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Cover Page Footnote
The author is indebted to an anonymous referee for helpful suggestions.
On the Behaviour of Entrepreneurial Factor Supply to the Firm

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ABSTRACT

This paper draws on an existing, but little used, approach to the choices governing the supply of ‘entrepreneurial’, in the sense of ‘residually remunerated’, resources to an enterprise, especially post start up. It focuses in particular on the hybrid ‘own factor demand/supply curve’ to the firm of Bronfenbrenner (1960), but attempts to treat such supply in conjunction with ‘contractual’ employment of resources, thus making use of gearing and portfolio concepts. To achieve this, it is found necessary for the hybrid schedule to be reinterpreted and recast as the locus of the relevant utility maximising choices. A model is presented which features combining all productive resources in a single factor F so as to concentrate on the choice between the entrepreneurial and contractual employment of these. The model identifies a variety of possible entrepreneurial resource supply responses to product market signals, including the possibility of divergence from the ‘normal’ expectation of a monotonically upward sloping supply curve. Such ‘unexpected’ behaviour is seen as rational and consistent, both with the needs of survival in distress and of safeguarding gains in the upside. Hence it is also efficient on welfare grounds. Analogies with the backward bending labour supply model and with risk/return choices in finance are pursued. The policy implication is that greater use of equity, especially in startups and distress situations, should be encouraged.

Keywords: Entrepreneurial, Factor Supply, Utility, Gearing, Backward Bend
JEL Codes: D01, D21, G3

I. Introduction

This paper deals with a specific aspect of factor supply namely the supply of resources to a firm by its owners. The centrepiece of the discussion is the hybrid

1 The author would like to thank an anonymous referee for helpful suggestions.
supply/demand schedule which traces the firm’s demand for/supply of own resources as a function of prospective returns. This schedule features in Bronfenbrenner’s (1960) rather neglected contribution. The discussion explores the nature and properties of the schedule. It leads us to question the standard expectation that such supply will be of the ‘normal’, monotonically upward sloping shape. Relatedly, it examines how far the ceteris paribus assumptions underlying supply curves are appropriate in this context.

The definition of entrepreneurship and risk is tied up with the definition of profit. Bronfenbrenner’s (1960) particular conceptual scheme defines ‘entrepreneurial’ resources, narrowly, as those resources supplied with only a residual (as opposed to contractually fixed) claim to remuneration. More broadly, we are here in the Knightian tradition which identifies as ‘entrepreneurial’ those resources the suppliers of which bear the risk, expecting to realise a ‘profit’ in return. Profit in the ‘in the economist’s’ sense is simply an expected risk premium i.e. the difference between the expected return accruing to such residually remunerated resources and the (lower) return attracted by the same resources when employed contractually. The expected return to factors supplied ‘entrepreneurially’, i.e. profit in the ‘accountants’ sense should normally include enough of a premium to remunerate the risk incurred by the entrepreneur, but that is of course not guaranteed.

The distinction between contractual and residual factor supply is firmly established in financial literature, notably in modelling the choice of debt vs equity in the financing decision. Curiously, the distinction has not ‘caught on’ in mainstream models of firm behavior in economics literature. Nor has the distinction been generalized to encompass the supply of ‘labor’, or ‘effort’, as well as ‘finance’, on ‘entrepreneurial’ terms. Yet, these are very real choices facing any enterprise and thus relevant in the modelling of firm behavior generally.

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2 There has not been, to the author’s knowledge any follow up to Bronfenbrenner (1960) in later literature, even though the basic diagram of his analysis did appear on the front cover of the Breit and Hochman (1968) well known collection of readings. A relatively recent attempt by Moro et al (2012) to model the optimization of alternative components of debt takes the equity as given and thus does not address the basic issue raised in this paper.

3 The partial exception is the literature on labor managed firms. That is generally based however on a rigid attribution of the entrepreneurial role to labor (as the residually remunerated input) reversing the stereotype of ‘capital’ as the entrepreneurial input in the conventional firm. Also the labor managed firm is typically modelled to maximize a residual income ratio, rather than absolute ‘profit. See Vanek (1970).
It is argued here that there is considerable unexploited potential for modelling firm behavior in the residual/contractual distinction, and especially in the analytical device of the firm’s hybrid factor supply/demand curve. The model presented here represents an attempt to draw on such potential. The conclusions regarding firm behavior will be seen to possibly depart from the usual expectation of monotonically upward sloping factor supply.\(^4\)

We should emphasize, for the sake of clarity, that in what follows the physical productivities of various resources are undifferentiated by the mode of their employment and remuneration. ‘Entrepreneurial factors’ are those supplied ‘entrepreneurially’ and are physically identical to the same resources which may be supplied ‘contractually’. In this sense the present discussion will bypass the extensive literature on the distinctive nature of entrepreneurship as a special ‘factor’ or ‘activity’.\(^5\) The greater part of what might be recognised as entrepreneurial ‘services’ is subsumed here under the various categories of necessary productive inputs, ranging from unskilled labour and simple equipment to higher specification labour and capital, administration, management and initiative/innovation activity at the other end of the spectrum.

The present approach is nevertheless still consistent with the conventional notion that some of the resources performing a more specialised entrepreneurial ‘role’ might be specific to the particular entrepreneur(s) and/or to the particular firm and thus impossible to obtain externally or replicate elsewhere. Such specificity could be seen as an aspect of the firm’s production function, or else as ‘the’ (unique) ‘entrepreneurial factor’ of some economics literature. Although the precise nature of such a factor does not need to be spelled out, it is what makes for eventual diminishing returns to the other inputs and would thus account for differences in the sizes and configurations of firms, set up optimally in all other respects.

II. The Market for Entrepreneurially Employed Factors

Consider the ‘internal’ aspect of supply to the enterprise of an entrepreneurial input, here labelled as \(F_e\), i.e. supply from the entrepreneur’s own resources, as demanded

\(^4\) It is recognized, nevertheless, that the uncertainty of the residual position is not be the only risk affecting the suppliers of resources to the firm, residual or contractual.

\(^5\) For a comprehensive discussion of the various views see Casson (2003).
by the entrepreneur himself. It is this that produces the hybrid supply/demand function. Figure 1 is a representation of the position as in Bronfenbrenner (1960), who explains:

**Figure 1: The Entrepreneurial Market for a Factor**

“The left hand panel...relates the *internal* supply and demand for an entrepreneurial service to its anticipated gross return. The supply and demand functions are identical since each entrepreneur as demander is buying services from himself as supplier. The combined supply and demand function is represented by a single curve DS. This curve slopes upwards in accordance with the general observation that high rates of gross profit result in increased business population, increased internal investment and similar signs of increased use of productive services under non contractual conditions....” (p 305).

The ‘hybrid’ curve is thus hypothesised to behave like a conventional supply curve. To complete the picture, the middle panel in Fig 1 shows the *external* market for input $F_e$ (with D and S curves sloping conventionally). The right hand panel is the *combined* internal/external market, i.e. the horizontal sum of the other two. This is drawn with the internal supply/demand dominant and resulting in upward sloping supply *and* demand curves, the latter still being, however, the steeper one. Bronfenbrenner (1960) suggests that this may be the typical situation in unincorporated businesses whose securities are not traded publicly.
We should note, finally, that the market for the same factor employed contractually (not shown explicitly in the graph) follows the normal pattern. That results in a lower expected per unit return \( r_c \) at equilibrium, shown in the diagram at a level below the equilibrium (expected) entrepreneurial per unit return \( r_e \).

An important point to note is that the above analysis conflates internal investment in existing firms with changes in business population (i.e. the formation or closure of firms). A continuum is defined, in effect, to encompass startup and/or closure and also post startup investment/disinvestment decisions. This characteristic is shared by Bronfenbrenner’s (1960) analysis with almost the totality of existing literature which indeed focuses predominantly on the latter aspect.\(^6\)

In what follows apparent weaknesses in the foregoing are addressed and an attempt is made to extend the analysis. The next section is devoted to a consideration of the opportunity cost stipulations typically underlying supply curves with a view to questioning their applicability. The following section presents a formal model which features the analytical device of bundling all productive resources in a single factor \( F \), to facilitate focusing on contractual vs entrepreneurial choices. The sequence of decisions envisaged is explained in Figure 2. The main conclusions are shown in Figures 3 and 4 which recast the hybrid supply/demand curve as a locus of optimal choices, based on the interface of entrepreneurial utility and external conditions. Subsequent sections explore the relevance of related ideas from the perspective of prospect theory and some evidence on relevant entrepreneurial investment choices. Analogies are drawn with similar styled analyses in economics and finance literature and issues concerning the applicability and limitations of the discussion are addressed.

III. Nature of the Hybrid Demand/Supply Curve

The above hybrid DS curve is, as seen, expected to be upward sloping. But upward sloping is the ‘normal’ expectation relating only to the supply aspect. Qua demand curve, the hybrid would be expected to be downward sloping! That is the prospective return \( r_e \) could be viewed as an opportunity cost, as well as a return, another hybrid concept. The entrepreneur would ‘demand’ less of his own resources for employment in the firm as their prospective returns, and hence the ‘costs’ of these rose, and vice versa. This suggests that the curve representing ‘identical’ supply and demand functions needs

\(^6\) For a useful summary see e.g. Ricketts (2003, pp 74-75). Also Casson (2003, 194-196).
to be reinterpreted and analysed further as a *locus of optimal positions* derived from the underlying choices.

It will be argued here that the owner/entrepreneur’s decisions with respect to the firm, both pre and post start up, are best viewed as *portfolio* ones, whereby resources to be committed to the firm are compared with alternative external uses. Simultaneously, the firm makes a *gearing* decision to combine own (and possibly external) resources to be supplied entrepreneurially with external (but possibly also own) resources to be employed contractually. Given these possibilities, which would naturally enter into the decision makers’ utility calculus, the *ceteris paribus* stipulations of conventional supply and demand analysis seem restrictive.

How well does the hybrid curve represent startup and post startup behavior? Consider the startup first. The formation of a new firm is a ‘long run’ decision, in the sense that all the costs are variable and externally determined. But rather than the usually hypothesised ‘unit by unit’ comparison of prospective returns with opportunity costs, *entrepreneurial* resources are committed for a start up on a *one off* comparison of the prospective firm with alternative firm projects. All such projects are compared *in their entirety* on the basis of optimal design and scale. Firms are created as they emerge successfully from such comparisons. Due to the ‘one off’ nature of the startup choice, a smooth upward sloping supply curve of own resources is not plausible in the sense of resources supply to the *individual* firm. Supply to the individual firm as an increasing function of prospective returns can only be spoken of in the more limited sense of the scale decision, higher returns calling for larger optimal scale. Even this general expectation would need to be qualified by the presence of any ‘specialised’ resources, the best utilisation of which might not be compatible with larger scale. The startup of the individual firm, and its counterpart closure decision, can thus not be represented by any continuous supply curve. The continuous upward sloping curve may still be plausible as

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7 It is thus possible that the decision making entrepreneurs may not commit all available own resources to the firm. However, the needs of the startup phase in particular may, realistically, require use of all own resources as well as any others that the entrepreneur(s) can muster!

8 Once ‘new’ external input suppliers are engaged entrepreneurially, they become embodied in the owner/decision making element of the firm, to the extent of their contributions.

9 We are thinking here of possibilities for contractual supply of own resources inside the firm, such as ‘close’ company directors receiving part of expected income as salary, or loans to the company by its directors. Such may well be dictated by tax considerations.
a representation of aggregate investment in startups, of which more can be expected as the general level of returns in the economy rises.

The same differentiation between the aggregate and the individual level applies post start up. But here too, opportunity costs present a difficulty. The ‘internalization’ involved once the firm is formed means that external opportunity costs become less relevant. Resources already committed are now, in large measure, ‘captive’ to the firm, as fixities set in, while additional resources could be more difficult to incorporate into the original design and scale. Rigidities and indivisibilities mean that external opportunity costs are again difficult to impute meaningfully to the firm as a ‘going concern’.

The argument then is that the portfolio decisions of the entrepreneur(s) would be governed primarily by considerations ‘internal’ to the firm, such as optimal scale pre start up, or performance post start up. Entrepreneurial choices are best understood in terms of a utility function, with prospective residual returns inside the firm and factor supply to the firm as its arguments. Related gearing possibilities of mixing own resources in the firm with some contractual employment of external resources would similarly be governed by the ‘internal’ requirements of optimal start up and subsequent operation. The firm is best thought of as seeking, at all times, to reach an optimal balance of preferences between (expected) ‘residual income’ and own resources supplied on entrepreneurial terms. The more constrained is the consideration of external opportunity costs the more relevant are ‘psychic’ preferences of residual income vs entrepreneurial factor supply. The gearing option means, however, that entrepreneurial supply preferences will not relate to entrepreneurial factor quantities in isolation, but in conjunction with quantities to be employed contractually.

This difficulty of meaningful imputation of opportunity costs also implies that the ‘residual income’ to enter the firm’s utility function will depart from the economist’s usual definition of profit. What we have here is ‘net enterprise income’ i.e. profit inclusive of (any) implicit costs of the entrepreneurial input(s), and allowing only for contractual costs. As suggested already, and also as in Bronfenbrenner (1960), this concept is more akin to the ‘accounting’ than the ‘economic’ definition of profit. Hence, the behaviour

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10 This happens to the extent that such markets become difficult to use, even with the help of intermediaries. See Casson (2003, pp 115-118). Considerations of trust and ease of monitoring are among the advantages of such internalization.
of the individual utility maximising firm will not necessarily align with the profit maximising firm of conventional theory and, as a consequence, may not conform to ‘normal’ supply behaviour, especially post start up.

As hinted earlier, however, ‘normal’ supply behaviour is more likely to apply at the aggregate level and as result of individual firms’ behaviour on balance. But the conflation of the different dimensions of entrepreneurial factor supply, individual and aggregate, pre and post start up, is arguably a serious weakness in existing literature.

IV. The Choice Problem: A Formal Statement

A. Assumptions

These can be grouped conveniently under headings (a) and (b) below

(a) It will be convenient analytically to bundle together the ensemble of necessary resources to be represented generically in a single productive factor, where \( F \) denotes units of the factor, and with components present in fixed proportions. This device, which is explained further in Figure 2 below, is meant to focus on the entrepreneurial/contractual choice abstracting from other aspects (e.g. the choice between capital and labour) which have no bearing on the present discussion.

Assume for simplicity a price taking firm with exogenous product price \( P \).\(^{11}\) Assume also that the firm faces competitive conditions in its demand for \( F \) for contractual employment. The amount of \( F \) to be employed contractually is denoted by \( F_c \), and has a unit cost of \( c \). That in turn is an increasing function of \( g \), the gearing level adopted by each firm.

The quantity of \( F \) can now be seen to derive from the firm’s production function, which is of the form \( Q = Q(F) \), the parameter \( P \) and the function \( c = c(g) \). These would determine the optimal product quantity (\( Q \)), hence also revenue (\( R \)), as well as the optimal quantities of the various categories of capital (\( K \)), and labour (\( L \)), therefore also the optimal proportions of these. The decisions facing the firm, at startup as well as later, are then i) what quantity of \( F \) to engage and ii) how to divide that quantity between the

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\(^{11}\) The assumption of an exogenous \( P \) is made in the interests of simplicity only. The more general formulation \( R = P(Q)Q \) may be used instead to indicate imperfectly competitive conditions.
contractual and entrepreneurial modes $F_c$ and $F_e$, where, of course, $F = F_c + F_e$. The variables $R, K$ and $L$ from this point onwards have no explicit role in the analysis.

This framework is thus meant to focus explicitly on the interdependence between the optimal amounts of $F_c$ and $F_e$, i.e. the gearing decision. Rather like the usual Debt/(Debt + Equity) ratio of financial literature, a gearing ratio is defined here, with respect to the composite factor $F$, as $g = F_c/(F_c + F_e)$. In principle the selection of gearing would be made simultaneously with the absolute level of employment of $F$. But in another sense the gearing decision is ‘subordinate’ to the scale decision of how much $F$ to employ overall, i.e. it does not prevent the optimal scale from being reached. Figure 2 provides an illustrated summary of the decisions involved.

Figure 2: Composition, amount and gearing of composite factor $F$

$Q = Q(L_{Innov}, L_{Mgr}, L_{Adm}, L_2, L_1; K_1, K_2, K_3, K_4)$

Composite factor $F$ consists of above inputs combined optimally

Determine simultaneously optimal quantity of $F$ and optimal gearing $g$ i.e. $F = F_c + F_e$ with $g = F_c/(F_c + F_e)$

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12 $F$ is thus only categorized as $F_c$ or $F_e$, differentiated only by the mode of employment. No further subcategories are identified here as between e.g. bank loans and trade credit, or senior and subordinated equity.

13 Correspondingly, $1-g$ will measure the entrepreneurial resources ratio, i.e. the supply of entrepreneurial resources as a % of the total.
The use of $F_e$ is thus now not studied in isolation (as in the hybrid curve of Figure 1) but in conjunction with the use of $F_c$.

(b) The entrepreneur maximises a utility function of the form

$$U = U[R_e, F_e]$$

with two arguments $R_e$ and $F_e$.

$R_e$ is the entrepreneurial remuneration function, i.e. the expected value of ‘net residual income’ or ‘net enterprise income’. That is exclusive only of contractual costs.

$$R_e = PQ(F) - c(g)F_c$$  \hspace{1cm} (1.1)

The $F_e$ argument represents the utility impact of supplying $F$ entrepreneurially. In terms of the utility trade off $F_e$ is thus the ‘bad’ that the entrepreneur sets against the ‘good’ of $R_e$. It is important to note that $F_e$ enters the utility function both directly as an argument of utility and indirectly as an influence on $R_e$. In the former sense, the disutility value of $F_e$ encompasses mainly its opportunity costs (in so far as possible to define) but also the net impact of factors that produce preferences for or against supplying resources on entrepreneurial terms.\(^{14}\)

Reflecting our assumptions the production function will have

$$\frac{\partial Q}{\partial F_c} = \frac{\partial Q}{\partial F_e} > 0$$

That is, the Marginal Physical Products of factor $F$ employed in either mode (which are by definition identical) are positive throughout. The implication for the net income $R_e$ is that a higher $F_e$, holding $F_c$ constant, involves a larger $R_e$ in absolute terms and possibly also per unit of $F_e$. However,

\(^{14}\) It should be noted here that the ‘portfolio’ option of not using own resources, certainly of not using all of one’s resources if that can be avoided, is not inconsistent with an overall preference for running one’s own business. See again Bronfenbrenner (1960, p 309).
That is the Marginal Product of factor \( F \) in either mode increases at a decreasing rate as more of \( F \), in the same or in the other mode, is applied.

In line with financial analysis the function \( c = c(g) \) will have \( \frac{\partial c}{\partial g} > 0 \) and

\[
\frac{\partial^2 c}{\partial g^2} > 0. 
\]

That is, the cost of using \( F \) contractually rises, and at an accelerating pace, with increases in gearing.

The utility function of (1) will have \( \frac{\partial U}{\partial R_e} > 0 \) and \( \frac{\partial U}{\partial F_e} < 0 \) That is, utility increases with a higher net income but decreases with each successive application of the factor \( F \) in the entrepreneurial mode. It is assumed also that \( \frac{\partial^2 U}{\partial R_e^2} < 0 \) while \( \frac{\partial^2 U}{\partial F_e^2} > 0 \). That is, the utility of net (residual) income increases at a diminishing rate, while the disutility of entrepreneurial factor supply increases with further applications of the factor in the entrepreneurial mode.

B. Results: The Optimal Position

For any exogenously given product price \( P \), the first order conditions (FOCs) for maximising the utility function of (1) require optimisation of the values of the two decision variables \( F_c \) and \( F_e \). Differentiating partially with respect to each and, after application of the chain function rule, we set equal to zero we obtain the stationary point.

\[
U_{F_c} = \frac{\partial U}{\partial R_e} [P \frac{\partial Q}{\partial F_c} - c(g) - F_c \frac{dc}{dg} \frac{F_e}{(F_c + F_e)^2}] = 0 \quad (2.1)
\]

\[
U_{F_e} = \frac{\partial U}{\partial R_e} [P \frac{\partial Q}{\partial F_e} - F_c \frac{dc}{dg} \frac{-F_e}{(F_c + F_e)^2} + \frac{\partial U}{\partial F_e}] = 0 \quad (2.2)
\]

These can be written more conveniently as
The first term on the left hand side (LHS) of the bracketed expressions of (3.1) and (3.2), which is a common one, will be recognized as the utility value of the effect on the residual income of the marginal revenue product of a unit increase in $F$, by definition the same for each ‘factor’ $F_c$ and $F_e$. The next two terms inside the bracket of (3.1) represent the marginal utility impact, through the residual income, of the cost of a unit increase in $F_c$. That is negative, as higher contractual costs reduce residual income. Similarly, the second term inside the bracket of (3.2) represents the marginal impact on utility, again through the residual income, of a unit increase in $F_e$. This is a positive effect, as more $F_e$ increases residual income. More generally, all the terms on the LHSs of (3.1) and (3.2) measure the utility impact of unit changes in each decision variable exerted indirectly through the financial effects on $R_e$, the first argument of the utility function of (1). In contrast, the term on the right hand side (RHS) of (3.2) measures the direct utility impact of a unit change in $F$. The derivative is negative by our earlier assumption, but the term on the RHS of (3.2) becomes positive due to the minus sign.

Given that $\frac{\partial U}{\partial R_e}$ is by assumption non zero, (3.1) can only be satisfied if the bracketed expression is zero. That allows (3.1) to be rewritten as

$$P \frac{\partial Q}{\partial F_c} = c(g) + F_c \frac{dc}{dg} \frac{F_e}{(F_c + F_e)^2}$$  \hspace{1cm} (3.1.1)

which shows effects in financial terms only. (3.1.1) is none other than the usual profit maximising condition equating Marginal Revenue Product to Marginal Factor Cost. The firm thus behaves conventionally as regards the contractual employment of $F$. Nevertheless, the amount of $F_c$ selected will not be quite the same as that of a pure profit maximizer, as that amount is also influenced by $F_e$, with which it is determined simultaneously.

(3.2) on the other hand, is still couched in terms of utility effects. Once again, however, the optimal position involves the quantity of $F_e$, as well as $F_c$, highlighting the
interdependence of the costs of $F$ in the two modes of employment. As already suggested, the optimum defined by the maximisation of (1) will not generally be the same as the profit maximum of conventional theory. Notwithstanding the simultaneity, preferences regarding the supply of $F$, to the firm can then be thought of as formulated against the background of specific quantities of $F$, as determined by (3.1.1). The firm’s ability to use $F$ to move towards the profit maximum provides it with leeway, as it were, to accommodate preferences regarding the use of $F$. The restraining influence on the use of $F$ is now the disutility associated with entrepreneurial supply, such disutility taking the place, in effect, of the external opportunity costs of conventional theory.

The position defined by the FOCs of (2.1) and (2.2) will represent a maximum provided that the second order conditions (SOCs) are satisfied. That is generally the case with economic functions of this nature.

C. Results: Comparative Statics

The ultimate task which we need to address here is to determine the firm’s responses, in terms of the amounts of contractual and entrepreneurial employment of $F$, and hence output $Q$, to a change in product price $P$. We require, in other words, to find the signs of the comparative static derivatives $\frac{dF_c}{dP}$ and $\frac{dF_e}{dP}$. To obtain these requires inspection of the total differentials of (2.1) and (2.2) with respect to $P$, $F_c$, and $F_e$. While the derivations of the second order partials are somewhat tedious, the findings may be stated rather simply.

It turns out that the solution of the relevant system of equations does not make it possible to attach definite signs to $\frac{dF_c}{dP}$ or $\frac{dF_e}{dP}$. I.e. these are of indeterminate sign. But indeterminacy here means precisely that the ‘normal’ responses of supplying more $F$ in both modes following a rise in $P$, and less of both in the event of a falling $P$, are not necessarily the only ones to expect.

These conclusions, are illustrated diagrammatically, in Figures 3 and 4. The preference map to represent the utility trade off between $R$ and $F_e$ consists of a set of positively sloped indifference curves. Given product price $P$, and the firm’s production function, we also have a ‘set of ‘residual income curves’ at each of which $R_e$ as an increasing function of $F_e$. Figure 3 shows three such curves, representing three different levels of $P$. The curves are drawn concave to the horizontal axis to reflect eventual
diminishing returns to the use of $F$. Taking the middle curve to represent e.g. the startup, the required optimum is clearly at its tangency with the highest possible indifference curve of the utility function of (1), namely at point A.

Figure 3 also illustrates the elementary comparative statics to trace what may be interpreted as post startup responses to changes in product market conditions. The two other positions shown are point B, following a favourable change in the product market (i.e. a higher $P$), and point D in the reverse scenario. Joining the points A, B and D together then traces the ‘normal’ upward sloping schedule for $F$, as an increasing function of its expected return.

The ‘residual income curves’ shown in Figure 3 will be recognized to bear a certain analogy with the ‘budget lines’ of consumer theory and also with the ‘capital market line’ of financial models. The resulting schedule however, is no longer our starting ‘internal’ DS schedule of the left hand panel of Figure 1. For it is not drawn on the usual ‘all other things equal’ assumption. Each curve corresponds not only to different product market conditions (as measured by $P$) but also to a different level of $F_c$, which is subject also to interaction with $F_e$. The ‘utility locus’ of Figure 3 measures amounts of $F_e$ which are determined simultaneously with the optimal amounts of $F_c$, the latter not shown explicitly. To remind also that $F_c$ affects not only the (dis)utility of supply directly but also indirectly the residual income $R_e$. To that extent the analogy of the residual income curves with exogenously given ‘budget lines’ is not a complete one.

The likely shape and steepness of entrepreneurial factor supply function, recast here as the entrepreneurial utility locus, may now be explored. When $P$ rises, calling for a move from A to B, changes in the relative utility values of $F_e$ and $R_e$, (measured by the slope $\frac{\partial U}{\partial R_e} / \frac{\partial U}{\partial F_e}$ at the higher residual income curve), may mean that expansion will be achieved mainly by an increase in $F_c$ and a lesser increase in $F_e$, thus through higher gearing. This may be the result of an improved choice set, once the firm had established itself. In the reverse case, where a fall in $P$ calls for contraction, it may again be that $F_c$ bears the brunt of the adjustment, this time to a lower scale, while the reduction in $F_e$ is smaller (i.e. lower gearing). In such circumstances the resulting schedule would be

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As indicated already, although $F_e$ in either mode of employment, is here the only variable productive input, the production function does embody some ultimate specificity, and hence fixity, making for diminishing returns. Additionally, under imperfect competition, there may be diminishing increments to $R_e$ due to lower price, as quantity of output increases.
relatively steep, although still upward sloping, i.e. entrepreneurial resource supply might be less responsive to changes in conditions than might be expected.

Furthermore, the schedule cannot be expected necessarily to be upward sloping throughout. The ‘unexpected’\textsuperscript{16} possibility of a negatively sloped supply response in respect of entrepreneurial factor employment $F_e$ is illustrated in Figure 4. Following a further rise in $P$ the required move is now from B to C. This would again be a move towards a preferred ‘portfolio’ position of higher gearing. But this time it would involve an absolute reduction in the amount of $F_e$ employed, (possibly coupled with some deployment of own resources elsewhere) to protect from further risk the gains made in the upside. The move from B to C thus introduces a backward bend in the upper reaches of the schedule.

This ‘unexpected’ response may also be visualized, and perhaps more plausibly, in the downside. Suppose that a relatively big deterioration of the original conditions calls for contraction from point D to point E (Figure 4). Here we have $F_e$, actually

\textsuperscript{16} This term is preferred as the opposite of ‘normal’, to avoid speaking of ‘anomalous’ or ‘perverse’ responses, such as might have been encountered in earlier literature.
increasing in contraction and such response is represented by a downward sloping segment in the lower part of the schedule of Figure 4. This would be the result of lower aversion to supplying own resources at lower expected residual income levels, reflecting the now reduced implicit cost, or value, of these. Indeed such behaviour in the downside may be dictated, above all else, by the desire to secure the survival of the enterprise, overcoming the disutility in supplying \( F_e \).

The winding schedule of Figure 4 thus represents a more extended locus of possible equilibrium positions than those shown in Figure 3, now encompassing two downward sloping segments. The existence or otherwise of these would hinge on the magnitude of \( \frac{\partial U}{\partial R_e} / \frac{\partial U}{\partial F_e} \) at the points of tangency with the relevant residual income curves, Whether the responses to changes in conditions are the normal or the above ‘unexpected’ ones depends on the gearing choices to be made at various times or stages.

Figure 4: The possibility of ‘unexpected’ supply responses
of development, notably pre- or post-startup. The indifference curves in both figures are drawn with curvatures such as to reflect changing relative valuations of returns and entrepreneurial supply disutility along the expansion and contraction paths. In the region of the lower residual income curves, a marginal increase in income may be enough to compensate for the disutility of an increase in factor supply (flat indifference curve). At the higher residual income curves, a relatively large change in income is needed to compensate for more factor supply (steeper indifference curve).

Figure 4 also depicts the more extreme possibility of the firm’s residual income curve falling in the negative range, implying loss of financial, and possibly also ‘human’ capital. Conventional theory would lead us to expect that the entrepreneur(s) would not wish to operate at all in that range. But negative residual returns do not automatically mean voluntary closure, or bankruptcy. The entrepreneurs may wish to continue in operation, at least for a while, in the hope of a turnaround. ‘Lifestyle’ considerations would significantly attenuate, in this range, the disutility in supplying $F_e$, possibly even turning into positive utility! The entrepreneur(s) now forego the return on $F_e$ and even accept some erosion of the capital values of their resources, to avoid bankruptcy. This is shown as the alternative path from D to G, again involving the ‘unexpected’ response of increased $F_e$ supply following an adverse change in the external environment.

V. Prospect Theory and Other Hypotheses

In comparing post- with pre-startup conditions in particular a hypothesis inspired by prospect theory may be relevant. This suggests that entrepreneurial factor supply decisions may depend on post start up performance relative to expectations formed at the ‘planning’ stage, that is the returns targeted at the initially optimal scale of operation selected at the start up. Performance, and hence prospective returns, above the ‘target’ of initial expectation, will normally call forth an increased supply of entrepreneurial resource to support expansion. But such increased commitment may at some point come into conflict with the reluctance to employ resource entrepreneurially, calling for reduced, rather than increased, entrepreneurial factor supply to the firm. Especially in the absence of a threat to enterprise survival, the entrepreneur’s appetite for risk could be attenuated. Own resources may be withdrawn from the firm for redeployment elsewhere, probably to be replaced with contractual ones inside the firm. Conversely, performance below initial expectation would normally call forth a reduction in entrepreneurial resource commitment. But a point might be reached where the threat
of extinction dictates the taking of more risk and the commitment of more resources on entrepreneurial terms.

The above is a version of a finding which suggests that above target performance would elicit low risk choices, and vice versa, in a S-shaped utility function (Kahneman and Tversky, 1979). Although this hypothesis is usually advanced to explain managerial behaviour, there seems no reason why it should not apply to owner/decision maker behaviour as well. Above initially targeted performance, that would translate into reluctance to commit more own resources in the firm (as well as, or instead of selection of low risk projects, not shown explicitly in the diagrams). Below target, the reverse would apply. Generalizing such possibilities would define a locus of preferred positions with different sections, namely a middle one of the usual upward sloping shape and the ‘backward bending’ shape hypothesized here at either or both of its extremes. The schedule of Figure 4 has a ‘normal’ upward sloping range from D to B. But outside that range it bends backwards to become downward sloping as we approach levels of prospective returns significantly lower or higher from initial expectations at A. In essence, the shape of the curve pivots around performance levels that simply matched the expectations on the basis of which the firm has been launched. Such a pivot may be thought of as a boundary between ‘normal’ and ‘unexpected’ entrepreneurial factor commitment responses.

Those firm responses referred to as ‘unexpected’, need not, however, and indeed should not be viewed as an aberration. Far from being aberrant, the decision makers of a firm experiencing success would be well justified in substituting contractual for own resources. Loans would be easier to obtain and employees would be easier to hire in the upside, while the entrepreneur would have diversification options for deploying own resources to alternative uses. In the downside contractual resources become unavailable while survival becomes the dominant priority for the use of and the now depleted own resources. So, once again, the entrepreneur(s) would be well justified in ‘choosing’ not only lower gearing but indeed a higher level of own resources from what can still be mustered, to commit entrepreneurially.

The above is, in any event, only one possible pattern. Other possibilities are that backward bending might occur only above, or only below initial expectations, or not all! Correspondingly, the curve of Figure 4 would be upward sloping also in the BC or DE ranges, or even throughout, having no DG segment. Indeed, behaviour in the upside and/or in the downside may instead follow, not only a ‘normal’ but an ‘accentuated normal’ response. That would be due to ‘status quo bias’ or increased optimism in the upside and pessimism in the downside. Of relevance here is the work of Moskwitz and
Vissig-Jørgensen (2002) who find that the majority of household investment in private companies is concentrated in a single privately held firm in which the household has an active management interest. Despite the lack of diversification involved, however, the returns to private equity are found to be surprisingly low. It would seem that some households at least are sensitive to the lure of very high returns, notwithstanding low probabilities of achieving these.

Related possibilities are suggested by the literature on bootstrap financing. Here decisions would be dominated by difficulties in securing long term finance (from debt holders or new equity partners) coupled with positive preferences for informal, short term financing from family, friends and clients. In their study of the bootstrapping phenomenon Winborg and Landstrom (2001) have indeed distinguished between ways of securing such finance and ways of minimising the need for finance of any kind, by saving costs.

In our present context overoptimistic behaviour would mean that we should always expect increases or larger increases of $F_i$ in expansion. Overly pessimistic behaviour would correspondingly lead us to expect decreases, or larger decreases in contraction. It could induce maximal entrepreneurial engagement of resources in the upside and minimal engagement in the downside. The schedule of Figure 4 would in effect acquire the shape of Figure 3, i.e. it would be upward sloping throughout but also flatter (more responsive) than it would be under less extreme assumptions. Bootstrapping on the other hand signals an overarching reliance on, or preference for equity, or quasi-equity. The model presented here cannot handle formally bootstrapping of the economising, or cost reducing variety, as the level of factor engagement is governed rigidly by the production function. If, however, such rigidity were relaxed we would again observe a quasi supply curve of the ‘normal’ shape where the downside was concerned.

VI. Aggregation

In transferring now these conclusions about the individual firm behaviour to the aggregate level, and hence summing individual ‘internal supply curves’ horizontally, we need to remember, always, that the ones presented here purport to describe entrepreneurial supply in conjunction with contractual, i.e. on different ceteris paribus assumptions. It needs to be remembered also that we are mainly concerned with supply to existing firms, rather than new startups or closures. With these provisos, if ‘normal’ supply were the dominant behavioural mode of the majority of firms, then aggregate supply would indeed be of the same conventional shape. This would subsume the
possibly ‘backward bending’ behaviour of some of the individual firms, only becoming less elastic overall than would be the case in the absence of any such behaviour. If on the other hand the winding schedule of Figure 4 were the typical one, that shape would be also be replicated on the aggregate. If, finally, exuberance in the upside and pessimism in the downside were the typical response, then the aggregate internal supply would be not only upward sloping but also more elastic than otherwise.

The present discussion thus purports to advance somewhat from the deceptive simplicity of the DS hybrid curve of Figure 1. To the extent that this will produce a schedule such as that of Figures 3 and 4, rather a conventionally defined supply curve, discovery of unusual features in its shape will not in itself call for any wholesale revision of factor supply analysis! It bears emphasizing, that the range of behavioural patterns identified relates to the individual firm and that aggregate supply will, on balance, probably still be of the ‘normal’ shape. But in so far as firms are justified in swapping entrepreneurial and contractual input employment in different circumstances we can expect some departure from ‘normal’ supply behaviour. We should indeed welcome, on grounds of economic welfare, the flexibility of responses that are envisaged.

VII. The Empirical Agenda

The discussion of the range of low returns in particular finds us in agreement with Bronfenbrenner’s (1960) observation of

“….the tendency of marginal firms to concentrate on entrepreneurial and avoid contractual inputs whenever possible. This is conventionally criticized as inefficient but may result from negative normal profit in these enterprises. It may be good marginalism for such firms to consider entrepreneurial services as costing considerably less than the contractual market prices for the same services would suggest.” (p. 307).

Another common observation, as e.g. in Moro et al (2012, p 89) is that the self-employed work for significantly lower monetary rewards, and/or longer hours than in at least some comparable employment situations. This is also consistent with our depiction of the D to E and B to C ranges in Figure 4. Conversely, it may not be difficult to find instances of greater application of contractual resources by successful firms in the expansion phase.
Observations such as the above are however superficial and cannot substitute for the rigorous empirical study that is called for, unfortunately outside the scope of this paper.\textsuperscript{17}

VIII. Analogies with Similar Models

Mention of backward bends, at least under one hypothesis in the foregoing, invites comparison with the usual analysis of labour supply. It will be recalled that basic labour supply behaviour, and the supposed backward bend of the labour supply curve, is usually explained in terms of income and substitution effects of a change in the wage rate. The choice involved here is between ‘income’ (or ‘other goods’) and ‘leisure’. In simple formulations the wage rate is shown on the vertical axis with ‘leisure’ on the horizontal, while more thorough treatments have (total) ‘income’ on the vertical axis and the wage rate as the slope of the relevant (downward sloping) budget line.\textsuperscript{18}

Our approach here has $R$ in place of the ‘income’ of labour supply. But our curve is upward sloping as the residual income increases with more application of $F$. The $F$ horizontal axis represents the opposite of ‘leisure’ as a disutility generating factor and hence our indifference curves are also upward sloping. Beyond these comparabilities, our present analysis attempts to widen the labour supply framework to involve not only ‘labour’ but any resources to be engaged entrepreneurially, including financial capital, as conventionally understood, and also ‘human’ or ‘social’ capital. As suggested already, factor supply decisions may involve family, and/or friends’ resources. In the generalized sense of ‘own resources’ then, the investment of own financial and/or of human/social capital in the enterprise shares some of the characteristics of basic ‘labour’ in that its supply creates disutility. The trade off with income is also not only ‘leisure’ but with the totality of utility generating factors inherent in ‘withholding’ or ‘conserving’ own resources. These would include a ‘leisure’ aspect but would embrace also e.g. lifestyle/independence choices which avoided ‘being tied down’ excessively in the

\textsuperscript{17} The reader is however referred to Winborg and Landstrom (2001), already mentioned on the issue of diversification of entrepreneurial investments. Also Winborg (2009).

\textsuperscript{18} See for example Estrin, Laidler and Dietrich (2012, pp 470-472)
business. Our discussion thus encompasses the possibility of being able to deploy resources elsewhere, at least in part, as a ‘portfolio diversification’ measure. Unlike here, labour supply is not usually treated as a ‘portfolio’ decision, although arguably it should be.

There are also analogies with the standard risk/return choice problem. This is usually presented in diagrams with the ‘expected rate of return’ on the vertical axis and some measure of ‘riskiness’ on the horizontal. Unlike in the income/leisure choice, utility now depends on a ‘good’ (i.e. income) and a ‘bad’ (i.e. risk), so the indifference curves are, as upward sloping. The ‘budget lines’, showing different measures of riskiness compensated by return, are also upward sloping, although the usual presentation aims to arrive at a unique ‘budget line’ known as the ‘securities market line’.

The preceding discussion has made reference to risk/return preferences, especially those likely to prevail under the hypothesis inspired by prospect theory. Our Figures 3 and 4, which feature the ‘bad’ of own resource supply on the horizontal axis can be said to belong to this general framework. The relevant choices involve the entrepreneur’s appetite for risk, itself likely to vary at different points on the expected returns scale, and opening the ‘portfolio’ possibility of committing some resources outside the firm to avoid ‘putting all eggs in the same basket’. A weakness of the present approach (and of others) is however that risk preferences are shown exclusively through entrepreneurial factor supply, rather than e.g. policies to select more or less, risky projects. Such preferences cannot be shown explicitly in the analytical framework adopted here.

IX. Qualifications and Policy Implications

The present analysis has focused on utility maximising resource supply decisions under residual remuneration (entrepreneurially) and optimal responses to the changes in product market conditions. The resulting trade-off between higher income and

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19 As seen earlier, however, ‘lifestyle’ and ‘independence’, may equally indicate an overarching desire to run one’s own business, thus pointing towards the maximal use of own resources by entrepreneurs who are so inclined.

20 See e.g Brealey, Myers and Allen (2014, pp 194-197). If riskiness is replaced by ‘safety’ on the horizontal axis, as e.g. in Frank (2008, p 512), then the indifference curves become downward sloping with a correspondingly downward sloping risk/return tradeoff curve.
entrepreneurial resource supply may produce responses such as those illustrated in Figure 4. But the implicit assumption here is that external changes are still of a magnitude that can be accommodated with relatively modest adjustments in scale. Bigger changes in the upside may dictate a more radical reappraisal of the firm in its present form and its replacement by another, in effect an alternative start up calculation. The downside may similarly dictate consideration of another radical alternative, namely closure. We may then qualify the present analysis as applying in the vicinity of the initial optimum. Our earlier assumption regarding the constancy of the composition of $F$ might also need to be relaxed if radical changes of scale were to be considered.

As suggested already, entrepreneurs in the upside can possibly be caught in (delusional?) overoptimism about the good times continuing indefinitely. Or sink in undue pessimism in the downside. Clearly, several behavioural patterns are possible here. Unlike in the labour supply case, where physical limitations would prevent higher returns from eliciting ever greater inputs of labour, it can be argued that there is nothing to stop overoptimistic entrepreneurs from throwing all their resources into an apparently successful venture. The premise of the present analysis however is, ultimately, that of rational behavior whereby the entrepreneur would weigh up alternatives and risk return trade-offs. An implicit, perhaps, aspect of such rationality, which should be highlighted, is the contrarian approach which recognises that neither success nor failure is for ever. That would dictate some conservation of resources following an initial success, and conversely, application of more resources in the hope of a turnaround.

Here we are essentially at the borders of ‘rational’ and ‘irrational’ behavior. Other possible examples of irrational behaviour would be

- persistence with decisions taken and refusal to acknowledge errors recognise change course
- predilection in favour of own/family resources and reluctance to engage with formal input markets;
- the reverse bias for not utilising own resources at all if resources are available externally

Both the purported ‘rational’ behaviour of the model, and the qualifications to it, are based on hypotheses, or even ‘hunches’, not capable of conclusive proof. The debate regarding the objectives of the firm, profit maximisation or other, is long standing as is the issue of what empirical evidence would constitute proof. There is also the
associated issue of whether optimisation models indicate what behaviour should be, even if though not representing actual behaviour in any plausible way.

We have sought here to highlight the ‘rationality’ of recognizing the implicit changes of value of the entrepreneurial resources post start up. In the upside, it is ‘rational’ to safeguard appreciating resources rather than throwing more of these in the ‘one basket’ of the firm. In the downside/distress it is similarly ‘rational’ to employ even more such resources in a bid to avert bankruptcy. This inherent flexibility of the ‘entrepreneurial supply price’ is perhaps the strongest feature of entrepreneurial resource supply, especially in adverse conditions! It enables competitive pricing of own resources, to mirror likely discounts in product price, and helps the enterprise to achieve its arguably least ambiguous objective, namely survival.

It bears emphasising that, apart from mixing own resources with contractual ones (gearing) as above, the owner/decision makers of the firm at any given moment have the option of engaging more external resources entrepreneurially (new equity), or of releasing such entrepreneurial resources in contraction. Expansion, or a rescue attempt, with new equity may well begin by drawing on the resources or family and/or the extended circle of business and social contacts. Following, inevitably, some dilution of control, and possible conflicts in the valuation of the interests of existing and new partners, the ‘reconstituted’ equity would then be faced with the ‘gearing’ choices outlined here.

In anticipation of the advantages of post startup behavior hypothesized here, use of equity for startups should be encouraged. Equity (possibly bootstrap) capital from own or family or associates’ resources and also own/family/working partners’ labor should be used, as far as possible, in preference to debt and employee labor. Contractual resources should be used as a complement to the equity, to optimize the mix, rather than as a first resort. Reduced emphasis on contractual resources should help to defuse some recent/current anxiety concerning the shortage of loans for small business.

### Conclusion

We have revisited a very neglected piece of analysis of choices governing the ‘internal’ supply/demand of entrepreneurial resources. We have mainly drawn on the analytical device of the hybrid ‘internal factor supply/demand curve’ of Bronfenbrenner (1960). We have introduced a further analytical device of the composite factor $F$, representing the bundle of all necessary resources, to facilitate focusing on the choice
between entrepreneurial and contractual modes of employing $F$. We have *recast* the hybrid own supply/demand curve as a locus of utility maximizing choices. Entrepreneurial factor supply has been redefined to be in conjunction with decisions about contractual factor supply. The resulting schedule shows entrepreneurial resource supply integrated with the gearing decision of obtaining similar resources contractually. The gearing decision has been treated as simultaneous with the scale decision in expansion and in contraction.

Drawing on prospect theory concerning possible attitudes to risk relative to targeted performance, the schedule is found not to necessarily be of the normal upward sloping shape throughout and to possibly involve a backward bending section. We have found a possible conflict between the demands of significant expansion calling for *more* $F$, when *less* $F$ may be preferred on utility grounds, and *vice versa* in contraction. The resulting possibilities of a downward sloping part in the shape of the curve in the upside or downside are seen as conducive to enterprise survival in distress and/or sensible management of the gains of successful performance. The implied behaviour is seen as welfare enhancing.

Possible alternative behavioural patterns have also been considered, such as possibly strong a preferences for, supplying own resources. Empirical work suggesting overoptimism among entrepreneurs, and pointing in the direction of exaggerated responses to external changes has been drawn upon.

We have distinguished supply to the individual firm from aggregate supply, focusing on the former. We have distinguished start up decisions from post start up operational ones. We have finally drawn the analogies with two similar modelling approaches in economics and finance; the standard economic model of income/leisure choices (which produces a backward bending labour supply curve), and the standard model of risk/return choices of financial literature.

**REFERENCES**


