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Could a higher inflation target enhance macroeconomic stability?


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Could a Higher Inflation Target Enhance Macroeconomic Stability?

by José Dorich, Nicholas LABELLE ST-PIERRE, Vadym Lepetyuk and Rhys R. Mendes
Could a Higher Inflation Target Enhance Macroeconomic Stability?

by

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Abstract

Recent international experience with the effective lower bound on nominal interest rates has rekindled interest in the benefits of inflation targets above 2 per cent. We evaluate whether an increase in the inflation target to 3 or 4 per cent could improve macroeconomic stability in the Canadian economy. We find that the magnitude of the benefits hinges critically on two elements: (i) the availability and effectiveness of unconventional monetary policy (UMP) tools at the effective lower bound and (ii) the level of the real neutral interest rate. In particular, we show that when the real neutral rate is in line with the central tendency of estimates, raising the inflation target yields some improvement in macroeconomic outcomes. There are only modest gains if effective UMP tools are available. In contrast, with a deeply negative real neutral rate, a higher inflation target substantially improves macroeconomic stability regardless of UMP.

Bank topics: Economic models; Inflation targets; Monetary policy framework
JEL codes: E32, E37, E43, E52

Résumé

Le phénomène d’atteinte de la valeur plancher des taux d’intérêt nominaux observée dans de nombreux pays ces dernières années a renouvelé l’intérêt porté aux avantages des cibles d’inflation supérieures à 2 %. Nous cherchons à savoir dans quelle mesure l’économie canadienne gagnerait en stabilité macroéconomique si la cible d’inflation était portée à 3 ou à 4 %. L’ampleur des avantages dépendrait fondamentalement de deux éléments : a) la possibilité de recourir à des instruments de politique monétaire non traditionnels et l’efficacité de ces instruments en contexte d’atteinte de la valeur plancher des taux nominaux; b) le taux d’intérêt neutre réel. En particulier, nous montrons que lorsque le taux d’intérêt neutre est de l’ordre de la tendance centrale des estimations, le relèvement de la cible d’inflation s’accompagne d’effets bénéfiques sur le plan macroéconomique. L’amélioration est cependant modeste quand des instruments de politique monétaire non traditionnels efficaces sont mis en œuvre. En cas de taux d’intérêt neutre réel fortement négatif, par contre, le relèvement de la cible contribue grandement à la stabilité macroéconomique, indépendamment des mesures non traditionnelles de politique monétaire mises en place.

Sujets : Modèles économiques; Cibles en matière d’inflation; Cadre de la politique monétaire
Codes JEL : E32, E37, E43, E52
Non-Technical Summary

Prior to the 2007–2009 global financial crisis, an inflation target of 2 per cent was widely viewed as high enough to make the effective lower bound (ELB) constraint on nominal interest rates largely irrelevant. Two important developments have contributed to challenge this view. First, in the aftermath of the financial crisis, the ELB proved to be a more persistent and severe constraint than anticipated. Second, the real neutral interest rate has fallen, implying lower nominal interest rates and less scope for conventional monetary policy easing in response to shocks, given the 2 per cent target. Motivated by these developments, we study the extent to which raising the inflation target to 3 or 4 per cent could improve macroeconomic stability in Canada.

Our quantitative analysis is carried out in the Terms-of-Trade Economic Model (ToTEM), the Bank of Canada’s main policy model. We consider different assumptions about the real neutral rate and the availability and effectiveness of unconventional monetary policy (UMP). In particular, we consider two UMP tools that can stimulate the economy at the ELB: forward guidance and quantitative easing. Before turning to our main quantitative analysis, we analyze these UMP tools in a simple New Keynesian model in order to make clear the key channels through which they operate.

We find that when the real neutral rate is positive, the availability of effective UMP eliminates most of the macroeconomic stabilization benefits associated with raising the inflation target to 3 or 4 per cent. On the other hand, if the real neutral rate is negative, an increase in the inflation target provides substantial macroeconomic benefits by reducing the impact of the ELB on the output gap and inflation, regardless of the availability or effectiveness of UMP. We also find that forward guidance is more powerful when the real neutral rate is sufficiently high, while quantitative easing becomes more effective if the real neutral rate is very low.
1 Introduction

Most advanced-economy central banks now target an inflation rate of 2 per cent, but in recent years this ubiquitous objective has come under increased scrutiny. Prior to the 2007–2009 global financial crisis, most observers viewed a 2 per cent inflation target as sufficiently high to make the constraint arising from the effective lower bound (ELB) on short-term nominal interest rates largely irrelevant. But, in the aftermath of the crisis, the ELB proved to be a more persistent and severe constraint on conventional monetary policy than anticipated. At the same time, there is now wide agreement that the real neutral rate of interest has fallen. For a given inflation target, this decline implies lower nominal interest rates and less scope for conventional policy easing in response to shocks. In this context, several academics and policymakers have called for higher inflation targets in order to reduce the likelihood that monetary policy will be constrained by the ELB.¹

In this paper, we assess the extent to which raising the inflation target to 3 or 4 per cent could lead to greater macroeconomic stability in Canada. In doing so, we take into account the uncertainty about two important considerations: (i) the extent of the decline in the neutral rate and (ii) the availability of effective unconventional monetary policy (UMP) tools at the ELB. An important contribution of this paper is that we analyze the benefits of a higher inflation target in an environment in which the central bank can make systematic, rule-based use of the two most prevalent types of UMP: forward guidance and quantitative easing (QE).

Our quantitative analysis is carried out in the Terms-of-Trade Economic Model (ToTEM), a large-scale dynamic stochastic general equilibrium (DSGE) model of the Canadian economy. While ToTEM is useful for producing quantitatively realistic results, its size and complexity may obscure the key channels at play in our analysis. For this reason, before turning to the quantitative results, we present a simple New Keynesian model with market segmentation to illustrate how forward guidance and QE operate in our framework.

¹ See Williams (2009), Blanchard, Dell’Ariccia and Mauro (2010), Ball (2014), Krugman (2014), Williams (2016) and Summers (2016).
There are several alternative definitions of neutral or natural rates of interest.\footnote{See Mendes (2014) for a discussion of alternative definitions.} We define the real neutral interest rate as the short-term real interest rate that would prevail in a flexible-price equilibrium after the effects of all business cycle shocks have dissipated. We further assume that there are no low-frequency shocks to trend growth or other trends in our models. Thus, in our analysis, this concept of the neutral rate is constant and equal to the steady-state short-term real interest rate. Our use of this longer-term concept is driven mainly by the fact that we wish to assess the impact of the shift in the average level of short-term interest rates.

Assumptions about the level of the real neutral rate can play a significant role in assessing the benefits of a higher inflation target. Prior to the crisis, macroeconomic models were usually calibrated to generate steady-state short-term real interest rates around 3 per cent.\footnote{For example, Christiano, Eichenbaum and Evans (2005) calibrate the household discount rate to yield a steady-state real interest rate of 3 per cent, while Adam and Bill's (2007) calibration implies a rate of 3.5 per cent.} However, most recent estimates for the United States are much lower than 3 per cent, ranging from zero to 1.5 per cent.\footnote{For example, Lubik and Matthes (2015) estimate the US real neutral rate to be roughly zero; Holston, Laubach and Williams (2017) put it at 0.4 per cent; Johannsen and Mertens (2016) estimate it to be 0.7 per cent; and Del Negro et al. (2017) put it between 1 and 1.5 per cent.} Some estimates are even lower. For example, in a model of secular stagnation, Eggertsson and Mehrotra (2017) estimate the real neutral rate to be -1.5 per cent. The US estimates may be relevant to Canada because, as explained in Mendes (2014), the foreign neutral rate can be an important determinant of the domestic neutral rate in a small open economy like Canada. There are fewer studies that focus directly on Canada, but they also suggest a decline: Mendes (2014) reports a range of estimates between 1 and 2 per cent for the Canadian real neutral rate, Holston, Laubach and Williams (2017) report a point estimate of around 1.25 per cent for Canada and the Bank of Canada (2017) estimates the real neutral rate to be between 0.5 and 1.5 per cent. Thus, current estimates of the real neutral rate span a wide range from -1.5 per cent to 2 per cent, with most estimates in positive territory. Consequently, it is important to consider the implications of alternative estimates of the neutral rate.

The availability and effectiveness of UMP tools is also an important consideration when assessing the benefits of a higher inflation target. In principle, if these tools were to do a sufficiently good job of substituting for conventional policy easing, then there would be no reason to raise the inflation target. Numerous empirical studies...
have used recent experiences with UMP to estimate their effectiveness. The literature has tended to focus on the
two most widely used types of unconventional monetary policy: forward guidance and QE.

Most studies of the effects of QE find that it has had sizable impacts on longer-term interest rates, but there
remains considerable uncertainty about the precise magnitudes.\(^5\) A survey by Reza, Santor and Suchanek (2015)
finds that estimates of the cumulative impact on longer-term rates of the Federal Reserve’s first three programs
range from 65 to 120 basis points. There is also a more limited but growing body of work on forward guidance.\(^6\)
Charbonneau and Rennison (2015) survey this literature and conclude that the evidence is “somewhat mixed,” but
that forward guidance “has generally been found to be effective in lowering expectations of the future path of policy
rates,” among other things.

Given the diversity of views in the literature about the level of the real neutral rate and the effectiveness
of unconventional monetary policy tools, we consider a range of different assumptions. We find that alternative
estimates of the neutral rate lead to starkly different conclusions about the implications of a higher inflation target.
If, consistent with the majority of the literature, we assume that the neutral rate remains positive, then the
availability of effective UMP is sufficient to eliminate most gains from raising the target. Effectively, UMP provides
substantial scope for stimulus at the ELB so that there is little additional benefit from raising the target. On the other
hand, if we assume that the neutral rate is negative, then a higher inflation target substantially improves
macroeconomic performance by reducing the impact of the ELB on the output gap and inflation, regardless of the
availability or effectiveness of UMP.

It is also important to note what we do not do in this paper. We do not analyze the welfare implications of
raising the inflation target and we therefore do not characterize the optimal rate of inflation. This choice is motivated

\(^5\) See, for example, Bauer and Rudebusch (2014), Christensen and Rudebusch (2012), Chung et al. (2012), Gagnon et al. (2011),
Dahlhaus, Hess and Reza (2014), Ihrig et al. (2012), Li and Wei (2013), Breedon, Chadha and Waters (2012), Caglar et al. (2011),
Bridges and Thomas (2012), Joyce et al. (2011), Joyce and Tong (2012), Kapetanios et al. (2012), McLaren, Banerjee and Latto

\(^6\) See, for example, Kool and Thornton (2012), Woodford (2013), He (2010), Filardo and Hofmann (2014), Campbell et al. (2012),
Chang and Feunou (2013), Femia, Friedman and Sack (2013) and Swanson and Williams (2014).
by two considerations. First, the costs associated with changing the inflation target are much broader than those captured by standard New Keynesian models.\footnote{For example, a change in the inflation target could give rise to a sizable arbitrary redistribution of wealth. See Box 4 in Bank of Canada (2016a).} Second, it is well known that the welfare costs of business cycles are small in standard models. Despite this fact, policymakers appear to place significant weight on macroeconomic stabilization. Therefore, to understand and inform the behaviour of actual policymakers, it is important to assess the implications for macroeconomic stabilization, as we do in this paper. Finally, it should be noted that even if we were to take account of the full range of costs associated with higher inflation, the basic message of our main result would remain intact. Taking account of the costs of higher inflation would only reinforce the conclusion that raising the target would not yield large benefits under the baseline assumptions about the real neutral rate and the effectiveness of UMP.

The remainder of the paper is organized as follows. Section 2 presents a small New Keynesian model that illustrates the role of forward guidance and QE in our analysis. Section 3 provides a brief summary of ToTEM, the model used in our quantitative analysis. Section 4 presents our main quantitative results on the benefits of a higher inflation target under different assumptions about the level of the real neutral rate and the availability and effectiveness of UMP. Section 5 concludes.

2 Forward Guidance and QE in a Simple New Keynesian Model

In this section, we present a simple New Keynesian model in which both forward guidance and QE can stimulate the economy at the ELB. This simple model is useful as an expositional device, but it is not empirically realistic. Hence, in Section 4 we provide a quantitative analysis in ToTEM.

2.1 A Simple New Keynesian Model

In the standard New Keynesian (NK) model, aggregate demand is a function of the expected path of future short-term interest rates, so there is a clear role for forward guidance. But the standard NK model has no role for QE, so we modify the household side of the model along the lines of Andrés, López-Salido and Nelson (2004) and Chen,
Cúrdia and Ferrero (2012). The model allows for a particular type of asset market segmentation, which allows for the long-term interest rate to affect aggregate demand distinct from the expected path of short-term rates. In particular, there is a fraction of “restricted” households, \(1 - \omega\), that can trade only in long-term bonds.\(^8\) The remaining fraction, \(\omega\), of “unrestricted” households trade in both short- and long-term bonds.

These modifications to the standard NK model result in an aggregate demand equation of the form

\[
x_t = E_t x_{t+L} - \frac{L}{\sigma} \left[ \omega \frac{1}{L} E_t \sum_{j=0}^{L-1} r_{r,t+j} + (1 - \omega) r_{L,t} - \bar{r} \right] + \nu_t,
\]

where \(x_t\) denotes the output gap, \(r_{r,t}\) is the short-term real rate, \(r_{L,t}\) is the long-term real rate, \(\bar{r}\) is the real neutral rate, \(\sigma\) is the inverse of the intertemporal elasticity of substitution, \(L\) is the duration of long-term bonds and \(\nu_t\) is a demand shock. Note that this equation collapses to the standard New Keynesian IS curve when \(\omega = 1\). The short- and long-term real rates are given by

\[
r_{r,t} \equiv i_t - E_t \pi_{t+1}
\]

\[
r_{L,t} \equiv \frac{1}{L} E_t \left[ \sum_{j=0}^{L-1} r_{r,t+j} \right] + t p_t,
\]

where \(i_t\) is the short-term nominal interest rate, \(\pi_t\) is the inflation rate and \(t p_t\) denotes the term premium.

Following Chen, Cúrdia and Ferrero (2012), we assume that the term premium is a function of the ratio of the market value of household holdings of long-term bonds to short-term bonds. Up to a first order approximation, this implies

\[
t p_t = \tau (\hat{b}_{L,t} - \hat{b}_{S,t}),
\]

\(^8\) These households could be motivated by a preferred-habitat motive. See Vayanos and Vila (2009) for details.

\(^9\) For simplicity, here we assume that the steady-state real short- and long-term rates are equal, which implies that the term premium is zero in steady state.
where $\tau$ is a parameter, $\tilde{b}_{L,t}$ is household holdings of long-term bonds and $\tilde{b}_{S,t}$ is household holdings of short-term bonds. Thus, the central bank can affect the term premium by conducting open market operations in the short- and long-term bond market. We do not explicitly model the open market operations. Instead, we assume that the term premium is one of the instruments used by the central bank when the economy is at the ELB and it is set according to a rule that we describe below. Given this rule, the ratio of long- to short-term bonds required to achieve the desired term premium is determined residually.

Aggregate supply is given by the standard NK Phillips curve. The monetary authority sets the short-term nominal interest rate according to the following equations:

$$i^d_t = \bar{r} + \theta_\pi (\pi_t - \bar{\pi}) + \theta_x x_t$$

(5)

$$i_t = \max[ELB, i^d_t],$$

(6)

where $i^d_t$ is the central bank’s desired short-term nominal interest rate, $\theta_\pi$ measures the response to inflation and $\theta_x$ measures the response to the output gap. The second equation requires that the actual short-term nominal interest rate is always greater than or equal to the ELB.

2.2 Modelling Forward Guidance and QE

To operationalize the forward guidance in the model, we need to make it systematic. We do this by assuming that (i) guidance is implemented whenever the ELB is reached and (ii) the guidance takes the form of thresholds for the output gap and inflation. In particular, we assume that whenever the short-term nominal interest rate reaches the ELB, the central bank commits not to raise this rate as long as the output gap is negative and the inflation rate is not more than 1 percentage point above the target. This approach to modelling forward guidance is similar to threshold-based guidance implemented by the Federal Reserve and the Bank of England.10

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10 While these central banks had an unemployment rate threshold in their forward guidance, we instead employ an output gap threshold to avoid the need to introduce unemployment into the model.
We model QE by assuming that when the short-term nominal interest rate reaches the ELB, the central bank conducts open market operations to set the term premium according to the following rule:

\[
    t_p_t = \frac{\lambda}{(1 - \omega)L} E_t \left[ \sum_{j=0}^{L-1} \min(i_{t+j}^d - i_{t+j}, 0) \right].
\]  

(7)

The parameter \( \lambda \) captures the degree to which the central bank increases QE as the ELB constraint becomes more severe.

The model also makes clear that the transmission of forward guidance and QE can be expected to be quantitatively different. To see this, combine equations (1) and (3) to get an expression for aggregate demand of the form

\[
    x_t = E_t x_{t+L} - \frac{L}{\sigma} E_t \left[ \sum_{j=0}^{L-1} rr_{t+j} \right] + (1 - \omega) t_p - \bar{r} + v_t.
\]

(8)

Note that the weight on the expected path of short-term rates (through which forward guidance operates) is larger than the weight on the term premium (1 on the path versus \( 1 - \omega \) on the term premium). Consequently, a 1-percentage-point movement in long-term real rates that is brought about through forward guidance will have a larger direct impact on aggregate demand than an equivalent movement in long-term real rates brought about by QE. This reflects the fact that changes in short-term real rates affect every agent in the model, whereas changes in the term premium affect only the restricted households.

2.3 Parameterization and Solution Method

This model is too simple to be quantitatively realistic. We calibrate it merely to illustrate some of the key qualitative features that also carry over to our quantitative analysis in ToTEM. We assume that one period of the model is one quarter and \( \beta \) is equal to 0.9975, which implies that the steady-state real neutral rate \( \bar{r} \) is 1 per cent in annualized terms. The intertemporal elasticity of substitution is 1. The share of unrestricted households is equal to 0.12, in line with that estimated by Dorich et al. (2013). The parameter \( L \) is set equal to 20, which means that the duration of long-term bonds is 5 years. As in Galí (2015), we set \( \theta^e \) equal to 1.5 and \( \theta^s \) equal to 0.125. The ELB is set equal
to -0.5 per cent in annualized terms, in line with Witmer and Yang (2015). The calibration of the structural parameters in the NK Phillips curve (except for $\beta$) is taken from Gali (2015). The calibration of $\lambda$ is discussed below. The demand shock follows an AR(1) process with a persistence parameter equal to 0.5. The standard deviation of the shock is chosen such that the economy is at the ELB about 8 per cent of the time.\(^{11}\) The model is solved using a collocation method. Appendix A covers the technical details of the solution method.

2.4 Criteria to Evaluate the Benefits of a Higher Target

Ultimately, the goal of a higher target would be to enhance economic stability by reducing the impact of the ELB on inflation and the output gap. It might seem natural to assess this by examining the performance of the economy in periods in which the ELB is binding. This, however, would not yield informative results because the periods in which the ELB is binding are not the same under different inflation targets.

Instead, our approach is to compare the performance of different inflation targets during (i) all simulated periods (“All” in the tables) and (ii) periods with large negative shocks (“LNS” in the tables). We define the LNS periods as those in which the ELB is binding when the inflation target is 2 per cent and the real neutral rate is 1 per cent. This approach allows us to focus on the same periods and shocks across the different cases.

2.5 Results

To illustrate the effects of forward guidance and QE, we show in Table 1 the simulation results for these two policies in isolation.\(^{12}\) In particular, this table presents the macroeconomic outcomes during LNS periods for the case of a 1 per cent real neutral rate and a 2 per cent inflation target. To make the results comparable, we calibrate the QE effectiveness parameter $\lambda$ to achieve the same average long-term real rate in the LNS periods as the one obtained with forward guidance. This implies that the long-term real rate gaps under only forward guidance and only QE are

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\(^{11}\) This probability was estimated using ToTEM simulations and assuming a nominal neutral rate of 3 per cent. See Section 4 for details.

\(^{12}\) We report inflation rate gaps and interest rate gaps as percentage-point deviations from their corresponding steady state values, expressed in annualized terms. The output gap is reported as per cent deviations of output from potential output. The five-year average expected inflation gap is constructed as the difference between the long-term nominal and real rate gaps.
both 11 basis points lower in annualized terms than in the case without UMP (-0.33 pp in the former two cases versus -0.22 pp without UMP).

Table 1: Macroeconomic outcomes in simple NK model during LNS periods (1% real neutral rate, 2% inflation target)

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th></th>
<th>One-Quarter-Ahead Expected Inflation Gap (pp)</th>
<th>Four-Year Average Expected Inflation Gap (pp)</th>
<th>Long-Term Real Rate Gap (pp)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inflation Gap (pp)</td>
<td>Output Gap (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without UMP</td>
<td>-2.72</td>
<td>-1.87</td>
<td>-1.45</td>
<td>-0.38</td>
<td>-0.22</td>
</tr>
<tr>
<td>Only FG</td>
<td>-1.14</td>
<td>-1.28</td>
<td>-0.26</td>
<td>0.26</td>
<td>-0.33</td>
</tr>
<tr>
<td>Only QE</td>
<td>-1.84</td>
<td>-1.41</td>
<td>-0.88</td>
<td>0.02</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

As explained earlier, for the same movement in the long-term real rate, forward guidance has a larger effect on the output gap than QE because forward guidance affects the interest rates faced by both types of households. Our simulations confirm this intuition. Forward guidance leads to a less negative output gap in the LNS periods. In this case, the average output gap is -1.28 per cent, whereas it is -1.41 per cent with QE and -1.87 per cent without any UMP.

The average inflation gap in the LNS periods is also less negative with forward guidance than with QE. Forward guidance increases the inflation gap from -2.72 pp in the case without UMP to -1.14 pp. QE increases the same gap only to -1.84 pp. The intuition behind the stronger effect of forward guidance on the inflation gap is as follows. First, given a positive relationship between current inflation and current output gap in the NK Phillips curve, the stronger effect reflects that forward guidance has a greater impact on current output gap than QE. Second, the larger effect of forward guidance on inflation also reflects its stronger effect on inflation expectations. Table 1 shows that forward guidance raises the one-quarter-ahead inflation expectation gap from -1.45 pp in the case without UMP to -0.26 pp. In contrast, QE only increases the same gap to -0.88 pp. Qualitatively, this difference in the impact of forward guidance and QE on inflation also holds in ToTEM, as we will discuss in Section 4.

Table 1 also reveals that expected inflation plays a more important role in influencing long-term real rates under forward guidance than under QE. Relative to the case without UMP, forward guidance raises the five-year
average expected inflation gap by 64 basis points (from -0.38 pp to 0.26 pp), whereas QE only raises it by 40 basis points (from -0.38 pp to 0.02 pp).

Finally, the results in Table 1 also illustrate the forward guidance puzzle commonly observed in the standard NK models. In the small NK model, we find that a relatively small reduction in the long-term real rate of 11 basis points leads to rather large improvements in the output gap and inflation. As we will show in Section 3, the forward guidance puzzle is mitigated in ToTEM, making it more suitable for numerical evaluation of benefits of higher inflation targets.

3 Brief Description of ToTEM

The model presented in the previous section is useful to understand how UMP works and to give us a rough sense of the role of UMP in the debate about raising the inflation target. However, the simple model lacks key ingredients that are empirically relevant to explain the Canadian data. To address this issue, we conduct our quantitative analysis in ToTEM, a quantitatively realistic, large-scale open economy DSGE model of the Canadian economy.13

ToTEM features more disaggregation than in prominent DSGE models used in the academic literature such as Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). The model includes producers of four distinct types of final products: core consumption goods, business investment goods, government goods and non-commodity export goods. ToTEM also contains a separate commodity-producing sector due to the importance of commodities in the Canadian economy.14 The model contains three types of households: (i) unrestricted lifetime-income households, (ii) restricted lifetime-income households and (iii) current-income households. The first two types of households are modelled in the same way as in Section 2. Consequently, ToTEM allows aggregate household spending to depend on both short- and long-term interest rates. Conventional monetary policy in ToTEM is governed by a Taylor rule with interest rate smoothing that reacts to the expected year-over-year inflation four quarters ahead

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13 For details on ToTEM, see Dorich et al. (2013).
14 Commodity production represented 18 per cent of total GDP in 2014. Moreover, the investment in the commodity sector accounted for 55 per cent of total investment and commodity exports represented 49 per cent of total exports.
and the output gap. To match the data, the model contains 33 structural shocks. Among these shocks, foreign demand and commodity-price shocks are the most important drivers of ELB episodes in ToTEM.

Important, ToTEM includes features that make it less susceptible to the so-called forward guidance puzzle – the fact that standard NK models exhibit excessively large reactions to anticipated monetary policy shocks. In particular, allowing for rule-of-thumb price setters as in Gali and Gertler (1999) and habit persistence in consumption significantly dampens the responses of output and inflation to these shocks. We can compare the effect of such
shocks in ToTEM against the same shocks in other well-known macroeconomic models. Figure 1 shows the response of inflation, output and consumption to an anticipated cut of 25 basis points in the short-term nominal interest rate four quarters ahead for three different models: ToTEM, the simple NK model and the SWBY model developed in Del Negro, Giannoni and Patterson (2015), which incorporates Blanchard’s (1985) and Yaari’s (1965) perpetual youth framework in a DSGE model along the lines of Smets and Wouters (2007). The latter model was specifically designed to address the forward guidance puzzle. The forward guidance puzzle is clearly evident in the response of the simple NK model. On the other hand, it is clear that forward guidance in ToTEM is even less powerful than in models like SWBY that are intended to address the puzzle. Thus, ToTEM is well suited to analysis involving forward guidance.

4 Main Quantitative Results

In this section, we first use ToTEM to show that the estimated probability of being constrained by the ELB has increased considerably in recent years. We then present our main results on the benefits of a higher inflation target under different assumptions about the availability of UMP and the level of the real neutral rate. 15

4.1 The Probability of Being Constrained by the ELB

The unconditional probability that monetary policy will be constrained by the ELB depends on both the level of the neutral rate and the level of the ELB itself. Figure 2 shows the relationship between the level of the ELB and the probability of being constrained by the ELB for different values of the neutral rate, under the assumption of a 2 per cent inflation target.

Views of both the ELB and the neutral rate have evolved in recent years. Between 2006 and 2017, the Bank of Canada’s estimates of the real neutral rate in Canada declined from about 3 per cent to about 1 per cent (Bank of Canada, 2017). 16 Estimates of the ELB in Canada have also declined. While the Bank of Canada set an ELB of 25 basis points in 2009, it now estimates the ELB to be around -50 basis points (Poloz, 2015; Witmer and Yang, 2015).

15 For details on the solution simulation methodology used to obtain our quantitative results, see Appendix B. The results presented in the Bank of Canada’s Renewal of the Inflation-Control Target: Background Information—October 2016 (Bank of Canada, 2016b) are explained in Appendix C.

16 The Bank of Canada reports a range of between 0.5 and 1.5 per cent for the Canadian real neutral rate.
As shown in Figure 2, with the values of the neutral rate (3 per cent) and the ELB (25 basis points) that would have prevailed in the mid-2000s, the ELB would have been binding only about 3 per cent of the time. Thus, this estimated probability is consistent with the then-widespread view that a 2 per cent target was sufficient to render the ELB irrelevant. Assuming that the neutral rate is now closer to 1 per cent raises the unconditional probability of being constrained by a 25-basis-point ELB to about 14 per cent. With an ELB of -50 basis points, the probability would stand at about 8 per cent. This estimated increase in the likelihood of ELB episodes motivates our interest in the possibility of a higher inflation target.

4.2 The Impact of a Higher Inflation Target

Through the Fisher relation, a higher inflation target would lead to higher nominal interest rates on average (for a given real neutral rate). This would create more space for conventional policy to ease before hitting the ELB. Here, we analyze the impact of raising the inflation target to 3 or 4 per cent under different assumptions about the availability of effective UMP tools and the level of the neutral rate. We maintain the assumption of a -0.5 per cent
ELB. Consistent with the literature on monetary transmission at low interest rates, we assume that only half of any rate changes in negative territory are passed on to households and firms in our simulations.\(^{17}\)

4.2.1 Results without Unconventional Monetary Policy Tools

We begin by examining the impact of raising the target in the absence of UMP tools. For the time being, we assume that the real neutral rate is 1 per cent, which is the same value we assumed in Section 2 (we explore other cases later). In this context, we find that raising the target could materially reduce the frequency with which the ELB is a binding constraint on the short-term nominal interest rate. As shown in Table 2, raising the inflation target to a level between 3 and 4 per cent would be sufficient to undo the increase in the ELB probability associated with the estimated decline in the neutral rate. Most of the ELB episodes that occur with the 2 per cent inflation target would not occur if the inflation target were increased to 4 per cent. Counting the disappearing episodes as being of zero duration, the average duration of all episodes would fall from 6.8 to 1.2 quarters if the inflation target were increased from 2 to 4 per cent. A higher inflation target would also considerably reduce the duration of the most severe ELB episodes. If the inflation target were increased from 2 to 4 per cent, the duration at the 90th percentile of the distribution would decrease from 15 to 5 quarters.

<table>
<thead>
<tr>
<th>Real Neutral Rate (%)</th>
<th>ELB (%)</th>
<th>Inflation Target (%)</th>
<th>Proportion of Time ELB Is Binding (%)</th>
<th>Average Duration (qrt)</th>
<th>ELB Duration at 90th Percentile (qrt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-0.5</td>
<td>2.0</td>
<td>8.4</td>
<td>6.8</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.0</td>
<td>3.9</td>
<td>2.9</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.0</td>
<td>1.6</td>
<td>1.2</td>
<td>5</td>
</tr>
</tbody>
</table>

The simulation results presented in Table 3 show that raising the inflation target to 3 or 4 per cent makes the average output and inflation gaps during LNS periods less negative (“LNS” periods are defined in the same manner as in Section 2). For instance, on average, raising the target to 4 per cent would improve the output gap by 0.57 pp during LNS periods, increasing it from -2.25 per cent to -1.68 per cent. Similarly, with that increase in the

\(^{17}\) See Witmer and Yang (2016) for a brief survey of this literature.
inflation target, the inflation gap would increase by 0.28 pp (from -1.10 pp to -0.82 pp). The volatility of the output gap and inflation would also be reduced with the increase of the inflation target. Therefore, assuming no UMP tools, a higher inflation target improves macroeconomic stability.

### Table 3: Macroeconomic outcomes under different inflation targets (1% real neutral rate)

<table>
<thead>
<tr>
<th>Inflation Target (%)</th>
<th>Inflation Target (%)</th>
<th>Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inflation Gap (pp)</td>
<td>Output Gap (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>All</td>
<td>LNS</td>
</tr>
<tr>
<td>Without UMP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td>-0.06</td>
<td>-1.10</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td>-0.02</td>
<td>-0.90</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td>-0.01</td>
<td>-0.82</td>
</tr>
<tr>
<td>Only FG</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td>-0.01</td>
<td>-0.80</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td>0.00</td>
<td>-0.72</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td>0.01</td>
<td>-0.72</td>
</tr>
<tr>
<td>Only QE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td>-0.03</td>
<td>-0.92</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td>-0.01</td>
<td>-0.83</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td>-0.01</td>
<td>-0.80</td>
</tr>
<tr>
<td>With UMP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td>0.00</td>
<td>-0.74</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td>0.01</td>
<td>-0.70</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td>0.01</td>
<td>-0.72</td>
</tr>
</tbody>
</table>

### 4.2.2 The Role of Unconventional Monetary Policy Tools

Maintaining the assumption of a 1 per cent real neutral rate, we introduce forward guidance and QE into the analysis. There are two differences in the way we model forward guidance here relative to what we did in Section 2. First, the output gap threshold is replaced by an unemployment threshold. Second, each time guidance is implemented, the central bank chooses the level of the unemployment threshold to minimize an ad hoc loss function defined as the sum of the squared deviations of inflation from target and output from potential. These changes make the way we...
model forward guidance closer to the way state-contingent guidance was actually implemented by the Federal Reserve and the Bank of England.

In this setup, the central bank commits to keep the short-term nominal interest rate at the ELB until the unemployment rate reaches the threshold as long as the current and expected inflation rates do not rise more than 1 percentage point above target. In our simulations, the value of the unemployment threshold is on average slightly below the natural rate of unemployment. As a result of this type of forward guidance, the simulated duration of ELB episodes increases by an average of two quarters.

We modify the QE rule in equation (7) to allow for a gradual decline of the effects of QE on the term premium. In particular, we set \( \lambda \) equal to 1 and introduce intrinsic persistence in the term premium so that QE reduces the term premium by 40 basis points, on average, during the LNS periods in the case in which both QE and forward guidance are used and the inflation target is 2 per cent.\(^{19}\) This reduction in the term premium is consistent with conservative estimates of the cumulative effects of the first two rounds of QE in the United States and the United Kingdom.\(^{20}\)

Overall, we find that forward guidance and QE provide scope for policy easing at the ELB. Consequently, the gains from raising the inflation target when the real neutral rate is 1 per cent and UMP is used are considerably smaller than in the case with the same neutral rate but without UMP. As displayed in Table 3, raising the inflation target from 2 per cent to 4 per cent in the presence of UMP would improve the output gap during LNS periods by 0.15 pp on average (from -1.66 per cent to -1.51 per cent), which is about a quarter of the gains in the case without UMP. The inflation gap would only increase by 0.02 pp on average (from -0.74 pp to -0.72 pp), which is minor relative to the improvement of 0.28 pp in the case without UMP. Furthermore, the reduction in volatility of both the output gap and inflation would be about 5 basis points.

\(^{19}\) The average decrease in the term premium during the ELB episodes is also 40 basis points.\(^{20}\) Reza, Santor and Suchanek (2015) survey the literature and report ranges for the impact of QE on long-term yields in the United States and the United Kingdom. The average of the lower bounds of these ranges is about 60 basis points. A reduction of about 40 basis points in the term premium is implied by the assumption that about two-thirds of the decline in yields is attributable to a decline in the term premium (consistent with results in Joyce et al. 2011).
Table 3 also shows the results if only forward guidance is available or only QE is available. The results with only forward guidance are closer to the combined UMP outcomes than those with only QE. In particular, with the 2 per cent inflation target, when both forward guidance and QE are employed, the average inflation gap during the LNS periods is -0.74 pp. When only forward guidance is used, this gap is -0.80 pp, whereas it is -0.92 pp when only QE is implemented. Thus, most of the gains associated with UMP are coming from forward guidance. This partly reflects the same intuition behind the stronger effect of forward guidance on inflation, as explained in Section 2. In addition, the persistence in the policy rate rule makes forward guidance relatively more powerful than QE. However, as we will see in the sensitivity analysis, forward guidance can lose its advantage if the neutral rate is sufficiently negative.

It is also important to verify that the movements in interest rates in the simulations are plausible. Table 4 shows how each of the policy options (only forward guidance, only QE and a combination) affect interest rates and the term premium. In isolation, forward guidance implies that the long-term nominal and real rates are lower by 10 and 25 basis points, respectively, relative to the no UMP case under a 2 per cent inflation target. This reduction in the long-term nominal rate is in line with the empirical evidence presented in Del Negro, Giannoni and Patterson.

<table>
<thead>
<tr>
<th>Inflation Target (%)</th>
<th>Inflation Gap (pp)</th>
<th>Output Gap (%)</th>
<th>Short-Term Nominal Rate Gap (pp)</th>
<th>Short-Term Real Rate Gap (pp)</th>
<th>Long-Term Nominal Rate Gap (pp)</th>
<th>Long-Term Real Rate Gap (pp)</th>
<th>Term Premium Gap (pp)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without UMP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>-1.10</td>
<td>-2.25</td>
<td>-3.50</td>
<td>-2.59</td>
<td>-1.40</td>
<td>-0.98</td>
<td>0.44</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.90</td>
<td>-1.85</td>
<td>-4.23</td>
<td>-3.49</td>
<td>-1.63</td>
<td>-1.27</td>
<td>0.44</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.82</td>
<td>-1.68</td>
<td>-4.58</td>
<td>-3.92</td>
<td>-1.73</td>
<td>-1.41</td>
<td>0.44</td>
</tr>
<tr>
<td><strong>Only FG</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>-0.80</td>
<td>-1.83</td>
<td>-3.50</td>
<td>-2.89</td>
<td>-1.50</td>
<td>-1.23</td>
<td>0.44</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.72</td>
<td>-1.58</td>
<td>-4.23</td>
<td>-3.68</td>
<td>-1.70</td>
<td>-1.44</td>
<td>0.44</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.72</td>
<td>-1.52</td>
<td>-4.60</td>
<td>-4.03</td>
<td>-1.78</td>
<td>-1.50</td>
<td>0.44</td>
</tr>
<tr>
<td><strong>Only QE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>-0.92</td>
<td>-1.87</td>
<td>-3.50</td>
<td>-2.75</td>
<td>-2.04</td>
<td>-1.68</td>
<td>-0.21</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.83</td>
<td>-1.70</td>
<td>-4.21</td>
<td>-3.54</td>
<td>-1.90</td>
<td>-1.57</td>
<td>0.16</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.80</td>
<td>-1.62</td>
<td>-4.58</td>
<td>-3.93</td>
<td>-1.83</td>
<td>-1.51</td>
<td>0.34</td>
</tr>
<tr>
<td><strong>With UMP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>-0.74</td>
<td>-1.66</td>
<td>-3.49</td>
<td>-2.93</td>
<td>-1.88</td>
<td>-1.62</td>
<td>0.04</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.70</td>
<td>-1.53</td>
<td>-4.22</td>
<td>-3.68</td>
<td>-1.84</td>
<td>-1.58</td>
<td>0.29</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.72</td>
<td>-1.51</td>
<td>-4.59</td>
<td>-4.03</td>
<td>-1.82</td>
<td>-1.55</td>
<td>0.40</td>
</tr>
</tbody>
</table>
They find that long-term nominal rates move, on average, by roughly 10 basis points during the first four quarters after the announcement.

Relative to the no UMP case, our term premium rule calls for a reduction of 65 basis points in the term premium when QE is used in isolation and the inflation target is 2 per cent. The long-term nominal rate decreases by 64 basis points, which is within the range of estimates in the literature.\footnote{Reza, Santor and Suchanek (2015) survey the literature and report ranges for the impact of QE on long-term yields in the United States and the United Kingdom. They show that the cumulative effect of the first three asset purchase programs in the United States on the yields of 10-year Treasuries range from 65 to 120 basis points. They also show that the effects of the Bank of England’s gilt purchases are estimated to be between 45 and 150 basis points. Given that this empirical evidence also captures a potential signalling channel effect from QE, we consider that a decline of 64 basis points in the long-term rate is a reasonable effect of QE through a decline in term premium in our simulations.}

When forward guidance and QE are combined in the 2 per cent inflation target environment, the long-term real rate decreases by 64 basis points relative to the case without UMP. This reduction is explained by both a decline in the expected path component of the long-term real rate (24 basis points) and a reduction in the term premium (40 basis points). The reduction in the expected path component is very similar to the one achieved by only forward guidance, whereas the decline in the term premium is smaller than that with only QE. This reveals that forward guidance affects the endogenous response of the term premium rule.

Table 4 can also be used to assess the effects of each of these policies on inflation and the output gap. Under a 2 per cent inflation target, a decline of 25 basis points in the long-term real rate with only forward guidance (relative to the case without UMP) is associated with an increase of 30 basis points in inflation and 42 basis points in the output gap.

The effects of QE in isolation on inflation and output gap under a 2 per cent inflation target relative to the no UMP case are 18 and 38 basis points, respectively. Most of the empirical literature focuses on the effects of QE on real GDP. In our simulation, the effect on real GDP is 80 basis points, which is in line with what has been found in empirical studies on the effects of QE in the United States and the United Kingdom.\footnote{See Reza, Santor and Suchanek (2015).}
ToTEM simulations also reveal that forward guidance has a greater impact than QE on the five-year average expected inflation, as described in the simple NK model in Section 2. For the inflation target of 2 per cent, forward guidance alone leads to an increase of 15 basis points in the five-year average expected inflation (relative to the case without UMP). In contrast, with QE only, the five-year average expected inflation increases by only 6 basis points. Given that long-term real rates move more under QE, inflation expectations play a more important role in moving long-term real rates under forward guidance.

To conclude this subsection, we illustrate the effects of UMP on the length of ELB episodes. To do so, we focus again on the case in which the inflation target is equal to 2 per cent. Figure 3 presents the histogram showing the distribution of durations with and without UMP. As we can see in the chart, UMP increases the average duration by increasing the frequency of longer ELB episodes due to the forward guidance executed at the ELB.

![Figure 3: Distribution of ELB duration (1% real neutral rate, 2% inflation target)](image)

4.2.3 Sensitivity Analysis

In this subsection we carry out a comprehensive sensitivity analysis regarding two important sources of uncertainty: (i) the value of the real neutral rate and (ii) the effectiveness of UMP.
Figure 4 shows the average inflation and output gaps with and without UMP during LNS periods for different inflation targets and real neutral rates (from -1.5 per cent to 1.5 per cent). This figure makes clear that the benefits of a higher inflation target change as the real neutral rate moves from positive to negative values. As in the previous section, for positive values of the neutral rate, accounting for UMP largely eliminates the gains obtained by raising the inflation target. However, for negative neutral rates like the one estimated with the Eggertsson and Mehrotra (2017) secular stagnation model (-1.5 per cent), raising the inflation target leads to substantial improvements.

Figure 4: Average inflation and output gaps during LNS periods for different real neutral rates
Figure 5 shows the average inflation and output gaps during LNS periods for a 2 per cent inflation target, different real neutral rates and different assumptions about the availability of UMP. These charts show that, when the neutral rate is positive, most of the UMP benefits come from forward guidance. In contrast, when the neutral rate is negative, most of the UMP benefits come through QE. The intuition for this result is that at very low neutral rates the ELB is binding in a very high proportion of periods. Thus, there is little scope for forward guidance to provide additional stimulus by extending the period of time at the ELB.

![Figure 5: Average inflation and output gaps during LNS periods for different real neutral rates and inflation target of 2%](image)

Finally, Figure 6 shows how the benefits of a higher target are affected by our assumptions on the effectiveness of QE, given our baseline estimate of the real neutral rate (1 per cent). In particular, this figure shows
how different values of the parameter $\lambda$ in the term premium rule affect the results. When $\lambda = 1$, the degree of effectiveness of UMP is equal to that in our baseline simulations. When $\lambda = 0$, QE is inactive, while forward guidance still applies. The figure shows that varying the effectiveness of QE when the real neutral rate is 1 per cent has only very modest implications. We conjecture that this result reflects that forward guidance is almost a perfect substitute for QE. When QE is less effective, the central bank uses forward guidance more aggressively to substitute for QE.

Figure 6: Average inflation and output gaps during LNS periods for different effectiveness of QE and real neutral rate of 1%
5 Concluding Remarks

In this paper we have assessed the impact of a higher inflation target on macroeconomic stability. While we conducted our quantitative analysis in a model of the Canadian economy, our main results are likely to have broader relevance. In particular, we show that if the real neutral rate is in line with standard estimates and UMP is effective, then there are only modest gains from raising the target. This result diminishes, but does not extinguish, the case for a higher inflation target. While there is empirical evidence that supports the effectiveness of UMP, there does remain uncertainty about its limits. A higher target could help to insure against this uncertainty.

In contrast, if the real neutral rate is negative, as suggested by some advocates of the secular stagnation hypothesis, our analysis shows that the availability of UMP becomes irrelevant and a higher inflation target substantially improves macroeconomic performance. Of course, a higher target would only yield the benefits we identify if it were perceived as credible. In a situation of chronically deficient demand such as that envisioned by secular stagnationists, it may not be possible for the central bank to credibly commit to achieve a higher inflation target in a timely manner.

Our results also make clear that the relative effectiveness of different types of UMP depends on the macroeconomic context. More specifically, forward guidance can be a powerful tool in environments in which the neutral rate is sufficiently high. But, if the neutral rate is negative, QE becomes relatively more effective.

In sum, our results show that the effectiveness of UMP and the level of the real neutral rate are key determinants of the benefits of a higher inflation target. The current state of knowledge leaves considerable room for future research to reduce uncertainty about both UMP and the neutral rate. In particular, research on the secular stagnation hypothesis is likely to be important for assessing the case for a higher inflation target.
References


22. Chung, H., J.P. Laforte, D. Reifschneider and J.C. Williams (2012) “Have we underestimated the likelihood and severity of zero lower bound events?” *Journal of Money, Credit and Banking* 44 (s1), 47–82


Appendix A: Methodology to Solve the Small New Keynesian Model

Abstracting from long-term rates, the small NK model with forward guidance and QE can be conveniently expressed with one-period-ahead expectations. In particular, aggregate demand can be expressed as:

\[ x_t = E_t x_{t+1} - \frac{1}{\sigma} [rr_t - \bar{r}r + \lambda \min(i_t^d - i_t, 0)] + u_t. \]  
\[ (A.1) \]

The Phillips curve is given by

\[ \pi_t = (1 - \beta)\bar{\pi} + \beta E_t \pi_{t+1} + \kappa x_t, \]  
\[ (A.2) \]

where \( \kappa = 0.1679 \). The interest rate is determined as follows:

\[ i_t = \begin{cases} \text{ELB, if } (i_{t-1} = \text{ELB} \text{ and } i_t = i_t^d) \text{ implies } (x_t < 0 \text{ and } \pi_t - \bar{\pi} \leq 0.0025) \text{ or } (i_t^d < \text{ELB}), \\ i_t^d, \text{ otherwise} \end{cases} \]  
\[ (A.3) \]

Thus, the model can be stated in five variables: \( x_t, \pi_t, rr_t, i_t, i_t^d \) and with five equations: (A.1), (A.2), (A.3), (2) and (5). In a model without forward guidance, (A.3) is substituted by (6). In a model without QE, \( \lambda = 0 \).

The long-term real and nominal rates can be computed after solving for the five variables in the model above. Rather than computing the long-term rates exactly, which would require computing expectations of the short-term rates up to \( L \) quarters ahead, we approximate the long-term rates by using the following recursive formulas with one-period-ahead expectations:

\[ rr_{t,L} = \frac{1}{L} \left( rr_t + \frac{\lambda}{1 - \omega} \min(i_t^d - i_t, 0) \right) + \left( 1 - \frac{1}{L} \right) E_t [rr_{t,L+1}] \]  
\[ (A.4) \]

\[ i_{t,L} = \frac{1}{L} \left( i_t + \frac{\lambda}{1 - \omega} \min(i_t^d - i_t, 0) \right) + \left( 1 - \frac{1}{L} \right) E_t [i_{t,L+1}] \]  
\[ (A.5) \]

---

23 Iterating (A.1) forward for \( L - 1 \) periods, we can obtain (8) with the term premium given by (7).

24 An alternative approach is relying on non-product monomial rules or similar low-cost methods for computing the \( L \)-period-ahead expectations.
We obtain numerical solutions of the NK models by the standard collocation method. In versions of the model without forward guidance, the state space consists of the current-period demand shock, which we discretize by 1,000 evenly spaced grid points. In a model with forward guidance, the state space additionally includes a binary indicator of the forward guidance commitment. To get a precise solution of this model, we rely on an endogenously constructed grid with more than 25,000 grid points. Between the grid points of the demand shock, we approximate the policy functions by linear interpolation. At each iteration, given the approximated policy functions, we evaluate one-period-ahead expectations by a quadrature with about 300 nodes and using model equations we compute an update of the policy functions.
Appendix B: Solution Simulation Methodology for ToTEM

We conduct stochastic simulations of ToTEM to generate artificial time series of key macroeconomic variables. When conducting these simulations, we draw shocks from a multivariate normal distribution. We exclude all policy shocks and measurement errors from the simulations, leaving 33 structural shocks. The variance-covariance matrix of the shocks was estimated using shock series backed out over 1995Q1–2015Q2. This historical range is a useful benchmark because the inflation rate in Canada has been stable around the 2 per cent target since 1995.

Solving the model poses several challenges. Given that we need to impose an occasionally binding ELB on nominal interest rates, we cannot simply linearize the model and apply standard methods for solving linear rational expectations models. Global solution methods are often used to solve smaller models with the ELB.25 These methods, however, are limited to models with a small number of state variables and are therefore not a feasible approach for solving ToTEM.

Our solution methodology involves two approximations. First, we linearize the structural equations of the model. This is a common practice in the ELB literature, as it simplifies the computational problem by making the ELB the only source of nonlinearity in the model.26 Second, following Reifschneider and Williams (2000), we assume that agents’ beliefs about the future path of the economy are equal to the model’s predictions under the assumption that there are no future shocks to the economy. Thus, agents in the model use a modal forecast to form expectations, while the true rational expectation would be a mean forecast. This is an approximation because, despite the fact that all exogenous shocks are assumed to be symmetrically distributed, the nonlinearity associated with the ELB can lead to differences between the mode and the mean of the distribution of endogenous variables. Nevertheless, the use of modal forecasts facilitates simulation of the model because it allows us to use perfect foresight methods to compute agents’ expectations.

25 In some cases, such as Adam and Billi (2006), the structural equations of the model are first linearized and then the linearized structural equations are solved together with a nonlinear policy rule that respects the ELB using a global method. Others, including Basu and Bundick (2012) and Nakata (2013), apply the global solution methods directly to the fully nonlinear model.

26 Others who have used linearization techniques when analyzing the ELB include Eggertsson and Woodford (2003), Adam and Billi (2006) and Christiano, Eichenbaum and Rebelo (2011).
To obtain reliable estimates of the impact of a higher inflation target on macroeconomic performance, we conduct a large number of random draws for every parameterization that we consider. We obtain all results in Section 4, except those reported in Tables 3 and 4 and Figure 3, based on 300 simulations with each simulation being 1,050 periods long. The statistics in Tables 3 and 4 are obtained based on 3,000 simulations of 1,050-period-long series. The histogram in Figure 3 is obtained based on 12,000 simulations of the same length. When computing summary statistics, we exclude the initial 225 periods of each simulation to randomize over the initial conditions. We also exclude the last 225 periods to avoid misreporting the statistics due to potential truncation of the last ELB episode.
Appendix C: Results presented in the Bank of Canada’s *Renewal of the Inflation-Control Target: Background Information—October 2016*

With a few exceptions, the methodology followed in this paper matches that used to obtain the results presented in the Bank of Canada’s *Renewal of the Inflation-Control Target: Background Information—October 2016* document (Bank of Canada, 2016b). In this Appendix, we present these exceptions and show how they shape those results.

As an alternative to the assumption of the neutral rate of 1 per cent, we can assume this rate to be 1.25 per cent, which is the midpoint of the neutral rate range of 0.75 to 1.75 per cent estimated by the Bank of Canada at the time of the renewal of the agreement on the inflation-control target in 2016. This value also roughly matches the estimate found by Holston, Laubach and Williams (2017). The 25-basis-point increase in the assumed neutral interest rate leads to slightly lower probabilities of hitting the ELB (see Figure C.1). Particularly, for the ELB of -50 basis points, the probability of being at the ELB is about 7 per cent, which is about 1.5 ppt lower than that estimated under the assumption of the neutral rate of 1 per cent.

Whether the ELB is -0.5 or 0.25 per cent, an increase in the inflation target from 2 per cent to 3 or 4 per cent significantly reduces the frequency and the duration of ELB episodes (see Table C.1). In particular, setting the target to 4 per cent makes the ELB periods about five times less likely to occur. This target also lowers the average duration of ELB episodes to 1 quarter when the ELB is -50 basis points and to 1.5 quarters when the ELB is 25 basis points. These average durations are computed by taking the inflation target of 2 per cent and the ELB of 25 basis points as the baseline and counting episodes that disappear with a higher inflation target or with a lower ELB as being of zero duration.

Instead of modelling QE according to rule (7), which requires evaluating the expected gravity of each ELB episode, we can consider a simpler alternative. More specifically, when the policy rate reaches the ELB, we assume that the central bank buys government securities in sufficient quantities to reduce the term premium by 40 basis points. The central bank is assumed to hold these securities for five years before gradually normalizing the balance sheet. With such assumptions on QE (and with the same modelling assumptions on forward guidance as described in Section 4.2.2), we also reach our main conclusion that with UMP tools the gains of raising the inflation target are
small. As shown in Table C.2, when the ELB is assumed to be 25 basis points, an increase in the inflation target from 2 to 3 per cent would narrow the output gap by 0.2 percentage points. However, when the ELB is assumed to be -50 basis points, the narrowing of the output gap associated with a 3 per cent target is only 0.1 percentage point. Moreover, raising the target further from 3 to 4 per cent does not lead to any significant narrowing of the output gap. The gains in terms of inflation performance are also relatively small.

![Figure C.1: Unconditional probability of binding ELB constraint for different neutral rates](image)

**Table C.1: Frequency and duration of ELB**

<table>
<thead>
<tr>
<th>Real Neutral Rate (%)</th>
<th>ELB (%)</th>
<th>Inflation Target (%)</th>
<th>Proportion of time ELB is binding (%)</th>
<th>Average Duration (qrt)</th>
<th>ELB Duration at 90th Percentile (qrt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>-0.50</td>
<td>2.0</td>
<td>7.1</td>
<td>4.0</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>3.0</td>
<td>3.2</td>
<td>1.9</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.0</td>
<td>11.7</td>
<td>7.3</td>
<td>16</td>
</tr>
<tr>
<td>0.25</td>
<td>3.0</td>
<td>4.0</td>
<td>5.7</td>
<td>3.2</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.5</td>
<td></td>
<td>1.5</td>
<td>6</td>
</tr>
</tbody>
</table>
Table C.2: Macroeconomic outcomes under different inflation targets (1.25% real neutral rate)

<table>
<thead>
<tr>
<th>UMP</th>
<th>Inflation Target (%)</th>
<th>Average (-0.5% ELB)</th>
<th>Average (0.25% ELB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inflation Gap (pp)</td>
<td>Output Gap (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>All</td>
<td>LNS</td>
</tr>
<tr>
<td>Without UMP</td>
<td>2.0</td>
<td>-0.1</td>
<td>-0.9</td>
</tr>
<tr>
<td>Without UMP</td>
<td>3.0</td>
<td>0.0</td>
<td>-0.8</td>
</tr>
<tr>
<td>Without UMP</td>
<td>4.0</td>
<td>0.0</td>
<td>-0.7</td>
</tr>
<tr>
<td>With UMP</td>
<td>2.0</td>
<td>0.0</td>
<td>-0.7</td>
</tr>
<tr>
<td>With UMP</td>
<td>3.0</td>
<td>0.0</td>
<td>-0.7</td>
</tr>
<tr>
<td>With UMP</td>
<td>4.0</td>
<td>0.0</td>
<td>-0.7</td>
</tr>
</tbody>
</table>