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# Long-Run Monetary and Fiscal Policy Trade-Off in an Endogenous Growth Model with Transaction Costs

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Abstract:

*In this paper, we study maximizing long-run economic growth trade-off in monetary and fiscal policies in an endogenous growth model with transaction costs. We show that both monetary and fiscal policies are subject to threshold effects, a result that gives account of a number of recent empirical findings. Furthermore, the model shows that, to finance public expenditures, maximizing-growth government must choose relatively high seigniorage (respectively income taxation), if “tax evasion” and “financial repression” coefficients are high (respectively low). Thus, our model may explain why some governments resort to seigniorage and inflationary finance, and others rather resort to high tax-rate, as result of maximizing-growth strategies in different structural environments (notably concerning tax evasion and financial repression). In addition, the model allows examining how the optimal mix of government finance changes in response to different public debt contexts.*

**Keywords:** endogenous growth, threshold effects, monetary policy, fiscal policy, public deficit, policy mix, tax evasion, financial repression, financial development

**JEL codes:** H6, E5, E6, O4

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## I. Introduction

The impact of monetary and fiscal policies on long-run economic growth is an open issue of macroeconomic theory. Concerning monetary policy, most of standard “neoclassical” exogenous growth models conclude to some neutrality or “superneutrality” of money in steady-state (see Fischer, 1979), with the notable exception of Stockman (1981) cash-in-advance model, in which a money expansion reduces steady-state levels of capital and output, when (part of) investment expenditures are subject to the money constraint. Concerning fiscal policy, results critically depend on the nature of government expenditures and on the way they are financed. Baxter & King (1993) introduce government consumption and public capital into the core Real Business Cycles model and show that increases in productive government spending can have very large effects on long-run output because they increase the marginal product of private capital. However, this result is obtained on that assumption that increases in productive government spending are financed by lump-sum taxes. Exploring different ways of government finance, Kamps (2004) shows that distorsionary taxes may discourage private investment while there is a trade-off between short-run and long-run effects of deficit-financed increases in productive public spending.

These results only concern “neoclassical” models without endogenous growth in steady-state. Yet, to establish a relation between monetary and fiscal policies and long-run economic growth one needs an endogenous growth model, which ensures the existence of a positive long-run economic growth path that can be affected by policy variables.

Concerning fiscal policy, standard endogenous growth models show the existence of a threshold in the tax-rate to long-run growth relation, in link with the pioneer work of Barro (1990). In his model, productive public expenditures are financed by a flat-tax rate on output. A higher tax rate produces a lower marginal net-of-taxes return of private capital, but also provides resources for public expenditures that enhance private capital productivity. Thus, one can find a tax rate that maximizes economic growth in the long-run. This tax ceiling sums up the trade-off between these two conflicting effects.

Concerning monetary policy, most of theoretical results on endogenous growth models conclude that it is harmful or at best neutral<sup>1</sup>, as in exogenous growth models. Empirical work is less conclusive. Barro (1995) shows that sustained high inflation rates, used as a proxy of expansionist monetary policies, are detrimental to long-run performances, but the robustness of this result is questioned by other studies (see, e.g. Levine & Renelt, 1992). Subsequent work seems to confirm the existence of a negative correlation between inflation and economic growth, but only for high-inflation countries (Bullard & Keating, 1995, Sarel, 1996,...), showing that the relation between inflation and economic growth is probably nonlinear<sup>2</sup>. In addition, a number of recent econometric results exhibit threshold effects of inflation on economic growth<sup>3</sup>, confirming the nonlinear relation between monetary policy and long-run growth. However, as Boyd, Levine & Smith emphasizes, “*the mechanism underlying this apparently nonlinear association remains to be unearthed*” (1997, p.1) and this “epidemic of thresholds” in monetary policy seems not to have found its theoretical support.

Recent econometric studies have the merit of unifying the analysis of both monetary and fiscal policies. One interesting paper is Adam & Bevan (2005), who find strong empirical

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<sup>1</sup> See, for example, Alogoskoufis & van der Ploeg (1991), Mino (1991), Wang & Yip (1992), Jones & Manuelli (1993) or Palivos & Yip (1995).

<sup>2</sup> See also the pioneer work of Thirlwall & Barton (1971).

<sup>3</sup> A nonlinear association between inflation and growth is detected by Fischer (1993), Bullard & Keating (1995), Chari, Jones & Manuelli (1995), Barro (1995), Boyd, Levine & Smith, (1997), Bruno & Easterly (1998), Kim & Willett (2000) and Adam & Bevan (2005).

evidences on the existence of non-linearities in the effects of both fiscal deficits and seigniorage tax rates on economic growth. Unfortunately, economic theory does not provide, to our knowledge, any global model for studying the effect of the fiscal and monetary policy-mix on long-run economic growth, even if some papers deal with fiscal issues (see, for example, Greiner & Semmler, 2000, Ghosh & Mourmouras, 2004), or monetary ones (Palivos & Yip, 1996). Therefore, the aim of this paper is to analyze the impact of different economic policies (money creation, tax policy, public spending and public debt) on long-run economic growth in a global framework.

In our framework, interactions between monetary and fiscal policies principally occur by the channel of the government budget constraint. To model monetary and fiscal policies in the simplest way, we start with Barro (1990) endogenous growth model with productive public spending. In this model, we introduce a non trivial government budget constraint, in order to study the effect of alternative ways of government finance on long-run economic growth. In our model, government takes a flat tax-rate from households' income, but also collects revenues from central bank money creation (seigniorage) and can borrow from households, generalizing the Barro government budget constraint where only taxes are allowed. In addition, we assume that part of tax revenues is subject to "tax evasion" (or collecting cost). Similarly to this "tax flight", we distinguish between central bank seigniorage (which is transferred to government) and private seigniorage (which is transferred to households). Effectively, in countries with developed financial systems, most of the seigniorage is retrieved by the banking system, and constitutes a "seigniorage flight" for the central bank. On the contrary, in financial repressed economies, most of the seigniorage is collected by the central bank and can be used for government finance. Therefore share of seigniorage that is collected by the central bank, which is closely (inversely) related to the "money multiplier", can be interpreted as a measure of 'financial repression'.

We introduce money by the way of a simple "transaction cost" technology, allowing to obtain a simple but general form of money demand, depending both on income and on nominal interest rate, and displaying the "cash-in-advance" technology of Stockman (1981) as a special case.

Our results are the following. First, our model exhibits a threshold effect on long-run economic growth in both monetary and fiscal policies. If the threshold effect of fiscal policy is standard in Barro-type models, the threshold in monetary policy is less usual. In our model, using seigniorage enables government to make more growth-enhancing productive spending. But there is also a negative effect, because increasing the rate of money creation raises transaction costs, which damages private investment incentives and economic growth. As a result, we find an inverted-U relation between the money growth rate and the long-run rate of economic growth, with a positive long-run growth-maximizing seigniorage rate. So, to maximize long-run economic growth, there are ceilings in both seigniorage and tax rates. These ceilings create a trade-off between the two instruments: to finance public expenditures, a maximizing-growth government must choose relatively high seigniorage (respectively income taxation) if "tax evasion" and "financial repression" coefficients are high (respectively low). Thus our model may explain why some governments resort to seigniorage and inflationary finance, and others rather resort to high tax-rate, as a result of maximizing-growth strategies in different structural environments (notably concerning tax collecting costs and financial development).

Moreover, our model allows dealing with the effect of public indebtedness on long-run economic growth and studying how the monetary and fiscal policy-mix is affected by the size of public debt. We show that a higher public debt (or public deficit) ratio is always growth

detrimental, and that it increases the maximizing-growth rates of seigniorage and income taxation.

The paper is organized as follows: section II presents the model and computes the steady-state rate of economic growth. In section III, we study a no-money case, which will serve as a benchmark. In section IV, we introduce money and we compute the threshold effects of monetary and fiscal policies on long-run economic growth. Section V brings some numerical results about the maximizing-growth policy mix, and section VI concludes the paper.

## II. The model

We consider a closed economy, with a private sector and monetary and fiscal authorities.

### *The private sector*

The private sector consists on a producer-consumer infinitely-lived representative agent, who maximizes the present value of a discounted sum of instantaneous utility functions based on consumption:

$$U = \int_0^{\infty} u(c_t) \exp(-rt) dt \quad (1)$$

$c_t > 0$  designs consumption and  $r > 0$  is the subjective discount rate. In order to obtain an endogenous growth path, we assume an isoelastic instantaneous utility function:

$$u(c_t) = \begin{cases} \frac{S}{S-1} \left( (c_t)^{\frac{S-1}{S}} - 1 \right), & \text{for } S \neq 1 \\ \text{Log}(c_t), & S = 1 \end{cases} \quad (2)$$

For the intertemporal utility  $U$  to be bounded, we also have to ensure that  $\left( \frac{S-1}{S} \right) g_c < r$ , with  $g_x$  the growth rate of the variable  $x$ <sup>4</sup>.

The representative household generates per capita output  $y_t$  using per capita private capital  $k_t$  and per capita productive public expenditures  $g_t$ , with population normalized to unity:

$$y_t = f(k_t, g_t) = A k_t^a g_t^{1-a} \quad (3)$$

where  $0 < a < 1$  is the elasticity of output to private capital<sup>5</sup>.

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<sup>4</sup> This condition corresponds to a no-Ponzi game constraint:  $g_c < r$ .

The flow of public spending ( $g_t$ ) enters the production function, like in Barro (1990). The condition  $0 < a < 1$  ensures the existence of a competitive equilibrium, since, at the representative agent level,  $g_t$  is exogenous and the production function exhibits decreasing returns to scale. In equilibrium, on the contrary,  $g_t$  is endogenously determined and the production function exhibits constant returns to scale, a necessary condition for a constant growth path to appear in the long-run.

In order to motivate a demand for money, we assume that all transactions, including consumption ( $c_t$ ), investment ( $z_t$ ) and government expenditures ( $g_t$ ), are subject to a “transaction cost” ( $\Phi$ ) and that money provides “liquidity services”, by reducing transaction costs. Thus, the form of the transaction cost function is:  $\Phi = \Phi(c_t + z_t + g_t, m_t)$ , where  $m_t = M_t / P_t$  is the stock of real balances. At the representative household level, the transaction cost affects consumption and investment decisions, since public spending is given ( $g_t$  is not a choice variable for the representative household). But in equilibrium, the transaction cost depends on revenue:  $\Phi = \Phi(y_t, m_t)$ , with usual properties<sup>6</sup>:  $\Phi_y > 0$ ,  $\Phi_{yy} > 0$ ,  $\Phi_m < 0$ ,  $\Phi_{mm} > 0$  (see Feenstra, 1986). Assuming an isoelastic function, we have:

$$\Phi(y_t, m_t) = y_t \frac{f}{m} \left( \frac{y_t}{m_t} \right)^m \quad (4)$$

where  $f$  is a positive parameter. Thus, in equilibrium, function  $\Phi(\cdot)$  express that a fraction  $\frac{f}{m} \left( \frac{m_t}{y_t} \right)^{-m}$  of output is “lost” in the process of financial intermediation (in line with Pagano, 1993 and Roubini & Sala-i-Martin, 1995). This fraction is inversely related to the ratio of real balances to output ( $m_t / y_t$ ), which is often used as an indicator of financial development in empirical works (see *e.g.* King & Levine, 1993, Gylfason & Herbertsson, 2001,...). Here, we simply use transaction technology  $\Phi(\cdot)$  to obtain a realistic demand for money, depending both on revenue and on nominal interest rate, as we shall see below.

The transaction technology is an essential feature of our model. Notice that, contrary to usual cash-in-advance (hereafter CIA) or transaction costs models in which money provides liquidity services to consumers only, in our model, money is used in all transactions and provides liquidity services also for investors and government. As in the CIA model of Stockman (1981), what is essential is that investment expenditures are subject to the transaction cost, so that monetary policy can affect capital accumulation in the long-run, as we shall see. The fact that transaction costs also affect government spending is inessential and only simplifies calculations, since we obtain a demand for money that depends on aggregate

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<sup>5</sup> This form of production function has been introduced by Barro (1990). For a general discussion on productive government expenditures, see Barro & Sala-i-Martin (1992) and, for empirical evidence, see Aschauer (1989) or Munnell (1992).

<sup>6</sup> We use the notation:  $x_y = \frac{\partial x(\cdot)}{\partial y}$ ,  $\forall x, y$ .

income. Thus, the transaction technology  $\Phi(\cdot)$  depicts the fact that, in a monetary economy, money is used in (almost) all transactions<sup>7</sup>.

Notice that relation (4) describes a rather general transaction technology, which allows studying several special cases. In particular, parameters  $\mathbf{m}$  and  $\mathbf{f}$  will serve to distinguish between three specifications: a “no-money” case, which corresponds to  $\mathbf{f}=0$ , a “cash-in-advance” model with cash requirement for all transactions, which corresponds to the case  $\mathbf{m} \rightarrow \infty$ , and intermediate cases in which  $\mathbf{f} > 0$  and  $0 < \mathbf{m} < +\infty$ .

In real variables, the budget constraint of the representative household is (a dot over a variable denotes its time derivative):

$$\dot{k}_t + \dot{b}_t + \dot{m}_t = r_t b_t + (1 - \mathbf{t}) y_t - \Phi(c_t + z_t + g_t, m_t) - c_t - \mathbf{d} k_t - \mathbf{p}_t m_t + x_t \quad (5)$$

where:  $r_t = R_t - \mathbf{p}_t$  is the real interest rate,  $R_t$  is the nominal interest rate and  $\mathbf{p}_t m_t$  represents the “inflation tax” on money. As in Barro (1990), we assume that households pay a flat tax rate on income ( $0 < \mathbf{t} < 1$ ). Households use their disposable income to consume ( $c_t$ ), invest ( $z_t = \dot{k}_t + \mathbf{d} k_t$ , with  $\mathbf{d}$  the rate of capital depreciation), and savings, in the form of money holdings ( $\dot{m}_t$ ) or purchase of government bonds ( $\dot{b}_t$ ), whose rate of return is the real interest rate  $r_t$ <sup>8</sup>.  $x_t$  is a transfer, to be defined below, from the financial sector.

### *Monetary and fiscal authorities*

In what follows, we are interested in the different ways of government finance, and specifically on the trade-off between income tax and seigniorage revenues. Clearly, in countries with a developed financial sector, most of seigniorage is recovered by private banks, and only the seigniorage collected on the monetary base (bank notes and bank reserves) is caught by monetary authorities. Thus, the degree of development of the banking sector or, more largely, of the financial sector may represent a “seigniorage flight” from central bank revenues. In addition, a number of LDC countries have inefficient tax systems, with high collecting cost or “tax evasion”. Thus, while attempting to explain the impact of monetary and fiscal policies on economic growth in a large group of countries, one has to study the impact of some “frictions”, such as tax evasion or financial repression, on the policy-mix trade-off in the long-run.

In order to take these facts into account while keeping the model simple, we consider that the nominal stock of money ( $M_t$ ) is the sum of “cash” money ( $L_t$ ) and deposits ( $D_t$ ). Monetary authorities set an exogenous nominal stock of high-powered money ( $L_t$ ), and we

<sup>7</sup> A demand for money depending on income is also obtained with a CIA constraint on investment (and public spending), in models with money in the production function (see Dornbusch & Frenkel, 1973 or Fischer, 1974, for example), or in models in which money is valued because government accepts it to pay taxes. Notice that empirical estimations of money demand are more often based on income than on consumption. In addition, in our specification, money demand depends not only from income but also on nominal interest rate.

<sup>8</sup> Private bonds may also been introduced without difficulty, knowing that at the macroeconomic level such bonds are not hold, because of the representative household-investor hypothesis. Thus, government is the only debtor in equilibrium.

suppose that the supply of money is linked to high-powered money by a standard “money multiplier” ( $1/h > 1$ ):  $M_t = \frac{1}{h} L_t$ . Coefficient  $h$  depends on the ratio of banknotes to the nominal stock of money and on bank reserve requirements, which we do not model explicitly. In what follows, we interpret the size of parameter  $h$  as an indicator of “financial repression”, or “financial underdevelopment”. Effectively, countries endowed with a developed financial sector have a modern banking system with a large money multiplier ( $1/h$ ). Furthermore, there is a strong empirical inverse relation between the reserve ratio, used as a proxy of financial repression, and the level of financial development (see the impressive empirical evidence found by Haslag & Koo, 1999, for example).

We are interested in monetary policies that set an exogenous stock of high-powered money growth rate ( $\dot{L}_t / L_t = w$ ), thus the rate of growth of the stock of money is also:  $\dot{M}_t / M_t = \dot{L}_t / L_t = w$ . Monetary authorities receive seigniorage on high-powered money (in real variables:  $wL_t / P_t = hwm_t$ ) and transfer it to government. The distinction between money ( $M_t$ ) and high-powered money ( $L_t$ ) simply serves to depict the fact that in developed economies, the stock of money used in transactions is much larger than the stock of central bank money than can be used for government finance. As we will see, coefficient  $h$  will have a crucial influence on the trade-off between money and tax financing.

To finance public expenditures<sup>9</sup>, government takes its resources from seigniorage, income taxes and public borrowing. In addition, we suppose that only a fraction  $h \leq 1$  of taxes on households is really collected by government. Parameter  $h$  measures the “cost of collecting taxes” or, more generally, “tax evasion”. Such an assumption allows studying the joint effect of financial repression and tax evasion on long-run policy trade-off, as do Roubini & Sala-i-Martin (1995). Contrary to their analysis, we allow for government debt. With  $b_t$  the stock of real bonds at time  $t$ , the government budget constraint is, in real variables:

$$\dot{b}_t = r_t b_t + g_t - h t f(k_t, g_t) - h w m_t \quad (6)$$

As relation (6) clearly shows, we model monetary and fiscal policies symmetrically, with a “seigniorage flight”  $(1-h)$  from the private banking sector, and a “tax flight”  $(1-h)$  from the public sector. In a more general version of the model,  $h$  and  $h$  coefficients should be endogenously determined; but, in order to obtain simple results, we choose to treat them as exogenous parameters.

Notice that, apart from tax evasion ( $h \leq 1$ ), relation (6) departs from the Barro (1990) government constraint ( $g_t = t y_t$ ) in two ways: first, we allow for money financing of public expenditures (as do, *e.g.*, Palivos & Yip, 1996 in a model without productive public expenditures), second, we allow for government borrowing.

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<sup>9</sup> In our model, we suppose that all (primary) public expenditures are productive. Effectively, interests on public debt constitute an unproductive expenditure, so we do not need to model explicitly unproductive primary expenditures.



In an endogenous growth setting, public debt must grow at the same rate as output (the balanced growth rate) in the long-run. A simple way to characterize this fact is to suppose that government adopts a constant public deficit-to-output ratio ( $d$ ):

$$\dot{b}_t = dy_t \quad (7)$$

Effectively, in an endogenous growth set-up, the issue of permanent deficits can be addressed, since the government intertemporal budget constraint does not force long-run public debt to be constant (as in models with no growth in steady-state), but only forces its long-run rate of growth (namely the deficit to public debt ratio) to be lower than long-run real rate of interest. In effect, since in the long-run all variables must grow at the same rate, the rate of growth of public debt will equalize the long-run balanced rate of economic growth, which must be lower than the real rate of interest for the solvability condition to be enforced. Let us denote the long-run balanced rate of economic growth as  $g$ , the solvability condition ( $\dot{b}_t / b_t = dy_t / b_t = g < r$ ) allows government running permanent public deficits as soon as  $d < rb / y$  in the long-run.

Notice that in steady-state, the deficit ratio target  $d$  corresponds to a debt ratio target  $q = d / g$ . Effectively, in the long-run  $\dot{b}_t / b_t = dy^* / b^* = g$ , so that:  $q = b^* / y^* = d / g$ , where  $b^*$  and  $y^*$  are steady-state values for public debt and output. So, we can interpret the fiscal rule (9) either as a permanent deficit rule ( $d$ ) or as a long-run target for the public debt to output ratio ( $q$ ). Since we are mainly interested in steady-state properties and there are much more long-run evidences concerning the linkage between public debt and economic growth or inflation than on public deficits, we will favor the second interpretation in what follows.

Finally, to complete the model, the banking sector transfers to households the real value of private seigniorage on deposits and revenues from “transactions costs”<sup>10</sup>, in equilibrium:  $x_t = w(1-h)m_t + \Phi(y_t, m_t)$ .

### Equilibrium

The representative agent maximizes (1) subject to (2)-(3)-(4)-(5),  $k_0$  given and the transversality condition:  $\lim_{t \rightarrow \infty} \left( \exp \left( - \int_0^t r_s ds \right) (k_t + b_t + m_t) \right) = 0$ . Since investment is subject to transaction costs, it is convenient to replace the budget constraint (5) by two constraints on two state variables:  $a_t \equiv m_t + b_t$  and  $k_t$ , using the definition of net investment:  $\dot{k}_t = z_t - dk_t$ . Thus, we can write the *current Hamiltonian*:

$$H_c = u(c_t) + I_{1t} \left[ r_t b_t + (1-t) f(k_t, g_t) - \Phi(c_t + z_t + g_t, m_t) - c_t - z_t - p_t m_t + x_t \right] \\ + I_{2t} (z_t - dk_t) + q_t (a_t - b_t - m_t) \quad (8)$$

<sup>10</sup> We consider that the cost of collecting taxes is a “deadweight loss” for the economy, but things would be unchanged if this cost was transferred to households in the form of a lump-sum transfer.

where  $I_1$  and  $I_2$  are the costate variables associated respectively with  $a_t$  and  $k_t$ , and  $q_t$  is the costate variable associated with the static constraint. First order conditions are:

$$/b_t \quad q_t / I_{1t} = r_t \quad (9.1)$$

$$/c_t \quad u_c(c_t) = I_{1t} (1 + \Phi_c) \quad (9.2)$$

$$/m_t \quad I_{1t} (-\Phi_m - p_t) = q_t \Rightarrow -\Phi_m = R_t \quad (9.3)$$

$$/z_t \quad I_{2t} = I_{1t} (1 + \Phi_z) \quad (9.4)$$

$$/a_t \quad \dot{I}_{1t} / I_{1t} = r - r_t \quad (9.5)$$

$$/k_t \quad \dot{I}_{2t} / I_{2t} = r + d - [(1 - t) f_k(k_t, g_t)] I_{1t} / I_{2t} \quad (9.6)$$

These relations have a standard interpretation.  $I_1$  is the shadow price of “financial wealth” ( $a_t$ ), which differs from the shadow price of capital ( $I_2$ ) in equation (9.4), since investment expenditures are subject to a transaction cost ( $\Phi_z$  is the marginal transaction cost on investment expenditures). Similarly, in equation (9.2), the marginal utility of consumption has to be distinguished from the shadow price of financial wealth, since there is a transaction cost on consumption expenditures (with  $\Phi_c$  the marginal transaction cost on consumption expenditures). Equation (9.3) states that the marginal cost of money (the nominal interest rate  $R_t$ ) must equalize its marginal return ( $-\Phi_m$ , since money economizes transaction costs).

With (3) and (4), first order conditions (9.2-3-6) become<sup>11</sup>:

$$R_t = f \left( \frac{y_t}{m_t} \right)^{1+m} \quad (10.1)$$

$$u_c(c_t) = (c_t)^{-1/s} = I_{1t} / q(R_t) = I_{2t} \quad (10.2)$$

$$\dot{I}_{2t} / I_{2t} = r + d - q(R_t)(1 - t) f_k(k_t, g_t) \quad (10.3)$$

where:  $f_0 = (f)^{\frac{1}{1+m}}$ ,  $f_1 = \frac{1+m}{m} f_0$  and:  $q(R_t) = \left[ 1 + f_1(R_t)^{\frac{m}{1+m}} \right]^{-1}$

Notice that the transaction technology introduces a wedge between the return of bonds ( $r_t$  in (9.5)) and the return of investment ( $q(R_t)(1 - t) f_k(k_t, g_t) - d$  in (10.3)), since there is no transaction cost on financial transactions. Such a wedge would also appear in a CIA model in which investment expenditures are subject to the money constraint. Our model obtains such a CIA constraint as a special case of our transaction technology. Effectively, if  $m \rightarrow \infty$ ,  $y_t \rightarrow m_t$ , and relations (10.2) and (10.3) become<sup>12</sup>:

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<sup>11</sup> Notice that:  $\Phi_c = \Phi_z = f \left( \frac{1+m}{m} \right) \left( \frac{y_t}{m_t} \right)^m$  and  $\Phi_m = -f \left( \frac{y_t}{m_t} \right)^{1+m}$ .

<sup>12</sup> Notice that:  $f_0 = f_1 = 1$  if  $m \rightarrow \infty$ .

$$u_c(c_t) = I_{1t}(1+R_t) = I_{2t} \quad (10.2b)$$

$$\dot{I}_{2t}/I_{2t} = r + d - \left[ \frac{(1-t)f_k(k_t, g_t)}{1+R_t} \right] \quad (10.3b)$$

Relations (10.2b) and (10.3b) are similar to those obtained in CIA models with cash requirement on investment<sup>13</sup>, like Stockman (1981) or Palivos & Yip (1995), for example. The advantage of our transaction technology is that it is more general, and that it gives rise to an interest-elastic money demand. Effectively, from (9.3), using transaction technology (4), we obtain a simple usual expression for money demand:

$$m_t = \frac{y_t}{(R_t/f)^{\frac{1}{1+m}}} = m \left( y_t, R_t \right) \quad (10.4)$$

If  $m=1$  (as in most of our simulations) for example, the demand for real balances is close to usual empirical evidences, with an income elasticity of 1 and an interest-elasticity of  $-0.5$ .

By differentiating (10.2) and (10.3) and after some simple manipulations we obtain a reduced form of the representative's agent program (time indexes will henceforth be omitted):

$$\frac{\dot{c}}{c} = S \left[ a(1-t)q(R)A(g/k)^{1-a} - r - d \right] \quad (11.1)$$

$$\frac{\dot{R}}{R} = \left( \frac{1+m}{m(1-q(R))} \right) \left[ r + d - a(1-t)q(R)A(g/k)^{1-a} \right] \quad (11.2)$$

The IS equilibrium is:

$$\frac{\dot{k}}{k} = A(g/k)^{1-a} - (c/k) - (g/k) - d \quad (11.3)$$

Money equilibrium determines the price level, obtained by the ratio between the (exogenous) nominal supply of money and the demand for real balances (10.4):

$$P = \frac{L}{hm \left( y, R \right)}.$$

Taking the time derivative of this expression, we get the evolution of the stock of real balances (with:  $p = R - r$ ):

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<sup>13</sup> Strictly speaking our model collapses to a CIA model with a money constraint on all transactions, including government spending. But since  $g$  is exogenous for representative households, first orders conditions are unchanged, whether government spending enters or not the CIA constraint. Thus, results would be qualitatively unchanged with a CIA constraint on investment and consumption only.

$$\frac{\dot{m}}{m} = w - p = w + r - R \quad (11.4)$$

Equalizing this relation to the time derivative of money demand (10.4), we obtain the equilibrium real rate of interest (which determines the equilibrium rate of inflation:  $p = R - r$ ):

$$r = R - w + \frac{\dot{m}}{m} = R - w + a \frac{\dot{k}}{k} + (1-a) \frac{\dot{g}}{g} - \frac{1}{1+m} \frac{\dot{R}}{R} \quad (11.5)$$

Finally, the law of motion of public debt is given by the deficit rule (8):

$$\frac{\dot{b}}{b} = dA(g/k)^{1-a} (b/k)^{-1} \quad (11.6)$$

and the equilibrium ratio of public expenditures comes from the government constraint (6) and the deficit rule (8):

$$h t A(g/k)^{1-a} + h w(m/k) - g/k + (\dot{b}/b - r)(b/k) = 0 \quad (11.7)$$

Relations (11.1-7) constitute a seven variables ( $c$ ,  $k$ ,  $g$ ,  $m$ ,  $R$ ,  $b$ , and  $r$ ) for seven equations dynamic system, which can be easily solved. Defining (as usual in growth literature) modified variables:  $c_k \equiv c/k$ ,  $b_k \equiv b/k$ ,  $g_k \equiv g/k$  and  $m_k \equiv m/k$ , *Appendix I* shows that the dynamics of the model can be resumed by a four variables reduced form driving the evolution of  $c_k$ ,  $g_k$ ,  $b_k$  and  $R$ . However, the steady-state solution can be derived without fully analyzing the dynamics of this reduced form, which we only use to study stability properties of the model in the neighborhood of steady-state (see *Appendix I*).

We obtain the steady-state endogenous growth solution by setting:  $\dot{c}_k = \dot{b}_k = \dot{g}_k = \dot{m}_k = \dot{R} = 0$  in system (11). The latter relation implies constant values of  $c_k$ ,  $b_k$ ,  $g_k$ ,  $m_k$  and  $R$  which means that the initial variables ( $c$ ,  $k$ ,  $g$ ,  $m$  and  $b$ ) grow at the same constant rate  $\mathbf{g} = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{m}}{m} = \frac{\dot{g}}{g} = \frac{\dot{b}}{b}$ , while  $R$ ,  $r$  (and  $\mathbf{p}$ ) are constant.

#### *Steady-state endogenous growth solution*

The long-run solution of the model can be expressed by two relations between  $\mathbf{g}$  and  $g_k = g/k$ . In steady-state, since the nominal interest rate is constant ( $\dot{R} = 0$ ), we obtain the simple Keynes-Ramsey relation, from (11.1) and (11.2):  $\dot{c}/c = \mathbf{g} = S(r - \mathbf{r})$ . Thus, we can extract the nominal interest rate from (11.4):  $R = w + r - \mathbf{g} = w + \mathbf{r} - \left(\frac{S-1}{S}\right)\mathbf{g}$ . Therefore, equation (11.1) provides a first relation between  $\mathbf{g}$  and  $g_k$ :

$$g_k^A(\mathbf{g}) = \left[ \frac{1}{\mathbf{a}(1-t)A} \left( \frac{\mathbf{g}}{S} + \mathbf{d} + \mathbf{r} \right) \left( 1 + \mathbf{f}_1 \left( \mathbf{w} + \mathbf{r} - \left( \frac{S-1}{S} \right) \mathbf{g} \right)^{\frac{m}{1+m}} \right) \right]^{\frac{1}{1-a}} \quad (12a)$$

The second relation is given by equation (11.7) in steady-state:  $htAg_k^{1-a} + hwm_k - g_k + (\mathbf{g} - r)b_k = 0$ . Using the definition of the real interest rate:

$r = \mathbf{r} + \mathbf{g}/S$  and expressing the money to capital ratio in (10.4) as:  $m_k = \mathbf{f}_0 Ag_k^{1-a} R^{\frac{-1}{1+m}}$ , the unique unknown of this expression is the debt to capital ratio  $b_k$ , which we can obtain in the fiscal rule (11.6). As we have seen, in the long-run we can extract the debt to capital ratio either from a deficit rule:  $b_k = dAg_k^{1-a} (\dot{b}/b)^{-1} = dAg_k^{1-a} / \mathbf{g}$  or from the corresponding debt rule:  $b_k = \mathbf{q} Ag_k^{1-a}$ , where the debt to output ratio  $\mathbf{q}$  is defined as:  $\mathbf{q} = d / \mathbf{g}$ . Here we favor the latter solution. Replacing the value of the debt ratio in the government budget constraint in the long-run provides another steady-state relation between  $\mathbf{g}$  and  $g_k$ :

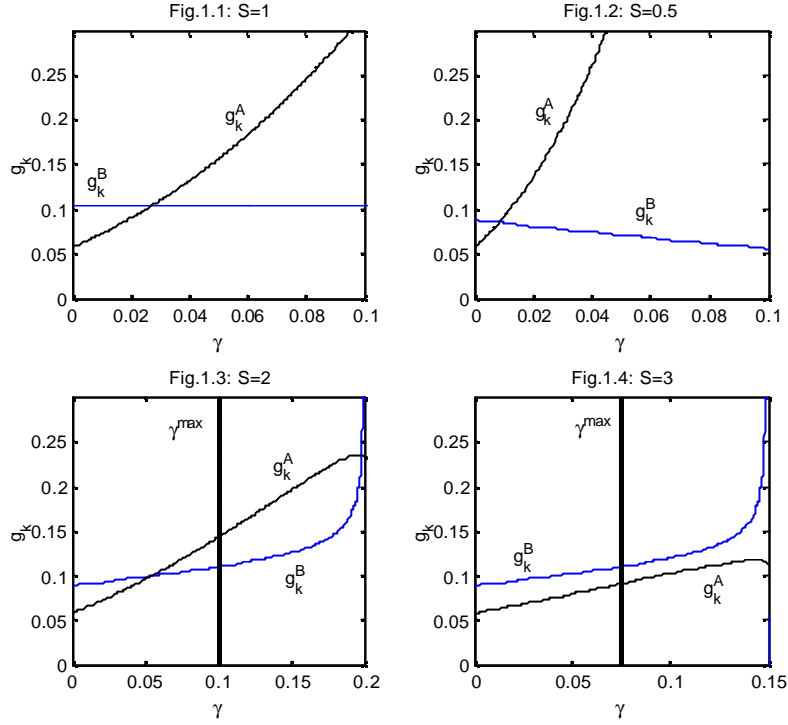
$$g_k^B(\mathbf{g}) = A^{1/a} \left[ ht + wh\mathbf{f}_0 \left( \mathbf{w} + \mathbf{r} - \left( \frac{S-1}{S} \right) \mathbf{g} \right)^{\frac{-1}{1+m}} - \left( \mathbf{r} - \left( \frac{S-1}{S} \right) \mathbf{g} \right) \mathbf{q} \right]^{1/a} \quad (12b)$$

By modeling a constant public debt to output ratio ( $\mathbf{q}$ ) rather than a deficit ratio ( $d$ ) in the long-run, we avoid multiplicity of steady-states. Effectively, in an endogenous growth model without money, Minea & Villieu (2005) show that a deficit rule conducts to two steady-state solutions, with a stable and an unstable equilibrium. Since we are only interested in long-run issues, we can focus on the stable solution only, and a constant debt to output ratio in the long-run allows ignoring difficulties associated to multiplicity. With the debt rule  $\mathbf{q}$ , the model is much simpler and conducts, for a large range of admissible parameters, to a unique (as we shall see) and stable (see *Appendix I*) steady-state solution.

Coefficient  $\mathbf{q}$  may be interpreted as a policy parameter, in which case government chooses to target a constant ratio  $\mathbf{q}$  between public debt and output in the long-run, or, more generally, it may simply describes the fiscal stance of government. In what follows,  $\mathbf{q}$  will serve us to study how the optimal trade-off between seigniorage and income tax is affected by the level of public debt (as a percentage of output) in the long-run.

We define a steady-state solution  $\mathbf{g}$  as the intersection of the curves (12a) and (12b) that we represent in a  $(\mathbf{g}, g_k(\mathbf{g}))$  plan. In equation (12a),  $g_k^A$  positively depends on  $\mathbf{g}$  if  $S \leq 1$ . This is also the case if  $S > 1$ , provided that  $\mathbf{g}$  is not “too large”. The  $g_k^B$  curve is positively or negatively linked to  $\mathbf{g}$  in relation (12b), depending on the sign of  $S - 1$ . These two functions are depicted in *Figure 1* for different values of the consumption elasticity of substitution  $S$ .

Figure 1 – Long-run steady-state growth rate



For:  $r = d = f = 0.05$ ,  $S = A = m = 1$ ,  $a = 0.6$ ,  $h = 0.5$ ,  $h = 0.6$ ,  $q = 0.5$

If  $S = 1$ ,  $g_k^B$  is independent from  $g$  in (12b), and we can extract a simple analytic expression for the long-run rate of economic growth:

$$g = \frac{aA^{1/a}(1-t) \left[ ht + hf_0 w(w+r)^{-\frac{1}{1+m}} - qr \right]^{\frac{1-a}{a}}}{1 + f_1(w+r)^{\frac{m}{1+m}}} - r - d \quad (13)$$

We suppose that this solution is positive (we will see below that this is the case for a large range of admissible parameters).

If  $S < 1$ , the  $g_k^B$  line is negatively sloped and the  $g_k^A$  line is positively sloped; thus there is still a unique point of intersection between the two curves. If  $S > 1$ , the  $g_k^B$  line is positively sloped, and the  $g_k^A$  eventually becomes a decreasing function of  $g$ , for large values of  $g$ . Thus, we cannot exclude that the  $g_k^A$  curves and the  $g_k^B$  curves intersects twice if  $S > 1$ , as in Figure 1.3. However, the higher point of intersection does not verify the solvability condition. Effectively, if  $S > 1$ , the solvability condition provides a maximum value for the long-run rate of economic growth:  $g^{\max} = Sr / (S - 1)$ . If  $S = 2$ , for example, we can see in Figure 1.3 that only the lower point of intersection verifies the condition:  $g < g^{\max}$ . Despite a large amount of simulation work, no simulation has allowed to find two long-run solutions that verify the solvability condition whatever the values of  $S$  (and of other parameters) are. As we can see on Figures 1, a steady-state solution exists for a large range of parameters,

provided that  $S$  is not “too high” (if  $S=3$ , *Figure 1.4* shows that the two curves do not intersect for admissible values of parameters, thus there is no steady-state solution).

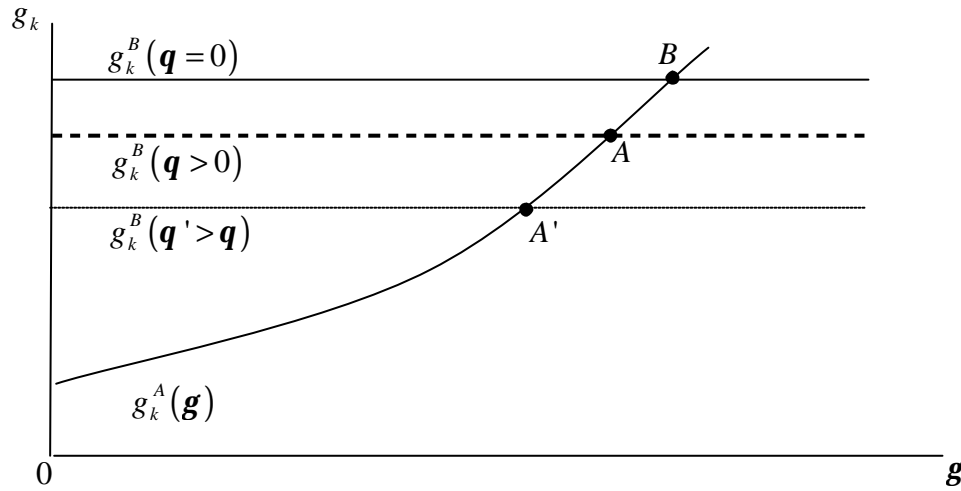
### III. Fiscal policy, public debt and long-run growth: a special case without money

In the no-money case ( $f=0$ ), used as a benchmark,  $g_k^A(g)$  and  $g_k^B(g)$  curves become:

$$g_k^A(g) = \left[ \frac{\frac{g}{S} + d + r}{aA(1-t)} \right]^{\frac{1}{1-a}} \quad (14a)$$

$$g_k^B(g) = A^{1/a} \left[ ht - \left( r - \left( \frac{S-1}{S} \right) g \right) q \right]^{1/a} \quad (14b)$$

*Fig. 2 – Steady-state rate of economic growth ( $f=0$ )*



Both curves are depicted in *Figure 2* for  $S=1$ . Without debt ( $q=0$ ), the  $g_k^B$  curve is independent from  $g$  even if  $S \neq 1$ , and we find the “Barro solution” at the point where  $g_k^B(q=0)$  and  $g_k^A$  intersect ( $B$  point in *Figure 2*). This solution is<sup>14</sup>:

$$\left. \begin{aligned} g_k^{Barro} &= A^{1/a} (ht)^{1/a} \\ g^{Barro} &= S \left[ a(1-t)(ht)^{1-a/a} A^{1/a} - d - r \right] \end{aligned} \right\} \quad (15)$$

<sup>14</sup>  $h=1$  in Barro (1990).

With a positive debt ratio ( $q > 0$ ), one can find a set of admissible parameters so that the system (14a-14b) has one solution conducting to a positive long-run growth rate (A point in Figure 2).

**Proposition 1:**

- (a) Any increase in the debt-to-GDP ratio reduces the long-run rate of economic growth. The highest long-run rate of economic growth, namely the Barro solution, corresponds to a balanced-budget rule, with zero public debt in the long-run.
- (b) There is an inverted-U relation between the tax rate ( $t$ ) and the long-run growth rate. With positive public debt, the long-run growth-maximizing tax rate is higher than the solution of Barro, namely the elasticity of output with respect to public capital in the production function.

*Proof:* (a) From (14b):  $\left. \frac{dg_k^B}{dq} \right|_{g \text{ given}} = \frac{-g_k^{\frac{1-a}{a}}}{aA} \left( r - \frac{S-1}{S} g \right) < 0$ , since the solvability condition imposes  $\frac{r}{g} - \frac{S-1}{S} > 0$ . Since  $g$  and  $g_k$  are positively linked in  $g_k^A(g)$ , we have:  $\frac{dg^H}{dq} < 0$ .

Raising the debt ratio makes the  $g_k^B$  curve move downward, as in Figure 2, without any change in the  $g_k^A$  curve. So any increase in the public debt ratio lowers the steady-state rate of economic growth. In particular, moving from a balanced-budget regime to a debt regime makes the equilibrium moves from B to A, with a lower long-run rate of economic growth ( $g^A < g^B$ ). Effectively, a quick examination of Figure 2 confirms that the rate of economic growth is always below the Barro solution (B point on Figure 2) obtained with a zero public debt. Of course, if the starting point is a situation with positive public debt, any increase in the debt ratio ( $q' > q$ ) still lowers the long-run rate of economic growth (from A to A' in Figure 2).

Proposition 1a has a very intuitive interpretation. In the long-run, public debt generates unproductive public expenditures, in the form of interest payments, which crowd out productive public expenditures, with an adverse effect on economic growth. The fact that the government is not allowed to play Ponzi games implies that the new resources associated with the growing public debt in the long-run ( $gb$ ) never can exceed its interest cost ( $rb$ ). Effectively, in steady-state, the rate of growth of public debt is:  $\dot{b}/b \equiv g < r \Leftrightarrow gb < rb$ , for the solvability condition to be enforced. Thus, in the government budget constraint (11.7), we have:  $htAg_k^{1-a} + hwm_k - g_k = (r-g)b_k > 0$ , which means that part of tax and seigniorage revenues must be affected to the payment of interest (net of additional debt revenues), with a crowding-out effect on productive public spending, explaining why the Barro solution can no longer be reached.

(b) Steady-state equilibrium is obtained for:  $g_k^A(g) = g_k^B(g)$ . The first order condition for  $t$  to be an optimum is, from the implicit function theorem:



$$\hat{t} = 1 - a + a \left( r - \frac{S-1}{S} g(\hat{t}) \right) \frac{q}{h} \geq 1 - a \quad (16)$$

Equation (16) only provides an implicit function for  $\hat{t}$  if  $S \neq 1$ . Nevertheless, for the solvability condition to be enforced, we have:  $\frac{r}{g} - \frac{S-1}{S} > 0$ , and the maximizing-growth tax rate  $\hat{t}$  is higher than the Barro solution  $t^B = 1 - a$ , for any positive level of the debt ratio. In addition, we can see that relation (16) also provides an explicit value for  $\hat{t}$  if  $S = 1$ , namely:  $\hat{t} = 1 - a + arq/h$ .

Similarly to Barro (1990), increasing taxes stimulates economic growth, by providing resources for growth-enhancing public spending, but simultaneously discourages private capital accumulation, with an adverse effect on growth. Notice that we find the Barro solution ( $t^B = 1 - a$ ) as a special case when government adopts a zero debt long-run target ( $q = 0$ ).

Furthermore, we can remark from relation (16) that the optimal tax rate is an increasing function of the debt ratio (if the consumption elasticity of substitution is close to 1). Effectively:  $\left. \frac{d\hat{t}}{dq} \right|_{S \rightarrow 1} = \frac{a}{h} \left( r - \frac{S-1}{S} g(\hat{t}) \right) > 0$ . The intuitive explanation of this result is as follows. Suppose that the starting point is the Barro long-run equilibrium with  $t^B = 1 - a$  and a zero public debt. If  $q$  jumps to some positive value, long-run economic growth will be lower, since the public-debt burden crowds out productive expenditures. To restore (part of) productive expenditures, government must increase the tax rate beyond the Barro value.

Let us now introduce money in the model.

#### IV. Threshold effects of monetary and fiscal policy on economic growth

In the general case with money and transaction costs ( $f > 0$  and  $m > 0$ ), equations (12a) and (12b) provide an implicit definition of the long-run rate of economic growth:

$$F(g, \cdot) = \frac{aA^{1/a} (1-t) \left[ ht + hf_0 w (w + s(g))^{\frac{-1}{1+m}} - s(g)q \right]^{\frac{1-a}{a}}}{1 + f_1 (w + s(g))^{\frac{m}{1+m}}} - r - d - \frac{g}{S} = 0 \quad (17)$$

where:  $s(g) = r - \left( \frac{S-1}{S} \right) g$ . The following proposition states the effects of monetary and fiscal policies on long-run growth.

##### Proposition 2:

- (a) Any increase in the debt ratio reduces the long-run rate of economic growth.
- (b) There is again an inverted-U relation between the tax rate ( $t$ ) and the long-run growth rate. The tax rate that maximizes long-run economic growth is an

increasing function of the debt ratio ( $q$ ) and a decreasing function of the money growth rate ( $w$ ).

- (c) There is an inverted-U relation between the monetary growth rate ( $w$ ) and the long-run rate of economic growth.

*Proof:* (a) Let us denote relation (17) as an implicit function:  $F(g, w, t, q) = 0$ . From the implicit function theorem, we have:  $\frac{dg}{dq} = -\frac{\partial F(\cdot)/\partial q}{\partial F(\cdot)/\partial g}$ . Since  $\partial F(\cdot)/\partial g < 0$  and

$\partial F(\cdot)/\partial q < 0$  (notice that the solvability condition imposes  $s(g) > 0$ ), we find:  $\frac{dg}{dq} < 0$ . As

in the previous section, raising  $q$  has no effect on the  $g_k^A(g)$  curve, but lowers the  $g_k^B(g)$  curve. Consequently, higher public debts still reduce the long-run rate of economic growth, because of the crowding-out effect of the debt interest burden.

(b) Notice first that the tax rate has a positive effect on both curves  $g_k^A(g)$  and  $g_k^B(g)$ <sup>15</sup>. Like in the previous section, these movements imply the existence of a threshold for the maximizing-growth tax-rate (see Fig.3 below for the analysis of the seigniorage threshold). Therefore, there is again an inverted-U shaped curve between taxes and economic growth, with the same explanation as in the previous section. To compute the tax rate ceiling, we proceed as in section III. From the implicit function theorem, we obtain the first order condition for the maximization of economic growth ( $\partial F(\cdot)/\partial t = 0$ ):

$$\hat{t} = 1 - a - \frac{a}{h} \left[ h f_0 w (w + s(\hat{t}, w, q))^{\frac{-1}{1+m}} - q s(\hat{t}, w, q) \right] \quad (18)$$

where:  $s(\hat{t}, w, q) = r - \left( \frac{S-1}{S} \right) g(\hat{t}, w, q)$ . Since  $s_t = s_w = s_q \rightarrow 0$  if  $S \rightarrow 1$ , we easily obtain:

$\left. \frac{\partial \hat{t}}{\partial w} \right|_{S \rightarrow 1} < 0$ . Thus, the tax ceiling  $\hat{t}$  is a decreasing function of the money growth rate, for

close to one values of the consumption elasticity of substitution. We can also compute the effect of changes in the debt ratio. Effectively, from (18), we obtain, for  $S \rightarrow 1$ :

$\left. \frac{\partial \hat{t}}{\partial q} \right|_{S \rightarrow 1} = r q s(\hat{t}, w, q) > 0$ . Thus, the tax ceiling is positively linked to the deficit ratio,

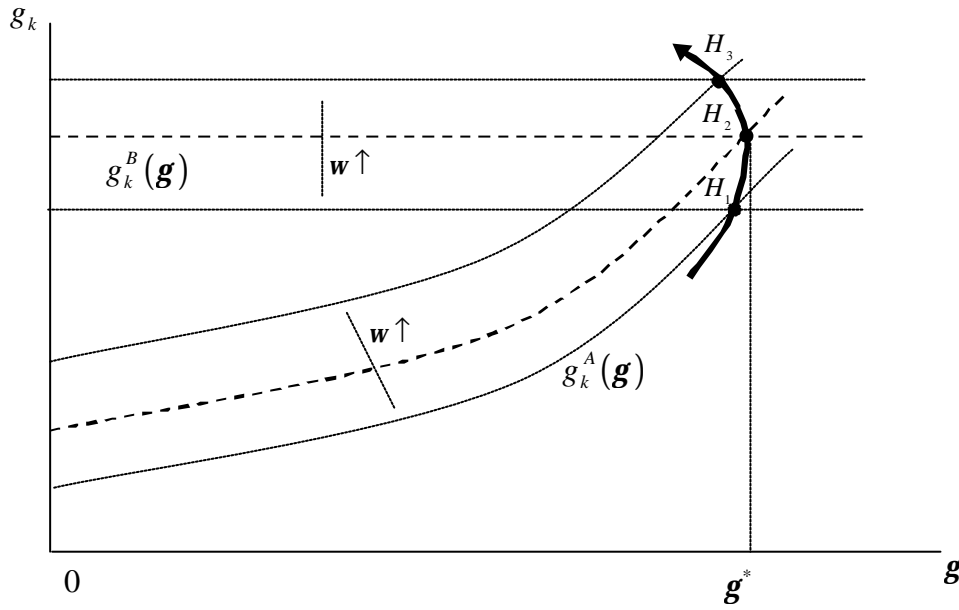
generalizing the section III finding in the model without money, with the same explanation.

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<sup>15</sup> Effectively we obtain:  $\left. \frac{dg_k^A}{dt} \right|_{g \text{ given}} = \frac{g_k}{(1-a)(1-t)} > 0$ , and:  $\left. \frac{dg_k^B}{dt} \right|_{g \text{ given}} = h A g_k^{1-a} / a > 0$ .

(c) An increase in the money growth rate makes both  $g_k^A(g)$  and  $g_k^B(g)$  curves move upwards<sup>16</sup>. As depicted in *Figure 3*, getting  $w$  higher first raises the long-run growth rate, from  $H_1$  to  $H_2$ , and then decreases it, from  $H_2$  to  $H_3$ .  $H_2$  point gives the maximum value of the long-run rate of economic growth in response to changes in  $w$  (for given values of other parameters). Thus, there is a threshold effect of monetary policy on economic growth. The economic intuition of this threshold is as follows. Any increase in the rate of seigniorage is devoted to productive public expenditure which is growth enhancing (numerator of (17)), but simultaneously raises the nominal interest rate, which increases transaction costs (denominator of (17)). The latter effect is harmful to long-run growth, since transactions are more “costly”. The trade-off between these two effects results in a ceiling for  $w$ , say  $\hat{w}$ .

*Fig.3—Effect of an increase in the money growth rate on long-run economic growth ( $S = 1$ )*



Observe that the adverse effect of the rate of money growth on long-run economic growth comes from the fact that investment expenditures are subject to transaction costs. In contrast, if transaction costs affect only consumption expenditures, seigniorage gives government additional resources, allowing more productive growth-enhancing public spending, unless the inflation tax has a negative effect on private investment. In this case, seigniorage would be similar to a flat-tax rate on consumption, and a maximizing-growth government should always resort to seigniorage only<sup>17</sup>. On the contrary, with a transaction

<sup>16</sup> Effectively, we obtain:  $\left. \frac{dg_k^A}{dw} \right|_{g \text{ given}} = \frac{f_1 m (w + s(g))^{-1} g_k}{(1-a)(1+m+f_1 m (w + s(g)))} > 0$ , and:

$$\left. \frac{dg_k^B}{dw} \right|_{g \text{ given}} = ((1+m)(w + s(g)) - w) A h f_0 g_k^{1-a} \frac{(w + s(g))^{-2-m}}{(1+m)} > 0.$$

<sup>17</sup> This proposition meets numerous works in endogenous growth models, showing the benefits of consumption taxes rather than taxes on output, which penalize capital accumulation (see, for example, Turnovsky, 1996).

cost on investment expenditures, there is a “Laffer curve” of seigniorage, which provides a trade-off between monetary and fiscal policies for maximizing long-run economic growth.

Notice that the seigniorage “Laffer curve” that our model exhibits is in line with abundant econometric literature on non-linear effects of inflation on economic growth. For example, Thirlwall & Barton (1971) identify positive effects on growth of lower than 8% inflation rates, and negative ones for higher than 10% inflation rates. Gylfasson (1991) associates high-growth countries with lower than 5% inflation rates, and low-growth economies to inflation higher than 20% ones, while Sarrel (1996) finds a breakpoint in the effect of inflation on growth for about 8% inflation rates. More recently, Adam & Bevan (2005) find thresholds effects in both monetary and fiscal policies to growth relation, a result especially consistent with our model.

It is rather difficult to find a simple analytical value of the money growth rate ceiling. Effectively, from the implicit function theorem, the first order condition for the maximization of economic growth ( $\partial F(.) / \partial \mathbf{w} = 0$ ) is:

$$G(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q}) = \frac{\left( \frac{\mathbf{a}}{1-\mathbf{a}} \right) \left[ h\mathbf{t} + \hat{\mathbf{w}} h f_0(\hat{\mathbf{w}} + s(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q}))^{\frac{-1}{1+m}} - \mathbf{q} s(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q}) \right]}{h \left( 1 + f_1(\hat{\mathbf{w}} + s(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q}))^{\frac{m}{1+m}} \right) \left( 1 - \frac{\hat{\mathbf{w}}}{(1+m)(\hat{\mathbf{w}} + s(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q}))} \right)} = 1 \quad (19)$$

where:  $s(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q}) = \mathbf{r} - \left( \frac{S-1}{S} \right) \mathbf{g}(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q})$ .

Relation (19) only provides an implicit definition of  $\hat{\mathbf{w}}$ . Fortunately, we can easily extract the sign of the partial derivatives of  $G(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q})$  with respect to its arguments:  $G(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q})$ , if  $S \rightarrow 1$ . Thus, we obtain an implicit relation for the money growth rate ceiling  $\hat{\mathbf{w}}$ :

$$\hat{\mathbf{w}} = \mathbf{w}(\mathbf{t}, \mathbf{q}, \mathbf{g}(\hat{\mathbf{w}}, \mathbf{t}, \mathbf{q})) \quad (20)$$

Using the same reasoning than above, we are able to compute the effect of the tax-rate and the debt ratio on the ceiling  $\hat{\mathbf{w}}$  for close to one values of the consumption elasticity of substitution, since  $\frac{\partial \hat{\mathbf{w}}}{\partial \mathbf{g}} = 0$  in eq. (20) if  $S \rightarrow 1$ . Thus we have:  $\frac{\partial \hat{\mathbf{w}}}{\partial \mathbf{t}} \Big|_{S \rightarrow 1} < 0$  and  $\frac{\partial \hat{\mathbf{w}}}{\partial \mathbf{q}} \Big|_{S \rightarrow 1} > 0$ .

The intuitive explanation of these findings is that seigniorage and income tax are two substitute ways for government finance, while seigniorage and borrowing are complementary, since public debt creates costly interest expenditures in the long-run.

An interesting feature of the model is that the long-run rate of economic growth may exceed the Barro solution<sup>18</sup> ( $g^{Barro}$ , defined in equation (14) above), since money provides resources for productive government spending. As a corollary, contrary to the no-money case, an economy with some positive level of public debt may resort to seigniorage revenues to finance its debt burden. Consequently, its long-run rate of economic growth will go beyond the no-debt no-money solution, if seigniorage revenues are high enough to offset the net burden of public debt. Observe that in such cases, even if the long-run rate of economic growth of an economy with public debt may exceed the Barro solution, the highest long-run rate of economic growth that can be reached for a given rate of money expansion is still the one associated with a balanced-budget rule ( $q = 0$ ).

## V. Maximizing-growth monetary and fiscal policy trade-off

In this section we search for maximizing-growth combinations of monetary and fiscal policies. Observe that relations (18) and (19) are the result of a maximization of growth by only one instrument, keeping the others given. Here, we consider simultaneous changes in the two tax instruments ( $t$  and  $w$ ) that maximize economic growth<sup>19</sup>.

Relations (17), (18) and (19) give only implicit functions for  $\hat{g}$ ,  $\hat{t}$  and  $\hat{w}$ , if  $S \neq 1$ . Since the qualitative properties of the model are not affected by the value of  $S$  (provided that this value is not too high, so that there is a steady-state that verifies the solvability condition), we focus in this section on the  $S = 1$  case<sup>20</sup>. If  $S = 1$ , relation (17) provides an explicit value for the long-run rate of economic growth, which we can inject in relations (18) and (19) to obtain the values of the tax-rate and the money growth ceilings  $\hat{t}$  and  $\hat{w}$  as functions of other parameters. We denote these “reaction functions” as (from (18)):  $\hat{t} = \hat{t}(w, q, \dots)$ , and (from (19)):  $\hat{w} = \hat{w}(t, q, \dots)$ . We obtain an explicit reaction function for  $\hat{t}$ , but only an implicit reaction function for  $\hat{w}$ , but simulations results show that these “reaction functions” are decreasing functions in the  $(w, t)$  plane, a result that we had already established in the previous section for  $S \rightarrow 1$ . These reaction functions are depicted in *Figure 4*<sup>21</sup>.

In *Figure 4A*, arrows indicate that long-run economic growth increases. Since the  $\hat{t} = \hat{t}(w)$  locus provides the higher value for economic growth in function of  $t$ , economic growth increases on the right hand-side and on the left hand-side of  $\hat{t} = \hat{t}(w)$  as soon as  $t$  gets closer to  $\hat{t}$  for a given  $w$ . Similarly, since the  $\hat{w} = \hat{w}(t)$  locus provides the higher value for economic growth in function of  $w$ , economic growth increases as soon as  $w$  gets closer to  $\hat{w}$  for a given  $t$ .

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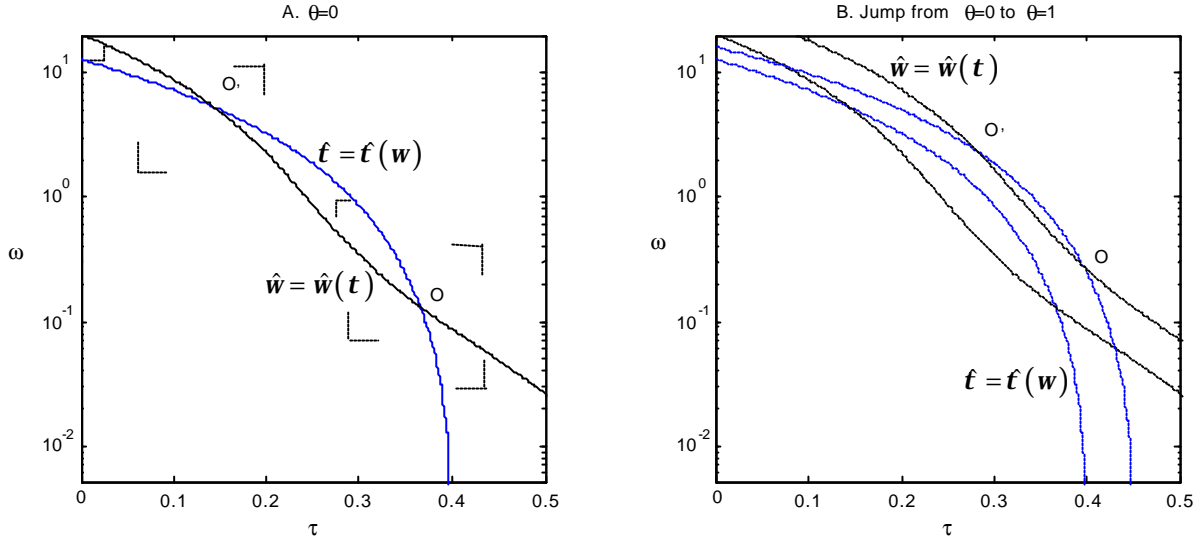
<sup>18</sup> Effectively, using (15) and (17), we show that:  $g > g^{Barro}$  if  $t < \frac{h f_0 w (w + s(g))^{-\frac{1}{1+m}} - q s(g)}{\left[ 1 + f_1 (w + s(g))^{\frac{m}{1+m}} \right]^{\frac{a}{1-a}} - h}$

<sup>19</sup> These two instruments are interesting to study jointly, first because they exhibit threshold effects in their relation with economic growth, while the optimal value for the debt ratio (in long-run growth terms) is always zero, and second because they are two alternative ways of taxation: the “inflation-tax” versus the “income-tax”.

<sup>20</sup> In addition the solvability condition is always verified if  $S = 1$ .

<sup>21</sup> Remark that we use a logarithmic scale for seigniorage, since the two curves intersect for two extreme values of  $w$ .

Fig.4—Reaction functions for maximizing-growth income-tax rate and money growth rate



For:  $r = d = f = 0.05$ ,  $S = A = m = 1$ ,  $a = 0.6$ ,  $h = 0.5$ ,  $h = 0.6$

Finally, we can express the “optimal” couple  $(t^*, w^*)$  which maximizes long-run economic growth, at the point where the two “reaction functions” intersect, since for an interior solution to exist, first order conditions (18) and (19) have to be simultaneously verified. In the most general case, as *Figure 4* shows, there are two candidate points ( $O$  and  $O'$ ) for an interior solution. But some second order conditions have also to hold for a candidate to be a maximum<sup>22</sup>. As we can remark on *Figure 4A*, these conditions are fulfilled only for the  $O$  point, while  $O'$  point is a “saddle path”, where economic growth is not maximized. Simulations of the  $g(w, t, q, \dots)$  function clearly confirm this statement for admissible values of parameters (see *Figures 5* and *6* below).

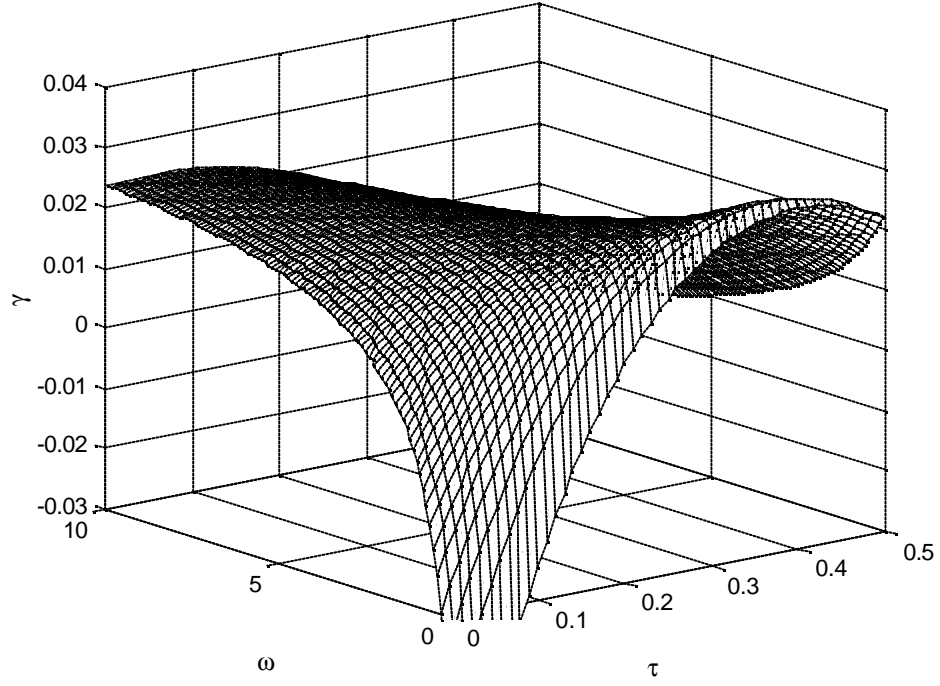
#### Some numerical results

To illustrate the model, we choose:  $S=1$ , in order to verify the solvability condition whatever the values of other parameters are,  $A=1$ ,  $r=0.05$ ,  $d=0.05$ ,  $a=0.6$ , namely Aschauer (1989) findings about the value of public capital elasticity in the production function,  $m=1$  in order to obtain a semi-elasticity of money demand to nominal interest rate of 0.5, consistent with empirical findings, and  $f=0.05$  in order to obtain “reasonable” values for transaction costs. Simulations show that qualitative properties of the model are strongly resistant to changes in these parameters.

<sup>22</sup> Formally, since  $g$  is function of two variables, we must have:  $g_{tt}(t^*, w^*) < 0$ ,  $g_{ww}(t^*, w^*) < 0$ , and:

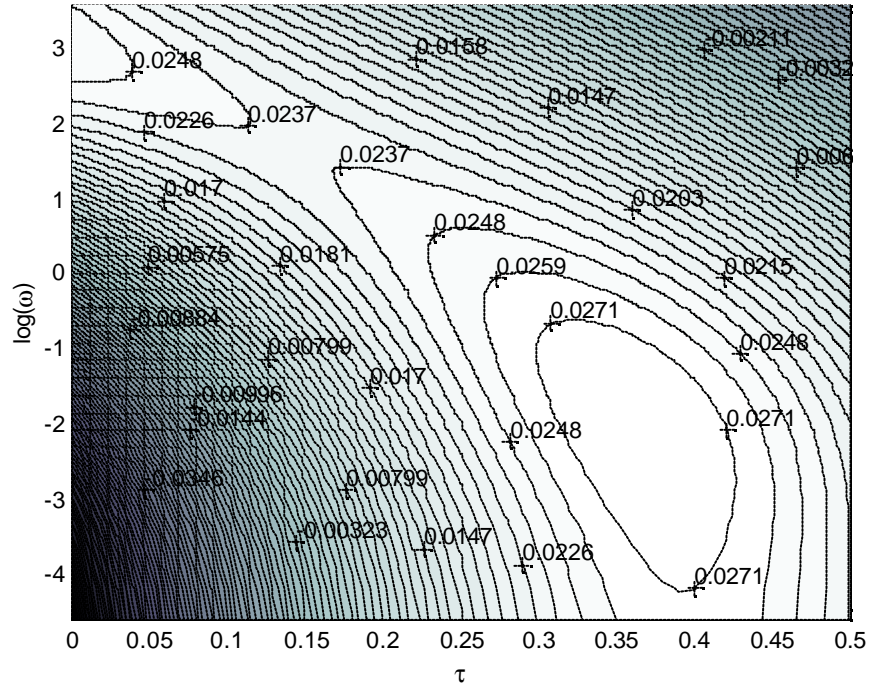
$$g_{tt}(t^*, w^*)g_{ww}(t^*, w^*) - (g_{tw}(t^*, w^*))^2 > 0.$$

Fig.5: Economic growth as a function of  $\mathbf{t}$  and  $\mathbf{w}$



For:  $\mathbf{r}=\mathbf{d}=\mathbf{f}=0.05$ ,  $\mathbf{S}=\mathbf{A}=\mathbf{m}=1$ ,  $\mathbf{a}=0.6$ ,  $\mathbf{h}=0.5$ ,  $\mathbf{h}=0.6$ ,  $\mathbf{q}=0$

Fig.6: Contour lines of economic growth as a function of  $\mathbf{t}$  and  $\mathbf{w}$



For:  $\mathbf{r}=\mathbf{d}=\mathbf{f}=0.05$ ,  $\mathbf{S}=\mathbf{A}=\mathbf{m}=1$ ,  $\mathbf{a}=0.6$ ,  $\mathbf{h}=0.5$ ,  $\mathbf{h}=0.6$ ,  $\mathbf{q}=0$

The optimum solution is more sensitive to “financial repression” and “tax evasion” parameters  $h$  and  $\mathbf{h}$ . For intermediate values of the “money multiplier” ( $1/h$ ) and tax-collecting costs ( $\mathbf{h}$ ), for example:  $h=0.5$  and  $\mathbf{h}=0.6$ , the highest rate of economic growth ( $\mathbf{g}^* = 2.8\%$ ) is obtained for the interior solution:  $\mathbf{t}^* = 36.6\%$  and  $\mathbf{w}^* = 12.8\%$ , as in *Fig.5*. *Fig.6* depicts contour lines of growth in the  $(\mathbf{t}, \mathbf{w})$  plane, namely combinations of the tax rate and the money growth rate leading to the same long-run rate of economic growth (“iso-growth” locus), which clearly show that the  $(\mathbf{t}, \mathbf{w})$  couple which maximizes growth is an interior solution.

For high or low values of the money multiplier and/or of the cost of collecting taxes, the two reaction functions eventually does not intersect on  $O$  point, or this point eventually coincides with negative values of  $\mathbf{t}$  or  $\mathbf{w}$ , which we exclude. In such cases, the highest rate of economic growth is obtained for a corner solution (since  $O'$  point never is a maximum).

*Figure 7* depicts sensitivity of the maximizing-growth solution to changes in  $h$  and  $\mathbf{h}$ . If tax evasion coefficient ( $\mathbf{h}$ ) or money multiplier ( $1/h$ ) are high, maximizing-growth governments have to use income taxation only, without resort to seigniorage:  $\bar{\mathbf{t}} > 0$  and  $\bar{\mathbf{w}} = 0$ . This corner solution is obtained at the tangency point between the highest contour line of economic growth and the  $\mathbf{w} = 0$  axis, as in the south-west plots of *Fig.7* below. This situation may depict most of developed countries, in which tax evasion is small and which possess large financial sectors with little financial repression. In our model, such countries must optimally (in terms of long-run economic growth) choose low inflation monetary policies, and government finance must be essentially based on income taxation.

On the contrary, in countries strongly repressed (high  $h$ ), money multiplier is low and central bank seigniorage taxes a large part of the stock of real balances. Additionally, countries with massive tax evasion (low  $\mathbf{h}$ ) will not be able to finance public spending in another way than by using seigniorage. The north-east plots of *Figure 7* illustrate these cases: to maximize long-run economic growth, government must only resort to seigniorage, with a zero tax-rate on income:  $\bar{\mathbf{t}} = 0$  and  $\bar{\mathbf{w}} > 0$ . This corner solution is obtained at the tangency point between the highest contour line of economic growth and the  $\mathbf{t} = 0$  axis.

For intermediate values of tax evasion and financial repression coefficients, the optimal strategy for government finance is a mix between income taxation and seigniorage, as in *Figure 6*.

These results can easily be interpreted. In financial developed countries, seigniorage represents only a very small part of government resources, and it is more efficient to finance public spending exclusively by a flat tax rate on income. In terms of economic growth seigniorage is more costly than the income tax, because of a “seigniorage flight” to private banks. In countries with narrow financial markets, on the contrary, seigniorage is an essential resource for government<sup>23</sup>. Furthermore, in LDC, the lack of financial market is generally coupled with large cost of collecting taxes, which makes seigniorage even more attractive, because fiscal evasion causes an “income-tax flight”. Thus our model may explain why some

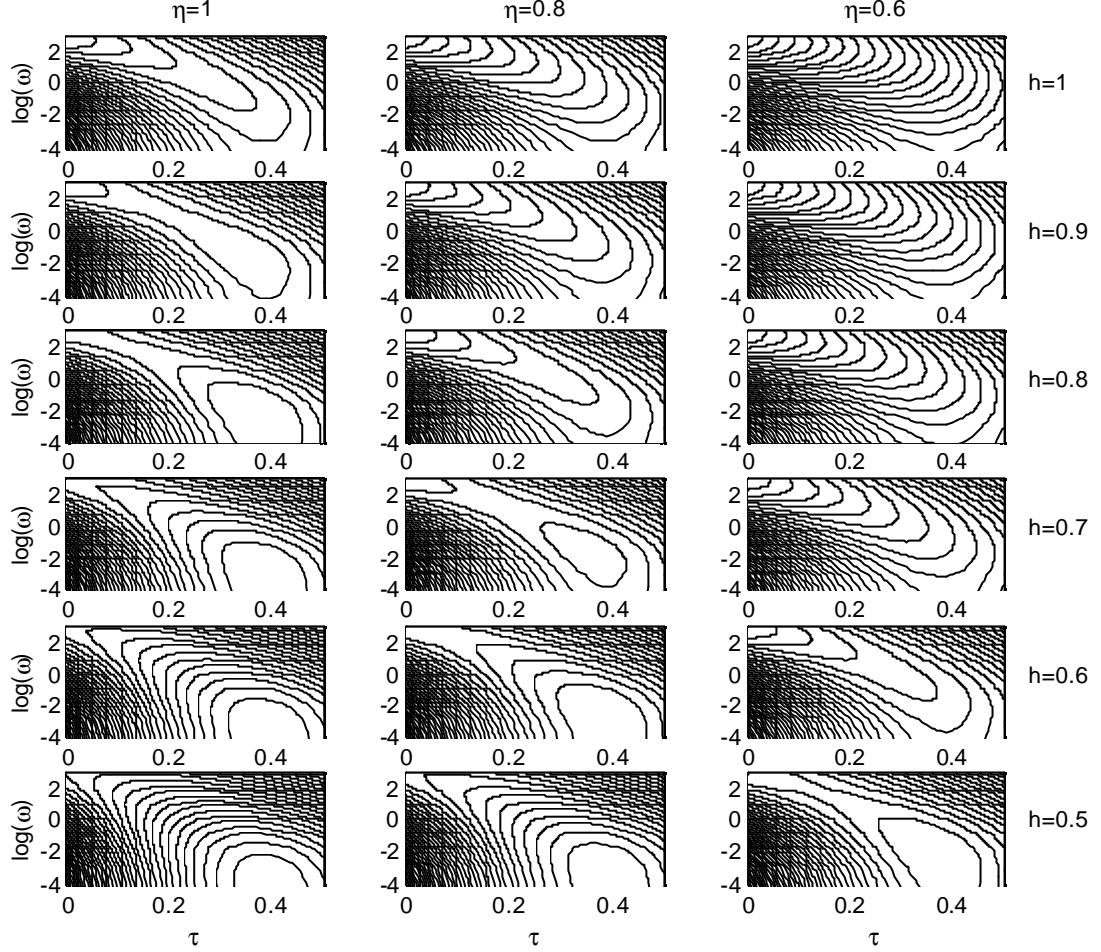
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<sup>23</sup> For example, Fischer (1982), and more recently, Cukierman, Edwards & Tabellini (1992) show that seigniorage revenues are an important source of government revenue in many countries, especially in developing countries, while McKinnon (1973) and Giovannini & De Melo (1993) maintain that financial repression is an important source of government revenue for many LDC countries.



governments resort to seigniorage and inflationary finance, and others rather resort to high tax-rates, as a result of maximizing-growth strategies in different structural environments (notably concerning tax evasion and financial repression).

Fig.7: Contour lines of economic growth—Sensitivity analysis



For:  $\mathbf{r} = \mathbf{d} = \mathbf{f} = 0.05$ ,  $S = A = \mathbf{m} = 1$ ,  $\mathbf{a} = 0.6$ ,  $\mathbf{q} = 0$

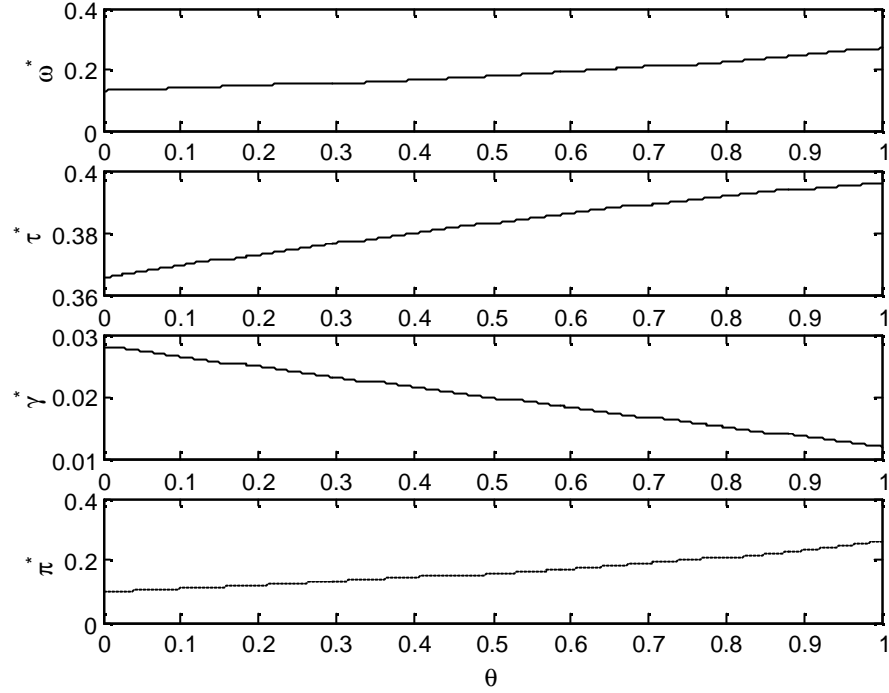
#### Impact of a change in the debt ratio

Simulations allow computing the effect of public debt on the two reaction functions and on the maximizing-growth couple  $(\mathbf{t}^*, \mathbf{w}^*)$ . As we can see on *Figure 4B*, the two reaction functions simultaneously move upwards when the public debt to income ratio is higher, with an increase in both optimal seigniorage  $\mathbf{w}^*$  and optimal income tax  $\mathbf{t}^*$ .

*Figure 8* depicts the optimal values (in term of long-run growth) of tax and seigniorage rates, and the associated long-run rates of economic growth and inflation, provided that an interior solution exists. As we can verify, public debts are positively related to seigniorage. From *Figure 8*, we can also remark that the long-run inflation rate corresponding to the maximized rate of economic growth  $(\mathbf{p}^* = \mathbf{w}^* - \mathbf{g}^*)$  is positively associated with the ratio of public debt to output. The correlation between public deficits,

inflation rates and seigniorage revenues is fully documented in empirical literature (see, e.g. Edwards & Tabellini, 1991 and Roubini, 1991 for evidence in LDC). But what our findings show is that this correlation may be the result of a maximizing economic growth policy.

Figure 8—Maximizing-growth couples  $(t^*, w^*)$  in function of the public debt ratio



For:  $r = d = f = 0.05$ ,  $S = A = m = 1$ ,  $a = 0.6$ ,  $h = 0.5$ ,  $h = 0.6$

For higher values of the debt ratio, or for other parameter values, corner solutions eventually appear, with only one active policy instrument. In such corner solutions, the impact of changes in the public debt ratio may be quite different, since there can be a “regime switch” in government finance for some parameter values. This case is studied in details in *Appendix 2* for the “cash-in-advance” specification of the model ( $m \rightarrow \infty$ ) in which analytical results can be obtained. In the CIA specification, the optimal government finance strategy is a “bang-bang” strategy with a debt ratio ceiling, say  $\bar{q}$  (whose value is computed in *Appendix 2*). High-debt countries ( $q > \bar{q}$ ) must resort only to seigniorage, while low-debt countries ( $q < \bar{q}$ ) must resort exclusively to income taxation. However, corner solutions do not fit well the data, since most of countries adopt interior solutions for government finance, with positive seigniorage and income tax rates. Furthermore, the seigniorage only strategy (which is the strategy that maximizes long-run economic growth in the CIA model of Palivos & Yip, 1995, for example) gives rise to incredibly large inflation rates. Nevertheless such corner solutions may eventually depict some extreme situations in high public debt countries with inflationary finance.

## VI. Conclusion

In an endogenous growth model with transaction costs, we have showed that, to finance public expenditures, a maximizing-growth government must choose relatively high seigniorage (respectively income taxation) if “tax evasion” and “financial repression” coefficients are high (respectively low). Thus our model may explain why some governments resort to seigniorage and inflationary finance, and others rather resort to high tax-rate, as a result of maximizing-growth strategies in different structural environments. Governments of countries with inefficient tax systems (high tax evasion) and/or low level of financial development may optimally choose seigniorage as a way of government finance, with associated high rates of inflation. In addition, our model shows that, if governments adopt maximizing-growth strategies, the resulting inflation rate will be positively associated to the public debt ratio.

This finding is in line with Roubini & Sala-i-Martin (1995) analysis, but in a rather different framework with explicit thresholds effects of both monetary and fiscal policies. Moreover, contrary to Roubini & Sala-i-Martin (1995), in our model such inflationary finance does not necessarily conduct to poor growth performances, but on the contrary may be the result of a maximizing-growth strategy for governments, because seigniorage allows making productive public expenditures, which are growth enhancing. Therefore, the threshold in the relation between seigniorage and economic growth may be very high, a result that conforms well to the existing empirical evidence (for example, Bruno & Easterly, 1998, observe a significantly negative correlation between inflation and real activity only when inflation is fairly high, about 40%).

Our model is also consistent with numerous empirical evidences, concerning the role of financial repression or financial development in economic growth. For example, Giovannini & De Melo (1993) provide evidences of a high positive correlation between seigniorage and financial repression and suggest that these two variables are complementary. Similarly, Boyd, Levine & Smith (1997) find evidences that inflation is negatively correlated with indicators of financial development, but that this relation is subject to substantial non linearities. Our model exhibits such a positive correlation between financial repression and seigniorage or inflation (owning to the fact that financial repression is negatively linked to financial development, as emphasize Haslag & Koo, 1999), but as a result of maximizing-growth economic policies. Concerning the relation between tax systems and financial repression, McKinnon (1973, 1991) and Fry (1988) presents evidence that inefficient tax systems and high tax evasion are associated with financial repression (see also Cukierman, Edwards & Tabellini, 1992). All these papers provide empirical evidence of a correlation between inefficient tax systems, large seigniorage, and high inflation rates, notably for developing countries, a correlation that our model establishes in a theoretical set-up.

The strong non-linear association between monetary and fiscal policies and economic growth that our model exhibits requires further empirical work, in the lines of Adam & Bevan (2005). Since the ceilings of seigniorage and income tax are functions of public debt and other parameters, our model puts the case for an empirical strategy that clearly purge regressions of policy indicators (such as the fiscal stance) or structural indicators (such as financial repression or financial development indicators), as do for example, Boyd, Levine & Smith (1997). Such empirical tests of our model constitute interesting research topics for future work.

## Appendix 1: Dynamics and local stability of the steady-state

To obtain the reduced form of the model, we define “intensive” variables:  $c_k \equiv c/k$ ,  $b_k \equiv b/k$ ,  $g_k \equiv g/k$  and  $m_k \equiv m/k$ . Subtracting (11.3) from (11.1), (11.2), (11.5) and (11.6) and (11.7), we get a four variables reduced form driving the evolution of  $c_k, R, b_k$  and  $g_k$ :

$$\frac{\dot{c}_k}{c_k} = S \left[ \mathbf{a} (1-t) q(R) A g_k^{1-a} - \mathbf{r} - \mathbf{d} \right] - (A g_k^{1-a} - c_k - g_k - \mathbf{d}) \quad (\text{A1})$$

$$\frac{\dot{R}}{R} = \left( \frac{1+m}{m(1-q(R))} \right) \left[ r + \mathbf{d} - \mathbf{a} (1-t) q(R) A g_k^{1-a} \right] \quad (\text{A2})$$

$$\frac{\dot{b}_k}{b_k} = \frac{d A g_k^{1-a}}{b_k} - (A g_k^{1-a} - c_k - g_k - \mathbf{d}) \quad (\text{A3})$$

$$\frac{\dot{g}_k}{g_k} = \frac{1}{1-a} \left( \mathbf{w} + r - R + \frac{\dot{R}}{(1+m)R} - (A g_k^{1-a} - c_k - g_k - \mathbf{d}) \right) \quad (\text{A4})$$

with:  $r = \frac{(\mathbf{h}t + \mathbf{d}) A g_k^{1-a} + \mathbf{h} \mathbf{w} m_k - g_k}{b_k}$  and  $m_k = \mathbf{f}_0 A g_k^{1-a} R^{\frac{-1}{1+m}}$  are endogenous variables.

*Local stability of steady-state*

	$\mathbf{h} = 0.25$	$\mathbf{h} = 0.5$	$\mathbf{h} = 0.75$
$h = 0.25$	-2.05; 0.29; 0.27; 0.0826	-3.91; 0.42; 0.27; 0.0488	-5.67; 0.49; 0.26; 0.0184
$h = 0.5$	-2.62; 0.28+0.05i; 0.0753	-4.46; 0.41; 0.28; 0.0425	-6.21; 0.49; 0.25; 0.0124
$h = 0.75$	-3.18; 0.29+0.07i; 0.0683	-5.01; 0.41; 0.28; 0.0362	-6.75; 0.49; 0.25; 0.0064

$$\mathbf{a} = 0.6; \mathbf{r} = 0.1; \mathbf{d} = 0.025; \mathbf{j} = 0.05; S = A = \mathbf{m} = 1; \mathbf{q} = 0.5; \mathbf{w} = 0.2; \mathbf{t} = 0.4$$

	$\mathbf{t} = 0.3$	$\mathbf{t} = 0.4$	$\mathbf{t} = 0.5$
$\mathbf{w} = 0.1$	-3.41; 0.26+0.02i; 0.0535	-4.46; 0.31; 0.26; 0.0450	-5.50; 0.34; 0.27; 0.0436
$\mathbf{w} = 0.2$	-3.54; 0.37; 0.27; 0.0493	-4.46; 0.41; 0.28; 0.0425	-5.37; 0.44; 0.29; 0.0424
$\mathbf{w} = 0.3$	-3.65; 0.47; 0.27; 0.0466	-4.50; 0.51; 0.29; 0.0410	-5.33; 0.55; 0.30; 0.0418

$$\mathbf{a} = 0.6; \mathbf{r} = 0.1; \mathbf{d} = 0.025; \mathbf{j} = 0.05; S = A = \mathbf{m} = 1; \mathbf{q} = 0.5; h = \mathbf{h} = 0.5$$

	$\mathbf{q} = 0.1$	$\mathbf{q} = 0.2$	$\mathbf{q} = 0.3$	$\mathbf{q} = 0.4$	$\mathbf{q} = 0.5$
$S = 0.5$	-24.04; 0.45; 0.28; 0.0091	-11.91; 0.43; 0.28; 0.0118	-7.87; 0.4; 0.29; 0.0144	-5.85; 0.39; 0.30; 0.0169	-4.65; 0.36; 0.31; 0.0193
$S = 1$	-23.69; 0.48; 0.27; 0.0191	-11.67; 0.46; 0.27; 0.0251	-7.67; 0.45; 0.27; 0.0310	-5.66; 0.43; 0.27; 0.0369	-4.46; 0.41; 0.28; 0.0425
$S = 0.5$	-23.33; 0.50; 0.26; 0.0301	-11.42; 0.49; 0.26; 0.0402	-7.44; 0.48; 0.26; 0.0505	-5.45; 0.46; 0.26; 0.0609	-4.24; 0.45; 0.27; 0.0713

$$\mathbf{a} = 0.6; \mathbf{r} = 0.1; \mathbf{d} = 0.025; \mathbf{j} = 0.05; A = \mathbf{m} = 1; \mathbf{t} = 0.4; \mathbf{w} = 0.2; \mathbf{q} = 0.5; h = \mathbf{h} = 0.5$$

In our model there are one state variable, namely  $b_k$  (as both  $b$  and  $k$  are state variables), and three jump variable, namely  $c_k, R$  and  $g_k$  (because  $c, R$  and  $g$  jump). As we obtain

one negative and three positive eigenvalues, Blanchard-Kahn conditions imply that the equilibrium is a saddle point, *i.e.* for any given value  $b_{k0}$ , jump variables initially adjust to values  $c_{k0}$ ,  $R_0$  and  $g_{k0}$ , to put the economy on the unique trajectory leading to the steady state.

## Appendix 2: The Cash-In-Advance version of the model

If  $m \rightarrow \infty$ , our model collapses with a CIA model with a money constraint on all expenditures<sup>24</sup>. The long-run rate of economic growth is determined by the following implicit relation:

$$H(\mathbf{g}, \mathbf{t}, \mathbf{w}, \mathbf{q}) = \frac{\mathbf{a} A^{1/a} (1-\mathbf{t}) [\mathbf{h}\mathbf{t} + \mathbf{h}\mathbf{w} - \mathbf{q}s(\mathbf{g})]^{\frac{1-a}{a}}}{1 + \mathbf{w} + s(\mathbf{g})} - \mathbf{r} - \mathbf{d} - \frac{\mathbf{g}}{S} = 0 \quad (\text{A5})$$

First order conditions for the maximization of long-run economic growth are:

$$\frac{\partial H}{\partial \mathbf{t}} = X [\mathbf{h}(1-\mathbf{a}) - \mathbf{h}\mathbf{t} - (\mathbf{a}\mathbf{h}\mathbf{w} - \mathbf{a}\mathbf{q}s(\mathbf{g}))] = 0 \quad (\text{A6})$$

$$\frac{\partial H}{\partial \mathbf{w}} = Y [(1-\mathbf{a})\mathbf{h}(1 + \mathbf{w} + \mathbf{r}) - \mathbf{a}(\mathbf{h}\mathbf{t} + \mathbf{h}\mathbf{w} - \mathbf{q}s(\mathbf{g}))] = 0 \quad (\text{A7})$$

where:  $X = A^{1/a} [\mathbf{h}\mathbf{t} + \mathbf{w}\mathbf{h} - \mathbf{q}s(\mathbf{g})]^{\frac{1-2a}{a}} [1 + \mathbf{w} + \mathbf{r}]^{-1}$  and  $Y = (1-\mathbf{t}) X [1 + \mathbf{w} + \mathbf{r}]^{-1}$ .

First order conditions (A6) and (A7) simultaneously give rise to one unique candidate couple  $(\tilde{\mathbf{w}}, \tilde{\mathbf{t}})$  for maximizing economic growth<sup>25</sup>. However, in the CIA case,  $(\tilde{\mathbf{w}}, \tilde{\mathbf{t}})$  couple never is a maximum, but correspond to  $O'$  point of *Figure 4*, namely a saddle path. Effectively, computing second order derivatives from relations (A6-A7), we obtain:

$$\frac{\partial^2 H}{\partial \mathbf{t}^2}(\tilde{\mathbf{t}}, \tilde{\mathbf{w}}) = -\frac{\mathbf{h}X}{\mathbf{a}} < 0, \quad \frac{\partial^2 H}{\partial \mathbf{w}^2}(\tilde{\mathbf{t}}, \tilde{\mathbf{w}}) = \frac{(1-\mathbf{t})X(1-2\mathbf{a})\mathbf{h}}{1 + \mathbf{w} + \mathbf{r}} < 0 \quad \text{and} \quad \frac{\partial^2 H}{\partial \mathbf{t} \partial \mathbf{w}}(\tilde{\mathbf{t}}, \tilde{\mathbf{w}}) = -\mathbf{h}X < 0.$$

Thus:  $\frac{\partial^2 H}{\partial \mathbf{t}^2} \frac{\partial^2 H}{\partial \mathbf{w}^2} - \left( \frac{\partial^2 H}{\partial \mathbf{t} \partial \mathbf{w}} \right)^2 = - \left[ \frac{\mathbf{h}(1-\tilde{\mathbf{t}})(1-2\mathbf{a}) + \mathbf{a}\mathbf{h}(1 + \tilde{\mathbf{w}} + \mathbf{r})}{\mathbf{a}(1 + \tilde{\mathbf{w}} + \mathbf{r})} \right] \mathbf{h}X^2$ ; and, since (A6) and

(A7) give rise to:  $\mathbf{h}(1 + \tilde{\mathbf{w}} + \mathbf{r}) = \mathbf{h}(1 - \tilde{\mathbf{t}})$ , we can compute:

$$\frac{\partial^2 H}{\partial \mathbf{t}^2} \frac{\partial^2 H}{\partial \mathbf{w}^2} - \left( \frac{\partial^2 H}{\partial \mathbf{t} \partial \mathbf{w}} \right)^2 = -\frac{\mathbf{h}(1-\tilde{\mathbf{t}})(1-\mathbf{a})\mathbf{h}X^2}{\mathbf{a}(1 + \tilde{\mathbf{w}} + \mathbf{r})} < 0. \quad \text{Thus, the } (\tilde{\mathbf{w}}, \tilde{\mathbf{t}}) \text{ couple is not a}$$

maximum for economic growth, and maximizing-growth solutions are always corner solutions, with only one active policy instrument.

<sup>24</sup> As we have seen, results would be qualitatively unchanged with a CIA constraint on investment and consumption only.

<sup>25</sup> If  $S = 1$ , for example, we find:  $\tilde{\mathbf{w}} = \frac{\mathbf{a}(\mathbf{h} - \mathbf{q}\mathbf{r}) - \mathbf{h}(1 + \mathbf{r})}{(1 - \mathbf{a})\mathbf{h}}$  and:  $\tilde{\mathbf{t}} = 1 - \mathbf{a} - \frac{\mathbf{a}}{\mathbf{h}} \left[ \frac{\mathbf{a}\mathbf{h} - \mathbf{q}\mathbf{r} - \mathbf{h}(1 + \mathbf{r})}{(1 - \mathbf{a})} \right]$ .

If  $S=1$ , for example, optimal couple is, depending on parameters:  $w^{*1}=0$  and (from (A6)):  $t^{*1}=1-a+\frac{aqr}{h}$ , or:  $t^{*2}=0$  and (from (A7)):  $w^{*2}=\frac{(1-a)h(1+r)+aqr}{(2a-1)h}$ , with the associated maximized solutions for economic growth  $g^{*1}$  or  $g^{*2}$ :

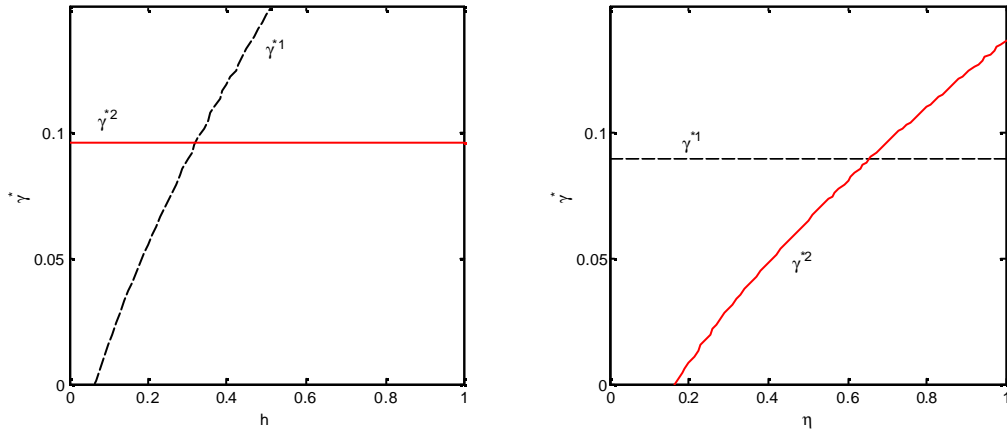
$$g^{*1} = \frac{aA^{1/a}(1-t^{*1})[ht^{*1}-qr]^{\frac{1-a}{a}}}{1+r} - r - d$$

and:

$$g^{*2} = \frac{aA^{1/a}[hw^{*2}-qr]^{\frac{1-a}{a}}}{1+w^{*2}+r} - r - d$$

Figure 9 shows that  $g^{*1}$  dominates  $g^{*2}$  if tax evasion coefficient  $h$  is low and/or if the money multiplier is low ( $h$  is large), and conversely. Thus, to maximize long-run economic growth, countries with a large financial system or with low costs of collecting taxes have to resort only to income tax, while countries with less financial developed sectors or with large tax flight must resort exclusively to seigniorage as government finance. These results are in accordance with previous section findings in the general case, but the CIA special case of the model brings their logic to extremes.

Figure 9—Maximizing-growth trade-off in the CIA case



For:  $r=d=0.025$ ,  $S=1$ ,  $a=0.6$ ,  $A=1$ ,  $h=0.3$ ,  $h=0.7$

Let us now turn our attention on the effect of the public debt to income ratio on the optimal choice of policy instruments. Denote  $g^{*1}$  and  $g^{*2}$  as a function of the public debt ratio  $q$ . We can find a ceiling value of  $q$ , say  $\bar{q}$  such as:  $g^{*1}(\bar{q})=g^{*2}(\bar{q})$ .  $\bar{q}$  is such as:

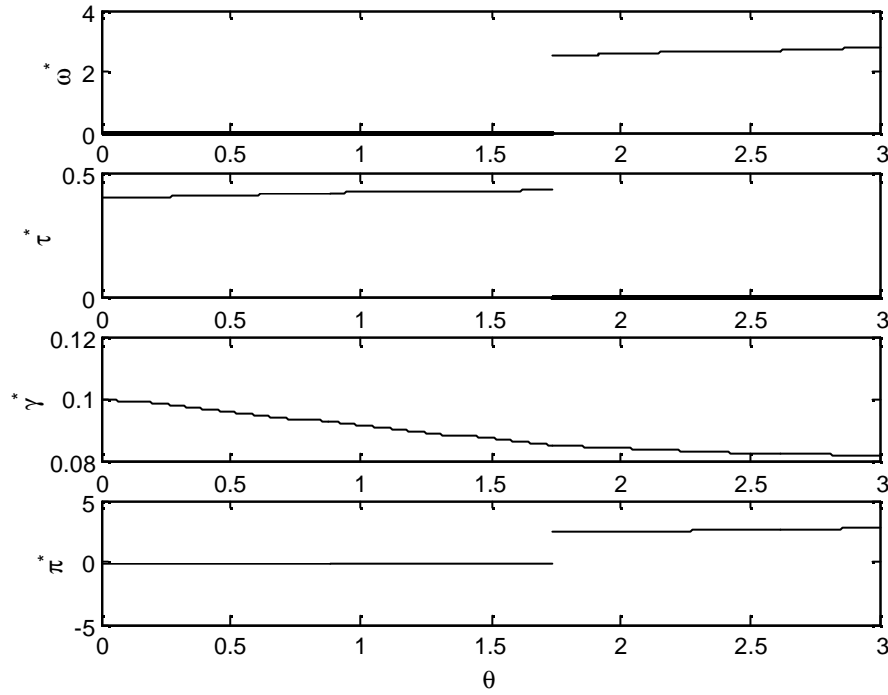
$$g^{*1}=g^{*2} \Leftrightarrow F(h-\bar{q}r) = \left[1 + \frac{\bar{q}r}{h(1+r)}\right]^{1-2a} \quad \text{where:} \quad F \equiv \left(\frac{a^2}{h}\right)^a (h(1+r))^{a-1} (2a-1)^{1-2a}.$$

Since  $qr \rightarrow 0$ , we can use a logarithmic approximation to find the value of the public debt

ceiling:  $\bar{q} = \frac{hF-1}{r} \left[ F + \frac{1-2a}{h(1+r)} \right]^{-1}$ . High-debt countries ( $q > \bar{q}$ ) must resort only to

seigniorage, while low-debt countries must resort exclusively to income taxation. Thus, the optimal policy mix is a “bang-bang” solution: in accordance with previous section results, optimal tax and seigniorage rates are positively linked to the public debt ratio, and long-run maximized rate of economic growth is negatively related to this ratio, but there a regime switch for  $q = \bar{q}$  ( $\bar{q} = 1.7$  in *Figure 10*). These findings are depicted in the following figure:

*Figure 10—Maximizing-growth couples  $(t^*, w^*)$  in function of the public debt ratio:  
The CIA case*



For:  $r = d = 0.025$ ,  $S = 1$ ,  $a = 0.6$ ,  $A = 1$ ,  $h = 0.3$ ,  $\bar{h} = 0.7$

If long-run maximizing growth solutions are corner solutions, as it is always the case in the CIA version of the model, there is a switch in the association between optimal economic growth, or inflation, and optimal seigniorage and tax rates. Such regime switches also appear for corner solutions in the general model. These corner solutions however often give rise to implausible long-run rates of seigniorage, as we can notice in *Figure 10* for  $q > \bar{q}$ .

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