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**Working Paper**

## Adverse selection with heterogeneously informed agents

Bank of Canada Staff Working Paper, No. 2018-7

**Provided in Cooperation with:**

Bank of Canada, Ottawa

*Suggested Citation:* Davoodalhosseini, S. Mohammad R. (2018) : Adverse selection with heterogeneously informed agents, Bank of Canada Staff Working Paper, No. 2018-7, Bank of Canada, Ottawa,  
<https://doi.org/10.34989/swp-2018-7>

This Version is available at:

<https://hdl.handle.net/10419/197860>

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Staff Working Paper/Document de travail du personnel 2018-7

# Adverse Selection with Heterogeneously Informed Agents



by S. Mohammad R. Davoodalhosseini

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Bank of Canada Staff Working Paper 2018-7

February 2018

# **Adverse Selection with Heterogeneously Informed Agents**

by

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## **Acknowledgements**

This paper was previously circulated under the titles “Public Information Acquisition, Endogenous Quality Choice and Welfare,” and “Directed Search toward Heterogeneously Informed Buyers.” I wish to gratefully acknowledge the help of Neil Wallace and Kala Krishna for their guidance and support for an earlier version, and Russel Wong and Jonathan Chiu for their comments on the current version. I would also like to thank Kalyan Chatterjee, Antonio Diez de los Rios, Manolis Galenianos, Veronica Guerrieri, David Jenkins, Ricardo Lagos, Shouyong Shi, Venky Venkateswaran, Randy Wright and the participants in several seminars and conferences at Penn State, EconCon (Princeton University), summer workshop on money, banking, payments, and finance (FRB Chicago), mid-west macro (Purdue University), North American summer meeting of the Econometric Society (University of Pennsylvania), and Bank of Canada for their helpful comments and suggestions. The views expressed in this paper are solely those of the author and no responsibility for them should be attributed to the Bank of Canada.

## Abstract

A model of over-the-counter markets is proposed. Some asset buyers are informed in that they can identify high quality assets. Heterogeneous sellers with private information choose what type of buyers they want to trade with. When the measure of informed buyers is low, there exists a unique and stable equilibrium, and interestingly, price, trading volume and welfare typically decrease with more informed buyers. When the measure of informed buyers is intermediate, multiple equilibria arise, and price, trading volume and welfare may decrease or increase with more informed buyers, depending on the equilibrium being played. A switch from one equilibrium to another can lead to large drops in liquidity, price, trading volume and welfare, like a financial crisis. The measure of informed buyers is then endogenized by allowing buyers to invest in a technology that enables them to identify high quality assets. In this case, the model features endogenous strategic complementarity in acquiring the information technology. Multiple equilibria still exist, with different measures of informed buyers, but a scheme of tax/subsidy on information acquisition sometimes leads to the unique equilibrium.

*Bank topics: Economic models; Financial markets; Financial system regulation and policies; Market structure and pricing; Financial stability*

*JEL codes: D40, D82, D83, G01, G10, G20*

## Résumé

La présente étude propose un modèle de marchés de gré à gré. Certains acheteurs ont accès à de l'information qui leur permet de repérer les actifs de haute qualité. Les vendeurs hétérogènes détenant de l'information privilégiée choisissent le type d'acheteurs avec lesquels ils veulent négocier. Lorsque la mesure des acheteurs bien informés est faible, il existe un équilibre unique et stable. De plus, il est intéressant de constater que les prix, le volume des opérations et le bien-être diminuent habituellement lorsque le nombre d'acheteurs bien informés augmente. Quand la mesure des acheteurs bien informés est moyenne, de nombreux équilibres se forment. Les prix, le volume des opérations et le bien-être peuvent alors diminuer ou augmenter s'il y a une hausse du nombre d'acheteurs bien informés, selon l'équilibre en question. Le passage d'un équilibre à l'autre peut entraîner une chute importante de la liquidité, des prix, du volume des opérations et du bien-être, comme en situation de crise financière. La mesure des acheteurs bien informés est alors

endogénéisée en donnant la possibilité aux acheteurs d'investir dans une technologie qui leur permet de repérer les actifs de haute qualité. Dans ce cas, le modèle présente une complémentarité stratégique endogène dans l'acquisition de la technologie de l'information. Il existe encore beaucoup d'autres équilibres, basés sur différentes mesures des acheteurs bien informés; toutefois, l'instauration d'une taxe ou d'une subvention sur l'acquisition d'information mène parfois à un équilibre unique.

*Sujets : Modèles économiques; Marchés financiers; Réglementation et politiques relatives au système financier; Structure de marché et fixation des prix; Stabilité financière*

*Codes JEL : D40, D82, D83, G01, G10, G20*

## Non-Technical Summary

Many assets, such as corporate bonds and derivatives, are traded over-the-counter (OTC). Sellers of assets are heterogeneous and may have private information about the quality of their assets as well as their own valuation of the assets. Buyers of assets may be heterogeneous too. Some of these buyers, such as venture capitalists, hedge funds, trusts, and investment banks, are more sophisticated so they may know more than other buyers about the quality of the assets for sale. I study a model of OTC markets, in which some buyers of assets are informed in that they can identify high-quality assets. Markets are segmented; that is, informed buyers trade in one market and uninformed buyers trade in another one. Heterogeneous sellers with private information choose what type of buyers they want to trade with.

Information has adverse effects in this environment. As the relative size of the informed segment of the market rises, the asset becomes less liquid and is traded with a lower price, and the resulting trading volume and welfare may be lower. Moreover, multiplicity of equilibria arises naturally when the size of the informed segment of the market is sufficiently high. This is due to the strategic complementarity in the behavior of sellers with a high-quality asset. The higher the measure of sellers with a high-quality asset who try to meet uninformed buyers, the higher the price that uninformed buyers are willing to pay, and consequently, the higher the incentive of other sellers to meet uninformed buyers. As a result of this strategic complementarity, changes in beliefs regarding the quality of assets in different segments of the market can lead to sudden and large drops in liquidity, price and trading volume, similar to a financial crisis.

To study the incentive of buyers to acquire information, I allow buyers to invest in an information technology that enables them to identify high-quality assets. In practice, financial institutions can hire economists to determine the underlying value of assets. Multiple equilibria still exist, with different measures of informed buyers, but a scheme of tax/subsidy on acquisition of the information technology sometimes leads to the unique and more efficient equilibrium and saves the economy from financial crisis. However, finding the optimal intervention requires the policy maker to know the details of the model, e.g., the type of the equilibrium being played and the cost distribution of the information technology.

# 1 Introduction

Many assets, such as corporate bonds and derivatives, are traded over-the-counter (OTC). In OTC markets, agents must search to find a trading partner, and terms of trade are determined through bilateral negotiations. Many of these markets suffer from adverse selection in that sellers have superior information about the value of the assets relative to buyers.<sup>1</sup> Some buyers—venture capitalists, hedge funds, trusts, and investment banks—however, are sophisticated, so they may identify the quality of assets better than other buyers. A natural, important question in these markets is whether more information is preferred or not. More specifically, what are the implications on markets if there are more sophisticated buyers?

To address these questions, I study a model of OTC markets where traders have heterogeneous information about the asset. Sellers have private information about the quality of their asset and their outside option. Some buyers, called *informed* buyers, can identify high-quality assets while other buyers, called *uninformed* buyers, cannot.<sup>2</sup> Informed and uninformed buyers trade in different markets. That is, markets are segmented. Sellers choose which type of buyers they want to meet. Meetings are bilateral. When a seller gets to match with a buyer, the terms of trade are determined through a bargaining protocol.<sup>3</sup> By studying the case where only some buyers are informed, I can address more detailed questions: What quality of assets are traded with each type of buyer, for what price and with what probability? What are the implications of more informed buyers for price, liquidity, trading volume and welfare? Finally, what are the implications of a less expensive information technology for price, liquidity and other indexes as the cost of acquiring information has decreased in recent decades (because of the development of information technology, for example)? There are two main insights from the paper summarized below.

The first insight is that more information has typically adverse effects on price, trading volume and welfare. This result has arisen in various environments (e.g., [Hirshleifer \(1971\)](#)),

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<sup>1</sup>There is evidence that markets for non-agency mortgage-backed securities were subject to adverse selection. See [Guerrieri and Shimer \(2014\)](#) and [Demiroglu and James \(2012\)](#) for more detailed discussion.

<sup>2</sup>The application on which I focus is OTC markets, but this type of buyers' heterogeneity can be seen in many markets that are subject to search-and-matching frictions with adverse selection, and there is nothing in the environment that limits the results only to OTC markets. For example, some universities can better identify good applicants, and banks may have heterogeneous ability in screening the projects presented to them for funding.

<sup>3</sup>Two extreme versions of this model are standard in the literature. If all buyers are informed, then the model would reduce to a simple version of standard search-and-matching models e.g., [Mortensen and Pissarides \(1994\)](#). If all buyers are uninformed, then the model would reduce to a version of [Akerlof \(1970\)](#) with bilateral meetings.

but the mechanism in my paper is new and different: The incentive of high-quality sellers to meet uninformed buyers decreases with more informed buyers. Thus, the price that uninformed buyers are willing to pay falls, encouraging high-quality sellers further to meet informed buyers. Consequently, there would be too much congestion in the market where informed buyers trade. Many high-quality sellers would be left unmatched, and average price, trading volume and welfare all decrease.

The second insight is that multiplicity of equilibria arises naturally in this environment on two levels. With a fixed measure of informed buyers, there are multiple equilibria with different prices when the measure of informed buyers is intermediate. Also, when the buyers endogenously choose to become informed, there are multiple equilibria with different measures of informed buyers. Fix the measure of informed buyers and suppose the economy has been at an equilibrium in which relatively many high-quality sellers meet uninformed buyers and the price that uninformed buyers are willing to pay is high. Now suppose the economy switches to the second equilibrium in which high-quality sellers meet mostly with informed buyers and, consequently, the price that uninformed buyers are willing to pay is low. In the latter equilibrium, since there are too many high-quality sellers who want to meet informed buyers, many of these sellers cannot be matched. Trading volume and welfare in the latter equilibrium are lower than those in the former one. The change from the former to the latter equilibrium shares many features of a financial crisis in that the price, liquidity, trading volume and welfare fall considerably. The mechanism through which multiplicity arises here is new and comes from the strategic complementarity between the choice of high-quality sellers in meeting different types of buyers. Now I explain my results in greater detail.

Two types of equilibria exist in this model: separating, in which uninformed buyers believe that only sellers with a low-quality asset visit them, and pooling, in which uninformed buyers believe some sellers with a high-quality asset will visit them as well. I focus on the more interesting case of pooling equilibrium for most of the paper.<sup>4</sup> Suppose informed buyers are relatively scarce. High-quality sellers generally prefer to meet informed buyers because uninformed buyers cannot differentiate between them and the low-quality sellers. As a result, there is too much demand from high-quality sellers to meet informed buyers and too little demand to meet uninformed buyers. Hence, there is a cutoff point for the outside option of high-quality sellers. Low-quality sellers, as well as high-quality sellers with outside options lower than the cutoff point, meet uninformed buyers, as these sellers prefer to sell faster even

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<sup>4</sup>The analysis in the separating equilibrium is simple. An increase in the measure of informed buyers unambiguously increases the incentive of high-quality sellers to meet with informed buyers. As a result, separating equilibrium exists only when the measure of informed buyers is sufficiently high.

with a lower price; and other high-quality sellers with outside options higher than the cutoff point prefer to wait longer, meet informed buyers and sell with a higher price.

There are two economic forces at play in the pooling equilibrium. First, an increase in the measure of informed buyers ( $\lambda$ ) implies, unsurprisingly, that high-quality sellers have more opportunities to trade with informed buyers and fewer opportunities with uninformed buyers, so the high-quality sellers around the cutoff point are encouraged to meet informed buyers rather than uninformed buyers. Second, there is a form of strategic complementarity in that an individual high-quality seller is more willing to meet an uninformed buyer when the aggregate measure of high-quality sellers who meet uninformed buyers increases. This is because the price that uninformed buyers are willing to pay is increasing in the average quality of assets that uninformed buyers face, which in turn is increasing in the measure of high-quality sellers who choose to meet uninformed buyers. If more high-quality sellers meet uninformed buyers, then other high-quality sellers would gain more by meeting uninformed buyers, and similarly, if fewer high-quality sellers meet uninformed buyers, then other high-quality sellers would gain less by meeting uninformed buyers. Thus, an increase in  $\lambda$  may imply that uninformed buyers are willing to pay a higher or a lower price depending on the beliefs that agents hold in equilibrium. This strategic complementarity between the choice of high-quality sellers in meeting different types of buyers, if stronger than the first force, can generate multiple equilibria.

When the measure of informed buyers is low, there is no separating equilibrium, but there exists a pooling equilibrium. The equilibrium is unique under a regularity condition. In this equilibrium, the cutoff point and subsequently the price that uninformed buyers are willing to pay decrease with the measure of informed buyers. Interestingly, welfare and trading volume may also decrease as the measure of informed buyers increases. Welfare may decrease because the price that uninformed buyers are willing to pay is lower, so too many high-quality sellers try to meet informed buyers. This implies that some sellers with intermediate outside options who used to meet with uninformed buyers with a high probability now try to meet with informed buyers and are matched with a lower probability. This is beneficial to these sellers individually because they can sell with a higher price. However, this imposes a negative externality on other high-quality sellers who could have benefited more (by being matched with a higher probability) from the increase in the number of informed buyers if those high-quality sellers had not changed their search strategy. Trading volume may decrease too, because those sellers who have switched are now matched with a lower probability.

When the measure of informed buyers is intermediate, multiplicity of equilibria arises

for a given measure of informed buyers as a result of the strategic complementarity. As more high-quality sellers meet with uninformed buyers, other high-quality sellers have more incentives to do the same. In this region, welfare and trading volume may decrease or increase with more informed buyers depending on the equilibrium being played.<sup>5</sup>

To study the incentive of buyers to become informed, I then endogenize the measure of informed buyers by allowing buyers to invest in an information technology that enables them to identify high-quality assets. For example, financial institutions can hire economists to help them determine better the underlying value of assets. Interestingly, even if the cost of acquiring the information technology is the same for all buyers, it may be the case that ex-post some buyers will be informed and some will be uninformed. More interestingly, this version of the model features endogenous strategic complementarity in acquiring the information technology under certain parameters. That is, the higher the measure of informed buyers, the higher the incentive of buyers to become informed. This endogenous complementarity leads to another dimension of multiplicity, with different measures of informed buyers, but a scheme of tax/subsidy on acquiring the information technology sometimes leads to the unique equilibrium. However, the choice of the scheme is sensitive to the cost distribution of the information technology.

This paper is related to several strands of the literature. It is related to recent papers by [Fishman and Parker \(2015\)](#) and [Bolton et al. \(2016\)](#), who study a similar type of heterogeneity in financial markets. The assumption common to my paper and theirs is that the type of buyers is known to sellers before trading. However, the reason that more information may be detrimental to welfare differs between my paper and theirs. The welfare loss in my paper comes from the fact that there is too much congestion in the market for informed buyers. The reason for congestion is not only that sellers receive too much surplus from the bilateral match, but also because the price that uninformed buyers are willing to pay decreases with a higher measure of informed buyers. Hence, the incentive of high-quality sellers to meet uninformed buyers decreases too. Therefore, for my mechanism, the interaction of price in the market for uninformed buyers and search frictions is essential.

Furthermore, the results in both [Fishman and Parker \(2015\)](#) and [Bolton et al. \(2016\)](#) hinge on the assumption that informed (or sophisticated) buyers are treated differentially in that sellers can first trade with informed buyers, and if they cannot trade with them, then they can trade with uninformed buyers, and uninformed buyers do not see whether the

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<sup>5</sup>When the measure of informed buyers is high, the only equilibrium is separating, and the economy would resemble an economy with complete information. I focus in this paper on the cases in which the measure of informed buyers is not too high.

seller has already tried to trade with an informed buyer or not.<sup>6</sup> In contrast, in my model, informed and uninformed buyers are treated equally. All sellers have only one chance of meeting, and they must choose to meet with only one type of buyer. Since informed buyers do not have any exogenous advantage in terms of matching in my paper, the search-and-matching framework is crucial for pooling equilibrium to exist. To have pooling equilibrium, some high-quality sellers should want to meet informed buyers and some should want to meet uninformed buyers. Here, sellers face a tradeoff between price and probability of matching, and their heterogeneous outside options lead them to self-select into different markets.

The paper is organized as follows. In Section 2, I describe the environment. In Section 3, I present my main results. I characterize the equilibria, and discuss the regions of  $\lambda$  in which there is uniqueness or multiplicity. I also conduct comparative statics along each equilibrium and also compare different equilibria for a given  $\lambda$ . In Section 4, I endogenize  $\lambda$ . In Section 5, I characterize the problem of the social planner with complete information. In Section 6, I discuss some of the assumptions and also the constrained planner's problem, and elaborate on the relationship between my paper and [Fishman and Parker \(2015\)](#), [Bolton et al. \(2016\)](#), and [Kurlat \(2016\)](#).<sup>7</sup> Section 7 concludes. Some proofs come in the main body and more technical ones are relegated to the Appendix.

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<sup>6</sup>Moreover, in [Bolton et al. \(2016\)](#), the trade with informed buyers is conducted through bargaining, while trade with uninformed buyers is conducted through exchange (competitive pricing).

<sup>7</sup>More broadly, my paper is related to the extended literature following [Akerlof \(1970\)](#) on private information and its role in the markets. See for example [Hirshleifer \(1971\)](#), [Rothschild and Stiglitz \(1976\)](#) and [Håkansson et al. \(1982\)](#). Akerlof's lemons model has been used in macroeconomic settings. See for example [Eisfeldt \(2004\)](#), [Daley and Green \(2012\)](#), [Tirole \(2012\)](#), [Kurlat \(2013\)](#), [Guerrieri and Shimer \(2014\)](#), [Carmargo and Lester \(2014\)](#), and [Chang \(2017\)](#). One strand of literature in monetary economics that studies counterfeiting is related to my paper if one interprets the low-quality assets as counterfeits. See [Nosal and Wallace \(2007\)](#) and [Li et al. \(2012\)](#) among others. For other papers in monetary economics with a similar buyers' heterogeneity, see [Williamson and Wright \(1994\)](#) and [Li \(1998\)](#). My paper is also related to directed search literature with information frictions, e.g., [Guerrieri, Shimer, and Wright \(2010\)](#), [Delacroix and Shi \(2013\)](#) and [Davoodalhosseini \(2017\)](#). Search is directed in my paper, but terms of trade are not posted; rather, they are determined by bilateral negotiations. For the mechanism that I use in the bilateral trading, I follow the literature on bargaining under asymmetric information using mechanism design: [Myerson \(1979\)](#), [Myerson and Satterthwaite \(1983\)](#), and in particular [Samuelson \(1984\)](#). For a recent contribution to this literature, see [Shimer and Werning \(2015\)](#).

## 2 Model

There is a measure 1 of sellers and a measure  $n > 0$  of buyers. Each seller possesses a single, indivisible good that is heterogeneous in the buyer's valuation,  $u_T$ , where  $T \in \{H, L\}$  and  $u_H \equiv h > u_L \equiv \ell > 0$ , and in the seller's outside valuation,  $v$ . The measure of sellers with  $T = H$  (high type) is  $k \in (0, 1)$  and with  $t = L$  (low type) is  $1 - k$ . Given  $T$ ,  $v$  is drawn from a conditional cumulative density function (CDF). If  $T = H$ , then  $F_H(v)$  denotes the continuously differentiable CDF of  $v$  over the interval  $[0, 1]$ . If  $T = L$ , then it is assumed for simplicity that all sellers of type  $L$  have outside valuation of 0. Therefore, the seller's/good's characteristics are summarized by a pair  $(T, v)$ , and both  $T$  and  $v$  are private information to the seller. Of course, if  $T = L$ , the buyer knows that the seller's outside valuation is 0, but if  $T = H$ , then there is still uncertainty about the seller's outside valuation.<sup>8</sup> Notation-wise, whenever I want to refer to  $T$ , I call it the *type* of the good or asset or type of the seller. For example, "H seller" refers to the seller with a high type good. Whenever I want to refer to  $v$ , I call it the *outside option* or *outside valuation* of the seller.

After learning their  $(T, v)$ , sellers direct their search to target a particular type of buyers to sell their goods. A fraction  $\lambda$  of buyers, called informed or *I* buyers, have perfect information to distinguish the type of the good, and the remaining fraction  $1 - \lambda$  of buyers, called uninformed or *U* buyers, cannot distinguish the type of the good. Buyer types are public information. Meetings between buyers and sellers are subject to search and matching frictions. After a buyer and a seller match, the terms of trade are determined by the trading protocol discussed below.<sup>9</sup>

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<sup>8</sup>The heterogeneity of sellers in their outside valuation is not crucial for the results. Even if all high-quality sellers have the same outside option, the results will go through. In that case, however, the high-quality sellers who meet uninformed buyers are similar to those who meet informed buyers. With sellers' heterogeneous outside valuations, the results clearly identify that sellers with low outside options meet uninformed buyers and those with high outside options meet informed buyers.

<sup>9</sup>Here, the search is directed to the type of buyers but the terms of trade are determined endogenously through bargaining, not competitive posting. For models of competitive search with buyers' heterogeneity in productivity, see Shi (2001), Eeckhout and Kircher (2010) and recently Davoodalhosseini (2015). In Lester (2011) and also Mei and Jiang (2012), buyers are heterogeneous in that some of them see the posted prices and pick a seller strategically while other buyers do not see the prices and search randomly. In my paper, however, the information of buyers is about fundamentals (quality of asset) not prices. Lester (2011) finds that having more informed customers may decrease or increase prices.

## 2.1 Trading Protocol

When a buyer and seller meet, they bargain over terms of trade. Choosing a specific bargaining protocol requires ad-hoc assumptions and may lead to multiplicity of equilibria in the bargaining game. To avoid these issues, I adopt a mechanism design approach. According to [Myerson \(1979\)](#), we can focus on the direct revelation game. In the direct revelation game, the seller reports his private information and then the buyer and seller are allocated a pair  $(q(\cdot), t(\cdot))$ , the probability that the seller gives the good to the buyer and the amount of transfers that the buyer gives to the seller, respectively. If the buyer is informed, the seller has private information only about  $v$ , so the allocation can be summarized as  $(q_{IH}(v), t_{IH}(v))$  for a high type seller, and  $(q_{IL}, t_{IL})$  for a low type seller. The informed buyer does not have any uncertainty about the low type sellers, so their allocation and transfer are not a function of  $v$ . If the buyer is uninformed, then the allocation can be summarized as  $(q_U(v), t_U(v))$ . The seller does not need to report his type  $T$ , i.e., it is not  $(q_{UT}(v), t_{UT}(v))$ , because given the assumption on the seller's outside options, knowing  $v$  is sufficient for the uninformed buyers to learn about all the private information of the seller. If  $v = 0$ , then the seller is of type  $L$  with probability 1, and if  $v > 0$ , then the seller is of type  $H$  with probability 1. The allocation is called implementable if it satisfies incentive compatibility of sellers and participation constraint of buyers and sellers in the bilateral meeting.

[Samuelson \(1984\)](#) studies a similar bargaining problem and considers two different cases, one in which the allocation maximizes the expected surplus of the buyer among all implementable allocations and one that maximizes the expected surplus of the seller among all implementable allocations. In this paper in the benchmark model, I assume the latter. Note that this allocation also maximizes the joint expected surplus of the match. If one instead assumes that the mechanism maximizes the buyer's payoff with a high probability, then the search patterns—what type of sellers with what outside options search for what type of buyers—might be different.<sup>10</sup>

Equilibria are constructed by working backward. The trading protocol pins down the seller's payoff given the seller's strategies of directed search. Expecting these payoffs, sellers choose the optimal strategies of directed search. Consider first the game in a bilateral match between an informed buyer and seller. I assume for simplicity that there are gains from trade for all possible  $(T, v)$  pairs.

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<sup>10</sup>In Section 4 when I endogenize the measure of informed buyers, I assume that the allocation maximizes the expected surplus of the buyer with a low probability and the expected surplus of the seller with a complementary probability.

**Assumption 1.**  $h > 1$ .<sup>11</sup>

As a result, when meeting an informed buyer, a seller of high type asks for the price of  $h$ , and similarly a seller of low type asks for the price of  $\ell$ , and all sellers give the good with probability 1 to the buyer, that is,

$$q_{IH}(v) = 1, t_{IH}(v) = h \quad \text{for all } v \in [0, 1], \text{ and } q_{IL} = 1, t_{IL} = \ell. \quad (1)$$

Now consider the interesting case between an uninformed buyer and a seller. Let  $b(v)$  denote the expected value of the good for an uninformed buyer given  $v$ , and  $\tilde{F}(v)$  denote the posterior CDF of sellers who have chosen to meet with an uninformed buyer. According to the mechanism design problem described above, the optimal allocation  $(q_U(v), t_U(v))$  must solve the following problem:

**Problem 1** (Optimal allocation in the match with an uninformed buyer and a seller).

$$\begin{aligned} & \max_{\{t_U(v), q_U(v)\}} \int [t_U(v) - q_U(v)v] d\tilde{F}(v) \\ & \text{subject to } t_U(v) - q_U(v)v \geq t_U(\hat{v}) - q_U(\hat{v})v \text{ for all } v, \hat{v} \in [0, 1] \quad (\text{IC}), \\ & \quad t_U(v) - q_U(v)v \geq 0 \text{ for all } v \in [0, 1] \quad (\text{IRS}), \\ & \quad \text{and } \int [q_U(v)b(v) - t_U(v)] d\tilde{F}(v) \geq 0 \quad (\text{IRB}). \end{aligned}$$

This mechanism maximizes the expected payoff of the seller subject to the following constraints: IC is the incentive compatibility constraint ensuring that the seller has incentive to report truthfully, and IRS and IRB are the individual rationality of the seller and the buyer, respectively.<sup>12</sup>

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<sup>11</sup>Technically speaking, it ensures that if the last constraint in Problem 1 is not binding, then the solution is  $q_U(v) = 1$  for all  $v \leq 1$ . If  $h < 1$ , all results still go through but some high type sellers may not enter either market.

<sup>12</sup>To split the surplus between a buyer and seller, I use a mechanism design approach instead of choosing a specific bargaining game. This is to avoid multiplicity of equilibria arising from details of the bargaining protocol, which usually occurs in models of bargaining with private information. Using equilibrium refinement/selection with multiplicity would be then inevitable, but these refinements or selections may be hard to justify. In an earlier version of the paper, I show that in a bargaining game where the seller makes a take-it-or-leave-it offer to the buyer, the multiplicity of equilibria arises because of the signaling effect. However, as long as the payoff that the seller receives is increasing in the fraction of sellers who are of high type, regardless of the equilibrium being selected, the results in the paper go through. The outcome of such bargaining game with a standard equilibrium selection yields the same outcome as the mechanism design problem adopted here.

Expecting these payoffs, sellers decide what location they want to visit. There are two locations for trade, one composed of only informed buyers, denoted by I submarket, and one composed of only uninformed buyers, denoted by U submarket. That is, these markets are segmented.<sup>13</sup> Buyers of one type cannot go to the other submarket. At each submarket, buyers and sellers are bilaterally matched according to a constant-return-to-scale matching technology. If the ratio of buyers to sellers at submarket  $M \in \{I, U\}$  is  $\theta_M \in (0, \infty)$ , then the probability that a buyer finds a match is  $\mathcal{Q}(\theta_m)$  and the probability that a seller finds a match is  $\mathcal{M}(\theta_M)$ . Meeting in pairs implies that  $\mathcal{M}(\theta) = \theta \mathcal{Q}(\theta)$ . As standard in the literature, I assume that  $\mathcal{M}(\cdot)$  is continuously differentiable almost everywhere, non-decreasing and concave;  $\mathcal{Q}(\cdot)$  is non-increasing; and  $\mathcal{M}(\theta) \leq \min\{1, \theta\}$ . On some occasions in the paper, I assume that  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  where  $\mu \leq 1$  in order to obtain sharp predictions.

## 2.2 Equilibrium Definition

Let  $s_{HM}(v)$  denote the probability that  $(H, v)$  sellers choose submarket  $M \in \{I, U\}$ . These sellers do not enter the market with probability  $1 - s_{HI}(v) - s_{HU}(v)$ . Similarly, let  $s_{LM}$  denote the probability that low type sellers choose submarket  $M \in \{I, U\}$ . It is assumed that sellers of the same type and outside option follow the same strategy.

**Definition 1** (Equilibrium Definition). *An equilibrium consists of submarket choices  $s_{HM}(v) : [0, 1] \rightarrow [0, 1]$  and  $s_{LM} \in [0, 1]$  for  $M \in \{I, U\}$ ; allocations  $q_{IL} \in [0, 1]$ ,  $q_{IH}(v)$  and  $q_U(v)$  both defined over  $[0, 1] \rightarrow [0, 1]$ ; transfer functions  $t_{IL} \in \mathbb{R}_+$ ,  $t_{IH}(v)$  and  $t_U(v)$  both defined over  $[0, 1] \rightarrow [0, h]$ ; such that the following conditions hold:*

*(i) **Sellers' submarket choice and participation constraint:** Given the allocation and the transfer functions, sellers choose the probability of going to the I submarket to maximize their expected payoff:*

$$s_{HI}(v) \in \arg \max_{r \in [0, 1]} \left( \pi_{IH}(v) - \pi_U(0) \right) r \text{ for all } v \in [0, 1],$$

$$s_{LI} \in \arg \max_{r \in [0, 1]} \left( \pi_{IL} - \pi_U(0) \right) r.$$

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<sup>13</sup>This assumption does not mean that these two markets are physically separated. Rather, it states that buyers' types are publicly known to sellers before sellers choose a buyer, so sellers can direct their search to a certain type of buyers.

where  $\pi_{IH}(v) = \mathcal{M}(\theta_I)(t_{IH}(v) - q_{IH}(v)v)$ ,  $\pi_U(v) = \mathcal{M}(\theta_U)(t_U(v) - q_U(v)v)$  and  $\pi_{IL} = \mathcal{M}(\theta_I)t_{IL}$ . Also, if  $\max\{\pi_{IH}(v), \pi_U(v)\} \leq 0$ , then  $s_{HU}(v) = s_{HI}(v) = 0$ , and if  $\max\{\pi_{IL}, \pi_U(0)\} \leq 0$ , then  $s_{LU} = s_{LI} = 0$ .

**(ii) Optimal mechanism:** Given  $\tilde{F}(\cdot)$  and  $b(\cdot)$ , induced by the submarket choices,  $(q_U(v), t_U(v))$  is a solution to the mechanism design Problem 1, and  $q_{IL}, t_{IL}, q_{IH}(v)$ , and  $t_{IH}(v)$  satisfy (1).

**(iii) Consistency conditions:** Variables  $\theta_I, \theta_U, \tilde{F}(\cdot), b(v)$  are given below:

$$\tilde{F}(v) = \frac{k \int_0^v s_{HU}(\nu) dF_H(\nu) + (1-k)s_{LU}}{k \int_0^1 s_{HU}(\nu) dF_H(\nu) + (1-k)s_{LU}} \text{ for } v \geq 0,$$

$$b(v) = h \text{ if } v > 0 \text{ and } b(0) = \ell,$$

$$\theta_I = \frac{\lambda n}{k \int_0^1 s_{HI}(\nu) dF_H(\nu) + (1-k)s_{LI}}, \quad (2)$$

$$\theta_U = \frac{(1-\lambda)n}{k \int_0^1 s_{HU}(\nu) dF_H(\nu) + (1-k)s_{LU}}. \quad (3)$$

In part (i), sellers with a given type and outside option choose a submarket that maximizes their expected payoff and are allowed to use a mixed strategy in choosing a submarket. In part (ii), I impose a restriction on the equilibrium, stating that if a seller receives a net payoff of zero from entering a submarket, then the seller does not enter that submarket.<sup>14</sup> Part (iii) ensures that the allocation and transfers are derived by solving the mechanism design problem given the seller's choice of submarket. Since the solution to the mechanism design problem for the meetings that involve an informed buyer is easily characterized, I incorporated that in the definition of equilibrium, where the net payoff from the match for H and L types in the I submarket were substituted by  $h - v$  and  $\ell$ , respectively. Part (iv) ensures that the market tightness at both submarkets and also the posterior distribution of outside options in the U submarket and the buyer's payoff from the match for the uninformed buyers are consistent with the optimal choices of sellers and the trading protocol.

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<sup>14</sup>This restriction is not crucial for our results. Rather, it is made to eliminate uninteresting inefficiencies that would arise in the equilibrium otherwise. This is because there are search frictions in this environment, so if one seller participates in the market and finds a match, then some other sellers may be left unmatched. This assumption ensures that if a seller finds a match, this seller expects a strictly positive payoff with probability 1. If one assumes that searching for a buyer costs sellers  $C > 0$ , and then let  $C$  go to 0, then this restriction will endogenously arise by a continuity argument, but here I simply impose it on the equilibrium definition.

### 3 Characterization

Throughout the paper, I assume that  $\lambda \leq k$ . If  $\lambda > k$ , then L sellers want to meet with I buyers only because there are more opportunities for trade in the I submarket. Analyzing this case is straightforward because the problem of private information would be absent in this case. There can be two types of equilibria: separating equilibrium, in which H sellers do not go to the U submarket, and pooling equilibrium, in which some H sellers go to the U submarket. I analyze the separating equilibrium first and then the pooling equilibrium, but I focus on the pooling equilibrium for most of the paper.

#### 3.1 Separating Equilibrium

In the separating equilibrium, buyers believe that sellers visiting them are of L type. Given this belief, the problem of private information is virtually non-existent in the separating equilibrium. The solution to the mechanism design problem is that the sellers in the I and U submarkets simply ask for prices  $h$  and  $\ell$ , and they give the good to the buyer with probability 1. The ratio of buyers to sellers in the I and U submarkets are given by  $\theta_I = \frac{\lambda n}{k}$  and  $\theta_U = \frac{(1-\lambda)n}{1-k}$ . Separating equilibrium exists if the H sellers with a zero outside option prefer the I submarket to the U submarket, i.e.,  $\mathcal{M}(\frac{\lambda n}{k})h \geq \mathcal{M}(\frac{(1-\lambda)n}{1-k})\ell$ . In that case, other H sellers also prefer the I to the U submarket. This condition is equivalent to  $\lambda \geq \lambda^*$ , where  $\lambda^*$  is given by

$$\mathcal{M}\left(\frac{\lambda^* n}{k}\right)h = \mathcal{M}\left(\frac{(1-\lambda^*)n}{1-k}\right)\ell. \quad (4)$$

The separating equilibrium in which H sellers go to the I submarket and L sellers go to the U submarket exists only for  $\lambda \in [\lambda^*, k]$ . An increase in  $\lambda$  in the separating equilibrium in this region will lead to an increase in the payoff of H sellers, because they will have more opportunities to trade, but it does not affect the sellers' submarket choices. These results are summarized in the following proposition.

**Proposition 1.** *The separating equilibrium, the equilibrium in which H sellers go to the I submarket and L sellers go to the U submarket, exists only for  $\lambda \geq \lambda^*$ . The separating equilibrium is unique. The payoff of H and L sellers are, respectively, given as follows:*

$$\pi_{IH}(v) = \mathcal{M}\left(\frac{\lambda n}{k}\right)(h - v), \pi_U(0) = \mathcal{M}\left(\frac{(1-\lambda)n}{1-k}\right)\ell.$$

### 3.2 Pooling Equilibrium

The payoff of H sellers in the I submarket is given by  $\pi_{IH}(v) = \mathcal{M}(\theta_I)(h - v)$ , which is a straight line with  $-\mathcal{M}(\theta_I)$  slope when plotted against  $v$ . The payoff of H sellers in the U submarket is given by  $\pi_U(v) = \mathcal{M}(\theta_U)\psi_U(v)$ , where

$$\psi_U(v) \equiv \max_{\hat{v}} \{t_U(\hat{v}) - q_U(\hat{v})v\} = t_U(v) - q_U(v)v,$$

therefore,

$$\psi_U(v) = \psi_U(v_H) + \int_v^{v_H} q_U(\nu) d\nu,$$

following a standard envelope argument in the mechanism design literature. Also,  $q_U(v)$  should be decreasing in any allocation that satisfies IC, thus  $\psi_U$  is a weakly convex function in  $v$ , and so is  $\pi_U(v)$ . As a result,  $\pi_{IH}(v)$  and  $\pi_U(v)$  can have at most two intersections. Therefore, three types of pooling equilibria are possible. These three cases are illustrated in Figure 1.

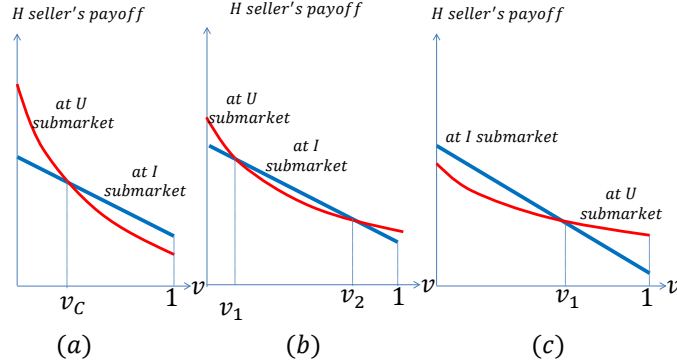


Figure 1: Illustration of pooling equilibria. It is argued in Proposition 2 that only case (a) is feasible.

I show in Proposition 2 that the only possible type of pooling equilibrium is case (a). In this equilibrium, there exists a cutoff point  $v_C \in [0, 1]$  such that H sellers with  $v \leq v_C$  and L sellers choose the U submarket and other H sellers choose the I submarket. I proceed by introducing the following assumption.

**Assumption 2.**  $kh + (1 - k)\ell < 1$ .

Assumption 2 implies that private information in this economy causes inefficiency. Even when all buyers are uninformed, buyers are not willing to trade with sellers with probability 1, because they would have to pay 1 but receive only  $kh + (1 - k)\ell$  on average. As a result, there might be a role for informed buyers to trade with sellers who cannot, or are not willing to, trade with uninformed buyers.

**Proposition 2** (Type of Pooling Equilibrium). *Suppose Assumptions 1 and 2 hold. The only type of pooling equilibrium possible is the equilibrium in which H sellers with outside options less than  $v_C$  and also all L sellers go to the U submarket and other H sellers go to the I submarket.*

In words, the tradeoff for H sellers is whether to sell with a higher price but low probability in the I submarket or with a lower price but high probability in the U submarket. The H sellers whose outside valuation of the asset is high prefer the former while other H sellers prefer the latter. The L sellers, of course, prefer to sell to the uninformed buyer because they sell with a higher price and higher probability compared with those if they sell to informed buyers. I must emphasize that this result may not hold with different bargaining protocols, for example when the buyer's payoff from the match is maximized, and other types of equilibria may become feasible.

A formal proof will be provided in the Appendix where I show cases (b) and (c) are not possible. Here, I study the bilateral meeting in the U submarket and then show why the proposed allocation in case (a) in Figure 1 can be an equilibrium. Denote by  $v_H$  the highest outside option of H sellers who come to the U submarket. Focusing on the bargaining problem, one can arrange Problem 1 as follows after incorporating the IC constraint according to Samuelson (1984).

**Problem 2.**

$$\begin{aligned} & \max_{\{q_U(v)\}} \int (b(v) - v) q_U(v) d\tilde{F}(v) \\ & \text{subject to: } q_U(\cdot) \text{ is decreasing,} \\ & \int (b(v) - v) q_U(v) d\tilde{F}(v) - \int \tilde{F}(v) q_U(v) dv \geq 0. \end{aligned}$$

Samuelson's result states that, to solve for the mechanism design problem, one simply needs to maximize the expected surplus from the match subject to the following constraints. First,  $q_U(\cdot)$ , the probability that the good is given to the buyer, must be decreasing to ensure that an incentive-compatible set of transfers can be found. Second, the net payoff to the seller with the highest outside option must be weakly greater than zero to ensure that the IRS for all sellers is satisfied. Transfers are set such that the IRB is also satisfied. Once the problem is solved, transfers can then be calculated from:

$$\psi_U(v_H) = t_U(v_H) - q_U(v_H)v_H = \int (b(v) - v) q_U(v) d\tilde{F}(v) - \int \tilde{F}(v) q_U(v) dv, \quad (5)$$

$$\psi_U(v) = t_U(v) - q_U(v)v = \psi_U(v_H) + \int_v^{v_H} q_U(\nu) d\nu \text{ for } v \leq v_H. \quad (6)$$

Samuelson shows that the solution to this problem is in general a three-stepped function for some  $v_a \geq 0$ ,  $v_b \in [v_a, 1]$  and  $q_0 \in [0, 1]$ :

$$q_U(v) = \begin{cases} 1 & 0 \leq v < v_a \\ q_0 & v_a \leq v \leq v_b \\ 0 & v_b < v \leq v_C \end{cases}.$$

It becomes clear from Samuelson's characterization that when the last constraint in Problem 2 is not binding, then  $q_U(v) = 1$  for all  $v \leq v_C$ ; otherwise, one could increase  $q$  for the second or third segment by a small amount to increase the value of the objective function in Problem 2, while all constraints continue to be satisfied.

I now argue that the last constraint in Problem 2 cannot be binding in the suggested equilibrium (the allocation illustrated in case (a) of Figure 1). That is, the sellers around the cutoff point who come to the U submarket receive a strictly positive payoff. Since H sellers at the left side of the cutoff point go to the U submarket, it must be the case that in the pooling equilibrium, the probability of matching in the I submarket is lower than that at U submarket, so the H sellers with zero outside option strictly prefer to go to the I submarket. As a result, L sellers also strictly prefer to go to the U submarket, as they receive the same payoff as the H sellers with zero outside option in the U submarket, but they would get strictly less payoff if they choose to go to the I submarket. Therefore, in the pooling equilibrium, all L sellers go to the U submarket, i.e.,  $s_{LU} = 1$ . Moreover, H sellers at the cutoff point (with  $v = v_C$ ) are indifferent between the I and U submarkets. Since these sellers receive a strictly positive payoff in the I submarket, the participation constraint of the maximization problem cannot be binding. As a result, we must have the following in the pooling equilibrium:

$$q_U(v) = 1 \text{ for } v \in [0, v_C].$$

Thus,  $\tilde{F}(v)$  is given by:

$$\tilde{F}(v) = \begin{cases} \frac{kF_H(v) + (1-k)\ell}{kF_H(v_C) + (1-k)} & 0 \leq v \leq v_C \\ 1 & v_C < v \end{cases},$$

and

$$\begin{aligned} \psi_U(v_C) &= \frac{k \int_0^{v_C} \left( b(\nu) - \nu - \frac{F_H(\nu)}{f_H(\nu)} \right) f_H(\nu) d\nu + (1-k)(\ell - v_C)}{kF_H(v_C) + 1 - k} = \frac{kF_H(v_C)h + (1-k)\ell}{kF_H(v_C) + 1 - k} - v_C, \\ \psi_U(v) &= \psi_U(v_C) + v_C - v = \underbrace{\frac{kF_H(v_C)h + (1-k)\ell}{kF_H(v_C) + (1-k)}}_{\text{price in the U submarket}} - v \text{ for } v \leq v_C, \end{aligned}$$

where the first equality in the latter equation follows from (6).

In words, in the pooling equilibrium, all L sellers and also H sellers with  $v \leq v_C$  go to the U submarket and trade with probability 1. The price in the U submarket is exactly equal to the average value of the good to the uninformed buyers. All H sellers who go to the U submarket receive a strictly positive payoff. If these sellers trade with probability less than 1 in equilibrium, their participation constraint must have been binding, in which case their net payoff from going to the U submarket would have been zero. However, this is in contradiction with the fact that they could have gone to the I submarket to receive a strictly positive payoff. All other H sellers go to the I submarket and are matched with probability less than 1.

### 3.3 Existence and the Number of Equilibria

Now that the search patterns are clear, I fully characterize the pooling equilibrium. Following the fact that  $(H, v_C)$  sellers are indifferent between the two submarkets, one can summarize the equilibrium conditions for the pooling equilibrium as follows:

$$\mathcal{M}(\theta_I)(h - v_C) = \mathcal{M}(\theta_U)(P(v_C)h + (1 - P(v_C))\ell - v_C), \quad (7)$$

where the tightness at each submarket is given by

$$\theta_I = \frac{\lambda n}{k(1 - F_H(v_C))}, \quad (8)$$

$$\theta_U = \frac{(1 - \lambda)n}{kF_H(v_C) + 1 - k}, \quad (9)$$

and  $P(v)$  is defined as the probability that a seller is of type H in the U submarket in the pooling equilibrium with cutoff point  $v$ , and is given by

$$P(v) \equiv \frac{kF_H(v)}{kF_H(v) + 1 - k}.$$

Therefore, there will be simply one equation and one unknown,  $v_C$ , for characterizing the pooling equation:

$$\mathcal{M}\left(\frac{\lambda n}{k(1 - F_H(v_C))}\right)(h - v_C) = \mathcal{M}\left(\frac{(1 - \lambda)n}{kF_H(v_C) + 1 - k}\right)(P(v_C)h + (1 - P(v_C))\ell - v_C). \quad (10)$$

It is generally hard to solve (10) analytically for  $v_C$ . To characterize  $v_C$ , it is convenient to define  $G(\lambda, v)$  as follows:

$$G(\lambda, v) \equiv -\mathcal{M}\left(\frac{\lambda n}{k(1 - F_H(v))}\right)(h - v) + \mathcal{M}\left(\frac{(1 - \lambda)n}{kF_H(v) + 1 - k}\right)(P(v)h + (1 - P(v))\ell - v). \quad (11)$$

Of course,  $v_C$  solves  $G(\lambda, v_C) = 0$ . Note that  $G(\lambda, 1) < 0$  because  $kh + (1 - k)\ell - 1 < 0$  according to Assumption 2. Therefore, if  $G(\lambda, v) \geq 0$  for some  $v$ , then the existence of the pooling equilibrium is guaranteed by intermediate value theorem. Now define

$$J(\lambda) \equiv \max_{v \in [0,1]} G(\lambda, v),$$

$$v(\lambda) \equiv \arg \max_{v \in [0,1]} G(\lambda, v),$$

and if there are multiple maximizers of  $G$ , just pick the largest one for  $v(\lambda)$ . Since  $G(\lambda, v)$  is continuous, the necessary and sufficient condition for the existence of a pooling equilibrium for a given  $\lambda$  is that  $J(\lambda) \geq 0$ . A simple envelope argument indicates that  $J(\lambda)$  is decreasing in  $\lambda$ . Therefore, if  $\lim_{\lambda \rightarrow 0} J(\lambda) \geq 0$ , then the pooling equilibrium exists. If it exists, then it exists only for  $\lambda \leq \hat{\lambda}$ , where  $\hat{\lambda}$  solves

$$J(\hat{\lambda}) = 0. \tag{12}$$

If  $\lim_{\lambda \rightarrow 0} J(\lambda) < 0$ , then the pooling equilibrium does not exist for any  $\lambda$ . If it exists and  $G(\lambda, 0) < 0$ , then there should generically exist multiple equilibria. See Figure 2 for illustration.

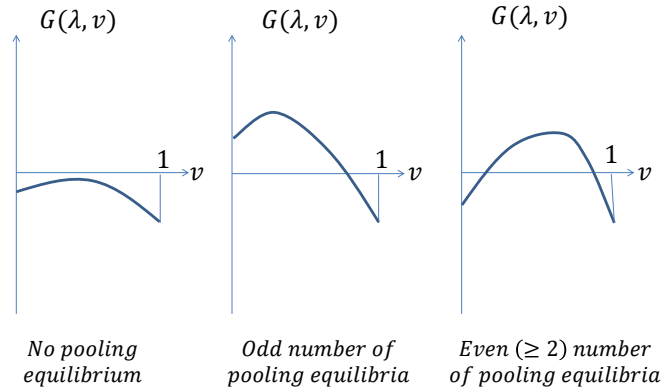


Figure 2: Determining the number of pooling equilibria.

These results are summarized in Proposition 3, for which the following definition and assumption are introduced. Define the net payoff of an uninformed buyer after getting matched by

$$H(v) \equiv P(v)h + (1 - P(v))\ell - v.$$

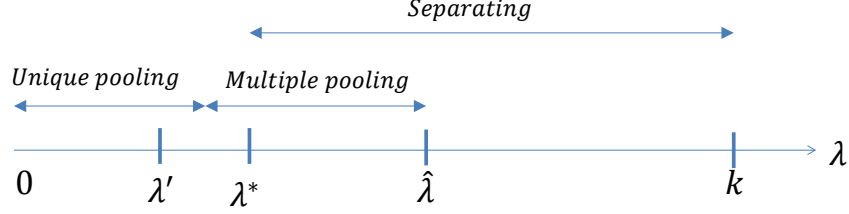


Figure 3: Illustration of the type and number of equilibria for different  $\lambda$ .

**Assumption 3.**  $H(v) = 0$  has exactly one root for  $v \in [0, 1]$ .

Remember that Assumption 2 states that even if all H sellers come to the U submarket, trade cannot happen with probability 1. This is because the average value of the good to buyers is less than the value of the good to the high type sellers with the highest  $v$ . Moreover, under Assumption 2,  $H(0) > 0$  and  $H(1) < 0$ . Assumption 3 encompasses Assumption 2 and requires  $H(v)$  to be decreasing in  $v$  only around the neighborhood of its root. By making Assumption 3, I avoid making an alternative, stronger assumption requiring  $H(v)$  be decreasing on the entire domain of  $[0, 1]$ . This alternative assumption is much harder to satisfy in the applications.

**Proposition 3** (Equilibrium existence, and uniqueness/multiplicity). *Under Assumptions 1 and 2, the following results hold:*

- (i) *Pooling equilibrium exists only for  $\lambda \in [0, \hat{\lambda}]$ .*
- (ii) *If  $\lambda \geq \lambda^*$ , then there exists generically an even number of pooling equilibria.*
- (iii) *Suppose Assumption 3 holds as well. Then there exists  $\lambda' \in [0, \hat{\lambda}]$  such that the pooling equilibrium is unique for  $\lambda \in [0, \lambda']$ .*

*Note that  $\lambda^*$  and  $\hat{\lambda}$  are defined by (4) and (12) respectively.*

Part (i) of Proposition 3 is clear. Part (ii) states that if pooling equilibrium exists for  $\lambda \geq \lambda^*$ , then there are multiple pooling equilibria. Based on this result, one may speculate that if pooling equilibrium exists, then it may be unique for sufficiently small values of  $\lambda$ . Part (iii) confirms this speculation.

Propositions 1–3 provide sharp predictions regarding the equilibria. First, when  $\lambda$  is close to 0, there is a unique equilibrium and the equilibrium is pooling. As  $\lambda$  grows larger, multiple equilibria arise. In part of this region, there exist multiple pooling equilibria together with the separating equilibrium. As  $\lambda$  passes another threshold, pooling equilibrium does not exist anymore and the only equilibrium will be separating. Figure 3 illustrates the regions for different equilibria. In the numerical examples that I study,  $\lambda' = \lambda^*$ .

As noted above, L sellers in the pooling equilibrium strictly prefer to go to the U submarket, so one simply needs to focus on the behavior of H sellers to understand the economic forces at play in this environment. Especially, one needs to focus on (10). For the sake of this explanation, fix the matching function to  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  and assume parameters are set such that  $\theta_U > 1$ , i.e., the probability of matching in the U submarket is fixed and does not change with a change in  $\lambda$ , so  $\mathcal{M}(\theta_U) = \mu$ .<sup>15</sup> This is the case in numerical Examples 1 and 2 below.

Suppose  $\lambda$  increases. To understand the economic forces at play, first assume that the price in the U submarket is kept fixed. An increase in  $\lambda$  implies that H sellers have more opportunities to trade in the I submarket and fewer opportunities in the U submarket, so the H sellers around the cutoff point are encouraged to switch from the U to the I submarket. Call this effect of an increase in  $\lambda$  on the cutoff point the *matching* effect. Under the matching effect, the cutoff point decreases with  $\lambda$ , and welfare tends to decrease with  $\lambda$ . This is because as  $\lambda$  increases, more sellers go to the I submarket but they trade with a lower probability compared with the equilibrium with a lower  $\lambda$  in which they go to the U submarket and trade with a high probability.

Second, assume that the probability of trade in the I submarket is kept fixed. Suppose that H sellers believe that the price in the U submarket is increasing with  $\lambda$  in the equilibrium. This belief encourages H sellers around the cutoff point to go to the uninformed submarket; as a result, the average value of the good to the buyers in the U submarket increases, so the sellers can charge a higher price. Alternatively, suppose H sellers believe that the price in the U submarket is decreasing with  $\lambda$ . This would discourage H sellers around the cutoff point from going to the U submarket, lowering the price that the sellers in the U submarket can charge. Therefore, there is a form of strategic complementarity in this environment between the behavior of H sellers; if more H sellers go to the U submarket, then other H sellers would gain more by going to the U submarket. Similarly, if fewer H sellers go to the U submarket, then other H sellers would gain less by going to the U submarket. The key in this argument is that, similar to the Akerlof's model of lemons market, the price in the U submarket is increasing in the average quality, and the average quality is determined by the decision of the H sellers at the cutoff point, which, in turn, is a function of the price

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<sup>15</sup>For example, if the measure of uninformed buyers is sufficiently high, i.e.,  $(1 - \lambda)n > 1$ , then  $\theta_U > 1$ . The assumption that the probability of matching in the U submarket is fixed helps us to understand the mechanism better. Even without this assumption, since  $\theta_U > \theta_I$  and  $\mathcal{M}$  is increasing and concave, if  $k$  is not too high, the change in  $\mathcal{M}(\theta_U)$  would be less than the change in  $\mathcal{M}(\theta_I)$  for a small change in  $\lambda$ . Therefore, the intuition provided here is robust and does not hinge on this assumption.

that they receive in the equilibrium. Call this effect of a change in  $\lambda$  on the cutoff point the *complementarity* effect. Under the complementarity effect, the cutoff point may decrease or increase with  $\lambda$ . Following an increase in  $\lambda$ , if agents believe that more (fewer) H sellers go to the U submarket, then we say that the complementarity effect is positive (negative).<sup>16</sup>

If the complementarity effect is negative, then both the matching effect and the complementarity effect act in the same direction, so an increase in  $\lambda$  lowers the cutoff point. If the complementarity effect is positive but not strong enough, an increase in  $\lambda$  still decreases the cutoff point. However, if the complementarity effect is positive and strong enough, then an increase in  $\lambda$  leads to an increase in the cutoff point.

A positive complementarity effect is destabilizing. I do not formally define stability here, but suppose a small fraction of sellers around the cutoff point make a mistake in choosing their submarket. The equilibrium is called stable if other sellers still prefer to follow their equilibrium strategy. If the complementarity effect is positive and strong enough in the pooling equilibrium, if more H sellers around the cutoff point go to the U submarket, then other H sellers are also encouraged to go to the U submarket, so this equilibrium is unstable. If the complementarity effect is negative or not strong enough, if more H sellers around the cutoff point go to the I submarket, then the probability of matching in the I submarket decreases, discouraging other sellers from following the same strategy, so this equilibrium is stable.<sup>17</sup>

Now that the underlying economic forces are clear, we can understand the intuition behind Proposition 3. Regarding Proposition 3(i), note that pooling equilibrium exists if some H sellers are indifferent between the two submarkets. As mentioned earlier, the probability of matching for sellers in the I submarket is always less than that in the U submarket in the pooling equilibrium. The price that sellers receive in the I submarket is higher than that in the U submarket, so for the pooling equilibrium to exist, the probability of matching in the I submarket should be sufficiently lower than that in the U submarket. If  $\lambda$  is sufficiently high, then the probability of matching in the I submarket is so high that no H seller wants to go to the U submarket. Thus, pooling equilibrium cannot exist for high values of  $\lambda$ .

Regarding Proposition 3(iii), consider the pooling equilibrium for a given  $\lambda$  and increase the fraction of informed buyers by a small amount. When  $\lambda$  is close to zero, then the

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<sup>16</sup>This explanation reveals that as long as the trading mechanism in the uninformed market is such that the price is increasing in the average quality of the asset in the uninformed market, then the results will qualitatively continue to hold.

<sup>17</sup>I referred to stability along one equilibrium here. Multiplicity of equilibria, of course, can lead to another type of instability, which simply results from switching between different equilibria.

probability of matching in the I submarket must be small (unless almost all H sellers go to the U submarket, in which case the H sellers with the highest outside option receive a zero payoff following Assumption 2, but they could go to the I submarket and receive a strictly positive payoff). In order for the H sellers around the cutoff point to be indifferent between the two submarkets, the payoff that they receive in the U submarket conditional on matching should be close to zero. By Assumption 3, the net payoff is decreasing in  $v_C$  if the net payoff is around zero. If the complementarity effect is positive, i.e., if more H sellers choose to go to the U submarket following an increase in  $\lambda$ , then the net payoff of H sellers around the cutoff point will decrease, pushing the cutoff point to the left, but this is in contradiction with more sellers going to the uninformed submarket. As a result, the complementarity effect cannot be positive in the pooling equilibrium when  $\lambda$  is small. Therefore, when  $\lambda$  is small, the complementarity effect is negative. Since both complementarity effect and matching effect work in the same direction, the pooling equilibrium should be unique.<sup>18</sup> Moreover, as mentioned earlier, the separating equilibrium exists only if  $\lambda$  is sufficiently high; therefore, when  $\lambda$  is sufficiently small, only one equilibrium exists and this equilibrium is pooling. When  $\lambda$  grows larger, this argument does not apply, because  $H(v_C)$  is not close to zero, so Assumption 3 cannot be used. In that case, the complementarity effect may be positive and multiple equilibria can arise. This explains Proposition 3(ii).

### 3.4 Comparison of Cutoff Points Along One Equilibrium

Now we turn to the comparative statics with respect to  $\lambda$  along one equilibrium.

**Proposition 4** (Comparative statics with respect to  $\lambda$ ). *Suppose Assumptions 1–3 hold. Take any  $\lambda > 0$  under which pooling equilibrium exists and rank the equilibria in terms of their cutoff point,  $v_C$ , from high to low. Then,  $v_C$  exhibits alternating comparative statics with respect to  $\lambda$  across different equilibria. More specifically,  $v_C$  is generically downward (upward) sloping in  $\lambda$  in the odd-ranked (even-ranked) equilibria.*

This result states that the cutoff point in different equilibria demonstrates opposing comparative statics with respect to  $\lambda$ . Combining Propositions 3 and 4, one concludes that when  $\lambda \leq \lambda'$ , the equilibrium is unique and stable, and the cutoff point decreases with  $\lambda$ . In this region, an increase in the measure of informed buyers leads to an increase in the

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<sup>18</sup>Both effects work in the same direction, so if we deviate from the equilibrium, both effects push further from the equilibrium, so it is not possible to have another equilibrium.

*Payoff of H sellers at the cutoff point*

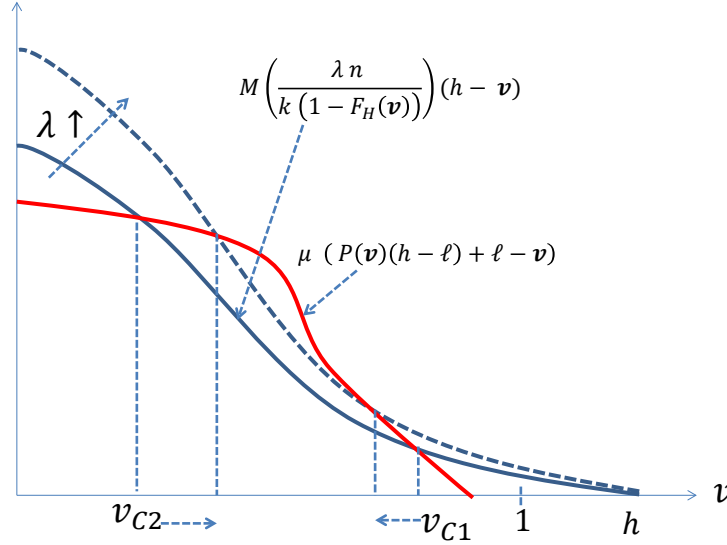


Figure 4: When  $\lambda$  increases, the odd-ranked cutoff points (such as  $v_{C1}$ ) decrease and the even-ranked cutoff points (such as  $v_{C2}$ ) increase.

measure of H sellers who go to the I submarket. Consequently, the price in the U submarket declines.

To understand how equilibrium behavior changes with respect to  $\lambda$ , again assuming that the matching function is  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  and the parameters are set such that  $\theta_U > 1$ , one can write the pooling equilibrium condition as

$$\mathcal{M}\left(\frac{\lambda n}{k(1 - F_H(v))}\right)(h - v) = \mu((h - \ell)P(v) + \ell - v).$$

The right-hand side (RHS) of this equation is independent of  $\lambda$  while the left-hand side (LHS) is increasing in  $\lambda$ . An increase in  $\lambda$  shifts the LHS upward, thus increasing the cutoff point for odd-ranked equilibria and decreasing it for even-ranked equilibria. See Figure 4 for illustration of RHS and LHS and how they change with  $\lambda$ . The payoff of the sellers at the cutoff point may increase or decrease with  $\lambda$  depending on the shape of the matching function and the distribution of outside options. A similar argument can be made to conduct comparative statics with respect to other variables. For example, if  $k$  increases, the LHS shifts downwards and the RHS shifts upwards. In this case, the direction of change in the cutoff point depends on the relative size of the shift of the two functions.

### 3.5 Comparison of Welfare and Trading Volume Along One Equilibrium

Define welfare to be the weighted average of surplus across all matches. Consider an allocation in which L sellers and also H sellers with  $v \leq v_C$  go to the U submarket and H sellers with  $v \in [v_C, 1]$  go to the I submarket. All sellers who match with a buyer trade with probability 1. Separating equilibrium is one such allocation with  $v_C = 0$ . Welfare for such allocation can be easily calculated by

$$W(v_C, \theta_I, \theta_U) = \mathcal{M}(\theta_I)k \int_{v_C}^1 (h-v)dF_H(\nu) + \mathcal{M}(\theta_U) \left( k \int_0^{v_C} (h-v)dF_H(\nu) + (1-k)\ell \right). \quad (13)$$

Note that  $\theta_I$  and  $\theta_U$  for such allocation are given by (8) and (9).

Define trading volume to be the total number of trades in both markets. Trading volume in the allocation described above can be calculated by

$$T(v_C, \theta_I, \theta_U) = \mathcal{M}(\theta_I)k(1 - F_H(v_C)) + \mathcal{M}(\theta_U)(kF_H(v_C) + 1 - k). \quad (14)$$

To study how welfare changes in the pooling equilibrium with  $\lambda$ , assuming differentiability whenever needed, I take a simple derivation of welfare with respect to  $\lambda$ :

$$\begin{aligned} \frac{dW}{d\lambda} = & \underbrace{\underbrace{\frac{\partial W}{\partial \theta_I}}_{\geq 0} \underbrace{\frac{\partial \theta_I}{\partial \lambda}}_{\geq 0} + \underbrace{\frac{\partial W}{\partial \theta_U}}_{=0} \frac{\partial \theta_U}{\partial \lambda}}_{\text{(direct) effect of change in } \lambda \text{ on market tightness through buyers}} \\ + & \underbrace{\left[ \underbrace{\frac{\partial W}{\partial \theta_I}}_{\geq 0} \underbrace{\frac{\partial \theta_I}{\partial v_C}}_{\geq 0} + \underbrace{\frac{\partial W}{\partial \theta_U}}_{=0} \frac{\partial \theta_U}{\partial v_C} \right] \frac{dv_C}{d\lambda}}_{\text{(indirect) effect of change in } \lambda \text{ on cutoff point and then market tightness through sellers}} \\ + & \underbrace{\underbrace{\frac{\partial W}{\partial v_C}}_{= (\mathcal{M}(\theta_U) - \mathcal{M}(\theta_I))(h - v_C)F'_H(v_C) \geq 0} \frac{dv_C}{d\lambda}}_{\text{effect of change in } \lambda \text{ on sellers' surplus at the cutoff point}}. \end{aligned} \quad (15)$$

Again, assuming that the matching function is  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  and the parameters are set such that  $\theta_U > 1$ , the probability of matching at U submarket is fixed, so  $\frac{\partial W}{\partial \theta_U} = 0$ . It is easy also to see that welfare is increasing in  $\theta_I$ , and  $\theta_I$  is increasing in  $\lambda$  and  $v_C$ . Therefore, if  $\frac{dv_C}{d\lambda} \geq 0$ , then welfare will be increasing in  $\lambda$ , but if  $\frac{dv_C}{d\lambda} < 0$ , then the overall welfare effect depends on the relative magnitude of the first line and the sum of second and third lines in (15).

In words, an increase in  $\lambda$  directly increases the market tightness in the I submarket and weakly decreases the market tightness in the U submarket. That is, there will be more trades in the I submarket and weakly fewer trades in the U submarket, *ceteris paribus*. In general, whether this direct effect on welfare is positive or negative depends on the parameters of the model, especially on the elasticity of the matching function. With the specific matching function adopted here and with the assumption on  $\theta_U$ , this effect is positive. (See the first line in (15).) Intuitively, in the pooling equilibrium there are relatively too few buyers in the I submarket compared with the U submarket, so a marginal increase in the number of buyers at the I submarket has a larger effect on welfare than a marginal decrease in the number of buyers in the U submarket. The number of sellers in each submarket also changes with  $\lambda$ . The change in the tightness in the U submarket does not change the probability of trade for sellers (because of the form of matching function), so I focus on the incentive of H sellers to go to the I submarket. If the cutoff point increases following an increase in  $\lambda$  (as is the case in the even-ranked equilibria), fewer sellers go to the I submarket. This leads to an increase in the probability of matching for all sellers in the I submarket and, in turn, increases welfare. (See the second line in (15).) Moreover, when H sellers at the cutoff point find it optimal to go to U instead of I submarket (as is the case in the even-ranked equilibria), then the surplus that these sellers create increases proportional to the difference in matching probabilities in the two submarkets. But the probability of matching is higher in the U submarket, so this also has a positive effect on welfare. (See the third line in (15).)

However, if the cutoff point decreases following an increase in  $\lambda$  (as is the case in the odd-ranked equilibria), then the effect on market tightness through the number of sellers and also the effect on the surplus created by the sellers around the cutoff point together work in the opposite direction relative to the effect on market tightness through the number of buyers. In this case, the overall welfare effect is ambiguous and depends on the relative magnitude of these effects. In the examples below, the overall welfare effect is negative in odd-ranked equilibria. These results are summarized in the following proposition.

**Proposition 5** (Welfare and trading volume along one equilibrium). *Suppose Assumptions 1–3 hold. Take any  $\lambda > 0$  under which pooling equilibrium exists and rank the equilibria in terms of their cutoff point,  $v_C$ , from high to low. Assume  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  and parameters are set such that  $\theta_U > 1$ . Then, welfare and trading volume in the even-ranked equilibria are increasing in  $\lambda$ . Welfare and trading volume in the odd-ranked equilibria can be increasing or decreasing in  $\lambda$ .*

The mechanism through which welfare effect may be negative in the odd-ranked equilibria

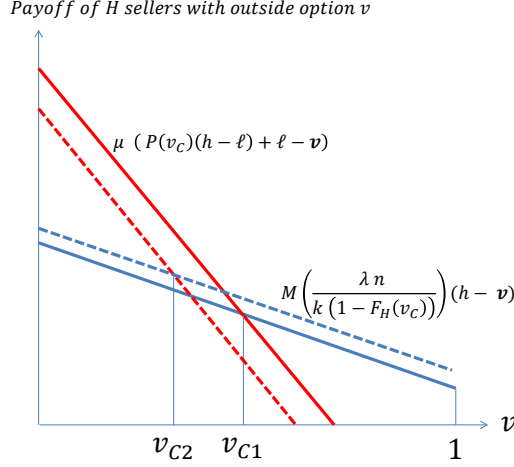


Figure 5: Comparison between the odd-ranked pooling equilibria with cutoff points  $v_{C1}$  and  $v_{C2}$  associated with  $\lambda_1$  and  $\lambda_2 > \lambda_1$ .

is interesting. When  $\lambda$  increases, there are more opportunities for H sellers to sell to the informed buyers (first effect). In the odd-ranked equilibria, agents believe that overall the price in the U submarket decreases. Therefore, H sellers have strong incentive to visit informed buyers. This implies that the probability of trade in the I submarket cannot increase as much because now more sellers choose to sell in the I submarket (second effect). Also, the marginal sellers previously sold with a higher probability in the U submarket, but they now sell with a lower probability in the I submarket, of course, expecting to receive a higher price if matched (third effect). In terms of welfare, second and third effects work in the opposite directions and may lead to an overall decrease in welfare, as is the case in the numerical examples below. Although welfare overall may decrease in the odd-ranked equilibria with  $\lambda$ , it does not mean that all sellers are equally affected. As  $\lambda$  increases, the payoff of sellers in the I submarket increases and the payoff of sellers in the U submarket decreases. See Figure 5 for illustration.

### 3.6 Comparison of Welfare and Trading Volume Across Equilibria

Comparing trading volume across different equilibria (with the same  $\lambda$ ) is relatively easy. The measure of buyers in the I and U submarkets is fixed. In any equilibrium, sellers trade with probability 1 conditional on matching in both submarkets. Also, the matching probability in the U submarket is higher than that at the I submarket. That is, there is more congestion at the I submarket. Removing one seller from the I submarket and adding that seller to the U submarket increases the total number of matches because the matching function is concave.

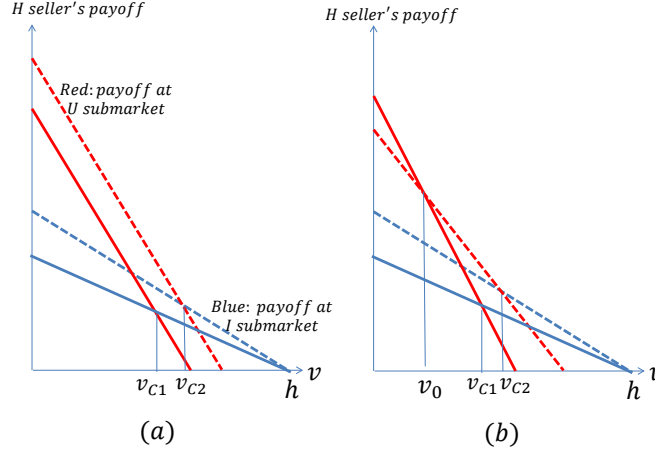


Figure 6: Comparison between the equilibria with cutoff points  $v_{C2}$  and  $v_{C1}$  for a given  $\lambda$ . All sellers are worse off in the latter compared with the former (graph (a)). All L sellers and also H sellers with  $v \in [0, v_0]$  are better off while other sellers are worse off in the latter compared with the former (graph (b)).

Therefore, trading volume is higher in the equilibrium with a higher cutoff point, because relatively more sellers go to the U submarket.<sup>19</sup>

Studying welfare and Pareto efficiency across equilibria requires a more careful discussion. Consider two equilibria for a given  $\lambda$  with cutoff points  $v_{C1}$  and  $v_{C2}$  with  $v_{C1} < v_{C2}$ . Denote the associated price at the U submarket by  $P_i$ , and the associated market tightness in the I submarket by  $\theta_{Ii}$  and in the U submarket by  $\theta_{Ui}$  for  $i \in \{1, 2\}$ . The payoff of H sellers with different outside options at different submarkets are depicted in Figure 6. The payoff of H sellers who go to I submarket is  $\mathcal{M}(\theta_I)(h - v)$ . Since  $\theta_{I2} > \theta_{I1}$ , the sellers who go to I submarket are better off in equilibrium 2 than in equilibrium 1. The payoff of sellers who go to the U submarket is given by  $\mathcal{M}(\theta_U)(P - v)$ . If  $\mathcal{M}(\theta_{U2})(P_2 - v) \geq \mathcal{M}(\theta_{U1})(P_1 - v)$  for all  $v \leq v_{C1}$ , then equilibrium 1 is Pareto dominated by equilibrium 2, and subsequently, the welfare level in equilibrium 1 is lower than that in equilibrium 2. This is the case, for example, if  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  and parameters are such that  $\theta_U > 1$ . This case is illustrated in Figure 6(a). In this case, the probability of matching for sellers in the U submarket is always equal to  $\mu$ . Also,  $v_{C2} > v_{C1}$ , thus more H sellers go to the U submarket in equilibrium 2. Consequently, the average price in the U submarket is higher, i.e.,  $P_2 > P_1$ , and therefore,  $\mathcal{M}(\theta_{U2})(P_2 - v) = \mu(P_2 - v) > \mu(P_1 - v) = \mathcal{M}(\theta_{U1})(P_1 - v)$ . In this case, all H sellers are

<sup>19</sup>Note that  $\theta_I < \theta_U$ . A higher  $v_C$  is associated with the case in which  $\theta_I$  is closer to  $\theta_U$ . Since  $\mathcal{M}$  is concave,  $\alpha\mathcal{M}(\frac{\lambda n}{\alpha}) + (1 - \alpha)\mathcal{M}(\frac{(1 - \lambda)n}{1 - \alpha})$  is decreasing in  $\alpha$  as long as  $\frac{\lambda n}{\alpha} < \frac{(1 - \lambda)n}{1 - \alpha}$  or  $\lambda < \alpha$ . This is true in both separating and pooling equilibria.

better off in equilibrium 2 relative to equilibrium 1. All L sellers are better off too, because they receive a higher price. Buyers' payoffs do not change as they receive a zero payoff from the match according to the trading protocol.

In contrast, if the probability of matching for sellers in the U submarket is significantly lower in equilibrium 2 than equilibrium 1 (as more H sellers go to the U submarket), then the H sellers who go to the U submarket may be worse off in equilibrium 2 than equilibrium 1. The welfare comparison across these two equilibria is ambiguous in this case. This case is illustrated in Figure 6(b). These results are summarized in the following proposition.

**Proposition 6** (Comparison across equilibria for a given  $\lambda$ ). *Suppose Assumptions 1–3 hold. Take any  $\lambda > 0$  and rank the equilibria in terms of their cutoff point,  $v_C$ , from high to low. Take two equilibria with cutoff points  $v_{C1}$  and  $v_{C2} > v_{C1}$  for a given  $\lambda$ . (Note that for the separating equilibrium,  $v_C = 0$ .)*

(i) *Then the trading volume is lower in the equilibrium with a lower cutoff point.*

(ii) *Furthermore, assume  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  and assume that parameters are set such that  $\theta_U > 1$  for both equilibria. Then, the allocation in the equilibrium with a lower cutoff point is Pareto dominated by the allocation in the other equilibrium.*

Under conditions specified in Proposition 6, the cutoff point is a sufficient statistic for trading volume and Pareto efficiency of different equilibria.

### 3.7 Illustrating the Results with Numerical Examples

I present one numerical example here and another example in the next section.

**Example 1.** *Parameters:  $h = 1.21$ ,  $\ell = 0.2$ ,  $k = 0.75$ ,  $n = 1$ ,  $F_H(v) = (1 - \gamma_H)v + \gamma_H v^2$ ,  $v \in [0, 1]$ , and  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$ ,  $\mu = 1$ .*

I conduct comparative statics with respect to two variables:  $\lambda$  and  $\gamma_H$ . In the comparative statics with respect to  $\lambda$ , I set  $\gamma_H = 0$ . In the comparative statics with respect to  $\gamma_H$ , I set  $\lambda = 0.15$ . It turns out that there are at most three equilibria for a given  $\lambda$ : pooling 1, pooling 2, and separating. The cutoff point in pooling 1 is lower than that in pooling 2. The comparative statics for pooling equilibria with respect to  $\lambda$  and  $\gamma_H$  are illustrated in Figures 7 and 8, respectively. A lower  $\gamma_H$  corresponds to the case where the outside option of H sellers is lower (in the first order stochastic sense), so they are less distressed. Again, to obtain concrete results, parameters are chosen such that  $\theta_U > 1$ , i.e., the probability of matching in the U submarket is fixed. In the pooling equilibria, the sellers at the cutoff

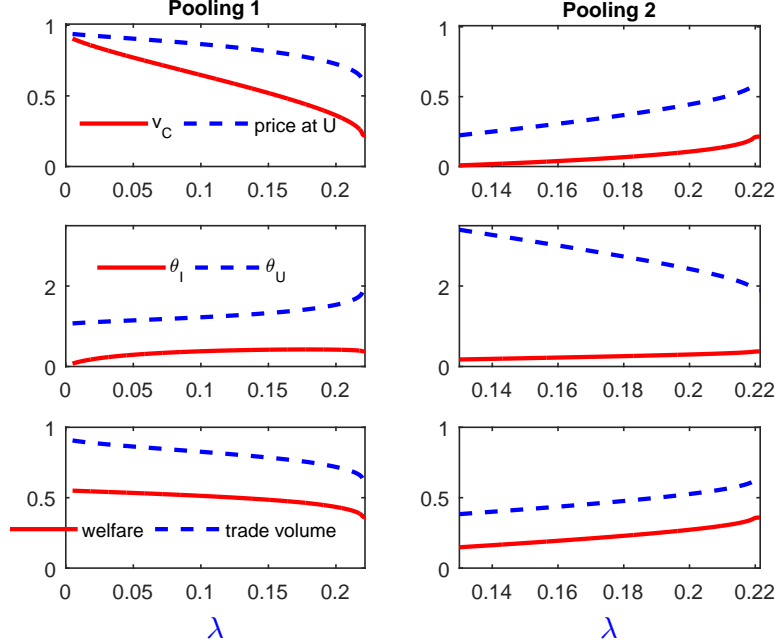


Figure 7: Comparative statics with respect to the measure of informed buyers in pooling equilibria in Example 1. The cutoff point and price in the U submarket in the first row, the ratio of buyers to sellers at I and U submarkets in the second row, and the welfare and trading volume in the third row are depicted. The left and right columns correspond to pooling 1 and pooling 2 equilibria.

point are indifferent between the two submarkets and they receive a higher payoff in the I submarket than the U submarket conditional on being matched. Hence, it must be the case that the sellers are matched with probability less than  $\mu$  in the I submarket. Therefore, the informed buyers are on the short side of the market, so they must be matched with probability  $\mu$  in the pooling equilibrium.

Regarding the comparative statics with respect to  $\lambda$  in Example 1, it turns out that  $\lambda^* = 0.22$  and  $\lambda' = \hat{\lambda} = 0.13$ . Pooling 1 equilibrium is the unique equilibrium for  $\lambda \in [0, \hat{\lambda}]$  and is stable. Pooling 1 and pooling 2 equilibria coexist for  $\lambda \in (\hat{\lambda}, \lambda^*]$ . Separating equilibrium, which exists for  $\lambda \geq \hat{\lambda}$ , is not shown to save space. The cutoff point, price in the U submarket, welfare and trading volume all increase in pooling 2 equilibrium consistent with the findings of Proposition 5. It is interesting to note, however, that these variables all decrease in pooling 2 equilibrium.

One may associate the introduction of new information technologies in recent years, especially in the financial markets, with an increase in the size of the informed segment of the market (increase in  $\lambda$  in the model). A higher number of financial institutions may now have access to information technologies enabling them to better learn the value of the

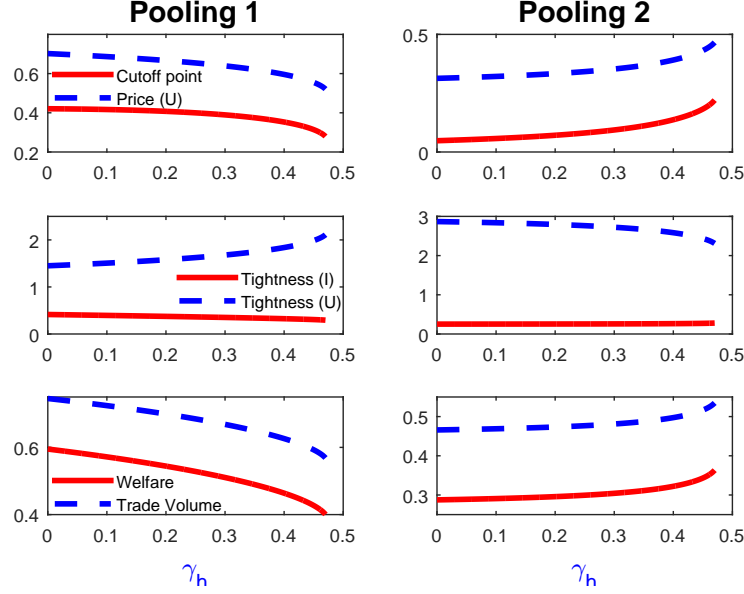


Figure 8: Comparative statics with respect to  $\gamma_H$  in pooling equilibria in Example 1. The description of graphs are the same as that in Figure 7.

assets that they want to buy or to better recognize the actual value of the projects that are presented to them for funding. The comparative statics with respect to  $\lambda$  in pooling 1 equilibrium show that the effects of these changes on markets in terms of composition of different types of assets in different segments of the markets (which is captured by the cutoff point in the model) can be counterintuitive. That is, with an increase in the measure of informed buyers, welfare and trading volume may decline.

Now suppose  $\lambda \in [\hat{\lambda}, \lambda^*]$ , e.g.,  $\lambda = 0.16$ , and suppose that equilibrium switches from pooling 1 to pooling 2. In pooling 2, fewer H sellers go to the U submarket, the price that uninformed buyers are willing to pay is lower, the probability of matching in the I submarket is lower, and welfare and trading volume are lower too. The change from pooling 1 to pooling 2 equilibrium, which can be triggered simply by a change in beliefs or non-fundamentals, shares many features of a financial crises in that the average price and liquidity of the asset, trading volume, and welfare decrease.

Regarding the comparative statics with respect to  $\gamma_H$ , notice that a decrease in  $\gamma_H$  corresponds to the situation in which H sellers are more willing to trade as their outside option decreases. Again, there are two pooling equilibria. As  $\gamma_H$  decreases, the cutoff point and consequently price decrease in the uninformed submarket in the pooling 2 equilibrium. (See upper right graph in Figure 8.) That is, sellers who are more distressed for small values of  $\gamma_H$  prefer to trade with informed buyers. This creates more congestion in the

informed submarket and leads to a lower price in the uninformed submarket. As a result, trading volume also decreases. This situation shares some features with what is called *market freeze*, where there is no trading in the equilibrium. (See [Chiu and Koepl \(2016\)](#) for a theoretic model of market freeze.) Unlike a market freeze, however, trading does not stop in equilibrium here, but it declines as the market is filled more with distressed sellers.

## 4 Endogenous Information Acquisition

In this section, I endogenize the buyers' decision to become informed. In the first subsection below, I investigate how the buyers' incentive changes when the relative measure of informed buyers changes. I also present another numerical example and use it along with Example 1 throughout the section to explain the intuition behind the results. In the second subsection, I introduce a cost that is to be incurred if the buyers want to become informed. In the real world, this cost includes hiring human resources (e.g., financial analysts), purchasing tools (e.g., software programs) or purchasing devices (e.g., computer hardware) to recognize the quality of assets or projects. Next, I compare the cost and benefit of becoming informed to solve for the equilibrium level of  $\lambda$ . Finally, I show how this simple model can be used to explain the fall in asset prices and liquidity during the recent financial crisis. I will also discuss some policy implications.

### 4.1 Incentive of Buyers to Become Informed

In the benchmark model, the mechanism in the bilateral meetings was designed as maximizing the seller's payoff. As the buyer does not have any private information, it is of no surprise that the buyer does not receive any surplus from the match. In order for us to study the incentive of buyers to become informed, the buyers need to receive some surplus from the match. Assume that the mechanism maximizes the buyer's payoff with probability  $\alpha \in [0, 1]$  and maximizes the seller's payoff with probability  $1 - \alpha$ , in both cases subject to IC and IRS and IRB constraints. In the benchmark model in Section 2,  $\alpha$  was equal to 0. Here,  $\alpha$  is assumed to be strictly positive but small enough so that the patterns of search remain the same although the cutoff points may change. That is, the L sellers and H sellers with a low outside option go to the U submarket, and the H sellers with a high outside option go to the I submarket. Also in the separating equilibrium, the L sellers go to the U submarket and the H sellers go to the I submarket.

When the mechanism maximizes the buyer's payoff, if the seller is of type L and the buyer

is informed, then there is effectively no private information, so the buyer extracts the whole surplus by receiving the good with probability 1 and paying zero. The interesting cases are when the buyer is informed and the seller is of high type or when the buyer is uninformed. In these cases, the seller has private information and the buyer wants to extract as much surplus as possible from the match. For example, when the buyer is informed and the seller is of high type, while the buyer knows his own valuation of the asset, the terms of trade still depend on the outside option of the seller. Thus, the buyer wants to trade off the probability of trade against the price. That is, the higher price the buyer pays, the higher probability that the buyer can obtain the good.

Samuelson (1984) shows that the solution to the mechanism design problem when the payoff of the party without private information (buyer) is maximized is basically a take-it-or-leave-it offer. That is, the buyer offers a price  $v$ . If the seller's outside option is smaller than  $v$ , the seller accepts the offer and gives the good to the buyer with probability 1. Otherwise, there is no trade. Formally, for the situation in which the buyer's payoff is maximized, let  $q_{IH}^B(v)$  denote the probability that an H seller with outside option  $v$  gives the asset to an informed buyer and let  $t_{IH}^B(v)$  denote the associated payment from the buyer to the seller. Similarly, denote by  $q_{IL}^B$  and  $t_{IL}^B$  the probability that an L seller gives the asset to an informed buyer and the associated payment from the buyer to the seller, respectively. Similarly, denote the probability that a seller gives the good to an uninformed buyer by  $q_U^B(v)$  and denote the associated payment by  $t_U^B(v)$ . The mechanism that maximizes the buyer's payoff is given by

$$(q_{IL}^B, t_{IL}^B) = (1, 0),$$

$$(q_{IH}^B(v), t_{IH}^B(v)) = \begin{cases} (1, v_{IH}^B) & v \leq v_{IH}^B \\ (0, 0) & v > v_{IH}^B \end{cases},$$

$$(q_U^B(v), t_U^B(v)) = \begin{cases} (1, v_U^B) & v \leq v_U^B \\ (0, 0) & v > v_U^B \end{cases}.$$

Let  $U_{IH}^B$  and  $U_U^B$  denote the expected payoff of an informed buyer when matched with an H seller and the expected payoff of an uninformed buyer, respectively, conditional on matching. These are functions of  $v_C$  but I drop this functionality to make the notation simpler. Similar to previous sections,  $v_C$  denotes the cutoff point and is a function of  $\lambda$  for a given equilibrium. The equilibrium search patterns are unchanged compared with the benchmark model as long as  $\alpha$  is small. Therefore,  $v_{IH}^B$  and  $v_U^B$ , which maximize the informed

and uninformed buyers' payoffs, respectively, are given by

$$U_{IH}^B \equiv \max_{v \in [v_C, 1]} \left\{ \underbrace{\frac{F_H(v) - F_H(v_C)}{1 - F_H(v_C)}}_{\text{prob. of trade}} \underbrace{(h - v)}_{\text{informed buyer's payoff if matched with H seller}} \right\}, \quad (16)$$

$$U_U^B \equiv \max_{v \in [0, v_C]} \left\{ \underbrace{\frac{kF_H(v) + 1 - k}{kF_H(v_C) + 1 - k}}_{\text{prob. of trade}} \underbrace{\left( \frac{kF_H(v)h + (1 - k)\ell}{kF_H(v) + 1 - k} - v \right)}_{\text{uninformed buyer's payoff if matched}} \right\}. \quad (17)$$

Regarding the informed buyer's problem, if the seller's outside option is less than  $v$ , which happens with probability  $\frac{F_H(v) - F_H(v_C)}{1 - F_H(v_C)}$  (since only H sellers with outside options greater than  $v_C$  go to the I submarket), then the buyer pays  $v$  and receives the good. If the seller's outside option is greater than  $v$ , then the buyer and the seller do not trade. A similar explanation applies to the uninformed buyer's problem.

To solve for the equilibrium measure of informed buyers,  $\lambda$ , one first needs to calculate the payoff of buyers from becoming informed,  $V_I^B(\lambda)$ , and the payoff of buyers if they remain uninformed,  $V_U^B(\lambda)$ . The net payoff of buyers from becoming informed for a given equilibrium when a measure  $\lambda$  of buyers have chosen to become informed is given by

$$\Delta V^B(\lambda) \equiv V_I^B(\lambda) - V_U^B(\lambda) = \alpha(\mathcal{Q}(\theta_I)U_{IH}^B - \mathcal{Q}(\theta_U)U_U^B),$$

where  $\theta_I$  and  $\theta_U$  denote the tightness in the I and U submarkets, respectively, calculated in the same way as in the benchmark model (from (8) and (9)). Note that with probability  $1 - \alpha$ , the mechanism maximizes the seller's payoff, in which case the buyer receives no surplus.

To understand how  $\Delta V^B(\lambda)$  changes with  $\lambda$ , one needs to know how  $V_I^B(\lambda)$  and  $V_U^B(\lambda)$  change with  $\lambda$ . These two functions are generally non-monotone without imposing further structure on the matching function. This is because the buyers' matching probability may move in an opposite direction relative to the buyers' net payoff if matched. To derive analytical results, I assume  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$  and also parameters are such that  $\theta_U > 1$ . In this case, if pooling equilibrium exists, the sellers who go to the U submarket are matched with probability  $\mu$ , and the sellers who go to the I market are matched with probability less than  $\mu$ . This implies that informed buyers are on the short side of the market and are matched with probability  $\mu$ . Hence,  $V_I^B(\lambda) = \alpha\mu U_{IH}^B$ . It is shown in the lemma below that  $U_{IH}^B$  is monotone in  $\lambda$  as long as  $v_C$  is monotone. However,  $V_U^B(\lambda)$  can be still non-monotone as shown in Examples 1 and 2.

**Lemma 1.** *Suppose Assumptions 1–3 hold. Assume that  $\mathcal{M} = \mu \min\{\theta, 1\}$  and parameters are such that  $\theta_U > 1$ . If  $v_C$  is decreasing (increasing) in  $\lambda$ , then  $V_I^B(\lambda)$  is increasing (decreasing) in  $\lambda$ .*

In the unique equilibrium for small values of  $\lambda$ , the cutoff point is decreasing in  $\lambda$  (according to Propositions 3 and 4). Therefore,  $V_I^B(\lambda)$  is increasing in  $\lambda$ . This result is interesting: When more buyers are informed, the payoff from becoming informed is higher. If  $V_I^B$  increases more than  $V_U^B$  with an increase in  $\lambda$ , then  $\Delta V^B(\lambda)$  will also increase with  $\lambda$ . In that case, the demand function for becoming informed is increasing in  $\lambda$ . This is a form of strategic complementarity in the information acquisition, and I have been able to derive it *endogenously* with a simple model.<sup>20</sup> In Example 2 below,  $V_I^B$  increases more than  $V_U^B$  in pooling 1 equilibrium.

How the expected payoff of uninformed buyers from the match,  $\mathcal{Q}(\theta_U)U_U^B$ , changes with a change in  $\lambda$  is generally ambiguous. This is because the monotonicity of  $\theta_U$  in  $\lambda$  depends on how large the changes in  $v_C$  are in response to a change in  $\lambda$ . Even if  $\theta_U$  is monotone,  $U_U^B$  may or may not be monotone. If the constraint in (17) is not binding, then  $U_U^B$  is decreasing in  $v_C$ , but if the constraint is binding, it may or may not be monotone depending on the distribution function,  $F_H$ .

I introduce Example 2 and characterize the equilibria for this. Next, I study the buyers' incentive to become informed for both Examples 1 and 2.

**Example 2.** *Parameters:  $h = 1.05$ ,  $\ell = 0.2$ ,  $k = 0.75$ ,  $n = 1$ ,  $\alpha = 0.05$ ,  $F_H(v) = (1 - \gamma_H)v + \gamma_H v^2$ ,  $v \in [0, 1]$  and  $\gamma_H \geq 0$ ,  $\mathcal{M}(\theta) = \mu \min\{\theta, 1\}$ ,  $\mu = 1$ .*

All parameters in Example 2 are the same as those in Example 1 except that  $h = 1.05$ . Also,  $\gamma_H = 0$  for both examples. In the results reported in the previous section for Example 1,  $\alpha$  was zero because the mechanism was maximizing the seller's payoff. In this section,  $\alpha = 0.05$  for both examples. Variables  $v_I^B$ ,  $v_U^B$  and  $v_C$  are depicted in Figure 9 (11) for Example 1 (2). Variables  $V_I^B$ ,  $V_U^B$ , are depicted in the upper graphs of Figure 10 (12) for Example 1 (2). In the lower graph,  $\Delta V^B(\lambda)$  is depicted for all equilibria. In Example 1,  $\Delta V^B$  for pooling 1 equilibrium is positive for  $\lambda \in [0.07, 0.22]$  and it is not monotone. In contrast, in Example 2,  $\Delta V^B$  for pooling 1 equilibrium is positive only for  $\lambda \in [0.16, 0.22]$  and it is monotone.

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<sup>20</sup>The following papers address strategic complementarities specially in financial markets: Garcia and Strobl (2011), Veldkamp and Wolfers (2007), Hellwig and Veldkamp (2009), Veldkamp (2006), Nikitin and Smith (2008). However, information acquisition in these papers is mostly about prices or aggregate variables and not about quality of the traded goods or assets, as is the case in my model.

In what follows, I focus on pooling 1 equilibrium, because it is stable and it is unique for sufficiently small values of  $\lambda$ . Consider Example 1 and the associated Figures 9 and 10. The constraint in (17) is binding for small values of  $\lambda$ , and the uninformed buyer's payoff increases with  $\lambda$ .<sup>21</sup> Regarding the informed buyers,  $V_I^B = Q(\theta_I)U_{IH}^B$  is constant for small values of  $\lambda$ , because informed buyers are matched with the highest possible probability and trade with probability 1 if matched (as the constraint is binding in (16)). Altogether, when  $\lambda$  is small,  $\Delta V^B$  decreases with  $\lambda$  in pooling 1 equilibrium in Example 1. See the lowest graph in Figure 10 when  $\lambda \in (0, 0.07)$ . When  $\lambda$  grows larger,  $V_I^B$  starts to increase because the constraint in (16) is not binding anymore. The cutoff point decreases in pooling 1 equilibrium, so more H sellers go to the I submarket. As a result, sellers who go to the I submarket have on average a lower outside option compared with those in the equilibrium with a lower  $\lambda$ . Thus, informed buyers will become more picky, as they can receive the good by making a lower price offer. In other words, the informed buyers sacrifice probability of trade in the bilateral meeting to receive the good with a lower price. In general it is ambiguous whether  $V_I^B$  increases more than  $V_U^B$ , but this is the case in Example 1 when  $\lambda \in [0.07, 0, 22]$ .

A similar explanation can be provided for Example 2. See Figures 11 and 12 associated with this example. The only difference with Example 1 is that the constraint of the informed buyer's problem is not binding here, so  $V_I^B$  is strictly increasing for all  $\lambda$  for which pooling 1 equilibrium exists. Moreover,  $V_I^B$  increases more than  $V_U^B$ , so  $\Delta V^B$  is increasing. However,  $\Delta V^B$  is negative for  $\lambda \leq 0.18$ , so buyers do not want to become informed in this region.

## 4.2 Characterizing the Equilibria of the Whole Game

In the benchmark model, whether buyers are informed or uninformed is exogenous. Now I endogenize the decision of buyers to become informed. Suppose buyers are ex-ante homogeneous, but they can incur cost  $d$  drawn from a distribution  $G$  to become informed. That is,  $G(d)$  denotes the measure of buyers whose cost of being informed is weakly less than  $d$ . Conversely, denote by  $\mathcal{G}(\lambda) \equiv \{d | G(d) = \lambda\}$  the  $\lambda$  quantile of the cost distribution. Since  $\lambda$  is endogenous in this section, it is easier to work with  $\mathcal{G}$  than  $G$  to characterize  $\lambda$ . Regarding the timing of actions, buyers first decide whether or not to become informed. Next, sellers observe the type of buyers, and the rest of the actions follow the benchmark model.<sup>22</sup>

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<sup>21</sup>If the constraint is binding in (17) at  $v = v_C$ , then the uninformed buyers trade with the seller who he meets with probability 1; therefore, the buyer pays  $v_C$  and receives the net payoff of  $\frac{kF_H(v_C)h + (1-k)\ell}{kF_H(v_C) + (1-k)} - v_C$ , which is decreasing in  $v_C$  in this example.

<sup>22</sup>The definition of equilibrium is also similar. The only point is that when  $\lambda = 0$ , then  $V_I^B(0)$  is not well-defined and would be determined by some out-of-equilibrium-belief restrictions. To define and charac-

Highest seller's outside option in pooling 1(left) and pooling 2 (right)

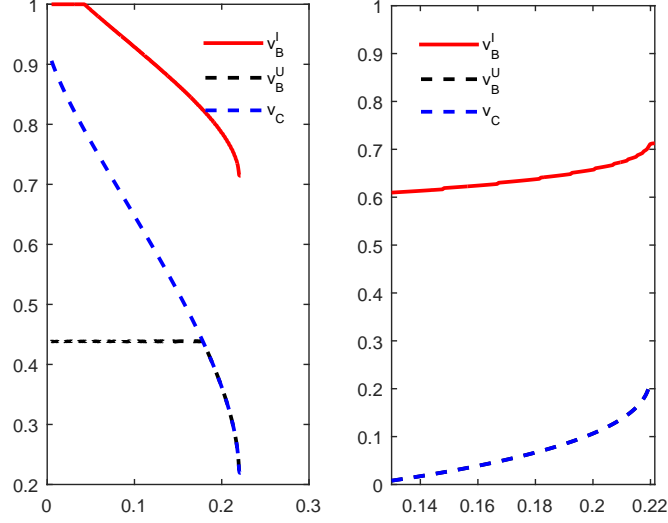


Figure 9: The highest outside option of sellers who trade with informed buyers ( $v_B^I \equiv v_{IH}^B$ ) and with uninformed buyers ( $v_B^U \equiv v_U^B$ ), and the cutoff point for going to the uninformed submarket ( $v_C$ ) are depicted here for pooling 1 (left) and pooling 2 (right) for parameters in Example 1.

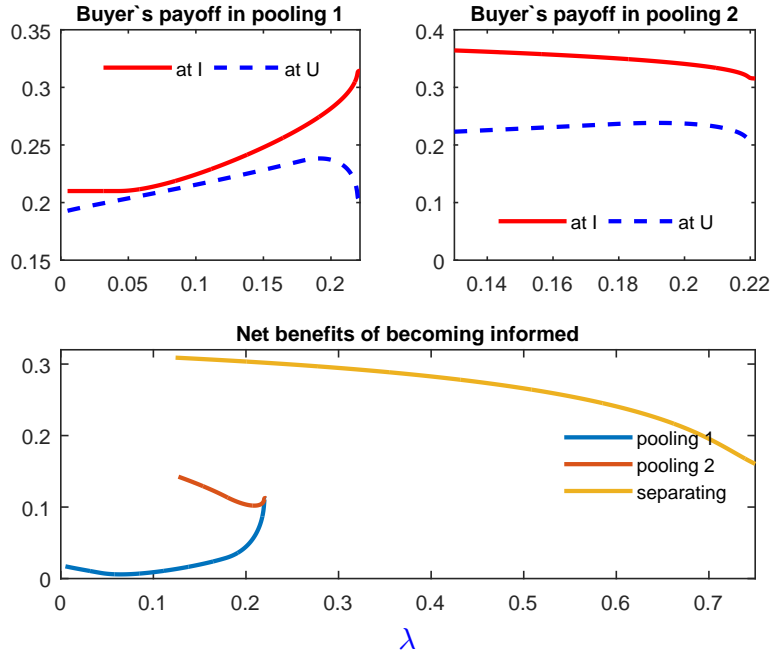


Figure 10: Buyer's payoff at informed and uninformed submarkets for pooling 1 equilibrium (upper left graph) and pooling 2 equilibrium (upper right graph), and net benefits of buyers from becoming informed in different equilibria (lower graph), for parameters in Example 1.

Highest seller's outside option in pooling 1(left) and pooling 2 (right)

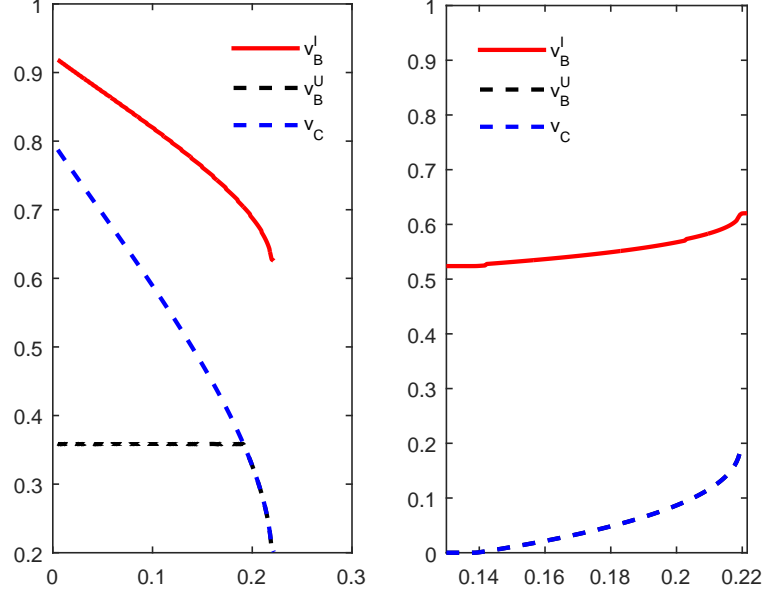


Figure 11: The description is the same as that in Figure 9 for parameters in Example 2.

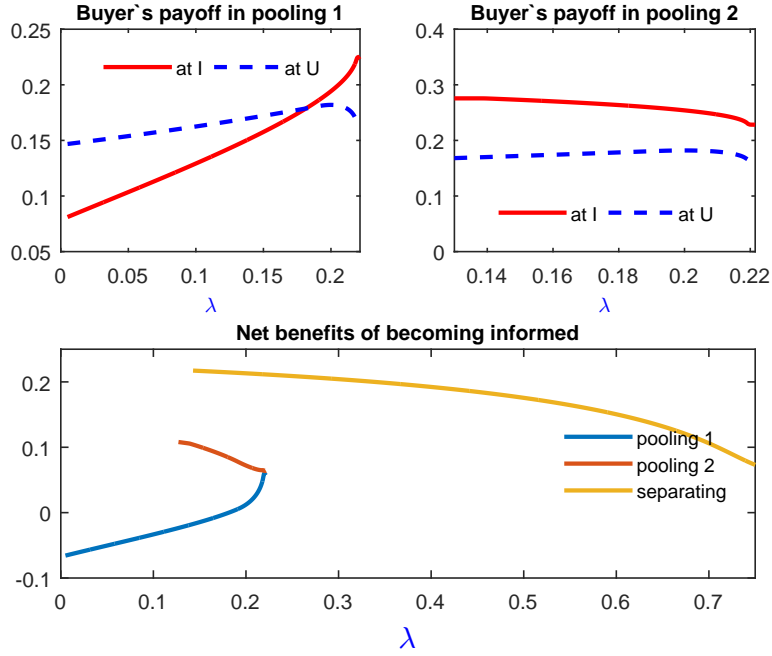


Figure 12: The description is the same as that in Figure 10 for parameters in Example 2.

Multiplicity of equilibria arises here on two levels. First, there is multiplicity resulting from different types of equilibria being played for a given  $\lambda$  (different types of pooling as well as separating). For example, if the cost of becoming informed for  $\lambda_a \in [\lambda^*, \hat{\lambda}]$  fraction of buyers is 0, and for  $1 - \lambda_a$  fraction of buyers is  $\infty$ , then there will be two pooling and one separating equilibria with parameters in Example 1. See Figure 13(a) or the lower graph in Figure 10 for illustration. Second, given a certain type of equilibrium, we can have multiple equilibria with different values of  $\lambda$ . In this section, I focus on this type of multiplicity. Especially, I focus on the pooling equilibrium with the highest cutoff point. In Examples 1 and 2, this is pooling 1 equilibrium.

It may seem trivial that buyers should always prefer to become informed. However, as mentioned in the last subsection, it is not generally true here. The reason is that although the probability of matching is higher for informed buyers than for uninformed buyers, and although the type of sellers that informed buyers attract is better on average, the sellers that are attracted to the I submarket are more expensive sellers (as only sellers with outside options higher than  $v_C$  come to the I submarket). In other words, the informed buyers attract higher quality sellers and find a match with a higher probability, but the price that they pay is higher. This implies that the benefits from becoming informed may be positive or negative. In Example 1,  $\Delta V^B(\lambda) > 0$  for all  $\lambda$ , but it is non-monotone. In Example 2,  $\Delta V^B(\lambda) < 0$  for sufficiently small values of  $\lambda$  and  $\Delta V^B(\lambda) > 0$  for others. (See Figure 13(b) or the lower graph in Figure 10.) Still there are multiple equilibria in Example 2.

Now consider Example 1 with another cost correspondence: the cost is equal to  $\bar{d}$  for all buyers, so  $\mathcal{G}(0) = [0, \bar{d}]$ ,  $\mathcal{G}(\lambda) = \bar{d}$  for  $\lambda \in (0, 1)$ , and  $\mathcal{G}(1) = [\bar{d}, \infty]$ . If  $\bar{d} < \min_{\lambda \in [0, \hat{\lambda}]} \{\Delta V^B(\hat{\lambda})\}$ , then there will be no pooling equilibrium. See Figure 14 for illustration. If  $\bar{d} > \min_{\lambda \in [0, \hat{\lambda}]} \{\Delta V^B(\hat{\lambda})\}$ , then there are 2 equilibria. The equilibrium with  $\lambda_c$  is stable and the one with  $\lambda_d$  is not.<sup>23</sup> Both equilibria have the feature that although the cost of becoming informed is the same for all buyers, but ex-post some buyers are informed and

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terize equilibrium, buyers need to know their payoff from becoming informed if all buyers are uninformed. This payoff can be defined by imposing some out-of-equilibrium-belief restrictions taken from, for example, Guerrieri et al. (2010). One may alternatively assume that the payoff of buyers in any equilibrium should be continuous in  $\lambda$ . I choose the latter. For example, when  $\lambda = 0$ , if a small  $\epsilon > 0$  measure of buyers become informed, then they get  $V_I^B(\epsilon)$ , which is well-defined. The continuity restriction requires that  $V_I^B(0) = \lim_{\epsilon \rightarrow 0} V_I^B(\epsilon)$ .

<sup>23</sup>Regarding the stability, similar to the notion of stability in the benchmark model, suppose a small fraction of buyers make a mistake in acquiring information technology. If other buyers still want to follow their equilibrium strategy, then the equilibrium is stable. Consequently, if the cost correspondence intersects  $\Delta V^B$  from below, then the equilibrium is stable.

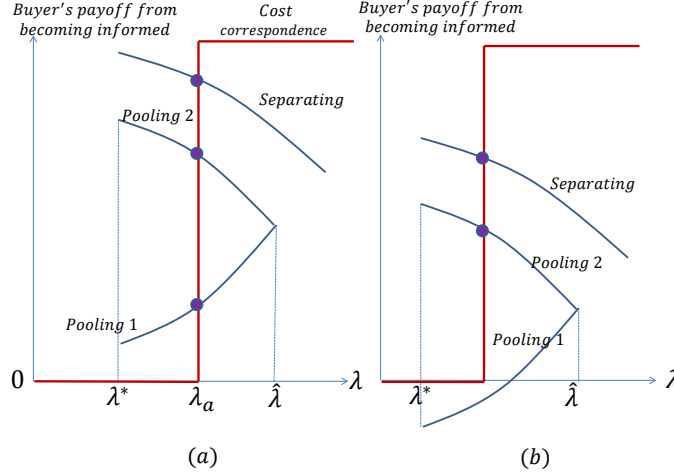


Figure 13: Illustration of equilibria in Example 1 (2) in the left (right) graph for the case in which the cost of becoming informed is 0 for a fraction  $\lambda_a$  of buyers and is  $\infty$  for other buyers.

some are not. Moreover, taxing information acquisition decreases  $\lambda_c$  and increases  $\lambda_d$ . Since the equilibrium with  $\lambda_c$  is stable, I focus on this equilibrium. Imposing tax on acquiring information lowers  $\lambda_c$  in this equilibrium. That is, fewer buyers become informed. As a result, the total cost spent on becoming informed decreases. Welfare (excluding information costs) is decreasing with  $\lambda$  in pooling 1 equilibrium as depicted in the lower left graph of Figure 7.<sup>24</sup> Therefore, this policy of taxing information acquisition is welfare enhancing.

If  $\bar{d} > \Delta V^B(0)$ , again conditional on pooling 1 equilibrium being played, there is a unique equilibrium, but this equilibrium is unstable. The policy of taxing information acquisition may not be helpful in this region, as it leads more buyers to acquire information and does not solve the problem of instability. The policy of subsidizing information acquisition is more helpful, because it decreases  $\lambda$ . Subsequently, the cost of becoming informed reduces and

<sup>24</sup>Abstracting from information acquisition costs, welfare and aggregate trading volume can be simply calculated as follows:

$$\begin{aligned}
 W(\lambda, v_C, v_H) &= k\mathcal{M}\left(\frac{\lambda n}{k(F_H(v_H) - F_H(v_C))}\right) \left( (1 - \alpha_B) \int_{v_C}^{v_H} (h - v) f_H(v) dv + \alpha_B \int_{v_C}^{v_I^B} (h - v) f_H(v) dv \right) \\
 &\quad + \mathcal{M}\left(\frac{(1 - \lambda)n}{kF_H(v_C) + 1 - k}\right) \left[ k \left( (1 - \alpha_B) \int_0^{v_C} (h - v) f_H(v) dv + \alpha_B \int_0^{v_U} (h - v) f_H(v) dv \right) + (1 - k)\ell \right], \\
 L(\lambda, v_C, v_H) &= k\mathcal{M}\left(\frac{\lambda n}{k(F_H(v_H) - F_H(v_C))}\right) ((1 - \alpha_B)(F_H(v_H) - F_H(v_C)) + \alpha_B(F_H(v_I^B) - F_H(v_C))) \\
 &\quad + \mathcal{M}\left(\frac{(1 - \lambda)n}{kF_H(v_C) + 1 - k}\right) (kF_H(v_C) + 1 - k).
 \end{aligned}$$

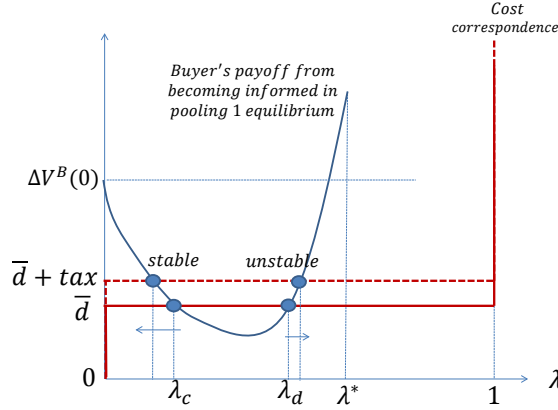


Figure 14: Illustration of equilibria in Example 1 when the cost of becoming informed is identical for all buyers.

the total surplus increases. Moreover, if the subsidy is sufficiently high, then the stable equilibrium may come into existence. That is, this policy of subsidizing information acquisition may be both stabilizing and welfare enhancing.

Several lessons can be learned from the exercise in this section. First, the intervention in the market in the form of taxation/subsidization in this environment is most of the time welfare enhancing and may even lead to stability of equilibrium. Second, the type of policy intervention, whether taxation or subsidization, depends on the details of the model, e.g., the equilibrium being played and the shape of the cost correspondence. Therefore, finding the optimal intervention is difficult. Third, it is difficult to avoid instability in this environment. For instance, as shown in Example 1, if  $\lambda \in (\lambda^*, \hat{\lambda})$ , then there are three equilibria and one of them (pooling 2) is unstable. Even for pooling 1 equilibrium, if  $\lambda$  is determined endogenously, the resulting equilibrium with endogenous  $\lambda$  may be unstable because the benefits of becoming informed may be increasing in  $\lambda$ . This is similar to situations in which the demand function is increasing in a certain region, so instability of equilibrium is natural if the slope of the cost correspondence is smaller than the slope of the benefits from becoming informed.

Some scholars argue that trading activity in OTC markets, especially markets for collateralized debt obligations, asset-backed securities and commercial paper was zero or close to zero during the recent financial crisis ([Gorton and Metrick \(2012\)](#)). Consider sellers in my model as originators of non-agency asset-backed securities who have heterogeneous outside valuations for their high-quality, say AAA, assets. Consider buyers in my model as investors

who want to buy asset-backed securities and may have heterogeneous costs of acquiring information technology that enables them to recognize the true quality of these securities. This technology can be acquired by hiring several specialists to evaluate the quality of assets. Suppose the economy prior to the financial crisis has been at the first equilibrium as described above (with  $\lambda_c$  measure of informed buyers), but as the result of a shock to beliefs (a non-fundamental shock), the equilibrium switched to the second one (with  $\lambda_d$  measure of informed buyers) in which more buyers acquire information and consequently, the measure of high type sellers who go to the U submarket decreases, and, importantly, the price in the uninformed submarket falls. The welfare and trading volume in the second equilibrium are lower than those in the first equilibrium (according to Proposition 6). More buyers invest in acquiring information technology, so more resources are wasted on that. Sellers who have high-quality assets prefer to wait longer (matched with a lower probability) to trade with informed buyers rather than selling to uninformed buyers faster (as depicted in the left graphs in Figure 7).<sup>25</sup> The chain of events explained above is reminiscent of the chains of events during the recent financial crisis.

## 5 Planner's Problem

In this section, I characterize the planner's problem when he does not face any informational frictions. The buyers are homogeneous from the planner's point of view. It is easy to see that the planner must choose a cutoff point for high type sellers, denoted by  $v_P$ , above which the seller does not participate, and a probability that the low type sellers should be matched, denoted by  $s_L$ . The planner's problem with complete information can be written as

$$\max_{(v_P, s_L) \in [0, 1]^2} \mathcal{M} \left( \frac{n}{kF_H(v_P) + (1 - k)s_L} \right) \left( k \int_0^{v_P} (h - \nu) dF_H(\nu) + (1 - k)s_L \ell \right).$$

Denote the solution to this problem by  $(v_P^*, s_L^*)$ .

The planner, having complete information, first wants to match those agents who create higher social surplus. Assume for simplicity that  $h - 1 > \ell$ . Since all H sellers create more surplus than L sellers (as  $h - v \geq \ell$  for all  $v \in [0, 1]$ ), the planner matches buyers with H sellers, beginning from the sellers with the lowest outside option  $v = 0$ . Adding one more seller imposes a negative externality on other sellers and positive externality on other buyers because of the matching technology. The planner keeps adding sellers to the market until

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<sup>25</sup>If the equilibrium is switched to pooling 2 or separating equilibrium, the consequences in terms of quality and welfare will be even more severe.

the value created by the marginal seller is equal to the negative externality that this seller imposes on others. If all H sellers are matched but still the value that an L seller creates exceeds the negative externality imposed on others by adding an L seller to the market, the planner adds L sellers too until either all L sellers are matched or the surplus created by an L seller is equal to the negative externality imposed on others by this seller's entry. This discussion is summarized in the following proposition.

**Proposition 7.** *Assume  $h - 1 \geq \ell$ . Also assume that  $\mathcal{M}$  and  $F$  are continuously differentiable, and  $1/\mathcal{Q}$  is convex. Define the elasticity of matching function by  $\eta(\theta) \equiv \frac{\theta \mathcal{M}'(\theta)}{\mathcal{M}(\theta)}$ . Under complete information, the solution to the planner's problem is given by*

$$\begin{cases} s_L = 0 \text{ and } v_P \text{ solves (19),} & n \leq n_1 \\ s_L \text{ solves (20) and } v_P = 1, & n_1 < n \leq n_2, \\ s_L = 1 \text{ and } v_P = 1, & n_2 < n \end{cases}$$

where  $n_1 = k\eta^{-1}\left(\frac{h-1}{\int_0^1 (h-\nu)dF_H(\nu)}\right)$ ,  $n_2 = \eta^{-1}\left(\frac{\ell}{k\int_0^1 (h-\nu)dF_H(\nu) + (1-k)\ell}\right)$ , and (19) and (20) are given in the proof.

This result also reveals that if  $h - \ell > 1$  and as long as buyers are sufficiently scarce ( $n < n_2$ ), the probability of matching for H sellers is higher than L sellers, opposite to the equilibrium allocation. This is intuitive because H sellers have a higher value for the planner so they are matched first, and if there are enough buyers remaining, then L sellers are matched. In contrast, in equilibrium, the informed buyers are more attractive for H sellers, so the probability of matching is endogenously lower for H sellers relative to L sellers.

## 6 Discussion

In this section I discuss some assumptions of the paper. I then make comments about the constrained planner's problem and explain how my paper is related to some papers in the literature.

### 6.1 Zero Outside Option for Low-Quality Sellers

I have assumed that the outside option of L sellers is identical and is lower than that of H sellers. If the outside option of some L sellers is greater than that of some H sellers, then the patterns of search may change. In particular, some L sellers may go to the I submarket

even when  $\lambda$  is small, so the fraction of L sellers in the U submarket would be varying with  $\lambda$ . However, this does not change the main insights of the paper. It can be shown that under some parameters, there will still be adverse effects of higher  $\lambda$  along some equilibria and there will be multiplicity of equilibria for some  $\lambda$ .

## 6.2 Ex-Ante Observability of Buyers' Types

Observing the type of buyers before agents are matched has quite different implications than observing it after matching. To show this, I assume in an alternative setting that buyers' types are unobservable until after matching. That is, search is random. The same trading protocol as in the benchmark model applies.

With random search, any seller can receive a strictly positive payoff from entering the market. This is because the H sellers matched with an informed buyer receive  $h - v > 0$  for all  $v \in [0, 1]$  and the L sellers matched with an informed buyer receive  $\ell > 0$ . The seller who is matched with an uninformed buyer receives at least a zero payoff. Therefore, all sellers enter the market, and changes in  $\lambda$  do not change the search decision of sellers. The average probability of trade for buyers increases monotonically with  $\lambda$  as informed buyers trade with probability 1 while uninformed buyers may trade with a lower probability. The terms of trade are independent from  $\lambda$ . Consequently, welfare weakly increases with  $\lambda$  too. This argument clearly shows that the assumption that the type of buyers is observable (or the markets for informed and uninformed buyers are segmented) is crucial for the results regarding multiplicity and adverse effects of an increase in  $\lambda$ .

Intuitively in the random search version, an increase in  $\lambda$  does not induce high-quality sellers to exit from the uninformed submarket; rather, sellers can find informed buyers with a higher probability. This is different from the predictions of the benchmark model in pooling 1 equilibrium in Example 1, in which such an increase will substantially increase the incentive of H sellers to meet with informed buyers, so the probability of matching with informed buyers for H sellers does not increase as much.

## 6.3 Who Are the Informed Buyers?

[Philippon and Skreta \(2012\)](#) divide the finance sector into three sub-sectors: credit intermediation, insurance and other finance. Other finance is small but includes sophisticated financial institutions, such as venture capitalists, hedge funds, trusts, and investment banks. These institutions can fit the descriptions of the informed buyers in my model. [Philippon and](#)

[Skreta \(2012\)](#) document that in other finance, the relative wage has increased enormously since 1980. This finding is consistent with the findings of my model in both Examples 1 and 2 in Section 4, where I show that in the unique stable equilibrium when measure of informed buyers is small, the payoff of informed buyers is increasing in the measure of informed buyers. Presumably, the cost of information technology in recent years has decreased, which has led to an increase in the size of the informed segment of the market.

## 6.4 Constrained Planner's Problem

The constrained planner faces the same information constraints that agents do, but the constrained planner can design the trading mechanism in the bilateral meetings so as to maximize the total welfare of the agents. If the buyer is informed, the planner can condition the allocation on the type of the seller, but the outside valuation of the seller is still private information, so the planner faces the IC constraint of the seller, and the IR constraint of the buyer and the seller. If the buyer is uninformed, the planner faces the IC constraint of the seller (both for the seller's type and outside valuation), and again the IR constraint of the buyer and the seller. Characterizing such a planner's problem is generally difficult. However, it is not difficult to characterize its solution within a certain class of mechanisms. Consider a single-stepped mechanism in which  $q(v) = 1$  for  $v \leq v_H$ , and  $q(v) = 0$  otherwise. It is easy to show that such a mechanism with optimally chosen  $v_H$  can lead to higher welfare than the pooling equilibrium. In the optimal mechanism, the planner sets  $v_H < 1$  for informed buyers. The key is that by setting  $v_H$  to be less than 1, the planner increases the probability of matching in the I submarket so that H sellers with lower outside options can be matched with a higher probability. In contrast, in equilibrium, since all H sellers with  $v \in [v_H, 1]$  go to the I submarket as well, some H sellers with  $v$  less than  $v_H$  who create more surplus cannot match. This simple mechanism can increase welfare compared with equilibrium. A simple way to implement this mechanism is to impose a price ceiling in the trades with informed buyers. This policy would restrict the entry to the informed market, thus increasing welfare.

## 6.5 Relationship Between My Paper and [Fishman and Parker \(2015\)](#), [Bolton et al. \(2016\)](#), and [Kurlat \(2016\)](#)

As mentioned in the Introduction, although my paper and [Fishman and Parker \(2015\)](#) share the same type of heterogeneity of buyers, there are important differences. One difference is that I have a search-and-matching framework, while they do not. I do not consider any

advantage for informed buyers in terms of how they trade with sellers, so this framework is crucial for my results. Another difference is that although the mechanism through which the price for uninformed buyers (“unsophisticated investors” in the language of [Fishman and Parker \(2015\)](#)) responds to the average quality of sellers in my paper is similar to theirs, welfare loss in my model and their model occurs for different reasons. In their paper, welfare is different *across* different equilibria (indeed, equilibria can be Pareto ranked), because in some of the equilibria investors try to value the assets more than the optimal level (which happens to be zero). In my paper, equilibrium is unique in a region of parameters and welfare may decrease in this region *along* one equilibrium because too many sellers try to meet with informed buyers. Even when there is multiplicity, welfare is different *across* different equilibria not because there is too much valuation (as valuation for informed buyers in my benchmark model is costless). In contrast, this is because in the equilibrium with a lower price there are too many high-quality sellers trying to meet informed buyers and many of them cannot find a match.

[Bolton et al. \(2016\)](#) have a model with an endogenous measure of informed and uninformed buyers and show that the size of the informed side of the market is inefficient. This result is similar to my result in Section 4. In addition to the differences above, the trading mechanism in their paper is exogenously different for informed and uninformed buyers. Informed buyers trade in an OTC market while uninformed buyers trade in a competitive market. Also, search is sequential in their paper; if a seller cannot find a match in the OTC markets, the seller will get a chance to sell in the competitive market.

My paper is closely related to [Kurlat \(2016\)](#) in terms of the environment, but he takes a notion of Walrasian competitive equilibrium while I take a search-and-matching framework. My focus in this paper is on the implications of the size of the informed segment of the market on various measures of market activity, while his focus is on the set of markets that endogenously arises in equilibrium, and he finds sufficient conditions under which fire sales or flights to quality occur. His equilibrium is typically unique, while a multiplicity of equilibria arises in my environment as a result of the strategic complementarity in the choice of sellers to meet uninformed buyers (and also in the choice of buyers to become informed).

## 7 Conclusion

In this paper I have studied the implications of buyers’ heterogeneity with respect to their information regarding the quality of assets in a model of OTC markets. In the model, sellers

have private information about the quality of their asset and their outside option. The results can be summarized as follows.

First, I show that when the measure of informed buyers is sufficiently small, the equilibrium is unique and stable. Surprisingly, welfare and trading volume may be decreasing in the measure of informed buyers in this region. That is, more information has adverse effects on the market. Second, I show that such markets are subject to multiplicity (and instability) of equilibria. Multiplicity arises as a result of the strategic complementarity between sellers in choosing the type of buyers that they want to trade with. More specifically, if more high-quality sellers choose to meet uninformed buyers, the price that uninformed buyers are willing to pay increases, which in turn increases the incentive of other high-quality sellers to meet uninformed buyers. This leads to multiplicity and instability of equilibria. Third, I endogenize the measure of informed buyers by allowing buyers to invest in an information technology by incurring a cost to *become* informed. The model with endogenous measure of informed buyers generates other interesting results and policy implications. Depending on the cost distribution for buyers, there might be unique or multiple equilibria. In the case of multiplicity, some equilibria are unstable. When the cost is identical for all buyers, i.e., buyers are ex-ante homogenous, buyers may be ex-post heterogeneous in that some choose to become informed and some do not. Furthermore, there generally exist tax/subsidy schemes on information acquisition that increase welfare and trading volume. These schemes can eliminate multiplicity and instability, but they require the planner to have precise information about the cost distribution, making such interventions difficult in practice.

I have abstracted from many aspects of OTC markets in this paper. For future research, the following extensions or modifications can be considered. First, in this paper, sellers are exogenously endowed with the asset and have private information about the asset's value and their outside option. Some assets that have been arguably prone to private information in the recent financial crisis were private-labeled mortgage-backed securities. One could consider a richer environment in which the process of secularization of assets is explicitly modeled and then study the optimal intervention, as well as welfare and stability implications of heterogeneity of buyers. Second, adding dynamics to the model would allow one to study other interesting questions. For example, one can investigate whether the informed buyers would endogenously emerge as intermediary. That is, they may buy the asset from only high-quality sellers not for the value that they receive from the consumption but to sell it at a future date to other buyers. Furthermore, one can study the agents' incentive to form long-term relationships, and how this incentive affects sellers' entry and submarket decisions.

Finally, since there are multiple equilibria in the static version, introducing dynamics would make it possible to study sunspot equilibria. One could study the dynamics of asset prices and other market indexes following a shock to non-fundamentals of the economy, and could probably offer a richer explanation for the chain of events during the recent financial crisis. These are all left for future research.

## Appendix: Omitted Proofs

*Proof of Proposition 2.* In the text, it was argued that  $\pi_{IH}(v)$  and  $\pi_U(v)$  have at most two intersections. I want to show here that the allocations illustrated in cases (b) and (c) of Figure 1 cannot be equilibrium allocations.

**Case (b): H sellers with outside options  $[0, v_1]$  and  $[v_2, 1]$  and L sellers go to the U submarket and other H sellers go to the I submarket.**

Suppose such equilibrium exists. The slope of  $\pi_U(v)$  is greater than that of  $\pi_{IH}(v)$  at  $v = v_1$ , so  $\mathcal{M}(\theta_I) < \mathcal{M}(\theta_U)q_U(v_1)$ . Similarly one obtains  $\mathcal{M}(\theta_I) > \mathcal{M}(\theta_U)q_U(v_2)$ . Together,  $q_U(v_2) < \frac{\mathcal{M}(\theta_I)}{\mathcal{M}(\theta_U)} < q_U(v_1)$ . In this type of equilibrium,  $\pi_U(1) > 0$ , so the participation constraint in Problem 2 is not binding, therefore,  $q_U(1) = 1$ . But  $q_U(v)$  is decreasing, so  $q_U(v) = 1$  for all  $v$ . This is a contradiction with  $q_U(v_2) < q_U(v_1)$ , so this type of equilibrium does not exist.

**Case (c): H sellers with outside options  $[v_2, 1]$  and L sellers go to the U submarket and other H sellers go to the I submarket.**

Again,  $\pi_U(1) > 0$ , so  $q_U(v) = 1$  for all  $v$ . Comparison of slopes at  $v = v_1$  yields  $\mathcal{M}(\theta_I) > \mathcal{M}(\theta_U)$ . This implies that  $\mathcal{M}(\theta_I)(h-v) > \mathcal{M}(\theta_I)(t_U(v) - q_U(v)v)$  for all  $v$ , because  $t_U(v) < h$  and  $q_U(\cdot) = 1$ . Therefore, all H sellers can receive a strictly higher payoff in the I submarket than the U submarket. Hence this cannot be an equilibrium allocation, because in the considered allocation, H sellers with high outside options go to the U submarket.  $\square$

*Proof of Proposition 3.* For part (i), it was shown in the text that  $\lim_{\lambda \uparrow 0} J(\lambda) \geq 0$  if and only if the pooling equilibrium exists. Notice that

$$\lim_{\lambda \uparrow 0^+} J(\lambda) \geq \lim_{\lambda \uparrow 0^+} G(\lambda, 0) = 0 + \mathcal{M}\left(\frac{n}{1-k}\right)\ell > 0.$$

Therefore, pooling equilibrium exists for  $\lambda$  sufficiently small. If  $\lambda > \hat{\lambda}$ , then  $J(\lambda) < J(\hat{\lambda}) = 0$ , so pooling equilibrium does not exist for  $\lambda > \hat{\lambda}$ .

For part (ii),  $G(\lambda, 0) < 0$  iff  $-\mathcal{M}(\frac{\lambda n}{k})h + \mathcal{M}(\frac{(1-\lambda)n}{1-k})\ell < 0$ . Also,  $G(\lambda, 1) < 0$ . Therefore, it must be the case that  $G(\lambda, v) = 0$  has generically an even number of roots. See Figure 2 for illustration.

For part (iii), I introduce the following functions in this proof to reduce and simplify the notation:  $A(\lambda, v) \equiv \frac{\lambda n}{k(1-F_H(v))}$  and  $B(\lambda, v) \equiv \frac{(1-\lambda)n}{kF_H(v)+1-k}$ . It follows that  $\frac{A_\lambda(\lambda, v)}{A(\lambda, v)} = \frac{f_H(v)}{1-F_H(v)}$  and  $\frac{B_\lambda(\lambda, v)}{B(\lambda, v)} = \frac{kf_H(v)}{kF_H(v)+1-k}$ . Also, define  $\eta(\theta) \equiv \frac{\mathcal{M}'(\theta)\theta}{\mathcal{M}(\theta)}$ . Now, we can write  $G$  function as follows:

$$G(\lambda, v) = -\mathcal{M}(A(\lambda, v))(h-v) + \mathcal{M}(B(\lambda, v))(P(v)(h-\ell) + \ell - v).$$

Suppose there are at least two solutions for  $G(\lambda, v) = 0$  for any given small  $\lambda > 0$ . For one of the solutions, we must have both  $G(\lambda, v) = 0$  and  $G_v(\lambda, v) > 0$ . (See the right graph in Figure 2.) Now note that

$$\begin{aligned}
0 < G_v(\lambda, v) &= -\frac{\mathcal{M}'(A(\lambda, v))A_v(\lambda, v)}{\mathcal{M}(A(\lambda, v))}\mathcal{M}(A(\lambda, v))(h-v) + \mathcal{M}(A(\lambda, v)) \\
&+ \mathcal{M}(B(\lambda, v))(P_v(v)(h-\ell)-1) + \frac{\mathcal{M}'(B(\lambda, v))B_\lambda(\lambda, v)}{\mathcal{M}(B(\lambda, v))}\mathcal{M}(B(\lambda, v))(P(v)(h-\ell)+\ell-v) \\
&= \left( \underbrace{\eta(B(\lambda, v))\frac{B_v(\lambda, v)}{B(\lambda, v)}}_{<0} - \underbrace{\eta(A(\lambda, v))\frac{A_v(\lambda, v)}{A(\lambda, v)}}_{>0} \right) \underbrace{\mathcal{M}(A(\lambda, v))}_{>0} (h-v) \\
&\quad + \underbrace{\mathcal{M}(A(\lambda, v))}_{\rightarrow 0} + \underbrace{\mathcal{M}(B(\lambda, v))(P_v(v)(h-\ell)-1)}_{< \epsilon < 0},
\end{aligned}$$

where the second equality comes from  $G(\lambda, v) = 0$ . The first term above (the whole expression in the third line) is negative. To derive a contradiction, I show below that when  $\lambda$  is close to 0, the second term is close to 0 too, and the third term is negative and bounded away from 0. Therefore, the sum of them cannot be strictly positive. Regarding the second term, when  $\lambda \rightarrow 0$ , we must have  $\mathcal{M}(A(\lambda, v)) \rightarrow 0$ . Otherwise,  $v$  must go to 1, in which case  $C(\lambda, v) \rightarrow k$ , so  $G(\lambda, v) \rightarrow -\mathcal{M}(A(\lambda, 1))(h-1) + \mathcal{M}(B(\lambda, 1))(kh + (1-k)\ell - 1) < 0$ , which is a contradiction with the fact that  $G(\lambda, v) \rightarrow 0$ ; therefore,  $v$  cannot go to 1. The facts that  $\mathcal{M}(A(\lambda, v)) \rightarrow 0$  and  $G(\lambda, v) = 0$  imply that  $P(v)(h-\ell) + \ell - v \rightarrow 0$ ; therefore,  $P_v(v)(h-\ell) + \ell - 1 < 0$  by Assumption 3. Thus, the third term is strictly negative away from 0. That is, it is less than  $-\epsilon$  for some  $\epsilon > 0$  sufficiently high. This completes the proof.  $\square$

*Proof of Prop. 4.* I write (10) in the following form:

$$\frac{\mathcal{M}\left(\frac{\lambda n}{k(1-F_H(v_C))}\right)}{\mathcal{M}\left(\frac{(1-\lambda)n}{kF_H(v_C)+1-k}\right)} = \frac{P(v_C)(h-\ell) + \ell - v_C}{h - v_C}$$

I plot both LHS and RHS against  $v$  in Figure 15. The LHS is increasing in  $v$ . The LHS is increasing in  $\lambda$ , but the RHS is independent of  $\lambda$ . For a given  $\lambda$ , when  $v_C \rightarrow 1^-$ , the  $RHS|_{v=1} < 0 < LHS|_{v=1}$ . Therefore, for the highest root and all the odd-ranked roots, the RHS intersects the LHS from above, and an increase in  $\lambda$  decreases the root. An increase in  $\lambda$  changes even-ranked roots in the opposite direction compared with odd-ranked roots. This completes the proof.

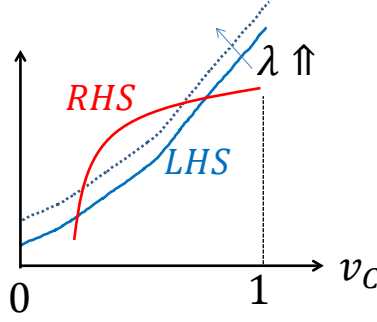


Figure 15: Illustration of the proof of Proposition 4 for comparative statics with respect to  $\lambda$ .

□

*Proof of Proposition 5.* The proof for welfare came in the text. For trading volume, the proof is similar.

$$\frac{dT}{d\lambda} = \underbrace{\frac{\partial T}{\partial \theta_I}}_{\geq 0} \underbrace{\frac{\partial \theta_I}{\partial \lambda}}_{\geq 0} + \underbrace{\frac{\partial T}{\partial \theta_U}}_{=0} \frac{\partial \theta_U}{\partial \lambda} + \left[ \underbrace{\frac{\partial T}{\partial \theta_I}}_{\geq 0} \underbrace{\frac{\partial \theta_I}{\partial v_C}}_{\geq 0} + \underbrace{\frac{\partial T}{\partial \theta_U}}_{=0} \frac{\partial \theta_U}{\partial v_C} \right] \frac{dv_C}{d\lambda} + \underbrace{\frac{\partial T}{\partial v_C}}_{\geq 0} \frac{dv_C}{d\lambda}. \quad (18)$$

If  $\frac{dv_C}{d\lambda} > 0$ , as is the case in the even-ranked pooling equilibria, then this whole expression is positive. □

*Proof of Lemma 1.* By assumption we have  $\mathcal{M}(\theta_U) = \mu$  and  $\mathcal{Q}(\theta_U) = \mu/\theta_U$ . In the pooling equilibrium,  $\mathcal{M}(\theta_I) < \mathcal{M}(\theta_U) = \mu$ . Thus,  $\theta_I < 1$ , so  $\mathcal{Q}(\theta_I) = \mu$ . Therefore, the probability of matching for informed buyers is independent of  $\lambda$ .

Suppose that the constraint in the buyer's maximization problem, (16), is binding at  $v = 1$ , so the informed buyers make the highest offer ( $v = 1$ ). Since they trade with probability 1 (with all H sellers who come to the I submarket) and the price that they pay is fixed, their payoff is independent of  $\lambda$ , so their net payoff is weakly increasing in  $\lambda$ . Now suppose that the constraint is not binding; then a simple envelope argument shows that  $U_I^B$  is decreasing in  $v_C$ . Finally, the constraint cannot be binding at  $v = v_C$ , because it would deliver the buyer a zero payoff. Therefore,  $U_{IH}^B$  must be weakly decreasing in  $v_C$ . This completes the proof. □

*Proof of Proposition 7.* Since  $h - 1 \geq \ell$ , all H sellers create more surplus than L sellers, so the planner first allocates H sellers, and then, if there are more opportunities for matching, allocates L sellers. The first order conditions with respect to  $v_P$  and  $s_L$  can be simplified as

follows:

$$FOC(v_P) : -\eta(\theta_P) + \frac{(kF_H(v_P) + (1-k)s_L)(h-v_P)}{k \int_0^{v_P} (h-\nu) dF_H(\nu) + (1-k)s_L \ell} \begin{cases} \geq 0, & v_P = 1 \\ = 0, & v_P \in (0, 1) \\ \leq 0, & v_P = 0 \end{cases} \quad (19)$$

$$FOC(s_L) : -\eta(\theta_P) + \frac{(kF_H(v_P) + (1-k)s_L)\ell}{k \int_0^{v_P} (h-\nu) dF_H(\nu) + (1-k)s_L \ell} \begin{cases} \geq 0, & s_L = 1 \\ = 0, & s_L \in (0, 1) \\ \leq 0, & s_L = 0 \end{cases} \quad (20)$$

where  $\theta_P = \frac{n}{kF_H(v_P)}$ .<sup>26</sup>

But  $h-1 \geq \ell$ , so  $h-v_P \geq \ell$ , therefore, the only possible solutions are as follows:

$$\begin{cases} v_P \leq 1, & s_L = 0 \\ v_P = 1, & s_L > 0 \end{cases}$$

One can summarize the FOCs to look exactly like those in the proposition. This completes the proof.

Note that the solution can be found in exactly the same manner for the other case when  $h-1 < \ell$ , but it involves a great deal of algebra so is omitted from the proposition to save space. Similarly as above, one would simply need to consider the following scenarios for

$$h-1 < \ell: \begin{cases} v_P \leq h-\ell, & s_L = 0 \\ v_P = h-\ell, & 0 < s_L < 1, \text{ and then use the FOCs to pin down } v_P \text{ and } s_L. \\ v_P \geq h-\ell, & s_L = 1 \end{cases}$$

□

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<sup>26</sup> Note that  $\eta(\theta)$  is decreasing in  $\theta$ , because  $\eta(\theta) = \frac{\mathcal{M}''}{q}(1 - \frac{\mathcal{M}'q'}{\mathcal{M}''q}) \leq 0$ , which is true by the assumption that  $\frac{\theta}{\mathcal{M}(\theta)}$  is a convex function, resulting from the following:

$$0 > -\left(\frac{\theta}{\mathcal{M}(\theta)}\right)'' = \frac{q''q - 2q'q'}{q^3} = \frac{\mathcal{M}''q - 2q'\mathcal{M}'}{\theta q^3},$$

where the second equality holds because  $\mathcal{M}'' = \theta q'' + 2q'$ .

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