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# The role of corporate saving over the business cycle: Shock absorber or amplifier?

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# The Role of Corporate Saving over the Business Cycle: Shock Absorber or Amplifier?



by Xiaodan Gao and Shaofeng Xu

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## The Role of Corporate Saving over the Business Cycle: Shock Absorber or Amplifier?

by

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### Abstract

We document countercyclical corporate saving behavior with the degree of countercyclicality varying nonmonotonically with firm size. We then develop a dynamic stochastic general equilibrium model with heterogeneous firms to explain the pattern and study its implications for business cycles. In the presence of financial frictions and fixed operating costs, a persistent negative productivity shock signals low future income and prompts firms to hold more cash in order to preserve financial flexibility and maintain normal operations. This countercyclicality exhibits a hump-shaped relation to firm size. Compared with medium-sized firms, small firms have a higher marginal product of capital and thus better investment opportunities, which compete for resources with cash, while large firms have more pledgeable assets and demand less cash. We find that, on average, firms accumulate cash by cutting investment and employment in recessions, which reduces aggregate output and increases economic fluctuations. Corporate saving, therefore, amplifies aggregate shocks.

*Bank topics: E20, E22, E32, G31, G32 JEL codes: Business fluctuations and cycles; Economic models* 

## Résumé

Dans la présente étude, nous montrons tout d'abord que les entreprises ont un comportement d'épargne contracyclique, qui varie de façon non monotone avec leur taille. Nous élaborons ensuite un modèle d'équilibre général dynamique et stochastique intégrant des entreprises hétérogènes pour expliquer cette tendance et en étudier l'incidence sur les cycles économiques. En présence de frictions financières et de coûts fixes d'exploitation, un choc de productivité négatif persistant laisse augurer de faibles revenus futurs et incite les entreprises à détenir plus de liquidités pour préserver leur souplesse sur le plan financier et continuer à fonctionner normalement. La relation entre ce comportement contracyclique et la taille des entreprises se traduit par une courbe en bosse. Comparativement aux moyennes entreprises, les petites entreprises bénéficient d'une productivité marginale du capital plus élevée, et donc de meilleures possibilités d'investissement, rendant l'accumulation de liquidités moins attrayante. Quant aux grandes entreprises, elles ont davantage d'actifs à donner en garantie et nécessitent donc moins de liquidités. Nous constatons qu'en moyenne, les entreprises accroissent leurs liquidités en réduisant leurs investissements et leur personnel en période de récession. La production globale s'en voit ainsi réduite, et les fluctuations économiques augmentent. C'est donc dire que l'épargne des entreprises amplifie les chocs globaux.

*Sujets : E20, E22, E32, G31, G32 Codes JEL : Cycles et fluctuations économiques; Modèles économiques* 

## **Non-Technical Summary**

An intriguing phenomenon, which has recently attracted considerable attention from academic researchers and policy makers, is the substantial cash reserves maintained by the U.S. corporate sector during the Great Recession. Neither the firms' saving behavior nor its implications for aggregate fluctuations are fully understood by the literature. The objective of this paper is to shed light on these puzzling issues.

We first examine U.S. corporate saving behavior using both aggregate and firm-level data. We document two empirical regularities. First, corporate saving is countercyclical, i.e., firms on average accumulate more internal funds during recessions. Second, the degree of countercyclicality exhibits a hump-shaped relation with firm size.

We then develop a dynamic stochastic general equilibrium model with heterogeneous firms to explain these empirical regularities. The model features financial frictions, in the form of costly external financing, and fixed operating costs. The combination of these two features encourages firms to funnel their cash flow into liquid assets rather than fully into physical capital or shareholder distributions. In the model, a persistent negative productivity shock signals low future income and prompts firms to hold more cash in order to preserve financial flexibility and maintain normal operations. This mechanism is tested and validated using Compustat data.

Finally, we use our model as a laboratory to examine the implications of corporate saving for business cycles. Our study suggests that corporate saving acts as an important channel through which financial frictions amplify aggregate shocks. For example, we find that in response to a negative productivity shock, firms invest less in capital, hire fewer workers, pay fewer dividends, and divert more resources into cash. Moreover, lower capital investment today translates into lower collateral value of capital in the future, which gives rise to an even stronger incentive for cash saving. As a result, corporate saving increases the decline in output during recessions and raises output volatility over the business cycle.

## 1 Introduction

An intriguing phenomenon, which has recently attracted considerable attention from academic researchers and policy makers, is the substantial cash reserves maintained by the U.S. corporate sector. During the recent recession, firms had a higher share of cash on their balance sheets than at any time for the past 30 years. For U.S. publicly traded industrial firms, the average cash-to-assets ratio soared to 23.5% in 2009. The staggering amount of cash reserves held by firms has been conjectured to be one of the main reasons for the slow recovery from the Great Recession.<sup>1</sup> However, intuition suggests that firms have more than enough cash to mitigate adverse shocks. To shed light on this fundamental yet puzzling issue, this paper examines the role of corporate cash saving over the business cycle.

We begin by documenting the cyclical behavior of corporate saving at the aggregate level. We find it to be countercyclical—that is, firms on average accumulate more internal funds during recessions. We then augment our analysis using data from Compustat. Our firm-level study confirms the countercyclicality of corporate saving. Moreover, the countercyclicality varies nonmonotonically with firm size: It first becomes more pronounced, then weakens as firm size increases.

To explain the stylized facts of countercyclical corporate saving and explore its role in the aggregate economy, we build a quantitative general equilibrium model featuring financial frictions and fixed operating costs. In the model, a continuum of homogeneous households solve a standard consumption-saving problem, while a continuum of heterogeneous firms face both aggregate and idiosyncratic productivity shocks. Firms combine labor and capital to produce outputs. They pay fixed operating costs before production starts and allocate post-production revenue among physical capital, internal cash balance, and dividend payments. To finance operating expenses and capital investments, firms can use internal cash and external funds. External finance takes the form of risk-free

<sup>&</sup>lt;sup>1</sup>See, for instance, "Corporate Hoarding and the Slow Recovery," by Paul Krugman, *New York Times*, February 8, 2013.

debt—which is subject to a borrowing constraint—and costly equity issuance. The assumed capital market frictions prompt firms to save. That is, firms accumulate cash to avoid tapping into expensive external financing in case of insufficient internal funds to finance operations and real investments. The model is carefully calibrated to match aggregate and firm-level U.S. data from 1960 to 2013.

The countercyclicality of corporate saving is induced by financial frictions and fixed operating costs. A persistent negative productivity shock signals low income in the future. To ensure sufficient internal funds to pay fixed operating costs and facilitate future operations, firms save more cash. This explains an increase in corporate cash accumulation during recessions.

Our model also successfully replicates the heterogeneity in saving patterns across firm size. Small firms have a sufficiently high marginal product of capital. Despite experiencing temporarily low aggregate productivity, these firms decide to draw down internal cash saving and maintain a high level of capital investment. As firm size increases, the effect of financial frictions outweighs that of the marginal product of capital, which generates countercyclical corporate saving. As firm size continues to rise, the degree of countercyclicality starts to fall. This less countercyclical saving behavior can be explained by firms' sound financial positions: Large firms have more pledgeable assets, and therefore need less cash in recessions.

We use our model as a laboratory to examine the implications of corporate saving for business cycles by comparing our benchmark model with a model that features a high cash-holding cost (i.e., a much weaker incentive to save cash). We find that corporate saving propagates aggregate shocks. In particular, in response to a negative productivity shock and therefore lower marginal products of capital and labor, firms in our benchmark model invest less in capital, hire fewer workers, pay fewer dividends, and divert more resources into cash holdings in order to preserve financial flexibility. Moreover, lower capital investment in this period translates into lower collateral value of capital in the future, which gives rise to an even stronger incentive for cash saving. Firms therefore further allocate resources from investment and employment to cash. As a result, an increase in corporate saving causes larger initial drops in capital investment, working hours, and outputs; amplifies aggregate fluctuations; and prolongs economic recovery. Our results suggest that corporate saving acts as an important channel through which financial frictions propagate aggregate shocks, in contrast to the common view of its role in absorbing adverse shocks as a liquidity cushion.

In this paper, we model debt market frictions as collateral constraints and abstract from risky debt and default decisions. This assumption is made to keep the model tractable. Considering risky debt in the model will strengthen the mechanism highlighted above. More specifically, during recessions, instead of being constrained by the amount of debt that can be borrowed, firms will face higher debt-financing costs caused by two factors: an increase in default risks and a rise in stochastic discount factor. The former is generated by a low productivity shock and thus insufficient future income for survival, while the latter is generated by a rise in the marginal utility of consumption. The increase in external borrowing costs will then induce a stronger precautionary saving motive relative to the model considered in this paper, and further push up corporate saving during recessions.

In addition, we abstract from endogenous exit in the model. As such, this paper does not consider the potential role of cash saving in reducing exit rates and thus, to some degree, in stabilizing aggregate output. We view this simplification as innocuous. Hathaway and Litan (2014) document that firm exit rate was relatively stable over the period 1978–2011, although a significant increase in exit rates was found during the Great Recession. Similar patterns of exit rates are shown by Tian (2018), who finds that exit rates are acyclical on average and are strongly countercyclical during the Great Recession. Given that we study the implications of corporate saving for an average recession, our exclusion of endogenous exit is unlikely to alter our model's central results.

This paper contributes to the literature along three dimensions. First, it establishes two important business cycle facts: Corporate saving is countercyclical, and the degree of countercyclicality varies with firm size. Previous studies examine the cyclical property of corporate external financing. Covas and Den Haan (2011) document that most U.S. publicly traded firms issue less debt and equity during recessions. Jermann and Quadrini (2012) find countercyclical equity financing and procyclical debt financing at the aggregate level.<sup>2</sup> In this paper, we focus on another critical source of investment financing: internal cash holdings—which, together with debt and equity financing, have important implications for the aggregate economy.

This paper is also related to the literature on saving behavior. Lane and Tornell (1998) document significantly procyclical aggregate saving rates in most OECD countries, including the U.S., which supports the hypothesis of "saving for a rainy day" to smooth consumption over time. On the other hand, Adema and Pozzi (2015) report a countercyclical household saving rate and argue that the countercyclical behavior can be explained by the buffer stock model proposed by Deaton (1991) in the presence of a borrowing constraint and nonstationary, positively autocorrelated income processes. Our paper bridges the gap by studying corporate saving behavior and distinguishes itself from Deaton (1991) in one key aspect: In the case of stationary and positively correlated income processes, our study is able to generate countercyclical cash saving, while Deaton (1991) predicts a procyclical pattern.

Second, this paper contributes to the literature on the importance of financial frictions in propagating aggregate shocks. Intuition suggests that corporate cash reserves can be used to mitigate adverse shocks, and therefore challenge the role of financial frictions in accounting for aggregate fluctuations. In contrast to this common belief, we find that corporate saving acts as a channel through which financial frictions amplify aggregate shocks and change economic outcomes. In previous studies, financial frictions affect the aggregate economy in two ways. First, borrowing constraints tighten budget constraints and reduce resources available for capital investment (Bernanke, Gertler, and Gilchrist, 1999; Jermann and Quadrini, 2012; Kiyotaki and Moore, 1997). Second, costly external financing—risky debt and/or expensive equity issuance—changes the macroeconomic equilibrium by altering optimal corporate policies (Arellano, Bai, and Kehoe, 2016; Gilchrist, Sim, and Zakrajsek, 2014). This paper considers both of these channels

<sup>&</sup>lt;sup>2</sup>See also Korajczyk and Levy (2003), Halling, Yu, and Zechner (2016), and Begenau and Salomao (2018).

and distinguishes itself by proposing an additional mechanism: Corporate cash saving generated by financial frictions further shifts resources away from physical capital and exacerbates economic conditions during recessions.

Third, this paper contributes to the corporate cash literature by providing insights into the macroeconomic implications of corporate saving. It answers a fundamental question about the role of corporate cash during economic downturns. While the corporate cash-hoarding phenomenon is well documented (Bates, Kahle, and Stulz, 2009), few studies have explored its aggregate implications. Khan and Thomas (2013) and Khan, Senga, and Thomas (2014) incorporate firms' cash holdings in a framework similar to ours. However, cash in their models is defined as firms' net worth at the beginning of each period, which is different from the cash-saving decision considered in our paper. Contemporaneous work by Xiao (2018) is the study most closely related to ours. Our paper differs in the following respects. First, we examine the role of corporate cash saving over business cycles that are generated by aggregate productivity shocks in general, while Xiao (2018) focuses on credit supply shocks and the Great Recession of 2008. Second, our proposed mechanism based on the significance of corporate saving is valid in the absence of persistently high uncertainties and debt substitution, elements that Xiao (2018) greatly emphasizes. Third, Xiao (2018) answers the question of whether corporate saving has played a role in slowing down the economic recovery since 2009, while the main goal of our paper is to explore the role of corporate saving in accounting for economic fluctuations. Last, the friction that generates the coexistence of debt and cash differs. In our model, firms simultaneously hold cash and debt because of collateral constraints and costly equity issuance costs, while the timing of shock realization in Xiao (2018) plays a key role in producing the simultaneous holdings.

The paper is organized as follows. Section 2 presents stylized facts of the countercyclicality of corporate cash saving. Section 3 lays out the model to provide an explanation for the empirical observation. Section 4 calibrates the model and demonstrates its ability to reproduce key moments of U.S. data. Section 5 explains the observed corporate-saving patterns and discusses the implications for aggregate fluctuations. Section 6 concludes.

## 2 Cyclicality of Corporate Saving

In this section, we document the cyclical behavior of corporate cash saving at both the aggregate level and disaggregated firm level. We follow the typical approach in the macroeconomic literature by examining the correlation between the cyclical components of corporate saving and real GDP (Covas and Den Haan, 2011).

#### 2.1 Aggregate-level Facts

For our aggregate-level analysis, we use the Flow of Funds data maintained by the Federal Reserve System, covering the period from 1952:Q1 to 2010:Q2. We construct two measures of corporate saving. One is defined as the sum of private foreign deposits (item 12), checkable deposits and currency (item 13), total time and saving deposits (item 14), money market mutual fund shares (item 15), federal funds and security repurchase agreements (item 16), credit market instruments (item 17), and mutual fund shares (item 24), while the other measure differs by excluding all the components of credit market instruments other than commercial paper (item 18) and Treasury securities (item 19).<sup>3</sup> We then scale both measures by business GDP and detrend both series with a band-pass (BP) filter that preserves cycles for two to eight years.<sup>4</sup> We examine both the nonfinancial business sector (F.101Q) and nonfinancial corporate business sector (F.102Q) and plot the detrended time series in Figure 1.

#### [Figure 1 about here.]

Evidently, all four measures of corporate saving in Figure 1 suggest the same pattern: Firms save hard during recessions, which are indicated by the gray shaded areas. In particular, during the recent recession episode, firms chose to increase their cash saving by more than 3% of business GDP. This finding is consistent with the observation that firms shunned capital investment but piled up cash during the Great Recession.

<sup>&</sup>lt;sup>3</sup>The first measure is firms' net acquisition of financial assets minus trade receivables and miscellaneous assets. <sup>4</sup>Note that we do not take logarithms of corporate-saving measures because some observations are negative. That is, firms dissave from time to time.

We further characterize the cyclical property of corporate saving by examining its correlation with aggregate real GDP. We use the first measure of corporate saving for nonfinancial corporate business and report the result in Table 1. It suggests that the correlation is -0.27, which is statistically significantly different from zero.

#### [Table 1 about here.]

We then perform two robustness tests. First, we examine the cyclical behavior of corporate saving at annual frequency and report the results in Panel B of Table 1.<sup>5</sup> Second, we allow for a different cyclical behavior during different time periods. We examine the patterns for the post-Great Moderation period, 1984–2010 and 1984–2007. The latter subperiod is examined to alleviate concern that the high correlation found in the data is driven by the Great Recession. Results are reported in Table 1. Overall, corporate saving exhibits a negative correlation with real GDP, which is in line with the cyclical patterns shown in Figure 1.

#### 2.2 Firm-level Facts

In this subsection, we use disaggregated data to re-examine the cyclicality of corporate saving. The sample is constructed from Compustat Fundamentals Annual, consisting of a panel of nonfinancial (SIC 6000–6999) and nonutility (SIC 4900–4999) publicly traded U.S. firms over the period 1971 to 2010. We delete observations with negative total assets or negative sales and drop the first five observations of each firm to limit the bias introduced by IPOs.

We analyze firms' cyclical cash-saving behavior using three approaches. First, we examine aggregate behavior by summing up the variables of interest across all firms in the sample. Second, we analyze micro-level behavior by sorting firms into several size-based portfolios and showing the cyclical behavior by firm size. And third, we

<sup>&</sup>lt;sup>5</sup>Note that for annual frequency, the GDP measure used to scale cash saving is computed by summing up the GDP in the following nonfinancial sectors: agriculture, forestry, fishing, hunting, mining, utilities, construction, manufacturing, wholesale trade, retail trade, transportation and warehousing, information, professional and business services, education services, health care, social assistance, arts, entertainment, and other services except government.

analyze the cyclical behavior of cash saving in each major industry sector. The second and third exercises provide a detailed look at the cash-saving behavior across firms of different sizes and different industries and examine whether the aggregate behavior is strongly influenced by a small group of firms. The measure of cash saving is defined as the difference between firms' end-of-period and beginning-of-period cash balances, while the cash balance is the sum of cash, cash equivalents, and short-term financial investments (item *che*).

#### 2.2.1 Aggregate Behavior with Firm-level Data

To study the aggregate behavior, we compute total cash saving of all firms in the sample for each year. We then scale the measure by its corresponding total capital stock net of depreciation (item *ppent*) and detrend the variable with a BP filter that preserves cycles for two to eight years.

Results are plotted in Figure 2. Consistent with the findings from the Flow of Funds data, corporate saving displays a countercyclical pattern, except for the 1981 recession, during which firms chose to dissave. We conjecture that the procyclical saving behavior in late 1980 and early 1981 arises from the anti-inflation measure undertaken during that period: The Federal Reserve tightened the money supply to address the increase in inflation. In the face of a decrease in the money supply, firms' cash holdings fell accordingly.

#### [Figure 2 about here.]

To test the robustness of our results at the firm level, we use alternative scale variables, including total gross capital stock (item ppegt), total sales (item sale), and total assets (item at). Although the fluctuation magnitude of detrended corporate saving varies across different measures, the patterns document the same phenomenon: Firms tend to accumulate more cash during recessions.

#### 2.2.2 Cyclicality of Cash Saving by Firm Size

Exploiting the property of disaggregated data, we next examine whether the aggregate behavior documented in the previous subsections is entirely driven by a certain group of firms by showing the saving behavior for firms of different sizes.

Following Covas and Den Haan (2011), we sort firms based on their beginning-ofperiod total assets and form seven groups: [0, 25%], [25%, 50%], [50%, 75%], [75%, 90%], [90%, 95%], [95%, 99%] and [99%, 100%]. For each group, we use the same method as before to study firms' saving behavior, which is measured as the ratio of changes in total cash holdings to total assets. Figure 3 depicts the dynamics of cash saving over business cycles for each group.

#### [Figure 3 about here.]

Our results suggest that in four out of six recessions—that is, 1974–1975, 1982–1983, 1990–1991, and 2008–2009—firms of different sizes, on average, behave in the same manner. They tend to accumulate internal cash during recessions, although in different magnitudes. During the 1980 recession, consistent with the aggregate behavior shown in Figure 2, all firms drew down their cash balances. During the 2001 recession, firms of different sizes exhibited heterogeneity in saving behavior: Firms with sizes below the  $75^{th}$  percentile decumulated their cash stocks, while larger firms chose to store more cash. The aggregate pattern during that period was driven by the saving behavior of the latter.

We report the correlation between corporate saving and GDP in Table 2, which presents two findings. First, the negative saving–GDP correlation holds for all firms, regardless of firm size. Second, corporate saving first becomes more countercyclical as firm size increases, then becomes less countercyclical when firm size reaches the top  $10^{th}$ percentile. In particular, the cash-saving policy of firms in the bottom  $25^{th}$  percentile is less sensitive to economic conditions, with the correlation not being significantly different from zero.

[Table 2 about here.]

#### 2.2.3 Cyclicality of Cash Saving by Industry

Lastly, we sort firms based on two-digit SIC codes and examine whether the countercyclicality of corporate saving is also found in each major industry sector other than utility, finance, and public administration.<sup>6</sup> Results are summarized in Table 3.

[Table 3 about here.]

As shown, corporate saving in the agriculture, forestry, and fishing sector is uncorrelated with real GDP, with the correlation moving from -0.02 to 0.01. Corporate saving in retail trade, services, and manufacturing sectors exhibits a very strong countercyclical pattern, while the countercyclicality becomes weaker for construction, wholesale trade, and mining. Overall, a negative correlation is found in most industries, which lends further support to the countercyclical nature of corporate saving.<sup>7</sup>

In summary, our aggregate-level and firm-level analyses are consistent and demonstrate that corporate saving is countercyclical. We next present a parsimonious model to explain the stylized fact documented in this section and explore its implication for the aggregate economy.

## 3 The Model

This section constructs a dynamic stochastic general equilibrium economy populated by a representative household; a continuum of heterogeneous firms differing in their capital, net leverage, and productivity; and a perfectly competitive financial intermediary. Firms face persistent shocks to their aggregate and idiosyncratic productivity, and they purchase capital and hire labor to produce a numeraire good used for both consumption and investment. Operating costs and capital expenditures can be financed internally by using operating income and cash balances, or externally by issuing debt and equity.

 $<sup>^{6}</sup>$ Public administration is excluded because none of the firms in our Compustat sample operates in this sector.

<sup>&</sup>lt;sup>7</sup>To rule out the possibility of a high correlation between industries and firm size, we also examine cash-saving behavior by firm size within manufacturing, retail trade, and services, respectively. A hump-shaped relationship between the countercyclicality of cash saving and firm size is confirmed for these three major industries. Results are available on request.

#### 3.1 Preferences, Technology, and Shocks

The economy is populated by a unit measure of identical infinitely lived households. The representative household has a periodic utility function,

$$u(C, N) = \log C + \phi (1 - N),$$
 (1)

where C and N denote, respectively, the household's consumption and hours worked, and  $\phi$  is a positive parameter that captures the household's preference for leisure. The household's time preference is characterized by a constant discount factor  $\beta$ .

There are a large number of firms. Each of them combines predetermined capital stock k and labor n to produce a homogeneous numeraire good y used for consumption and investment. All firms operate their production via a decreasing returns-to-scale (DRS) Cobb-Douglas technology described by the function

$$y = (z\varepsilon)^{1-(1-\alpha)\theta} \left(k^{\alpha} n^{1-\alpha}\right)^{\theta}, \qquad (2)$$

where variables z and  $\varepsilon$  represent, respectively, the aggregate and firm-specific components of exogenous stochastic factor productivity. The parameter  $\alpha \in (0, 1)$  is the share of capital in production, and  $\theta \in (0, 1)$  captures the DRS. Firms must pay fixed operation costs  $\chi > 0$  in advance of operation each period.

We assume that the processes of aggregate and individual productivity are two Markov chains that evolve independently. More precisely, the aggregate productivity  $z \in \mathbf{Z} = \{z_1, ..., z_{n_z}\}$ , with the transition probability  $\Pr(z' = z_m | z = z_l) = \pi_{lm}^z \ge 0$  satisfying  $\sum_{m=1}^{n_z} \pi_{lm}^z = 1$  for  $l = 1, ..., n_z$ . Similarly, the individual productivity  $\varepsilon \in \mathbf{E} = \{\varepsilon_1, ..., \varepsilon_{n_\varepsilon}\}$ , where  $\Pr(\varepsilon' = \varepsilon_j | \varepsilon = \varepsilon_i) = \pi_{ij}^{\varepsilon} \ge 0$  satisfies  $\sum_{j=1}^{n_\varepsilon} \pi_{ij}^{\varepsilon} = 1$  for  $i = 1, ..., n_\varepsilon$ . Here, a prime denotes a variable in the subsequent period.

To prevent firms from effectively outgrowing financial frictions in the long run, we introduce exogenous entry and exit to the model. In particular, each firm faces a fixed probability  $\xi \in (0, 1)$  of exiting the market following production in each period. Exiting

firms are replaced by the same number of entrants whose initial states will be detailed later.

#### 3.2 The Firm's Problem

At the beginning of each period, an individual firm is characterized by its capital stock,  $k \in \mathbf{K} \subset \mathbb{R}_+$ ; its net debt obligation from previous period,  $b \in \mathbf{B} \subset \mathbb{R}$ ; and its realized idiosyncratic productivity,  $\varepsilon_i \in \mathbf{E}$ . The variable b is defined as the difference between the firm's holdings of debt  $b_d$  and cash  $b_c$  in the previous period, i.e.,  $b = b_d - b_c$ , where b > (<)0 means that the firm is a net borrower (saver). We denote the distribution of firms over  $(k, b, \varepsilon_i)$  using probability measure  $\mu$  over the  $\sigma$ -algebra generated by the open subsets of the product space  $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E}$ . Evolution of the firm distribution is determined by a mapping  $\Gamma$  from the aggregate state of the economy  $(z_l, \mu)$  such that  $\mu' = \Gamma(z_l, \mu)$ .

It is important to note that our model is able to generate the coexistence of debt and cash holdings within firms. We introduce net debt b into the model to reduce the model's state space by one dimension. Net debt b fully captures the influences of a firm's previous-period financing choices over its current-period decisions; as such, we do not have to separately track the holdings of debt and cash in the firm-level state space, which greatly simplifies computation of the model.

After observing its individual state  $(k, b, \varepsilon_i)$  and the aggregate state  $(z_l, \mu)$ , the firm makes choices to maximize the expected discounted value of its future dividends. First, it repays its existing debt, pays fixed operating costs, and operates production. If it receives an exit shock, it sells its capital and distributes revenue, together with cash stocks, to shareholders as it departs. Conditional on survival, the firm can issue new debt or equity to finance debt repayment and operating costs. It also makes its investment and cash-saving choices for the next period. The decision process of a firm in state  $(k, b, \varepsilon_i; z_l, \mu)$  is described in detail as follows.

#### 3.2.1 Production

We first examine the production decisions of a firm that has already paid off its debt and fixed operating costs. The problem is to find a profit-maximizing employment level n such that

$$\pi\left(k,\varepsilon_{i};z_{l},\mu\right) = \max_{n}\left\{\left(z_{l}\varepsilon_{i}\right)^{1-(1-\alpha)\theta}\left(k^{\alpha}n^{1-\alpha}\right)^{\theta} - w\left(z_{l},\mu\right)n\right\},\tag{3}$$

where  $w(z_l, \mu)$  is the market-determined wage rate that depends on the aggregate state of the economy. Note that the firm's labor hiring and output are independent of its existing net debt obligation and its current-period investment and financing choices. Solving problem (3) yields the following labor demand and output functions:

$$n\left(k,\varepsilon_{i};z_{l},\mu\right) = A\left(z_{l},\mu\right) \times z_{l}\varepsilon_{i}k^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}, \ y\left(k,\varepsilon_{i};z_{l},\mu\right) = A\left(z_{l},\mu\right)^{(1-\alpha)\theta} \times z_{l}\varepsilon_{i}k^{\frac{\alpha\theta}{1-(1-\alpha)\theta}},$$

$$(4)$$

where  $A(z_l,\mu) = \left(\frac{w(z_l,\mu)}{\theta(1-\alpha)}\right)^{\frac{1}{(1-\alpha)\theta-1}}$ . The firm's profit function then can be rewritten as:

$$\pi\left(k,\varepsilon_{i};z_{l},\mu\right) = \left(A\left(z_{l},\mu\right)^{(1-\alpha)\theta} - w\left(z_{l},\mu\right)A\left(z_{l},\mu\right)\right) \times z_{l}\varepsilon_{i}k^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}.$$
(5)

#### 3.2.2 Investment and Financing

We next analyze how the firm chooses its investment and financing strategies to optimize its problem. Before discussing these decisions, we describe two types of frictions embedded in our model—one real and one financial—that would impede a firm's capital reallocation.

The first type of friction is that undertaking investment incurs adjustment costs, including linear adjustment costs,  $\gamma_0 > 0$ ; convex adjustment costs,  $\gamma_1 > 0$ ; and the partial irreversibility of capital stock,  $\gamma_2 \in (0, 1)$ . The total cost of changing capital from k to k' is given by

$$I(k,k') = \gamma_0 k \mathbf{1}_{i \neq 0} + \frac{\gamma_1}{2} \left(\frac{i}{k}\right)^2 k + (\mathbf{1}_{i \geq 0} + \gamma_2 \mathbf{1}_{i < 0}) i, \text{ where } i = k' - (1 - \delta) k.$$
(6)

Here,  $\delta$  denotes the capital depreciation rate, *i* measures the gross investment, and  $\mathbf{1}_A$  is an indicator function that takes value one when event *A* occurs and zero otherwise. In the expression I(k, k'), the third term captures the idea that the resale value of installed capital is smaller than its purchase price, reflecting the illiquidity of long-lived capital assets, as in Abel and Eberly (1996).

The second type of friction arises from capital market imperfections. In the economy, to fund operational expenses and capital expenditures, the firm can use its internal funds or finances externally by issuing debt and equity. Acquiring funds from external sources, however, is subject to imperfections in both debt and equity markets. In the economy, the debt market is represented by a perfectly competitive financial intermediary, which takes deposits and makes loans in the form of one-period risk-free debt. Firms that want to borrow face collateralized borrowing constraints

$$b'_d \le \gamma_2 \left(1 - \delta\right) k',\tag{7}$$

where  $b'_d$  represents the amount of debt firms are willing to take on for the next period. It guarantees that firms will not default on their debt obligations, since the amount of debt due in the next period never exceeds the value of collateral. Meanwhile, frictions also exist in the equity market. Equity finance is costly: Raising new equity reduces the value of existing shares by more than the amount of newly issued shares. Following the existing corporate finance literature (Hennessy and Whited, 2007), we denote D < 0as equity issuance and  $D \ge 0$  as dividend payment, and model the loss in the value of existing shares due to the newly issued equity D by

$$\Psi\left(D\right) = \left(\psi_0 - \psi_1 D\right) \mathbf{1}_{D<0},\tag{8}$$

where  $\psi_0 > 0$  denotes the fixed equity issuance costs and  $\psi_1 > 0$  represents the linear costs proportional to the amount of issuance.

#### 3.2.3 The Firm's Problem

We now proceed to formally formulate a firm's optimization problem. If the firm learns that it will not operate beyond the current period, it repays its debt, pays operating costs, and operates production. Since it will carry zero capital or debt into the future, its dividend payments to shareholders equal its profit and the liquidation value of its capital and cash balances, minus expenditures on debt payment and operating costs.

If the firm is not hit by the exit shock, its problem is more involved and can be viewed as having two stages: the pre-production stage and the post-production stage. In the pre-production stage, the firm repays debt and pays fixed operating costs  $\chi$  in advance of production. It can issue new debt and roll over existing debt. If funds remain insufficient to cover expenses, the firm can issue equity. Unused funds at this stage will be carried forward to the post-production stage as

$$\hat{D}_1(b'_d, b; z_l, \mu) = q(z_l, \mu) b'_d - b - \chi,$$
(9)

where  $b = b_d - b_c$ , with  $b_d$  and  $b_c$  representing, respectively, the amount of debt and cash firms carry from the preceding period. The first term on the right-hand side of equation (9),  $q(z_l, \mu) b'_d$ , represents the proceeds from debt issuance this period, where  $q(z_l, \mu)$ equals the inverse of the risk-free gross interest rate given by

$$q(z_l,\mu) = \sum_{m=1}^{n_z} \pi_{lm}^z \omega(z_l,\mu,z_m,\mu').$$
(10)

Here,  $\omega(z_l, \mu, z_m, \mu')$  is the stochastic discount factor of the representative household when the future aggregate state is  $(z_m, \mu')$ .

In the post-production stage, the firm produces output and makes capital investment and cash-saving decisions. If current-period revenue is insufficient to meet investment expenditures, the firm again issues equity; otherwise, it distributes dividends. Equity flow in the second stage is

$$\tilde{D}_{2}(k', b'_{d}, b'_{c}, k, b, \varepsilon_{i}; z_{l}, \mu) = \pi (k, \varepsilon_{i}; z_{l}, \mu) + \tilde{D}_{1}(b'_{d}, b; z_{l}, \mu) \mathbf{1}_{\tilde{D}_{1} \ge 0} - I(k, k') - (1+\eta)q(z_{l}, \mu)b'_{c}$$
(11)

where  $D_1 \mathbf{1}_{D_1 \ge 0}$  represents the unused funds from the pre-production stage, and  $q(z_l, \mu) b'_c$ denotes the firm's savings in the form of risk-free bonds. To ensure the existence of an upper bound on optimal corporate saving, we impose a cost on it, which is captured by  $\eta$ . For simplicity, we assume that the holding costs are paid to the representative household.

We summarize the firm's problem as follows. Let  $\tilde{V}$  be the value of a firm prior to the realization of the exit shock. By construction, we have

$$\tilde{V}(k,b,\varepsilon_i;z_l,\mu) = \xi \left[\pi \left(k,\varepsilon_i;z_l,\mu\right) - b - \chi - I\left(k,0\right)\right] + (1-\xi) V\left(k,b,\varepsilon_i;z_l,\mu\right).$$
(12)

Here, V is the value of the firm conditional on its continuing to operate in the subsequent period, and is defined by the following Bellman equation:

$$V(k, b, \varepsilon_{i}; z_{l}, \mu) = \max_{k', b'_{d}, b'_{c}} \left\{ \begin{array}{c} (D_{1} - \Psi(D_{1})) \,\mathbf{1}_{D_{1}<0} + D_{2} - \Psi(D_{2}) \\ + \sum_{m=1}^{n_{z}} \sum_{j=1}^{n_{\varepsilon}} \pi_{lm}^{z} \pi_{ij}^{\varepsilon} \omega\left(z_{l}, \mu, z_{m}, \mu'\right) \tilde{V}\left(k', b', \varepsilon_{j}; z_{m}, \mu'\right) \end{array} \right\}$$
(13)

subject to

$$D_{1} = \tilde{D}_{1} \left( b'_{d}, b; z_{l}, \mu \right), D_{2} = \tilde{D}_{2} \left( k', b'_{d}, b'_{c}, k, b, \varepsilon_{i}; z_{l}, \mu \right),$$
(14)

$$b'_{d} \leq \gamma_{2} (1 - \delta) k', \ b'_{d} \geq 0, \ b'_{c} \geq 0, \ k' \geq 0, \ \mu' = \Gamma (z_{l}, \mu).$$
 (15)

We denote  $k'(k, b, \varepsilon_i; z_l, \mu)$ ,  $b'_d(k, b, \varepsilon_i; z_l, \mu)$ , and  $b'_c(k, b, \varepsilon_i; z_l, \mu)$ , respectively, as the firm's optimal policy functions for capital, debt, and cash holdings that solve problem (13). These, together with the optimal choices over labor  $n(k, \varepsilon_i; z_l, \mu)$  and output  $y(k, \varepsilon_i; z_l, \mu)$  given by (4), constitute the full set of decision rules for a firm with the state  $(k, b, \varepsilon_i; z_l, \mu)$ .

#### 3.2.4 Optimal Cash Policy

Solving the firm's problem, we can derive the condition for optimal cash holdings as follows:

$$1 + \mathbf{1}_{D_2 < 0} \psi_1 = \frac{1}{1 + \eta} \sum_{j=1}^{n_{\varepsilon}} \pi_{ij}^{\varepsilon} [\xi + (1 - \xi)\Theta'] + \lambda_{b'_c},$$
(16)

where  $\Theta = \mathbf{1}_{D_1 < 0} (1 + \mathbf{1}_{D_1 < 0} \psi_1) + (1 - \mathbf{1}_{D_1 < 0}) (1 + \mathbf{1}_{D_2 < 0} \psi_1)$ , and  $\mathbf{1}_{D_i < 0}$  is an indicator function that equals one if the subscript condition is satisfied and zero otherwise, with i = 1, 2.

The left side of equation (16) gives the marginal cost of carrying one additional dollar of cash into the subsequent period. If the firm distributes dividends  $(\mathbf{1}_{D_2<0} = 0)$ , this cost is the cost of forgone distributions, which equals one. If the firm issues equity  $(\mathbf{1}_{D_2<0} = 1)$ , the marginal cost is given by one plus the linear cost of equity issuance.

The right side represents the marginal benefit and has two components. The first is the expected present value of an additional dollar of cash net of its holding cost  $\eta$ . With probability  $\xi$ , the firm will exit next period and pay this extra dollar of cash as dividends to shareholders. With probability  $1 - \xi$ , this additional dollar of cash will serve to relax financial constraints in the future. In other words, firms hold cash to save equity financing costs in case there are liquidity shortages, and therefore need to issue equity at either the pre-production stage,  $\mathbf{1}_{D'_1<0}(1 + \mathbf{1}_{D'_1<0}\psi_1)$ , or the post-production stage,  $(1 - \mathbf{1}_{D'_1<0})(1 + \mathbf{1}_{D'_2<0}\psi_1)$ , in the subsequent period. This motive for accumulating cash echoes the precautionary motives in dynamic models such as that of Riddick and Whited (2009). The second component is embodied in the Lagrange multiplier on the nonnegativity constraint on cash holdings,  $\lambda_{b'_c}$ .

As suggested by equation (16), corporate cash can be valuable when the probability of being liquidity constrained in the subsequent period is high and external financing is costly.

#### 3.3 The Household's Problem

The representative household holds her wealth in firm shares and risk-free bonds. She maximizes her lifetime expected utility by choosing consumption C, working hours N, new bond holdings B', and the number of new shares  $\{\Lambda'(k', b', \varepsilon_j; z_l, \mu)\}$ , given the wage rate  $w(z_l, \mu)$ , risk-free bond price  $q(z_l, \mu)$ , dividend-inclusive equity prices  $\{p_0(k, b, \varepsilon_i; z_l, \mu)\}$ , and ex-dividend equity prices  $\{p_1(k', b', \varepsilon_j; z_l, \mu)\}$ .

The optimization problem can be characterized by the following Bellman equation:

$$U(B,\Lambda;z_{l},\mu) = \max_{C,N,B',\Lambda'} \left\{ u(C,N) + \beta \sum_{m=1}^{n_{z}} \pi_{lm}^{z} U(B',\Lambda';\mu',z_{m}) \right\},$$
 (17)

subject to

$$C + q(z_l, \mu) B' + \int p_1 \Lambda' \left( d\left[ k' \times b' \times \varepsilon_j \right] \right) = w(z_l, \mu) N + B + \int p_0 \Lambda \left( d\left[ k \times b \times \varepsilon_i \right] \right) + T,$$
  
$$\mu' = \Gamma(z_l, \mu), \qquad (18)$$

where T represents the lump-sum transfer payment received by households, which is funded by firms' cash-holding costs. We use  $C(B, \Lambda; z_l, \mu)$ ,  $N(B, \Lambda; z_l, \mu)$ , and  $B'(B, \Lambda; z_l, \mu)$  to denote the optimal policy functions of consumption, working hours, and risk-free bond holdings, respectively, and let  $\Lambda'(k', b', \varepsilon_j; z_l, \mu)$  be the number of equity claims issued by firms with state  $(k', b', \varepsilon_j)$  in the next period.

### 3.4 Market Clearing

In the economy, we have four markets to clear—good market, labor market, bond market, and equity market—and specify them below.

The clearing condition for the good market can be written as

$$C(B,\Lambda;z_{l},\mu) = \int_{\mathbf{S}} \left[ y\left(k,\varepsilon_{i};z_{l},\mu\right) - (1-\xi) I\left(k,k'\left(k,b,\varepsilon_{i};z_{l},\mu\right)\right) - \xi\left(k_{0}+I\left(k,0\right)\right) \right] \mu\left(d\left[k\times b\times\varepsilon_{i}\right]\right) \right]$$
(19)

That is, aggregate output equals aggregate consumption plus capital investment and capital adjustment costs.

The labor market clears when aggregate labor demand from firms is equal to labor supply provided by households:

$$N(B,\Lambda;z_l,\mu) = \int_{\mathbf{S}} n(k,\varepsilon_i;z_l,\mu) \,\mu\left(d\left[k \times b \times \varepsilon_i\right]\right).$$
(20)

Similarly, the bond market clears when total deposits by households and firms are equal to firms' total debt borrowing from the financial intermediary:

$$B'(B,\Lambda;z_l,\mu) = \int_{\mathbf{S}} b'(k,b,\varepsilon_i;z_l,\mu)\,\mu\left(d\left[k\times b\times \varepsilon_i\right]\right),\tag{21}$$

where  $b'(k, b, \varepsilon_i; z_l, \mu) = b'_d(k, b, \varepsilon_i; z_l, \mu) - b'_c(k, b, \varepsilon_i; z_l, \mu).$ 

Lastly, the equity market clears when the following condition holds:

$$\Lambda'(k',b',\varepsilon_j;z_l,\mu) = \int_{\{(k,b,\varepsilon_i)\in\mathbf{S}:(k'(k,b,\varepsilon_i;z_l,\mu),b'(k,b,\varepsilon_i;z_l,\mu))=(k',b')\}} \pi_{ij}^{\varepsilon} \mu\left(d\left[k \times b \times \varepsilon_i\right]\right).$$
(22)

#### 3.5 Recursive Competitive Equilibrium

We now define the recursive competitive equilibrium of the economy.

**Definition 1** A recursive competitive equilibrium consists of prices  $\{w, q, p_0, p_1\}$ , value function U, and optimal policy functions  $(C, N, B', \Lambda')$  for the representative household; value function V and optimal policy functions  $(n, y, k', b'_d, b'_c)$  for firms; and a transition function  $\Gamma$  for firm distribution  $\mu$  with  $\mu' = \Gamma(z_l, \mu)$  such that

- given prices {w,q,p<sub>0</sub>,p<sub>1</sub>}, the value function U and policy functions (C, N, B', Λ') solve the household's utility maximization problem (17);
- 2. given prices {w,q,p<sub>0</sub>,p<sub>1</sub>}, the value function V and policy functions (n, y, k', b'<sub>d</sub>, b'<sub>c</sub>) solve firms' value maximization problem (13);
- 3. markets clear; and

the transition function Γ is consistent with individual behavior, i.e., for any Borel set Q × {ε<sub>j</sub>} ⊂ S, one has

$$\mu'(Q \times \{\varepsilon_j\}) = (1 - \xi) \int_{\{(k,b,\varepsilon_i) \in \mathbf{S}: (k'(k,b,\varepsilon_i;z_l,\mu),b'(k,b,\varepsilon_i;z_l,\mu)) \in Q\}} \pi_{ij}^{\varepsilon} \mu\left(d\left[k \times b \times \varepsilon_i\right]\right) + \xi \mathbf{1}_{(k_0,0) \in Q} M\left(\varepsilon_j\right)$$
(23)

#### 3.6 Computation

We compute the recursive competitive equilibrium of the model using the nonlinear method developed by Khan and Thomas (2008). This method extends the traditional Krusell and Smith (1998) approach and has been applied widely to solve business cycle models with heterogeneous firms in the literature.

Following the insights of Khan and Thomas (2008), we first reformulate the firm's problem by subsuming the efficiency conditions associated with the household's problem. Let  $\bar{C}$  and  $\bar{N}$  denote the market-clearing values of household consumption and hours worked, respectively, and define  $p(z_l, \mu)$  as the intertemporal price of consumption good, i.e.,

$$p(z_l,\mu) = u_1(\bar{C},\bar{N}) = \bar{C}^{-1},$$
 (24)

where  $u_1(\bar{C}, \bar{N})$  is the partial derivative of utility with respect to its first argument.

From the optimality conditions for the household's problem, in equilibrium we have that

$$w(z_{l},\mu) = -\frac{u_{2}(\bar{C},\bar{N})}{u_{1}(\bar{C},\bar{N})} = \phi p(z_{l},\mu)^{-1}, \qquad (25)$$

$$\omega\left(z_{l},\mu,z_{m},\mu'\right) = \beta \frac{u_{1}\left(\bar{C}',\bar{N}'\right)}{u_{1}\left(\bar{C},\bar{N}\right)} = \beta \frac{p\left(z_{m},\mu'\right)}{p\left(z_{l},\mu\right)},\tag{26}$$

$$q(z_{l},\mu) = \beta \sum_{m=1}^{n_{z}} \pi_{lm}^{z} \frac{u_{1}(\bar{C}',\bar{N}')}{u_{1}(\bar{C},\bar{N})} = \beta \sum_{m=1}^{n_{z}} \pi_{lm}^{z} \frac{p(z_{m},\mu')}{p(z_{l},\mu)}.$$
(27)

Equation (25) states that the wage equals the household's marginal rate of substitution between leisure and consumption. Equation (26) means that a firm's state-contingent discount factor coincides with the household's stochastic discount factor. Equation (27) gives the inverse of the risk-free gross interest rate.

By the above three equations, the marginal utility function  $p(\cdot)$  is sufficient to deduce the evolutions of equilibrium prices in the economy. As in Khan and Thomas (2008), we reformulate problem (13) in terms of marginal utility, i.e.,  $V_1(\cdot) = p(z_l, \mu) V(\cdot)$  and  $\tilde{V}_1(\cdot) = p(z_l, \mu) \tilde{V}(\cdot)$ , in which case the problem can be rewritten as

$$V_{1}(k,b,\varepsilon_{i};z_{l},\mu) = \max_{k',b'_{d},b'_{c}} \left\{ \begin{array}{l} p(z_{l},\mu)\left(\left(D_{1}-\Psi(D_{1})\right)\mathbf{1}_{D_{1}<0}+D_{2}-\Psi(D_{2})\right) \\ +\beta\sum_{m=1}^{n_{z}}\sum_{j=1}^{n_{\varepsilon}}\pi_{lm}^{z}\pi_{ij}^{\varepsilon}\tilde{V}_{1}\left(k',b',\varepsilon_{j};z_{m},\mu'\right) \end{array} \right\}, \quad (28)$$

subject to constraints (14) and (15). With the above reformulation, the equilibrium allocations in the economy are fully characterized by problem (28) and conditions from (24) to (27).

To solve problem (28) with a high-dimensional state variable  $\mu$ , we employ the approximation method developed by Krusell and Smith (1998). Firms are assumed to be boundedly rational—that is, they only use partial information regarding the distribution to predict the law of motion for state variables. More precisely, this paper postulates that the aggregate capital stock K is a sufficient statistic to represent firm distribution  $\mu$  in order for firms to deduce prices, and assumes that the laws of motion for capital stock (K) and marginal utility price (p) take the following functional forms:

$$\ln K' = \alpha_K (z_l) + \beta_K (z_l) \ln K, \qquad (29)$$

$$\ln p = \alpha_p \left( z_l \right) + \beta_p \left( z_l \right) \ln K, \tag{30}$$

where  $\{(\alpha_K(z_l), \beta_K(z_l), \alpha_p(z_l), \beta_p(z_l))\}_{l=1}^{n_z}$  are the forecasting coefficients associated with the aggregate productivity  $z_l$ .

Given the specifications above, the competitive equilibrium of the economy can be approximated by an otherwise identical equilibrium, except that firms solve the following optimization problem with partial information:

$$V_{2}(k, b, \varepsilon_{i}; z_{l}, K) = \max_{k', b'_{d}, b'_{c}} \left\{ \begin{array}{c} p(z_{l}, K) \left( \left( D_{1} - \Psi(D_{1}) \right) \mathbf{1}_{D_{1} < 0} + D_{2} - \Psi(D_{2}) \right) \\ +\beta \sum_{m=1}^{n_{z}} \sum_{j=1}^{n_{\varepsilon}} \pi_{lm}^{z} \pi_{ij}^{\varepsilon} \tilde{V}_{2}\left(k', b', \varepsilon_{j}; z_{m}, K'\right) \end{array} \right\}, \quad (31)$$

where the evolutions of K and  $p = p(z_l, K)$  are specified in equations (29) and (30), respectively. The resulting model moments generated based on the approximate policy functions are simulated over a long period of time and compared with those perceived by firms. If they are close enough, then one obtains a good approximation of the equilibrium. Otherwise, one could try different functional forms for the forecasting functions, or include additional moments of the distribution  $\mu$ .

## 4 Calibration

In this section, we take the model described above to data by calibrating it to match aggregate and firm-level U.S. data from 1960 to 2013. Model parameters are grouped into two sets. The first set consists of parameters whose values are standard in the literature or can be estimated directly from data without solving the model. The second set includes parameters that are estimated jointly by minimizing the distance between selected data moments and simulated model moments implied by the stationary equilibrium of the model. Time period t corresponds to one year.

#### 4.1 Parameterization

We calibrate the time discount factor  $\beta$  to match a long-run annual real interest rate of 3%, which is the average during the sample period and implies a discount factor of 0.97. We set leisure preference  $\phi$  to 2.33, such that the household spends one-third of her time in market work.

We use a sample of nonfinancial nonutility firms constructed from Compustat to calibrate the curvature  $\theta$  of firms' production function and the stochastic process of

idiosyncratic productivity shocks  $(\{\varepsilon_i\}, \{\pi_{ij}^{\varepsilon}\})$ . More precisely, we estimate the regression model specified as

$$Y_{i,t} = \beta_0 + \beta_1 k_{i,t} + firm_i + year_t + \epsilon_{i,t}, \qquad (32)$$

where the dependent variable  $Y_{i,t}$  is the logarithm of operating income for firm *i* scaled by the GDP deflator during period *t*, and the independent variables include a constant term; the logarithm of capital stock  $k_{i,t}$  scaled by GDP deflator; firm dummies,  $firm_i$ ; and time dummies,  $year_t$ . The error term  $\epsilon_{i,t}$  corresponds to the logarithm of idiosyncratic productivity  $\varepsilon_i$  in the model. Estimating equation (32) yields  $\hat{\beta}_1 = 0.66$ . Note that by construction, the coefficient of capital stock captures the curvature of the profit function  $\pi (k, \varepsilon_i; z_l, \mu)$  in (5), i.e.,  $\beta_1 = \frac{\alpha \theta}{1-(1-\alpha)\theta}$ . Capital share  $\alpha$  is set at 0.3, a value standard in the literature, and implies  $\theta = 0.86$ . We then collect the fitted residuals and estimate an AR(1) model for a continuous Markov process of idiosyncratic productivity,

$$\hat{\epsilon}_i' = \rho_{\varepsilon} \hat{\epsilon}_i + \eta_i',$$

with  $\eta'_i \sim N(0, \sigma_{\varepsilon}^2)$ . Estimation results yield  $\rho_{\varepsilon} = 0.42$  and  $\sigma_{\varepsilon} = 0.52$ . We discretize this estimated AR(1) process into a three-state Markov chain with transition matrix:

$$\Pi^{\varepsilon} = \left[ \begin{array}{rrrr} 0.46 & 0.38 & 0.16 \\ 0.29 & 0.42 & 0.29 \\ 0.16 & 0.38 & 0.46 \end{array} \right].$$

To estimate the stochastic process of aggregate productivity  $(\{z_l\}, \{\pi_{lm}^z\})$ , we begin by assuming that aggregate TFP follows an AR(1) process  $\log z' = \rho_z \log z + \eta'_z$  with  $\eta'_z \sim N(0, \sigma_z^2)$ . The values of  $\rho_z$  and  $\sigma_z$  are estimated from Solow residuals measured using data on real GDP, private capital, and employment hours, which gives  $\rho_z = 0.902$ and  $\sigma_z = 0.011$ . We then discretize the resulting productivity process into a two-state Markov chain with transition matrix:

$$\Pi^z = \left[ \begin{array}{cc} 0.95 & 0.05 \\ 0.05 & 0.95 \end{array} \right].$$

We set the annual depreciation rate of capital stock to 0.13, the average value from Compustat during the sample period. Estimates for fixed equity issuance costs  $\psi_0$  vary widely in the literature (Cooley and Quadrini, 2001; Gomes, 2001; Hennessy and Whited, 2007). We choose a conservative value by setting it to be 0.007. The internal cash balance held by firms earns interest, which is taxable. The average effective corporate tax rate for the period 1960–2013 was 0.28. For an interest rate of 3%, the cost of holding cash is roughly 85 basis points. We therefore set  $\eta$  to be 0.008.

Moreover, we set exogenous exit rate  $\xi = 0.03$  to match the average value from Compustat. More precisely, we treat it as an exit if firms are deleted from Compustat and their current sales drop by more than one-third from the previous year. We then follow Davis and Haltiwanger (1992) and set the fraction of the steady-state aggregate capital stock held by each entrant at 5%.

The remaining parameters are calibrated by matching selected data moments constructed from Compustat. We target the following moments in the stationary equilibrium (i.e., without aggregate productivity shocks): average cash-to-assets ratio, average debtto-assets ratio, average investment-to-capital ratio, average equity issuance-to-assets ratio, and the standard deviation of the investment-to-capital ratio. More specifically, average cash ratio is informative about fixed operating costs  $\chi$ . A larger  $\chi$  strengthens the transaction motive for cash demand and prompts firms to accumulate more internal cash to facilitate future operations. The average and standard deviation of the capital investment rate helps to identify linear and quadratic capital adjustment costs,  $\gamma_0$  and  $\gamma_1$ . An increase in linear adjustment costs pushes up the real price of capital and affects average investment rates, while an increase in quadratic adjustment costs encourages firms to smooth capital investment, which reduces within-firm volatility. The resale price of capital  $\gamma_2$  can be inferred from the average leverage ratio, given the borrowing constraint specified in condition (7). Lastly, linear equity issuance costs  $\psi_1$  influence firms' choice of equity financing. The average equity issuance ratio thus provides information about  $\psi_1$ . Table 4 summarizes the parameter choices.

[Table 4 about here.]

#### 4.2 Model Moments

#### 4.2.1 Steady State

Table 5 reports model-implied moments in the stationary equilibrium and their empirical counterparts, including the first and second moments of cash holdings, debt financing, and capital investment. Panel A shows the targeted moments used for the joint calibration of  $\gamma_0$ ,  $\gamma_1$ ,  $\psi_1$ , and  $\chi$ , and Panel B presents the nontargeted moments to validate our model.

We construct data moments using a sample of nonfinancial nonutility U.S. firms in Compustat from 1960 to 2013. Corporate cash balance in the model is measured as the sum of cash, cash equivalents, and short-term investments (item *che*). Debt financing is measured as debt in current liabilities (item *dlc*). Capital investment and equity issuance are measured by the items *capx* and *sstk*, respectively. To minimize the effect of outliers, we drop observations with nonpositive assets and winsorize all variables at the bottom and top 1% level. We construct simulated moments by solving the stationary equilibrium of the model. We simulate an economy consisting of 10,000 firms for 45 periods and drop the first 10 periods to limit the effects of initial conditions, and repeat this 1,000 times.

#### [Table 5 about here.]

Results are reported in Table 5. As shown, the calibration targets are well matched. The nontargeted moments, which include both within-firm moments (standard deviation of variables of interest and their serial correlations) and cross-sectional moments (the distributions of variables of interest), are overall close to their corresponding data moments. Specifically, the model-implied serial correlation of cash ratio, leverage ratio, and capital investment rate are lower than the actual values. The low persistence is generated by the exogenous exit rate in the model—that is, firms put a lower weight on future firm value and care more about current-period net equity flow, which reduces the persistence of real and financing variables. In addition, the high persistence of cash in the data is possibly due to its role in smoothing R&D investments in practice, which is absent in our model; the low persistence of capital investment in the model is partially generated by the low quadratic adjustment costs, which are calibrated to match the volatility of investment rates in the data.

Model-implied distributions resemble the data relatively well, except for the 75<sup>th</sup> percentile of cash-to-assets and debt-to-assets ratios. Compared with the data, the model generates a thinner right tail of cash distribution and a fatter right tail of debt distribution. That is, fewer firms in the model tend to have high cash ratios and low leverage ratios. This model-data discrepancy can be explained by the fact that in reality, high-tech firms have limited access to debt financing as a result of the low collateral value of R&D and aggressively accumulate internal cash balances to finance their risky R&D projects; this is not captured by our model.

#### 4.2.2 Business Cycles

As a further validation, we study the dynamics of economic variables implied by our full model, which is driven by aggregate productivity shocks, and compare them with the aggregate data.

We simulate our model with a sequence of aggregate and idiosyncratic productivity shocks and compute the responses of agents in the economy. We then HP-filter all variables with smoothing parameter 100 and compute business cycle moments: relative standard deviations of real variables and their contemporaneous correlations with output. Results are reported in Table 6.

[Table 6 about here.]

As shown, our model successfully reproduces the second-moment properties of U.S. aggregate data. The relative volatilities and cyclical behavior of all real variables—consumption, investment, working hours, and capital stock—are close to the data. The exception is the volatility of working hours, which is smaller compared with the volatility of output. We conjecture that the low volatility of working hours is generated by the low labor-elasticity assumption made in our model; however, allowing for a high labor-supply elasticity by changing households' utility function would substantially increase computational complexity without significantly improving model performance in other aspects.

## 5 The Role of Corporate Saving

In this section, we use the parameterized model to explain the observed cyclical pattern of corporate saving and explore its implications for aggregate fluctuations.

#### 5.1 Model-implied Corporate Saving Behavior

#### 5.1.1 Countercyclicality of Aggregate Corporate Saving

We first examine the model-implied cyclical behavior of financial variables and present it in Table 7. We follow Jermann and Quadrini (2012) to construct debt borrowing and equity issuance at the aggregate level. As shown, the model generates procyclical debt financing and countercyclical cash saving and equity financing, which are in line with their empirical counterparts.

#### [Table 7 about here.]

Using firm-level data, Covas and Den Haan (2011) find that the countercyclicality of aggregate equity issuance is driven by the largest firms in the economy, and most firms exhibit procyclical equity financing. Introducing countercyclical equity issuance costs to generate procyclical aggregate equity financing in our model will not alter our main conclusions. This is because procyclical equity issuance reflects worsened capital market conditions and tends to produce an even stronger incentive for firms to save during recessions, which renders the countercyclicality of corporate saving more pronounced and reinforces the role of corporate cash over business cycles.

#### 5.1.2 Suggestive Evidence for Model Mechanisms

The countercyclicality of corporate saving in our model is mainly induced by fixed operating costs in the presence of costly external financing. To demonstrate this mechanism, we compare two models with different fixed operating costs  $\chi$ : (i) our benchmark model ( $\chi = 0.09$ ) and (ii) a model with a lower operating cost ( $\chi = 0.06$ ). As shown in Table 8, in an economy with a lower operating cost, the countercyclicality of corporate saving disappears. Intuitively, a negative productivity shock signals low income in the future. To ensure sufficient funds to pay fixed operating costs and facilitate operations, firms allocate resources toward cash stock, which generates an increase in corporate cash accumulation during recessions. When fixed operating costs fall, the precautionary saving motive becomes weaker, and cash saving becomes less countercyclical.

To test our proposed explanation, we rank firms based on their fixed operating costs, which we measure as the once-lagged ratio of their selling, general, and administrative costs (item xsga) to sales (item sale). We classify firms in the top third of the distribution as the ones with high operating leverage, and those in the bottom third as the ones with low operating leverage. We then examine whether and how the correlation between corporate cash saving-to-assets ratio and real GDP varies when operating leverage changes. Results are reported in Table 8.

#### [Table 8 about here.]

During the whole sample period 1971–2010, both firms with high operating leverage and those with low operating leverage exhibit countercyclical cash saving, but the former show a more pronounced pattern. To limit the impact of the Great Recession, which is beyond the scope of this paper,<sup>8</sup> we also use the subsample 1971–2007 and find countercyclical cash saving for the former and acyclical cash saving for the latter. These empirical facts lend support to the role of fixed operating costs in generating countercyclical cash-saving behavior.

#### 5.1.3 Heterogeneous Corporate Saving Behavior

We next exploit the feature of firm heterogeneity in our model and examine corporate saving behavior for different-sized firms. We measure firm size by total assets and construct four equal groups. Results are reported in Table 9.

#### [Table 9 about here.]

Our model successfully replicates the hump-shaped relationship between the countercyclicality of cash-saving behavior and firm size, although our model-implied turning point of the pattern differs from the data. The smallest firms appear to have a weak countercyclical cash-saving pattern. This is due to their high marginal product of capital, which counterbalances the negative impact of recessions on capital investment and limits the resources available for cash saving. As firm size increases, the effects of recessions and financial frictions outweigh that of the marginal product of capital, which produces a strong countercyclical movement in corporate saving. As firm size continues to increase, the degree of countercyclicality starts to fall. This less countercyclical saving behavior can be explained by firms' less binding borrowing constraint: Large firms have more pledgeable assets. As a result, they are less likely to be financially constrained and need less precautionary cash.

Overall, we demonstrate that our model is able to reproduce key empirical regularities at both firm level and aggregate level. This strengthens the model's reliability as a laboratory to perform counterfactual experiments, which we examine next.

 $<sup>^8 \</sup>rm Xiao~(2018)$  explains cash-accumulation behavior during the Great Recession.

#### 5.2 Implications for Business Cycles

In this subsection, we use our validated model to examine (i) the effects of corporate saving on economic fluctuations in general and (ii) its particular role during recessions. To this end, we solve a version of the model in which firms face a higher cash-holding cost and thus have weaker incentives to allocate resources to cash stocks. We then compare the resulting equilibrium with that of our baseline model.

#### 5.2.1 Aggregate Fluctuations

We begin by investigating the effects of corporate saving on aggregate fluctuations. We simulate the economy for our benchmark model ( $\eta = 0.8\%$ ) and the model with a higher cash-holding cost ( $\eta = 8\%$ ) and compare the volatility of each real variable and their correlations with output. The ratio of the moment implied by the counterfactual ( $\eta = 8\%$ ) to that implied by our baseline model ( $\eta = 0.8\%$ ) illustrates the role of corporate saving. A ratio greater than 1 indicates that the moment implied by the counterfactual is larger. Results are reported in Table 10.

#### [Table 10 about here.]

Our results reveal that corporate saving does not act as a cushion to stabilize our economy. Instead, it raises the volatility of most real variables—output, investment, hours worked, and capital stock. In particular, the relative volatility of output implied by the counterfactual to that implied by our benchmark model is 0.919, which suggests that lowering corporate savings by one-third dampens aggregate output fluctuations by 9.1%.

The amplification role of corporate saving arises from firms' precautionary concerns, which can be partially inferred from the changes in the degree of cyclicality of investment and employment reported in the second column of Table 10. To better understand the intuition behind these results, we next explore the role of corporate saving during recessions.

#### 5.2.2 The Response to a Negative Productivity Shock

In this subsection, we examine the effects of corporate saving on the impulse responses of key economic variables—output, consumption, capital investment, and labor hiring—to a negative aggregate productivity shock. Figure 4 plots the results. The blue solid line with circles shows the dynamics for our benchmark model ( $\eta = 0.8\%$ ), while the black dashed line shows the patterns for a model with higher cash-holding costs ( $\eta = 8\%$ ).

#### [Figure 4 about here.]

The impulse responses confirm the role of corporate saving in affecting the degree of cyclicality of investment and employment shown in the last subsection. Compared to the model with a higher cash-holding cost, firms in the benchmark model have stronger incentives to accumulate internal cash. In response to a negative productivity shock—and therefore lower marginal products of capital and labor—firms reallocate resources from investment and employment to cash stock. Lower investment reduces the collateral value of capital in the subsequent period. To fund fixed operating costs and facilitate operations, firms further shift resources toward cash. As such, firms in our baseline model make larger cuts in capital investment and labor inputs, which in turn translate into a larger drop in outputs. Quantitatively, the initial drops in output, investment, and working hours in our benchmark model are 4.5%, 47%, and 69% larger than the counterfactual, respectively.

Moreover, the greater economic deterioration in the early stage of the recession lengthens the recovery time, as shown by the dynamics of investment, labor, and output. These results suggest that corporate saving propagates aggregate shocks.

#### 5.2.3 Heterogeneous Impulse Responses

Lastly, we study the heterogeneity in responses to a negative productivity shock across small and large firms, which has important policy implications. We classify firms in the top half of the size distribution as large firms and those in the bottom half as small firms. The corresponding impulse responses are plotted in Figure 5.

#### [Figure 5 about here.]

During recessions, small and large firms behave in a qualitatively similar way along most dimensions, except for cash saving and equity financing. Firms cut capital investment and working hours, which, together with low productivity, pushes down output. A lower collateral value of future capital stock reduces debt capacity. Larger firms respond by substituting equity financing for debt financing, while small firms are less likely to be able to afford fixed equity issuance costs and choose to reduce both types of external finance.<sup>9</sup> Quantitatively, a high marginal product of capital largely prevents small firms from cutting capital investment and reduces resources available for cash saving. This explains the different cash-saving patterns across firms, echoing results shown in subsection 5.1.3.

#### 5.2.4 Financial Frictions and Business Cycles

Previous studies emphasize two mechanisms by which financial frictions amplify shocks. One is through debt borrowing constraints, which limit the resources available for real capital investment. The other is through external financing costs, which alter firms' optimal investment and production decisions.

Our result suggests a new channel: precautionary corporate saving. Facing worsened credit conditions during recessions, firms have stronger incentives to accumulate internal cash to avoid raising expensive external funds. The precautionary saving induced by financial frictions does not dampen the adverse effects of negative shocks; instead, it competes for resources with capital investment and lowers future income, which generates a deeper recession and a longer recovery.

## 5.3 Policy Implications

Our results have important implications for designing effective expansionary monetary policies during recessions. First, our finding of firm heterogeneity in impulse responses

<sup>&</sup>lt;sup>9</sup>These heterogeneity responses in debt and equity financing are consistent with the data; see Covas and Den Haan (2011).

speaks to the question of whether liquidity injections—i.e., quantitative easing—help the real economy during recessions. In response to a negative aggregate productivity shock, large firms tend to reallocate assets from capital investment to cash savings, whereas small firms adhere to their production plans by drawing down existing cash balances to smooth investment. The difference stems from firms' heterogeneity in the marginal product of capital. Our finding, as such, suggests that targeting small firms and providing them with cheap credit during recessions can benefit the real economy.

Second, our result shows that imposing a high cost on holding cash can effectively reduce firms' saving incentives and encourage investment and employment during economic downturns. This finding supports commercial banks' move to introduce charges for large firms' cash deposits and suggests that it can be an effective way to resolve the corporate cash-hoarding issue and speed up economic recovery from recessions.

## 6 Conclusion

The past three decades have witnessed a sharp increase in corporate cash holdings. In this paper, we examine the macroeconomic implications of corporate saving by exploring its role over the business cycle.

We document two empirical regularities regarding corporate saving using both aggregate and firm-level data. First, firms tend to accumulate more internal funds during recessions. Second, there is a hump-shaped relationship between the degree of countercyclicality of corporate saving and firm size.

We next develop a general equilibrium model to account for the stylized facts. The countercyclical corporate saving can be explained by financial frictions and fixed operating costs. During recessions, the presence of financial frictions and fixed operating costs prompts firms to cut investment and shift more resources toward cash so as to retain sufficient internal funds for future operations. Reduced capital investment further lowers future income and borrowing limits, which reinforces the precautionary motive for cash saving. As a result, firms accumulate more cash in response to a negative aggregate productivity shock.

The countercyclicality varies nonmonotonically with firm size. Compared with medium-sized firms, small firms have a higher marginal product of capital and tend to decumulate their cash balances to smooth investment during recessions, while large firms have more collateral assets and therefore demand less cash in response to a negative shock.

Overall, corporate saving leads to a deeper recession, lengthens the entire economic recovery process, and raises output volatility. Our results, therefore, suggest that corporate saving acts as an important channel through which financial frictions propagate aggregate shocks.

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Figure 1: Cyclical Behavior of Corporate Cash Saving: Aggregate-level Facts (1952:Q1–2010:Q2). The figure plots the cyclical dynamics of corporate cash saving over time. The sample is constructed from Flow of Funds, covering the period from 1952:Q1 to 2010:Q2. We use two measures of corporate cash saving for both the nonfinancial business sector and nonfinancial corporate business sector.



Figure 2: Cyclical Behavior of Corporate Cash Saving: Firm-level Facts (1971–2010). The figure plots the cyclical dynamics of corporate cash saving over time. The sample is constructed from Compustat, covering the period from 1971 to 2010. We use four measures of corporate saving for nonfinancial nonutility publicly traded U.S. firms.



Figure 3: Cyclical Behavior of Corporate Saving: By Firm Size (1971–2010). The figure plots the cyclical dynamics of corporate cash saving over time for firms of different sizes. The sample is constructed from Compustat, covering the period from 1971 to 2010. We measure corporate saving as the ratio of changes in total cash to total assets.



Figure 4: Effects of a Negative Productivity Shock. The figure plots the impulse responses of output, consumption, investment, employment, wage rate, and interest rate to a negative aggregate productivity shock. We compare two models: (i) our benchmark model ( $\eta$ =0.8%) and (ii) a model with a high cash-holding cost ( $\eta$ =8%).



Figure 5: Firm Heterogeneity in Impulse Responses. The figure plots the impulse responses of output, investment, employment, debt issuance, cash saving, and equity issuance to a negative aggregate productivity shock. We compare two groups of firms: (i) small firms (blue line with dots) and (ii) large firms (black line with stars).

#### Table 1: Business Cycle Properties of Corporate Saving

Table 1 reports the correlation between the cash saving of nonfinancial corporate businesses and aggregate real GDP. We consider both quarterly and annual data for sample periods 1952–2010, 1984–2010, and 1984–2007. Two-sided p-values are reported in parentheses.

	1952–2010	1984–2010	1984–2007
Panel A: Quarterly			
$corr(\Delta Cash/GDP_{corporate}, GDP_{total})$	-0.27	-0.51	-0.26
	(0.00)	(0.00)	(0.01)
Panel B: Annual			
$corr(\Delta Cash/GDP_{corporate}, GDP_{total})$	-0.26	-0.35	-0.07
	(0.04)	(0.07)	(0.71)

#### Table 2: Business Cycle Properties of Corporate Saving: By Firm Size

Table 2 reports the correlation between corporate saving and real
GDP for different firm sizes. The sample is constructed from Com-
pustat, covering the period from 1971 to 2010. We use four measures
of corporate saving for nonfinancial nonutility publicly traded U.S.
firms. Two-sided <i>p</i> -values are reported in parentheses.

Firm size (percentile)	$\frac{\sum \Delta c_t}{\sum net \ K_t}$	$\frac{\sum \Delta c_t}{\sum gross \ K_t}$	$\frac{\sum \Delta c_t}{\sum Sales_t}$	$\frac{\sum \Delta c_t}{\sum Assets_t}$
All firms	-0.58	-0.56	-0.59	-0.55
	(0.00)	(0.00)	(0.00)	(0.00)
	× /			
[0,25]	-0.13	-0.15	-0.13	-0.11
	(0.42)	(0.36)	(0.43)	(0.50)
[25,50]	-0.32	-0.32	-0.33	-0.34
	(0.04)	(0.04)	(0.04)	(0.03)
[50,75]	-0.48	-0.47	-0.51	-0.50
	(0.00)	(0.00)	(0.00)	(0.00)
		0.00	0.00	
[75,90]	-0.67	-0.66	-0.68	-0.67
	(0.00)	(0.00)	(0.00)	(0.00)
	0.00	0.64	0.00	0.64
[90,95]	-0.00	-0.04	-0.08	-0.04
	(0.00)	(0.00)	(0.00)	(0.00)
[05 00]	-0.50	_0.48	-0.51	-0.46
[90,99]	(0.0)	(0.00)	(0.01)	(0.01)
	(0.0)	(0.00)	(0.01)	(0.01)
[99.100]	-0.32	-0.31	-0.34	-0.27
	(0.04)	(0.05)	(0.03)	(0.09)
	(0.01)	(0.00)	(0.00)	(0.00)

#### Table 3: Business Cycle Properties of Corporate Saving: By Industry

Table 3 reports the correlation between corporate saving and real GDP for each two-digit SIC sector (except utility, finance, and public administration). The sample is constructed from Compustat, covering the period from 1971 to 2010. We use four measures of corporate saving. Two-sided p-values are reported in parentheses.

Industry	$\frac{\sum \Delta c_t}{\sum net \ K_t}$	$\frac{\sum \Delta c_t}{\sum gross \ K_t}$	$\frac{\sum \Delta c_t}{\sum Sales_t}$	$\frac{\sum \Delta c_t}{\sum Assets_t}$
Agriculture, Forestry & Fishing	0.01	0.00	-0.02	-0.00
	(0.95)	(1.00)	(0.90)	(1.00)
Mining	-0.16	-0.18	-0.29	-0.18
	(0.32)	(0.27)	(0.07)	(0.27)
Construction	-0.07	-0.06	-0.14	-0.12
Construction	(0.67)	(0.71)	(0.30)	(0.12)
	(0.01)	(0.11)	(0.00)	(0.40)
Manufacturing	-0.46	-0.44	-0.48	-0.41
	(0.00)	(0.00)	(0.00)	(0.01)
Transportation & Communications	-0.35	-0.34	-0.35	-0.38
	(0.03)	(0.03)	(0.03)	(0.02)
Wholesale Trade	0.16	0.15	0.10	0.10
Wholesale Hade	(0.20)	(0.26)	-0.10	-0.10
	(0.32)	(0.30)	(0.04)	(0.04)
Retail Trade	-0.64	-0.62	-0.67	-0.69
	(0.00)	(0.00)	(0.00)	(0.00)
	~ /			
Services	-0.59	-0.59	-0.59	-0.57
	(0.00)	(0.00)	(0.00)	(0.00)
Nonclassifiable	-0.43	-0.43	-0.46	-0.49
	(0.01)	(0.01)	(0.00)	(0.00)

#### Table 4: Model Parameterization

Parameter	Value
Preference	
discount factor $(\beta)$	0.97
leisure preference $(\phi)$	2.33
Technology and shocks	
capital share $(\alpha)$	0.30
curvature of production function $(\theta)$	0.86
persistence of idiosyncratic shock $(\rho_{\varepsilon})$	0.42
standard deviation of idiosyncratic shock $(\sigma_{\varepsilon})$	0.52
persistence of aggregate shock $(\rho_z)$	0.902
standard deviation of aggregate shock $(\sigma_z)$	0.011
Capital adjustment and depreciation	
linear capital adjustment costs $(\gamma_0)$	0.039
quadratic capital adjustment costs $(\gamma_1)$	0.255
resale price for disinvestment $(\gamma_2)$	0.399
capital depreciation rate $(\delta)$	0.13
Financial and operation frictions	
fixed equity issuance costs $(\psi_0)$	0.007
linear equity issuance costs $(\psi_1)$	0.15
fixed production costs $(\chi)$	0.09
cash holding costs $(\eta)$	0.008
Market exit	
exogenous market exit rate $(\xi)$	0.03

Table 4 summarizes the parameter values used to solve the model. The sample period covers 1960 to 2013.

#### Table 5: Model Moments

Table 5 presents moments of particular interest. Data moments are calculated based on aggregate data and a sample of nonfinancial nonutility firms in Compustat from 1960 to 2013. Model moments are constructed by solving the stationary equilibrium of our baseline model.

Moments	Data	Model
Panel A: Targeted moments		
share of working hours	0.330	0.329
average cash-to-assets ratio	0.156	0.159
average debt-to-assets ratio	0.096	0.105
average investment rate	0.159	0.182
within-firm standard deviation of investment rate	0.133	0.144
average equity-to-assets ratio	0.093	0.068
Panel B: Nontargeted moments		
(i) Cash to assets $(b_{c,t+1}/(k_{t+1}+b_{c,t+1}))$		
within-firm standard deviation	0.111	0.147
within-firm serial correlation	0.779	0.196
$25^{th}$ percentile of the distribution	0.031	0.108
$50^{th}$ percentile of the distribution	0.084	0.133
$75^{th}$ percentile of the distribution	0.216	0.175
(ii) Short-term debt to assets $(b_{d,t+1}/(k_{t+1}+b_{c,t+1}))$		
within-firm standard deviation	0.124	0.094
within-firm serial correlation	0.694	0.310
$25^{th}$ percentile of the distribution	0.005	0.000
$50^{th}$ percentile of the distribution	0.028	0.084
$75^{th}$ percentile of the distribution	0.091	0.190
(iii) Investment to total capital $(i_t/k_{t+1})$		
within-firm serial correlation	0.499	0.266
$25^{th}$ percentile of the distribution	0.063	0.081
$50^{th}$ percentile of the distribution	0.111	0.149
$75^{th}$ percentile of the distribution	0.198	0.226

#### Table 6: Business Cycle Moments: Real Variables

Table 6 summarizes aggregate moments from the full model. We feed both aggregate and idiosyncratic productivity shocks into the model, and compute the volatility of each real variable and their correlations with output.

	$\sigma_x/\sigma_Y$		corr	(x, Y)
Variables $x$	Data	Model	Data	Model
Consumption $(\log C)$	0.910	0.835	0.904	0.977
Investment $(\log I)$	4.090	2.456	0.866	0.908
Working hours $(\log N)$	1.200	0.235	0.835	0.898
Capital stock $(\log K)$	0.584	0.505	0.444	0.250

#### Table 7: Business Cycle Moments: Financial Variables

Table 7 summarizes aggregate financial moments from the full model. We feed both aggregate and idiosyncratic productivity shocks into the model and compute the correlations between financial variables and output.

	corr(x, Y)	
Variables $x$	Data	Model
Corporate saving rate $(\Delta B_c/Y)$	-0.162	-0.099
Debt borrowing rate $(\Delta B_d/Y)$	0.532	0.295
Equity issuance rate $(\Lambda/Y)$	-0.359	-0.704

#### Table 8: Cyclicality of Corporate Saving: By Fixed Costs

Table 8 presents the correlation between corporate saving-to-assets ratio and real GDP for firms with different fixed costs. We first report model-implied moments. We show the cash-GDP correlation implied by our benchmark model ( $\chi = 0.09$ ) in column (1) and that implied by a counterfactual ( $\chi = 0.06$ ) in column (2). We then report data moments for 1971–2010 and 1971–2007, using the sample from Compustat. We use nonfinancial nonutility publicly traded U.S. firms and identify the corresponding two groups based on xsgato-sales ratio. We classify firms at the top one-third percentile as the ones with high fixed costs, and firms at the bottom one-third percentile as the ones with low fixed costs. Two-sided *p*-values are reported in parentheses.

1		
	(1)	(2)
Sample	High fixed costs	Low fixed costs
Model	-0.10	0.24
1971 - 2010	-0.55 (0.00)	-0.36 (0.02)
1971-2007	-0.49 (0.00)	-0.07 (0.68)

#### Table 9: Cyclical Saving Behavior of Different-sized Firms

Table 9 reports the correlation between corporate saving-to-assets ratio and real GDP for different firm sizes. Data moments are calculated from Compustat, covering the period from 1971 to 2010. Model moments are constructed based on our simulated data. Firm size is measured by total assets.

Firm size (percentile)	Data	Model
[0,25]	-0.16	0.79
[25, 50]	-0.32	-0.76
[50,75]	-0.59	-0.50
[75,100]	-0.54	0.07

#### Table 10: Implications for Aggregate Fluctuations

Table 10 summarizes the implications of corporate saving for aggregate fluctuations. We solve our benchmark model ( $\eta = 0.8\%$ ) and a model with a higher cash-holding cost ( $\eta = 8\%$ ). We then compare the model-implied volatility of each variable and their contemporaneous correlations with output by measuring the relative behavior of the counterfactual model ( $\eta = 8\%$ ) compared with our baseline model ( $\eta = 0.8\%$ ). A ratio greater than 1 indicates that the moment implied by the counterfactual is larger.

	Relative volatility	Relative cyclicality		
Output $(\log Y)$	0.919	_		
Consumption $(\log C)$	1.100	1.011		
Investment $(\log I)$	0.608	0.968		
Working hours $(\log N)$	0.556	0.867		
Capital stock $(\log K)$	0.579	0.927		