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Inequality and growth:
A joint analysis of demand and supply

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Abstract

The paper contributes to the literature of income inequality and economic growth in two directions. First, it analyzes the impact of income inequality on economic growth both through the supply of human capital and the incentive to invent induced by the demand for better quality goods. Secondly, I decompose Gini-coefficient in two variables, which have different effect on the economic performance. Thus, our result suggests that the empirical research on the base of Gini-coefficient can not generate an overall relationship between income inequality and economic growth.

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1. Introduction

The relationship between a country’s income distribution and its economic growth is far from consensus. Some cross-country studies (e.g., Berg and Sachs 1988, Persson and Tabellini 1994, Alesina and Rodrik 1994, Clarke 1995) show that Gini-coefficient, as a measure of income inequality, has a negative impact on long run growth rates. Nonetheless, there also is evidence that Gini-coefficient has a positive impact on short or medium run growth rates (Forbes 2000), or that the relationship between the income distribution and the long run growth rate is non-linear (Chen 2003, Banerjee and Duflo 2003). The ambiguous empirical results imply us that there is not an overall relationship between income inequality and economic growth (Barro 2000). Hence, it is more important for economists to develop theoretical models, to illustrate the possible different effects of income inequality on economic growth in different circumstances.

The existing theoretical wisdom has proposed either a negative or a positive relationship between initial inequality and economic growth. The contribution of this paper is to show that both are extreme cases in an integrating simple model. We deepen the analysis of relationship between income inequality and economic growth in two directions. First, in a simple model with two types of individuals, the poor and the rich, we decompose Gini-coefficient in two variables, namely the relative income of the poor and the population of the poor. We argue that they may have different, even opposite, effects on the economic growth under certain conditions.

Secondly, we combine the supply of production factors and the demand for the new quality goods in a general equilibrium model. Thus, income inequality has two channels to affect the economic performance: the supply side and the demand side. As most literature argues, income inequality reduces the aggregate of human capital investment, given a neoclassical production function of investment and imperfect capital market. Consequently, inequality has a negative effect on the supply of the consumption goods. Hence, we name this effect as “the supply side effect”. The main arguments of the supply side effect are included in the survey of Benabou 1996. On the other hand, following the literature of endogenous growth theory with quality-improving innovation (Aghion and Howitt 1998, Zweimüller et al. 2000), we argue that the engine of economic growth is innovation, which can improve the quality of goods and, in turn, the utility of consumers. After successful innovation, there exists a monopolist, who
can produce this new invented good alone, hence, the incentive of innovation is the monopolistic profit. Income distribution can affect the demand for the new invented goods, in turn, the price and profit of monopolist. Hence, we name it as “the demand side effect”.

As we assume that there are only two types of individuals, the monopolistic supplier of the new invented good could set the price either at the separating level, i.e., only the rich is able to buy it, or the pooling level, at which even the poor can buy, too. Because income distribution has different effects on the profit in these two cases, in general, the relationship between inequality and economic growth is non-linear. Inequality may give rise to a higher incentive for firms to innovate because rich consumers can pay more for high quality goods than the poor. However, on the other side, the relatively small market share of high quality goods implied by inequality impedes the spread of better quality goods.

This paper shows that in a separating equilibrium, a lower relative income of the poor is good for innovation, and a larger population share of the poor is bad for innovation. This result is consistent with Foellmi and Zweimüller (2002) and Shen (2004). But in Foellmi et al. (2002) they introduce hierarchic preferences\(^2\), and the innovation induces new goods but not the improvement of quality. And Shen (2004) considers the interdependent relationship between the relative income of the poor and the population share of the poor. In the pooling equilibrium, the lower relative income of the poor is bad for innovation, and the population of the poor has no effect on innovation. The threshold value which distinguishes these two equilibria depends on the strength of the supply side effect. If the supply side effect is strong enough, the relative income of the poor has a dominant negative effect on innovation.

This paper integrates two main strands of theory relating growth and income inequality. Recent surveys of the supply side effect are Benabou (1996) and Aghion et al. (1999), where three broad categories corresponding to the main feature stressed: imperfect financial market, political economy and social unrest. And the demand side effect is illustrated in papers of Zweimüller et al. (2000, 2002, and 2004).

The rest of this paper is organized as follows. Section 2 outlays the basic framework. Section 3 analyzes the equilibrium. And finally, section 4 concludes.

\(^2\) “A hierarchy of wants implies that goods can be ranked according to their priority in consumption” (Foellmi and Zweimüller 2002)
2. The Model

We consider a closed economy with two types of individuals: the poor and the rich. They work for firms and consume products of firms. There are two kinds of goods: standard good and quality good. The quality improves over time due to innovation. Hence, the innovation rate represents the growth rate of economy in the current model. There are two channels, through which inequality can effect innovation: the aggregate supply of production factor, here, labor, and the demand for the best quality good. Income distribution can be decomposed to two parameters: the population share of the poor and the income of the poor. They have also different effect on economic performance.

2.1 The Environment

Time is discrete, indexed by \( t = 1, 2, ..., \) and at each point in time there is a continuum of individuals who live for one period. The population size is constant over time and normalized as 1. Individuals, within as well as across generations, are identical in their preference. However, they may differ in their family wealth and thus, due to absence of perfect financial market, in their capability to invest in human capital. For simplicity, we assume that there are two kinds of individuals: the poor and the rich, their population shares are \( \beta \ (0 < \beta < 1) \) and \( 1 - \beta \), respectively. The average wealth is denoted by \( V \). The poor individual has \( A_p = dV \), where \( d \ (0 < d < 1) \) measures the wealth of the poor relative to the average level \( V \). As a result, the rich has \( A_r = \frac{1 - d\beta}{1 - \beta} V \). For simplicity, we assume that the family wealth should not be eaten and can be transferred from generation to generation, thus, there is no social mobility in this simply model.

At birth, individual \( i \) receives a mount of dividend \( \theta A_i \), where \( \theta \) is a constant dividend rate. She invests it in human capital inelastically. Human capital is measured by the labor units. We assume that the production function of labor is \( L_i = a + H_i = a + h(\theta A_i) \), where \( a \ (a > 0) \) is constant, represents the basic supply of labor without any human capital investment, and human capital \( H_i = h(\theta A_i) \) is a strictly concave increasing function satisfying the neoclassical boundary conditions, and \( h(0) = 0 \). This transformation from wealth to human capital is
realized by a benevolent education sector, which needs $s$ units of labor and keeps budget balance in equilibrium.

We assume a simplistic view regarding the production of consumption goods. Labor is the single productive factor, and every individual inelastically supplies her all labor units to the competitive labour market. Consequently, wage $w$ is the same for all. The rich have more labor units than the poor due to the higher education level, as a result, income $y_i = wL_i$, $i = p, r$.

There are two kinds of goods, referred to as standard good and quality good, respectively. Let $x$ be the quantity of the standard good, which has a constant quality (normalized to 1) and is traded in a competitive market. Hence, the price $P_x$ is equal to its marginal cost, which is also normalized to 1. The marginal cost of the standard good can be expressed as $wb$, where the unit labor demand is $b$, which measures how many units of labor are needed to produce one unit of the standard good. We get $P_x = wb = 1$.

The quality goods are firstly invented by researchers. Each innovation introduce a $k$-times better quality good than the existing best one in the next period: $q_j = kq_{j-1}$, $k > 1$, $j = 0, -1, \ldots$. The authority to produce this best quality will be sold to one monopolist through an auction. We assume that researchers prefer to sell the authority to the incumbent as long as its offer is at least same as others’. After successful innovation the current best quality good can be produced under perfect competition with a price equal to the marginal cost from the next period on. We assume that all quality goods have the same marginal cost, denoted by $wa$, where $a$ is again the unit labor demand. The assumption that the unit labor demand of quality good $a$ is equal to $L(0)$ simplifies the calculation without loss of generality. Thus, we have single monopolist who produces the best quality in every period. In order to keep this priority, it has to buy the new innovation from researchers at a price equal to the current value of the future monopolistic profit. This monopolistic firm is owned by individuals. The dividend which it generates in every period is the profit $\pi$ net of the payment to the researchers $R$. Hence, we have:

$$\delta V = \pi - R$$ (1)
Following the work by Aghion and Howitt (1992), innovation is a random process, which will be introduced in section 2.3.

2.2 The Pricing Decision of monopolist

Every individual works and consumes. There is no saving. All income is spent either on the consumption of the standard good or the quality good. Every individual can consume one and only one unit of the quality good $q_j$. There is no limitation to the consumption of the standard good $x_i$ except for the budget constraint, i.e., $y_i = 1 \cdot x_i + P_j \cdot 1 \quad j = 0, -1, \ldots$, where the price of standard goods is 1, the price of the quality $j$ is denoted by $P_j$. The preference for consumption of the standard good and the quality good is given by the following utility function:

$$u_i(x_i, q_j) = \ln x_i + \ln q_j \quad i = p, r \quad \text{and} \quad j = 0, -1 \quad (2)$$

substituting budget constraint, it can also be expressed as: $u_i = \ln(y_i - P_j) + \ln q_j$.

Firms have all the above information but they are unable to distinguish between individuals by income. The strategy, which firms can pursue, is to choose a price while quality is fixed. We concentrate only on the steady state where prices are constant over time. For simplicity, we assume that the consumer prefers better quality goods if both quality goods yield the same utility.

First of all, only the most recent old quality good ($q_{-1}$) can be sold at the price $wa$ in the competitive market of quality goods $q_j \quad j < 0$. Hence, the highest price that the monopoly can offer has to satisfy:

$$\ln(y_i - P_0) + \ln q_0 = \ln(y_i - wa) + \ln q_{-1} \quad (3)$$

The left hand side of this equation is the utility when individuals buy the best quality good $q_0$ and the right hand side is the utility when they consume the second best quality good $q_{-1}$. Only if the best quality good can yield at least the same utility as the second best quality good
to consumers, the consumer prefers buying it. Substituting \( q_o = kq_{-1} \) and rearranging the equation, we get the highest price of the best quality good:

\[
\bar{P}_o = (1 - \frac{1}{k})y_i + \frac{wa}{k} \quad i = p, r
\]  \hspace{1cm} (4)

Hence, the monopolist has two possible price strategies. It can either set price so high to attract only the rich consumers (separating price), or, so low to occupy the whole market (pooling price). The instantaneous profits are as follows:

\[
\pi^{sep} = (1 - \beta)(1 - \frac{1}{k})(y_r^{sep} - wa)
\]  \hspace{1cm} (5)

\[
\pi^{pool} = (1 - \frac{1}{k})(y_p^{pool} - wa)
\]  \hspace{1cm} (6)

The monopolist sets the separating price in steady state, if 1) given the separating strategy before, she has no incentive to deviate;

\[
(1 - \beta)H_r^{sep} \geq H_p^{sep}
\]  \hspace{1cm} (7)

2) the profit of separating strategy in steady state is larger than that of the pooling.

\[
(1 - \beta)H_r^{sep} \geq H_p^{pool}
\]  \hspace{1cm} (8)

2.3 Innovation

As mentioned before, the quality improves over time due to innovation. Following the work by Aghion and Howitt (1992), we assume that the innovation is random and arrives according to a Poisson process with parameter \( \phi \). The researcher can employ \( n \) workers to reach the Poisson arrival rate \( \phi \), i.e., \( \phi = \lambda n \), where \( \lambda \) is the productivity of workers in research. This assumption of innovation means that the success of research depends only on current input, not upon past research. The flow of research cost is \( wn \). And the flow of research benefit is the payment of monopolist \( R \). We assume free entry, which is the traditional assumption of quality-improving model, to get the research arbitrage equation (Aghion and Howitt 2004).
Hence, $wn = R$. Facing the potential competition of the new entrant, the incumbent has to pay the present value of future profits $B$ when innovation takes place. Hence, the flow payment is $R = \phi B$, where $B$ is:

$$B = \sum_{t=1}^{\infty} \left( \frac{\pi}{(1 + \theta)^t} \text{prob} \{ \text{no innovation before } t \} \right) = \sum_{t=1}^{\infty} \left( \frac{\pi (1 - \phi^t)^{-1}}{(1 + \theta)^t} \right)$$

leads to

$$B = \frac{\pi}{\phi + \theta} \quad (9)$$

where $t$ is a time index, $\phi^t = \lambda n^t$ is the expected future arrival rate of innovation, $n^t$ is the expected future number of workers in the research sector, and $\theta$ is interest rate. In steady state, all agents are perfect foresight. Consequently, $\phi = \phi^\circ$ (or, $n = n^\circ$). This leads to:

$$w = \frac{\pi}{\lambda} \quad \left( \frac{1}{\phi + \theta} \right) \quad (10)$$

The underlying intuition is similar to Aghion and Howitt (1992). The left hand side of equation (10) represents the flow cost of research per efficient worker, which decreases in the productivity of research workers $\lambda$. The effect of $\lambda$ on $\phi$ is positive, because the researcher employs more workers to do research, if the productivity of workers increases.

The effect of interest rate is ambiguous. First, it is a discount factor. Hence, the higher $\theta$, the lower is the benefit of research. Therefore, the innovation rate decreases in the interest rate. The other channel through which the interest rate can affect the innovation rate is, that it raises the initial income of individuals. Hence, the higher $\theta$, the larger the human capital. Consequently, the larger the monopolistic profit. It is the positive effect on the innovation rate.

We are now in a position to calculate $V$. Using (1), (9), $R = \phi B$ and $\phi = \phi^\circ$ we have:

$$V = \frac{\pi}{\phi + \theta} \quad (11)$$
In this simple model, only the monopolistic firm has a positive value, while it can generate a positive dividend in every period. The value of monopolist is the discounted future profit. The discount factor is the sum of the interest rate and the innovation rate. Interpretation of $V$ could be versatile. As we assumed at the beginning of this section, it is the private wealth. Hence, current model discusses the general effect of inequality on innovation. We can also suppose that $V$ is the social wealth, and the government distributes it unequally among rural and urban. The stylized example is China, where the disparity between urban and rural residents is ensured by the Chinese household registration (Hukou) system. (Yang and Zhou 1999) Lack of the free immigration between urban and rural areas, Chinese government invests more public goods, such as education, social insurance and infrastructure, in cities than in the rural areas. The result of this paper is helpful to understand this disparity.

3. **Equilibrium**

Rearranging (10), we have two possible innovation rates in the separating and the pooling cases, respectively.

$$
\phi^{\text{sep}} = \lambda (1 - \frac{1}{k})(1 - \beta) H_r^{\text{sep}} - \theta \\
\phi^{\text{pool}} = \lambda (1 - \frac{1}{k}) H_p^{\text{pool}} - \theta
$$

Using $H_r = h(\theta V \frac{1 - d \beta}{1 - \beta})$, $H_p = h(\theta V d)$ and substituting (11), we have

$$
H_r^{\text{sep}} = h(\frac{\theta \pi^{\text{sep}}}{\phi^{\text{sep}} + \theta} \frac{1 - d \beta}{1 - \beta}) \\
H_p^{\text{pool}} = h(\frac{\theta \pi^{\text{pool}}}{\phi^{\text{pool}} + \theta} d)
$$

Again, after substituting (5) and (6), we have

$$
H_r^{\text{sep}} = h(\frac{\theta}{\phi^{\text{sep}} + \theta} (1 - \frac{1}{k}) w H_r^{\text{sep}} (1 - d \beta))
$$

$$
H_p^{\text{pool}} = h(\frac{\theta}{\phi^{\text{pool}} + \theta} (1 - \frac{1}{k}) w H_p^{\text{pool}})
$$
As the single production factor, labor should have a market clearing function, i.e., the supply of labor should be equal to the demand for labor, in equilibrium. The total labor supply is $\beta L_p + (1 - \beta)L_r$, which is equal to $a + \beta H_p + (1 - \beta)H_r$. The demand for labor consists of four parts. First, the research sector needs $n$. Secondly, the quality good sector needs $a$, because every consumer consume one unit of quality good. Thirdly, the standard good sector needs $b(\beta x_p + (1 - \beta)x_r)$. And finally, the education sector needs $s$, which is equal to $\frac{\partial V}{w}$ in equilibrium. Hence, the total demand for labor is $n + a + b(\beta x_p + (1 - \beta)x_r) + s$, which is exactly the total supply of labor. (details see Appendix 1)

**Proposition 1**

If the production function of human capital $h(\bullet)$ satisfies the neoclassical boundary conditions, i.e., $h' > 0$, $h'' < 0$, $h'(0) = +\infty$, $h'(+\infty) = 0$ and $h(0) = 0$, we have unique solutions for the innovation rates $\phi^{\text{sep}}$ and $\phi^{\text{pool}}$ in both the separating and pooling cases. They satisfy equations (12)-(15) and depend on parameters $k, w, \theta, \alpha, \lambda, \beta, d$. If condition $(1 - \beta)H_r^{\text{sep}} \geq H_p^{\text{pool}}$ holds, the monopolist chooses the separating strategy in steady state. And if $(1 - \beta)H_r^{\text{sep}} \geq H_p^{\text{sep}}$, the monopolist has no incentive to deviate.

In order to get an analytic result, we have to specify the function of $h$. Consistent with the literature, we assume that $h(\theta A) = (\theta A)^\alpha$, $0 < \alpha < 1$. $\alpha$ is an indicator measuring the effect of inequality on the supply of human capital. The lower $\alpha$, the smaller is the aggregate human capital given the income distribution $(d, \beta)$. Hence, the larger is the supply side effect of income inequality on economic performance. Suppose $\alpha = 1$, income distribution has no effect on the aggregate supply of human capital.

Thus we can solve out the innovation rates and the effect of income distribution on them:

**Proposition 2**

The effect of inequality on the innovation rate is non-linear and ambiguous. Given assumption $h(\theta A) = (\theta A)^\alpha$, $0 < \alpha < 1$ we have:
1) The effect of \( d \) on \( \phi \) is negative for \( d \in [0, d^*] \) and positive for \( d \in [d^*, 1] \). The threshold value \( d^* \in (0,1) \) satisfies 
\[
(1 - \beta)^{1/\alpha} \left( \frac{1}{d^*} - \beta \right)^{\alpha} = 1.
\]

2) The effect of \( \beta \) on \( \phi \) is negative for \( \beta \in [0, \beta^*] \). In the pooling case \( \beta \in [\beta^*, 1] \), \( \beta \) has no effect on \( \phi \). The threshold value \( \beta^* \in (0,1) \) satisfies 
\[
(1 - \beta^*)^{1/\alpha} \left( \frac{1}{d} - \beta^* \right)^{\alpha} = 1.
\]

**Proof:** see Appendix 2.

Gini-coefficient (here, \((1-d)\beta\)) has no overall effect on the innovation rate, because \( d \) and \( \beta \) have different effects. In particular, their effects are offset each other in the separating case. In this sense, it is important for us to decompose Gini-coefficient in the empirical research. It is also of interest to know the different effects of the relative poorness and the population share of the poor, because they imply the different policy recommendation. In the framework of urban-rural disparity, we argue that the policy decreasing the rural population is expected to induce a higher innovation rate, whereas the policy increasing the rural income could retard the innovation rate in the separating case.

Income inequality can affect not only the supply of production factor (here, labor), but also the demand for the new better quality. The parameter \( \alpha \) measures the strength of the supply side effect. Suppose \( \alpha \to 0 \), the supply side effect becomes greater. The threshold value \( d^* \to 0 \), hence, the effect of \( d \) on \( \phi \) is overall positive. Vice versa, if \( \alpha \to 1 \), the supply side effect is smaller, \( d^* \to \frac{1}{1+\beta} \), i.e., \( \text{Gini}^* \to \frac{\beta^2}{1+\beta} \). Since \( \frac{1}{1+\beta} > \frac{1}{2} \), we could argue that \( d \) has negative effect on the innovation rate over the most range. In reality, Gini-coefficient lies between 0.25 to 0.65 for most countries and most periods, according to the most data. (Deininger and Squire 1996, Barro and Lee 1994). Suppose \( \beta = 0.75 \), then \( d \) is between 0.13 and 0.67, whereas \( d^* = 0.58 \). Hence, the separating case \( d < 0.58 \) is more possible than the pooling case \( d > 0.58 \). In following figure 1, we show different effects of the relative wealth of the poor on the innovation rate in both the extreme cases.
4. Conclusion

This paper investigates the ambiguous relationship between income inequality and economic growth, in a framework of quality-improving growth model. Our contribution is to deepen the analysis of this relationship in two lines. First, we argue that Gini-coefficient, which is used by most of empirical research in this branch, could include too much variables, which have different effects on the economic growth. Hence, we need to decompose Gini-coefficient into different variables. The current model supplies an example that divides Gini-coefficient into the relative income of the poor and the population share of the poor. We show that they have even contradictive effect under certain condition.

Secondly, we combine two sides of market in one simple model, the supply of production factors and the demand for the consumption goods. Thus, in this simple model, we can observe two different channels, through which income distribution can affect economic performance. We find that the positive effect of equality on growth is more possible in the case, where the supply side effect is dominant. Vice verse, inequality has possibly positive effect on economic growth if the demand side is dominant.

Figure 1
Appendix

Appendix 1

The total labor demand is \( n + a + b(\beta x_p + (1 - \beta) x_r) + s \). As the standard good expenditure is the income of individuals net of the price of the quality good, we have \( x_i = y_i - P_i \). Substituting it in the demand for labor and rearranging, we have \( n + a + b(\beta y_p + (1 - \beta) y_r - P) + s \), where \( P \) is the aggregate expenditure in the quality good. Using the fact \( \pi = P - wa \) and \( wb = 1 \), we have \( n + b(\beta y_p + (1 - \beta) y_r) - b \pi + s \). Because of the education equilibrium condition \( s = \frac{\theta V}{w} = \frac{\theta \pi b}{\theta + \phi} \) and the research arbitrage equation \( nw = R = \frac{\phi \pi}{\phi + \theta} \), we have \( n + s - b \pi = 0 \). Thus, the total demand for labor is equal to \( b(\beta y_p + (1 - \beta) y_r) \), which is exactly the total supply of labor, \( a + \beta H_p + (1 - \beta) H_r \).

Appendix 2

\[
H^{sep}_r = (\frac{\theta}{\phi^{sep} + \theta}(1 - \frac{1}{k})wH^{sep}_r (1-d \beta))^{\alpha} \Rightarrow H^{sep}_r = (\frac{\theta w}{\phi^{sep} + \theta}(1 - \frac{1}{k})(1-d \beta))^{\frac{\alpha}{1-\alpha}}
\]
\[
\phi^{sep} = \lambda(1-\frac{1}{k})(1-\beta)(\frac{\theta w}{\phi^{sep} + \theta}(1 - \frac{1}{k})(1-d \beta))^{\frac{\alpha}{1-\alpha}} - \theta
\]
\[
\Rightarrow (\phi^{sep} + \theta)^{\frac{1}{1-\alpha}} = \lambda(1-\frac{1}{k})^{\frac{1}{1-\alpha}} (\theta w(1-d \beta))^{\frac{\alpha}{1-\alpha}}
\]
\[
\Rightarrow \phi^{sep} = (1-\frac{1}{k})[\lambda(1-\beta)]^{-\alpha} (\theta w(1-d \beta))^\alpha - \theta
\]
\[
\frac{\partial \phi^{sep}}{\partial d} < 0 \text{ and } \frac{\partial \phi^{sep}}{\partial \beta} < 0.
\]

\[
H^{pool}_p = (d - \frac{\theta w}{\phi^{pool} + \theta}(1 - \frac{1}{k}))^{\frac{\alpha}{1-\alpha}}
\]
\[
\phi^{pool} = \lambda(1-\frac{1}{k})H^{pool}_p - \theta = \lambda(1-\frac{1}{k})(d - \frac{\theta w}{\phi^{pool} + \theta}(1 - \frac{1}{k}))^{\frac{\alpha}{1-\alpha}} - \theta
\]
\[
(\phi^{pool} + \theta)^{\frac{1}{1-\alpha}} = \lambda(1-\frac{1}{k})^{\frac{1}{1-\alpha}} (d \theta w(1-\frac{1}{k}))^{\frac{\alpha}{1-\alpha}}
\]
\[
\Rightarrow \phi^{pool} = (1-\frac{1}{k})\lambda^{-\alpha} (d \theta w)^\alpha - \theta
\]
\[ \frac{\partial \phi_{\text{pool}}}{\partial d} > 0 \text{ and } \frac{\partial \phi_{\text{pool}}}{\partial \beta} = 0 \]

As we know, the conditions of separating price are (7) and (8), which are same as follows after substituting \( H_i \):

\[
(1 - \beta) \left( \frac{\theta \nu}{\phi_{\text{sep}} + \theta} \right) \left( 1 - \frac{1}{k} \right) \frac{1 - d \beta}{\phi_{\text{sep}} + \theta} \geq \left( d - \frac{\theta \nu}{\phi_{\text{pool}} + \theta} \right) \left( 1 - \frac{1}{k} \right) \frac{1 - d \beta}{\phi_{\text{pool}} + \theta}
\]

\[
\Leftrightarrow (1 - \beta) \left( \frac{1 - d \beta}{\phi_{\text{sep}} + \theta} \right)^{1 - \alpha} \geq \left( \frac{d}{\phi_{\text{pool}} + \theta} \right)^{1 - \alpha} \Leftrightarrow (1 - \beta)^{1 - \alpha} \left( \frac{1}{d} - \beta \right)^{\alpha} \geq 1
\]

We denote \( d^* \) satisfying \( (1 - \beta)^{1 - \alpha} \left( \frac{1}{d^*} - \beta \right)^{\alpha} = 1 \). For \( d \in [0, d^*] \), the monopolist chooses the separating strategy and in the case of \( d \in [d^*, 1] \), she sets the pooling price. The threshold value \( d^* \) increases in \( \alpha \).
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