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Assessing Income Tax Perturbations

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Assessing Income Tax Perturbations

Abstract

Taking a piecemeal tax reform approach to tax analysis in the spirit of Feldstein (1976), we establish a framework for assessing perturbations of the income tax schedule. It decomposes a reform into a change in tax level and a structural reform part. Focussing on the latter, the analysis singles out four effects: A social efficiency effect measured as the change in tax revenue due to behavioural changes, distributional impact due to mechanical effects, total distributional effects, and overall welfare effects conditional on inequality aversion. When applying our approach to changes in the piecewise linear income tax in Norway during 2016-2018, we identify the cut-off value for the inequality aversion for which the reform is just welfare preserving. For lower inequality aversion the decision makers have accepted a reform which enhances social efficiency at the expense of redistribution in favour of the better-off households.


Keywords: income tax, tax reform, tax perturbation.

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1 Introduction

There are two major strands of research in the normative tax analysis of public economics. One approach is to characterise the optimal taxes starting with a clean sheet. This is known as the tax design problem, and is normally what we have in mind when we think about optimum taxation. The other is the tax reform approach, highlighted in particular by Feldstein (1976), who argued that optimal tax reform must take as its starting point the existing tax system. According to Feldstein (op.cit., p.90), "in practice, tax reform is piecemeal and dynamic in contrast to the once-and-for-all character of tax design." In the wake of Feldstein’s emphasis on tax reform analysis, a series of papers addressed in a theoretical framework the effects of small commodity tax reforms, often called tax perturbations (e.g. Dievert, 1978; Dixit, 1975; Guesnerie, 1977). It seems that piecemeal income tax reforms have not received the same attention. Nor has there been much interest in empirical analysis of actual slow and piecemeal tax reforms in the spirit of Feldstein. The purpose of this paper is to present a scheme for assessing income tax perturbations in a way that easily lends itself to empirical application.

The approaches taken in the tax reform literature and those pursued in the tax design literature are not entirely different in nature. Both strands of normative tax analysis typically consider small tax manipulations and capture allocative efficiency effects, distributional effects, and distributional preferences. Obviously, these concepts are central also in the current paper. In the tax design problem, optimal taxes are characterised by (first order) conditions requiring small departures from the optimum to yield no welfare effect. This means that the point of departure is an endogenous optimum, which is typically not solved for. Tax reform analysis takes as its point of departure the actual allocation, which in that sense is exogenous. The empirical question is then what are the welfare effects of a particular reform from this allocation.

Beyond contributing in general to the literature on "piecemeal" tax reforms modelled as tax perturbations, our contribution lies in the way we approach the reform analysis. A key concern is to find a simple approach that lends itself to empirical applications. In particular, we are interested in characterising structural tax reforms (to be explained below), distinguished from changes in tax level. For this purpose we find it convenient to measure the tax level by the burden imposed on the tax payers. We then cleanse out the pure level effect of a tax reform by means of a hypothetical lump sum tax or transfer to leave us with the changes in marginal tax rates at the various income levels. We define the latter changes as the structural reform. If the prevailing political preferences in society had been expressed by a social welfare function, we could assess a tax reform by applying this welfare function to the pre- and post-reform allocations in the economy. In practice, no explicit social welfare function of this kind is available, and we have to take a different route.

\footnote{There are, however, a few important contributions to optimal income tax design using numerical simulations to derive more or less empirically-based tax schedules; in particular, Saez (2001) and a sequence of contributions by Tuomala summarised in Tuomala (2016).}
We shall adopt a step-wise approach to deal with the various effects of the tax reform sequentially. First, we shall single out the pure efficiency effects measured by the behavioural-induced changes in tax revenue. The rationale for this approach is that, departing from a tax-distorted initial allocation, efficiency effects are determined by the pre-existing tax wedges and the behavioural responses to the tax reform. Secondly, we shall trace the distributional effects, and, by applying a particular class of welfare weights, we shall explore how the inequality aversion determines the total distributional effect. Our next step is then to consider how the overall welfare effect depends on the inequality aversion, which allows us to infer the range of distributional preferences that are implicit in political support for or rejection of the reform. By exploring implicit preferences assumed to be revealed by tax reforms, we add to the studies of implicit preferences previously based on the assumption that the actual policy is optimal, known as the inverse optimum problem. We then apply our procedure to changes in the income tax schedule in Norway implemented during the period 2016-2018, enabling us to achieve results with substantial empirical content.

A recent paper with resemblance to our paper is Golosov, Tsyvinski and Werquin (2014). The paper models a non-linear income tax and capital taxes in a life-cycle model with age-dependent behaviour. It addresses departures from the existing tax system such as introducing non-linear capital taxes and introducing joint taxation of various forms of income. Our paper takes a different approach. It considers a much simpler tax structure and focuses on tax perturbations within the existing tax system.

We describe our theoretical approach in Section 2. Section 3 presents the Norwegian tax perturbations used in the empirical illustration of our framework in Section 4. Section 5 concludes.

2 A scheme for assessing tax perturbations

2.1 Mechanical, efficiency and welfare effects

Consider a population of agents who choose labour supply for given wage rates and tax parameters. Denote the wage rate by \( w \) and labour supply by \( h \). The tax function for labour earnings is given by \( T(y, \theta) \), where \( y \) is income and \( \theta \) is a vector of tax parameters \( (\theta = \theta_1, \theta_2, ..., \theta_j,...) \), which may include tax rates and bracket limits of a piece-wise linear tax system. Let the initial tax function be defined by the parameter vector \( \theta^1 \). We may simplify the notation by writing \( T_1(y) = T(y, \theta^1) \). A tax perturbation is then defined by a vector of increments, as \( d\theta = d\theta_1, d\theta_2, ..., d\theta_j, ... \), generating a new tax function \( T_2(y) = T(y, \theta^2) = T(y, \theta^1 + d\theta^1) \). Assume there is a distribution of agents with density function \( f(w) \). The tax reform will have mechanical effects, behavioural effects and welfare effects in the taxonomy of Saez (2001) and Brewer, Saez and Shephard (2010). A mechanical effect is the effect on the tax liability for unchanged behaviour, i.e. fixed labour supply and consequently fixed income.
For some initial income $y$, the mechanical effect is $M(y) = T_2(y) - T_1(y)$. The behavioural effect on tax payment is the change due to behavioural changes, which in this case are labour supply responses. In formal terms, the behavioural effect is then $B(w) = T_0'(y(w, \theta^1)) \, wd\!h$ where $dh$ is the change in labour supply induced by the tax reform. The welfare effect is the sum of welfare-weighted real income effects of the tax reform. We note that real income losses are equal to the mechanical effects when, due to the envelope properties, there are no first order effects of behavioural changes.

Consider an agent with wage rate $w$ reflecting his marginal product of labour. His marginal disutility of labour is $s$ in monetary terms. Where the induced change in labour supply is $dh$, there is a social efficiency gain $\omega dh = w'T'dh = T'dy$, where $dy$ is the change in gross income. As is well known, changes in behaviour consist of substitution effects and income effects. We can write the labour supply function as $h((1 - T')w; I) = h(\omega; I)$ where $\omega$ is the marginal after-tax wage rate and $I$ is a measure of real income. Then the change in labour supply induced by a tax perturbation is $dh = -h_{\omega}wdT' + h_I dI$, where the former term is a substitution effect and the latter is an income effect.

In a piece-wise linear income tax regime we should allow for the fact that the function has kinks and is not differentiable everywhere. Consider a kink where the discontinuous tax rate jumps from $t_1$ to $t_2$. Denote by $S$ the marginal disutility in monetary terms incurred by increasing gross income by one unit, implying that $dh = 1/w$. For agents located optimally at the kink, $(1 - t_2) \leq S \leq (1 - t_1)$, otherwise moving to one of the segments on either side of the kink would be beneficial. Opting for an extra unit of income, the agent would incur a cost $S$, and the net gain would be $1 - t_2 - S \leq 0$. Choosing to earn one unit less, the agent would forego $1 - t_1$ after tax, but would escape a cost $S$, and the net gain would be $S - (1 - t_1) \leq 0$. The social efficiency effect of a one unit increase in gross income is then $1 - S$, and $t_1 \leq 1 - S \leq t_2$. The social gain from an induced income change lies between $t_1 dy$ and $t_2 dy$.

The tax change that we observe for an actual tax perturbation depends on the nature of the tax reform. Suppose the bracket with tax rate $t_1$ is extended, inducing agents at the kink to increase their earnings. Then we observe a tax increase $t_1 dy$. We note that $t_1 dy < (1 - S) dy$ for $t_1 < 1 - S \leq t_2$, implying that for these agents (with low marginal cost of working) observed increase in tax payments underestimates the social gain. The social efficiency effect is not fully captured by the behavioural effect on tax revenue, unlike what we found in the differentiable case. We observe that for some agents the discrepancy is negligible, while the maximum one is $(t_2 - t_1) dy$. Compared to a change $t_1 dy$

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2We should note that "welfare effects" in the terminology taken from the references above refer to "distributional effects" rather than overall welfare effects.
or $t_2 dy, (t_2 - t_1) dy$ is small and closer to a second order effect since tax rate discontinuities are not huge. Where an induced income change is due to a mechanical income effect (a larger or smaller tax burden), with no change in the kink, an income rise will generate an increase in tax revenue $t_2 dy$, which overestimates the social gain. A lowering of income will reduce tax payments by $t_1 dy$ where $dy$ now denotes the decline in income, which underestimates the social loss.

We should, however, note that for almost all taxpayers we assume that the Envelope Theorem applies and it is sufficient to consider changes in tax revenue. Kinks are rather exceptions. How important this caveat is depends on the number of kinks, how sharp the kink is (the size of $t_2 - t_1$) and the number of taxpayers at the kink. Since an important question has been to what extent there is bunching at kink points, we should note that in our context bunching is not a separate concern, but relevant only to the extent that it affects the number of persons, which may be a concern even in the absence of bunching since the special effects discussed above pertain to all persons at the kink. We shall return to this issue in section 2.3 and also in the empirical illustration in Section 4.2.

Also paying attention to the extensive margin of labour supply, we may assume that there is a cost of working, $k$, and a distribution of $k$ across the population is characterised by the density $g(k)$. Assume an agent pays the tax $T_0$ when not working and obtains an income net of tax, $y - T$, if working. The net private gain from working is then $y - k - (T - T_0) = 0$, while the net social gain is $y - k = T - T_0$, which is the change in tax revenue.

A further effect arises due to indirect taxes. A commodity tax drives a wedge between the marginal valuation of a commodity and the cost of producing it. Increased demand will then yield a social efficiency gain due to the preexisting distortion. Analogous to what we found in the case of income taxation, a rise (fall) in indirect tax revenue induced by behavioural changes reflects a social efficiency gain (loss). The effective tax is made up of both the income tax and indirect taxes, as analysed in Edwards, Keen and Tuomala (1994), and we need to allow for changes in both sources of revenue. This will be done in the empirical part, but for ease of exposition we shall confine attention to income taxes in the theoretical discussion.

In order to take indirect taxes into account, one has to decide how to treat savings since in a particular period the indirect tax base will be smaller the larger is the savings rate. However, a single-period perspective would be too narrow since postponed consumption will be taxed in later periods. We therefore model consumption as if there are no savings.

A mechanical effect by itself has no social efficiency effect. Social inefficiencies are due to misallocation of resources and only mitigated or exacerbated when there is a reallocation, which by assumption does not take place when there is a pure mechanical effect. However, a tax reform may affect the tax revenue due to income responses to mechanical effects. When an agent incurs a loss of income, he will typically increase his labour supply due to standard income effects.
Two issues are crucial when making decisions about tax policy. One is the choice of tax level. The other is the design of tax structure. The former is the question of how far the government should go in appropriating private resources. A pure increase in the tax level could take place by raising a uniform lump-sum tax. This would have a mechanical effect, and resources would become available for the public sector in two ways – both because private agents consume less and that they typically supply more labour (assuming leisure is a normal good).

The tax structure is defined by how the marginal tax rate varies across income, typically determined by number of tax brackets, bracket limits and marginal tax rate in each bracket. The choice of tax structure will be governed by distributional and social efficiency concerns. In this paper, we focus exclusively on the tax structure. We are not concerned with the overall resource allocation between the public and the private sector of the economy.

A tax reform will normally affect both the tax level and the tax structure. In accordance with our focus, we shall single out structural changes for further scrutiny. We do this in the following way. We introduce a lump sum element in the tax function allowing us to cleanse out the level effect. The new tax function can be written \( T_3(y) = T_2(y) - \alpha \), where we can interpret \( \alpha \) as a pure level parameter in the tax function changing neither marginal taxes nor bracket limits. This is a hypothetical tax schedule in the sense that it is not observed in practice. We shall interpret the aggregate mechanical effect of a tax reform as the level effect. We therefore compute the change in \( \alpha \) (initially set equal to zero) that offsets the average mechanical effect of the tax reform in question.

Since mechanical effects reflect the income losses of the taxpayers, assuming no aggregate mechanical effect (after adjusting \( \alpha \)) implies that we are left with redistributive and efficiency effects. These effects are not independent. When there is a transfer from agent \( i \) to agent \( j \), there will be income effects on behaviour that in turn will change the agents' tax payments and tax revenue for the government. To what extent there is a positive net effect on tax revenue depends on the agents' marginal propensities to pay tax, where an agent's marginal propensity to pay taxes is given by \( T'wdh/dI \). As there are pre-existing distortions of labour supply a behavioural-induced rise (fall) in tax revenue is beneficial (harmful). We can interpret this as a social efficiency effect of redistribution.\(^3\) In addition, the tax reform will obviously generate substitution effects. Our aggregate measure of the social efficiency impact will be the sum of these efficiency effects. It may also be of interest to observe which households and income groups that contribute (positively or negatively) to the efficiency effect.

Now taking a formal approach, write the indirect utility function \( V(w^i, \theta, \alpha) \), where \( i \) indicates agent. Simplifying the notation, we can write \( V_i(\theta, I + \alpha) \equiv V(w^i, \theta, I + \alpha) \). Taking \( \theta_1^i \) as our point of departure, we consider the tax reform \( d\theta = d\theta_1, d\theta_2, ..., d\theta_j, ... \), and \( d\alpha \) to cleanse out the level effect, as dis-

\(^3\)It is common in tax analysis to make use of Diamond's (1975) marginal social valuation of income for an agent, which is the sum of the direct effect on the agent and the marginal propensity to pay taxes affecting government revenue. The two effects are rarely distinguished. In our presentation we separate the two effects.
cussed above. Denote by $\gamma^i$ agent $i$’s marginal utility of income, and let $g^i = \sum_j d\theta_j \frac{\partial V^i}{\partial \theta_j}/\gamma^i + d\alpha$ be the gain in terms of income obtained by agent $i$ due to the structural tax reform $d\theta_j$, $d\alpha$. As discussed above, the private income gain (loss) for an agent is in most cases equal to a mechanical revenue loss (gain) for the government since both are defined absent behavioural changes. However, as discussed above, this does not apply to agents located at kinks, who experience a non-zero first order effect of a change in labour supply.

Now write total welfare as the welfare derived from private income plus the value of government revenue in terms of welfare:

$$\sum_i V^i (\theta, I + \alpha) + \mu R (w^1, ..., w^n, \theta, \alpha),$$

where $\mu$ is the shadow value of government revenue. The welfare effect of the structural tax reform under consideration can then be expressed as

$$dW = \sum_i \gamma^i g^i + \mu \sum_j \frac{\partial R}{\partial \theta_j} d\theta_j + \mu \frac{\partial R}{\partial \alpha} d\alpha.$$  \hspace{1cm} (1)

We can now distinguish the various effects of the structural tax reform. The social efficiency effect, measured in terms of government revenue, is given by the behavioural terms, $\sum_j \frac{\partial R}{\partial \theta_j} d\theta_j + \frac{\partial R}{\partial \alpha} d\alpha$. The first step is to estimate this effect. By our definition of constant tax level, implemented by $d\alpha$, it follows that $\sum_i g^i = 0$ if we neglect that the kinks may give rise to welfare effects. However, each element in the sum may be positive or negative, and there will be winners and losers. In the following we assume that the effects of kinks are negligible for the reasons discussed above.

Our next objective is to describe the distributional effects, or welfare effects in the terminology of Saez (2001) and Brewer, Saez and Shephard (2010). Since welfare effects in this terminology is associated with welfare-weighted redistribution and does not include social efficiency effects, we shall subsequently use the term welfare effect to capture both redistribution and efficiency. In order to assess the distributional effects, we let the welfare weight be a function of disposable income, denoted by $z$. We choose the functional form

$$\gamma^i = \kappa (z^i)^{-\beta},$$

where $\kappa > 0$. The welfare weight is decreasing in $z$ given that $\beta > 0$. This is a widely used function for generating welfare weights (see e.g. Ahmad and Stern, 1984, Evans, 2005 and Layard, Mayraz and Nickell, 2008). We have that $-\beta$ is the elasticity of the welfare weight with respect to disposable income: $el_{z^i} \gamma^i = -\beta$, and $\gamma^i = \left(\frac{z^i}{\kappa}\right)^{-\beta}$. It follows that $\beta$ is a measure of inequality

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4Where the tax base is the result of an optimal choice by the agent we can neglect any change in the tax base due to the envelope property.

5We assume from the outset that the cardinalisation (in particular the concavity) of the indirect utility function is chosen such that it reflects the inequality aversion of the government.
When assigning welfare weights to different households one may want to allow for differences in household size. The standard method to compare different households is to deflate the income of larger households by using an income equivalence scale. This implies dividing the household disposable income by an equivalence scale, $e$, for example as obtained by the following parametric characterisation (Banks and Johnson, 1994; Jenkins and Cowell, 1994): 

$$e = (n_q + \lambda n_c)^\eta$$

where $n_q$ is the number of adults, $n_c$ is the number of children in the household, $\lambda$ is the weight attached to children and $\eta$ represents the economies of scale. We shall as our main alternative simply use $e = \sqrt{n}$, where $n$ is the (total) number of household members, but will also report results for other scales (belonging to this parametric characterisation). Where an equivalence scale is used the $z$-variable will be disposable income adjusted for household size. The redistributional effect of the tax reform is welfare enhancing (diminishing) if $\sum_i \gamma_i g_i > 0$ ($< 0$), characterised as a distributional gain or loss. Deploying our weight function, we have that $\sum_i \gamma_i g_i = \sum_i \kappa (z_i)^{-\beta} g_i$. We note that the sign is independent of the value of $\kappa > 0$.

Having identified both distributional and efficiency effects, a final question is whether the overall welfare effect is beneficial or harmful. We then need to assign a value to the social efficiency gain (or loss) in terms of a behavioural-induced rise (decline) in tax revenue. Additional tax revenue could be used for many purposes (schools, health care, etc.) and in general it is an open question how they should be valued. We shall now assume that government revenue could be recycled to the taxpayers through a lump sum transfer. To pursue this approach, suppose that an amount $r$ of government funds is available for transfers to the taxpayers and denote by $L$ a uniform lump-sum transfer. Since a lump-sum transfer will affect tax revenue through income effects we can write the behavioural effect of $L$ on aggregate tax revenue as $\varphi(L)$. Then $L$ must satisfy: 

$$L = \frac{1}{N} r + \frac{1}{N} \varphi(L)$$

This means that $\frac{dL}{dr} = \frac{1}{N} + \frac{1}{N} \varphi \frac{d\varphi}{dr}$, and $\frac{dL}{dr} = \frac{1}{1 + \varphi} \frac{1}{N}$. When a lump sum transfer discourages labour supply, we have $\varphi' < 0$ and $n \frac{dL}{dr} < 1$. Since an initial positive lump sum transfer diminishes labour supply with a negative impact on tax revenue, the ultimate transfer that can be financed is less than the initial one. Thus, there is a revenue "leakage".

When one unit of income is equally distributed among the taxpayers as lump-sum transfers each taxpayer receives $1/N$ units. Denote by $m^i$ the additional tax that agent $i$ will pay when receiving a one unit transfer. We call this agent $i$'s marginal propensity to pay tax. The induced additional tax payments then amount to $\sum_i m^i \frac{1}{N} = \bar{m}$, and $\varphi' = \bar{m}$, which is the average marginal propensity to pay tax. Substituting for $\varphi'$, $dL = \frac{1}{1 - \frac{1}{N} \bar{m}} \frac{1}{N} dr$. We note that when a transfer to an agent has a negative impact on labour supply and shrinks the income tax base, the marginal propensity to pay tax is negative. Now letting the efficiency gain of the perturbation in our model accrue to the taxpayers as a uniform lump sum transfer, we set $dr = dR_b$, and $dL = \frac{1}{1 - \frac{1}{N} \bar{m}} \frac{1}{N} dR_b$. The overall welfare effect

\[^6\text{Not to be confused with the inequality aversion parameter of the Atkinson index (Atkinson, 1970).}\]
is then
\[ d\Omega = \sum_i \gamma^i g^i + n\overline{\gamma} dL = \sum_i \gamma^i g^i + \overline{\gamma} - \frac{1}{1 - \frac{1}{N}} dR_b. \] (3)

This enables us to find the cut-off value of \( \beta \), denoted \( \beta^* \), for which the perturbation is just welfare preserving, \( d\Omega = 0 \). To establish a link to the shadow value of government revenue, \( \mu \), introduced above, we see that \( \mu = \overline{\gamma} - \frac{1}{1 - \frac{1}{N}} \). It is determined both by the mean value of the welfare weights and the revenue leakage.

If we want to quantify the distributional gain (loss) or welfare effect of a perturbation for some value of \( \beta \), it is convenient to normalise the welfare measure by setting the average welfare weight equal to unity, \( \frac{1}{N} \sum_i \gamma^i = \overline{\gamma} = 1 \). We have \( \frac{1}{N} \sum_i \gamma^i = \frac{1}{N} \kappa \sum_i (z^i)^{-\beta} = 1 \), implying that \( \kappa = \frac{1}{N} \sum_i (z^i)^{-\beta} \). Then
\[ \gamma^i = \frac{1}{N} \sum_i (z^i)^{-\beta} (z^i)^{-\beta}. \] (4)

A marginal unit of income accruing to agent \( i \) is then valued as equal to \( \gamma^i \) units of equally distributed income.

2.2 A simple illustration

To provide a simple illustration of our approach, assume there is a linear income tax. This is actually the simplest possible special case of a stepwise linear income tax to be considered below. Assume there is a continuum of agents with different wage rates and normalise the population to unity. The wage rate distribution is described by the distribution function \( F(w) \) with density \( f(w) = F'(w) \). The initial linear income tax is written as
\[ T_1 = t_1 y + b. \]

Consider a "small" tax reform (perturbation) which simply increases the marginal tax rate from \( t_1 \) to \( t_2 \):
\[ T_2 = t_2 y + b. \]

We then define
\[ T_3 = t_2 y + b + a, \]
and the initial tax revenue is
\[ R = t_1 \int_0^\infty wh ((1 - t_1) w, I) f(w)dw + b + a. \]

Consider an increment \( dt_1 \). The aggregate mechanical effect is
\[ dR = dR^m = dt_1 \overline{\gamma}, \]
where $\bar{y}$ is average taxable income. The mechanical effect of increasing $a$ is

$$dR^m_a = da.$$  

There will be an offsetting effect when

$$dR^m_1 + dR^m_\infty = 0,$$

and it follows that

$$da = -dt_1 \bar{y},$$

and the transition from $T_1$ to $T_3$ is then the change in tax structure, which in this case is simply a strengthening of the tax progressivity. The behavioural effect is

$$dR^b = \int_{0}^{\infty} \left(-t_1 \omega dt_1 - t_1 \omega (\bar{y} - \bar{y}) dt_1\right) f(w) dw$$

where $\omega = (1-t) w$.

We can interpret $m = tw \bar{I}$ as an agent’s marginal propensity to pay tax, i.e. the additional tax revenue generated by giving an extra unit of income to the agent. As discussed above, we can interpret a behaviour-induced change in tax revenue as a social efficiency effect,

$$dR^b = \int_{0}^{\infty} \left(-t_1 \omega dt_1\right) + \int_{0}^{\infty} \left(-m(w) (\bar{y} - \bar{y}) dt_1\right) f(w) dw,$$

and

$$dR^b = \int_{0}^{\infty} \left(-m(w) \omega dt_1\right) - \text{cov}(m(w), y(w)) dt_1.$$  

The total efficiency effect is made up of the aggregate substitution effect $\int_{0}^{\infty} \left(-t_1 \omega dt_1\right)$ and the efficiency effect of redistribution $-\text{cov}(m(w), y(w)) dt_1$. For $dt_1 > 0$ the effect is positive if $\text{cov}(m(w), y(w)) < 0$. Then people with higher income have lower marginal propensity to pay tax. They incur a loss when the marginal tax rate is increased and there is no aggregate (or average) loss or gain since the aggregate mechanical effect is zero, while people with lower incomes gain. With normal responses high income taxpayers supply more labour, increase their income and face a larger tax liability whereas low-income people show opposite responses. A negative covariance implies that the losers increase their tax payments more than the winners diminish their tax payments: the efficiency gains outweigh the efficiency losses.
The welfare effect of increasing \( t_1 \) is

\[
d\Omega = \int_0^\infty \gamma(z(w)) (\overline{y} - y(w)) \, dt_1 f(w) \, dw + \mu \, dR^b,
\]

which is zero where the tax perturbation has no welfare effect.

### 2.3 The piece-wise linear income tax

A piece-wise linear income tax is defined by three properties: number of tax brackets (steps), the bracket limits and the marginal tax rate in each bracket. Denote by \( Y_j \) the upper limit of bracket \( j \). Assume there are \( J \) brackets and let \( Y_J = \infty \). Denote by \( t_j \) the marginal tax rate in bracket \( j \). It is helpful to start out by considering the \( J \) brackets as potential brackets in the sense that in the pre-reform situation it may be that \( t_{j-1} = t_j \) for some values of \( j \), while part of the reform is to differentiate \( t_{j-1} \) and \( t_j \) so that in the post-reform situation we have two proper tax brackets. In practice tax systems exhibit marginal tax progressivity in the sense that \( t_j \geq t_{j-1} \) and \( t_j < t_{j-1} \) for at least some values of \( j \).

For a given number of (potential) tax brackets, a tax reform can change the properties of a bracket in two ways. It can change the bracket limits, and it can change the marginal tax rate \( t_j \) in bracket \( j \). Suppose there is an increment \( dt_j \). This will have three effects. It increases the marginal and average tax rate on incomes in bracket \( j \), and taxpayers in the brackets above will face a lump-sum tax increase \( (Y_j - Y_{j-1}) \, dt_j \). The rise in the marginal tax rate in bracket \( j \) will discourage labour supply through the substitution effect while the increase in the average tax rate and the lump sum tax in the brackets beyond \( Y_j \) will, under standard assumptions, stimulate labour supply through the income effect. However, we may note that there are agents whose behaviour is unaffected because they are located at kink points, i.e. at bracket limits where the tax rate is discontinuous. Then imagine an increase \( dY_j \) in the bracket limit \( Y_j \). An implication is that those who are initially located at the kink can enhance their utility by increasing labour supply. We should note that there are people at the kink whose marginal after-tax wage rate in bracket \( j \) exceeds the marginal valuation of leisure. Extending bracket \( j \) then implies that, unlike in the tangency case, an above-zero first order effect on utility is achieved by increasing labour supply. This effect is clearly more important the more bunching there is at the kink. Moreover, the tax liability will decline by \( (t_{j+1} - t_j) dY_j \) at the income level \( Y_j + dY_j \) and beyond. Under standard assumptions the effect is to encourage labour supply due to the income effect.

It is common to model the tax schedule as comprising a universal transfer \( T_0 \).

\(^7\)In practice people with zero or very low earnings typically receive transfers that vary according to the circumstances facing the respective taxpayers. There are unemployment benefits, disability benefits, sick benefits, welfare benefits, etc.
at income $y$ is $ty - T_0$. This is zero for $y = y = \frac{T_0}{t}$, $T_0$ will then be a further tax parameter set by the government. In case all active workers have earnings above $y$ we can however model the tax schedule as having a zero marginal tax rate $t_1$ for $y < y = Y_1$. Even if this is not strictly true, we may for simplicity confine attention to cases where we neglect workers with very low earnings and focus on the tax brackets $2, 3, \ldots$, with endpoints $Y_2, Y_3, \ldots$ above $y$.

The optimal piece-wise linear income tax with two tax brackets is characterised by Apps, Long and Rees (2014) and Andrienko, Apps and Rees (2016). A number of trade-offs will determine the optimal tax schedule. A higher marginal tax rate in a bracket will increase the tax distortion but will shift more of the tax burden to those in tax brackets beyond the one we consider, and we have a standard equity-efficiency trade-off. Letting the higher tax rate kick in at a higher income level will stimulate the labour supply of agents at the initial kink and enhance both their utility, as discussed above, and their tax payment. These are positive efficiency effects. Agents located beyond the kink will face a tax cut and again there is an equity-efficiency trade-off.

Considering two adjacent tax brackets we may contemplate increasing the tax level in the higher bracket. It follows from our discussion above that there are two ways to implement such a tax rise. One can lower the upper limit of the lower bracket to let the higher tax rate kick in at lower income level than before, or one can raise the tax rate in the lower bracket. There are a number of differences between these alternatives. The tax rate change will increase the tax rate moderately for all the people in the lower bracket, increasing both the tax liability and the distortion on everybody. The limit change will affect only the agents with the highest incomes in the lower bracket, facing them with a steeper tax rate and a larger distortion than in the alternative. In addition, the limit change will discourage the labour supply of agents at the kink whose marginal after-tax wage rate exceeds the marginal valuation of leisure. However, there will be a smaller increase in tax liability among lower-bracket agents and in that respect more redistribution. At the optimum there must be indifference between the alternatives implying that the disadvantages of a steeper tax schedule for a subset of agents and the utility loss of agents at the kink, when the limit is changed, are no worse than facing more people with a moderately steeper tax schedule and achieving less redistribution in the alternative.

Within a standard optimal tax framework welfare can obviously be enhanced by increasing the number of tax brackets, approaching a continuous tax schedule as the polar case. On the other hand, avoiding complexity is often highlighted as a virtue of tax reforms. In practice there is a fairly small number of tax brackets. For this to be rational, there must be other concerns than those captured by the optimal tax model. Salience may be a factor in the sense that taxpayers prefer a fairly simple tax system that they can relate to. They may not be able to fine-tune their behaviour in response to a large number of tax brackets and tax rates. Neither may policy makers be able to carry out the fine-tuning that would in principle allow them to benefit from additional tax brackets beyond the
actual number.\textsuperscript{8} Numerical examples also indicate that there are diminishing returns to the number of tax brackets: the welfare gain from adding another bracket rather quickly becomes small (Andrienko, Apps and Rees, 2016).

In this paper, there is no assumption about optimality. Our interest is confined to the question whether a reform is efficiency or welfare enhancing. It may neither bring the schedule to its optimum nor be the most efficient step towards the optimum.

3 The Norwegian tax reform 2016–2018

Between 2016 and 2018 the tax schedule for labour earnings in Norway was subject to a number of perturbations that we shall study from the perspectives of social efficiency and distribution as outlined above. As a precursor to the exposition of these tax perturbations, it is useful to give a brief account of the preceding development of the tax system.

The Norwegian tax system went through two major reforms in 1992 and 2006. The former introduced the dual income tax in Norway, while the latter upheld the system with important modifications, see Sørensen (2005). The dual income tax, which proliferated throughout the Nordic countries in the early 1990s, combines a low proportional tax on capital income with a progressive piece-wise linear tax on labour income. The 1992-reform in Norway abolished double taxation of dividends, as taxpayers receiving dividends were given full credit for taxes paid at the corporate level. Correspondingly, the capital gains taxation exempted gains attributable to retained earnings taxed at the corporate level. Notably, the post-1992 tax system has adhered to the principle that the tax rate on personal capital income should equal the corporate tax rate. Throughout the period from 1992 to 2014 the tax rate applied to both tax bases was 28 percent. The tax schedule for labour earnings has been designed as consisting of the same basic tax rate as that of capital income, supplemented by a surtax schedule with a number of brackets and a 7.8 percent national insurance contribution.

The tax design of 1992 proved vulnerable to tax motivated organizational shifts, see Thoresen and Alstadsæter (2010). As the wedge between the top marginal tax rate on labour income and capital income increased over time, taxpayers faced stronger incentives to transform labour income into capital income for tax purposes.\textsuperscript{9} This phenomenon, known as the income-shifting problem, motivated the tax reform of 2006. The main innovation was to introduce a surtax on capital income from businesses (including dividends and capital gains) exceeding a risk-free rate of return, perceived as the normal rate of return. The rate of return was coined “the rate of return allowance”. This move brought the effective marginal tax rate on the above-normal return up to 48.2 percent.

\textsuperscript{8}Computing the actual payments that are due is hardly a concern with modern computer technology. Hence it may seem paradoxical that more tax brackets were used at the time when this may have been a concern.

\textsuperscript{9}Christiansen and Toumala (2008) discuss implications of income-shifting for tax design.
In parallel, the top marginal tax rate on wage income was cut from 55.3 to 47.8 percent. Narrowing the gap between these tax rates would then erode the tax saving from income shifting.

The background for the 2016–2018 reform, at the core of our interest, is different from the considerations underlying the 1992 and 2006 reforms. It is the level of the basic tax rate applicable to capital and labour income and business profits that creates tensions now. A tax on corporate profits of 28 percent was low at the time of its introduction in 1992 (Ministry of Finance, 2015), but in subsequent decades Norway was left behind in the international tax competition (Devereux, Griffith and Klemm, 2002), ending up with statutory tax rates well above the average in the OECD, in EU-28 (Ministry of Finance, 2015) and among neighbouring countries. To improve the international tax position of Norway, the main element of the 2016–2018 reform was a gradual lowering of the corporate tax rate. The government had initiated the move towards lower corporate tax already in 2014 when the rate was cut from 28 to 27 percent, and further steps have taken it down to 23 percent in 2018. Due to the link to the basic tax rate on general income of persons, described above, the immediate consequence was a cut in the flat-rate part of the tax on labour earnings as well as the tax rate on personal capital income. Together these tax rate cuts obviously implied a significant loss of tax revenue, and a major challenge was to offset at least a substantial part of the revenue foregone. The main move has been to introduce more steps in the schedule for the progressive part of the tax on labour income, previously referred to as the surtax schedule. To distinguish the new schedule from the old one, the term “step-tax” was adopted to reflect the larger number of steps in the new step-wise linear income tax. The small tax changes to generate tax revenue make an ideal illustration of tax perturbations. In Figure 1 the new schedule (as of 2018) is compared to the schedule of 2015. The figure clearly illustrates that there are two additional steps in 2018, compared to 2015, and that the 2018 rates in general are below those of 2015. In the interval from approximately NOK 100,000 to around NOK 210,000, the basic allowance has not reached its maximum, which means that the withdrawal rate, 45 percent, is effective. Thus, the marginal tax rate is 20.85 percent. The first step generates revenue at lower levels by introducing the first step, an addition of 1.4 percentage points. The next step kicks in when the basic allowance is exhausted, and now it increases to 3.3 percentage points (or alternatively, 1.9 percentage points on top of 1.4), which brings the marginal tax rate in the interval from approximately NOK 230,000 to around NOK 600,000, very close to the pre-reform schedule, see Figure 1. The two last steps are basically replicating the two-tier surtax schedule of the pre-reform

10 The Norwegian dual income tax schedule maintained the link between the corporate tax and the tax on capital income. In Sweden the correspondence between taxation of the corporate sector and capital income in the personal income tax schedule was left just a few years after the 1991-reform (Bastani and Waldenström, 2018).

11 Use average exchange rates for 2015 to convert income and wage measures to euros and US dollars: 1€=NOK 8.95 and 1$=NOK 8.07.

12 The social insurance rate (8.2%) plus the new tax rate on ordinary income (23%) multiplied by 0.55 ( = 1 − 0.45).
system, with somewhat lower rates.

4 Empirical implementation

4.1 Model tools

Our theoretical framework offers a rather general model of a population of agents supplying labour, which might comprise both wage earners and self-employed. Our theoretical illustration is restricted by available data and estimates, and is confined to wage earners. We make use of tax simulation models developed for Norwegian policy-making, the so-called LOTTE model system, see Aasness, Dagsvik and Thoresen (2007). We use the labour supply module of the model system to simulate labour supply decisions in the benchmark and in the alternative schedule, the 2015- and the 2018-system, respectively. The labour supply model is based on a discrete choice random utility framework, related to the model presented in van Soest (1995). The labour supply model employed here is a "job choice model", see Dagsvik et al. (2014) and Dagsvik and Jia (2016). Insofar as it gives fundamental importance to the notion of job choice, this approach differs from standard discrete choice models of labour supply, as the one in van Soest (1995). The representation of the error term of the model is
based on drawings from the extreme value distribution, which means that each individual is assigned a specific discrete choice.\textsuperscript{13} As we soon will come back to, the reform implies that some of the individuals find a new optimal choice of working hours, but most stay at their pre-reform choice.

The model is estimated by micro data from the Norwegian Labor Force Survey, deriving three separate submodules: a joint model for married couples and two separate models for single females and males. It is exploited that the labour supply module, LOTTE-Arbeid, interacts with the non-behavioural tax-benefit module, LOTTE-Skatt, which means that we have access to a detailed description of the Norwegian tax schedule.

Since the model is a discrete choice model, it deviates somewhat from the theoretical model, which assumes that continuous choices are possible. But we interpret the empirical model as a resonable approximation to the theoretical one.\textsuperscript{14} Taking the theoretical model as our benchmark, the empirical model will err in both directions. When the ideal continuous choices are not available in the discrete case, various agents will end up on either side of the hypothetical continuous optimum, depending on which deviation is the least harmful. Where no behavioural change is found according to our empirical estimates, this can be taken as evidence that any desirable continuous adjustment would be minor and could safely be neglected.

Moreover, we shall also account for the interaction between different tax bases by also controlling for the effect working through the indirect taxation. More precisely, when we calculate the efficiency effect of the perturbation, see Equation (3), we use the module LOTTE-Konsum (Aasness, Dagsvik and Thoresen, 2007) to calculate the indirect tax part of a change in disposable income, resulting from the labour supply effects. This raises the question of the marginal propensity to consume. Here, we simply assume that agents do not save, thus the MPC is 1.\textsuperscript{15} Revenue effects of labour supply adjustments also account for payroll tax revenues being affected. Norway has a differentiated payroll tax schedule, based on geography, which means that tax rates range from 0 to 14.1 (in 2018); we apply an average tax rate, at approximately 13.2 percent.

\subsection{4.2 Empirical estimates}

Recall that we apply a step-wise procedure to identify the welfare effects of the reform, distinguishing between the mechanical effect, the behavioural effect, and effects on overall welfare. The first effect, the mechanical effect, as described by the first part of Equation (3), is the change in tax burden when behavioural effects are ruled out by the envelope theorem. We therefore obtain estimates

\textsuperscript{13}Another tax simulation procedure for discrete choice models used in practical work is based on predictions of labor supply behavior, ignoring individual level information about error terms and only employing the sample information on probabilities.

\textsuperscript{14}Alternatively, one might argue that choices are indeed discrete, and it is the theoretical model that should be perceived as an approximation to reality.

\textsuperscript{15}See Thoresen, Aasness and Jia (2010) for further discussion on this.
of the mechanical effect by keeping labour supply behaviour fixed, as given by the tax rules of 2015, and derive individual tax burden differences caused by the reform by applying the tax rules of 2015 and 2018 on the same fixed income.\footnote{The 2018 tax rule is deflated to the 2015-level by using a wage growth index.} As the 2018-schedule diminishes the tax burden compared to the 2015-schedule, we control for the tax level effect by imposing a hypothetical lump sum tax that would offset the average tax cut. Each household would then be charged approximately NOK 6,000 lump sum\footnote{This was equivalent to approximately 750 US dollars or 670 euros at the average exchange rates in 2015.}. We are then left with purely redistributive effects, where those given a tax relief above NOK 6,000 by the actual reform are winners, and others are losers due to the structural reform. Whereas, the (net) changes in tax burdens are measured in actual values, note that \( z^i \) of Equation (2) is measured in terms of equalized income, where we have used the square-root of the number of household members as the equivalence scale.\footnote{We have also derived empirical estimates based on a framework founded on individual income; thus, no income accumulation across household members and therefore no need for equivalences scales. Of course, this gives other estimates of the inequality aversion in the benchmark case (no effects of the reform on distribution and welfare) – estimates that we soon will return to.}

Figure 2 presents the distribution of the net gain, defined by the difference in tax burden between the two schedules minus the lump sum tax, when households are ranked by pre-reform equivalized disposable income. The actual reform involves tax cuts in all parts of the piece-wise linear schedule, see Figure 1, but the substantial reductions occur at the high end of the distribution. The diagrams of Figure 2 reflect this: taxpayers with negative or small positive overall effect are predominantly found at the low end of the income distribution, whereas large gains are mostly found at the top end.

By definition, pure redistribution means that the sum of gains equals aggregate losses. We refer to the welfare effect of pure redistribution as the (total) distributional effect. Obviously, this effect is zero if all (positive and negative) income changes are given equal weight in the welfare assessment. It is trivial that this would happen only if there is no inequality aversion, i.e. the value of \( \beta \) is zero. We denote this threshold value by \( \beta^* \). For other values of \( \beta \), there will be a strictly positive or negative distributional effect. The \( \beta \)-function shows the "equity" effect of the reform for more or less inequality aversion.\footnote{By including negative values of \( \beta \), we also show for completeness the less plausible cases where there is equality aversion.} We shall soon return to what this \( \beta \)-function may look like (in Figure 4).

As discussed in the theoretical part, the efficiency effects of the structural reform are determined by the labour supply effects. As above, we clean out the level effect to obtain estimates of the behavioural responses to structural changes. The labour supply effects, measured in annual working hours, are presented Table 1.

In total, these effects imply that the tax revenue (from the personal income tax, the payroll tax and indirect taxation) increases by approximately NOK 2

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\(16\) The 2018 tax rule is deflated to the 2015-level by using a wage growth index.

\(17\) This was equivalent to approximately 750 US dollars or 670 euros at the average exchange rates in 2015.

\(18\) We have also derived empirical estimates based on a framework founded on individual income; thus, no income accumulation across household members and therefore no need for equivalences scales. Of course, this gives other estimates of the inequality aversion in the benchmark case (no effects of the reform on distribution and welfare) – estimates that we soon will return to.

\(19\) By including negative values of \( \beta \), we also show for completeness the less plausible cases where there is equality aversion.
Figure 2: Direct distributional effects of the reform

Table 1: Average yearly working hours according to the respective 2015 and 2018 tax schedules

<table>
<thead>
<tr>
<th></th>
<th>2015</th>
<th>2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Married female</td>
<td>1732</td>
<td>1747</td>
</tr>
<tr>
<td>Married male</td>
<td>1982</td>
<td>1985</td>
</tr>
<tr>
<td>Single female</td>
<td>1810</td>
<td>1822</td>
</tr>
<tr>
<td>Single male</td>
<td>1793</td>
<td>1805</td>
</tr>
</tbody>
</table>
billion. This is the behavioural-induced change in tax revenue, which is our measure of the social efficiency gain.

Further, in Figure 3 we show how the gains in NOK, due to labour supply responses, distribute on working hours in the three different subgroups. We see that most individuals do not alter their choice of working hours. All three diagrams display modest gains. In the appendix, we also consider how efficiency gains vary across levels of education. We find that the effect of the reform is somewhat larger for the highly educated taxpayers, as shown in Figure A1.

The empirical estimates of efficiency gains are based on our theoretical model neglecting the challenges posed by kinks. As discussed above, the error committed by assuming away the effects of kinks depends on the discontinuous change in the tax rate at the kink and the number of taxpayers affected by the kink. We have argued above that the former effect is minor. The latter effect is potentially important. Its magnitude will depend on the wage distribution of agents and the extent to which people bunch at the kink. To what extent there actually is bunching at the kinks is an empirical question. Obviously, bunching cannot be taken literally in a strict sense. One would not realistically expect there to be a large number of fine-tuning optimisers earning exactly the income at a kink of the tax schedule. As discussed in the literature (Chetty, 2012), there are optimisation errors or frictions. It is therefore common to consider potential
bunching in a small interval around the kink rather than at the exact kink. In the appendix, Figure A2 and Table A1, we consider the distribution of taxpayers around the kinks where the respective surtax rates kicked in according to the tax rules of 2015. Allowing for the fact that taxpayers fail to hit the exact kink, we can treat the taxpayers close to the kinks as if they were bunching at the kink and therefore subject to the efficiency measurement problems discussed in detail in Section 2.3. Figure A2 shows no signs of bunching. Moreover, and more important, the fraction of taxpayers around each of the thresholds is small, as seen in Table A1, given that there are approximately 2.6 million individuals with wage income above NOK 50,000 (approx. 5,600 euros and 6,200 US dollars).

We see that only a tiny fraction of taxpayers in the respective tax brackets earn income in the neighbourhood of the respective kinkpoints. Relying on this observation and the discussion of tax rates around the kinks in Section 2.3, we conclude that we can neglect the error we commit by not allowing for the initial kinks in the tax schedule when estimating welfare effects.

Finally, by using $d\Omega = \sum \gamma' g + n \gamma dL$ we combine the mechanical effects and the efficiency effect to find the cut-off value of $\beta$, denoted $\beta^*$, for which the perturbation is just welfare preserving, $d\Omega = 0$. Now, the revenue from the efficiency effect of the reform is given back to the agents in terms of lump sum transfers. In this transformation we also control for the labour supply effects working through the income effect on recipients of lump sum transfers. Figure 4 describes how an estimate of $\beta^*$ is obtained, where we also display the "distributional change curve". Including the efficiency part implies that we obtain a new curve for the welfare change with the same shape as the distributional change curve, but moved upward by the same vertical increment all along the scale. Since the tax reform enhances social efficiency, there is a positive effect counteracting the negative effect of redistribution according to inequality-averse preferences. The allocative efficiency gain will be the overriding effect even for strictly positive inequality aversion ($\beta > 0$) as long as it falls short of a cut-off value where the welfare loss due to unfavourable redistribution and the social efficiency gain just cancel out. This "no-effect-of-the-reform" benchmark occurs for the inequality aversion $\beta^* = 1.2$, corresponding to the "no-effect-of-the-reform" benchmark.$^{20}$

To put our result in perspective, it is of interest to note that the literature on the inequality aversion parameter has taken a number of approaches, ranging from presentation of purely illustrative examples to estimations and discussions of what may be "appropriate" values. Various strands of literature conceive of the $\beta$-parameter (in our notation) as either directly reflecting political preferences or originating from various more or less related sources, which can be pure political preferences or measures of individual utility, possibly adopted by political decision makers. In either case, $\beta$ is usually interpreted as the elasticity of people’s marginal (social) utility of income. Going a long way back, Dalton (1939) argued that $\beta$ was greater than 1. A study of the British income taxa-

$^{20}$This estimate is little influenced by the choice of equivalence scale. Table A2 in the appendix reports estimates of $\beta^*$ for other assumptions about equivalence scale.
Figure 4: Distributional gains and welfare gains as a function of $\beta$
tion, reported in Stern (1977), suggested that a value around 2 seemed to give
tax rates not too dissimilar to those existing in the UK. Taking an inverse opti-
mum (or implicit preference) approach, Christiansen and Jansen (1978) found
a value close to 0.9 in their preferred version. Evans (2005) provides a survey
of previous estimates, and itself offers an estimate of 1.4. Based on a number
of surveys, Layard, Mayraz and Nickell (2008) arrived at a preferred estimate
equal to 1.26. Applications in cost-benefit analyses have used many different
values. The guidance of the the UK Treasury has a preference for using 1,
but one can find cases where analysts have used values up to 2 or 2.5. With
these findings in mind, we may conclude that a value of around 1.2 finds its
place towards the lower end of the range of values appearing in the literature
but without deviating substantially from numbers that are quite common. It
follows that the considered tax reform is welfare enhancing only for a moderate
inequality aversion.

We interpret the cut-off value $\beta^* = 1.2$ as conveying information about the
politicians’ implicit distributional preferences, where acceptance of the reform
is taken as evidence that the decision-makers have a lower inequality aversion,
and rejection of the reform indicates a higher inequality aversion. This reform
approach to reveal implicit preferences bears close resemblance to the inverse
optimum approach referred to above, which is used to infer the preferences that
are consistent with the actual policy, assuming that the latter is optimal given
the preferences. The inference from reform analysis is less accurate since it
does not yield a single estimate$^{21}$, but only conveys information about a range
of preferences, such as the implicit inequality aversion being less than $\beta^*$. In
either case, a number of assumptions must be satisfied for the inference to be
meaningful: the underlying model and the estimates of behavioural responses
derived by the analyst must be sufficiently reliable and shared by the politicians,
who must also not be governed by other concerns.

5 Conclusion

We have established a scheme for assessing an income tax perturbation, which
we can envisage as a piecemeal tax revision in the spirit of Feldstein (1976). In
practice, a tax reform will consist of both a change of tax level and a change of
tax structure, i.e. slope and progressivity of the tax schedule. We cleanse out the
level effect by adjusting a hypothetical lump sum tax to isolate the structural
change. We conceive of the impact of the structural change as consisting of
distributional effects and social efficiency effects, which taken together yields an
overall welfare effect. These effects are closely related to the tax perturbation
effects that are referred to as mechanical effects, behavioural effects and welfare
effects in the terminology of Saez (2001) and Brewer, Saez and Shephard (2010).
Mechanical effects are effects on tax payments and tax revenue in the absence
of any change in labour supply and commodity demand. Invoking envelope

$^{21}$Of course, also a single estimate is "inaccurate" in the sense that it has a confidence
interval.
properties, behavioural changes have no first order effects on utility, and the mechanical effects are identical to the real income effects, or utility effects in monetary terms, experienced by the taxpayers. These effects will therefore reflect the direct distributional gains and losses for various taxpayers. To find the ensuing welfare impact one would have to assign welfare weights to the respective gains and losses.

Since there are pre-existing tax distortions, behavioural effects will affect social efficiency. Both direct and indirect taxes cause under-consumption of all commodities apart from leisure. Where a tax reform enhances labour supply and consumption of taxed commodities, a more efficient allocation is achieved. The increase in tax revenue due to behavioural changes is a measure of the allocative efficiency gain, while revenue foregone would reflect a loss. The overall efficiency effect, capturing both allocative efficiency and welfare effects of redistribution, depends on the value of the use of additional tax revenue. An option is to recycle the extra tax revenue through a uniform lump sum transfer. We can then find the gains and losses of various taxpayers due to the combined distributional and efficiency effects, and we can find the welfare weights that yield a positive or negative overall welfare impact.

We have applied the theoretical approach outlined above to the actual tax perturbations implemented in Norway in the period 2016–2018. We use households as units. Household welfare is assumed to depend on disposable income per consumer unit, where the number of units is determined by an equivalence scale. The welfare weights assigned to marginal income are then determined by equivalent income. We subscribe to the widely held view that additional income is more highly valued if accruing to a larger household than if given to a smaller household with the same income. We can interpret the key parameter that determines the welfare weight corresponding to each (equivalent) income level as a measure of inequality aversion.

The structural reform in Norway redistributes income in favour of better-off households. This means that there is an equity loss according to distributional preferences exhibiting inequality aversion. On the other hand, the tax reform induces behavioural changes that increase tax revenue and enhance allocative efficiency. In this sense, we face the frequently highlighted trade-off between equity and efficiency. The combination of a distributional loss and an allocative efficiency gain obviously yields an overall welfare gain only if a sufficiently moderate inequality aversion prevails. Our empirical finding is that the overall welfare gain created by the reform is positive if the value of the inequality parameter is less than 1.2, which is considered as an inequality aversion in the medium range. One finds both lower and higher values in various contexts in the literature. Assuming that a tax reform is implemented only if it is considered beneficial according to the prevailing political preferences, we can infer from a revealed preference perspective that the inequality aversion underlying the political reform decision is less than the threshold value of 1.2.
References


A Table and figure appendix
Table A1: Number of taxpayers around the two kink point points of the surtax schedule, 2015

<table>
<thead>
<tr>
<th>Kink point ( (x) )</th>
<th>( (x - 2000, x - 1000) )</th>
<th>( (x - 1000, x) )</th>
<th>( (x, x + 1000) )</th>
<th>( (x + 1000, x + 2000) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>560,000</td>
<td>3683</td>
<td>3762</td>
<td>3792</td>
<td>3721</td>
</tr>
<tr>
<td>890,000</td>
<td>666</td>
<td>700</td>
<td>642</td>
<td>634</td>
</tr>
</tbody>
</table>

Table A2: Estimates of threshold value for inequality aversion for alternative choice of equivalence scale

<table>
<thead>
<tr>
<th>Household Size</th>
<th>Benchmark eq. scale</th>
<th>Alternative equivalence scale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Square root scale</td>
<td>Per person</td>
</tr>
<tr>
<td>1 adult</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2 adults</td>
<td>1.4</td>
<td>2</td>
</tr>
<tr>
<td>2 adults, 1 child</td>
<td>1.7</td>
<td>3</td>
</tr>
<tr>
<td>2 adults, 2 children</td>
<td>2.0</td>
<td>4</td>
</tr>
<tr>
<td>2 adults, 2 children</td>
<td>2.2</td>
<td>5</td>
</tr>
<tr>
<td>( \beta^* )</td>
<td>1.203</td>
<td>1.529</td>
</tr>
</tbody>
</table>

Note: A selection of equivalence scales used in the literature

Figure A1: Labour supply effects with respect to education

Notes: Box plot of labour supply effects of reform, measured in NOK, for individuals who have adjusted their labour supply because of the reform. Band inside the box represents the median and whisker boundaries defined by interquartile range.
Figure A2: Description of wage income density, 2015