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Induced Price Leadership and (Counter-)Spying Rivals’ Play under Incomplete Information

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Abstract

We analyze spying out a rival’s price in a Bertrand market game with incomplete information. Spying transforms a simultaneous into a robust sequential moves game. We provide conditions for profitable espionage. The spied at firm may attempt to immunize against spying by delaying its pricing decision if its cost is low. This, however, adversely affects beliefs and becomes self-defeating. The spy may also be a counterspy or be fooled to report strategically distorted information. This gives rise to an intriguing signaling problem that admits only partially separating equilibria. Surprisingly, counter-espionage may aggravate the price leadership induced by spying. Altogether, our analysis offers an explanation and generalization of robust Stackelberg-Bertrand games.

JEL-Codes: L120, L130, L410, D430, D820.

Keywords: industrial espionage, price leadership, Stackelberg games, collusion, antitrust policy, incomplete information.
1 Introduction

Economic espionage is a common and widely despised activity by governments and firms. Its most common form concerns the illicit appropriation of essential inputs or technology. Prominent historical examples range from stealing the blueprints of the British Cartwright power loom by the American industrialist Francis Cabot Lowell to the smuggling of tea’s secrets - plants, seeds, and fermentation techniques - by the Scottish botanist Robert Fortune, which spurred one of the greatest episodes of early globalization, orchestrated by the British East India Company, and the downfall of China’s tea monopoly.¹

Another form of economic espionage - which is the focus of the present paper - concerns the disclosing of operational information about a rival firm’s pricing or sales.² While spying on technology hurts the spied at firm or nation, the disclosure of operational information may benefit both the spying and the spied at firms, although the detailed analysis reveals some more intricate distinctions.

In the present paper we analyze the impact and stability of spying out a rival’s play in a Bertrand market game with differentiated products that are substitutes where firms’ unit costs are their private information.

As a benchmark we first consider a game of complete information. There, spying out the rival’s price induces a sequential game, with the spied at firm as Stackelberg leader and the spying firm as follower, that benefits both firms, albeit it benefits the spying firm more. This is an immediate implication of the well-known second-mover advantage in Bertrand games with substitutes.

However, this equilibrium is not robust. If the spy’s observation is subject to noise, the spy’s signal is ignored and the unique equilibrium of the game is the same as that of the simultaneous moves game without espionage. This indicates that, in the present framework (that does not admit mixed strategy equilibria) espionage can only be explained in a robust manner if one introduces incomplete information.

Incomplete information not only remedies this robustness problem, it also gives rise to an enhanced role of espionage. Observing the rival’s price induces a sequential game and also eliminates the spying firm’s uncertainty. If one knows the rival’s price, incomplete information concerning its cost is inconsequential.

However, as the spied at firm knows that the rival observes its price, it may respond by adjusting its price downward if its cost is low and upward if that cost is high. Therefore, it is not clear, a priori, whether the spying firm altogether benefits from espionage.

In turn, the spied at firm benefits from having its private information disclosed if its cost is high and may be hurt if its cost is low. This is due to the fact that a high cost is associated with a high price which, when revealed by the spy, induces the rival to respond with a high price, and vice versa.

This suggests that, if the spied at firm’s cost is low, it should perhaps immunize against spying and delay its pricing decision until the spying firm has set its price or fire the spy if his identity has

¹Ben-Atar (2004) offers an inspiring review of the role of economic espionage in the development of the U.S. economy during the 18th and early 19th century. A vivid account of the tea smuggling out of China into British-ruled India is in Rose (2011).
²For a detailed review of the different kinds of economic espionage see Nasheri (2005).
been disclosed. Again, it is not clear, whether these reactions are profitable, and indeed, we find that they induce an adverse updating of beliefs that makes these attempts self-defeating.

The analysis becomes more intricate once we take into account that the spy may be a counterspy who serves the interests of the spied at firm and reports strategically distorted information or, equivalently, if the identity of the spy has been exposed, firm 1 fools him to report distorted information. In that case, the spying firm cannot be sure that the reported price is actually the true price. It then needs to draw an inference from the reported price about the type of spy and make a prediction about the true price set by the spied at firm conditional on facing a counterspy. This gives rise to an intriguing signaling problem that admits no fully revealing equilibrium.

Altogether, we identify two partially revealing equilibria: one that satisfies the intuitive criterion, yet exhibits a disturbing discontinuity, and one that exhibits no such discontinuity, yet violates the intuitive criterion. If one selects the equilibrium that violates the intuitive criterion the presence of the counterspy simply preserves but weakens the price leadership induced by the spy. However, if one selects the equilibrium that survives the intuitive criterion, the presence of the counterspy can surprisingly enhance the price leadership induced by the spy and yield a higher expected price of firm 1 than the expected Stackelberg leader price.

The present analysis builds on various strands of the literature, in particular the literature on commitment and observability and on second-mover advantages in supermodular games.

In a seminal contribution Bagwell (1995) questions the value of commitment in sequential games. He considers a Stackelberg-Cournot duopoly and shows that when the second-mover observes a noisy signal of the first-mover’s action, the unique pure strategy equilibrium coincides with the simultaneous moves equilibrium even if that noise is arbitrarily small.\(^3\) In the same spirit, Morgan and Várady (2013) show that the value of commitment is destroyed if observing the first-mover’s action is costly, even if that cost is arbitrarily small, which however requires a particular sequence of moves which one may judge as somewhat restrictive.

The fundamental implication of these results is that in a sequential game spying out a rival’s choice of action can only serve a useful purpose if the spied at player’s choice cannot already be inferred from its equilibrium strategy. This is the case if, in equilibrium, the first-mover plays a mixed strategy or if his strategy depends on some payoff relevant parameter that is this player’s private information.\(^4\)

Type dependent strategies are behaviorally indistinguishable from mixed strategies. Therefore, there are two frameworks to explain espionage in a robust manner: games of complete information that have equilibria in which the first-mover randomizes and games of incomplete information.

In the present paper we rely on incomplete information to analyze espionage. Inspired by Maggi (1999), who was the first to stress the importance of incomplete information in explaining the value of commitment in sequential games, we focus on incomplete information and pure strategy equilibria that are robust with respect to arbitrarily small noise or cost of spying. Under complete information robustness fails; under incomplete information robustness is assured.

\(^3\)However, due to the assumed discrete action sets, his game has also two equilibria in mixed strategies, one of which approximates the Stackelberg-Cournot and the other the Cournot equilibrium outcome.

\(^4\)If players are not sophisticated, this assessment needs to be qualified. This mirrors in the experimental literature which reports that subjects are willing to buy costly or noisy information if the cost and noise are sufficiently small (see, for example, Morgan and Várady, 2004, Güth, Müller, and Spiegel, 2006).
Games of incomplete information may, of course, also have mixed strategy equilibria. However, as Milgrom and Weber (1985) show, for every mixed strategy one can generally find an equivalent pure strategy that induces the same probability distribution of actions.\footnote{For a casual review of this “purification theorem” see Fudenberg and Tirole (1991, Theorem 6.2). Milgrom and Weber (1985) require finite action sets; however, recent extensions require only compactness (see Podczeck, 2009).}

The present paper is also related to the literature on second-mover advantages in games in which prices are strategic complements (see, for example, Gal-Or, 1985, Dowrick, 1986, Amir and Stepanova, 2006). Like that literature we assume general demand functions that are super-modular in the price vector. However, while this literature considers games of complete information (and does not address the robustness issue raised by Bagwell, 1995), incomplete information is essential for our analysis.

There is a small literature on espionage in particular market games. While most contributions consider spying out rivals’ type, some contributions also consider spying out rivals’ actions, as we do in the present paper.

The two kinds of espionage are fundamentally different. Whereas spying out rivals’ type allows players to sharpen their prediction of rivals’ strategy, spying out rivals’ action changes the order of moves and transforms a simultaneous into a sequential moves game. As a byproduct of observing a rival’s action a player may also learn about its rival’s type. Indeed, in the present model, after observing the rival’s price, its cost can also be inferred (provided the rival’s strategy is monotone). However, this inference is irrelevant because knowing the rival’s cost in addition to its price adds no value.

Like the present paper, Solan and Yariv (2004), Barrachina, Tauman, and Urbano (2014) consider spying out actions. They consider an entry game played by an incumbent and a potential entrant. In Barrachina, Tauman, and Urbano (2014) the incumbent may expand production capacity to deter entry and the potential entrant can spy out the first-mover’s investment in capacity. In equilibrium, the incumbent randomizes his investment and the entrant engages in spying even though spying is costly. Whereas in Solan and Yariv (2004) the incumbent commits to either fight or accommodate entry and the entrant’s spy imperfectly observes that decision.

Similar to the present paper, spying out the incumbent’s action confers the spied at firm a commitment device that yields first- and second-mover advantages. However, whereas these contributions rely on complete information coupled with mixed strategies, the present model admits no equilibrium in mixed strategies and thus relies on incomplete information coupled with pure strategies.

Spying out rivals’ type is considered in several contributions. Wang (2016) considers a linear Cournot duopoly where one firm has private information concerning its unit cost while the other firm can spy out the rival’s unit cost which results in a noisy observation. Similarly, Kozlovskaya (2018) assumes that firms can learn about demand by conducting their own market research and by spying out the results of rivals’ market research.
Zhang (2015) and Chen (2017, Ch. 2) analyze spying out rivals’ types in two-player contests where types are represented by valuations for the given prize. In some cases both the spying and the spied at firm benefit; mutual spying may then be viewed as a form of information sharing.

Apart from these contributions, the larger literature on industrial espionage has addressed a range of complementary issues. Whitney and Gaisford (1999) analyze spying of technology where the spy contributes to lower the cost of the spying firm, inspired by the relationship between Airbus and Boeing. Matsui (1989) analyzes an infinitely repeated two-person zero-sum game where, at the outset, one or both players will be perfectly informed of each other’s supergame strategy with a given small probability and then have the chance to revise their strategies.

The plan of the paper is as follows: Section 2 states the model. Section 3 explains why, in the present context, incomplete information is essential for a robust explanation of spying. Section 4 disentangles the informational and the strategic impact of spying and states conditions that assure that spying is profitable. Section 5 explores what happens if the spied at firm attempts to immunize against the adverse impact of spying by delaying its pricing decision or by firing the spy when its cost is low. These attempts are shown to adversely affect beliefs to such an extent that they become self-defeating. Section 6 takes into account that the spy may be a counterspy, solves the resulting signaling game, and shows that the presence of the counterspy may surprisingly aggravate the price leadership induced by the spy. We close with a discussion in Section 7. Various proofs are relegated to the Appendix.

2 Model

Consider a duopoly with firms 1 and 2 (where 1 is mnemonic for first-mover and 2 for second-mover) that play a Bertrand market game with differentiated products that are substitutes, subject to linear cost functions. Firm 2 has access to the services of a spy who observes the unit price chosen by firm 1, \( p_1 \), and who may report his observation to firm 2 before it sets its own price, \( p_2 \). The presence of the spy is common knowledge although his identity is unknown.

The time-line of the base model is as follows:\(^8\)

1. “Nature” independently draws unit costs, \( x_1 \) and \( x_2 \), from the probability distribution \( F \). The realized unit costs are firms’ private information.
2. Firm 1 sets its unit price, \( p_1 \); the spy observes it, firm 2 does not.
3. The spy reports his observation to firm 2.
4. Firm 2 sets its unit price, \( p_2 \), and payoffs are realized.

In a first extension we allow firm 1 to immunize against the adverse impact of being spied at by delaying its pricing decision or by firing the spy if his identity has been exposed.

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\(^6\)See also Baik and Shogren (1995) who pioneered the analysis of spying in contests.

\(^7\)Zhang (2015) also considers that the spy may be a double spy. This resonates with our analysis of counter-espionage which, however, raises very distinct signaling issues. Double spying is also considered in Ho (2008) who, however, focuses on the design of contracts that deter double spying in an agency setting.

\(^8\)We do not specify whether the spy is procured before the game is played or offers his service in the course of the game, because this does not matter for our analysis.
In a second major extension we take into account that the spy may be a counterspy who serves the interests of firm 1 and reports strategically distorted information or, equivalently, firm 1 may have fooled the spy to report distorted information.

Firms’ demand functions \( Q_i(p_i, p_j) \) are twice continuously differentiable with \( \partial_{p_i} Q_i(p_i, p_j) < 0 \), \( \partial_{p_j} Q_i(p_i, p_j) > 0 \), \( \partial_{p_i, p_j} Q_i(p_i, p_j) \geq 0 \), \( \partial_{p_i, p_j} Q_i(p_i, p_j) \leq 0 \). These properties assure that firms’ profit functions, \( \pi_i(p_i, p_j, x_i) := (p_i - x_i) Q_i(p_i, p_j) \), are strictly supermodular in the price vector (see, for example, Vives, 2005) and best replies are unique. By a well-known result due to Topkis (1978), this implies that firms’ best reply functions are non-decreasing. It also follows that “single-crossing” resp. “increasing differences” conditions (see Athey, 2001, Van Zandt and Vives, 2007, p. 344), \( \partial_{p_{i, x_i}} \pi_i(p_i, p_j, x_i) > 0 \), \( \partial_{p_p, p_i} \pi_i(p_i, p_j, x_i) > 0 \), are satisfied.

For simplicity we also assume strict concavity of \( \pi_1(p_1, R(p_1, x_2), x_1) \) in \( p_1 \), with \( R(p_1, x_2) := \arg\max_p \pi_2(p, p_1, x_2) \). This rules out equilibria in mixed strategies.10

The probability distribution \( F \) is continuous with support \([\alpha, \beta]\), \( 0 \leq \alpha < \beta \) and expected value \( \bar{x} := E[X] \).

The parameters of the demand functions and the probability distribution of unit costs are such that no firm is ever crowded out of the market as firms play the duopoly game.

### 3 Why incomplete information is essential

At the outset we emphasize that, in the present model, incomplete information is essential to explain espionage. We briefly examine the case of complete information and explain why spying does not occur in robust equilibria that pertain when the spy’s observation is subject to noise.

If firms have the same unit cost, the presence of the spy transforms the game from a simultaneous to a sequential moves game and makes the spied at firm Stackelberg leader and the spying firm Stackelberg follower. As is well-known, in a symmetric Stackelberg game with substitutes both firms are better off than in the simultaneous moves game and the Stackelberg follower is even better off than the Stackelberg leader (see Gal-Or, 1985, Dowrick, 1986).

If the game is asymmetric, the firm that has a substantially lower unit cost may have a first-mover advantage. In general, at least one firm has a second-mover advantage but it is not always the case that both firms have a second-mover advantage (see Amir and Stepanova, 2006).

In either case spying is a quasi-collusive scheme that supports higher equilibrium prices that benefit both the spying and the spied at firm.

However, these equilibria are not robust. If the spy’s observation is subject to noise, his observation is ignored and the unique equilibrium of the game is that of the simultaneous moves Bertrand game, no matter how small the noise. This result follows from a prominent result in the literature on the value of commitment in sequential games that is due to Bagwell (1995) who, however, considers a model with discrete action sets that admits also mixed strategy equilibria (one of which approaches the Stackelberg equilibrium and thus restores the value of commitment).

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9 Throughout this paper we write \( \partial f(x, y) \) for \( \partial f(x, y)/\partial x \) and \( \partial^2 f(x, y) \) for \( \partial^2 f(x, y)/\partial x \partial y \) and denote random variables by capital and realizations by lower case letters.

10 In lieu of ruling out mixed equilibria, one can use the fact that in games of incomplete information a mixed strategy equilibrium can generally be purified, and then focus on pure strategy equilibria.
We mention that Morgan and Várdy (2013) have similarly shown that firm 2 is not willing to pay for information and the only equilibrium that survives if information is costly is that of the simultaneous moves Bertrand game, no matter how small the cost. However, this requires the additional assumption that firm 1 does not know whether firm 2 has procured the service of the spy when it sets its price, whereas the Bagwell test requires no such qualification.\footnote{We thank a referee for stressing this important distinction between Bagwell (1995) and Morgan and Várdy (2013).} Like the present paper Morgan and Várdy (2013) consider a continuum of actions and concavity of the leader’s reduced form payoff function which rules out mixed strategy equilibria.

The fundamental conclusion from these results is that espionage can only serve a useful purpose if the spied at firm’s choice of action cannot already be inferred from its equilibrium strategy. This is the case if the first-mover plays a mixed strategy or his strategy depends on some payoff relevant parameter that is this firm’s private information. Type dependent strategies are behaviorally indistinguishable from mixed strategies. Therefore, there are two frameworks to explain espionage in a robust manner: games of complete information that exhibit equilibria in which the first-mover randomizes and games of incomplete information.

The present model admits no mixed strategy equilibrium. Therefore, in the present model, espionage can only be explained if firms are subject to incomplete information.

Games of incomplete information may, of course, also have mixed strategy equilibria. However, as we documented already in the introduction, for every mixed strategy one can generally find an equivalent pure strategy that induces the same probability distribution of actions. Therefore, focusing on pure strategy equilibria involves no loss of generality.

4 Espionage under incomplete information

Now suppose each firm has private information concerning its unit cost. In that case, firms’ pure strategies are functions of their privately observed cost, \(p_i(x_i)\). Correctly predicting the equilibrium strategy of firm 1, i.e., the function \(p_1(x_1)\), does not reveal its choice of action. Therefore, observing the first-mover’s price is valuable (unless \(p_1(x_1)\) is flat), even if using the spy is subject to a small cost or if the spy’s observation is subject to small noise.

In a seminal contribution, Maggi (1999) analyzes the value of commitment in games of incomplete information, where firms’ unit costs are their private information. His main finding is that, if the noise is arbitrarily small, the equilibrium is arbitrarily close to the equilibrium in the corresponding sequential game with perfect observability. This explains why the equilibrium of a game in which the spy perfectly observes the action of the first-mover is robust with respect to introducing small noise; it is also clear that it is robust with respect to introducing a small cost. This justifies that, in the following, we assume that the spy perfectly observes the price of firm 1 at zero cost.

As a benchmark we first solve the simultaneous moves Bertrand game to which we refer as game \(G^B\). The symmetric Bayesian equilibrium strategies of that game, \(p^B(x_i)\), solve the requirements:

\[
p^B(x_i) = \arg \max_p (p - x_i) E_{X_j} \left[ Q_i(p, p^B(X_j)) \right], \quad i \in \{1, 2\}.
\]

(1)

Existence of a monotone equilibrium strategy is assured by the fact that profit functions exhibit “increasing differences”.

11 We thank a referee for stressing this important distinction between Bagwell (1995) and Morgan and Várdy (2013).
In the case of linear demand, $Q_i(p_1, p_2) := 1 - p_1 + s p_2$, with $0 < s < 1$, and $\beta < 1$, the equilibrium prices, their expected value, and firms’ expected profits, $\Pi^B_i(x_i)$, are:

$$p^B_i(x_i) = \frac{2 + \bar{s} x_i}{2(2 - s)} + \frac{1}{2} x_i, \quad p^* := E[p^B(X)] = \frac{1 + \bar{s}}{2 - s}$$

(2)

$$\Pi^B_i(x_i) := (p^B_i(x_i) - x_i) E_x_i[Q_i(p^B(x_i), p^B(X_i))] = (p^B_i(x_i) - x_i)^2.$$ 

(3)

Now we solve the sequential game with espionage where the presence of the spy is common knowledge, although the identity of the spy has not been exposed. We refer to the resulting game as game $G^S$ (where $S$ is mnemonic for “Stackelberg-Bertrand”).

The second-mover’s payoff is strictly concave in its own action, $p_2$, for all prices quoted by firm 1: $\partial_{p_2} p_2 \pi_2(p_2, p_1, x_2) < 0$. This implies that the second-mover will never randomize in equilibrium. Similarly, the first-mover’s expected payoff is strictly concave in $p_1$ which implies that he does not randomize either. Existence of a pure strategy equilibrium in monotone increasing strategies is assured by the “increasing differences” assumption. Therefore, we can focus on pure strategy equilibria, without loss of generality.

Firms’ pure strategy equilibrium strategies $(p^S_1(x_1), p^S_2(x_2, x_1))$ solve the requirements:

$$p^S_2(x_2, p_1) = \arg\max_p (p - x_2) Q_2(p, p_1)$$

$$p^S_1(x_1) = \arg\max_p (p - x_1) E_{x_1}(Q_1(p, p^S_2(X_2, p)))$$

and the equilibrium expected payoffs are:

$$\Pi^S_1(x_1) := (p^S_2(x_2, x_1) - x_1) E_{x_2}(Q_1(p^S_1(x_1), p^S_2(X_2, p^S_1(x_1)))$$

$$\Pi^S_2(x_2) := E_{x_1}(Q_1(p^S_2(x_2, x_1) - x_2) Q_2(p^S_2(x_2, p^S_1(x_1)), p^S_1(x_1))$$.

Compared to the Bertrand game with two-sided private information, there are two effects: 1) firm 2 becomes second-mover in a sequential game (strategic effect), and 2) it becomes common knowledge that firm 2 is no longer subject to uncertainty about its rival’s cost (information effect).

In order to disentangle these effects, we introduce a hypothetical Bertrand game with one-sided private information, to which we refer as game $G^b$, where firm 2 knows its rival’s cost but firm 1 does not know the cost of firm 2. Therefore, as one compares game $G^S$ with game $G^b$, one captures exclusively the strategic effect of moving from the simultaneous to the sequential moves game.

The equilibrium solution of the game $G^b$ must solve the requirements:

$$p^b_1(x_1) = \arg\max_p (p - x_1) E_{x_2}(Q_1(p, p^b_2(X_2, x_1)))$$

$$p^b_2(x_2, x_1) = \arg\max_p (p - x_2) Q_2(p, p^b_1(x_1)),$$

and the interim and ex ante equilibrium expected payoffs are:

$$\Pi^b_1(x_2, x_1) := (p^b_2(x_2, x_1) - x_2) Q_2(p^b_2(x_2, x_1), p^b_1(x_1))$$

$$\Pi^b_2(x_2) := E_{x_1}(\Pi^b_2(x_2, X_1))$$
The following result shows that there are first- and second-mover advantages provided one controls for the information effect:

**Proposition 1.** In equilibrium both firms are better off in game \( G^S \) than in \( G^b \):

\[
\Pi_1^S(x_1) \geq \Pi_1^b(x_1), \quad \Pi_2^S(x_2) \geq \Pi_2^b(x_2), \quad \forall (x_1, x_2).
\]

**Proof.** In four steps:

1) By playing \( p_1^b(x_1) \) player 1 can induce the equilibrium outcome of game \( G^b : (p_1^b(x_1), p_2^b(x_2, x_1)) \), because \( p_2^b(x_2, x_1) = p_2^b(x_2, p_1^b(x_1)) \). However, player 1 can do better. Therefore,

\[
\Pi_1^b(x_1) = (p_1^b(x_1) - x_1)E_{\xi_1}[Q_1(p_1^b(x_1), p_2^b(x_2, x_1))]
\]

\[
\geq (p_1^b(x_1) - x_1)E_{\xi_1}[Q_1(p_1^b(x_1), p_2^S(x_2, x_1))]
\]

\[
= \Pi_1^S(x_1).
\]

2) Because \( (p_1^b(x_1), p_2^S(x_2, x_1)) \) is an equilibrium of game \( G^b \), unilateral deviations do not pay; therefore,

\[
\Pi_1^S(x_1) \geq (p_1^S(x_1) - x_1)E_{\xi_1}[Q_1(p_1^S(x_1), p_2^S(x_2, x_1))].
\]

Combining (5) and (6) gives:

\[
E_{\xi_1}[Q_1(p_1^S(x_1), p_2^S(x_2, x_1))] \geq E_{\xi_1}[Q_1(p_1^b(x_1), p_2^S(x_2, x_1))].
\]

3) Next we prove that \( p_1^S(x_1) \geq p_1^b(x_1) \), for all \( x_1 \). Suppose, \( p_1^S(x_1) < p_1^b(x_1) \) for some \( x_1 \). If this were the case, because the reaction function of player 2 is increasing in \( p_1 \), one would have:

\[
p_2^S(x_2, p_1^S(x_1)) < p_2^S(x_2, p_1^b(x_1)) = p_2^b(x_2, x_1).
\]

However, because \( Q_1 \) is increasing in \( p_2 \), (8) contradicts (7).

4) Having shown that \( p_1^S(x_1) \geq p_1^b(x_1) \) for all \( x_1 \), it follows immediately that \( \Pi_2^S(x_2) \geq \Pi_2^b(x_2) \).

However, in order to assess whether firm 2 overall benefits from using the spy one has to combine the strategic effect captured in Proposition 1 with the information effect.

A priori it is not clear whether firm 2 altogether benefits from using the spy. Although firm 2 benefits from observing the rival’s price, \( p_1 \), because that removes its uncertainty about the behavior of firm 1, it may however be hurt by the fact that firm 1 knows about this and adjusts its price strategy.

In non-strategic decisions it is a well established principle, known as Blackwell’s Theorem, that more information is better. However, in a strategic game context, it is not uncommon that more information actually hurts the player who obtains that information if other players know about this change of information and respond to it.\(^{12}\)

\(^{12}\)The only class of games where this seemingly paradoxical property can never occur is the class of zero sum games with common beliefs of players (see Bassan, Scarsini, and Zamir , 1997).
However, the following conditions assure that firm 2 altogether benefits from using the spy. Together with Proposition 1, this provides sufficient conditions for a second-mover advantage in models of price leadership under incomplete information.

**Proposition 2.** In equilibrium, the spying firm altogether benefits from spying: $\Pi_2^S(x_2) > \Pi_2^B(x_2)$, if its interim equilibrium expected payoff function satisfies the conditions:

(i) $\tilde{\Pi}_2^b(x_2, x_1)$ is convex in $x_1$ for all $x_2$, and
(ii) $\tilde{\Pi}_2^b(x_2, \tilde{x}) \geq \Pi_2^B(x_2)$, for all $x_2$.

**Proof.** The convexity assumption implies that in the game $G^b$ firm 2 benefits from uncertainty concerning $x_1$. Combining the two conditions gives:

$$\Pi_2^b(x_2) = E_{X_1}[\tilde{\Pi}_2^b(x_2, X_1)] \geq \tilde{\Pi}_2^b(x_2, \tilde{x}) \geq \Pi_2^B(x_2). \tag{9}$$

Using the fact that $\Pi_2^S(x_2) > \Pi_2^B(x_2)$ (by Proposition 1) we conclude:

$$\Pi_2^S(x_2) - \Pi_2^B(x_2) \equiv \Pi_2^S(x_2) - \Pi_2^b(x_2) + \Pi_2^b(x_2) - \Pi_2^B(x_2) > 0. \tag{10}$$

As we show in Appendix A, the conditions stated in Proposition 2 are satisfied for a class of demand functions that include linear demand as a special case.

In turn

**Proposition 3.** Firm 1 benefits from being spied at if its cost is sufficiently high but may be hurt if its cost is sufficiently low.

**Proof.** To show that firm 1 benefits if its cost is high, it is sufficient to show that:

$$\Pi_1^B(\beta) = (p_1^B(\beta) - \beta)E_{X_2}[Q_1(p_1^B(\beta), p_2^B(X_2))]$$

$$< (p_1^B(\beta) - \beta)E_{X_2}[Q_1(p_1^B(\beta), p_2^S(X_2, p_2^B(\beta)))]$$

$$\leq \arg \max_{p} (p - \beta)E_{X_2}[Q_1(p, p_2^S(X_2, p))] \equiv \Pi_1^S(\beta).$$

The first inequality applies because $p_2^S(x_2, p_2^B(\beta)) > p_2^B(x_2)$, which follows from the fact that $p_2^B(\beta) > p_2^B(x_1)$ for all $x_1 < \beta$.

To show that firm 1 may be hurt if its cost is low consider the linear example below. \hfill \Box

The intuition for this result is that firm 1 is happy to reveal its price if its cost and thus price is high because it thus triggers firm 2 to respond with a high price; conversely, it may not wish to reveal a low price because this triggers a low price response by firm 2. However, the chain of effects is complicated by the fact that firm 2 responds as well.

In the case of linear demand we find specifically:

$$p_1^S(x_1) = \frac{1}{2} \left( \frac{2 + s(1 + \tilde{x})}{2 - s^2} + x_1 \right), \quad p_2^S(p_1, x_2) = \frac{1}{2} (1 + sp_1 + x_2) \tag{11}$$

10
\[ \Pi_1^\delta(x_2) - \Pi_2^\delta(x_2) = \frac{s^3(1 - (1 - s)\bar{x})}{16(2 - s)^2} \frac{(2 - s^2)(1 - x_2) + 8s(x_2 + \bar{x}) + s^3(4x_2 - 5\bar{x}) + s^4\bar{x}}{2(\bar{x} - x_2)} \]

\[ \Pi_1^\delta(x_1) - \Pi_2^\delta(x_1) = \frac{s^3(2(1 - (1 - s)\bar{x}) - (2 - s^2) ((2 - s)x_1 - \bar{x} - 1)^2)}{8(2 - s)^2(x_2^2 - s^2)} \] (12)

There, \( \Delta(x_1) := \Pi_1^\delta(x_1) - \Pi_2^\delta(x_1) \), is a quadratic, concave function of \( x_1 \) which is positive at \( x_1 = \beta \). If \( \Delta(\alpha) > 0 \), \( \Delta(x_1) \) is positive everywhere. If \( \Delta(\alpha) < 0 \), there exists a unique \( \hat{x} \in (\alpha, \beta) \) at which \( \Delta(\hat{x}) = 0 \). Therefore, \( \Delta(x_1) \geq 0 \iff x_1 \geq \hat{x} \). Because \( \Delta(\hat{x}) \) is positive, it follows that \( \hat{x} < \bar{x} \).

One can easily construct examples where \( \Delta(x_1) \) is positive for all \( x_1 \) and where there is a threshold \( \hat{x} \in (\alpha, \beta) \). As an example consider the uniform distribution and \( (s, \beta) = (1/2, 1/2) \). Then, \( \Delta(x_1) > 0 \) everywhere if \( 1/20(14 - 3\sqrt{14}) < \alpha < 1/2 \) and \( \hat{x} \in (\alpha, \beta) \) if \( \alpha \in [0, 1/20(14 - 3\sqrt{14})] \).

**Remark 1.** If firm 1 believes that a spy is present only with positive probability \( \lambda \) less than one, firm 1 chooses a price strictly in between the Bertrand and Stackelberg leader price and the benefit of spying is, of course, reduced. In the linear model, the equilibrium prices are:

\[ p_1^\lambda(x_1) = \frac{2 + s}{4 - (1 + \lambda)s^2} + s\left(\frac{(2 + s)(1 - \bar{x})}{8 - 2(1 + \lambda)s^2} + x_1\right) < p_1^\delta(x_1) \]

\[ p_2^\lambda(x_2, p_1^\lambda(x_1)) = \frac{2 + s}{4 - (1 + \lambda)s^2} + s\left(\frac{(2 + s)(1 - \bar{x})}{8 - 2(1 + \lambda)s^2} + x_1\right) < p_2^\delta(x_2, p_1^\delta(x_1)) \], if firm 2 has a spy

\[ p_2^\lambda(x_2) = p_2^\delta(x_2, p_1^\lambda(\bar{x})), \text{ otherwise.} \]

There, the first inequality follows from the fact that \( \bar{x} < 1/1-s \) which follows from the requirement that equilibrium prices must be higher than the unit cost, in particular, \( p_1^\lambda(\bar{x}) > \bar{x} \).

**Remark 2.** What if both firms may have access to a spy? In that case only one firm can have the opportunity to respond to the price reported by its spy before it sets its own price. Of course, one could think of a model, where it is determined at random who is chosen to respond, but it cannot happen that both firms respond.

### 5 What if firm 1 can delay its pricing decision (or fire the spy)?

Because firm 1 may be hurt by being spied at if its cost is low it may attempt to immunize against spying if its cost is low by delaying its pricing decision until after firm 2 has set its price. Equivalently, it could fire the spy if his identity has been exposed. We now examine whether firm 1 can actually benefit from immunizing against spying.

We refer to the game that is played if firm 1 delays pricing (or fires the spy) if its cost is low as game \( G_d \) (where \( d \) is mnemonic for “delay”). Naturally, firm 2 knows whether firm 1 has delayed pricing (or fired the spy).

If firm 1 delays pricing it thus reveals information about its cost, which in turn induces firm 2 to adjust its price. Taking this signaling effect into account, it turns out that complete unraveling occurs and it never pays to delay pricing at any level of \( x_1 \).

**Proposition 4.** There is no equilibrium in which firm 1 delays its pricing decision (or fires the spy) with positive probability.
Proof. From the above we know that firm 1 will only contemplate to delay pricing if its cost is sufficiently low. Suppose, per absurdum, that there is an equilibrium in which firm 1 delays pricing with positive probability when its cost is low. Then this attempt to neutralize the spy must occur in some measurable set \( M \subseteq [\alpha, \beta] \). Let \( m = \sup M \). We may assume that \( m \in M \) (otherwise one can consider an element in \( M \) arbitrarily close to \( m \)).

If firm 1 delays pricing at \( x_1 = m \), firm 2 learns that \( x_1 \in M \). In that case, the equilibrium strategies, denoted by \((p^d_1(x_1), p^d_2(x_2))\), of the subsequent Bertrand game solve the conditions:

\[
p^d_1(x_1) = \arg\max_p \int_\alpha^\beta Q_1(p, p^d_2(x_2))(p - x_1)dF(x_2)
\]

\[
p^d_2(x_2) = \arg\max_p \int_M Q_2(p, p^d_1(x_1))(p - x_2)dF_M(x_1),
\]

where \( F_M \) is the conditional distribution of \( F \) on \( M \).

If firm 1 deviates, does not delay pricing at \( x_1 = m \), and sets the price it would choose in the Bayesian game after delaying pricing, i.e., \( p = p^d_1(m) \), firm 2 optimally responds with the price \( p_2 = p^d_2(x_2, p^d_1(m)) \) which satisfies:

\[
p^d_2(x_2, p^d_1(m)) = \arg\max_p (p - x_2)Q_2(p, p^d_1(m))
\]

\[
= \arg\max_p (p - x_2) \int_M Q_2(p, p^d_1(m))dF_M(x_1)
\]

\[
> \arg\max_p (p - x_2) \int_M Q_2(p, p^d_1(x_1))dF_M(x_1) = p^d_2(x_2),
\]

by the strict supermodularity of the profit function (which is preserved under integration). Therefore, by not delaying its pricing decision, firm 1 induces firm 2 to respond with higher price, which benefits firm 1. This contradicts the existence of such an equilibrium. \( \square \)

Essentially, if firm 1 delays pricing if its cost is in set \( M \subseteq [\alpha, \beta] \), when firm 2 observes that pricing has been delayed, it updates its beliefs and adjusts its price in such a way that the high cost types in set \( M \) would always like to reveal their type by not delaying pricing. After successive application of this reasoning, complete unraveling occurs.

Altogether we conclude from Sections 4 and 5:

Corollary. Firm 2 benefits from using the spy if the conditions stated in Proposition 2 are satisfied, and firm 1 can never benefit from immunizing against being spied at by delaying its pricing decision or firing the spy.

6 What if the spy may have been fed with distorted information?

Now suppose the spied at firm may have fed the spy with strategically distorted information because the spy is a counterspy (or, equivalently, because the identity of the spy has been exposed and firm 1 fools him to report distorted information).\(^{13}\) In that case, firm 2 cannot be sure that the reported

\(^{13}\)Counter-espionage figures prominently in military history. See, for example, the fascinating story of Duško Popov who enlisted in Germany’s military intelligence service during WWII to spy on the British military but actually fed the German “Abwehr” with disinformation in the service of the British “MI6” (see Popov, 1974, Loftis, 2016).
price is actually the true price and it needs to draw an inference from the reported price about the type of spy and make a prediction about the true price if the spy happens to be a counterspy. Naturally, the counterspy hides his type and the reported price is only an imperfect signal of the type of spy.

To keep the analysis tractable, we switch to a binary model with \( X_i \in \{0, x\}, x \in (0, 1) \), and linear demand: \( Q_i(p_i, p_j) := 1 - p_i + sp_j \), with \( s \in (0, 1) \).

Let \( p_0 \in (0, 1) \) be the prior probability that the spy is a counterspy and \( h \in (0, 1) \) the prior probability that firms’ cost is equal to \( x \). After observing the price reported by the spy, firm 2 updates its beliefs and chooses its own price.

For ease of exposition we refer to firm 1 with counterspy as type \( c \) and without counterspy as type \( n \), and to firm 1 type \( n \) with cost 0 as type \( nl \) and with cost \( x \) as type \( nh \).

The posterior beliefs concerning the type of spy and the cost of firm 1 type \( c \) are denoted by \( \rho(p_1) \) and \( \mu(p_1) \) (and if there is no risk of confusion simply by \( \rho \) and \( \mu \)). If firm 1 is type \( n \), its reported price is also its true price. In that case, the payoff of firm 2 does not depend on \( x_1 \). Belief updating concerning the cost of firm 1 matters only for predicting the behavior of firm 1 type \( c \).

We consider perfect Bayesian Nash equilibria, \( (p_1^*(x_1), p_1^*(x_1), p_1^*(x_1), p_2^*(x), \rho, \mu) \). There, \( p_1^*(x_1) \) denotes the prices set by firm 1 type \( n \), \( p_1^*(x_1) \) the prices set by firm 1 type \( c \), \( p_2^*(x_1) \) the prices reported by the counterspy, and \( p_2^*(p_1, 0, \rho, \mu) \) the prices set by firm 2.

As a benchmark we refer to the simultaneous moves game without spy as “Bertrand game” and the sequential game with spy but without counterspy \( (\rho_0 = 0) \) as “Stackelberg game”; their equilibrium strategies were already stated in equations (2) and (11), which apply also to the present binary model.

We employ the following solution procedure: As a working hypothesis assume the game has a partially separating equilibrium where the price reported by the counterspy is independent of \( x_1 \), \( p'(0) = p'(x) =: p' \). Consistent with such an equilibrium we stipulate the belief updating rule:

\[
\mu(p_1) := \Pr \{ X_1 = x \mid p_1 \text{ is reported and firm 1 is type } c \} = h. \tag{14}
\]

We use these to construct partially separating equilibria and show that the hypothesis confirms.\(^{14}\)

All omitted proofs are relegated to Appendix B.

In a first step we solve the duopoly subgames that are played between firm 1 type \( c \) and firm 2 after the spy has reported a price \( p_1 \) and firm 2 updated its beliefs. There, type \( c \) secretly sets its price, \( p_1^*(x_1) \), whereas the price of firm 1 type \( n \) coincides with the reported price.

**Lemma 1** (Duopoly subgames). *Given any price reported by the spy, \( p_1 \), updated beliefs \( \rho \) and \( \mu \) (as stipulated in (14)), firm 1 type \( c \) and firm 2 simultaneously choose their prices (firm 1 type \( n \) is bound to set \( p_1 \)). Their equilibrium strategies solve the conditions, for \( x_1, x_2 \in \{0, x\} \):

\[
p_1^*(p_1, x_1, \rho, \mu) = \arg \max_{p} E_{X_1} \left[ \pi_1(p, p_2(p_1, x_2, \rho, \mu), x_1) \right] \tag{15}
\]

\[
P_2(p_1, x_2, \rho, \mu) = \arg \max_{p} \left( \rho E_{X_1} \left[ \pi_2(p, p_1^*(p_1, x_1, \rho, \mu), x_2) \right] + (1 - \rho) \pi_2(p, p_1^*(x_1, \rho, \mu), x_2) \right). \tag{16}
\]

\(^{14}\) We mention that the game also admits pooling equilibria and partially separating equilibria where the price reported by firm 1 type \( c \) depends on its cost.
The solutions are linear and increasing in \( p \) (see (B.1), (B.2) in Appendix B).

Define the “expected best reply functions” of firm 2:

\[
\bar{R}(p_1, \rho, \mu) := E_X[\bar{P}_2(p_1, X_2, \rho, \mu)], \quad R(p_1, \rho) := \bar{R}(p_1, \rho, h).
\]

Due to the linearity of \( P_2(p_1, X_2, \rho, \mu) \) in \( p_1 \) and of \( \pi_1 \) in \( p_2 \), in the following we always write the expected profit of firm 1 type \( n \) as \( \pi_1(p_1, R(p_1, \rho), x_1) \) and firm 1 type \( c \) as \( \pi_1(p_1(x_1), R(p_1, \rho), x_1) \).

**Lemma 2.** For all \( \rho \in [0, 1) \) the expected best reply functions of firm 2 are increasing in \( p_1 \), with a slope that is decreasing in \( \rho \) and flat if \( \rho = 1 \); they have the common fixed-point \( p^* := E[p^h(X)] \):

\[
\begin{align*}
\partial_{\rho_1} R(p_1, \rho) &> 0 \quad \text{and} \quad \partial_{\rho_1} R(p_1, \rho) < 0, \quad \forall \rho \in [0, 1) \\
R(p^*, \rho) &> p^*, \quad \forall \rho \in [0, 1] \\
R(p_1, 1) &> p^*, \quad \forall p_1.
\end{align*}
\]

Evidently, the most favorable belief for firm 1 is \( \rho = 0 \) if \( p_1 > p^* \) and \( \rho = 1 \) if \( p_1 < p^* \).

**Proposition 5.** In the assumed partially separating equilibrium firm 1 type \( c \) mimics firm 1 type \( nh \) and reports the price \( p^* = p^h_1(x) > p^h_1(0) \).

To complete the construction of a partially separating equilibrium, we now adapt the two-step procedure introduced by Cho and Sobel (1990) and Sobel (2009) to compute \( p^h_1(0), p^h_1(x) \). In a standard signaling game this procedure finds the unique strategically stable equilibrium.

**Two-step procedure:** Consider the belief system (14) together with:

\[
\rho(p_1) = \begin{cases} 
0 & \text{if } p_1 \leq p^h_1(0) \\
\frac{\rho_0}{\rho_0 + (1-\rho_0) h} & \text{if } p_1 \in (p^h_1(0), p^h_1(x)] \\
1 & \text{if } p_1 > p^h_1(x),
\end{cases}
\]

which are consistent with equilibrium strategies and Bayes’ rule.

**Step 1:** Set \( p^h_1(x) \) at the efficient level, \( p^h_1(x) = \arg \max_p \pi_1(p, R(p, \rho_1), x) \), and define the associated expected payoff: \( \Pi^h_1(x) := \pi_1(p^h_1(x), R(p^h_1(x), \rho_1), x) \).

**Step 2** Set the most profitable \( p^h_1(0) \) subject to the requirements that firm 1 type \( nh \) does not benefit from switching to that price and firm 1 type \( c \) does not benefit from reporting \( p^* = p^h_1(0) \) rather than \( p^* = p^h(x) \):

\[
p^h_1(0) = \arg \max_p \pi_1(p, R(p, 0), 0)
\]

subject to \( \pi_1(p, R(p, 0), x) \leq \Pi^h_1(x) \) and \( R(p^h_1(0), 0) \leq R(p^h_1(x), \rho_1) \).

**Proposition 6.** The strategies \( p^h_1(x_1), p^h_1(p_1, x_1, \rho), p^* = p^h_1(x) \), together with the belief systems (14), (21) are a partially separating equilibrium. The equilibrium price \( p^h_1(x_1) \) continuously approaches the Stackelberg leader price, \( p^1_1(x_1) \), from below as \( \rho_0 \) goes to zero.
That equilibrium is either an interior or a corner solution, as illustrated in Fig. 1. There, $I_1(x)$, $I_1(0)$ are the indifference curves of firms 1 type $nh$ and $nl$. The lines (part solid, part dashed) are the expected best replies of firm 2, $R(p_1, \rho_1), R(p_1, 0), R(p_1, 1) \equiv p^\ast$. In both cases, the first constraint in (22) restricts firm 1 type $nl$ to choose a point on the graph of the $R(p_1, 0)$ line, up to the point where it intersects the indifference curve $I_1(x)$. In case (a) that constraint binds; in case (b) the solution is at the unconstrained efficient level.

The solid parts of the expected reaction functions display how the reported price impacts the beliefs of firm 2 and thus its expected price. As the reported price $p_1$ moves from $p^\ast$ to $p_1^\ast(0)$, the expected best response of firm 2 moves along the $R(p_1, 0)$ function. If $p_1$ is further increased up to $p_1^\ast(x)$, that expected best response jumps down to the $R(p_1, \rho_1)$ function, and as $p_1$ is further increased it jumps down again to the $R(p_1, 1) \equiv p^\ast$ function. This shows clearly that it never pays for firm 1 type $c$ to deviate from the asserted equilibrium and report a price $p_1 \neq p_1^\ast(x)$ (optimality for type $nl$ and $nh$ is obviously satisfied by construction).

Like other signaling problems, the present model has multiple equilibria because the perfect equilibrium does not pin down the beliefs of firm 2 for out-of-equilibrium prices $p_1$. To “refine” away implausible equilibria the literature proposed various equilibrium refinements. The most commonly used refinement is the intuitive criterion by Cho and Kreps (1987).

Adapted to the present context, the idea of that equilibrium refinement is that an out-of-equilibrium price, $p_1^\ast$, the belief system should prescribe $\rho(p_1^\ast) = 0$ if the following conditions are satisfied:15

\[
\begin{align*}
\pi_1(p_1^\ast, R(p_1^\ast, 0), x_1) &> \Pi_1^0(x_1), \text{ for some } x_1 \in \{0,x\} \\
\max_p \pi_1(p, R(p_1^\ast, 0), x_1) &< \Pi_1^0(x_1), \text{ for all } x_1 \in \{0,x\},
\end{align*}
\]

Figure 1: Equilibrium prices $(p_1^\ast(0), p_1^\ast(x))$. 

Adapted to the present context, the idea of that equilibrium refinement is that an out-of-equilibrium price, $p_1$, should be viewed as a signal that the spy is not a counterspy if, conditional on this belief, only a firm 1 type $n$ (type $nl$ or $nh$) has an incentive to deviate to that price. Stated formally:

**Intuitive Criterion** For any out-of-equilibrium price, $p_1^\ast$, the belief system should prescribe $\rho(p_1^\ast) = 0$ if the following conditions are satisfied:

\[
\begin{align*}
\pi_1(p_1^\ast, R(p_1^\ast, 0), x_1) &> \Pi_1^0(x_1), \text{ for some } x_1 \in \{0,x\} \\
\max_p \pi_1(p, R(p_1^\ast, 0), x_1) &< \Pi_1^0(x_1), \text{ for all } x_1 \in \{0,x\},
\end{align*}
\]

15If $p_1^\ast < (2 + \pi + (2 + \pi) x)/(4 - \pi^2)$, the most favorable belief system for firm 1 is $\rho = 1, \mu = 1$; therefore, in this case, replace $R(p_1^\ast, 0)$ by $R(p_1^\ast, 1, 1)$. 

15
where $\Pi_1(x_1) := \pi_1(p_1'(x_1), R(p_1'(x_1), \rho_1), x_1)$, $\Pi_1(x_1) := \pi_1(p_1'(x_1), R(p_1'(x_1), \rho(p_1'(x_1))), x_1)$ denote the equilibrium expected payoffs of firm 1 with and without counterspy.

The equilibrium summarized in Proposition 6 has some merit (see the discussion below); yet, it violates the intuitive criterion, for the following reasons.

First, consider the corner solution, displayed in Figure 1(a). Select an out-of-equilibrium price $p_1'$ slightly higher than $p_1''(0)$. That price is more profitable for firm 1 type $n_l$ if it triggers the belief $\rho = 0$, but not for firm 1 type $c$, because $R(p_1', 0) < R(p_1'', x_1, p_1)$. Therefore, by the intuitive criterion, the belief system should prescribe $\rho(p_1') = 0$, which destroys that equilibrium. This holds true not only for the belief system (21) but for any belief system that supports that equilibrium.

Next, consider the interior solution displayed in Figure 1(b). Let $p_A$ be the price at which the indifference curve of type $nh$ intersects the $R(p_1, 0)$ function and $p_B > p_A$ the price at which the belief $\rho = 0$ triggers the same expected response as the price $p_1''(x)$.

Select any out-of-equilibrium price $p_1' \in (p_A, p_B)$. That price is more profitable for firm 1 type $nh$ if it triggers the belief $\rho(p_1') = 0$, but not for firm 1 type $c$. Therefore, by the intuitive criterion, the belief system should prescribe $\rho(p_1') = 0$, which destroys that equilibrium. Again this holds true not only for the belief system (21) but for any belief system that supports that equilibrium.

We now propose an alternative two-step procedure which finds an equilibrium that is compatible with the intuitive criterion. In order to distinguish that equilibrium we denote its equilibrium prices of firm 1 type $n$ by $\tilde{p}_1'(x_1)$.

To prepare that alternative procedure, consider the pair of prices, $(\tilde{p}_1(0), \tilde{p}_1(x))$, that make both firm 1 type $nh$ and type $c$ indifferent between the price $\tilde{p}_1(x)$ combined with the belief $\rho_1$ and the price $\tilde{p}_1(0)$ combined with the belief $\rho = 0$:

$$
\pi_1(\tilde{p}_1(x), R(\tilde{p}_1(x), \rho_1), x) = \pi_1(\tilde{p}_1(0), R(\tilde{p}_1(0), 0), x) \quad (25)$$

$$
R(\tilde{p}_1(x), \rho_1) = R(\tilde{p}_1(0), 0). \quad (26)
$$

These prices have a unique solution $\tilde{p}_1(x) > \tilde{p}_1(0) > p^*$; $\tilde{p}_1(x)$ is increasing and $\tilde{p}_1(0)$ decreasing in $\rho_1$, and $\lim_{\rho_1 \to 0} (\tilde{p}_1(x) - \tilde{p}_1(0)) = 0$ (see (B.10) and (B.11) in Appendix B).

**Alternative two-step procedure:** Consider the belief system (14) together with:

$$
\rho(p_1) = \begin{cases} 
0 & \text{if } p_1 \leq p_1''(0) \\
\frac{\rho_0}{\rho_0 + (1-\rho_0)p_1} =: \rho_1 & \text{if } p_1 = p_1''(x) \\
1 & \text{otherwise},
\end{cases} \quad (27)
$$

which are consistent with equilibrium strategies and Bayes’s rule.

**Step 1** Set $\tilde{p}_1'(x) = \tilde{p}_1(x)$.

**Step 2** Set the most profitable $\tilde{p}_1''(0)$ subject to the requirement that firm 1 type $nh$ does not benefit from switching to that price, i.e.,

$$
\tilde{p}_1''(0) = \arg \max_p \pi_1(p, R(p(0), 0), s.t. \quad p \leq \tilde{p}_1(0). \quad (28)
$$
Proposition 7. The strategies $\hat{p}_1^a(x_1)$, $p^e(x_1)$, $p^e = \hat{p}_1^a(x)$, together with the belief systems (14), (27) are a partially separating equilibrium. That equilibrium is consistent with the intuitive criterion, yet exhibits a discontinuity at $p_0 = 0$.

The solution is either a boundary solution, illustrated in Figure 2(a), or an interior solution, illustrated in Figure 2(b) (there $I_c$ is an indifference curve of firm 1 type).

Again, the solid parts of the expected reaction functions summarize how the reported price $p_1$ impacts the beliefs of firm 2 and thus its expected price. As $p_1$ moves from $p^*$ to $p_1^o(0)$, the expected best response of firm 2 moves along the $R(p_1,0)$ function. If $p_1$ is further increased that expected best response jumps down to the $R(p_1,1) \equiv p^*$ function that can only be displayed in Figure 2(b) because $p^*$ is outside the displayed range of Figure 2(a). As $p_1$ reaches $p_1^a(x)$ the expected best reply jumps up to $R(p_1,p_1)$ and then, as $p_1$ is further increased, jumps down again to $R(p_1,1) \equiv p^*$. This shows clearly that for firm 1 type $c$ it never pays to deviate from the asserted equilibrium and report a price $p_1 \neq p_1^a(x)$. Optimality for type $nl$ and $nh$ is obviously satisfied and, adapting the reasoning spelled out above, it is also obvious that this equilibrium satisfies the intuitive criterion.

The discontinuity at $\rho = 0$ occurs for the following reasons, illustrated for the case of the interior solution in Figure 2(b): As $\rho_0$ (and thus $p_1$) is reduced, $\hat{p}_1(0)$ increases and $\bar{p}_1(x)$ decreases until these two prices coincide as $\rho_0$ approaches zero. At $\bar{p}_1 := \lim_{\rho_0 \to 0} p_1^a(x)$ the indifference curve of firm 1 type $nh$ has a minimum (the marginal rate of substitution is equal to zero) and its graph intersects the best response line $R(p_1,0)$. Therefore, the Stackelberg leader price, $p_1^a(x)$, that maximizes the payoff of firm 1 type $nh$ on $R(p_1,0)$, exceeds $\bar{p}_1$.

The intuitive criterion is an important plausibility test of an equilibrium. However, as a caveat, the discontinuity of the equilibrium that satisfies the intuitive criterion is disturbing. It does not appear reasonable that the outcome in a game with a one in a million chance of facing a counterspy differs

Figure 2: Equilibrium prices $(\hat{p}_1^a(0), \hat{p}_1^a(x))$ that survive the intuitive criterion
significantly from that in a game without counterspy.\footnote{Similar criticism of the intuitive criterion has been expressed by Mailath, Okuno-Fujiwara, and Postlewaite (1993) when they assess separating vs. pooling equilibria.} Therefore, the equilibrium that survives the intuitive criterion does not appear to be plausible if $\rho_0$ is “small”.

Conversely, the equilibrium summarized in Proposition 6 that fails the intuitive criterion converges to the Stackelberg equilibrium as $\rho_0$ goes to zero.

This suggests that both kinds of equilibria have merit and one should perhaps select the equilibrium that survives the intuitive criterion if the prior probability $\rho_0$ is sufficiently large and instead select the equilibrium that violates the intuitive criterion if $\rho_0$ is close to zero.

We conclude with an assessment of the overall impact of the presence of a counterspy. One may expect that the introduction of the counterspy essentially preserves but weakens the price leadership induced by the spy. This conjecture confirms if one selects the equilibrium that violates the intuitive criterion:

**Proposition 8.** Consider the equilibrium summarized in Proposition 6 which violates the intuitive criterion. If the spy is a counterspy with positive probability less than one, the price leadership induced by the presence of a spy is weakened but does not vanish, i.e.,

\[
p^* < \rho_0 E_X\left[p^n_1(\hat{p}^n_1(x), X_1, \rho_1)\right] + (1 - \rho_0)E_X\left[p^n_1(X_1)\right] < E_X\left[p^n_1(X_1)\right]. \tag{29}
\]

However, if one selects the equilibrium that survives the intuitive criterion (summarized in Proposition 7), the presence of the counterspy may surprisingly enhance the price leadership induced by the spy and yield a higher expected price of firm 1 than the expected Stackelberg leader price.

A case in point is the parameter profile $(s, h, \rho_0, x) = (0.4, 0.04, 0.026, 0.4)$, which yields:

\[
\rho_0 E_X\left[p^n_1(\hat{p}^n_1(x), X_1, \rho_1)\right] + (1 - \rho_0)E_X\left[p^n_1(X_1)\right] = 0.66246 > 0.66193 = E_X\left[p^n_1(X_1)\right] > 0.635 = p^*.
\]

In that case the expected price of firm 1 is higher than the expected Stackelberg leader price because $\hat{p}^n_1(x)$ exceeds $p^n_1(x)$ and $\rho_1$ is “small”, so that the price $\hat{p}^n_1(x)$ that exceeds $p^n_1(x)$ is indeed the true price with high probability.

7 Discussion

The results of the present paper indicate that spying may serve as a quasi-collusive scheme that supports high prices. This suggests that antitrust authorities should keep an eye on spying activities and perhaps probe them as potential antitrust violations.

The significance of this antitrust issue is underscored if one embeds the analysis in a repeated game context. Considering an infinitely repeated Bertrand game, Mouraviev and Rey (2011) show that once price leadership has been achieved, simple trigger strategies supports collusive pricing, essentially for all levels of the discount rate, whereas simultaneous pricing supports collusion only when the discount rate is sufficiently low.

Our analysis assumes that firms compete in a Bertrand market game. If Bertrand is replaced by Cournot competition it is well-known that, under complete information, the first-mover is better off than the second-mover who in turn is worse off than in the corresponding simultaneous moves.
game (see, for example, Gal-Or, 1985, Dowrick, 1986, Amir and Grilo, 1999). In that case it is only the spied at firm that benefits from the presence of a spy. However, in the presence of incomplete information, spying has the benefit of removing uncertainty about the rival’s cost. It remains to be seen whether this information benefit may outweigh the strategic disadvantage.

In our analysis the firm that engages a spy is given exogenously. This is appropriate insofar as firms’ cost is their private information.

17 There is a somewhat related literature on the endogenous timing in oligopoly games in which equilibrium refinements such as risk dominance play a key role (see, for example, Hamilton and Slutsky, 1990, van Damme and Hurkens, 1996, 2004).

A Appendix

Here we show that the conditions stated in Proposition 2 are satisfied for the demand functions $Q_t(p_1, p_2) = 1 - p_1 + sp_j + \theta p_i p_j$ with $\theta \geq 0$ (which includes linear demand as a special case).

Assuming these demand functions we find the following equilibrium prices of the games $G^b$ and $G^b$:

\[
p^b_i(x_i) = \frac{\gamma(x) - \sqrt{\gamma(x)^2 - 4\theta(sx + 2)}}{4\theta} + \frac{x_i}{2} \quad (A.1)
\]

\[
p^b_1(x_1) = \frac{1}{4\theta} \left( \gamma(x_1) - \sqrt{\gamma(x_1)^2 - 8(\theta + s)\frac{\lambda(x_1)}{\lambda(x)}} + 4s(2 - \theta x_1) \right) + \frac{x_1}{2} \quad (A.2)
\]

\[
p^b_2(x_2, x_1) = \frac{1}{4\theta} \left( \gamma(x) - \sqrt{\gamma(x)^2 - 8(\theta + s)\frac{\lambda(x)}{\lambda(x_1)}} + 4s(2 - \theta x) \right) + \frac{x_2}{2} \quad (A.3)
\]

\[
\partial_{x_1} p^b_1(x_1) = \frac{4\theta(\theta + s)^2\lambda(x)}{\left(\lambda(x_1)\lambda(x)(\lambda(x_1)\lambda(x) - 8(\theta + s))\right)^{3/2}} \quad (A.4)
\]

where $\gamma(x) := 2 - s - \theta x$, $\lambda(x) := 2 + s - \theta x$.

Using l’Hôpital’s rule one can confirm that, as $\theta$ goes to zero, one obtains the solutions of $p^b$ and $p^b_1, p^b_2$ that apply in the case of linear demand.

For $\theta > 0$ the above solutions apply only if $\theta$ is bounded from above. In order to assure existence of a solution of $p^b_i(x_i)$ we require that:

\[
\theta < \tilde{\theta} := \frac{1}{\beta^2} \left( 4 + 2\beta + \beta s - \sqrt{8(2 + 2\beta + \beta s + \beta^2 s)} \right) > 0. \quad (A.5)
\]

This parameter restriction also assures that $\partial_{x_1} p^b_1(x_1) > 0$ for all $x_1$. Further upper-bound restrictions apply to assure existence of $p^b_1, p^b_2$. 
Assuming that \( \theta \) is sufficiently small to assure that these upper-bound restrictions are satisfied, we find that \( \Pi^b_2(x_2, \bar{x}) = \Pi^b_2(x_2) \), for all \( x_2 \) which confirms condition (ii) in Proposition 2.

By the envelope property, the total derivative of the interim equilibrium expected payoff function \( \Pi^b_2 \) with respect to \( x_1 \) is:

\[
\frac{d}{dx_1} \Pi^b_2 = \left( p^b_2 - x_2 \right) \partial_{p_1} Q_2 \cdot \partial_{x_1} p^b_1.
\]  

Therefore, \( \Pi^b_2 \) is convex in \( x_1 \) if

\[
0 < \frac{d^2}{(dx_1)^2} \Pi^b_2 = \partial_{x_1} p^b_2 \cdot \partial_{p_1} Q_2 \cdot \partial_{x_1} p^b_1 + \left( p^b_2 - x_2 \right) \left( \partial_{p_1} Q_2 \cdot \partial_{x_1} p^b_1 + \partial_{p_1} Q_2 \cdot \partial_{x_1} p^b_1 \right) \partial_{x_1} p^b_1 + \left( p^b_2 - x_2 \right) \partial_{p_1} Q_2 \cdot \partial_{x_1} p^b_1.
\]  

Given the properties of the assumed demand functions and the solutions of \( p^b_1, p^b_2 \), all terms in the first and second lines of (A.7) are positive, and the third line is positive if \( \partial_{x_1} p^b_1 \geq 0 \), which holds true because the parameter restriction (A.5) implies \( \partial_{x_1} p^b_1 \geq 0 \). Therefore, the interim equilibrium expected payoff functions is convex in \( x_1 \) for all \( x_2 \), as asserted.

### B Appendix

Here we prove results in Section 6. The application of the intuitive criterion and the proofs of the (dis)continuity of equilibria are in the main text.

**Proof of Lemma 1** Solving the system of first order conditions of (15)-(16) yields the following solutions that are linear in \( p_1 \) and in firms’ own cost (second order conditions are satisfied):

\[
p^*_1(p_1,x_1,\rho,\mu) = \frac{1}{8 - 2s^2 \rho} \left( y_0 + y_1 p_1 \right) + \frac{x_1}{2}, \quad x_1 \in \{0,x\}.
\]  

\[
P_2(p_1,x_2,\rho,\mu) = \frac{1}{8 - 2s^2 \rho} \left( \delta_0 + \delta_1 p_1 \right) + \frac{x_2}{2}, \quad x_2 \in \{0,x\},
\]  

with \( y_0 := 4 + s(2 + x(2h + s\rho \mu)) \), \( y_1 := 2s^2(1 - \rho) \), and \( \delta_0 := 4 + s\rho(2 + x(2\mu + hs)) \), \( \delta_1 := 4s(1 - \rho) \). Both \( p^*_1 \) and \( p_2 \) are increasing in \( p_1 \) and in the own unit cost as well as in \( \mu \). Of course, \( p^*_1(p_1,x_1,1) = p^b_1(x_1) \), independent of \( p_1 \), i.e., the expected reaction function of firm 2 is flat if \( \rho = 1 \).

**Proof of Lemma 2** Using (B.2) one obtains

\[
R(p_1,\rho) = \frac{(1 + \bar{x})(2 + s\rho)}{4 - s^2 \rho} + \frac{2s(1 - \rho)}{4 - s^2 \rho} p_1.
\]  

Evidently, \( \partial_{p_1} R(p_1,\rho) > 0 \) and \( \partial_{\rho} \partial_{p_1} R(p_1,\rho) = -\frac{2(4 - x^2)}{(4 - s^2 \rho)^2} < 0 \), for all \( \rho < 1 \), \( R(p_1,1) = \rho^* \), and \( R(p^*,\rho) = \rho^* \) for all \( \rho \).
Proof of Proposition 5  The proof builds up in several steps (Lemmas B.1 to B.4):

Lemma B.1. Suppose \( p' \notin \{ p_1^*(0), p_1^*(x) \} \). Then, after observing \( p' \) firms play the unique equilibrium strategies of the benchmark Bertrand game, \( (p^B(x_1), p^B(x_2)) \).

Proof. If firm 2 observes \( p' \notin \{ p_1^*(0), p_1^*(x) \} \) it infers that firm 1 has a counterspy with probability one. Therefore firm 2 ignores the spy’s report and, because this fact is common knowledge, firms play \( (p^B(x_1), p^B(x_2)) \).

Lemma B.2. Suppose \( p' \notin \{ p_1^*(0), p_1^*(x) \} \). Then, \( p_1^*(x) = p^* \).

Proof. In two steps: 1) we show that \( p' \notin \{ p_1^*(0), p_1^*(x) \} \Rightarrow p_1^*(x) \leq p^* \); 2) we show that \( p_1^*(x) \) cannot be less than \( p^* \).

1) Suppose, per absurdum that \( p_1^*(x) > p^* \). Then, \( p_2(p_1^*(x), x_2, 0) > p^B(x_2) \). Therefore, firm 1 with counterspy is better off setting \( p_1^*(x) \). Therefore, one must have \( p_1^*(x) \leq p^* \).

2) Because \( \pi_1(p_1, R(p_1, 0), x_1) \) is strictly increasing in \( p_1 \) for all \( p_1 \) below the Stackelberg leader price and \( p^* \) is below that price, it follows that firm 1 type \( n \) is better off by raising its price to \( p^* \). Therefore, the inequality cannot apply.

Lemma B.3. The game does not admit an equilibrium with \( p' \notin \{ p_1^*(0), p_1^*(x) \} \).

Proof. Suppose there is such an equilibrium. Then, by Lemma B.2, \( p_1^*(x) = p^* \). Firm 1 type \( n \) can raise its profit by raising its price to \( p^B(x) > p^* \) because, by Lemma 2,

\[
\pi_1(p^B(x), R_2(p^B(x), \rho), x) \geq \pi_1(p^B(x), R_2(p^*, \rho), x) = \pi_1(p^B(x), p^*, x) > \pi_1(p^*, p^*, x).
\]

Lemma B.4. \( p_1^*(0) \neq p_1^*(x) \Rightarrow p_1^*(x) > p_1^*(0) \).

Proof. Let \( \tilde{Q} := Q(p_1^*(x), R(p_1^*(x), \rho)) \) and \( Q := Q(p_1^*(0), R(p_1^*(0), \rho')) \), where \( \rho, \rho' \) denote the updated beliefs after observing \( p_1^*(x) \), resp. \( p_1^*(0) \). Suppose, per absurdum, that \( p_1^*(x) < p_1^*(0) \).

Then, by definition of an equilibrium, one must have:

\[
(p_1^*(x) - x) \tilde{Q} \geq (p_1^*(0) - x) Q \tag{B.4}
\]

\[
\Rightarrow \tilde{Q} > Q. \tag{B.5}
\]

By the same reasoning and using (B.4):

\[
(p_1^*(0) - 0) \tilde{Q} \geq (p_1^*(x) - 0) \tilde{Q} \tag{B.6}
\]

\[
\Rightarrow (p_1^*(0) - x) \tilde{Q} + (x - 0) Q \geq (p_1^*(x) - x) \tilde{Q} + (x - 0) \tilde{Q} \tag{B.7}
\]

\[
\Rightarrow \tilde{Q} \geq \tilde{Q}. \tag{B.8}
\]

This is a contradiction.

As a final step of the proof of Proposition 5, note that in the assumed partially separating equilibrium one must have \( p_1^*(0) \neq p_1^*(x) \). By Lemma B.3 \( p' \in \{ p_1^*(0), p_1^*(x) \} \) and therefore by Lemma B.4 \( p_1^*(x) > p_1^*(0) \). Suppose \( p' = p_1^*(0) \), then firm 1 type \( c \) can increase its profit by raising \( p' \) to \( p_1^*(x) \), because \( P_2(p_1^*(x), x_2, 0) > P_2(p_1^*(0), x_2, \rho) \), and we conclude that \( p' = p_1^*(x) \).
Proof of Proposition 6  Here we supplement the proof in the main text and explain 1) why we need to impose the constraint \( R(p_r^H(0),0) \leq R(p_r^H(x),\rho_1) \) in step 2 of the two-stage procedure and 2) why \( p_r^H(x_1) \) continuously approaches the Stackelberg leader price \( p_r^H(x_1) \) as \( \rho_0 \) goes to zero.

1) In Lemma B.3 and Lemma B.4 we show that \( p^r = p_r^H(0) \) cannot be part of an equilibrium, because firm 1 type \( c \) would then benefit from switching to \( p_r^H(x) \). However, for an equilibrium one must also assure that firm 1 type \( c \) cannot benefit by switching from \( p^r = p_r^H(x) \) to \( p^r = p_r^H(0) \). This requires that by thus switching firm 1 cannot trigger a higher expected response of firm 2, which requires that \( R(p_r^H(0),0) \leq R(p_r^H(x),\rho_1) \).

2) The solution of \( p_r^H(x) \) is:

\[
p_r^H(x) = \frac{4(1+x) + s(2 - sx(2 - \rho_1) + \bar{x}(2 + s\rho_1))}{8 - 2sx(2 - \rho_1)}. \tag{B.9}
\]

\( p_r^H(x) \) is decreasing in \( \rho_1 \) and \( p_r^H(x) \) converges to \( p_r^H(0) \) as \( \rho_1 \) goes to zero. Moreover, \( p_r^H(0) \) is equal to \( p_r^H(0) \) if \( p_r^H(0) \) is an interior solution and otherwise converges to \( p_r^H(0) \) as \( \rho_1 \) goes to zero. Therefore, that equilibrium continuously approaches the Stackelberg equilibrium as \( \rho_1 \) goes to zero.

Supplement to the alternative two-step procedure  The unique solution of the prices, \( \rho_1(0), \rho_1(x) \), that satisfy the conditions (25)-(26) is:

\[
\rho_1(0) = \frac{4 + s(1 + \bar{x})(2 - \rho_1) - 2\rho_1 - sx(2 - (2 - h)\rho_1)}{(4 - s^2)(2 - \rho_1)} \tag{B.10}
\]

\[
\rho_1(x) = \rho_1(0) + \frac{\rho_1(x - \bar{x})}{2 - \rho_1} \tag{B.11}
\]

These prices are used in the construction of the equilibrium that satisfies the intuitive criterion.

Evidently, \( \rho_1(0) \) is decreasing and \( \rho_1(x) \) is increasing in \( \rho_1 \) and, as \( \rho_0 \) (and thus \( \rho_1 \)) approaches zero, these prices coincide at the level \( \rho_1 = \frac{(1 + sx(1 + h))/(4 - s^2)}{2 - \rho_1} \), illustrated in Figure 2(b). Obviously, at \( \rho_1 = \bar{\rho}_1 \) the marginal rate of substitution (slope of the indifference curve) of firm 1 type \( nh \) is equal to zero.

Proof of Proposition 8  The proof is in three steps:

1) Because \( p^r = p_r^H(x) \) we find, using (11), (B.1), and (B.9): \( p_r^H(p_r^H(x),\rho_1) < p_r^H(x_1) \).

2) \( p_r^H(x) \) is decreasing in \( \rho_1 \) and approaches \( p_r^H(x) \) from below as \( \rho_1 \) goes to zero. Similarly, \( p_r^H(0) \) approaches \( p_r^H(0) \) as \( \rho_1 \) goes to zero (in the case of the boundary solution \( p_r^H(0) \) is already equal to \( p_r^H(0) \) and does not change as \( \rho_1 \) is reduced to zero).

Combining 1) and 2) implies \( \rho_0E_{X_1}[p_r^H(p_r^H(x),X_1,\rho_1)] + (1 - \rho_0)E_{X_1}[p_r^H(X_1)] < E_{X_1}[p_r^H(X_1)] \).

3) Similarly, one can show that \( p_r^H(p_r^H(x),x_1,\rho_1) > p_r^H(x) \).

Combining 1) to 3) proves (29).
References


