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Working Paper

Dismiss the gap? A real-time assessment of the usefulness of Canadian output gaps in forecasting inflation

Bank of Canada Staff Working Paper, No. 2018-10

Provided in Cooperation with:

Bank of Canada, Ottawa

Suggested Citation: Pichette, Lise; Robitaille, Marie-Noëlle; Salameh, Mohanad; St-Amant, Pierre (2018) : Dismiss the gap? A real-time assessment of the usefulness of Canadian output gaps in forecasting inflation, Bank of Canada Staff Working Paper, No. 2018-10, Bank of Canada, Ottawa, <https://doi.org/10.34989/swp-2018-10>

This Version is available at:

<https://hdl.handle.net/10419/197863>

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Staff Working Paper/Document de travail du personnel 2018-10

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Bank of Canada Staff Working Paper 2018-10

March 2018

Dismiss the Gap? A Real-Time Assessment of the Usefulness of Canadian Output Gaps in Forecasting Inflation

by

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Acknowledgements

We thank Simon van Norden for useful and detailed comments and suggestions. We also thank, for their comments, Patrick Blagrove and participants in the following conferences: the June 2017 Canadian Economic Association conference in Antigonish; the May 2017 conference of the Société canadienne de science économique in Ottawa; and the October 2017 XIII Conference on Real-Time Data Analysis, Methods and Applications in Madrid. Finally, we thank Meredith Fraser-Ohman for helpful editing suggestions.

Abstract

We use a new real-time database for Canada to study various output gap measures. This includes recently developed measures based on models incorporating many variables as inputs (and therefore requiring real-time data for many variables). We analyze output gap revisions and assess the usefulness of these gaps in forecasting total CPI inflation and three newly developed measures of core CPI inflation: CPI-median, CPI-trim and CPI-common. We also study whether labour-input gaps, projected output gaps, and simple combinations of output gaps can add useful information for forecasting inflation. We find that estimates of excess capacity (the extent to which the economy is below potential) were probably too large around the 2008-2009 recession, as they subsequently tended to be revised down. In addition, we find that, when forecasting CPI-common and CPI-trim, some gaps appear to provide information that reduces forecast errors when compared with models that use only lags of inflation. However, forecast improvements are rarely statistically significant. In addition, we find little evidence of the usefulness of output gaps for forecasting inflation measured by total CPI and CPI-median.

Bank topics: Inflation and prices; Potential output; Econometric and statistical methods

JEL codes: C53, E37

Résumé

Nous utilisons une nouvelle base de données canadiennes en temps réel afin d'étudier diverses mesures de l'écart de production, dont des mesures élaborées récemment qui se fondent sur des modèles intégrant un grand nombre de variables (et nécessitant ainsi des données en temps réel pour un grand nombre de variables). Nous analysons les révisions de l'écart de production et évaluons l'utilité de ces écarts pour prévoir l'inflation mesurée par l'IPC global et l'évolution de trois nouveaux indicateurs de l'inflation fondamentale, en l'occurrence l'IPC-méd, l'IPC-tronq et l'IPC-comm. Nous cherchons aussi à savoir si les écarts du facteur travail, les écarts de production projetés et de simples combinaisons d'écarts de production peuvent éclairer utilement les prévisions d'inflation. Nous constatons que les estimations de la marge de capacités excédentaires (c.-à-d. le degré de sous-performance de l'économie en regard de son potentiel) étaient probablement trop élevées au moment de la récession de 2008-2009, étant donné qu'elles ont ensuite été généralement revues à la baisse. Par ailleurs, nous observons que, lorsqu'on projette l'IPC-comm et l'IPC-tronq, certains écarts paraissent fournir de l'information qui permet de réduire les erreurs de prévision par rapport aux modèles recourant uniquement à des valeurs retardées de l'inflation. Cependant, les améliorations obtenues dans la qualité des prévisions sont rarement significatives statistiquement. En outre, nous trouvons peu d'éléments qui montrent l'utilité des écarts de production pour prévoir l'inflation mesurée par l'IPC global et par l'IPC-méd.

Sujets : Inflation et prix; Production potentielle; Méthodes économétriques et statistiques

Codes JEL : C53, E37

Non-Technical Summary

The link between inflation and the output gap is key to the conduct of monetary policy in many central banks. However, the output gap is unobservable and highly uncertain. This paper examines whether output gaps, measured in real time, help improve forecasts of inflation when compared with simple models including only lags of inflation. Our paper makes three key contributions: i) it uses a novel database that enables the construction of real-time output gap estimates based on various models, including models incorporating a large number of variables as inputs, ii) it assesses the ability of different measures of output gap and labour gap, as well as combinations of output gaps, to improve forecasts of inflation, and iii) it is the first paper using real-time data to analyze the usefulness of output gaps to forecast Canada's new core CPI measures.

We find that most output gap estimates do not appear to add much information to simple models with lags of inflation in forecasting total CPI inflation and CPI-median. However, some output gaps, for instance those estimated with a simplified version of the Integrated Framework, help reduce forecast errors for CPI-common and CPI-trim inflation. But these improvements are rarely statistically significant, which partly reflects the fact that the sample of real-time data we could use to study out-of-sample forecasts is relatively small (about 10 years). The issue should be revisited as more data become available.

Our analysis of data revisions suggests that all output gap measures tended to overestimate excess capacity during the global financial crisis and in subsequent years. This is consistent with findings from other studies such as Grigoli et al. (2015), Martin, Munyan and Wilson (2015), and Dovern and Zuber (2017). Developing output gap measures that do better following recessions is an important subject for future research.

Our goal in this paper is not to develop the best possible forecasting models. It is instead to assess the information content of output gaps against simple benchmarks. We leave it to future work to examine how various sources of information, possibly including output gaps, could be combined to provide optimal forecasts.

1. Introduction

The link between inflation and the output gap is central to the conduct of monetary policy. When real gross domestic product (GDP) exceeds potential output³ (the output gap is positive), inflationary pressures are expected, causing inflation-targeting monetary authorities to tighten their policies.⁴

However, the usefulness of output gaps for forecasting inflation has been questioned. Orphanides and van Norden (2005) conclude that while output gaps constructed *ex post* appear to help predict inflation in the United States, real-time output gaps (i.e., output gaps estimated with the data available to policy makers) do not. Instead, they find that, in real time, models including output gaps often do not forecast better than simple models using only past inflation, and generally do worse than simple models incorporating past inflation and output growth. They show that the difference of information content between *ex post* and real-time output gaps partly reflects the large and persistent revisions to output gap estimates.

More recently, Edge and Rudd (2016) find that these gaps do not help forecast inflation in the United States for the period after the mid-1990s. Unlike Orphanides and van Norden (2005), Edge and Rudd (2016) find that this is true for both real-time and revised (*ex post*) output gaps. Revisions therefore do not seem to be the cause of the lack of information content of output gaps in their study.

The conclusion that real-time output gaps do not help forecast inflation is not limited to United States data. Indeed, Cayen and van Norden (2002) examine the issue with Canadian data and they, too, conclude that most output gaps do not improve the inflation forecasts of simple autoregressive models of inflation. Champagne, Poulin-Bellisle and Sekkel (2016) assess the information content of the output gaps used in the Bank of Canada's staff projection (a mix of models and judgment) as well as a few other simple output gap measures, and draw the same conclusion. However, contrary to Orphanides and van Norden (2005) and Cayen and van Norden (2002) but similar to Edge and Rudd (2016), they do not find that the information content of real-time output gaps is worse than that of output gaps measured with the latest vintage of data.

³ Potential output can be defined as the level of output that can be sustained in an economy without adding to inflationary pressures.

⁴ This paper focuses on the links between output gaps and inflation. However, output gap measurement is important for a variety of policies. See, for example, Ley and Mish (2013) for a discussion on fiscal policy.

Grigoli et al. (2015) study International Monetary Fund (IMF) staff output gap estimates and other output gap measures for several countries and conclude that output gaps do not provide useful information. In addition, they note that revisions to output gap measures tend to be particularly large following economic shocks—such as recessions—and that after negative shocks initial IMF output gap estimates are biased toward finding too much excess capacity. Martin, Munyan and Wilson (2015) and Dovern and Zuber (2017) find similar biases.

Guérin, Maurin and Mohr (2015) revisit the issue with European data from the 2000s. They find that some output gaps can somewhat improve inflation forecasts when compared with simple models using lags of inflation. This is particularly true for two- and four-quarter-ahead forecasts using output gaps calculated on the basis of simple unobserved component methods. They also find that simple combinations of output gaps tend to be less volatile than output gaps generated with a single approach, but that they do not necessarily improve inflation forecasting results.

The real-time data used in the studies discussed above have usually been limited to real GDP or output gap data produced by policy institutions. To our knowledge, output-gap measurement methods requiring many input variables subject to data revisions, such as production-function or multivariate-filter methods, have not been compared and analyzed with real-time data.⁵

In this paper, we assess the information content of various output gap measures for Canada using real-time data. This includes an assessment of the Extended Multivariate Filter (EMVF, also called the “statistical method” in the Bank of Canada *Monetary Policy Report* [MPR]) and of a simplified version of the Integrated Framework (IF, also called the “structural method” in the MPR).⁶ These output gap measures have been used in policy making but have not yet been assessed with real-time data, because of the large number of required variables and unavailability of these data in real time. For our project, we combine various real-time data sources—including a real-time database recently developed at the Bank of Canada—to enable such an examination. We assess the information content of simplified IF (SIF) and EMVF output gaps and compare these measures with others, including a state-space approach adapted from the work of Blagrove et al. (2015), which we call the basic multivariate filter (BMVF), and a different

⁵ However, some papers study the effects of output gap uncertainty in detailed macro models. See, for example, Orphanides et al. (2000).

⁶ Differences between the IF and the SIF are discussed below. See Pichette et al. (2015) for details about the EMVF and the IF.

state-space approach inspired by the BMVF, recently developed by Pichette, Robitaille and Bernier (forthcoming) called the Multivariate State-Space Framework (MSSF). We also include in the comparison simple mechanical filters that have often been considered in previous studies as well as the Bank of Canada staff assessment of the output gap studied in Champagne, Poulin-Bellisle and Sekkel (2016).⁷

Like Orphanides and van Norden (2005), we emphasize comparisons with simple models made of lags of inflation. We also investigate whether using simple real GDP growth rates instead of output gaps gives better inflation forecasts.⁸

As another experiment, we examine whether projected output gaps add useful information about future inflation. Output gaps forecasted for, say, $t + 1$, might be helpful in forecasting $t + 2$ inflation. Orphanides and van Norden (2005) limit their analysis to output gaps that could be estimated for the period up to the latest available quarter of real GDP, $t - 1$, which is one quarter before the inflation forecast is made. To project our output gap measures, we need to forecast some of the variables used as inputs in output gap models. For this, we use Bank of Canada staff monitoring (which covers the present and the next quarter) and simple assumptions about subsequent growth rates.⁹

While Orphanides and van Norden (2005) focus on total consumer price index (CPI) to measure inflation, we consider forecasts of both Canadian total CPI inflation and of recently developed measures of Canadian core inflation: CPI-trim, CPI-median and CPI-common.¹⁰ To our knowledge, this is the first paper providing an analysis of the usefulness of real-time output gaps to forecast Canada's newly developed measures of core inflation. We cannot take into account inflation data revisions for these core measures because only a few recent vintages are available. We nevertheless factor in data revisions in the case of total CPI inflation, but these revisions are very small—only because of seasonality adjustment—and do not affect the results.

⁷ Some of the output gap methods examined in this study (SIF, EMVF, BMVF and MSSF) were developed or modified over our forecasting sample. This means that, for these methods, the present study approximates a real-time examination. Further discussion is available in Section 2.

⁸ St-Amant and van Norden (1998) argue that using output growth in this way can be interpreted as implicitly defining an estimated output gap as a one-sided filter of output growth with weights based on the estimated coefficients in the inflation forecasting model. See also van Norden (1995).

⁹ We tested the robustness of our conclusions to changes in these simple assumptions.

¹⁰ See Bank of Canada (2016) for more details.

Output gaps may be better at forecasting core CPI inflation than total CPI inflation. This is because core measures should be less affected by short-run factors such as changes in commodity prices and idiosyncratic shocks (tax changes, weather effects on some prices, etc.). They should therefore be more clearly linked with longer-run determinants of inflation such as output gaps. And indeed, an analysis abstracting from data revisions found that the new measures of core CPI inflation—particularly CPI-common—are more correlated with output gap measures than total CPI inflation (Schembri 2017). In this paper, we examine whether output gaps estimated with real-time data are better at forecasting the new core measures.

We also examine whether gaps based purely on labour market information, which we call labour input gaps, provide better information about future inflation than output gaps. In addition, we examine the properties of simple combinations of output gaps and study whether they can improve inflation forecasts in Canada.

The rest of our paper proceeds as follows. We present our methodology in Section 2. In Section 3, we provide information about the output gap methods we consider and the revisions affecting our data. Our results regarding the usefulness of output gaps to forecast inflation are presented in Section 4. We conclude in Section 5.

Our main findings can be summarized as follows:

- Output gaps appear to have a potential to reduce forecast errors for CPI-common and CPI-trim. However, the improvement over models with only lags of inflation is rarely statistically significant, perhaps reflecting the small size of our sample of forecasts.¹¹ A longer sample will be needed to reach definitive conclusions.
- We find no evidence of the usefulness of output gaps for forecasting total CPI and CPI-median.
- Unlike Orphanides and van Norden (2005), but like other more recent studies, we find that our results concerning the usefulness of output gaps to forecast inflation are similar when data revisions are not factored in.
- Although output gaps estimated with the SIF tend to produce relatively low mean-square forecast errors, the improvement relative to our benchmark is not statistically significant. This is in part

¹¹ To assess statistical significance, we use an approach developed by Clark and McCracken (2009).

because the SIF is subject to substantial revisions. Yet these revisions have been smaller in recent years, partly due to improved data quality, and we expect that this will persist going forward.

- Our analysis of data revisions reveals that all the methods considered in our study tended to overestimate the amount of excess capacity (the extent to which the economy was below potential) during the period around the global financial crisis and in subsequent years. This is consistent with the findings, for various countries and output gap measures, of Grigoli et al. (2015), Martin, Munyan and Wilson (2015) and Dovern and Zuber (2017).
- We do not find that labour input gaps perform better than output gaps at forecasting inflation. Also, simple combinations of gaps do not seem to help much in terms of reducing the size, bias and volatility of revisions or in reducing inflation forecast errors. Also, adding forecasted output gaps does not change our conclusions, qualitatively.

2. Methodology

Our approach builds on that of Orphanides and van Norden (2005), whose basic idea was to study whether output gaps could add useful information to simple forecasting models based on past inflation and to simple forecasting models based on past inflation and output growth. If output gaps cannot pass such minimal criteria, they are unlikely to help predict inflation.

We follow this approach but add to it an examination of whether output gap projections—obtained with Bank of Canada staff monitoring and (when needed) simple assumptions about subsequent growth of the variables used in output gap models—could improve inflation forecasts. We also add a preliminary examination of the information content of labour input gaps and of output gap combinations. In addition, while Orphanides and van Norden (2005) studied forecasts of total CPI inflation in the United States, we consider forecasts of not only Canadian total CPI inflation but also of measures of Canadian core CPI inflation.

Let $\pi_t^h = \log(P_t) - \log(P_{t-h})$ denote inflation over h quarters ending in quarter t . We examine forecasts of inflation at various horizons (i.e., $h = 1$, $h = 2$, and $h = 4$). Forecasting inflation over short-term horizons alleviates the need to control for the Bank's reaction function, as it has been estimated that the impact of monetary policy decisions on inflation in Canada takes six to eight quarters to materialize (Bank of Canada 2012). We assume that output gaps are estimated and projected on the basis of the information

available toward the end of each quarter, shortly after the publication of Canadian National Accounts by Statistics Canada. This is when Bank of Canada staff present their assessment of the current state of the economy to the Bank's Governing Council.¹²

Our basic approach consists of three steps. First, like Orphanides and van Norden (2005), we estimate simple models of inflation of the form

$$\pi_{t+h}^h = \alpha + \sum_{i=1}^m \beta_i \cdot \pi_{t-i}^1 + e_{t+h}, \quad (1)$$

where the number of lags, m , is determined for all vintages with the Schwarz information criterion as estimated using the first available vintage (2007Q1). We re-estimate (1) in real time with data starting in 1992Q4¹³ in every quarter from 2007Q1 to 2017Q1 (we use an expanding window for the estimation), and use the resulting models to perform inflation forecasts. We use these forecasts as our main benchmarks in assessing models with output gaps. Our out-of-sample period starts in 2007Q1 because we do not have real-time data before 2007 for some important variables.

Second, we examine forecasting models of the form

$$\pi_{t+h}^h = \delta + \sum_{i=1}^m \eta_i \cdot \pi_{t-i}^1 + \sum_{j=1}^n \gamma_j \cdot y_{t-j} + \varepsilon_{t+h}. \quad (2)$$

The number of lags of inflation, m , is the same as in equation (1).¹⁴ n is the number of lags of the output gap, which is again determined with the Schwarz criterion as estimated with the first available vintage (2007Q1). As with equation (1), we re-estimate equation (2) in real time in every quarter from 2007Q1 to the end of our sample and perform inflation forecasts. Equation (2) follows the approach used by Orphanides and van Norden (2005) to assess the information content of historical output gaps. In such models, the most recent quarter of estimated output gap available to perform forecasts is the same as

¹² These projections are an input for Bank of Canada monetary policy decisions. See Murray (2013) for a discussion about the Bank's monetary policy decision-making process.

¹³ The sample starts in 1992Q4 because this was the first quarter for which the Bank of Canada set a specific inflation target (Bank of Canada 1991). Papers have found that the inflation process (mean, persistence, volatility) changed in Canada with the adoption of inflation targeting (Demers 2003).

¹⁴ Results are not sensitive to this assumption, as root-mean-squared error (RMSE) ratios between equations (1) and (2) are similar when we allow the number of lags of inflation to differ between equations (1) and (2).

the latest available quarter of real GDP. This means that the most recent historical output gap at the time a forecast of inflation is produced (t) is the previous-quarter output gap ($t - 1$).¹⁵

Third, and unlike Orphanides and van Norden (2005), we consider cases where projected output gaps are included. This implies that we also examine forecasting models of the form

$$\pi_{t+h}^h = \theta + \sum_{i=1}^m \varsigma_i \cdot \pi_{t-i} + \sum_{j=1}^n \psi_j \cdot y_{t-j} + \sum_{k=1}^p \varphi_k \cdot y_{t+h-k}^* + v_{t+h}. \quad (3)$$

The specification of equation (3) includes historical output gaps (y_t) and projected output gaps (y_t^*).¹⁶ Parameter p cannot exceed h and is determined with the Schwarz criterion as estimated with the first available vintage (2007Q1). The number of lags of inflation, m , and of output gaps, n , based on historical data is the same as in equations (1) and (2). To produce the projected output gaps, we use Bank of Canada staff monitoring of the variables used in the estimation of output gaps (e.g., SIF output gap projections require forecasts about population growth, investment, interest rates, hours worked, etc.). This gives us estimates of these variables for periods t and $t + 1$. For the subsequent quarters, we simply assume that the growth rates of the variables are equal to their historical means.^{17,18} It is important to note that these projected variables incorporate only the information available at the time inflation forecasts were made. No information obtained after period t is included in these projections.

We estimate equations (1), (2) and (3) for different definitions of Canadian inflation: total CPI inflation (the Bank of Canada target) and the three measures of core inflation announced by the Bank of Canada in 2016. We use ordinary least squares to estimate these equations.

To assess the models' forecast performance, we calculate and compare their root-mean-squared errors (RMSEs). Comparisons of forecasts with equations (2) and (1) give us information about whether output gaps estimated with historical data add information to simple models including only lags of inflation.

¹⁵ Note, however, that forecasted information for up to eight quarters is used in estimating the $t - 1$ output gap in two cases: i) with annual models to complete a given year; and ii) when using certain filters to address end-of-sample limitations.

¹⁶ Results are qualitatively similar when equation (3) does not include historical output gaps (y_t), as RMSE ratios between equations (1) and (3) remain very much alike.

¹⁷ There are some exceptions to this simple assumption. For example, projections for consensus forecasts—which are included in the BMVF and the MSSF—were imputed based on the latest available consensus forecasts.

¹⁸ We also checked the robustness of our results to this simple assumption by using Bank of Canada staff projections, in addition to staff monitoring, instead of the simple assumption that the variables return to their historical mean growth rates. We also considered a case where neither staff monitoring nor staff projections were used. In both cases, we obtained similar results.

Comparisons of equations (3) with (1) tell us whether having historical and projected output gaps helps improve inflation forecasts.

It is well known that there is considerable uncertainty surrounding output gap estimates. Some researchers have found that combining output gaps can help reduce that uncertainty. For instance, Guérin, Maurin and Mohr (2015) find that simple combinations of output gaps can reduce the volatility of output gap estimates. We therefore perform an assessment of whether combining output gaps can help reduce uncertainty and improve inflation forecasts with Canadian data. For this, we consider two of the simple weighting schemes also used by Guérin, Maurin and Mohr (2015): a measure obtained as the simple arithmetic average of our output gaps, and a measure obtained as the median estimate of all our output gaps. We also use a simple data reduction technique giving us their common component. We add to this an examination of the performance of the simple mean of the SIF and the EMVF (published on the Bank of Canada website), as well as the mean of the SIF and the MSSF (Pichette, Robitaille and Bernier, forthcoming).

Labour market variables tend to be less revised than variables related to GDP. It is therefore possible that labour input gaps (i.e., measures of the gap between potential and actual number of hours worked) should be used instead of output gaps in forecasting inflation. Three of our output gap measures incorporate labour input gap measures: the SIF, the EMVF and the MSSF. We replace the output gap with these in equations (2) and (3) to see whether they perform well in forecasting inflation.

Our conclusions about the usefulness of output gaps in forecasting inflation could potentially be different if we considered other variables that may affect Canadian inflation. As a robustness check, we therefore estimate versions of equation (2) where we add such variables. We focus on the exchange rate, commodity prices and unit labour costs (Schembri 2017; Bank of Canada 2017).¹⁹

As mentioned in the introduction, some of the output gap methods examined in this study (SIF, EMVF, BMVF, MSSF) were developed or modified over our forecasting sample. At the same time, developers of these methods could observe partially revised total CPI inflation. But they could not observe the new core measures of inflation and they did not have real-time data to perform a rigorous evaluation of gaps. For these cases, the present study should be seen as an approximation of a real-time examination.

¹⁹ We added one lag of each.

3. Output gaps

In this section, we first briefly describe the output gap methods we consider. Our focus is on methods developed in support of Canada's monetary policy and on some simple methods often used in the literature. Appendix A provides a list of the variables used for each method. Second, we discuss the real-time data we used to estimate the output gaps (Appendix A provides more detailed information) and the revisions to these output gaps.

3.1 Measures of output gap

Since 2014, one of the main methods used at the Bank of Canada to estimate and project potential output (and the output gap) has been the IF.²⁰ The IF is a production-function approach that decomposes potential output into trend labour input and trend labour productivity. Trend labour input is itself further decomposed into the working-age population, trend employment rate and trend average weekly hours. Estimates and projections of the working-age population are based on Statistics Canada data and Bank of Canada staff monitoring. Trend employment rate and trend average weekly hours are obtained with estimated models taking into account cohort (trend employment) and age-group (trend employment and trend weekly hours) fixed effects. These models also include variables such as interest rates, the job-offer rate, wealth, education level, an employment disincentive index and the share of the service sector in the economy. Trend labour productivity is the sum of capital deepening (capital stock per hour worked) and trend total factor productivity (trend TFP). Trend TFP is measured with a mechanical filter and judgment.²¹

Because real-time data do not exist for some of the variables used in the IF, in this project we use a simplified version of it. There are three differences between the SIF and the full model. First, while the model used by Bank staff has two interest rates (short- and long-term), the SIF has the short-term interest rate only. Second, the SIF does not include wealth. Third, while the size of the service sector affects employment rates and average hours worked of specific age groups and genders differently in the full model, in the simplified version the size of that sector enters as an aggregate variable. However, over the

²⁰ For a more detailed description of the IF, see Pichette et al (2015). The trend labour input part of the IF is derived from the work of Barnett (2007).

²¹ Although not a structural method *sensu stricto*, the IF allows for some interpretation of potential output developments. For instance, it allows for some analysis of how demographic developments and shocks to physical capital investment can affect potential output.

full sample, the SIF and the IF provide very similar estimates (see Figure B1 in Appendix B). We are therefore confident that the conclusions reached with the simplified version apply to the full model.

Another main method used at the Bank of Canada to estimate the output gap is the EMVF. The EMVF is a mix of mechanical filtering (Hodrick-Prescott filter), end-of-sample conditions and conditioning information applied to labour productivity and to components of labour input (participation rate, unemployment, average hours worked). This approach was first developed by Butler (1996), and it has been used at the Bank for many years as the main tool to measure past and present potential output. Pichette et al. (2015) proposed a modified version of the EMVF. As with the IF, output gaps based on this method are available on the Bank's website. One feature of the EMVF is that rather than filtering output directly, it filters components of output (productivity, average hours worked, etc.), which implies that real-time data for all variables used in the estimation of these components of potential output need to be taken into account in constructing real-time output gaps.²²

In this paper, we also examine output gap estimates used in the Bank of Canada's staff economic projections. This measure reflects the judgment of staff and is based on a large set of information, including the IF (since 2014) and the EMVF, output gap measures based on other methods, various labour market indicators, and some survey indicators, such as the information gathered in the *Business Outlook Survey* on firms' ability to meet an unanticipated increase in demand and on the intensity of labour shortages.²³ The real-time properties of Bank staff's output gaps are examined by Champagne, Poulin-Bellisle and Sekkel (2016).

The BMVF, proposed by Blagrove et al. (2015), is a state-space model requiring information about only a few variables to estimate potential output (real GDP growth, CPI inflation, unemployment rate, and output and inflation expectations from consensus forecasts). In this method, real GDP is subjected to three types of shocks: a level shock to potential, a shock to potential growth, and a demand shock to the output gap. A Phillips curve, Okun's law and consensus forecasts for output growth and inflation are added to help identify potential output. Consensus forecasts are included to improve the precision of end-of-sample estimates, the basic idea being that a forecast that output growth will be strong could indicate that current output is below potential. A weak inflation forecast would have similar implications.

²² Pichette et al. (2015) note various shortcomings of the EMVF, including the fact that, being a mix of mechanical filtering, conditioning information and end-of-sample conditions, it is difficult to interpret.

²³ For more details on these survey indicators, see Bank of Canada (2015).

Since the BMVF requires information about only a few observed variables, it does not incorporate some of the main determinants of potential output. Building on this method, Pichette, Robitaille and Bernier (forthcoming) propose an alternative measure of potential output, the MSSF, constructed by decomposing potential into the product of trend productivity and trend labour input. Each component of GDP is modelled as a stochastic process similar to the one assumed in the BMVF, with three types of shocks. The current version of the MSSF is an attempt to rewrite the EMVF. Therefore, the variables needed to construct the various vintages of this new approach are largely the same as for the EMVF.

We also consider two simple mechanical filters: the filter proposed by Hodrick and Prescott (1997) (HP filter, with a smoothing parameter set at 1,600) and the band-pass filter proposed by Christiano and Fitzgerald (2003) (BP filter).

Finally, a model including lags of real GDP growth instead of output gaps in the forecasting equations is considered. This is consistent with an approach discussed by van Norden (1995) and St-Amant and van Norden (1998).

3.2 Data and output gap revisions

An important contribution of this paper is the development of a database that includes real-time data for many variables. Data were collected from a variety of sources, including Bank staff projection and monitoring databases, to constitute a new Bank of Canada real-time database. Our project also used data obtained directly from Statistics Canada. Appendix A provides detailed information about the sources of our real-time data.

Since most of the data in the output gap estimations are subject to revisions—in some cases, very large revisions—it was essential to build this database. The capital stock (Chart A1) and the level and trend of the job offer rate²⁴ (Chart A2 and Chart A3), which are used in the estimation of the IF output gap, are examples of variables that have been subject to significant revisions. Those large revisions throughout our sample contributed to large revisions to the SIF output gap. Data revisions originated from multiple

²⁴ The job offer rate aims to capture labour demand variations in the IF. It can be defined as the measure of unmet demand in the economy. The job offer rate in the latest vintage of the IF is constructed by splicing the Help-Wanted Index from Statistics Canada, the Labour Shortage series from the Business Conditions Survey produced by Statistics Canada, the job vacancy rate published by the Canadian Federation of Independent Businesses, and the job vacancy rate from Statistics Canada.

sources ranging from new information gathered by Statistics Canada (such as annual surveys and censuses) to methodological changes.

Table 1 summarizes some key statistical properties of cumulative revisions to the various measures of output gap after eight quarters. This guarantees that each observation has undergone at least eight revisions. This hypothesis, which is based on the idea that the data must be close to final after eight revisions, has been used in the literature (e.g., Jacobs and Sturm 2004).²⁵ The table shows the mean, the absolute mean, the standard deviation, two measures of the noise-to-signal ratio (NSR, where NSR_1 is the ratio of the standard deviation of the revision to that of the estimate of the output gap after eight revisions, and NSR_2 is the ratio of the root mean square of the revision to the standard deviation of the estimate of the gap after eight revisions), the correlation between the output gap after eight revisions and the first release, and the frequency at which the sign of the output gap changes between the first release and the data after eight revisions. Charts comparing the real-time estimates with the data after eight revisions and the latest vintage are shown in Appendix C.

For all approaches, output gap estimates are, on average, revised up, implying some downward biases in the first estimates. This is highlighted by the similarity between mean and absolute mean revisions, and it is particularly evident for the EMVF, which is significantly revised, consistently on the upside. This tendency to overestimate the size of excess supply following a recession is consistent with other papers' results (e.g., Grigoli et al. 2015).

The standard deviation provides a signal about the volatility of the revisions. Revisions to the SIF are the most volatile across all approaches. This partly reflects the large revisions to the data feeding into this method, including capital stock and job offer rate data. However, revisions to some of these data may be smaller going forward. For instance, since 2016 Statistics Canada has been publishing new job vacancy data that are used in calculating the job offer rate and should be more reliable. In fact, we calculated that SIF revisions became smaller and less volatile after 2010, with the absolute mean and standard deviation of revisions falling in the more recent period (Table A1) from the much higher values calculated over our entire sample (Table 1 below). In general, these tables show that even though most methods have seen smaller and less biased estimates in recent years, the improvement has been particularly important in the case of the SIF.

²⁵ For most approaches used, statistics after eight revisions are similar to those that use the latest available vintage.

The two NSR measures shown in Table 1 indicate that, in real time, the MSSF provides the estimates least subject to revisions. The correlation and the frequency of changes in the sign of the output gaps suggest that the MSSF and the staff projection gaps are more likely to provide the same signal in real time and eight quarters after their first estimates.

Table 1: Properties of revisions after eight quarters (sample: 2006Q4 to 2014Q4)

	Mean	Absolute mean	Standard deviation	NSR ₁	NSR ₂	Correlation	Freq. of opposite signs
MSSF	0.36	0.45	0.37	0.22	0.31	0.98	9%
BMVF	0.64	0.73	0.71	0.43	0.58	0.94	18%
HP filter	0.44	0.62	0.68	0.51	0.61	0.89	42%
BP filter	0.59	0.66	0.63	0.53	0.72	0.88	52%
EMVF	0.97	0.97	0.55	0.38	0.78	0.94	45%
SIF	0.52	1.23	1.43	0.94	0.99	0.47	30%
Staff projection gaps	0.48	0.71	0.69	0.47	0.57	0.88	6%
Mean of output gaps	0.57	0.58	0.59	0.42	0.58	0.93	24%
Mean of EMVF and SIF	0.75	0.83	0.93	0.65	0.83	0.76	33%
Mean of MSSF and SIF	0.44	0.63	0.75	0.50	0.57	0.88	21%
Median of output gaps	0.69	0.69	0.57	0.42	0.65	0.93	42%
Principal component	0.84	0.92	1.04	0.44	0.57	0.94	24%

In general, these revisions are similar to those from econometric models for Europe (Marcellino and Musso 2011) on a similar sample. They are slightly less pronounced than statistical detrending procedures applied to U.S. data and slightly more pronounced than revisions to the Federal Reserve Board staff estimates of the output gap in the United States (Edge and Rudd 2016).²⁶

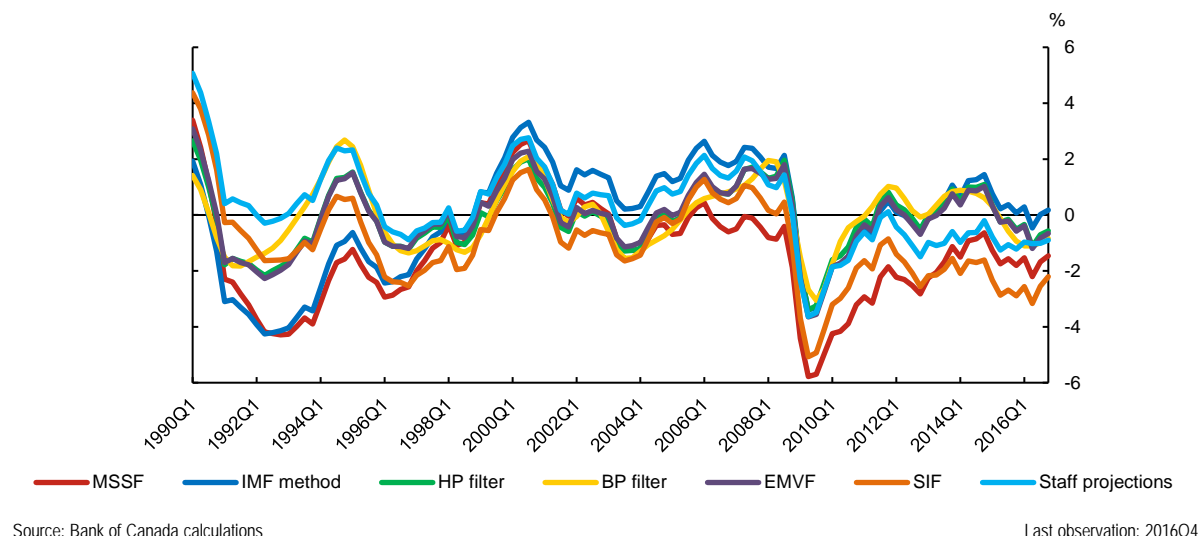
Chart 1 plots the output gap, estimated using the first six methods reported in Table 1. It shows the series constructed using the latest available vintage (2017Q1). It is worth noting that methods using mechanical filters (HP filter, BP filter and EMVF) produce output gaps that are less persistent. While the SIF and the MSSF show more excess supply since the 2008-2009 crisis, the BMVF and the MSSF indicate that the 1990s recession was somewhat deeper than suggested by the other estimates. It should be noted, however, that all output gap estimates are subject to considerable uncertainty and it is likely that many differences between output gap estimates are not statistically significant.²⁷

²⁶ The sample covered by Marcellino and Musso (2011) is from 2002Q1 to 2010Q2, while that of Edge and Rudd (2016) covers the period from 1998Q1 to 2006Q4.

²⁷ Providing confidence intervals for all output gap estimates is beyond the scope of this paper, since it would be complicated by the fact that many methods are only partly estimated.

Chart 1: Output gap estimates using the latest available vintage (2017Q1)

Quarterly data



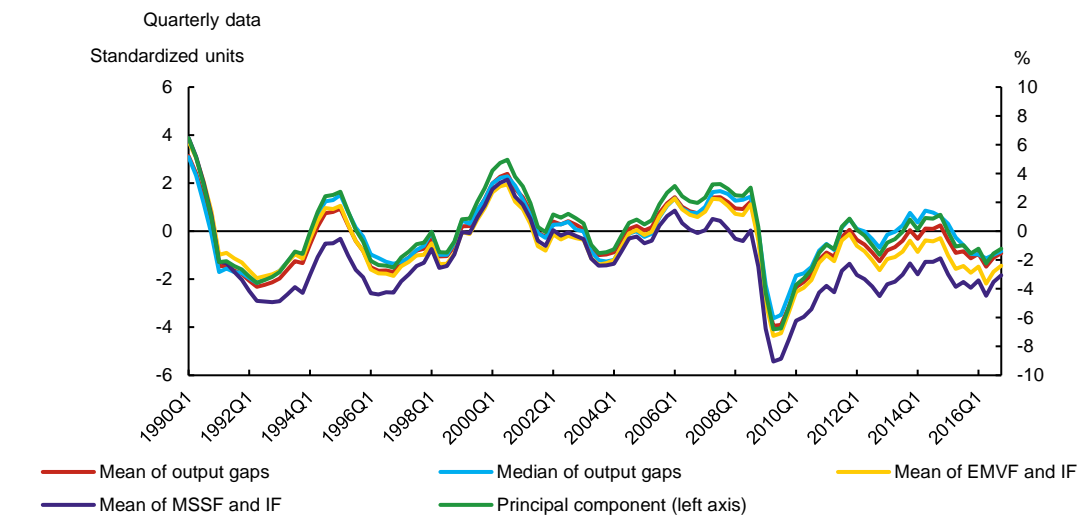
In estimating the equations, we use real-time seasonally adjusted total CPI inflation. Revisions to total CPI inflation on a quarter-over-quarter basis are minor and are due to seasonality. Total CPI inflation on a year-over-year basis is never revised because it is not seasonally adjusted and as such, real-time data are identical to the latest available vintage. On the other hand, for the newly constructed measures of core inflation used in our analysis (Chart D1), we focus on the latest available data since we have only a few vintages to work with.²⁸

Different combinations of the various output gap estimates (MSSF, BMVF, HP filter, BP filter, EMVF, SIF, and staff projection gaps) are shown in Chart 2. Common movements from the aforementioned output gap estimates are extracted using a data-reduction technique—principal component analysis (PCA). Only the first principal component is reported here, as it is the one that maximizes its contribution to the variance of the set of output gap estimates. On average in the different vintages, the first principal component accounts for 83 per cent of the total variance.²⁹

²⁸ Both year-over-year and quarter-over-quarter measures of core inflation are revised. For CPI-trim and CPI-median, revisions are due to seasonality, as they are based on seasonally adjusted components. In the case of CPI-common, revisions are due to the statistical technique used, as the factor model is estimated over all available historical data.

²⁹ The principal component cannot be interpreted as an output gap per se since it is expressed in standardized units. It is thus presented on a different axis.

Chart 2: Output gap combinations using the latest available vintage (2017Q1)



Source: Bank of Canada calculations

Last observation: 2016Q4

4. Inflation forecasting results

This section presents empirical results to assess whether the output gap improves forecasts of inflation at various horizons h . This is done through a real-time out-of-sample forecasting exercise, in which each equation is estimated for a sample spanning the 1992Q4 to 2006Q4 period using the real-time vintage (i.e., 2007Q1), and a forecast is produced for 2007Q1+ h . One observation is then added to the estimation period and the subsequent vintage of data is used to estimate the forecasting model and to compute the out-of-sample forecast. This process is repeated up to the moment where we obtain a forecast for 2016Q4. A key point in a typical real-time forecasting exercise is the choice of “actual data” from which forecast errors are calculated. As mentioned in the previous section, revisions to total inflation are minor, diminishing the importance of this choice. Table 2 compares the performance of real-time out-of-sample forecasts of total inflation using the latest available vintage as actuals.³⁰ A simple model with only lags of inflation—equation (1)—is taken as a benchmark. It is compared with the previously described output gap estimates to forecast inflation, which rely on historical output gap data only—equation (2)—or on a mix of historical and projected output gap data—equation (3). A comparison is also conducted with models including lags of inflation and real GDP growth. Statistical significance between the RMSE of equation (1)

³⁰ Similar results were obtained using the first data released of total inflation as “actual data.”

and equations (2) and (3) is assessed using an approach developed by Clark and McCracken (2009). This test compares the forecasting performance of nested models in the presence of data revisions.³¹

Table 2: Comparison of total inflation forecasts at various horizons

Model	Equation 2			Equation 3		
	$h = 1$	$h = 2$	$h = 4$	$h = 1$	$h = 2$	$h = 4$
MSSF	1.02	1.03	1.19	1.01	1.02	1.16
BMVF	1.00	1.01	1.11	0.98	0.97	1.07
HP filter	1.00	1.00	1.05	0.98	0.95	1.00
BP filter	0.99	1.13	1.00	0.98	1.26	1.06
EMVF	1.00	1.00	1.07	0.99	0.97	1.04
SIF	0.95	0.95	1.01	0.93	0.88	0.87
Staff projection gaps	0.99	0.99	1.06	0.97	0.95	0.99
GDP growth	1.01	0.98	0.92	1.00	0.98	0.92
Mean of output gaps	0.99	0.99	1.08	0.96	0.94	0.99
Mean of EMVF and SIF	0.97	0.96	1.03	0.94	0.90*	0.90
Mean of MSSF and SIF	0.97***	0.96***	1.06	0.95	0.92***	0.96
Median of output gaps	0.99	0.99	1.08	0.98	0.95	1.01
Principal component	0.99*	0.99	1.07	0.97	0.94	0.99
Labour input gap – MSSF	1.01	1.02	1.22	0.99	0.97	1.20
Labour input gap – SIF	1.03***	1.05	1.17	1.02	1.00	1.21
Labour input gap – EMVF	1.01	1.01	1.18	0.98	0.95	1.17

Notes:

1) The table reports the RMSE ratio between the forecasts obtained from equations 2 and 3 and equation 1 (benchmark model).

2) Results for the test of equal forecast accuracy are reported as follows: *** p-value < 0.01; ** p-value < 0.05; * p-value < 0.1.

3) The sample size varies according to the forecasting horizon: $h = 1$, 2007Q2-2016Q4; $h = 2$, 2007Q3-2016Q4; $h = 4$, 2008Q1-2016Q4.

The first observation apparent from the table is that most approaches for estimating the output gap do not provide accuracy gains compared with the benchmark model. The SIF estimates of the output gap most often give the lowest RMSEs, but improvements against the model with only lags of inflation are never statistically significant. This lack of statistical significance may be due to the large revisions affecting SIF gaps (Table 1). These tend to be penalized by the significance test. However, combining the SIF with the MSSF appears to bring statistically significant improvement to one- and two-quarter-ahead forecasts, both with and without forecasted gaps in the latter case. The fact that combining these methods greatly reduces the size and volatility of gap revisions (Table 1), when compared with SIF gaps, likely accounts for the statistically significant results.³² Except for the SIF/MSSF combination, the inclusion of both historical and projected output gaps and the use of combinations of methods do not seem to improve inflation

³¹ Technical details are available in Appendix E.

³² The matrix F , calculated in the context of the test developed by Clark and McCracken (2009), plays an important role in the significance of the mean of SIF and MSSF. As explained in Appendix E, the size of this matrix is often associated with the size of the revisions.

forecasts compared with the benchmark, nor does replacing output gap by GDP growth, as a measure of business cycles.

Results for CPI-median (Table 3) are similar to those for total CPI, but in this case the only exception is the inclusion of the BMVF historical estimates of the output gap to forecast four quarters ahead. Otherwise, output gaps do not appear to provide useful information, especially since the RMSE ratios are all close to one.

Table 3: Comparison of core inflation (CPI-median) forecasts at various horizons

Model	Equation 2			Equation 3		
	$h = 1$	$h = 2$	$h = 4$	$h = 1$	$h = 2$	$h = 4$
MSSF	1.04	1.05	1.07	1.03	1.06	1.07
BMVF	0.97	0.96	0.93***	0.96	0.96	0.93
HP filter	1.03	1.04	1.01	1.01	1.02	0.98
BP filter	1.05	1.05	1.11	1.08	1.17	1.10
EMVF	1.07	1.09	1.07	1.07	1.11	1.11
SIF	1.02	1.05	0.99	1.05	1.10	1.06
Staff projection gaps	1.00	0.99	0.95	0.98	0.99	0.93
GDP growth	1.04	1.01	0.99	1.06	1.07	0.98
Mean of output gaps	1.02	1.04	1.00	1.03	1.04	0.99
Mean of EMVF and SIF	1.02	1.04	0.98	1.04	1.07	1.01
Mean of MSSF and SIF	1.02	1.04	1.02	1.03	1.06	1.02
Median of output gaps	0.99	0.99	0.96	1.00	0.99	0.97
Principal component	1.03	1.04	1.01	1.03	1.05	1.00
Labour input gap – MSSF	0.99	0.99	1.00	0.96	0.96	1.02
Labour input gap – SIF	1.02	1.06	1.09	1.02	1.11	1.25
Labour input gap – EMVF	0.99	0.99	1.00	0.95	0.94	0.99

Notes:

1) The table reports the RMSE ratio between the forecasts obtained from equations 2 and 3 and equation 1 (benchmark model).

2) Results for the test of equal forecast accuracy are reported as follows: *** p-value < 0.01; ** p-value < 0.05; * p-value < 0.1.

3) The sample size varies according to the forecasting horizon: $h = 1$, 2007Q2-2016Q4; $h = 2$, 2007Q3-2016Q4; $h = 4$, 2008Q1-2016Q4.

Results are different in the cases of CPI-common and CPI-trim (Tables 4 and 5). Most RMSE ratios are below one for predicting CPI-common inflation, in particular for models and combinations including SIF gaps and for staff projection gaps. Yet, there are only a few results that are statistically significant. This includes models with the MSSF's labour input gap and the EMVF gap.

Table 4: Comparison of core inflation (CPI-common) forecasts at various horizons

Model	Equation 2			Equation 3		
	$h = 1$	$h = 2$	$h = 4$	$h = 1$	$h = 2$	$h = 4$
MSSF	0.91	0.89	0.95	0.99	0.92	0.94
BMVF	0.91	0.84	0.83	1.02	0.89	0.84
HP filter	0.86	0.87*	0.95	0.95	0.91	0.96
BP filter	0.90	0.90	0.93	0.94	0.95	0.92
EMVF	0.85	0.82***	0.92	0.96	0.89	0.93
SIF	0.87	0.74	0.72	1.00	0.80	0.72
Staff projection gaps	0.83	0.77	0.75	0.94	0.85	0.75
GDP growth	1.02	1.03	0.87	1.07	0.98	0.86
Mean of output gaps	0.84	0.78	0.81	0.94	0.84	0.81
Mean of EMVF and SIF	0.82	0.73	0.73	0.96	0.80	0.73
Mean of MSSF and SIF	0.84	0.76	0.77	0.94	0.81	0.78
Median of output gaps	0.84	0.85	0.82	0.94	0.88	0.81
Principal component	0.83	0.79	0.83	0.93	0.84	0.83
Labour input gap – MSSF	0.91	0.87***	0.90	0.93	0.92	0.91
Labour input gap – SIF	0.99	0.92	0.96	1.05	1.00	1.05
Labour input gap – EMVF	0.88*	0.91	0.94	0.91	0.97	0.94

Notes:

1) The table reports the RMSE ratio between the forecasts obtained from equations 2 and 3 and equation 1 (benchmark model).

2) Results for the test of equal forecast accuracy are reported as follows: *** p-value < 0.01; ** p-value < 0.05; * p-value < 0.1.

3) The sample size varies according to the forecasting horizon: $h = 1$, 2007Q2-2016Q4; $h = 2$, 2007Q3-2016Q4; $h = 4$, 2008Q1-2016Q4.

In the case of CPI-trim, the MSSF, the staff projection gaps, the mean and the median of all gaps, as well as the principal component are significant at certain horizons for equation (2), while for the HP filter, the mean of all gaps and the principal component are significant at certain horizons for equation (3). Overall, however, accuracy gains seem to be smaller than with the CPI-common, as the RMSE ratios are larger.

Table 5: Comparison of core inflation (CPI-trim) forecasts at various horizons

Model	Equation 2			Equation 3		
	$h = 1$	$h = 2$	$h = 4$	$h = 1$	$h = 2$	$h = 4$
MSSF	0.97*	0.94***	0.94***	0.98	0.96	0.92
BMVF	0.95	0.93	0.91	0.95	0.93	0.87
HP filter	0.98	0.97	0.97	0.95	0.93	0.93***
BP filter	0.99	0.95	1.00	1.01	1.05	0.99
EMVF	0.98	0.95	0.94	0.97	0.95	0.93
SIF	0.95	0.94	0.91	0.97	0.96	0.92
Staff projection gaps	0.94	0.92***	0.90	0.91	0.89	0.82
GDP growth	1.01	1.00	0.95	1.06	1.04	0.96
Mean of output gaps	0.94	0.91	0.89***	0.93	0.89	0.83***
Mean of EMVF and SIF	0.92	0.88	0.84	0.92	0.89	0.79
Mean of MSSF and SIF	0.92	0.87	0.83	0.93	0.88	0.80
Median of output gaps	0.95***	0.93***	0.92	0.93	0.89	0.86
Principal component	0.95	0.92	0.90***	0.93	0.90	0.84**
Labour input gap – MSSF	0.98	0.97	0.97	0.93	0.93	0.98
Labour input gap – SIF	1.11	1.16	1.21	1.09	1.21	1.39
Labour input gap – EMVF	0.98*	0.97	0.97	0.90	0.90	0.93

Notes:

1) The table reports the RMSE ratio between the forecasts obtained from equations 2 and 3 and equation 1 (benchmark model).

2) Results for the test of equal forecast accuracy are reported as follows: *** p-value < 0.01; ** p-value < 0.05; * p-value < 0.1.

3) The sample size varies according to the forecasting horizon: $h = 1$, 2007Q2-2016Q4; $h = 2$, 2007Q3-2016Q4; $h = 4$, 2008Q1-2016Q4.

We performed pseudo out-of-sample forecasts of total and core inflation. The pseudo out-of-sample forecasting exercise is similar to the real-time exercise except that all recursive estimations are conducted using the same vintage of data—the latest available vintage (2017Q1). Although the forecasting performance of different models should always be compared using real-time out-of-sample procedures, pseudo out-of-sample forecasts have long been used and can be compared with those of the literature. Consistent with real-time out-of-sample forecasts, pseudo out-of-sample forecasts of total inflation failed to provide evidence of the usefulness of output gaps to forecast total CPI.³³ This is in contrast with results from Orphanides and van Norden (2005) and Cayen and van Norden (2002) but in line with Edge and Rudd (2016), who do not find that the information content of real-time output gaps is worse than that of output gaps measured with final data. Like the real-time out-of-sample forecasts, the pseudo out-of-sample ones suggest that some output gap estimates may be more useful to forecast CPI-common and CPI-trim than total inflation.

Overall, our results are mixed: while output gaps do not seem to add much useful information in forecasting total CPI and CPI-median inflation, some may improve forecasts of CPI-common and CPI-trim inflation. Like Guérin, Maurin and Mohr (2015), we find that the gaps that are the least revised are not necessarily those resulting in the best RMSE compared with the benchmark equation. Output gap estimates from methods that are revised using new information, even substantially, may do better in terms of RMSE than those from methods that provide persistently inaccurate assessment. However, the test of predictive accuracy developed by Clark and McCracken (2009) penalizes variables that are heavily revised, which makes it more difficult for them to be found statistically significant. Ultimately, the MSSF, the staff projection gaps and the mean of the SIF and the MSSF, which were the least revised output gaps, are those that provide statistically significant accuracy gains in forecasting inflation.

However, all those results should be interpreted with caution because of the limitations of the Clark and McCracken test. Clark and McCracken (2009) present results from Monte Carlo simulations in which they assess the finite-sample performance of the test. For a sample of 40 observations, they find that, at the size of 5 per cent, the test falsely rejects the null hypothesis, which assumes that the mean squared errors (MSEs) for two forecasting equations are equal, 10 to 14 per cent of the time. Tables 2 through 5 present the results for 384 hypothesis tests—for about 10 per cent of the cases, the null is rejected. This is not

³³ Results of the pseudo out-of-sample forecasting exercise are not shown in this paper but are available upon request.

higher than what we would expect given the size of the test. In many cases, rejections are at the 1 per cent level, but still, it is reasonable to think that many of them might be spurious. On the other hand, the results of these Monte Carlo simulations for the power of the test in Clark and McCracken (2009) suggest that the probability of making a type-2 error is relatively high. This means that there might be cases in the above tables where the test did not reject the null while it is false. For instance, some equations forecasting CPI-common inflation in Table 4, where we found relatively low RMSE ratios, could perform significantly better than the benchmark, but the limited power of the test prevents us from correctly rejecting the null. That might simply be attributable to the small sample size of our experiments.

In general, we do not find that labour input gaps perform better than output gaps at forecasting inflation. We find that simple combinations, such as means or medians of output gaps, tend to do relatively well, but these methods do not always outperform single output gaps. We do not find that adding projected output gaps improves the results.

As a robustness check, we also estimated versions of equation (2) where we added other variables that have been found helpful to explain inflation in Canada (Schembri 2017; Bank of Canada 2017): the exchange rate, commodity prices and unit labour costs. Including these variables left our main results qualitatively unchanged.³⁴

4. Conclusion

The link between inflation and the output gap is key to the conduct of monetary policy in many central banks. However, the output gap is unobservable and highly uncertain. This paper examines whether output gaps, measured in real time, help improve forecasts of inflation when compared with simple models including only lags of inflation. Our paper makes three key contributions: i) it uses a novel database that enables the construction of real-time output gap estimates based on various models, including models incorporating a large number of variables as inputs, ii) it assesses the ability of different measures of output gap and labour gap, as well as combinations of output gaps, to improve forecasts of inflation, and iii) it is the first paper using real-time data to analyze the usefulness of output gaps to forecast Canada's new core CPI measures.

³⁴ Results are available upon request.

We find that most output gap estimates do not appear to add much information to simple models with lags of inflation in forecasting total CPI inflation and CPI-median. However, some output gaps help to reduce forecast errors—as measured by RMSEs—for CPI-common and CPI-trim inflation, although these forecast improvements are rarely statistically significant.

We also find that, in general, equations with projected output gaps and equations with labour input gaps do not perform better at forecasting inflation than equations with lags of output gaps.

The SIF does relatively well in general in terms of pure forecasting accuracy. This is interesting, given that its output gap estimates have been revised the most over our forecasting period. However, the importance of its revisions and the limited power of the test of equal predictive accuracy may explain why it was not found to provide statistically significant improvement over our benchmark. This situation may unwind, since the SIF should be less revised going forward, as has been the case since 2011.

Our analysis of data revisions reveals that all measures tended to overestimate excess capacity during the global financial crisis and in subsequent years. This is consistent with findings from other studies such as Grigoli et al. (2015), Martin, Munyan and Wilson (2015), and Dovern and Zuber (2017). Developing output gap measures that do better following recessions should be a subject for future research.³⁵

Monitoring with future data the performance of the output gaps examined in this paper will also be important, given that the sample of forecasts we used is small.

Our goal in this paper was not to develop the best possible forecasting models. It was instead to assess the information content of output gaps against simple benchmarks. We leave it to future work to examine how various sources of information, possibly including output gaps, could be combined to provide optimal forecasts.

³⁵ Models with stochastic volatility, such as those examined in Mertens (2014), might be worth considering in that context. That author obtains, with US data, output gap estimates that seem closer to “final” estimates.

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Appendix A: Details about the data

Data sources description:

- VINT: The Bank of Canada started collecting vintages of various series in the early 2000s. These series are stored in a special database and are accessible to Bank staff. Most of the real-time data used in the paper come from this source.
- IMP_VINT: In addition to the VINT database, the Bank of Canada recently created a second database that contains hundreds of new real-time series.
- MON: Bank of Canada short-term forecast databases.
- STAT CAN: Data provided from Statistics Canada through a special request.
- CF: Consensus forecasts from Consensus Economics.
- NR: The data are not revised.

Models:

Multivariate State-Space Framework (MSSF)

	Definition	Source	Real-time data source
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
Gross domestic product (consensus forecast)	Consensus forecasts of gross domestic product (the next five years)	Consensus Economics	CF
Inflation	Total consumer price index inflation (annual)	Statistics Canada — Consumer Price Index	MON
Inflation (consensus forecast)	Consensus forecasts of total consumer price index inflation (the next year)	Consensus Economics	CF
Labour productivity	Total gross domestic product divided by total hours worked		
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
Total hours worked	Total actual hours worked for all jobs: Seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Average hours worked	Total hours worked divided by employment		
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Total hours worked	Total actual hours worked for all jobs: seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON

Basic Multivariate Filter (BMVF)

	Definition	Source	Real-time data source
Unemployment rate	Number of people unemployed (labour force minus employed) divided by labour force	Statistics Canada – Labour force survey estimates	VINT & MON
Labour force	Total labour force (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
Gross domestic product (consensus forecast)	Consensus forecasts of gross domestic product (the next five years)	Consensus Economics	CF
Inflation	Total consumer price index inflation (annual)	Statistics Canada – Consumer Price Index	MON
Inflation (consensus forecast)	Consensus forecasts of total consumer price index inflation (the next year)	Consensus Economics	CF

Hodrick-Prescott filter (HP filter)

	Definition	Source	Real-time data source
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON

Band-Pass filter (BP filter)

	Definition	Source	Real-time data source
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON

Extended Multivariate Filter (EMVF)

	Definition	Source	Real-time data source
Average hours worked	Total hours worked divided by employment		
Total hours worked	Total actual hours worked for all jobs, seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Employment	Employment (x 1,000); both sexes; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
NAIRU			
Employment disincentive index	Employment insurance disincentive index assuming constant unemployment rate of 7.5% for all regions	Department of Finance Canada	NR
Payroll taxes (excluding CPP)	Payroll taxes (excluding CPP) as a share of wages, salaries, and supplementary income		
Employer and employee contribution to employment	Employer and employee contributions to employment insurance (x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	IMP_VINT & VINT

insurance			
Employers' contributions to worker's compensation	Employers' contribution to workers' compensation (x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	IMP_VINT & VINT
Compensation of employees paid by resident entities	Compensation of employees paid by resident entities (x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	IMP_VINT & VINT
Provincial payroll taxes	Provincial payroll taxes (x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	IMP_VINT & VINT
Employer and employee contributions to industrial employees' vacations	Employer and employee contributions to industrial employees' vacations (x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	IMP_VINT & VINT
Unemployment rate	Number of people unemployed (labour force minus employed) divided by labour force		
Labour force	Total labour force (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Working-age population	Number of persons of working age, 15 years of age and over (x 1,000)	Statistics Canada – Labour force survey estimates	IMP_VINT & VINT & MON

Simplified Integrated Framework (SIF)

	Definition	Source	Real-time data source
Average hours worked	Total hours worked divided by employment		
Total hours worked	Total actual hours worked for all jobs, seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Education	Ratio of university degree holders to working-age population		
University degree holders	University degree holders	Statistics Canada – Labour force survey estimates	IMP_VINT & VINT
Working-age population	Number of persons of working age, 15 years of age and over (x 1,000)	Statistics Canada – Labour force survey estimates	IMP_VINT & VINT
Employment disincentive index	Employment insurance disincentive index	Department of Finance Canada	NR
Enrollment rate	Full-time students aged 15-24 years divided by total population aged 15-24 (students + non-students)		
Full-time students (15-24)	Full-time students; aged 15-24 years; unadjusted for seasonality	Statistics Canada – Labour force survey estimates	VINT
Total non-students (15-24)	Non-students; aged 15-24 years; unadjusted for seasonality	Statistics Canada – Labour force survey estimates	VINT
Total students (15-24)	Students; aged 15-24 years; unadjusted for seasonality	Statistics Canada – Labour force survey estimates	VINT
Nominal interest rate	Bank of Canada Bank Rate	Bank of Canada	MON
Trend inflation	The trend in the consumer price index inflation rate	Internal	MON
Trend nominal interest rate	The trend in the nominal interest rate	Internal	MON
Job offer rate			
Help-wanted index	Changes in the demand for labour relative to a base year	Statistics Canada – Help-Wanted Index survey	NR
Labour shortage	Production difficulties due to skilled labour shortage from the Business Conditions Survey	Statistics Canada – Business Conditions Survey for the Manufacturing Industries	IMP_VINT

CFIB – job vacancy rate	Canadian Federation of Independent Business job vacancy rates	Canadian Federation of Independent Business Economics	NR
SEPH – job vacancy rate	Survey of Employment, Payrolls and Hours, job vacancy statistics; job vacancy rate (%); unadjusted for seasonality	Statistics Canada – Job Vacancy Rate	VINT
Labour input	Total actual hours worked for all jobs, seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Share of service-producing sector	Ratio of employment in the service-producing sector to total employment		
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT & MON
Service-producing sector employment	Employment (x 1,000); service-producing sector; both genders; aged 15 years and over	Statistics Canada – Labour force survey estimates	IMP_VINT & VINT
Working-age population	Number of persons of working age, 15 years of age and over (x 1,000)	Statistics Canada – Labour force survey estimates	IMP_VINT & VINT
Average hours worked, single age groups	Average hours worked by employee; males, females, and both genders; single age group (%)	Statistics Canada – Labour force survey estimates (special request)	STAT CAN
Employment rate, single age groups	Employment rate; males, females, and both genders; single age group	Statistics Canada – Labour force survey estimates (special request)	STAT CAN
Working-age group, single age groups	Number of persons of working age; males, females, and both genders; single age group (x 1,000)	Statistics Canada – Labour force survey estimates (special request)	STAT CAN
Total non-residential investment	Total non-residential investment (x 1,000,000); chained (2007) \$	Statistics Canada – Stock and Consumption of Fixed Non-residential Capital	STAT CAN & IMP_VINT & VINT
Business fixed investment	Business fixed investment; chained (2007) \$; seasonally adjusted at annual rate	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
Capital cost	Capital cost (\$ x 1,000,000); business sector	Statistics Canada – Productivity Measures and Related Variables (National and Provincial)	STAT CAN & VINT
Government consumption	The sum of government consumption expenditure and government gross fixed capital formation		MON
Government consumption expenditure	General government final consumption expenditure (x 1,000,000); chained (2007) \$; seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
Government gross fixed capital formation	General government gross fixed capital formation (x 1,000,000); chained (2007) \$; seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
Nominal gross domestic product	Gross domestic product (\$ x 1,000,000); business sector; value-added	Statistics Canada – Productivity Measures and Related Variables (National and Provincial)	STAT CAN & VINT
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON
Net capital stock	Geometric (infinite) end-year net stock; chained (2007) \$; total assets (x 1,000,000)	Statistics Canada – Stock and Consumption of Fixed Non-residential Capital	STAT CAN & IMP_VINT & VINT

Staff projection gaps

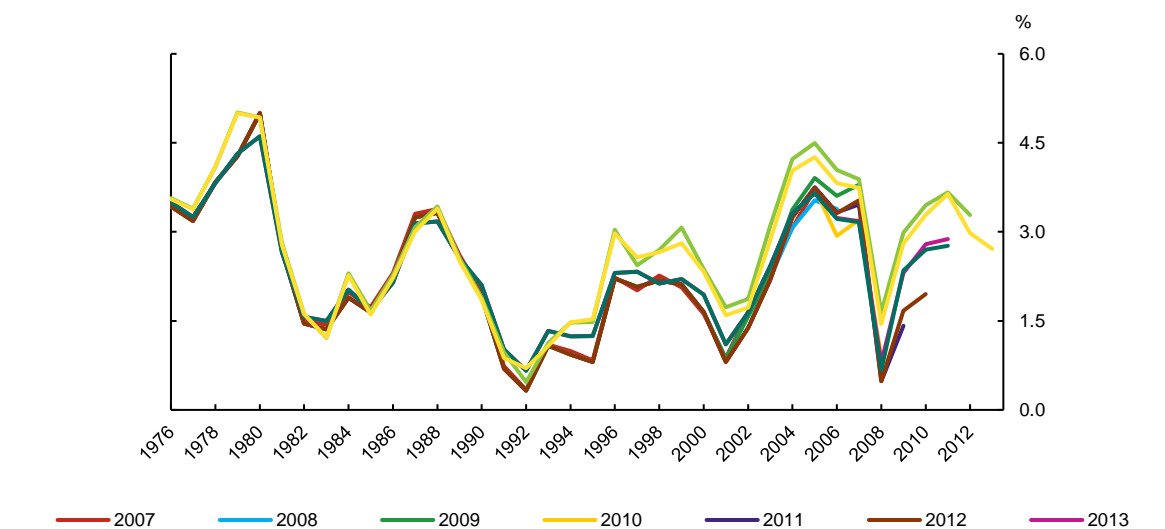
	Definition	Source	Real-time data source
Output gap	-	Internal	MON

Gross domestic product growth (GDP growth)

	Definition	Source	Real-time data source
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure-based; chained (2007) \$	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts (IEA)	MON

Chart A1: Large revisions to the growth rate of the capital stock

Year-over-year growth rate, annual data



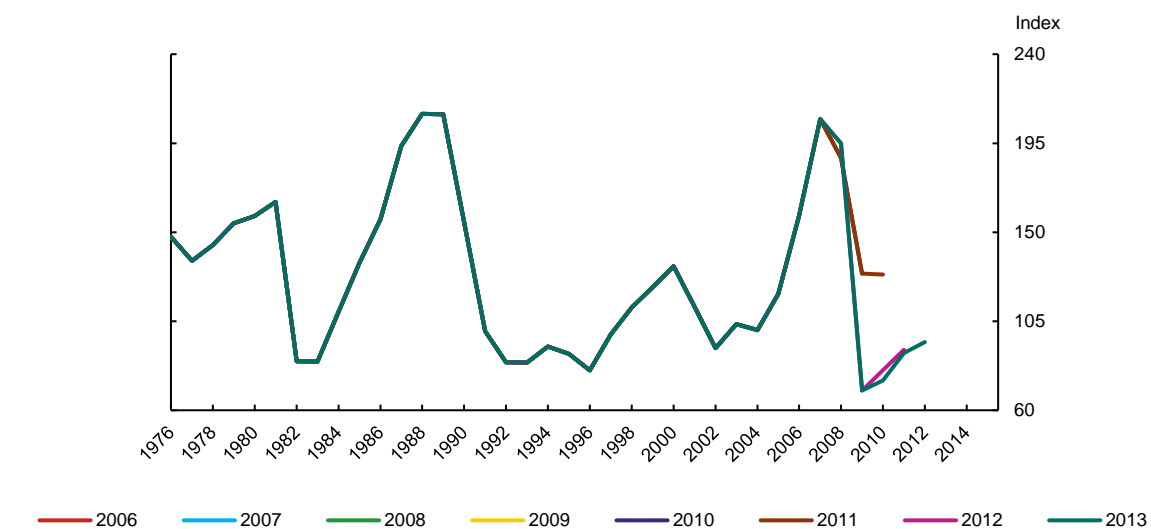
Note: The series in the chart represent a vintage for a specific year.

Sources: Statistics Canada and Bank of Canada databases

Last observation: 2013

Chart A2: Large revisions to the job offer rate

Annual data



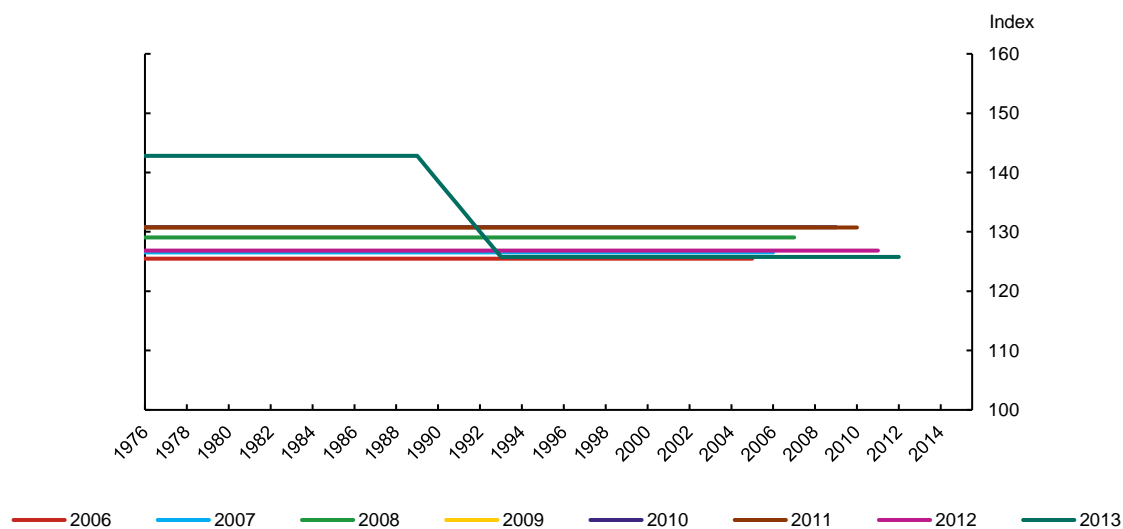
Note: The series in the chart represent a vintage for a specific year.

Sources: Statistics Canada, Conference Board of Canada, Canadian Federation of Independent Businesses, and Bank of Canada calculations

Last observation: 2016

Chart A3: Large revisions to the trend of the job offer rate

Annual data



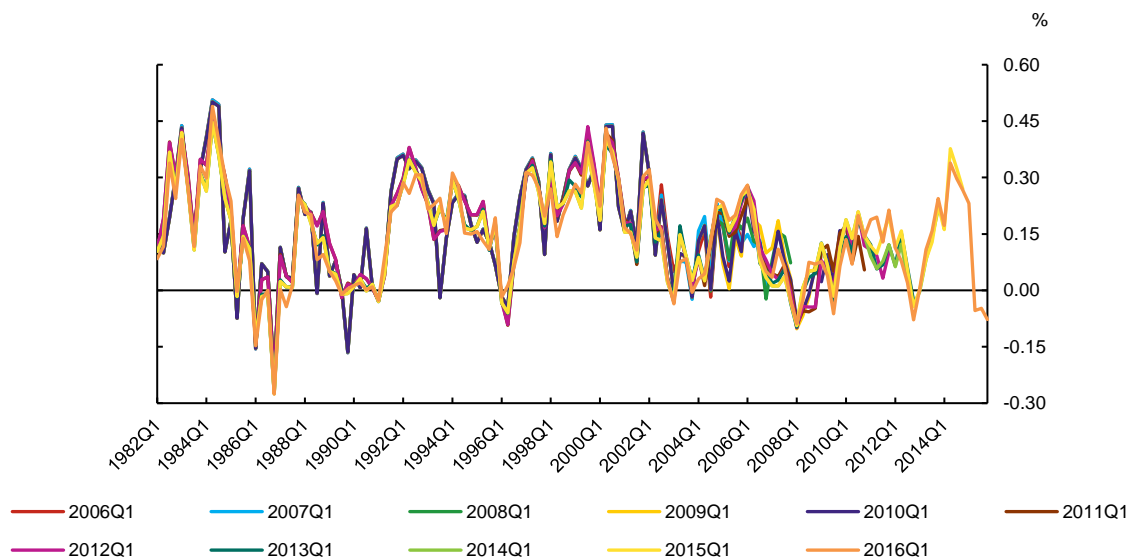
Note: The series in the chart represent a vintage for a specific year.

Source: Bank of Canada calculations

Last observation: 2016

Chart A4: Revisions to the growth rate of labour productivity (GDP/hours)

Year-over-year growth rate, quarterly data



Sources: Statistics Canada and Bank of Canada calculations

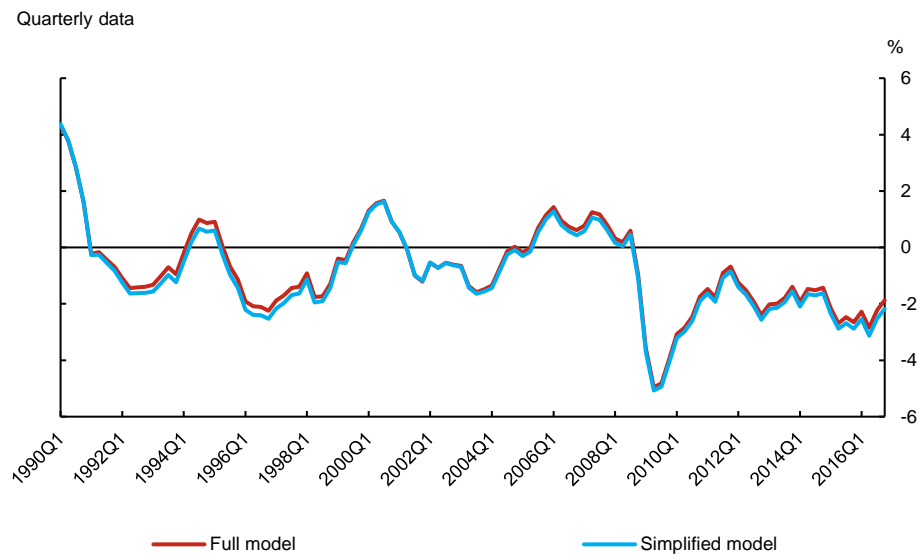
Last observation: 2016

Table A1: Output gap revisions (2011Q1 to 2014Q4)

	Mean	Absolute mean	Standard deviation	NSR ₁	NSR ₂	Correlation	Freq. of opposite signs
MSSF	0.47	0.47	0.19	0.27	0.72	0.96	0%
BMVF	0.53	0.53	0.30	0.46	0.92	0.91	38%
HP filter	0.38	0.43	0.34	0.76	1.12	0.73	56%
BP filter	0.48	0.48	0.27	0.95	1.91	0.43	56%
EMVF	0.71	0.71	0.30	0.72	1.83	0.70	75%
SIF	-0.26	0.58	0.61	0.78	0.82	0.63	13%
Staff projection gaps	-0.09	0.38	0.45	1.06	1.05	0.30	0%

Appendix B: Simplified and full integrated framework

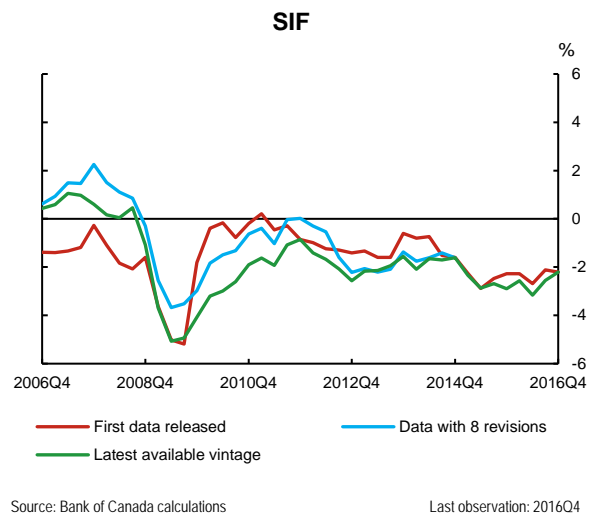
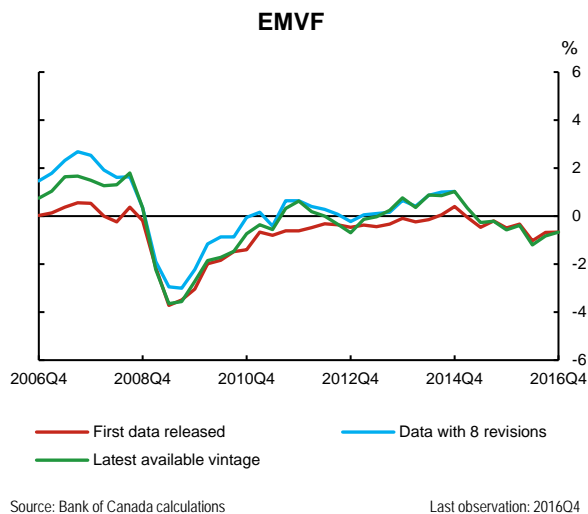
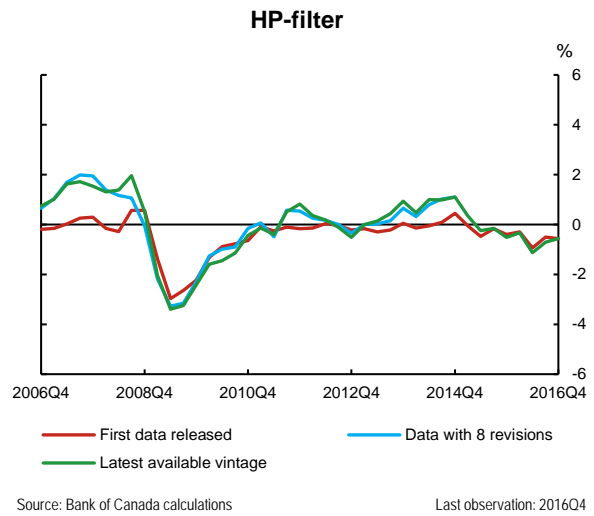
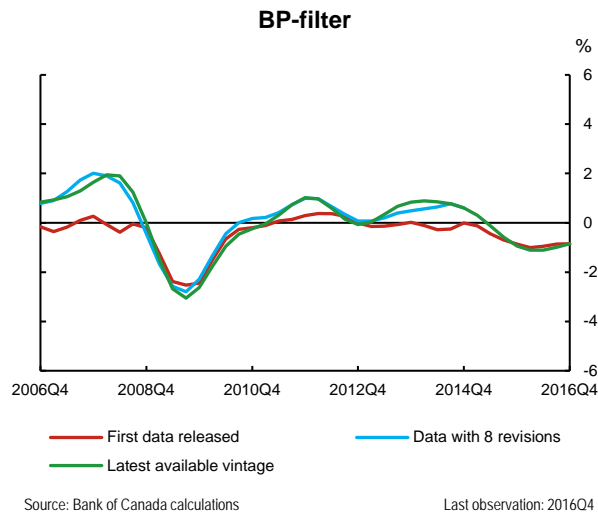
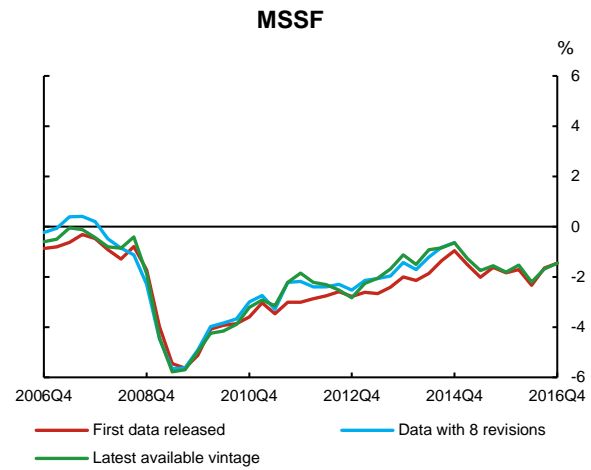
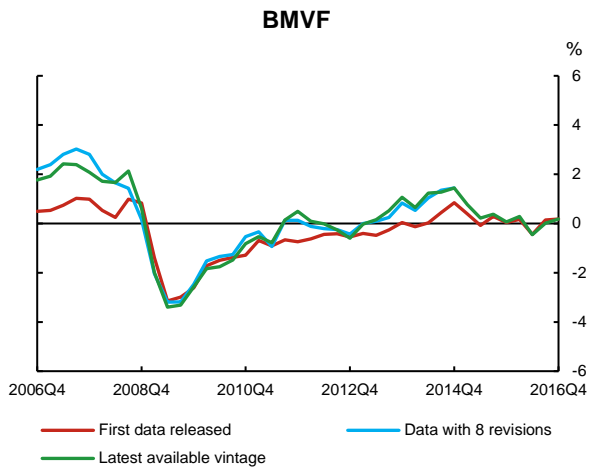
Chart B1: The simplified and full versions of the IF give very similar output gaps



Source: Bank of Canada calculations

Last observation: 2016Q4

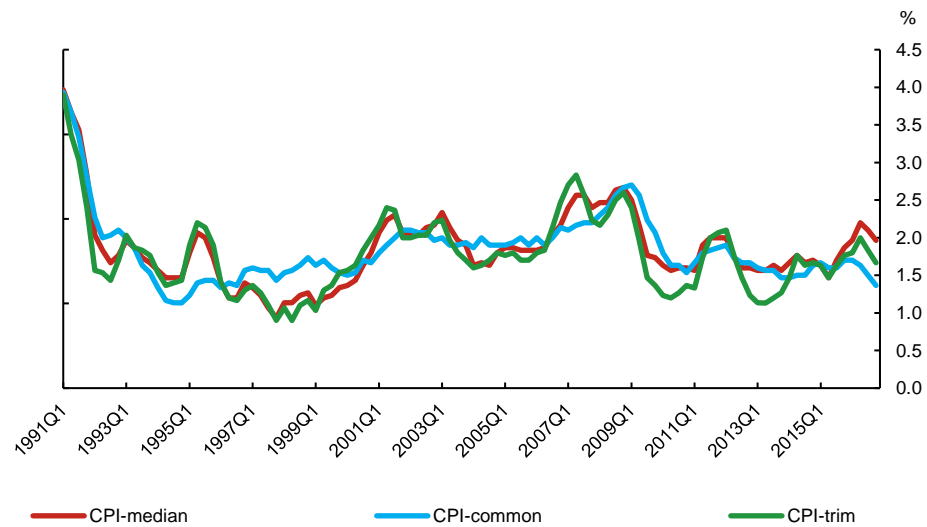
Appendix C: Output gap estimates



Appendix D: New measures of core inflation

Chart D1: New measures of core inflation, year-over-year

Quarterly data



Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2016Q4

Appendix E: Clark and McCracken test for comparing forecasting performance

This appendix details the Clark and McCracken (2009) test for nested models estimated with real-time data. We use this test to compare the forecasting performance of equations (2) and (3) with that of a simple equation made of only lags of inflation—equation (1).

We use two forecast models, $y_{s+\tau}(t) = \mathbf{x}'_{1,s}(t)\boldsymbol{\beta}_1(t) + u_{1,s+\tau}(t)$ (model 1) and $y_{s+\tau}(t) = \mathbf{x}'_{s,t}(t)\boldsymbol{\beta}_2(t) + u_{2,s+\tau}(t)$ (model 2), with $y_s(t)$ the value of the time t vintage of the observation s realization of y , where $t \geq s$. The observables are subject to revision over a finite number of periods, r . When the revision process is completed, we drop the notation indexing the vintage and simply let $y_s(t) = y_s$. k_1 is the number of parameters from equation (1) and k_2 is the number of parameters from equation (2). Since the two models are nested, $k_{22} = k_2 - k_1$ is the number of excess parameters.

The Clark and McCracken test statistic S for comparing predictive accuracy of nested models with real-time data is given by

$$S = \sqrt{(P - \tau - 1)} \frac{MSE_1 - MSE_2}{\sqrt{\Omega}},$$

where $P - \tau - 1$ is the number of forecasts, τ is the forecast horizon, MSE_i is the mean square error of model i , and Ω is the long-run variance of the scaled forecasting loss differential, which is estimated as follows:

$$\Omega = 2(1 - \pi^{-1} \ln(1 + \pi)) F(-\mathbf{J}\mathbf{B}_1\mathbf{J}' + \mathbf{B}_2) \mathbf{S}_{hh}(-\mathbf{J}\mathbf{B}_1\mathbf{J}' + \mathbf{B}_2) \mathbf{F}'.$$

In this equation:

$$\pi = \frac{P}{R},$$

R is the sample size at the initial forecast origin,

$$\mathbf{J}' = (\mathbf{I}_{k_1 \times k_1}, \mathbf{0}_{k_1 \times k_{22}}),$$

$$\hat{\boldsymbol{\beta}}_i = (T^{-1} \sum_{s=1}^{T-\max(\tau, r)} \mathbf{x}_{i,s} \mathbf{x}'_{i,s})^{-1},$$

$$T = R + P - \tau,$$

and

$$\hat{\mathbf{F}} = 2[P^{-1} \times \sum_{s=R}^T \hat{u}_{2,s+\tau}(t) \mathbf{x}'_{2,s}(t)].$$

Various scenarios make the matrix $\mathbf{F} = \mathbf{0}$, thus permitting the use of conventional tests. For example, this is the case when there are no revisions. The long-run covariance $\hat{\mathbf{S}}_{hh}$ of $h_{s+\tau}$ is estimated using the Newey–West (1987) HAC estimator with a bandwidth of 2τ .

$$\hat{h}_{s+\tau} = (y_{s+\tau} - \mathbf{x}'_{2,s} \hat{\boldsymbol{\beta}}_{2,T}) \mathbf{x}_{2,s}$$