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## Non-Performing loans fiscal costs and credit expansion in China

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# **Non-Performing Loans, Fiscal Costs and Credit Expansion in China**

by

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## Abstract

This paper studies how the credit expansion policy pursued by the Chinese government in an effort to stimulate its economy in the post-crisis period affects bank–firm loan contracts and the macroeconomy. We build a structural model with financial frictions in which the optimal loan contract reflects the trade-off between leverage and the probability of default. Credit expansion is introduced in the form of the government's partial guarantee on bank loans to (i) general production firms or (ii) infrastructure producers. We show that in the case of general credit expansion, more persistent credit shocks lead to higher credit multipliers at all horizons, as the benefits of persistently alleviating firms' borrowing constraint outweigh the costs associated with higher non-performing loans. We also show that a more persistent targeted credit expansion raises the production of infrastructure goods. However, higher infrastructure production not only boosts the public capital stock and generates positive externalities, it also crowds out private investment and consumption. With a short-lived targeted credit easing, the expansionary channel of public capital dominates, boosting output. As the credit expansion becomes more persistent, the contractionary channel of lower private investment starts to outweigh the expansionary channel in the medium term.

*Bank topics: Fiscal policy; Credit and credit aggregates; International topics*  
*JEL codes: E62; E44*

## Résumé

Dans cette étude, nous nous intéressons à l'incidence, sur les prêts octroyés aux entreprises par les banques et sur le plan macroéconomique, de la politique d'expansion du crédit menée par la Chine pour stimuler l'économie après la crise. Nous construisons un modèle structurel incorporant des frictions financières qui suppose que le prêt optimal reflète l'arbitrage entre le ratio de levier et la probabilité de défaillance. Les autorités chinoises favorisent l'expansion du crédit en garantissant, en partie, des prêts consentis par les banques : 1) aux producteurs en général ou 2) aux entreprises qui participent à des projets d'infrastructure. Nous montrons que, dans le premier cas de figure, des chocs de crédit plus persistants entraînent un effet multiplicateur du crédit plus important pour tous les horizons étudiés, la constante atténuation des contraintes d'emprunt subies par les entreprises offrant des avantages qui l'emportent sur les coûts associés à un accroissement des prêts non productifs. Nous montrons également qu'une expansion du crédit ciblée et continue fait augmenter la production de biens d'infrastructure. Cependant, une telle production ne fait pas que stimuler le stock de capital public et

générer des externalités positives, elle a aussi un effet d'éviction sur l'investissement privé et la consommation. Dans le cas d'un assouplissement ciblé et de courte durée du crédit, l'effet expansionniste du capital public domine, dynamisant la production. Mais à moyen terme, à mesure que l'expansion du crédit se poursuit, l'action restrictive de la faiblesse de l'investissement privé commence à dépasser l'action expansionniste du capital public.

*Sujets : Politique budgétaire ; Crédit et agrégats du crédit ; Questions internationales*

*Codes JEL : E62 ; E44*

## Non-technical Summary

In the wake of the recent global financial crisis, the Chinese government has actively influenced the credit market to stimulate its economy. China's post-crisis stimulus package includes two components: (1) a general credit expansion to the broad economy under guided policy from the central bank that disproportionately favors state-owned firms, to whom the government provides guarantees for loans, and (2) a fiscal stimulus program targeting infrastructure projects, and in large part financed by off-balance-sheet companies that are capitalized and owned by local governments. The total debt associated with these infrastructure projects grew rapidly following the 2008 crisis, from about 8 trillion RMB in 2007 to about 45 trillion RMB in 2015. These two stimulus measures have been viewed by many as playing an important role in uplifting the Chinese economy; however, the subsequent increase in non-performing loan ratios suggests that rapid credit expansion can raise financial challenges in the medium term.

In this paper, we construct a structural model with financial frictions and infrastructural goods production to study the transmission mechanism and effectiveness of Chinese credit expansion. We model the financial frictions in a similar way to Bernanke, Gertler, and Gilchrist (1999), and depart from the standard framework to account for the role of the government. When firms default, the government provides a partial guarantee, or bailout, on bank loans, encouraging financial intermediaries to lend and leading to a credit expansion. The bailout costs then need to be funded by taxes. The credit instrument can be general (extended to general production firms) or targeted (extended to infrastructure goods producers). Infrastructure production contributes to the public capital stock and boosts productivity of firms in general.

We define the credit multiplier as the present value of additional GDP produced by an increase in the present value of bailout costs over the same horizon. In the case of general credit expansion, a more persistent expansion leads to higher credit multipliers, as the benefits of persistently alleviating firms' borrowing constraints outweigh the costs associated with higher non-performing loans. In the case of targeted credit expansion, a more persistent targeted credit expansion also raises the production of infrastructure goods; therefore the key mechanism in general credit expansion is still at play. However, higher infrastructure production not only boosts the public capital stock and generates a positive externality, but also crowds out private investment and consumption. With a short-lived targeted credit easing, the expansionary channel of public capital dominates, boosting output. As the credit expansion becomes more persistent, the contractionary channel of lower private investment starts to outweigh the expansionary channel in the medium term.

Our findings contribute to the stream of research on unintended consequences of credit easing policies, suggesting that broad-based and persistent credit expansion policies can be beneficial, while targeted bailouts to public goods producers might be inefficient, since the positive externality of persistent bailouts raising public capital stock can be outweighed by the negative crowding-out impacts.

## 1 INTRODUCTION

Financial intermediaries play an important role in shaping the real economy through channeling credit from savers to borrowers. Government policy can either facilitate or hinder this credit allocation channel. In the wake of the recent global financial crisis, the Chinese government actively influenced the credit market in an effort to stimulate its economy.

As shown in figure 1, China’s post-crisis stimulus package includes two components – general credit expansion to the broad economy, and a fiscal stimulus program that targeted specific sectors. On the general credit expansion, total loan quota, which are lending targets for commercial banks, were doubled in 2009 compared with 2008.<sup>1</sup> Compliance with the new lending targets was achieved by the central bank relaxing bank financing constraints through adjusting the reserve requirement ratio and base lending rate, as well as through tools including “window guidance” and pledge supplementary lending.<sup>2</sup> The outstanding bank loans to firms increased by 5.6 trillion RMB in 2009, more than doubling the growth in the previous two years, with the service sector and the manufacturing sector absorbing the majority of the loan increase (see Cong, Gao, Ponticelli and Yang, 2017). The stimulus-driven credit expansion disproportionately favored state-owned firms,<sup>3</sup> to whom the government provides guarantees for loans. On fiscal stimulus, the lion’s share of the program was financed by off-balance-sheet companies, known as “Local Government Financing Vehicles” (LGFVs), that borrowed and spent on behalf of local governments. As local governments themselves were prohibited by law from borrowing at the time, the LGFVs were created to circumvent the restriction and borrow from banks to fund local infrastructure projects.<sup>4</sup> In 2009 and 2010, the LGFV companies borrowed 3.6 trillion RMB to finance the stimulus programs (see Bai, Hsieh and Song, 2016). In addition, both central and local governments added 1 trillion RMB to on-balance spending.

While the stimulus package has been viewed by many as playing an important role in boosting the Chinese economy, the non-performing loan ratios rose as banks extended their lending. Figure 2 shows that banks’ lending to the manufacturing, wholesale and retail, mining, and construction sectors rose steadily, from 39% in 2009 to 47% in 2012. The corresponding non-performing loan ratios have also increased steadily since 2011. We further uncover the trend and cyclical facts regarding bank loans following the econometric

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<sup>1</sup>Chinese commercial banks are subject to the loan quota imposed by the People’s Bank of China. The loan quota was increased from 4.9 trillion RMB in 2008 to almost 10 trillion RMB in 2009.

<sup>2</sup>Window guidance refers to the dialogue between the central bank and commercial banks on the general orientation of monetary policy and the pace of lending activities. Pledge supplementary lending is a lending facility under which the central bank directly provides loans with favorable interest rates to policy banks for their re-lending.

<sup>3</sup>Using confidential loan data from the 19 largest Chinese banks (data covering 80% of bank lending in China) Cong, Gao, Ponticelli and Yang (2017) found that, during the stimulus years, the effect of an increase in credit supply on firm borrowing was 36% larger for state-owned firms relative to private firms.

<sup>4</sup>LGFVs emerged in China in the 1990s when the tax sharing system was introduced, which reduced the allocation of the tax share of local governments from almost 80% to about 40%. In addition, the 1994 Budget Law prohibited local governments from incurring budget deficits. Local governments responded by looking for access to financial resources through off-balance-sheet LGFVs. A typical arrangement is for a local government to transfer ownership of a piece of land to an LGFV, which then uses the land as collateral to borrow from banks and issue bonds.



methodology developed by Chang, Chen, Waggoner, and Zha (2016).<sup>5</sup> Similar to Chang, Chen, Waggoner, and Zha (2016), we find that the government’s stimulation of investment after 2008 is visible as a rapid run-up of investment in the trend movement (figure 3). The cyclical patterns support positive but declining boosting effects from credit expansion in either form, confirmed by the 10-year moving window correlations of cycle components of bank loans and GDP (figure 4). Credit expansion that boosts economic growth in the short run can sow seeds for financial challenges in the medium and long run.<sup>6</sup> In particular, when these financial challenges bear fiscal costs, it is important to understand the distortion of credit expansion in bank-firm loan contracts and the economic impacts that follow.

In this paper, we construct structural models with financial frictions to study the mechanism through which credit expansion can distort bank-firms’ loan contracts, and to what extent such distortions can undermine the effectiveness of the stimulus package. We model the financial frictions in a similar way to Bernanke, Gertler, and Gilchrist (1999) and Chang, Liu, Spiegel, and Zhang (2016), and depart from the standard framework to account for the role of the government capturing aspects of Chinese fiscal policy. Before production takes place, firms have to pay working capital through their own net worth and external bank loans. Some firms may have to default on their loans if their idiosyncratic productivity turns out to be lower than the break-even threshold. The government, however, would provide a partial guarantee, or bailout, on bank loans, which changes the incentives for financial intermediaries to lend and firms to borrow. According to the model, a more generous guarantee on bank loans leads to a credit expansion. In the context of the Chinese stimulus package, we examine the impacts of expanding credits through two alternative ways: (1) credit expansion to general intermediate goods firms, and (2) targeted credit expansion to infrastructure producers (the LGFVs).

We find that in the case of general credit expansion, a more persistent expansion leads to higher credit multipliers at all horizons. This is because a more generous bailout policy encourages banks to lend more but at a higher break-even threshold. More loans help to alleviate the working capital constraint, raising firms’ output. A higher break-even threshold, however, raises the non-performing loans and reduces firms’ net worth, which could tighten the future working capital constraint. Our quantitative analysis shows that the overall impact on the economy depends on the persistence of credit expansion. With a short-lived credit expansion, the contractionary channel of non-performing loans dominates. Lower, instead of higher, expected return on capital prompts households to cut investment. Even

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<sup>5</sup>Specifically, we estimate a six-variable quarterly Bayesian vector autoregression (BVAR) over the period 1996Q1 to 2016Q4 and follow the Bayesian reduced-rank time-series method developed by Chang, Chen, Waggoner, and Zha (2016). The six variables are: log real household consumption, log real total business investment, log real GDP, log real labor income, new short-term bank loans and bill financing, and new medium- and long-term bank loans.

<sup>6</sup>Chinese commercial banks classify loans into five categories: pass, special-mention, substandard, doubtful, and loss, with only the last three being considered NPLs according to China’s official reports. When deciding whether a loan is an NPL, Chinese banks are allowed to consider whether they expect to suffer a loss if the borrower defaults. A loan that is more than 90 days past due in the United States is classified as an NPL, even if its collateral value is estimated to be greater than the loan; but in China this loan is categorized as a special-mention loan but not an NPL. However, in this paper we follow the worldwide standard approach and include the category of special-mention loans.

though relaxing the working capital constraint raises output initially, higher bailout costs from credit easing wipe out the initial gain, lowering GDP even in the short term. On the other hand, if credit expansion is persistent, the expansionary channel of relaxing the working capital constraint dominates. Higher expected return on capital raises investment persistently, boosting GDP over the long term, although part of the gain is wiped out by higher bailout costs.

When the credit expansion is targeted to LGFVs, either short-lived or highly persistent credit expansions can lead to higher credit multipliers. The key difference, when compared with the general credit expansion case, is that the partial government guarantee only covers bank loans on infrastructure projects that produce public capital. A more persistent targeted credit expansion raises the production of infrastructure goods; therefore the key mechanism in general credit expansion is still at play in the case with LGFVs. Higher infrastructure production boosts the public capital stock and, therefore, the productivity of firms in general. Since public capital is a public good, however, firms don't take the positive externality into consideration when making allocation decisions, and the targeted credit easing crowds out private investment and consumption. While lower private investment reduces output, lower consumption may prompt households to work more. With a short-lived credit easing, the expansionary channel of public capital dominates, boosting output. As the credit expansion becomes more persistent, the contractionary channel of lower private investment starts to outweigh the expansionary channel in the medium term. If the credit easing is highly persistent, however, the expansionary channel of higher labor supply kicks in over the medium and long run, driving up output.

Our paper contributes to the recent debate on the effectiveness of China's stimulus package in 2008. Wen and Wu (2014) argue that the bold and decisive fiscal stimulus program was the key for China to recover from the Great Recession quickly. Ouyang and Peng (2015), on the other hand, estimate that the program raised the annual real GDP growth in China by about 3.2%, but only temporarily. More recent papers, including Bai, Hsieh, and Song (2016) and Cong, Gao, Ponticelli, and Yang (2017), find that the stimulus-driven credit expansion has reduced the GDP growth in the medium and long run through resource misallocation. Our paper contributes to this debate by showing that the persistence and the format of credit policy play an important role in determining its impact on aggregate economy.

In addition, our paper is related to the literature on credit allocation and its macroeconomic implications. Empirical studies suggest that banks' credit allocation can affect firms' production and employment. For instance, Chodorow-Reich (2013) finds that the bank-firm relationship has a significant impact on employment, particularly for small or medium firms. Caballero, Hoshi, and Kashyap (2008) find that Japanese big banks engaged in bad loan restructuring and kept credit flowing to "zombie" firms, which depresses economic growth. Structural models, for instance those developed in Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997), are useful vehicles to explain those empirical findings. Our paper is closely related to Chang, Liu, Spiegel, and Zhang (2016), which studies the optimal reserve requirement policy in China by focusing on a two-sector model with both state- and private-owned firms. We differ from that paper by examining the Chinese stimulus package

from the perspective of fiscal policy. In particular, we model the Chinese stimulus package through the lens of general credit expansion and targeted credit expansion to LGFVs, emphasizing the interaction among credit policy, non-performing loans and fiscal costs.

The paper proceeds as follows. Section 2 first develops the baseline model with general credit expansion. Section 3 introduces the calibration and quantifies the impact of credit easing at various degrees of persistence in the baseline model. We introduce the LGFV model in Section 4, and the findings are discussed in Section 5. Finally, Section 6 concludes.

## 2 BASELINE MODEL: CREDIT EXPANSION

In this section, we lay out the baseline model with a broad credit expansion across the economy – the government provides a partial guarantee on bank loans issued to all intermediate goods firms. The economy is characterized by: (1) a continuum of infinitely lived households; (2) a continuum of retailers; (3) a continuum of intermediate goods firms facing working capital constraints; (4) financial intermediaries; and (5) fiscal and monetary authorities.

Households provide capital and labor services to intermediate goods firms and leave deposits with financial intermediaries. Retailers are monopolistically competitive and produce retail goods using intermediate goods as inputs. Intermediate goods firms, which use capital and labor as input for production, face idiosyncratic shocks on their productivity. Importantly, they face a working capital constraint and have to pay wages and capital rents before production takes place. Those expenses are financed through firms' own net worth and external loans made by financial intermediaries. Depending on the realizations of the idiosyncratic shocks on productivity, some firms may be unable to repay their loans. The government, however, can provide a partial guarantee on bank loans, which changes the incentives for financial intermediaries to lend and firms to borrow. In this baseline model, the government collects lump-sum taxes to pay for the bailout costs and government consumption.

**2.1 HOUSEHOLDS** The economy is populated with a continuum of identical households. Every household consumes, works, invests in physical capital, and deposits at a financial intermediary. The preference is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t - \psi \frac{h_t^{1+\sigma}}{1+\sigma} \right), \quad (2.1)$$

where  $\frac{1}{\sigma}$  is the Frisch elasticity of labor supply. The household maximizes the lifetime utility by choosing consumption,  $C_t$ , investment,  $I_t$ , labor supply,  $h_t$ , and deposits,  $D_t$ , subject to the budget constraint

$$C_t + I_t + \frac{D_t}{P_t} = W_t h_t + r_t^k K_{t-1} + \frac{R_{t-1} D_{t-1}}{P_t} + \Upsilon_t - T_t. \quad (2.2)$$

Households rent their labor and private capital to the intermediate goods firms with the real returns of  $W_t$  and  $r_t^k$ . Financial intermediaries pay the risk-free nominal interest rate,

$R_t$ , on deposits. They receive the lump-sum transfer,  $\Upsilon_t$ , from various sources, including monopoly profits from retailers and net worth transfers from intermediate goods firms that don't survive. At the same time, they pay lump-sum taxes,  $T_t$ , to the government.

In addition, investment in new capital incurs an adjustment cost,  $\frac{\Omega_k}{2} \left( \frac{I_t}{I_{t-1}} - g_I \right)^2 I_t$ , as the investment growth rate deviates from its steady-state rate,  $g_I$ .  $\Omega_k$  characterizes the size of the cost. Capital evolves with a depreciation rate of  $\delta$  following the law of motion

$$K_t = (1 - \delta)K_{t-1} + \left( 1 - \frac{\Omega_k}{2} \left( \frac{I_t}{I_{t-1}} - g_I \right)^2 \right) I_t. \quad (2.3)$$

Defining the Lagrangian multipliers associated with equation (2.2) and equation (2.3) as  $\Lambda_t$  and  $q_t^k \Lambda_t$ , respectively, the household's utility maximization implies the following optimality conditions:

$$W_t = \psi h_t^\sigma C_t \quad (2.4)$$

$$\frac{1}{R_t} = \beta E_t \frac{C_t}{C_{t+1}} \frac{1}{\pi_{t+1}} \quad (2.5)$$

$$q_t^k = \beta E_t \frac{C_t}{C_{t+1}} (q_{t+1}^k (1 - \delta) + r_{t+1}^k) \quad (2.6)$$

$$1 = q_t^k \left( 1 - \frac{\Omega_k}{2} \left( \frac{I_t}{I_{t-1}} - g_I \right)^2 - \Omega_k \left( \frac{I_t}{I_{t-1}} - g_I \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t q_{t+1}^k \frac{C_t}{C_{t+1}} \Omega_k \left( \frac{I_{t+1}}{I_t} - g_I \right) \left( \frac{I_{t+1}}{I_t} \right)^2. \quad (2.7)$$

**2.2 RETAILERS** There is a continuum of retailers indexed by  $i \in [0, 1]$  in the economy. They purchase intermediate goods at the price of  $P_t^w$  and produce differentiated retail goods  $Y_t(i)$ . Final goods  $Y_t$  used for consumption and investment are CES aggregates of retail goods such that

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (2.8)$$

where  $\epsilon$  is the elasticity of substitution among retail goods. The demand curve for each retail good  $Y_t(i)$  is thus given by

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t. \quad (2.9)$$

Retailers face Rotemberg adjustment costs in changing prices of the form  $\frac{\Omega_p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} \frac{1}{\pi} - 1 \right)^2 Y_t$ , such that price changes in excess of steady-state inflation rates are costly. The retail goods firm  $i$  maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left( \frac{P_{t+j}(i) - P_{t+j}^w}{P_{t+j}} Y_{t+j}(i) - \frac{\Omega_p}{2} \left( \frac{P_{t+j}(i)}{P_{t+j-1}(i)} \frac{1}{\pi} - 1 \right)^2 Y_{t+j} \right),$$

subject to the demand function (2.9). The first-order condition is

$$\frac{1}{x_t} = \frac{(\epsilon - 1)}{\epsilon} + \frac{\Omega_p}{\epsilon} \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} - \beta \frac{\Omega_p}{\epsilon} E_t \frac{C_t}{C_{t+1}} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{Y_{t+1}}{Y_t}, \quad (2.10)$$

where  $x_t = \frac{P_t}{P_t^w}$ . Equation (2.10) represents the New Keynesian Phillips curve (NKPC) under Rotemberg pricing, which would, upon linearization, correspond to the standard NKPC under Calvo pricing. Monopoly profits, which are transferred to households, are given by

$$\Upsilon_t^R = Y_t - \frac{1}{x_t} Y_t - \frac{\Omega_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t. \quad (2.11)$$

**2.3 INTERMEDIATE GOODS FIRMS** A continuum of intermediate goods firms indexed by  $j$  use capital and labor as inputs for production. Each firm  $j$  faces an idiosyncratic productivity shock  $\omega_t^j$  that is i.i.d drawn from a log normal distribution,  $F(\cdot)$ , with a mean of 1. As a result, each firm's output can vary depending on the realization of  $\omega_t^j$ ,

$$Y_t^j = \omega_t^j (K_{t-1}^j)^{1-\alpha} \left( A_t (h_t^{e,j})^{1-\theta} (h_t^j)^\theta \right)^\alpha, \quad (2.12)$$

where  $A_t$  represents the aggregate productivity shock, which has a deterministic growth trend,  $g$ , and a stationary component,  $A_t^m$ , such that  $A_t = g^t A_t^m$ .  $A_t^m$  follows an AR(1) process.  $h_t^{e,j}$  is the demand for managerial labor and  $h_t^j$  is the demand for household labor.

Intermediate firms face a working capital constraint. They have to pay for wages and capital rents before production takes place. Following the literature, we assume each firm starts with the same level of beginning-of-period capital,  $K_{t-1}$ . Therefore, all intermediate goods firms face the same ex ante cost minimization problem as such:

$$\min W_t h_t + r_{kt} K_{t-1} + W_t^e h_t^e \quad (2.13)$$

$$s.t. \quad Y_t = E_j(\omega_t^j) (K_{t-1})^{1-\alpha} \left( A_t (h_t^e)^{1-\theta} h_t^\theta \right)^\alpha. \quad (2.14)$$

Assuming  $\nu_t$  is the Lagrangian multiplier associated with the production function, the optimality conditions are

$$W_t = \nu_t \alpha \theta \frac{Y_t}{h_t} \quad (2.15)$$

$$W_t^e = \nu_t \alpha (1 - \theta) \frac{Y_t}{h_t^e} \quad (2.16)$$

$$r_{kt} = \nu_t (1 - \alpha) \frac{Y_t}{K_{t-1}}. \quad (2.17)$$

In a model without the working capital constraint, the competitive market drives profits to zero for intermediate goods firms, and therefore  $\nu_t = \frac{1}{x_t}$ . In this model, however, the working capital constraint renders a wedge between  $\nu_t$  and  $\frac{1}{x_t}$ . Combining the (2.15), (2.16), and (2.17), we have

$$\frac{1}{\nu_t} = \left( \frac{1 - \alpha}{r_{kt}^k} \right)^{1-\alpha} \left( A_t \left( \frac{\alpha(1 - \theta)}{W_t^e} \right)^{1-\theta} \left( \frac{\alpha\theta}{W_t} \right)^\theta \right)^\alpha. \quad (2.18)$$

To finance those expenses on wages and capital rents, firms resort to their own beginning-of-period net worth,  $N_{t-1}$ , which is assumed to be the same across firms, and external debt. Again, all firms would borrow the same amount of debt,  $B_t$ , for the given state of the economy, as the idiosyncratic productivity shock is i.i.d.:

$$\frac{N_{t-1} + B_t}{P_t} = W_t h_t + W_t^e h_t^e + r_t^k K_{t-1}. \quad (2.19)$$

Therefore,

$$\frac{Y_t}{x_t} = \tilde{A}_t \frac{N_{t-1} + B_t}{P_t}, \quad (2.20)$$

where  $\tilde{A}_t$  is the overall return on working capital, given by

$$\tilde{A}_t = \frac{1}{x_t} \left( \frac{1 - \alpha}{r_t^k} \right)^{1-\alpha} \left( A_t \left( \frac{\alpha(1-\theta)}{W_t^e} \right)^{1-\theta} \left( \frac{\alpha\theta}{W_t} \right)^\theta \right)^\alpha. \quad (2.21)$$

**2.4 FINANCIAL INTERMEDIARIES** We model financial intermediaries in a similar way to Bernanke, Gertler, and Gilchrist (1999) and Chang, Liu, Spiegel, and Zhang (2016). At the beginning of each period, a risk-neutral financial intermediary (FI) obtains household deposit,  $D_t$ , at the interest rate of  $R_t$ . It lends to intermediate goods firms, which choose the level of debt prior to the realization of idiosyncratic firm-specific productivity shocks, and charges a rate of  $Z_t$ . The optimal contract is then characterized by a threshold on idiosyncratic productivity,  $\bar{\omega}_t$ , such that the intermediate goods producer with the cutoff productivity is just able to repay the external debt  $B_t$ :

$$\bar{Y}_t P_t^w = Z_t B_t.$$

$\bar{Y}_t$  is the firm production with the cutoff idiosyncratic productivity,

$$\bar{Y}_t = \bar{\omega}_t (K_{t-1})^{1-\alpha} \left( A_t (h_t^e)^{1-\theta} (h_t)^\theta \right)^\alpha.$$

Note that  $\bar{Y}_t = \bar{\omega}_t Y_t$ , as all intermediate goods firms face the same ex ante cost minimization problem and make the same resource allocation decisions. Intermediate goods firms' working capital constraint is given by the following:

$$\tilde{A}_t (N_{t-1} + B_t) = Y_t P_t^w.$$

We thus have

$$\bar{\omega}_t = \frac{Z_t B_t}{\tilde{A}_t (N_{t-1} + B_t)}. \quad (2.22)$$

When  $\omega_t \geq \bar{\omega}_t$ , the firm repays the loan and FI receives the payoff of  $Z_t B_t$ . When  $\omega_t < \bar{\omega}_t$ , the firm cannot pay the contractual return and has to default. In this case, the FI pays a monitoring cost, defined as a fraction ( $m_t$ ) of the firm's realized total revenue, to observe the

realized idiosyncratic productivity shock and collect the firm's production. The government can partially guarantee bank loans, in which case the FI receives a fraction of the monitoring costs,  $l_t m_t \tilde{A}_t \omega_t (N_{t-1} + B_t)$ , from the government bailout funds. Overall, the expected nominal income for the lender is given by

$$\begin{aligned} & (1 - F(\bar{\omega}_t)) Z_t B_t + \int_0^{\bar{\omega}_t} \left( (1 - m_t) \tilde{A}_t \omega_t (N_{t-1} + B_t) + l_t m_t \tilde{A}_t \omega_t (N_{t-1} + B_t) \right) dF(\omega) \\ = & \tilde{A}_t (N_{t-1} + B_t) \underbrace{\left( [1 - F(\bar{\omega}_t)] \bar{\omega}_t + (1 - m_t + l_t m_t) \int_0^{\bar{\omega}_t} \omega dF(\omega) \right)}_{g(\bar{\omega}_t)}. \end{aligned} \quad (2.23)$$

As a result, the FI would lend to firms if the following participation constraint holds:

$$\tilde{A}_t (N_{t-1} + B_t) g(\bar{\omega}_t) \geq R_t B_t, \quad (2.24)$$

which illustrates the FI's supply of debt.

Given the participation constraint, firms choose  $\bar{\omega}_t$  and  $B_t$  to maximize their expected income, which is

$$\tilde{A}_t (N_{t-1} + B_t) \underbrace{\left( \int_{\bar{\omega}_t}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}_t)) \bar{\omega}_t \right)}_{f(\bar{\omega}_t)}. \quad (2.25)$$

$\tilde{A}_t f(\bar{\omega}_t)$  can be interpreted as firms' expected return on their total asset  $N_{t-1} + B_t$ . The following first-order condition characterizes the optimal contract,

$$\frac{N_{t-1}}{N_{t-1} + B_t} = - \frac{g'(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \frac{\tilde{A}_t f(\bar{\omega}_t)}{R_t}, \quad (2.26)$$

which illustrates firms' demand for external debt.

In addition, we follow the literature and assume that only a share ( $\zeta$ ) of intermediate goods firms survive at each period. This assumption ensures that firms won't accumulate enough net worth such that they do not need to resort to external debt for financing. Therefore, the end-of-period aggregate net worth,  $N_t$ , depends on profits from surviving firms and managerial labor income, which can be described as follows:

$$N_t = \zeta \tilde{A}_t (N_{t-1} + B_t) f(\bar{\omega}_t) + P_t W_t^e h_t^e. \quad (2.27)$$

The net worth for firms that don't survive,  $(1 - \zeta) \tilde{A}_t (N_{t-1} + B_t) f(\bar{\omega}_t)$ , is transferred to households in a lump-sum way.

**Loan Demand and Supply with Credit Shocks** This section explains how credit expansion through a higher bailout ratio  $l_t$  affects the incentive for financial intermediaries to lend and firms to borrow.

In a competitive market for financial intermediaries, their zero profit condition implies that the following constraint has to hold for any loan contract,

$$\tilde{A}_t(N_{t-1} + B_t)g(\bar{\omega}_t) = R_t B_t.$$

Note that the risk-free rate on deposit depends on the average return on intermediate goods firms (as shown in the definition of  $g(\bar{\omega}_t)$ ). This is a source of inefficiency, as a benevolent planner would prefer that the risk-free rate corresponds to the marginal return on firms. If we define the leverage ratio as  $lev_t = \frac{N_{t-1} + B_t}{N_{t-1}}$ , then the loan supply constraint becomes

$$lev_t = \frac{1}{1 - \frac{\tilde{A}_t}{R_t}g(\bar{\omega}_t)}. \quad (2.28)$$

As  $g(\bar{\omega}_t)$  is an increasing function of  $\bar{\omega}_t$ , a higher productivity cutoff raises the return on working capital relative to the deposit rate, which raises the FI's willingness to lend and therefore the leverage ratio.

Given the FI participation constraint, on the other hand, intermediate goods firms maximize their expected income given by

$$\tilde{A}_t(N_{t-1} + B_t)f(\bar{\omega}_t) = N_{t-1} \underbrace{\frac{1}{1 - \frac{\tilde{A}_t}{R_t}g(\bar{\omega}_t)}}_{\text{leverage}} \underbrace{\tilde{A}_t f(\bar{\omega}_t)}_{\text{firms' expected return}}. \quad (2.29)$$

For the given return on working capital,  $\tilde{A}_t$ , and the existing net worth,  $N_{t-1}$ , firms' expected income depends on the leverage ratio and the expected return on their assets, both of which depend on the productivity cutoff,  $\bar{\omega}_t$ . Moreover,  $f'(\cdot) < 0$  and  $g'(\cdot) > 0$  imply that a higher cutoff imposes tradeoffs for firms: it raises the leverage but reduces the expected return. Therefore, firms choose  $\bar{\omega}_t$  optimally to balance the tradeoffs and maximize the overall income. Its first-order condition becomes

$$\underbrace{\frac{\frac{\tilde{A}_t}{R_t}g'(\bar{\omega}_t)}{1 - \frac{\tilde{A}_t}{R_t}g(\bar{\omega}_t)}}_{\text{elasticity of leverage w.r.t } \bar{\omega}_t} = \underbrace{\frac{f'(\bar{\omega}_t)}{f(\bar{\omega}_t)}}_{\text{elasticity of firms' expected return w.r.t } \bar{\omega}_t}. \quad (2.30)$$

Now consider a scenario of where the government raises the bailout ratio,  $l_t$ . Since  $f(\cdot)$  is independent from  $l_t$ , this policy change doesn't affect the the elasticity of firms' expected return w.r.t  $\bar{\omega}_t$ , i.e., the right-hand side of first-order condition. However,  $g(\cdot)$  depends on  $l_t$ , and so does the elasticity of leverage w.r.t  $\bar{\omega}_t$ . A more generous bailout policy raises the elasticity of leverage w.r.t  $\bar{\omega}_t$ , as the FI is willing to lend more. As a result, firms are willing to accept a loan contract with a higher cutoff on productivity, which, according to the FI's loan supply constraint, implies higher loans.

**2.5 GOVERNMENT POLICY AND THE RESOURCE CONSTRAINT** The central bank conducts monetary policy following a standard Taylor rule,

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\psi_\pi} \left(\frac{GDP_t}{GDP_{t-1}g}\right)^{\psi_y}, \quad (2.31)$$



where  $g$  is the target GDP growth rate, and parameters  $\psi_\pi$  and  $\psi_y$  capture the magnitude of the monetary policy reaction.

The fiscal authority collects taxes to pay for the government consumption,  $G_t^c$ , and bailout costs,  $S_t$ . In this baseline model, we assume that the government collects lump-sum taxes and is unable to borrow. The government budget, therefore, is balanced in each period,

$$T_t = \underbrace{\tilde{A}_t(N_{t-1} + B_t)l_t m_t \int_0^{\bar{\omega}_t} \omega_t dF(\omega)}_{S_t} + G_t^c. \quad (2.32)$$

The government can affect the economy using two instruments: government consumption and the bailout ratio  $l_t$ . Both instruments are assumed to be exogenous:

$$\ln \frac{G_t^c}{G} = \rho_g \ln \frac{G_{t-1}^c}{G^c} + \epsilon_t^g \quad (2.33)$$

$$\ln \frac{l_t}{l} = \rho_l \ln \frac{l_{t-1}}{l} + \epsilon_t^l, \quad (2.34)$$

where  $\epsilon_t^g \sim N(0, \sigma^g)$ ,  $\epsilon_t^l \sim N(0, \sigma^l)$ .

Final goods are used for private consumption, investment, government consumption, monitoring costs, and price adjustment costs. If we let  $GDP_t$  denote total output net of adjustment and monitoring costs, then the market clearing condition implies

$$Y_t = GDP_t + \frac{\Omega_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t + \tilde{A}_t \frac{N_{t-1} + B_t}{P_t} m_t \int_0^{\bar{\omega}_t} \omega dF(\omega) \quad (2.35)$$

$$GDP_t = C_t + I_t + G_t. \quad (2.36)$$

### 3 QUANTITATIVE ANALYSIS: BASELINE MODEL

**3.1 CALIBRATION** We calibrate the baseline model as described below (also shown in Table 1). Some parameters are calibrated in line with previous macro studies. For example, the quarterly subjective discount factor is given a value of 0.99. The inverse of the Frisch elasticity is set to be 2. The elasticity of substitution among intermediate goods  $\epsilon$  is set to 10, yielding an average markup of 11%. The price adjustment cost parameter  $\Omega_p$  takes a value of 22, which implies that retailers, on average, change prices approximately once every three quarters. The estimated length of price contracts is in line with the literature. For instance, Lubik and Schorfheide (2005) report estimates of the price stickiness parameter ranging from 0.74 to 0.78 in their two-country benchmark model, corresponding to an average duration of price contracts of approximately three quarters. The depreciation rate is given a value of 0.025 such that the average annual depreciation rate is 10%. The investment adjustment cost parameter  $\Omega_I$  is given a value of 1, in line with macro estimates. Monetary policy coefficients are chosen to match values generally associated with the Taylor rule such that  $\psi_\pi$  and  $\psi_y$  are set to be 1.5 and 0.5 respectively.

Other parameters are calibrated to match the Chinese data. The capital share is calibrated to  $\alpha = 0.5$ , consistent with empirical evidences from Brandt, Hsieh, and Zhu (2008)

and Zhu (2012). Steady state inflation rate is calibrated to 2.2%, corresponding to the past 20-year average (1996-2006). The steady-state balance growth rate is set at 1.0125, which implies an annual growth rate of 5% at the steady state. The share of government spending to GDP is set to be 0.14 to match the average share of government spending in GDP over 1980 to 2016. Similar to Chang, Liu, Spiegel, and Zhang (2016), the share of household labor is set to .94.

Like Bernanke, Gertler, and Gilchrist (1999), we assume that the idiosyncratic productivity shocks are drawn from a log normal distribution. Hsieh and Klenow (2009) estimate that the standard deviation of revenue productivity across manufacturing firms in China was 0.74 in 1998, 0.68 in 2001 and 0.63 in 2005. We calibrate the log normal standard deviation  $\sigma_\omega$  to be 0.7. Regarding the parameters related to bank-firm loan contracts, we follow Bernanke, Gertler, and Gilchrist (1999) and set the fraction of output loss from bankruptcy  $m$  to be 0.12. The entrepreneur's survival rate is set at 0.93.

For the technology shock parameters, we calibrate the AR(1) coefficient to be 0.95 and the standard deviation to be 0.01. For the government spending shock, we fit the data on the log share of government spending to GDP (1980 to 2016) to an AR(1) process and calibrate the persistence parameter to be 0.84 and the standard deviation to be 0.05 accordingly. There is little guidance provided from the literature regarding the parameters characterizing the bailout shock. We calibrate the model to three alternative scenarios whether the bailout shock is short-lived ( $\rho_l=0$ ), highly persistent ( $\rho_l=0.95$ ) or in between ( $\rho_l=0.9$ ). In all three cases, we leave the standard deviation of the bailout shock as 0.3. The steady-state bailout ratio  $\bar{l}$  is set to be 0.2, which means the financial intermediary can recover 20% of the monitoring costs from the government at the steady state.

**3.2 CREDIT SHOCKS** Credit easing can relax the working capital constraint for firms, potentially boosting production; on the other hand, it also increases the default rate and NPL ratio, cutting firms' net worth and raising bailout costs. To what extent credit expansion can boost the economy depends on which channel dominates. Figure 5 shows that the bailout shock persistence plays a significant role.

We first consider a one-time increase in the bailout ratio from 0.2 to 0.5, whose impulse responses are shown as blue lines in figure 5. As explained in the previous section, credit easing leads firms to borrow more from the financial intermediaries. Higher external debt relaxes the working capital constraint, raising wages and capital rent at period  $t = 1$ . Households work more, raising their consumption and output initially. On the other hand, credit easing also raises the productivity cutoff and forces more firms to default, raising the bailout costs. The rise in bailout costs wipes out the initial gain in output shortly after, lowering GDP slightly in the short run. A higher default rate also lowers firms' net worth. Since agents expect credit easing to last for only one period, the expected return on capital drops, together with firms' net worth, prompting households to cut investment. Therefore, a temporary credit easing reduces, instead of boosting, investment.

Investment dynamics change significantly, however, if credit shocks are more persistent. The orange lines in figure 5 illustrate the impulse responses under a very persistent credit shock that is expected to last for 12.5 years (with  $\rho_l = 0.98$ ). In this case, firms can sustain

higher borrowing from the financial intermediaries for a long period of time, leading to a hump-shaped path for external debt. Access to external debt relaxes the working capital constraint persistently, raising the expected return on capital. Although firms' net worth is lower than their steady state, households increase their investment. The boost from investment raises the total output. Part of the gain in output is again wiped out by higher bailout costs. Higher investment, however, still persistently boosts GDP over the medium and long term.

The positive impact on GDP over the medium and long term, however, requires credit shocks to be highly persistent. The red lines in figure 5 show that if the credit expansion is expected to last for only 10 quarters (with  $\rho_l = 0.9$ ), the output jumps up following the shock. High non-performing loans, however, lead to high bailout costs over the medium term. As a result, GDP drops below its steady state shortly after the initial boost and only converges back to its steady state over the long run.

Even though highly persistent credit shocks raise investment and GDP, consumption is persistently lower than its steady state. In this baseline model, a more generous bailout policy doesn't improve welfare. In addition, credit shocks act like a supply shock initially: they relax the working capital constraint for firms, increasing the aggregate supply; therefore, inflation drops initially.

**3.3 FISCAL MULTIPLIERS** To quantify the impact of credit shocks on the aggregate economy, we compute credit multipliers defined in a similar way to conventional government spending multipliers. Following Uhlig (2010) and Leeper, Traum, and Walker (2017), we adopt present-value multipliers, which embody the full dynamics associated with exogenous policy actions and properly discount future macroeconomic effects. The present value of additional GDP over a  $k$ -period horizon produced by an exogenous change in the present value of credit expansion is

$$\text{mul}_t^l(k) = \frac{E_t \sum_{j=0}^k \prod_{i=0}^j (1 + r_{t+i})^{-1} \Delta GDP_{t+j}}{E_t \sum_{j=0}^k \prod_{i=0}^j (1 + r_{t+i})^{-1} \Delta S_{t+j}}. \quad (3.1)$$

Figure 6 shows that credit multipliers crucially depend on the persistence of credit shocks. With an i.i.d credit shock (blue line), the present-value accumulative multiplier is positive in the short term before turning negative after about five quarters, reaching -2.5 after five years. A more persistent credit expansion ( $\rho_l = 0.9$ , red line) leads to a positive multiplier on impact close to 3. Over the long run, however, the multiplier drops and eventually is close to 0 after five years. If the credit expansion is highly persistent ( $\rho_l = 0.98$ , orange line), the accumulative multiplier is around 3.5 upon impact and remains positive at close to 2 after two years.

The distinct multiplier patterns are consistent with the impulse responses in figure 5. Credit easing can relax the working capital constraint and boost production; on the other hand, it also increases the default rate and bailout costs. If credit shocks are short lived, the second channel dominates, discouraging investments. With highly persistent credit expansion, the first channel dominates, raising investment.

Since advanced economies largely use conventional instruments of government spending for fiscal stimulus, we also compute the present-value multipliers for government spending for comparison:

$$\text{mul}_t^g(k) = \frac{E_t \sum_{j=0}^k \prod_{i=0}^j (1 + r_{t+i})^{-1} \Delta GDP_{t+j}}{E_t \sum_{j=0}^k \prod_{i=0}^j (1 + r_{t+i})^{-1} \Delta G_{t+j}}. \quad (3.2)$$

The green line in figure 6 shows the government spending multiplier over different time horizons. For each dollar the government spends, credit expansion dominates the conventional instrument of government spending when the bailout is persistent. Persistent credit expansion helps to alleviate the working capital constraint and therefore market inefficiency, and is thus more useful than government spending. If the credit expansion is a one-time move, it outperforms the government spending tool in the short term but underperforms over the medium and long term (government spending is entirely useless in this model as in most fiscal multiplier papers).

We further examine the sensitivity of credit multipliers with respect to the size of investment adjustment costs. Figure 7 shows the government spending multiplier and credit multipliers with credit shocks at various levels of persistence when the investment adjustment cost is set to zero. Whether the credit shock is i.i.d, persistent, or highly persistent, zero friction in adjusting investment leads to positive credit multipliers upon impact, and higher multipliers than the counterparts in the baseline calibration at all horizons. This is consistent with the transmission mechanism: since credit shocks boost the economy through the investment channel, a frictionless environment helps to channel the credit expansion to the real economy, raising the multipliers.

**3.4 EXTENSION: DISTORTIONARY TAXES** The baseline model with lump-sum taxes serves as a good benchmark to illustrate the mechanism through which credit policy impacts the economy. Taxation, however, is often distortionary. If the government has to raise distortionary taxes to finance higher bailout costs, then credit easing may come with a higher price tag. In this section, we assume that retail firms have to pay a proportional sales tax,  $\tau_t$ , and therefore their optimization problem becomes

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left( \frac{(1 - \tau_{t+j}) P_{t+j}(i) - P_{t+j}^w}{P_{t+j}} Y_{t+j}(i) - \frac{\Omega_p}{2} \left( \frac{P_{t+j}(i)}{P_{t+j-1}(i)} \frac{1}{\pi} - 1 \right)^2 Y_{t+j} \right). \quad (3.3)$$

Their first-order condition becomes

$$\frac{1}{x_t} = \frac{(\epsilon - 1)(1 - \tau_t)}{\epsilon} + \frac{\Omega_p}{\epsilon} \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} - \beta \frac{\Omega_p}{\epsilon} E_t \frac{C_t}{C_{t+1}} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{Y_{t+1}}{Y_t}. \quad (3.4)$$

The government budget constraint becomes

$$\tau_t Y_t = \tilde{A}_t (N_{t-1} + B_t) l_t m_t \int_0^{\bar{\omega}_t} \omega_t dF(\omega) + G_t. \quad (3.5)$$

As shown in figure 8, whether the credit shock is i.i.d, persistent, or highly persistent, a distortionary tax leads to positive credit multipliers upon impact, but to lower multipliers

than the counterparts in the baseline case at all horizons. When the government collects distortionary taxes to finance bailout costs, credit multipliers turn negative in the medium to long term, unless the credit expansion is highly persistent. This is because the sales tax drives up the relative price of retail goods to intermediate goods, thus tightening the working capital constraint and offsetting the expansionary impact from credit easing. The ranking remains the same in the sense that credit expansion dominates the conventional instrument of government spending at all horizons except when the credit expansion is short-lived.

## 4 ALTERNATIVE MODEL: LGFV

In this section, we modify the baseline model by introducing infrastructure producers, which play the role of LGFVs in the real world. Specifically, LGFVs use land and final goods as inputs to produce infrastructure, which is authorized public investment and contributes to the stock of public capital. Public capital is a public good: even though it enters intermediate firms production, these firms don't pay rents on public capital; instead the government is the buyer of infrastructure products. Importantly, LGFVs are assumed to face working capital constraints and idiosyncratic shocks on their productivity, similar to the setup of intermediate firms in the baseline model. Working capital is financed through LGFVs' own net worth and external loans made by financial intermediaries. Upon the realizations of the idiosyncratic productivity shocks, the government can step in and cover partial bank losses when LGFVs are unable to repay their loans. Households, retailers, and the central bank function exactly the same way as in the baseline model. In the following, we describe what has changed in the alternative model.

**4.1 INTERMEDIATE GOODS FIRMS** Intermediate goods firms are perfectly competitive.<sup>7</sup> Let  $K_t^g$  denote the aggregate public capital, with  $\alpha_g$  being the elasticity of output with respect to public capital, indicating the productivity of public capital (similar to Leeper, Walker and Yang, 2010). The production function is given by

$$Y_t = (K_{t-1})^{1-\alpha} (A_t h_t)^\alpha (K_t^g)^{\alpha_g}. \quad (4.1)$$

Cost minimization implies the following optimality conditions:

$$W_t = \frac{1}{x_t} \frac{\alpha Y_t}{h_t} \quad (4.2)$$

$$r_{kt} = \frac{1}{x_t} \frac{(1-\alpha)Y_t}{K_{t-1}}. \quad (4.3)$$

where  $x_t$  is the inverse of the relative price of intermediate goods with respect to final goods.

**4.2 LGFVs** There is a continuum of LGFVs indexed by  $j$ . They build infrastructure using final goods,  $y_t^{gj}$ , and land,  $L_t^j$ , as inputs. Each producer faces an idiosyncratic productivity shock,  $\omega_t^j$ , that is i.i.d drawn from a log normal distribution  $F(\cdot)$ :

$$I_t^{gj} = \omega_t^j (y_t^{gj})^{\theta_g} (L_t^j)^{1-\theta_g}. \quad (4.4)$$

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<sup>7</sup>In order to highlight the channel through LGFVs, we have dropped the working capital constraint for intermediate goods firms in the alternative model.

LGFVs face a working capital constraint. They need to pay for the inputs before the idiosyncratic shock is realized and production takes place. Ex ante, they face the same cost minimization problem as the following:

$$\min y_t^g + p_t^L L_t \quad (4.5)$$

$$s.t. \quad I_t^g = E_j(\omega_t^j)(y_t^g)^{\theta_g}(L_t)^{1-\theta_g}, \quad (4.6)$$

where  $p_t^L$  is the relative price of land with respect to the final good price and land supply is assumed to be exogenously given. Assuming  $\nu_t^g$  is the Lagrangian multiplier associated with the production function, cost minimization entails

$$1 = \nu_t^g \theta_g \frac{I_t^g}{y_t^g} \quad (4.7)$$

$$p_t^L = \nu_t^g (1 - \theta_g) \frac{I_t^g}{L_t}. \quad (4.8)$$

To finance those expenses, LGFVs resort to their own beginning-of-period net worth,  $N_{t-1}$ , and external debt:

$$\frac{N_{t-1} + B_t}{P_t} = y_t^g + p_t^L L_t. \quad (4.9)$$

Therefore,

$$\frac{I_t^g}{x_t^g} = \tilde{A}_t \frac{N_{t-1} + B_t}{P_t}, \quad (4.10)$$

where  $x_t^g$  is the inverse of the relative price of infrastructure goods with respect to final goods,  $P_t/P_t^g$ .  $\tilde{A}_t$  is the overall return on working capital, given by

$$\tilde{A}_t = \frac{1}{x_t^g} \left( \frac{1 - \theta_g}{p_t^L} \right)^{1-\theta_g} (\theta_g)^{\theta_g}. \quad (4.11)$$

**4.3 FINANCIAL INTERMEDIARIES** The optimal contract is characterized by a threshold on idiosyncratic productivity,  $\bar{\omega}_t$ , such that the infrastructure producer with the cutoff productivity is just able to repay the external debt  $B_t$ :

$$\bar{I}_t^g P_t^g = Z_t B_t.$$

$\bar{I}_t^g$  is the infrastructure production with the cutoff idiosyncratic productivity,  $\bar{I}_t^g = \bar{\omega}_t I_t^g$ .

The government can partially guarantee bank loans as a fraction of the monitoring costs,  $l_t m_t \tilde{A}_t \omega_t (N_{t-1} + B_t)$ , when firms default. The optimal contract is given by

$$\frac{N_{t-1}}{N_{t-1} + B_t} = - \frac{g'(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \frac{\tilde{A}_t f(\bar{\omega}_t)}{R_t}. \quad (4.12)$$

To capture the real world, LGFVs are assumed to own land, and therefore the return on land enters their net worth. The end-of-period aggregate net worth  $N_t$  is

$$N_t = \zeta \tilde{A}_t (N_{t-1} + B_t) f(\bar{\omega}_t) + P_t p_t^L L_t. \quad (4.13)$$

**4.4 FISCAL POLICY AND RESOURCE CONSTRAINT** The fiscal authority collects lump-sum taxes to pay for the government consumption,  $G_t^c$ , government investment in infrastructure,  $G_t^I$ , and bailout costs:

$$T_t = \tilde{A}_t(N_{t-1} + B_t)l_t m_t \int_0^{\bar{\omega}_t} \omega_t dF(\omega) + G_t^c + G_t^I. \quad (4.14)$$

Infrastructure investment is made through purchasing the infrastructure goods produced by the LGFVs such that  $G_t^I = \frac{I_t^g}{x_t^g}$ . Public capital evolves according to the following law of motion

$$K_t^g = (1 - \delta^g)K_{t-1}^g + I_t^g \quad (4.15)$$

The government can affect the economy through multiple instruments. In the following section, we mainly focus on the instrument of bailout ratio  $l_t$ , which follows an  $AR(1)$  process,

$$\ln \frac{l_t}{l} = \rho_l \ln \frac{l_{t-1}}{l} + \epsilon_t^l \quad (4.16)$$

where  $\epsilon_t^l \sim N(0, \sigma^l)$ .

In this model, total output is split between private consumption, private investment, government consumption, government investment, monitoring costs, and price adjustment costs:<sup>8</sup>

$$GDP_t = C_t + I_t + G_t^c + y_t^g \quad (4.17)$$

$$Y_t + y_t^g - G_t^I = GDP_t + \frac{\Omega_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t + \tilde{A}_t \frac{N_{t-1} + B_t}{P_t} m_t \int_0^{\bar{\omega}_t} \omega dF(\omega). \quad (4.18)$$

## 5 QUANTITATIVE ANALYSIS: LGFV MODEL

The calibration of the alternative model largely follows the baseline model. Since the infrastructure producers now face the working capital constraint and are subject to government bailout, the parameters characterizing bank-firm loan contracts are calibrated to this relationship. Gao, Ru, and Tang (2017) explore the default probability of local governments in China using individual loan data, and find that the LGFV default rate is 1.7% among all loans that matured between 2007 and 2013. We calibrate the fraction of output loss from bankruptcy to be 0.12 and the LGFV survival rate to be 0.84 such that the implied default probability is 1.7%, consistent with the empirical evidence. Regarding the parameters characterizing the stock of public capital, we follow Leeper, Walker, and Yang (2010) to calibrate the ratio of public to private capital  $K^g/K = 0.31$  and the public capital depreciation rate  $\delta_g = 0.02$ .

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<sup>8</sup> $GDP_t$  denotes total final goods net adjustment and monitoring costs.



**5.1 AGGREGATE ECONOMY AND CREDIT SHOCKS** Credit easing relaxes the working capital constraint for LGFVs, raising both their production of infrastructure goods,  $I_t^g$ , and their demand for final goods,  $y_t^g$ . On the one hand, higher production increases the public capital stock, boosting the productivity of intermediate goods firms. On the other hand, everything else equal, higher demand for final goods crowds out consumption and private investment. Lower consumption may induce households to work more through the wealth effect, while lower private investment can reduce output. To what extent the targeted credit easing can boost the economy depends on which channel dominates. Figure 9 illustrates that the bailout shock persistence matters.

With a one-time increase in the bailout ratio, the expansionary channel through higher public capital dominates, and the total output jumps up on impact. The blue lines in figure 9 show the impulse responses with an i.i.d. increase in the bailout ratio from 0.1 to 0.3. Credit easing relaxes the borrowing constraint for one period. As a result, higher infrastructure production at  $t = 1$  raises public capital and therefore productivity for intermediate goods firms. On the other hand, LGFVs also demand more final goods, crowding out private investment and consumption. Households work more, while private investment dips initially. Even though higher external debt raises the NPL, the gain in output overall dominates the rise in bailout costs.

For a more persistent credit expansion, the contractionary channel of lower private investment starts to dominate in the medium term. The red lines in figure 9 illustrate the impulse responses if the credit shock is expected to last 10 quarters (with  $\rho_l = 0.9$ ). The output surges on impact, as higher public capital raises productivity for intermediate goods firms. Over the medium term, however, higher demand on final goods from LGFVs crowds out private investment, and the initial expansionary impact on output fades quickly. On top of that, higher bailout costs from the more persistent credit expansion pull the GDP into negative territory over the medium term.

If the increase in the bailout ratio is highly persistent, the expansionary channel through higher labor supply dominates in the medium and long term. The orange lines in figure 9 illustrate the impulse responses if the credit expansion is expected to last 12.5 years (with  $\rho_l = 0.98$ ). In this case, consumption dips on impact and remains low for a prolonged period of time. Over the medium and long run, households work more and output stays above its steady-state level. Even though the highly persistent credit expansion leads to persistently high bailout costs, the gains in output dominate so that the GDP responses remain positive.

**5.2 FISCAL MULTIPLIERS** The present-value output multipliers, shown in the top left panel of figure 10, are nonlinear with respect to the bailout shock persistence, which is consistent with the impulse responses in figure 9. GDP multipliers are positive at all horizons with either i.i.d or highly persistent credit shocks. With an i.i.d targeted credit shock (blue line), the present-value accumulative multiplier is 1.3 upon impact, declining somewhat in the medium term, before reaching 1.4 over the long term. A highly persistent targeted credit expansion leads to a credit multiplier of 0.8 upon the shock, which declines to close to 0.4 after five years. The multiplier is the lowest when  $\rho_l = 0.9$ , which declines quickly from around 0.7 upon impact to negative after four years. This nonlinearity is driven by



the investment multipliers, shown in the bottom left panel, as the contractionary channel of lower private investment dominates in the medium term when  $\rho_l = 0.9$ . On the other hand, the infrastructure multipliers are monotonically increasing with respect to the shock persistence, which is in line with the insight we have gained in the baseline model. The flip side of monotonically increasing infrastructure multipliers is that consumption multipliers are monotonically decreasing with respect to the shock persistence, as higher infrastructure production crowds out consumption.

## 6 CONCLUSION

Our paper contributes to the stream of research that studies the drivers and consequences of China's credit boom. Several recent papers have analyzed the unintended consequences of the credit easing in part due to the 2008 stimulus plan. We contribute to the discussion by taking into account the coordinated implementation of fiscal and monetary policies in China and examining how a government guarantee works differently when it is offered to the general economy or to the infrastructure producers, LGFVs. We show that: (1) When credit expansion is in the form of a government partial guarantee offered to general intermediate firms, the more persistent the bailout shock is expected to be, the higher the credit multiplier is, as persistent credit expansion relaxes the working capital constraint, thus driving up private investment and GDP; (2) When targeted at the LGFVs, however, either short-lived or highly persistent credit expansions lead to higher credit multipliers.

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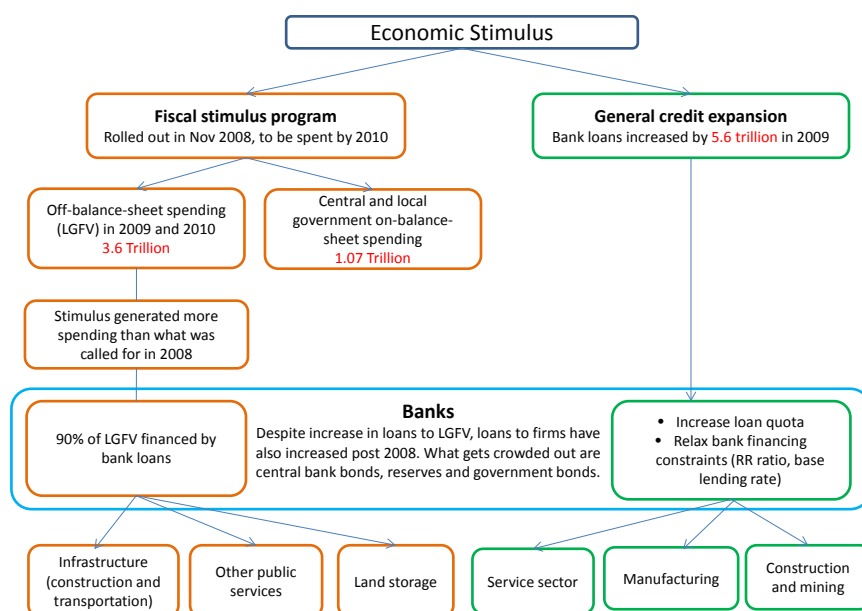
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Table 1: Calibration

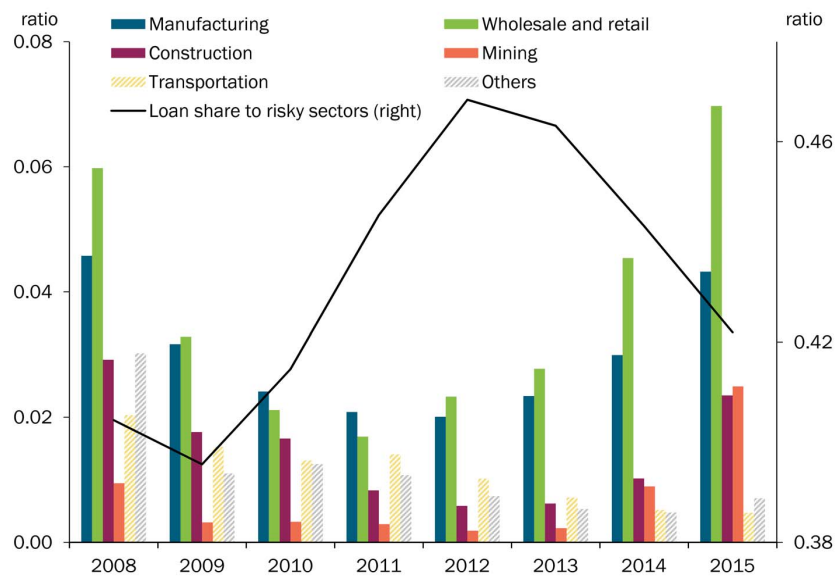
Parameter	Description	Values
$\beta$	Discount factor	0.99
$\psi$	Weight of disutility of working	23
$\sigma$	Inverse elasticity of labor supply	2
$\Omega_k$	Capital adjustment cost	1
$\delta$	Private capital depreciation rate	0.025
$\delta_G$	Public capital depreciation rate	0.02
$\epsilon$	Elasticity of substitution between retail goods	10
$\Omega_p$	Price adjustment cost	22
$\psi_\pi$	Response coefficient to inflation in Taylor rule	1.5
$\psi_y$	Response coefficient to GDP growth in Taylor rule	0.5
$\alpha$	Capital income share	0.5
$\theta$	Share of household labor	0.94
$\theta_g$	Share of final goods	0.9
$\pi$	Steady-state inflation rate	1.0055
$g$	Steady-state balance growth rate	1.0125
$g_I$	Steady-state growth rate of investment	1.0125
$\sigma_\omega$	Std for log-normal distribution	0.7
$m$	FI monitoring cost	0.12
$\zeta$	Firm survival rate	0.93
$G/GDP$	Steady-state government spending ratio	0.14
$K^g/K$	Steady-state ratio of public to private capital	0.31
$\bar{l}$	Steady-state government bailout ratio	0.2
$\rho_A$	Persistence of TFP shock	0.95
$\rho_g$	Persistence of government spending shock	0.84
$\rho_l$	Persistence of government bailout shock	0/0.9/0.98
$\sigma_A$	Standard deviation of TFP shock	0.01
$\sigma_g$	Standard deviation of government spending shock	0.05
$\sigma_l$	Standard deviation of government bailout shock	0.3

Figure 1: China's Economic Stimulus Package



**Notes:** Source: Cong, Gao, Ponticelli, and Yang (2017), Bai, Hsieh, and Song (2016).

Figure 2: NPL Ratios and Loan Share



Source: Bank annual reports for BoC and ICBC; and Bloomberg

Figure 3: Trend Patterns: Business Investment

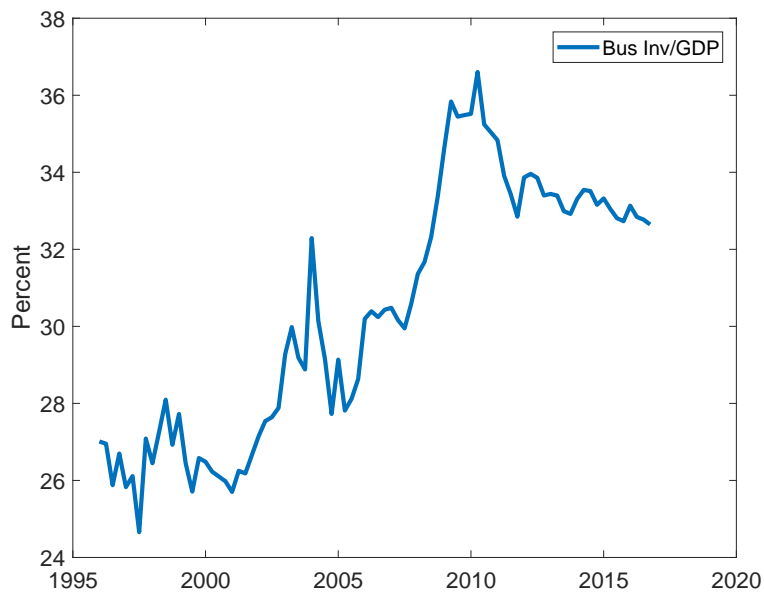


Figure 4: Correlations between Cycle Series with the Moving Window of 10 Years

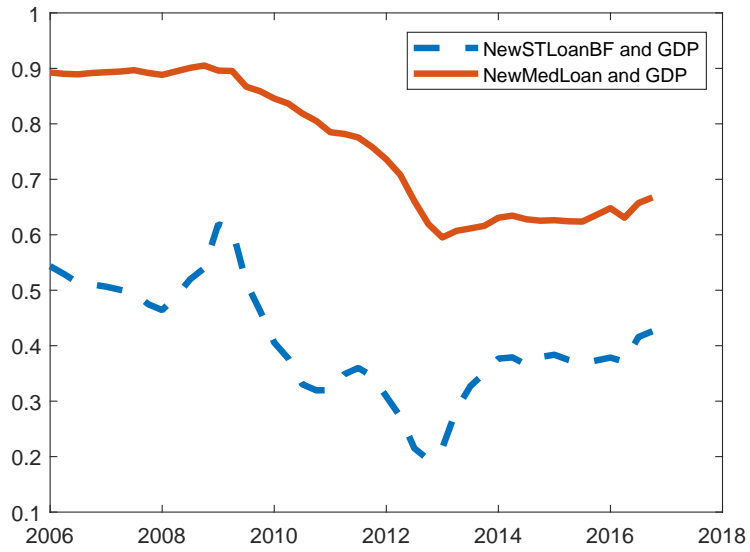


Figure 5: Impulse Responses under Credit Shocks with Different Persistence

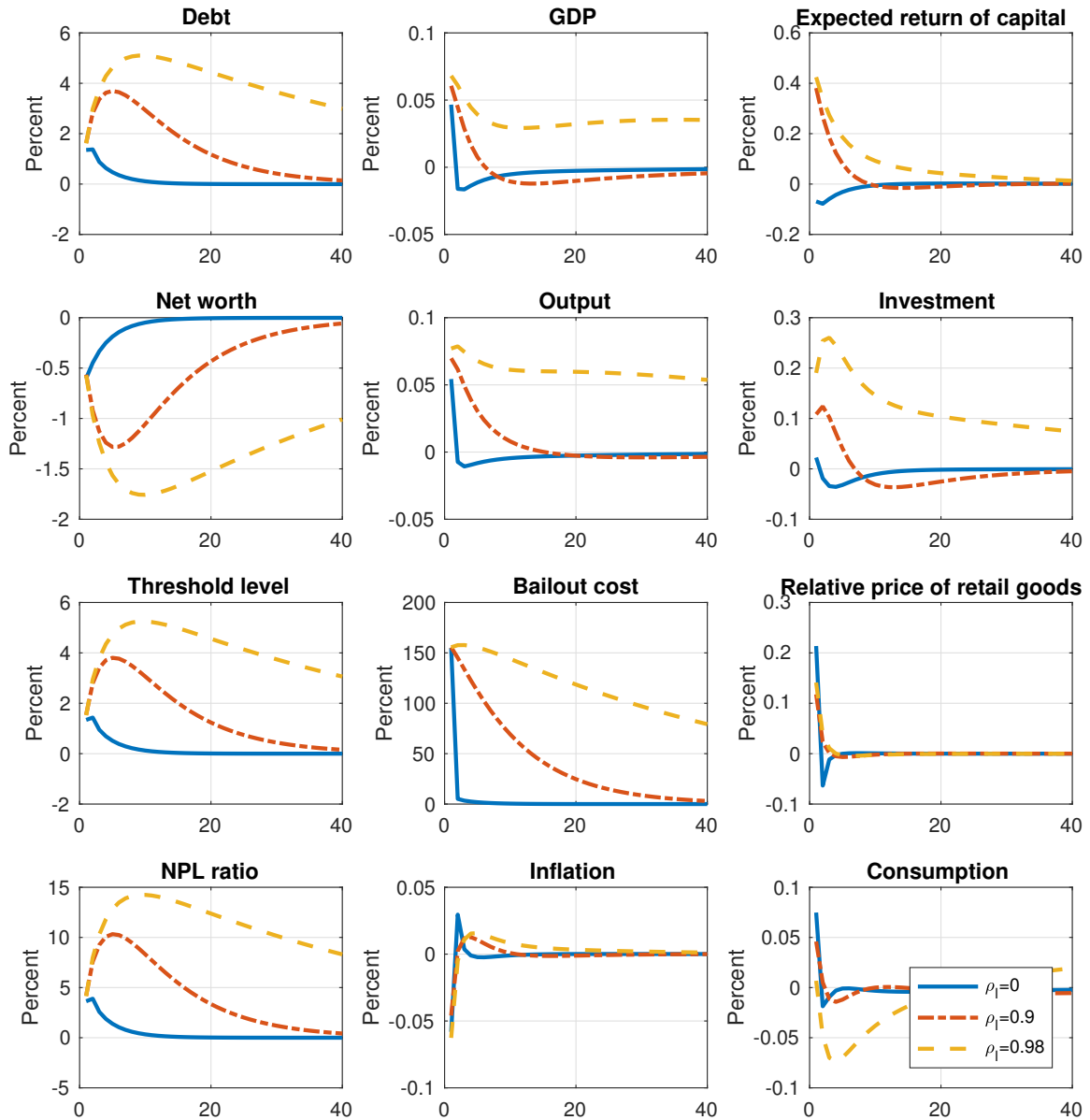


Figure 6: Credit Multipliers with Different Shock Persistence vs. Government Spending Multipliers

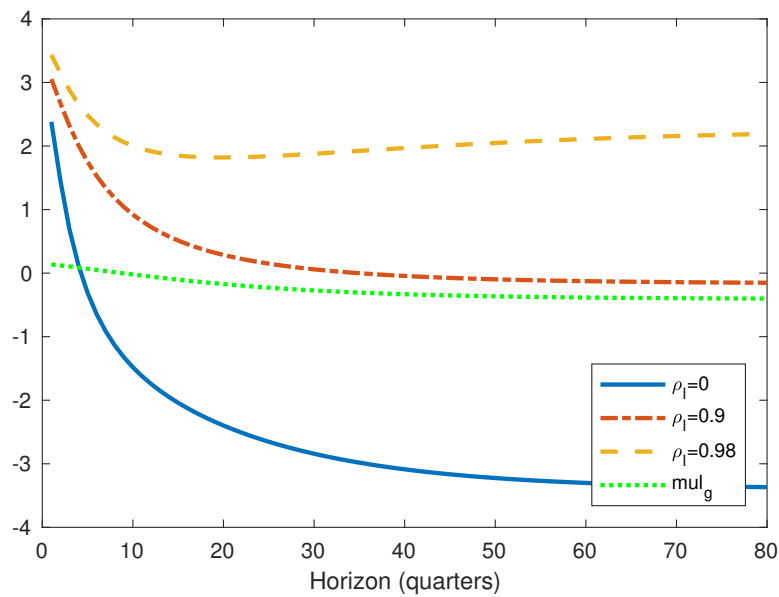


Figure 7: Fiscal Multipliers: Without Investment Adjustment Cost

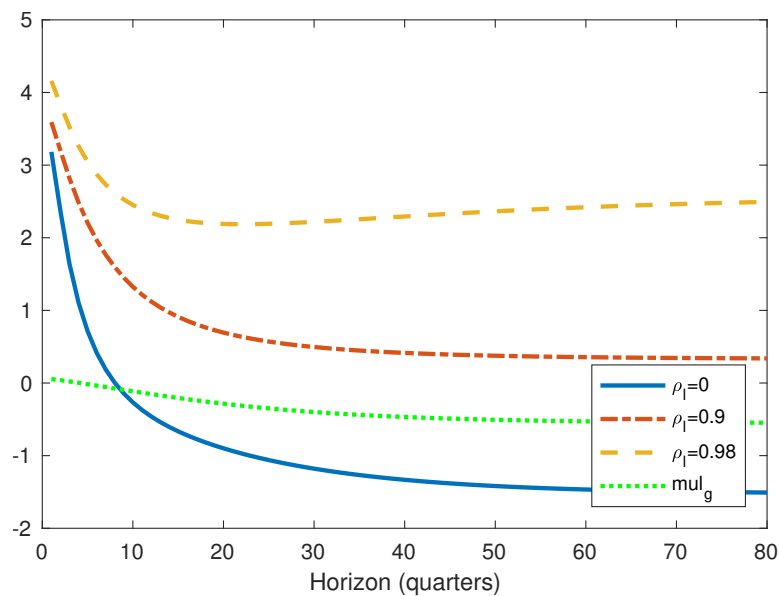




Figure 8: Fiscal Multipliers with Distortionary Taxes

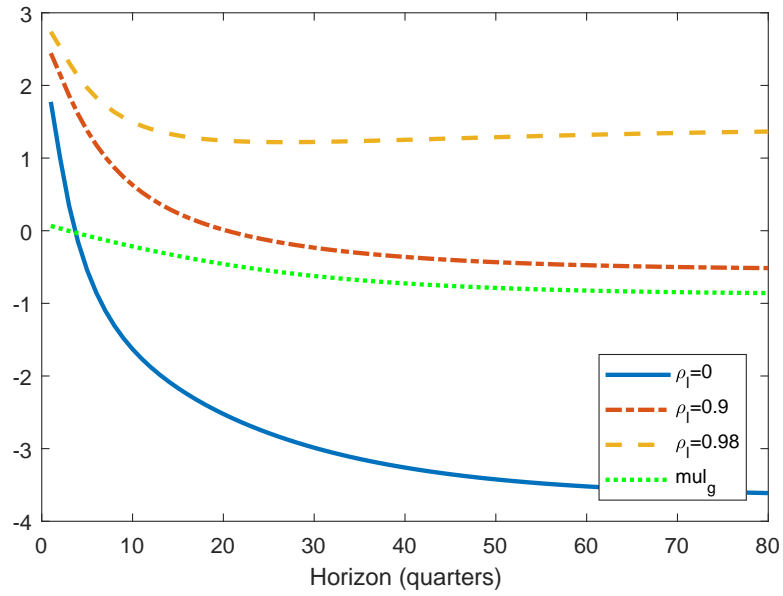


Figure 9: Impulse Responses under Credit Shocks with Different Persistence

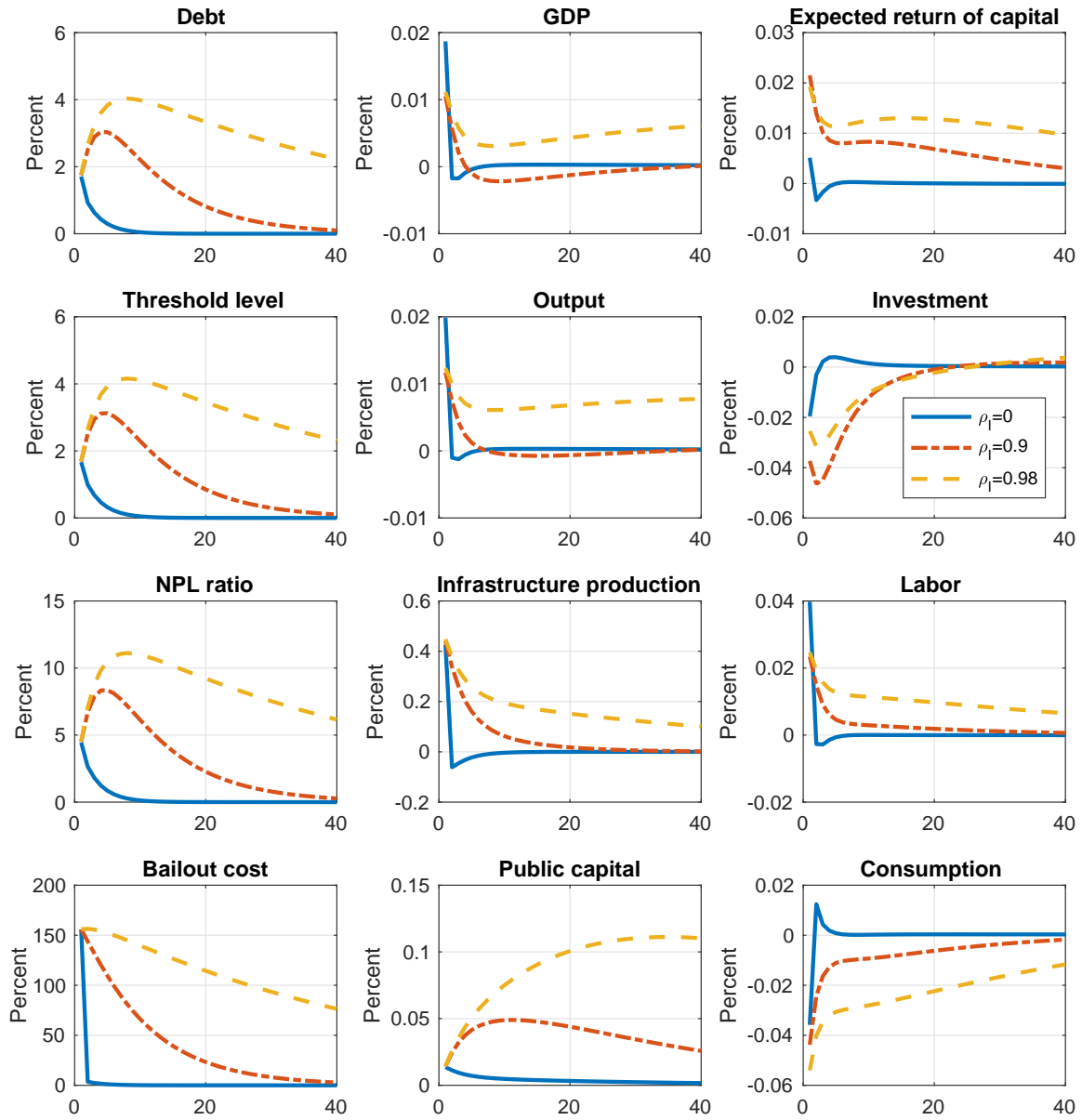
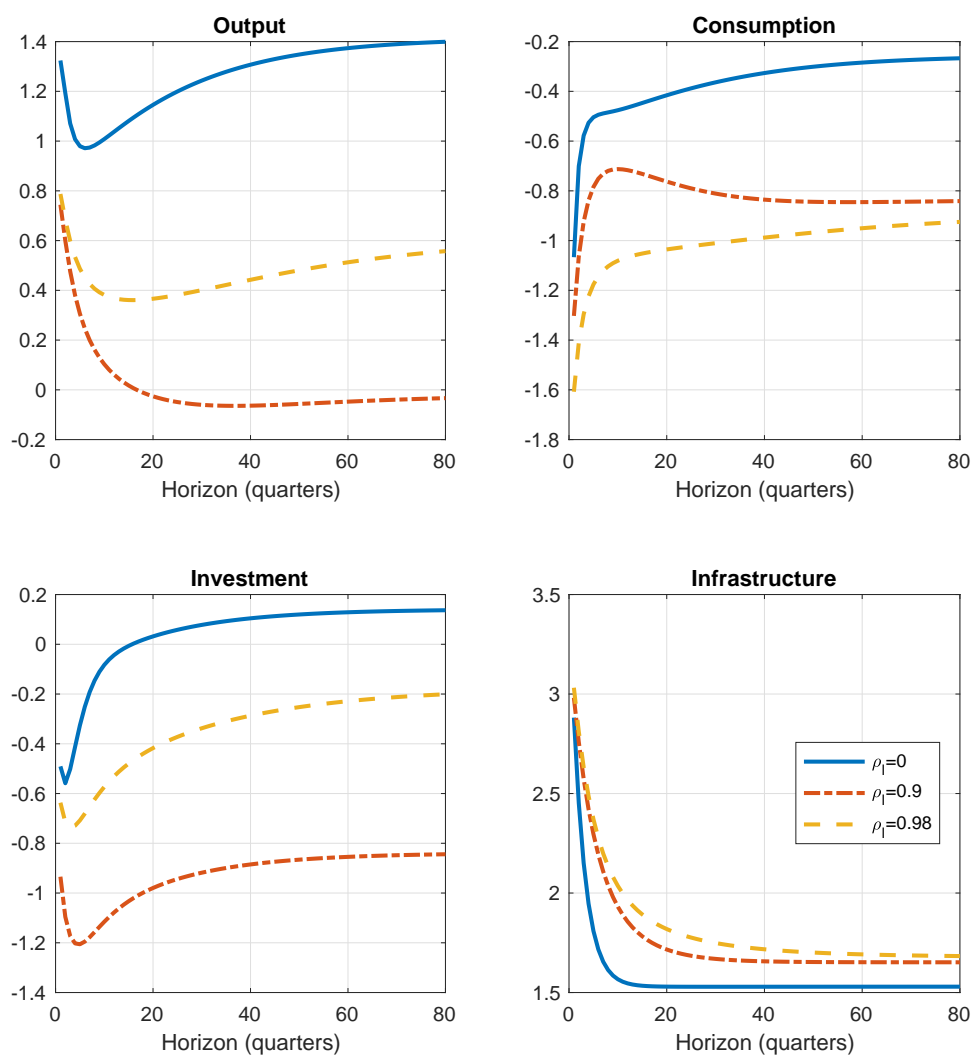


Figure 10: Credit Multipliers with Different Shock Persistence vs. Government Spending Multipliers



## APPENDIX

## A MODEL SUMMARY: BASELINE

On a balanced growth path, we make the following transformation:  $xx_t = \frac{XX_t}{g^t}$  where  $XX = (C_t, Y_t, I_t, H_t, K_t, W_t, W_t^e, GDP_t, T_t)$ ; and  $yy_t = \frac{YY_t}{g^t P_t}$  where  $YY = (N_t, B_t)$ . Assume  $h_t^e = 1$ .

## Households

$$k_t g = (1 - \delta)k_{t-1} + \left(1 - \frac{\Omega_k}{2} \left(\frac{i_t g}{i_{t-1}} - g_I\right)^2\right) i_t \quad (\text{A.1})$$

$$w_t = \psi h_t^\sigma c_t \quad (\text{A.2})$$

$$\frac{1}{R_t} = \beta E_t \frac{c_t}{c_{t+1} g} \frac{1}{\pi_{t+1}} \quad (\text{A.3})$$

$$q_t^k = \beta E_t \frac{c_t}{c_{t+1} g} (q_{t+1}^k (1 - \delta) + r_{t+1}^k) \quad (\text{A.4})$$

$$1 = q_t^k \left(1 - \frac{\Omega_k}{2} \left(\frac{i_t g}{i_{t-1}} - g_I\right)^2 - \Omega_k \left(\frac{i_t g}{i_{t-1}} - g_I\right) \frac{i_t g}{i_{t-1}}\right) + \beta E_t q_{t+1}^k \frac{c_t}{c_{t+1} g} \Omega_k \left(\frac{i_{t+1} g}{i_t} - g_I\right) \left(\frac{i_{t+1} g}{i_t}\right)^2 \quad (\text{A.5})$$

## Firm

$$y_t = \int_0^\infty \omega_t k_{t-1}^{1-\alpha} (h_t^\theta A_t^m)^\alpha dF(\omega) = k_{t-1}^{1-\alpha} (h_t^\theta A_t^m)^\alpha \quad (\text{A.6})$$

$$w_t h_t = \alpha \theta \left(\frac{n_{t-1}}{\pi_t g} + b_t\right) \quad (\text{A.7})$$

$$w_t^e = \alpha(1 - \theta) \left(\frac{n_{t-1}}{\pi_t g} + b_t\right) \quad (\text{A.8})$$

$$k_{t-1} r_t^k = (1 - \alpha) \left(\frac{n_{t-1}}{\pi_t g} + b_t\right) \quad (\text{A.9})$$

$$\tilde{A}_t = \frac{1}{\frac{n_{t-1}}{\pi_t g} + b_t} \frac{y_t}{x_t} \quad (\text{A.10})$$

$$R_t b_t = \tilde{A}_t \left(\frac{n_{t-1}}{\pi_t g} + b_t\right) g(\bar{\omega}_t) \quad (\text{A.11})$$

$$\frac{n_{t-1}}{n_{t-1} + b_t g \pi_t} = -\frac{g'(\bar{\omega}_t) \tilde{A}_t}{f'(\bar{\omega}_t) R_t} f(\bar{\omega}_t) \quad (\text{A.12})$$

$$n_t = \zeta \tilde{A}_t \left(\frac{n_{t-1}}{\pi_t g} + b_t\right) f(\bar{\omega}_t) + w_t^e \quad (\text{A.13})$$

$$\frac{1}{x_t} = \frac{(\epsilon - 1)(1 - \tau_t)}{\epsilon} + \frac{\Omega_p}{\epsilon} \left(\frac{\pi_t}{\pi} - 1\right) \frac{\pi_t}{\pi} - \beta \frac{\Omega_p}{\epsilon} E_t \frac{c_t}{c_{t+1}} \left(\frac{\pi_{t+1}}{\pi} - 1\right) \frac{\pi_{t+1}}{\pi} \frac{y_{t+1}}{y_t} \quad (\text{A.14})$$

## Policy and ARCs

$$\ln \frac{R_t}{R} = \psi_\pi \ln \frac{\pi_t}{\pi} + \psi_y \ln \frac{gdp_t}{gdp_{t-1}} \quad (\text{A.15})$$

$$t_t + \tau_t y_t = l_t \left( \frac{n_{t-1}}{\pi_t g} + b_t \right) \tilde{A}_t s(\bar{\omega}_t) + g_t \quad (\text{A.16})$$

$$\ln \frac{g_t}{\bar{g}} = \rho_g \ln \frac{g_{t-1}}{\bar{g}} + \epsilon_t^g \quad (\text{A.17})$$

$$gdp_t = g_t + c_t + i_t \quad (\text{A.18})$$

$$y_t = c_t + i_t + g_t + \frac{\Omega_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 y_t + \tilde{A}_t \left( \frac{n_{t-1}}{\pi_t g} + b_t \right) m_t d(\bar{\omega}_t) \quad (\text{A.19})$$

Assume  $\omega_t$  is drawn from a log-normal distribution,  $\ln(\omega_t) \sim N(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$ ;  $\Phi(\cdot)$  and  $\phi(\cdot)$  are the standard normal cdf and pdf respectively; and  $z \equiv (\ln(\bar{\omega}) + 0.5\sigma_\omega^2)/\sigma_\omega$ .

$$\begin{aligned} f(\bar{\omega}_t) &= \int_{\bar{\omega}_t}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}_t))\bar{\omega}_t \\ &= 1 - \Phi(z_t - \sigma_\omega) - \bar{\omega}[1 - \Phi(z)] \end{aligned} \quad (\text{A.20})$$

$$f'(\bar{\omega}_t) = -\frac{\phi(z_t - \sigma_\omega)}{\sigma_\omega \bar{\omega}} - (1 - \Phi(z_t)) + \phi(z_t)/\sigma_\omega \quad (\text{A.21})$$

$$\begin{aligned} g(\bar{\omega}_t) &= [1 - F(\bar{\omega}_t)]\bar{\omega}_t + (1 - (1 - l_t)m_t) \int_0^{\bar{\omega}_t} \omega dF(\omega) \\ &= (1 - \Phi(z_t))\bar{\omega}_t + (1 - (1 - l_t)m_t)\Phi(z_t - \sigma_\omega) \end{aligned} \quad (\text{A.22})$$

$$g'(\bar{\omega}_t) = 1 - \Phi(z_t) - \phi(z_t)/\sigma_\omega + \frac{(1 - (1 - l_t)m_t)\phi(z_t - \sigma_\omega)}{\sigma_\omega \bar{\omega}_t} \quad (\text{A.23})$$

$$s(\bar{\omega}_t) = l_t m_t \int_0^{\bar{\omega}_t} \omega dF(\omega) = l_t m_t \Phi(z_t - \sigma_\omega) \quad (\text{A.24})$$

$$d(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} \omega dF(\omega) = \Phi(z_t - \sigma_\omega) \quad (\text{A.25})$$

The (20x1) vector of endogenous variable is

$$[y_t, c_t, i_t, g_t, gdp_t, k_t, h_t, n_t, b_t, \tilde{A}_t, \bar{\omega}_t, w_t, w_t^e, r_t^k, R_t, \pi_t, x_t, g_t^c, t_t, q_t^k].$$