m_{c,t}$ and as a grown up $m_{c,t+1}$. To establish the intergenerational link between body masses we use the fact the a child after weaning equals μ times the mass of the mother (Charnov, 1991, 1993):

$$m_{t+1} = a\epsilon c_t + (1-d)\mu \cdot m_t, \tag{6}$$

where m_t is the mass of the child's parent, m_{t+1} is the mass of the former child when it becomes itself a parent; the parameters a and d are defined as in the last section. Thus adult body mass is a compound of energy intake during childhood and "inherited" body mass.

The mass of an adult is predetermined, and remains constant for the remaining part of his or her life. Hence, we are focusing on the irreversible component of body mass. During adulthood, individuals are subject to subsistence requirements. Subsistence consumption depends on body size and on fertility. In particular, we use the fact that rearing up a child from conception to weaning requires a fraction ρ of the mother's metabolic energy E_t (Prentice and Whitehead, 1987; Sadurskis et al., 1988). Thus with B denoting energy used up by the mother's own body,

$$E_t = \rho n E_t + B_t \quad \Rightarrow \quad \epsilon \bar{c}_t = E_t = \frac{B_t}{1-\rho n} = \frac{B_0 m_t^b}{1-\rho n}, \tag{7}$$

where $\bar{c}$ is subsistence consumption, measured in terms of goods. The last equality follows from employing Kleiber's law, $B_t = B_0 m_t^b$.

3.2. **Individual's Optimization.** A parent maximizes utility U derived from child quality and quantity, where quality is in the Beckerian (1960) sense measured by total expenditure for consumption (i.e. nutrition) of children, C.[11] For simplicity we impose a logarithmic form for

[11]Economists tend to think about schooling as a proxy of the quality *outcome of* child expenditure (see, for example, Rosenzweig and Evenson (1977) and Hanushek (1992). A recent study that supports our theory more directly is provided by Hagen et al. (2006). They show an inverse relationship between family size and alternative measures of the nutritional status (mass and height) of children for a community living at subsistence level. Interestingly, a trade-off between fertility and child energy expenditure is not a particularly human characteristic but is known to be operative in other animals as well (see, for example, Smith and Fretwell, 1974).

the utility function:

$$U(C_t, n_t) = \log(C_t) + \gamma \log(n_t) \tag{8}$$

with γ denoting the weight of child quantity in utility. Child expenditure is constraint by parental income y and subsistence consumption $\bar{c}_t$, i.e. $y_t = \bar{c}_t + C_t$. Combining this budget constraint with equation (7) leads to a single constraint:

$$y_t - C_t - \frac{B_o m_t^b}{\epsilon(1-\rho n)} = 0. \tag{9}$$

Accordingly, parents maximize (8) s.t. (9), by choosing C_t and n_t.

The first order conditions can be condensed to a single equation describing the quantity-quality trade-off:

$$\frac{\gamma}{n_t} = \frac{1}{C_t} \cdot \frac{\rho \cdot B_o m_t^b}{\epsilon(1-\rho n)^2}. \tag{10}$$

Subsequently, equations (9) and (10) can be solved for optimal child quantity and quality:

$$C_t = y_t - \frac{1}{2}\left[(1-\gamma)s_t - z_t\right], \tag{11a}$$

$$n_t = \frac{1}{\rho} + \frac{1}{2\gamma\rho y_t}\left[(1-\gamma)s_t - z_t\right], \tag{11b}$$

for $y_t > s_t$, with $s_t \equiv B_0 m_t^b/\epsilon$ denoting consumption needs of a childless (non-pregnant) adult and $z_t \equiv \sqrt{s_t^2(1-\gamma)^2 + 4\gamma s_t y_t}$. If $y_t < s_t$ income is not sufficient to fuel adult's metabolism and at the corner solution where $n_t = 0$ the population becomes extinct within a generation. If $y_t > s_t$ we see immediately that $C_t > 0$. For consistency we need that simultaneously $n_t > 0$ requiring that whenever there is child consumption there has to be a family. For that end we have to impose $\gamma \geq 1$, i.e. the utility-weight on quantity must be at least as great as that on quality.

3.3. **Production.** Total income, Y, is determined at the macro-level by a body mass adjusted technology:

$$Y_t = A\left(1+m_t\right)^{\phi} X^{\alpha} L_t^{1-\alpha} = y_t \cdot L_t,\ \phi \in (0,1)\,,\ \alpha \in (\phi, 1)\,. \tag{12}$$

Here, ϕ is thought of parameterizing the return on health. L denotes population size and X land. Since land is assumed to be constant (and $\alpha > \phi$) the technology implies stagnation in the long-run at an equilibrium population density L/X unless general productivity A is growing without bound.

3.4. **Steady State.** Generally, population evolves according to

$$L_{t+1} = n_t L_t, \tag{13}$$

where n_t is given by equation (11b). At a stationary Malthusian equilibrium, however, we observe $n^* = 1$ and thus from (11)

$$y^* = \frac{\gamma + (1-\gamma)\rho}{\epsilon\gamma(1-\rho)^2} \cdot B_0 m^{*b}. \tag{14}$$

Note that equilibrium income does not depend on the state of technology and in particular not on general productivity A. It is pinned down by metabolic constants, family preferences, and the energy exchange rate. Accordingly, steady state GDP per capita depends on human biology and geographic circumstances (in as much as they determine the diet and thus ϵ).

Equilibrium consumption per child is obtained as $c^* = C^*/n^* = \rho y^*/[\gamma+(1-\gamma)\rho]$. Substituting it into (6) and solving for equilibrium body mass we get

$$m^* = \frac{a\epsilon\rho}{[\gamma + (1-\gamma)\rho]\,[1-(1-d)\mu]} \cdot y^*. \tag{15}$$

Thus the model predicts that in the long-run equilibrium, richer countries should be populated by taller people, ceteris paribus. Yet stature depends also on the type of food consumed, via ϵ. For given income people are heavier the better food is exchanged into energy (the higher ϵ). We would thus expect that historical instances of imported new and energy rich food, like beans following the age of discovery, to make people taller. However, inserting (14) into (15) provides equilibrium body mass as pinned down solely by preferences and biological fundamentals:

$$m^* = \left[\frac{a\rho B_0}{\gamma\,[1-(1-d)\mu]\,(1-\rho)^2}\right]^{1/(1-b)}. \tag{16}$$

Finally use (15) and (12) to obtain equilibrium population density:

$$\left(\frac{L}{X}\right)^* = \left\{\frac{a\epsilon\rho A}{[\gamma + (1-\gamma)\rho]\,[1-(1-d)\mu]}\frac{(1+m^*)^\phi}{m^*}\right\}^{1/\alpha}. \tag{17}$$

3.5. **Comparative Statics.** The energy exchange rate (ϵ) changes when new forms of diet occur because, for example, new plants or animals are cultivated or imported so that more (or less) energy can be extracted from one unit of food consumption. Interestingly, inspection of (16) shows that ϵ does not affect equilibrium body mass. The model predicts that an improving

energy exchange rate makes people not bigger in equilibrium but – as shown by (14) – poorer, measured in units of physical goods.

The intuition for this seemingly puzzling effect becomes clear through inspection of (17) showing that population density rises, when ϵ goes up. The chain of effects is as follows. When the ϵ rises, people of the next generation become larger and thus more productive. With higher income they expand their family, and population grows temporarily. With the population growing, however, productivity is decreasing. The productivity loss more and more compensates the initial efficiency gain through the energy exchange rate. In the long-run the demo-economy stabilizes at a constant population and lower income. A lower level of income resulting from the production side is nevertheless sufficient to support a larger population because of the improved energy exchange rate. In other words, the standard of living in terms of calories consumed – and thus body mass – is the same as at the initial state before the change in diet. Income, measured in terms of calories produced is unaltered, but lower measured in terms of goods. Yet population size is higher than before. This result demonstrate that measuring subsistence needs in terms of income can be misleading. Different levels of income can support different equilibria of subsistence. The natural unit of measurement for subsistence needs is the amount of calories consumed.

The impact from technology on the number of individuals mirrors the standard results within a Malthusian model. An improvement of technology (higher productivity A, lower dependence on limited land, i.e. lower α, or higher efficiency in using brawn, i.e. increasing ϕ) leads eventually "only" to a larger spread of human genes, i.e. higher equilibrium population density, but leaves income unaltered. We also see that the long-run impact on body mass is zero. Accordingly, even if technology did advance during the last two millennia, the model predict this should not have led to a secular upward trend in size, as available evidence suggest. It is important to point out, however, that we are analyzing discrete improvements of technology. We will later modify the model towards permanent technological progress and obtain quite different results.

The model reveals also some interesting comparative statics with respect to γ. Suppose that the weight for child quantity increases permanently reflecting permanent change of preferences in favor of larger families.[12] From (16) we observe immediately that body mass is lower at the

[12]Galor and Moav (2002) suggest that the composition of the population, in the dimension of relative weight attached to child quality, may have changed during the epoch of Malthusian stagnation. Within the context of the present non-evolutionary model, such selection effects can be (crudely) captured by a change in γ.

new long-run equilibrium, $\partial m^*/\partial\gamma < 0$. Using this result and taking the derivative of (15) we obtain that income at the new equilibrium is also lower, $\partial y^*/\partial\gamma < 0$.

The chain of effects is the following. With γ going up, family size (n) increases temporarily and population density increases permanently. With higher expenditure for child quantity parents spend less (calories) for child quality and the next generation of adults is lighter. They have lower productivity and smaller children at birth which both affects adult size, productivity, and birth weight for the subsequent generation negatively. Again, the next generation will have lower income and for that reason prefer to adjust optimal family size downwards. The negative income effect operates until n approaches its equilibrium again and population stays constant. Thus the model predicts that an intrinsically higher preference for large families makes people smaller and poorer, but that land becomes more densely populated.

An organism consumes more energy during exercise, as mentioned above. In principle this can be accounted for by a higher species specific constant B_0 or - as some authors argue – by a larger exponent b for exercising animals (Darveau et al., 2002). Anyway, inspection of (16) shows that body mass (of humans of the same genetic make up) increases with B_0 and b, implying that more exercise, i.e. heavier work or longer work hours per day, lead ceteris paribus to bigger individuals in equilibrium. We also see from (15) and (17)that more energy consuming people produce more income per capita and live on a less densely populated area.

The reason is the following. A permanent increase in energy consumption induces the first generation to have less children. This occurs because body mass and thus income and energy supply is given for this generation, and increasing energy demand of a parent's own metabolism leaves relatively less energy for fertility. Being less numerous and smaller, income of the next generation rises and parents spend more on fertility and energy consumption per child. This induces an adjustment process of population growth towards zero from below (and thus a permanently less densely populated area) and a permanent increase of body mass. Increasing body mass and decreasing population density both support a permanent rise of income and so that an equilibrium of heavier people is indeed sustainable.

3.6. **Subsistence Dynamics.** In order to go as far possible with analytical results we focus now on the special case of equal weights in parental utility. Later on, numerical experiments for the general case ($\gamma > 1$) will complement our results.

With $\gamma = 1$ the model simplifies tremendously, in particular, because we get from (11) a simple expression for consumption per child, $c_t = \rho y_t$. Using it in (6) we see that body mass evolves according to

$$m_{t+1} = a\epsilon\rho y_t + (1-d)\mu m_t. \tag{18}$$

Inserting (11b) and (12) into (18) and (13) provides a reduced form of the model in terms of two-dynamical system for the evolution of body mass and population size (land, X, has been normalized to one):

$$m_{t+1} = F(m_t, L_t) = a\epsilon\rho A(1+m_t)^{\phi} L_t^{-\alpha} + (1-d)\mu m_t, \tag{19a}$$

$$L_{t+1} = G(m_t, L_t) = \frac{L_t}{\rho} - \frac{1}{\rho}\sqrt{\frac{B_0 m_t^b}{\epsilon(1+m_t)^{\phi}A}} \cdot L_t^{1+\alpha/2}. \tag{19b}$$

For phase diagram analysis we calculate the isoclines where $\Delta m \equiv m_{t+1} - m_t = 0$, i.e.

$$L = \left(\frac{(1+m)^{\phi} a\epsilon\rho A}{[1-(1-d)\mu]\, m}\right)^{1/\alpha}, \tag{20}$$

and where $\Delta L \equiv L_{t+1} - L_t = 0$, i.e.

$$L = \left(\frac{(1-\rho)^2 eA(1+m)^{\phi}}{B_0 m^b}\right)^{1/\alpha}. \tag{21}$$

The isoclines intersect once at the unique equilibrium and the $\Delta m = 0$-locus lies above the $\Delta L = 0$-locus *iff*

$$\frac{(1-\rho)^2\epsilon A(1+m)^{\phi}}{B_0 m^b} < \frac{(1+m)^{\phi} a\epsilon\rho A}{[1-(1-d)\mu]\, m} \quad \Rightarrow \quad m < m^*. \tag{22}$$

The isoclines and the implied arrows of motion are shown in Figure 3. From inspection of the figure one is tempted to identify the subsistence equilibrium as a globally stable focus. Yet, because time is discrete, adjustment dynamics are less obviously assessed. Besides monotonous convergence towards the equilibrium there exist two other possibilities. One is adjustment in damped cycles as illustrated by the gray trajectory in Figure 3.[13] Here, the first move out equilibrium is in southeastern direction. It may have resulted, for example, after a society resting at equilibrium experienced a negative shock of A (a natural disaster, a crop failure). As a consequence of low birth rates and malnutrition the next generation of adults is less heavy

[13]Strictly speaking only the endpoints at the kinks of the trajectory are values assumed in discrete time. We have connected them with a continuous line for better visibility.

and less numerous. With less than equilibrium population and the Malthusian mechanism at work, however, these people show productivity and thus income above equilibrium level. As a consequence of their relatively well-being the next generation is large and well fed. As drawn in Figure 3, overshooting occurs so that the next generation is situated above the $\Delta L = 0$-line. Adjustment is in damped cycles initially and then followed by monotonous convergence towards the equilibrium.

Figure 3: Subsistence Dynamics: Phase Diagram I

Figure 4: Subsistence Dynamics: Phase Diagram II

A second possibility, which can unfortunately not be ruled out in discrete time, is that overshooting causes instability of the subsistence equilibrium. This case is drawn in Figure 4 where a similar shock as before triggers society to leave the equilibrium permanently in explosive cycles.

An analytical discussion of stability is needed to rule out the implausible explosive behavior. The elements of the Jacobian matrix J evaluated at the steady-state are given by

$$\begin{aligned}
J_{11} &= \phi\left[1-(1-d)\mu\right]\frac{m^*}{(1+m^*)} + (1-d)\mu \\
J_{12} &= -\frac{\alpha}{L^*}\left[1-(1-d)\mu\right]m^* \\
J_{21} &= -\frac{(1-\rho)L^*}{2\rho m^*(1+m^*)}\left[m^*(b-\phi)+b\right] \\
J_{22} &= 1-\frac{1-\rho}{\rho}\frac{\alpha}{2}.
\end{aligned}$$

Stability requires that the eigenvalues of J are less than one in absolute terms, or alternatively that $|\det(J)| < 1$ and $|tr(J)| < 1 + \det(J)$. Stability is not easily assessed. We therefore employ the fact that there is much less uncertainty about parameter values in biology and allometry than generally in the social sciences. We thus proceed by calibrating the biological part of the model before we assess sensitivity of stability with respect to the economic parameters.

For any given $t = \tau$ we can extract d from (4).

$$d = -\log\left(\frac{1-(m_\tau/m_s)^{1/4}}{1-(m_0/m_s)^{1/4}}\right)\cdot\frac{4}{\tau}. \tag{23}$$

For calibration we use the WHO's standardized weight-for-age curve for US males and females.[14] Both sexes hit the 50 kg-line by the age of $\tau = 14$ implying $m_\tau = 50$. A female grown up weighs on average $m_s = 59$ kg. Child weight after weaning is $m_0 = 9$ kg (implying $\mu = 0.15$). Inserting the data in (23) provides $d = 0.63$ and thus $a = d \cdot m^{*1/4} = 1.69$. Following Prentice and Whitehead (1987) we set $\rho = 0.15$ implying that a woman pregnant with one child must consume 1.18 times the energy of a non-pregnant woman. Given these values we adjust the metabolic constant B_0 so that m^* in (16) equals 50 kg. We set $b = 0.75$ accordig to Kleiber's law.

Given the biological parameters, only α and ϕ remain to be decisive for stability (since L^* cancels out in the computation of $\det(J)$). It turns out that of the two stability conditions the

[14] We downloaded this data at http://www.who.int/nutgrowthdb/referen ce/en/.

condition $|tr(J)| < 1 + \det(J)$ constraints stability. For the calibrated version it assumes the simple form

$$0 < \alpha < \alpha_{crit} \equiv 0.43 + 0.33\phi.$$

As is well known, a standard Malthusian model ignoring human metabolism would require $0 < \alpha < 1$ for stability, i.e. decreasing marginal returns of labor on limited land.

Stability of the Malthusian fixed point is less easily obtained in the model where subsistence is endogenous. Note that instability means that one generation inherits so much body mass that they cannot support their own metabolism and simultaneously raise children. The particular human community becomes extinct.

Still, for a large range of reasonable parameters the Malthusian equilibrium is asymptotically stable. From Clark's (2006) estimates we use $\alpha = 0.25$. If we associate ϕ with the return to an extra cm in height, we can use estimates reported in Weil's (2005) to approximate $\phi = 0.025$. This renders an equilibrium safely within the range of stable equilibria.

Next, suppose equilibrium income is 400 (international Dollars) per year. Thus, during the period of adulthood measured by the length of the fecundity period (assumed as 20 years) equilibrium income is \$ 8000. This pins down ϵ to 0.023. Finally, we have one further parameter, A, which could be used to calibrate a particular equilibrium population size (or density). Yet, we found it more informative to report population density as relative deviation from stationary population. Parameters and steady-state values are summarized in Table 1.

Table 1: Subsistence Bio-Economy

a	b	d	μ	ρ	ϵ
1.69	0.75	0.63	0.15	0.15	0.023
α	ϕ	A	X	m^*	y^*
0.25	0.025	1	1	50	400

Our first numerical experiment is a permanent rise of productivity A by 10 percent. This could be conceived as use of a new agricultural technique (e.g. fertilizer), or a shock to climate (e.g. the end of the little ice age). It is worth noting that adjustment dynamics are qualitatively identical for a permanent rise of the energy exchange rate ϵ, which could be conceived as the cultivation of a new crop, or import of a new crop. In a phase diagram the parameter change leads to an upward shift of both the $\Delta m = 0$-locus and the $\Delta L = 0$-locus leaving the intersection at m^*

star unchanged. The old equilibrium lies south east of the new equilibrium capturing the fact that people are too light and too few with respect to the improved conditions. As a result the next generation of children will have higher birthweight and more nourishment at their disposal. It will have higher productivity and thus income when grown up and will itself have more and heavier children. Adjustment dynamics explained in connection with Figure 3 set in.

Figure 5: Adjustment Dynamics: Permanent Technology Shock

Parameters from Table 1 and $\gamma = 1$ (solid lines), $\gamma = 1.5$ (dashed lines), and $\gamma = 2$ (dotted lines).

Solid lines in Figure 5 show the resulting adjustment dynamics for benchmark parameters. We also report simulation results where $\gamma > 1$. These cases are represented by dotted and dashed lines. The sensitivity analysis reveals that the results are structurally stable against variation of the preference parameter. A larger γ makes adjustment more volatile but otherwise structurally similar. Generally, higher productivity or cultivation of more energy-providing food leads to a *temporary* rise in body mass (height) and a *permanent* rise of population size (and density). Over the first five generations ($\approx$ 100 years) we observe cyclical adjustment behavior.

The data series for height, as depicted in Figure 1, exhibits considerable persistence. To capture this in the simulation, we conduct a business-cycle theory related experiment, though the period of our cycles is of course much longer (one generation) than conventional business cycles. Specifically, suppose technology can assume two values, A_{high} and A_{low}, and follows a Markov process. With probability p next period's technology level coincides with the current period's and with $(1-p)$ it changes. By its nature, this stochastic process would imply that technology, occasionally, "regresses". That is, periods where technological knowledge is neglegted and lost. Aiyar et al. (2006) document this phenomenon for pre-industrial times, and provide microfoundations for it, within a Malthusian setting.

Figure 6 shows an example of a trajectory for benchmark parameters with $A_{high} = 1.05$, $A_{low} = 0.95$, and $p = 0.85$. If we assume that a period is 20 years long, then the trajectory shows the demo-metabolic history of a society from year 0 to year 1800. It corresponds quite well with the actually observed history, as discussed in the Introduction. That is, the trajectory displays long periods of smooth development, interrupted by abrupt changes and cyclical recovery.

Figure 6: Adjustment Dynamics: Markov Process

4. Convex-Concave Production and the Malnutrition Trap

One seeming criticism of our modelling so far could be that the production function does not take into account a lower limit of energy intake below which productive (or any other) activity becomes impossible and labor supply goes to zero. This assumption is visualized in Figure 7 where the per-capita production function $y(m, A, L, X)$, hits the m axes at a positive value. The figure shows the most frequently discussed case in the literature (see e.g. Leibenstein, 1957, Bliss and Stern, 1978) where production is convex-concave, or s-shaped in m although this is not essential. The essential feature is that output produced lies below output (energy) needed to feed a given body mass for m below a critical $\tilde{m}$. Here, production required is found from energy intake according to Kleiber's law divided by the energy exchange rate. It is thus unambiguously concave over the whole range of m.

The sad fact visible in Figure 7 is that people with initial mass below $\tilde{m}$ are not capable to produce the energy required in order to maintain their own body size. They emaciate up to a lower bound m^{**}, where they are unable to perform any physical activity and are thus relegated to living off the commons or begging (Dasgupta, 1997). Although Figure 7 is a time-less graph,

Figure 7: Output Produced and Production (Energy) Required

it suggests two stable equilibria at m^* and m^{**}. From this inspection some authors have argued in favor of a malnutrition-driven poverty-trap.

It is important to underline that we are not questioning the view that such poverty-traps exist at the individual level. The indirect empirical evidence for this is overwhelming. Fogel, (1994) argues convincingly that one-fifth of the population was situated close to m^{**} in 19th century England and France. However, we are challenging the possibility of a *general* equilibrium at m^{**}. In order to get this argument straight it is helpful to introduce the following definitions. An *equilibrium of subsistence* (Malthus, 1798) is defined by zero population growth. An *equilibrium of destitution* (Dasgupta, 1993) is defined by zero labor supply.

From these definitions one may already suspect that destitution cannot be a stable general equilibrium. It will now be proven. The new convex-concave production function changes the shape of the isoclines. In particular, both are now upward sloping at low m hitting the m axes at a some finite equilibrium of destitution m^{**}. The equilibrium of subsistence is observed at the intersection at some $m^* > m^{**}$. The crucial element of the proof is that it remains to be true that the $\Delta m = 0$-locus lies above the $\Delta L = 0$-locus *iff*

$$m^{**} < m < m^*.$$

To see that this must be so, replace $(1 + m)^\phi$ in (22) by a general $f(m)$ with $f(m^{**}) = 0$ and note that the condition is independent from any positive $f(m)$.

Figure 8: Subsistence Dynamics with Convex-Concave Production: Phase Diagram

With (22) still being valid the phase diagram of Figure 8 results. This renders the equilibrium of destitution unstable. The intuition why a situation where the whole society is destituted cannot be a *dynamic* equilibrium is straightforward and follows from the Malthusian mechanism. The destituted people will only have few children. This will make the next generation (of low population density) very productive. They can afford to nourish their children comparatively well so that the following generation of adults has body mass above m^{**}. An adjustment process towards m^* (possibly cyclical) is initiated. Thus, while subsistence as a general equilibrium phenomenon can prevail over centuries, destitution cannot.

5. The Take-Off from Subsistence

While our two-dimensional model of the Malthusian equilibrium can be fruitfully applied to the evolution of human history it does not – like Malthus original one-dimensional theory – hold for industrial and modern societies. Modelling the full transition towards such a society is beyond the scope of this paper. We can, however, show that our model captures one particular feature of the take off from subsistence: a permanent yet bounded increase of human body mass (or height, see Steckel, 1995, Clark, 2006).

The take off is initiated by a permanent increase of productivity growth. For simplicity we assume $A_{t+1} = (1 + g)A_t$ where g is a constant rate of TFP growth. Clearly, $g > 0$ renders the Malthusian equilibrium unstable. In order to discuss the new dynamics we introduce the

auxiliary variable $x_t \equiv L_t^\alpha / A_t$ and thus

$$x_{t+1} = \frac{L_{t+1}^\alpha}{A_{t+1}} = \frac{n_t^\alpha}{1+g} \cdot x_t. \tag{24}$$

with an equilibrium where $n_t = (1+g)^{1/\alpha}$, which is larger than one for $g > 0$. Permanent technological progress triggers population growth.

In order to obtain the new two-dimensional dynamic system we insert the definition of x_t into y_t and n_t from (11b) into (24). Thus (18) and (24) are rewritten as

$$m_{t+1} = \frac{a\epsilon\rho(1+m_t)^\phi}{x_t} + (1-d)\mu m_t \tag{25a}$$

$$x_{t+1} = \frac{1}{(1+g)\rho^\alpha} \cdot \left[1 - \sqrt{\frac{B_0 m_t^b x_t}{\epsilon(1+m_t)^\phi}}\right]^\alpha \cdot x_t. \tag{25b}$$

The $\Delta m = 0$ and $\Delta x = 0$ loci are given by

$$x = \frac{a\epsilon\rho}{1-(1-d)\mu} \cdot \frac{(1+m)^\phi}{m}$$

$$x = \frac{\epsilon\left[1-(1+g)^{1/\alpha}\rho\right]^2}{B_0} \cdot \frac{(1+m)^\phi}{m^b}.$$

Inspection shows that the implied phase diagram is structurally identical to the $L - m$ diagram in Figure 3. In particular, there exists a unique non trivial positive intersection at some (x^*, m^{***}) which is stable for standard parameters. A permanent increase of g, however, shifts the equilibrium to the south-west reflecting higher labor productivity (lower $x = L^\alpha/A$) and higher body-mass m. Perpetual technological progress and income growth leads to a permanent yet finite increase of body mass. Equilibrium mass is obtained as

$$m^{***} = \left[\frac{a\rho B_0}{\gamma\left[1-(1-d)\mu\right](1-(1+g)^{1/\alpha}\rho)^2}\right]^{1/(1-b)}. \tag{26}$$

Body mass is increasing with the growth rate and is – as comparison with (16) shows – larger than at the Malthusian equilibrium of economic stagnation.

Figure 9 shows adjustment dynamics implied by our benchmark parameterization after a permanent rise from $g = 0$ to $g = 0.02$. Implied fertility change is recovered from (11b). The Figure shows that technological progress triggers a monotonous increase of body mass and an initially cyclical and then monotonous increase of population growth. The first (generation-) periods of transition correspond well with the empirical facts for Europe in the 19th. Economic

growth is accompanied by a gradual but steady increase of body mass, i.e. height (see Steckel, 1995, 2001) and an initially steep and later flattening increase of population growth (Kremer, 1993). At later periods, however, results become less plausible because population growth fails to revert its trend. Yet, this result occurs naturally because the model neglects that rising income eventually triggers a change of fertility behavior, thereby initiating a demographic transition.

Figure 9: Take-Off from Subsistence

The variable g_L denotes annual population growth per generation in percent.

6. Conclusion

The present paper has developed a bio-economic model with the aim of studying the growth process during "Malthusian stagnation" and the process of "Take-off" (Galor and Weil, 2000). In particular, the model contributes to the literature by describing the long-run evolution of the representative individual's body size, and her subsistence requirements. The links between childhood nutrition, adult body size and subsistence requirements are based on deep microfoundations, drawing on recent work in the field of biology. The theory involves a two-dimensional Malthusian equilibrium concept, featuring a constant number and size of individuals.

The model demonstrates how human biology and preferences determine long-run body size. In a Malthusian state, and without evolutionary changes, the model predicts an absence of a secular trend in body size over time. Yet, if the economy is perturbed by shocks (be that technological, climatic, or diet related) oscillatory adjustment to steady state, in body size and population, may prevail. The state of stagnation in body size comes to an end, however, if technological progress accelerates. In response to such a change in the economic environment, average body size rises gradually towards a biologically determined upper limit. These predictions are consistent with available evidence pertaining to body size (height) from 1 A.D. to the 19th century.

The model does not allow for a full demographic transition whereby fertility ultimately declines. Such an extension could, however, be provided by incorporating another dimension of "child quality": human capital. Following Galor and Weil (2000), this extension would involve the feature that if the underlying productivity growth rate accelerates, investments in human capital eventually rises and fertility declines. The associated rise in income per capita would support increases in body mass, which elevates subsistence consumption; declining fertility works in the opposite direction however. Studying subsidence dynamics during (and after) the demographic transition, in a bio-economic setting as developed above, is left for future research.

The framework could also be fruitfully extended to include the impact of disease on subsistence requirements and body mass. There is considerable evidence to suggest that illness importantly affect basal metabolism; reconvalescence is energy intensive, and thus elevates subsistence requirements. Hence, the frequency of disease shocks could prove to be an important determinant of body size, population size and productivity in a Malthusian regime. Furthermore, this extension would allow for a comparative analysis of mortality and morbidity; in the end, morbidity (associated with disease shocks) may have had a much larger impact on the trajectory for productivity in pre-industrial times than changes in mortality. This extension is not straight forward, however. While bodily temperature (fever) should work so as to speed up biochemical processes, and among them metabolism, diseases differ in terms of their impact on metabolism nonetheless. Currently there appears to be little consensus in the context of how this link is to modelled in a unifying manner (Hoffer, 2003).

Introducing endogenous subsistence, in the manner described above, may also be important in addressing more contemporary issues. For example, it may inform the ongoing convergence debate. That is, the debate as to whether data support the club-convergence hypothesis, or rather conditional convergence. The former view involves a vision of the growth process where multiple equilibria arise while the latter is associated with a unique steady state equilibrium. A prominent explanation for multiple equilibria involves the introduction of subsistence consumption into a neoclassical growth framework (e.g. Azariadis, 1996). Recently, this approach has been criticized on quantitative grounds in an interesting paper by Kraay and Raddatz (2006). The authors demonstrate that an unconditional "s-shaped" association between average savings and per capita income, which should arise due to a minimum consumption threshold, is not borne out in the data. However, if the threshold is better conceived as a structural characteristic,

which should be controlled for in the analysis, their tests are no longer conclusive. The present theory implies that subsistence consumption is to be conceived as a structural characteristic, and that this level may be subject to substantial variation across countries and time. As an illustration, consider two societies: H and L. In society H average height is 169 cm and $n = 2$, whereas the comparable data for society L is 166 cm and $n = 1$. Assuming, in line with Table 1, that $\rho = 0.15$, we find that lifetime energy requirements of an individual should be nearly 25% higher in society H.[15] Of course, it is an open question whether allowing subsistence to be endogenous still admits multiple equilibria to arise in otherwise standard neoclassical growth models in the first place. These issues also appear to be worthwhile topics for future research.

[15]For this calculation we assume the body mass index (weight divided by height in cm squared) is the same. If so, the difference in energy requirements can be calculated as (using equation (7): $E_H/E_L = \left(\frac{\text{height}_H}{\text{height}_L}\right)^{2b} \left(\frac{1-\rho \cdot n_L}{1-\rho \cdot n_H}\right)$, where we use as height input 1.69 and 1.66, and put $b = 3/4$.

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