

Korol, Tomasz

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EVALUATION OF THE FACTORS INFLUENCING BUSINESS BANKRUPTCY RISK IN POLAND¹

TOMASZ KOROL²

Abstract

This article is devoted to the issue of assessing the causes of business failure. The presented studies answer two research questions – what are the causes of corporate bankruptcies in Poland and how to more effectively predict the scale of bankruptcies in the country. The author has conducted a study to analyze the specific endogenous and exogenous causes of company bankruptcy depending on the type of the bankruptcy with consideration of the three different phases of the crisis in enterprises. The research is based on 185 companies (60 bankrupt and 125 non-bankrupt firms) listed on the Warsaw Stock Exchange. For each company the author has calculated 14 different financial ratios for each of the six years before classifying the firm as bankrupt or non-bankrupt. The author has also done research to identify specific macroeconomic variables affecting the bankruptcy process of the companies in Poland. This provided specification of the manner and how strongly the various factors affect the quantity and intensity of bankruptcy applications in this region of Europe. Additionally, with the use of selected macroeconomic variables the author programmed a fuzzy logic model to forecast the general intensity of bankruptcies in Poland. This study is the first attempt at using fuzzy logic to predict the risk of bankruptcy in a macro aspect. The results demonstrate the great potential of this method. The received effectiveness of the fuzzy logic model is at the level of 84%.

JEL classification: A10, D22, E17, G01

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2 Gdańsk University of Technology, Faculty of Management and Economics, tomasz.korol@zie.pg.gda.pl.

INTRODUCTION

The latest global financial crisis showed that even the best enterprises must constantly monitor their financial situation. The globalization process increased uncertainty in the ability of firms to exist. The nature and structure of the current dynamic world and linkages between all of the world's financial and economic markets mean that nowadays, in times of uncertainty, risks, and incomplete information, crisis becomes a feature of modern business, not a state of emergency. No company, even during a period of prosperity, can be certain of its future. The global financial crisis, which began in the second half of 2008, caused the number of companies in danger of bankruptcy to significantly increase around the world. Additionally, due to the steady, structural increase in the number of bankruptcies around the world - as a result of increased global competition - a careful analysis of the risk of bankruptcy of businesses has become even more important today than it was in the past.

In most cases, bankruptcy is a continuous process, where it is possible to distinguish several stages – from the emergence of the first signs of financial crisis, through blindness and ignorance towards the financial and nonfinancial symptoms of crisis in a firm, to inappropriate activities that lead to the final phase of the crisis, which is bankruptcy. The bankruptcy process may even take up to 5-6 years. This is not a sudden phenomenon, impossible to predict. Therefore, the earlier warning signals are detected, the more time managers have for preparing and reacting in subsequent phases of a crisis. For this reason, the goal of the author's research presented in this article is to analyze the causes of insolvency of companies, taking into consideration three different phases of crisis and three different types of bankrupt firms. The research is based on 185 stock exchange listed companies (60 bankrupt and 125 non-bankrupt) from the years 1999-2007 in Poland. For each company the author has calculated 14 different financial ratios for each of the six years before bankruptcy or before the enterprise was classified as non-bankrupt. The second objective of this paper is to develop a fuzzy logic model to predict the intensity of bankruptcies in the economy. Such a model can be used as a tool to evaluate the risk of doing business in the country in a macro aspect. The idea of using fuzzy logic to forecast macroeconomic risk of bankruptcy is new in the world of the literature. Until now the researchers have used fuzzy

sets only to predict the risk of bankruptcy of individual enterprises (for example: Korol & Korodi, 2011). The purpose of this model will be to identify the overall risk of corporate bankruptcies in the country (the number of such bankruptcies per 10 000 operating enterprises). Such a model will enable the prediction of not only the effect, namely the risk of bankruptcies in Poland (in annual terms), but also the very reasons affecting the number of bankruptcies (for example, level of unemployment, exchange rates USD/PLN, etc.).

The paper is organized as follows. Section 2 describes the typical phases of going bankrupt and the basic types of bankrupt enterprises. The author has conducted a review of more than 100 studies from around the world, presenting the latest trends in classification of this phenomenon in literature. In Section 3, the author presents the results of the research on the causes of bankruptcies of enterprises. The analyses of the endogenous and exogenous factors influencing the financial failure of firms are presented. In Section 4, the author presents two econometric models. The goal of these models is to designate the macroeconomic variables affecting bankruptcies in Poland. Section 5 is devoted to present the author's fuzzy logic model forecasting the risk of financial failures of firms in a macro aspect. The last Section is devoted to conclusions.

PHASES OF FINANCIAL CRISIS AND BASIC TYPES OF BANKRUPTCY

The first author who distinguished the phases of crisis leading to bankruptcy was P. Fitzpatrick. In 1934 he characterized five stages of crisis based on an evaluation of bankrupted American enterprises (Fitzpatrick, 1934). According to the latest trends, taking into account the increasingly globalized and dynamic environment, only 3 phases of crisis exist. H. Ooghe and S. Prijcker in their 2006 studies identified them as (Ooghe & Prijcker, 2006):

- 1) first phase – “initial gaps” - executives make mistakes due to a lack of sufficient knowledge, skills, or experience,
- 2) second phase - “negative signals” - problems are further deepened as a result of an entire series of erroneous decisions,
- 3) third phase - “ financial stage” - financial crisis worsens and, as a consequence, the firm goes bankrupt.

There are many different types of bankruptcy

classifications found in the literature. The original method of classifying failed companies into four different types of bankruptcy was proposed by three authors – B. Richardson, S. Nwankwo, and S. Richardson. In their study they used “frog” metaphors to describe the different types of bankruptcy. According to them, the use of such metaphors made the characteristics of different types of bankruptcy livelier and easier to remember. Thus, they characterized the following types of “frogs” or failed companies (Richardson, Nwankwo & Richardson, 1994):

1) “boiled frog” – firms that have been operating in the market for at least several years. A characteristic feature of such a company is self-satisfied managers. The company is “blind” to changes in their business environment. The Executive Board inertly leads the company using old strategic, tactical, and operational plans. Hence, the analogy to “boiled frogs.” The authors give the example of inserting a frog into boiling water. The frog will immediately feel the scorching water and attempt to jump out of it. On the other hand, if the frog is inserted into a container with cold water that is gradually heated to a boiling temperature, the frog will show ignorance. As a consequence of this ignorance, the frog will die in the boiling water.

2) “sunk frog” - firms led by highly ambitious managers who, after achieving great market successes, are strongly motivated to pursue aggressive expansion into new markets and fields of activity. Initial success and arrogance underlie this type of bankruptcy. Dynamic company growth in all directions cause that at some point the additional operational scope is not linked to the key resources that ensured the company’s success previously. Therefore, the authors have used the sunk frog analogy, caused by the excessive ambition to take over an entire pond, wanting to be in all its parts at once. This leads to the proverbial frog getting exhausted and sinking.

3) “tadpole” – unsuccessful start of a firm. This type of bankruptcy applies to young companies (hence the tadpole analogy, as they die before they are able to become a full-grown frog). There are a number of potential causes of bankruptcy in such enterprises: overly optimistic planning assumptions on sales, profits, acquiring market share, and the attractiveness of the firm’s products, low level of entrepreneurship of owners or managers.

4) “toad” – firms operating for at least a dozen years. It addresses megalomaniac companies that focus all their time on successes achieved in the past. On the one hand, managers are megalomaniacs, and on the other

hand they are apathetic to changes in their business environment. The company’s management process is based more on the faith and convictions of managers for ongoing success, rather than on real analysis. At the same time, such companies have growing ambitions to extend the business in all directions.

THE CAUSES OF BUSINESS BANKRUPTCIES

When one starts a business, thoughts are always on achieving growth and not entering the bankruptcy process. So, why do companies go bankrupt? According to the literature, it is not possible to designate one reason that would be 100% responsible for the bankruptcy of a company. A firms’ failure is the result of a whole set of factors. These factors very often overlap with one another, even if their sources of origin are different - endogenous and exogenous.

There is consensus in the literature regarding the classification of the causes of business bankruptcies. The causes are divided into:

1) exogenous causes that consist of phenomena related to the overall economic situation of a country, which include the fiscal, monetary, and exchange rate policies of government authorities. Companies cannot influence these factors, however, they affect their financial situations, such as their ability to pay, liquidity, etc. It is possible to distinguish both phenomena affecting all or almost all companies in the same way, and the phenomena that affect only certain firms or the factors that affect in opposite ways different groups of companies. An economic recession in a country can be given as an example of the first group of events. An exchange rate change can be given as an example of the second type of factor that affects firms differently;

2) endogenous causes, which can be divided into three main groups (Lizal, 2005):

a. the neoclassical group associated with the inappropriate, inefficient allocation of assets. From this point of view, liquidation of an inefficient company positively affects the effectiveness of a country’s economy, and the bankruptcy process can be likened to J. Schumpeter’s theory - “creative destruction”,

b. the financial group associated with an inadequate financing structure. The level of liquidity, and appropriate and rational use of financial leverage play a very important role in this group of bankruptcies,

c. causes associated with poor governance – in this case, a bankrupt company has the appropriate structure of liabilities and assets, but is not adequately managed. Managers are incompetent, and do not have adequate knowledge and/or experience.

Table 1 presents an analysis of the causes of company failures according to the phases of crisis and type of bankruptcy. In practice there is no single type of bankrupt company. Therefore, in the opinion of this article's author, it is necessary when assessing enterprise bankruptcy to analyze the endogenous and exogenous causes of bankruptcy in the various phases of crisis, also taking into account the type of bankruptcy. It should be noted that the goal of the research is to analyze specific causes of bankruptcy of companies depending on the type of bankruptcy and taking into account the dimension of time (phases of crisis), rather than static analysis, e.g.

identifying which of them is the most common or most important.

To analyze and evaluate the roots of bankruptcy, the author has used three phases of crisis according to the classification developed by H. Ooghe and S. Prijcker in 2006. The various phases of crisis are characterized by different duration, and occur at different stages of a company's "life" depending on the type of bankruptcy. The author has also used three types of bankruptcies:

- 1) type I representing young companies that had an unsuccessful business start,
- 2) type II represents the type of companies that have a phase of rapid growth and equally rapid decline phase,
- 3) type III represents firms that have been operating in the market for a dozen or more years, characterized by apathy and blindness to the changes occurring in the environment.

Table 1: Endogenous and Exogenous Causes of Business Bankruptcies Regarding the Crisis Phase and Type of Bankruptcy

| | | I PHASE | II PHASE | III PHASE | |
|--------|-------------------|---|---|--|--|
| I TYPE | Exogenous causes | The economic cycle (recession) | | | |
| | | Interest rates | | | |
| | | Currency exchange rate | | | |
| | | Inflation rate | | | |
| | | Lack of customers | | | |
| | | Dissatisfaction of customers | | | |
| | Endogenous causes | Lack of experience of managers | Underestimation of the level of costs | Enormous financial losses | |
| | | Inadequate knowledge of managers regarding the specific industry sector in which the company operates | Absence of a cost control system | Lack of possibility of improving financial liquidity | |
| | | Absence of leadership | Huge capital expenses | Inability of restructuring the company because of the huge debts of the firm, the unavailability of additional sources of financing and incompetence of managers | |
| | | Inadequate financial experience | Low level of revenues from sales | | |
| | | Lack of capital | | | |
| | | Absence of business plan | Unsuccessful attempt at getting market share | | |
| | | Insufficient level of technical knowledge | The increase of financial costs | | |
| | | Lack of key success factors | Absence of advisors from consulting companies | | |
| | | Haste | | | |
| | | Investments in inappropriate projects | | | |
| | | Low level of diversification of products/markets (depending on the success of one project) | | | |

| | | | | |
|--|-------------------|---|--|---|
| II TYPE | Exogenous causes | The economic cycle | | |
| | | | Inflation rate | |
| | | | The increase of competitive pressure | |
| | | | The collapse of the demand for products | |
| | | Currency exchange rate | | |
| | | Distrust of customers | | The refusal of banks to continue financing the company |
| | Endogenous causes | Inadequate management structure for increasing size of the company | Overestimation of profits | The increase of losses due to unsuccessful expansion of the company on the market |
| | | | Excessive investment expenditures | |
| | | High profits during first years of operations – high overestimation of future demand for products in connection with plans of company expansion in the market | Inadequate financial experience | Unawareness of the problems with lack of financial liquidity |
| | | | Blindness of managers | |
| | | Lack of expertise (arrogance of managers) | Lack of efficient financial control | The increase of level of inventories after unsuccessful market expansion |
| | | Managers’ ignorance regarding the distrust of customers | Unsuccessful expansion of the company | |
| | | Irresponsible increase of company debt obligations (financial leverage effect) | Excessive amount of fixed assets | Loss of trust of customers and employees |
| | | Excessive optimism of managers | The increase of labor costs | |
| | | Inadequate price policy | | |
| III TYPE | Exogenous causes | Aging industry | | |
| | | The increase of competitive pressure | | |
| | | Inappropriate economic policy of government | | |
| | | The lack of market protection by government | | |
| | | Globalization | | |
| | | Change of technology | The loss of trust of customers | |
| | Endogenous causes | Lack of commitment and motivation of managers | The gradual decrease of revenues and profits | The loss of profits, liquidity and solvency |
| | | The loss of strategic advantage due to changes in competitors’ companies | The decrease of sales | The increase of inventories |
| | | General apathy | Unsuccessful, slow restructure of company | Specialists departing from the company |
| | | Slow or no process of product innovation | Chaos in the operational part of the company | |
| | | Routine in management (faith and convictions of company success achieved in the past) | Self-delude of managers | |
| | | | The search for consensus, avoiding radical changes | |
| | | Reluctance to invest in new technologies | Relatively lower lever of product quality | |
| High level of fixed costs | | | | |
| Inflexible, “stiff” process of decision-making | | | | |

Sources: Self-study based on the analysis of 60 bankrupt Polish stock exchange listed companies during the years 1999-2007 and the review of literature from around the world Asgharian, 2003; Baldwin, Gray, Johnson & Proctor, 1997; Bhattacharjee, Highson, Holly & Kattuman, 2003; Brown, James & Mooradian, 1994; Carter & Van Auken, 2006; Castellanos, 2001; Dahiya & Klapper, 2007; Dyrberg, 2005; Dyrberg, 2004; Faria & Carneiro, 2001; Haswell & Holmes, 1989; Hunter & Isachenkova, 2006; Kaiser, 2001; Kash & Darling, 1998; Kleinman & Anandarajan, 1999; Koke, 2002; Laitinen & Gin Chong, 1998; Lukason & Hoffman, 2014; Nocetti, 2006; Ooghe & Pijcker, 2006; Owen, 2001; Pearce & Michael, 2006; Richardson, Nwankwo & Richardson, 1994; Sun, Li, Huang & He, 2014; Watson & Everett, 1998; Yu-Chiang & Ansell, 2007.

Type I Bankruptcy

Responsible for the first bankruptcy type are mainly endogenous (internal) factors. The broad competences of managers play a big role in new, young companies. Such firms do not have a solid customer or supplier base. They are vulnerable to competitor retaliation. At the same time, very often such companies have limited access to financing sources for their operations, investments, and marketing activities, all of which would enable acquiring market share and market presence. In the first phase of the crisis, following managerial incompetence, factors that lead to the failure of these companies can be distinguished as:

- 1) poor decision-making process – lack of strong business management, long worker wait time periods for managerial decisions,
- 2) inability to take on and control different types of risk,
- 3) lack of general and expertise knowledge of managers in the company - managers do not have enough information and knowledge/experience about the industry in which they operate. Problems with the drafting of a proper business plan (managers are often not aware of the need to develop necessary issues for the business plan),
- 4) inexperience of management, marketing, and financial executives,
- 5) weak capacity to manage a team, to resolve conflicts in teams, and communicate with employees.

The result of the mentioned incompetence of managers is making poor project investment decisions and a low level of diversification of products and/or markets. Often, executives of such companies depend on the success of only one project. Managers are excessively optimistic about the success of the project. They do not pay attention to the details of running the business. They want to enter the market as quickly as possible. Additionally, the inability to acquire funding leads to serious company undercapitalisation, which also reduces the chance of “appearing” on the market.

The financial crisis deepens due to the consequences of errors committed in the first phase of the crisis. The lack of a proper business plan and vision results in the absence of any strategic advantage over competitors. The firm also experiences problems attracting potential customers. Overestimated sales revenue levels, increased financial costs (when a company is unable to obtain financing

sources, it will be willing to accept more expensive loans), and often persistent adherence to a decision once taken (further capital expenditures on inappropriate, low profit projects) leads to the deterioration of a company’s financial situation.

In the last phase of the crisis it is already too late to improve the economic and financial situation of such companies. It is practically impossible to improve financial liquidity. Banks refuse to finance additional business activity because of the huge share of debts financing total assets and poor financial results.

The exogenous factors to a small extent (especially in comparison with the second and third types of bankruptcy) affect the collapse of such companies. Recession, high interest rates, high inflation, high competition, or an unfavourable exchange rate can, of course, further exacerbate the negative financial and economic problems of a company. It should be noted that in the case of the first type of bankruptcy, the main source of problems is in the company itself. Even in the case of a positive impact from macroeconomic factors (for example an economic boom), such companies only have a chance of prolonging the whole bankruptcy process for a few years. However, positive macroeconomic factors alone are unable to protect such companies from bankruptcy.

Type II Bankruptcy

In the case of the second type of bankruptcy, external factors have a much greater influence on the failure process than in the case of the previous type of bankruptcy. Paradoxically, initially favourable macroeconomic conditions (e.g. business cycle, exchange rate) may have contributed to the emergence of the first phase of the crisis, because managers of the second type of bankrupt companies are characterized by excessive optimism and ambitions of becoming the leading company in the market. Favourable market conditions may only “heat up” their excessive expectations.

In the initial period these types of companies are characterized by very rapid growth. High profits received in the first years of a company’s existence stimulate managers to aggressive expansion and at the same time it assures them of their correct decisions. It is worth adding that, according to H. Ooghe and S. Prijcker, the situation of the second type of bankruptcy does not necessarily relate only to new companies. According to them, in some cases, these are companies that have been operating

on the market for several years, awaiting favourable economic conditions (Ooghe & Prijcker, 2006). Favourable macroeconomic conditions create euphoria and excessive optimism. Excessive optimism may lead to management arrogance and expertise on the capital structure, financial plans, and future market conditions being ignored. In some cases, there is even a lack of any expertise.

In order to be able to finance an aggressive expansion, managers of such companies irresponsibly take large loans at a very rapid pace. The huge and fast growth in company size leads to a situation where management structures become inadequate. This causes the company's decision process to be more chaotic. Often such charismatic managers prefer an autocratic style of management. They are reluctant to delegate authority to other employees, or listen to advice. According to B. Richardson and S. Nwankwo, a leader of such a firm tends to encircle himself with "servile clones" (Richardson et al., 1994). Nowadays, when the nature of managing a company is characterized by multidisciplinary complexity, management should be based on a team of people with different skills and competences. If people surrounding the leader are of similar knowledge, it will lead to a weakening of a company's management efficiency, which very quickly grows. Moreover, such companies often expand to markets that are not connected to core firm activities and are unrelated to the company and its resources. These types of investments only absorb additional capital.

Errors committed in the first phase of the crisis inevitably lead to its second phase. During this phase, further over-investment and spending takes place. At the same time, financial inexperience, a lack of effective financial control, inappropriate management structures, staff cost increases, and manager blindness will lead the company to a critical point – a collapsing event. Although the revenues of companies may continue to grow, profits fall at a rapid pace (profits decline due to the increasing financial costs, personnel costs, and cost of operating excessive fixed assets). Managers, in the conviction that declining profits are the result of a temporary deterioration in economic conditions in the market, can even begin to apply the practice of "creative accounting" in order not to lose their profitable business image.

Further continuation of a firm's "blindness" leads to the third phase of the crisis, in which there are increased losses and finally a loss of liquidity. Sales decreases

lead to an increase in inventories. Only at this stage are banks able to properly assess the financial situation of such companies and refuse to continue financing their activities.

It is worth pointing out that this type of bankruptcy is particularly vulnerable to a country's changing economic situation. In the situation when a company has huge debts and uses high financial leverage, a collapse in demand for a company's products, increase in competitive pressure, unfavourable exchange rate, or the start of a recession in the economy have disastrous consequences and further aggravates the problems. It will speed up the whole bankruptcy process.

Type III Bankruptcy

Exogenous causes have the greatest impact on the process of failure of companies that belong to the third type of bankruptcy. This situation concerns companies that already have a well-established position in the market, or even companies that have had spectacular successes in the past. Firms that have been in existence for several dozens of years feel safe. Those who invest in or are employees of such companies have the opinion that they are guaranteed stable profits or employment. It can be said that these companies are "blinded" by the success they once achieved. They do not analyze the changing environment.

It can be said that routine management in combination with such external factors as:

- 1) aging industry in which the company operates,
 - 2) increase in competitive pressure,
 - 3) increase in the number of substitutes to the products offered by the company,
 - 4) changing production technology,
 - 5) lack of market protection by government,
- are the main causes of the beginning of the first phase of the crisis.

Apathy, lack of commitment and motivation of the leaders of such companies make them blind to gradual market changes. Apathy to changes leads a company to lose strategic advantages in the market. A non-existent or slow product innovation process tends to move existing customers to the competition. As a consequence, the crisis in a company is exacerbated. The revenues from sales drastically decline. In the situation where there is an absence of decisive corrective activities the company bankrupts. B. Richardson and S. Nwankwo explain that

very often managers of such companies prefer to explain the decline in sales and revenues by some temporary disturbances in the market, rather than a lasting trend (Richardson et al., 1994). Managers believe that the implemented strategies that had helped them to be successful in the past will also work this time around. These managers will look for consensus rather than radical changes. In consequence, the second phase of the crisis ends with the slow restructuring of the company.

The last phase is a sharp drop in profitability – especially in the case of firms producing durables. Customers who have not switched to the competition in the first and second phases of the crisis, now in the fear of subsequent problems with their product service, or even the lack of services, resign from further purchases of products from such a company. This results in an increase of inventories, and problems with liquidity. The “nail in the coffin” is the best professionals leaving the company, as they view the company as indifferent to the situation.

In Table 1, among the external causes affecting the bankruptcy process of companies belonging to the third type of bankruptcy, there are no such factors as recession, exchange rates, interest rate levels or inflation. In the opinion of the author of this article, companies that have already achieved market success and have been functioning over a long period of time are more “immune” to such factors than other firms. For example, by having better access to different sources of financing, such companies are certainly better prepared for interest rate increases than companies that belong to the first and second type of bankruptcy.

To summarize the evaluation of causes of the business bankruptcies with the use of the presented classification, it can be said that the exogenous factors, such as globalization, play a fundamental role in the third type of bankruptcy. It can also be indicated that a few of the exogenous factors, such as the economic growth rate, have some role in the case of the second type of bankruptcy. In contrast, exogenous factors are of least importance in the case of the first type of bankruptcy. In any type of bankruptcy a huge role is played by endogenous factors, because one cannot forget that a key factor in this process is man – more precisely, his skills, competence, knowledge, entrepreneurship, and interdisciplinarity.

Of course, not all bankruptcies must follow one of the above schemas. The aim of these analyses was to characterize the typical patterns of bankruptcy, and not to

analyze all possible paths of bankruptcy. Also, one cannot forget about such important factors as the demographic characteristics of enterprises. A company’s age, size, sector – its size, degree of innovation, characteristics (high tech vs. low tech) can affect the vulnerability of companies to the causes of bankruptcy presented in Table 1.

ECONOMETRIC MODELS FOR MACRO FORECASTING

The bankruptcy prediction has been studied extensively in financial literature for the last 51 years (since 1966 when Altman published the multivariate analysis model forecasting corporate bankruptcy risk – for example: Chen, Ribeiro, Vieira & Chen, 2013; Iturriaga & Sanz, 2015; Jardin, 2015; Lin, Liang, Yeh & Huang, 2014; Tsai, 2014). Despite the large interest of microeconomists in the phenomenon of bankruptcies of firms, macroeconomists have not paid much attention to this issue. Thus, most models developed for forecasting bankruptcy are based solely on the micro level, using only financial ratios. In the opinion of this article’s author, such a “micro” approach is obsolete in the face of the dynamic changes and global financial crisis the world is experiencing. The conducted research allowed the author to designate the macroeconomic variables affecting the increase of bankruptcy risk in Poland (both in absolute scale and as a relative scale). This allows specification of the manner and how strongly the various factors affect the quantity and intensity of bankruptcy applications. It is noteworthy that the constructed equations can be useful tools for the simulation of this phenomenon.

The author has used 17 different Polish macroeconomic variables from the period 1991-2005, such as the dynamics of GDP, inflation rate, dynamics of GDP per capita, dynamics of domestic demand per capita, dynamics of investment, average annual rates of interest, reserve requirements, PLN/USD exchange rates, and unemployment rates. In both models created, the variables were selected by the use of a method developed by Hellwig. Hellwig’s method involves the selection of explanatory variables that are strongly correlated with the dependent variable, but poorly correlated with each other.

The first econometric model was estimated by the use of the ordinary least squares method. This model

describes the number of bankruptcy applications in the courts dependent on: unemployment rate, dynamics of GDP per capita in Poland and PLN/USD exchange rate. This model is as follows:

$$\text{Number of bankruptcies} = -13\,120.2 + 562.24 * \text{unemployment rate} + 125.614 * \text{dynamics of GDP per capita} - 2\,028.3 * \text{USD_PLN exchange rate} \pm 845.9218$$

This model is well fitted to empirical data (coefficient of determination of 94.19%). Therefore, there is little influence of other variables that are not included in the model on the number of bankruptcy applications during the analyzed period. All selected independent variables were found to be highly statistically significant ($p < 0.05$). The following conclusions can be made with this model:

- 1) when the unemployment rate increases by 1 percentage point, the number of applications submitted in the courts increases by an average of 562.24 cases per year (*ceteris paribus*),
- 2) when the dynamics of GDP per capita increases by 1 percentage point, the number of applications submitted in the courts increases by an average of 125.614 cases per year (*ceteris paribus*),
- 3) when the PLN/USD exchange rate increases by 1 zloty for each 1 dollar, the number of applications submitted in the courts decreases by 2 028.3 cases per year (*ceteris paribus*).

The conclusion from the above model is that a very strong Polish Zloty boosts the risk of bankruptcy in the Polish economy. An interesting fact is also that the unemployment rate has been identified as one of the most important predictors of the quantity of bankruptcy applications. Companies in the face of financial crisis first lay off employees. For this reason, the unemployment rate is an effective early symptom of the increased risk of firm bankruptcy. The number of bankruptcy applications does not fully reflect the scale of bankruptcies in Poland. The growth of dynamics of GDP per capita affects the increase in the number of bankruptcy applications in the country. This does not mean, however, that the risk of conducting business is greater. The growth of GDP will lead to an increase in the number of firms operating in the economy. It is logical that, with more companies operating, more bankruptcy cases will be registered. This is why the author has estimated a second econometric model, which explains the variability of bankruptcy

intensity (an indicator representing the relative scale of bankruptcies in Poland – number of bankruptcies per 10 000 operating enterprises in Poland) depending on: dynamics of domestic demand per capita in Poland, dynamics of investment in fixed assets in Poland, PLN/USD exchange rate. This model is as follows:

$$\text{Intensity of bankruptcies} = 196.8819 + 2.8335 * \text{dynamics of domestic demand per capita} - 1.7499 * \text{dynamics of investment in fixed assets} - 54.92 * \text{USD_PLN exchange rate} \pm 20.9299$$

This model is also well fitted to empirical data (coefficient of determination of 85.07%). All selected independent variables were highly statistically significant ($p < 0.05$). The following conclusions can be made with this model:

- 1) when the domestic demand per capita increases by 1 percentage point, bankruptcy intensity in Poland grows on average by 2.8333 cases per 10,000 operating companies,
- 2) when the dynamics of investment in fixed assets increases by 1 percentage point, bankruptcy intensity decreases by 1.7499 bankrupt companies per 10,000 operating companies,
- 3) when the PLN/USD exchange rate increases by 1 zloty for each 1 dollar (depreciation of the zloty), bankruptcy intensity decreases by 54.92 cases per 10,000 operating firms.

It can be seen from the above model that exchange rate changes strongly affect the bankruptcy intensity of companies in Poland. The estimated model shows that a strong zloty affects the increase of bankruptcy risk. This is an even more important conclusion as the studies, carried out by P. Znamirowski and P. Perz on the 500 largest companies in Poland, showed that 70% of companies are exposed to currency risk, which directly affects their financial results (Perz & Znamirowski, 2003). It is also worth noting that investments in fixed assets positively influence the decrease of bankruptcy risk.

FUZZY LOGIC MODEL FOR EVALUATING THE MACRO RISK

The process of business failure is affected by many internal and external factors that cannot be precisely and unambiguously defined. Additionally, very often those

factors are overlapping, as shown in section 3 of this article Table 1). Also, the mere allegation that a company is at risk of bankruptcy must be considered imprecise, and in fact rarely in economic reality are there companies that can be considered as 100% bankrupt. It is difficult to accurately determine the degree of bankruptcy threat using traditional statistical or econometric methods. With the use of fuzzy logic vague and ambiguous concepts can be defined, such as “high risk of bankruptcy” or “low risk of bankruptcy”

The concept of fuzzy sets was introduced by Zadeh in 1965 (Zadeh, 1965). The fuzzy set “A” in a non-empty space X ($A \subseteq X$) can be defined as:

$$A = \{(x, \mu_A(x)) | x \in X\}$$

where $\mu_A : X \rightarrow [0,1]$ is a function for each element of X that determines the extent to which it belongs to set A . This function is called a membership function of fuzzy set A .

Classical set theory assumes that any element (company) fully belongs or completely does not belong to a given set (bankrupt or non-bankrupt set of companies). In turn, in the fuzzy set theory an element (company) may partially belong to a certain set, and this membership may be expressed by means of a real number in the interval $[0,1]$. Thus, the membership function $\mu_A(x) : U \Rightarrow [0,1]$ is defined as follows:

$$\forall_{x \in U} \mu_A(x) = \begin{cases} f(x), & x \in X \\ 0, & x \notin X \end{cases}$$

where: $\mu_A(x)$ –function defining membership of element x to set A , which is a subset of U ; $f(x)$ - function

receiving values from the interval $[0,1]$. The values of this function are called the degrees of membership.

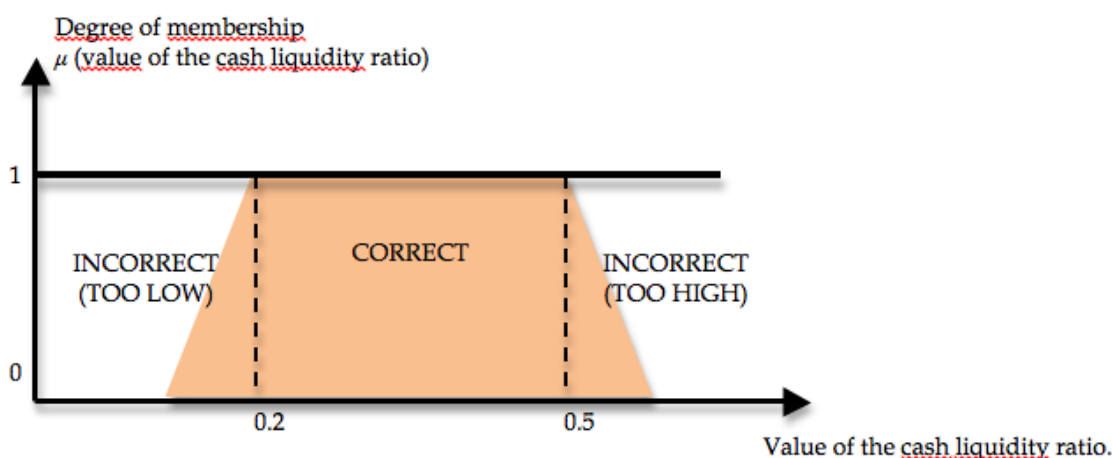
A membership function assigns the degree of membership of each element $x \in X$ to a fuzzy set A , where we can distinguish three situations:

- 1) $\mu_A(x) = 1$ means full membership of element x to the fuzzy set A ,
- 2) $\mu_A(x) = 0$ means that no element x belongs to fuzzy set A ,
- 3) $0 < \mu_A(x) < 1$ means partial membership of an element x to the fuzzy set A .

Membership functions are usually presented in graphic form. A trapezoidal function $\mu_A(x)$ is often used (see Figure 1). The graph shows information from the literature about the accepted values of the ratio of cash liquidity (cash ratio). The correct values for this ratio are values in the interval $[0.2, 0.5]$, and incorrect values are in the range of $(0; 0.2) \cup (0.5, \infty)$. When this ratio is lower than 0.2 it is considered that the company's cash liquidity is too low; in turn, when this amount is higher than 0.5 it is said that the company has excess liquidity, which is also rated as a negative phenomenon (in the case of excess liquidity such companies have too much cash, which is rated as inefficient management of the company's assets).

In this case, using the classical set theory to evaluate this financial ratio, there is a sharp boundary between the two sets of ratio values 0.2 and 0.5. If one company recorded a cash liquidity ratio, e.g. at 0.19, it would be classified as an incorrect value - negative, while the second company recorded this ratio at the level of 0.2, this would

Figure 1: An Example of the Trapezoidal Membership Function for the Cash Liquidity Ratio



be regarded as the correct value – positive assessment of bankruptcy risk, even though the financial ratio of the two firms differ only by 0.01. The interpretation of the values of individual ratios (e.g. liquidity) is further complicated by the fact that different literature sources give different reference limit values for individual financial ratios.

Application of a fuzzy set changes the assessment of the problem. A cash liquidity ratio with a value of 0.19 is considered as partly correct and partly invalid. The degree of membership in both sets depends on the shape of the membership function.

With such defined subsets, the boundary between the values considered to be positive or negative, is fuzzified – a certain ratio value is “partially good” and “partially bad.” There is no such possibility in the case of classical logic, i.e. bivalent, in which the value of the ratio is “good” or “bad”. Therefore, the use of classical logic in assessing macroeconomic risk of bankruptcies in the country can affect negatively the effectiveness of the forecasts. This is why the author has programmed a fuzzy logic model which forecasts the variability of bankruptcy intensity (an indicator representing the relative scale of

bankruptcies in Poland).

The set of rules which is used by the fuzzy decision model contains 25 rules for forecasting the risk in the economy for operating the business.

The structure of this fuzzy logic model is presented in Figure 2. The model consists of five inputs (macroeconomic variables) and one rule block. The model’s output is a variable representing a forecast of the intensity of bankruptcies in the country. This variable ranges from 0 to 1, while it is assumed that the threshold value separating the “prosperity” and “recession” period for companies is 0.5 (output variable values below 0.5 mean the risk of bankruptcies for companies operating in the country increases, while those above 0.5 represent a relatively safe period from bankruptcy for enterprises).

For each entry variable to the model, the author identified two fuzzy sets (which are subsets of a set of values of the entry variable): “positive” and “negative”, and their corresponding membership functions. The fuzzy sets and the shape of membership functions have been arbitrarily designated by the author. In order to create a

Figure 2: Structure of the Fuzzy Logic Model

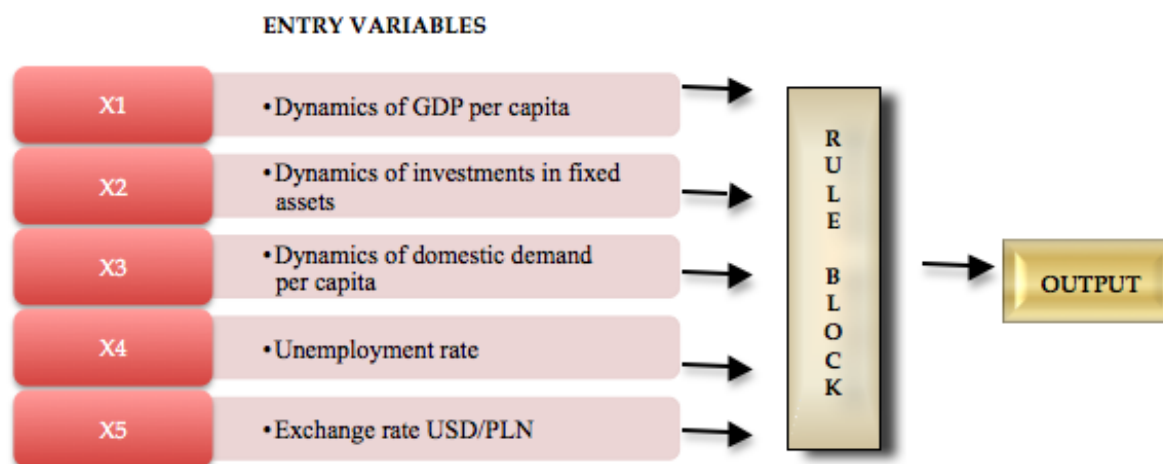


Table 2: The Threshold Values for Membership Functions Used in the Fuzzy Logic Model

| Variable Symbol | Threshold value for membership function | Description of variable |
|-----------------|---|---|
| X1 | 103% | Dynamics of GDP per capita in Poland |
| X2 | 100% | Dynamics of investments in fixed assets in Poland |
| X3 | 103% | Dynamics of domestic demand per capita in Poland |
| X4 | 9% | Unemployment rate in Poland |
| X5 | 2.3 PLN/USD | Exchange rate USD/PLN |

rule block consisting of a set of rule decisions, the author had to set the critical values for membership functions in the model. These values are presented in Table 2.

A set of rules for forecasting the intensity of bankruptcies is following:

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4 \geq 9\%$ and $X5 \leq 2.3$ then 0

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4 \geq 9\%$ and $X5 > 2.3$ then 0

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4 < 9\%$ and $X5 > 2.3$ then 0

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 > 103\%$ and $X4 < 9\%$ and $X5 > 2.3$ then 1

If $X1 \leq 103\%$ and $X2 > 100\%$ and $X3 > 103\%$ and $X4 < 9\%$ and $X5 > 2.3$ then 1

If $X1 > 103\%$ and $X2 > 100\%$ and $X3 > 103\%$ and $X4 < 9\%$ and $X5 > 2.3$ then 1

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 > 103\%$ and $X4 \geq 9\%$ and $X5 > 2.3$ then 0

If $X1 \leq 103\%$ and $X2 > 100\%$ and $X3 \leq 103\%$ and $X4 \geq 9\%$ and $X5 > 2.3$ then 0

If $X1 > 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4 \geq 9\%$ and $X5 > 2.3$ then 0

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4 < 9\%$ and $X5 \leq 2.3$ then 0

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 > 103\%$ and $X4 \geq 9\%$ and $X5 \leq 2.3$ then 0

If $X1 \leq 103\%$ and $X2 > 100\%$ and $X3 \leq 103\%$ and $X4 \geq 9\%$ and $X5 \leq 2.3$ then 0

If $X1 > 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4 \geq 9\%$ and $X5 \leq 2.3$ then 0

If $X1 \leq 103\%$ and $X2 > 100\%$ and $X3 > 103\%$ and $X4 < 9\%$ and $X5 \leq 2.3$ then 1

If $X1 \leq 103\%$ and $X2 \leq 100\%$ and $X3 > 103\%$ and $X4 < 9\%$ and $X5 \leq 2.3$ then 0

If $X1 \leq 103\%$ and $X2 > 100\%$ and $X3 \leq 103\%$ and $X4 < 9\%$ and $X5 \leq 2.3$ then 0

If $X1 \leq 103\%$ and $X2 > 100\%$ and $X3 > 103\%$ and $X4 \geq 9\%$ and $X5 \leq 2.3$ then 0

If $X1 > 103\%$ and $X2 \leq 100\%$ and $X3 > 103\%$ and $X4 \geq 9\%$ and $X5 \leq 2.3$ then 0

If $X1 > 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4 < 9\%$ and $X5 \leq 2.3$ then 0

If $X1 > 103\%$ and $X2 \leq 100\%$ and $X3 \leq 103\%$ and $X4$

$< 9\%$ and $X5 > 2.3$ then 1

If $X1 > 103\%$ and $X2 > 100\%$ and $X3 \leq 103\%$ and $X4 < 9\%$ and $X5 > 2.3$ then 1

If $X1 > 103\%$ and $X2 > 100\%$ and $X3 \leq 103\%$ and $X4 \geq 9\%$ and $X5 > 2.3$ then 1

If $X1 > 103\%$ and $X2 > 100\%$ and $X3 > 103\%$ and $X4 \geq 9\%$ and $X5 \leq 2.3$ then 1

If $X1 > 103\%$ and $X2 > 100\%$ and $X3 \leq 103\%$ and $X4 < 9\%$ and $X5 \leq 2.3$ then 1

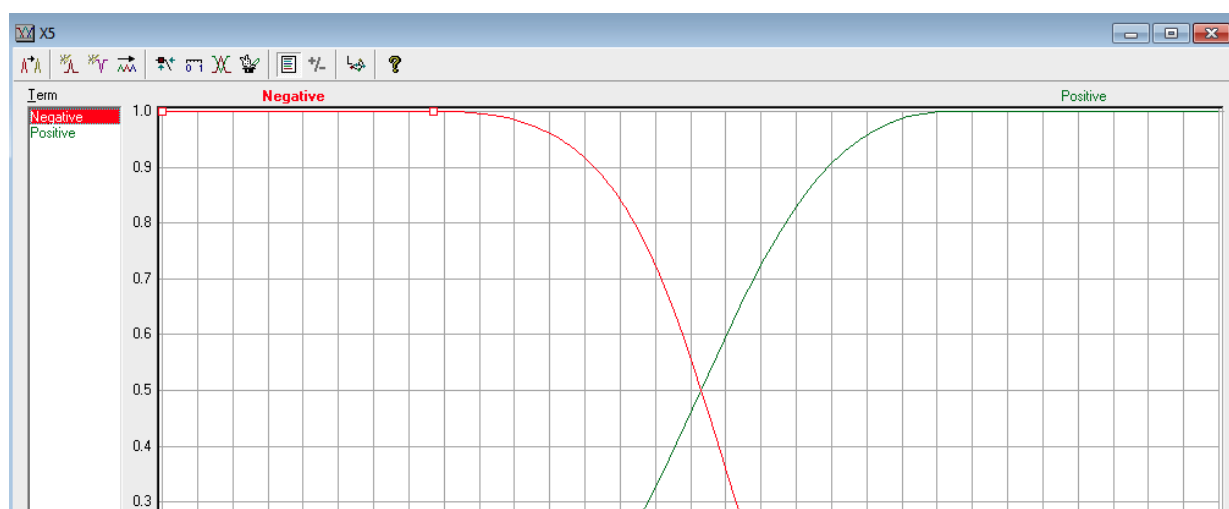
If $X1 > 103\%$ and $X2 \leq 100\%$ and $X3 > 103\%$ and $X4 < 9\%$ and $X5 > 2.3$ then 1

From the above set of rules, it can be concluded that in the case of four variables ($X1$, $X2$, $X3$ and $X5$) their high value decreases the level of the risk of bankruptcies. For example, the threshold for variable $X2$ – “the dynamics of investments in fixed assets in Poland” is 100%. If the enterprises expect prosperity, the dynamics of such investments are bigger than 100%. In such prosperity the scale of corporate bankruptcies decreases. In the case of expectations of recession in the economy, the firms will reduce investments (the dynamics will be lower than 100%) thus the overall risk of conducting business in Poland will increase. The exemplary form of the membership functions is presented in Figure 3 for the variable - “ $X5$ – exchange rate of PLN/USD”¹. It can be seen that a too strong Polish Zloty influences negatively the risk of bankruptcies in the macro aspect (the increase of intensity of financial failures among the enterprises). In the case of variable $X4$ – unemployment rate in Poland, the threshold is set at 9%. It means that the values higher than 9% of this variable will influence a higher rate of risk for bankruptcy among firms. The higher this value is, the bigger increase of intensity of corporate bankruptcies will occur.

The author tested the developed fuzzy logic model using the data representing the number and intensity of bankruptcies in Poland in the years 1991-2013. The effectiveness of the model is at the level of 84%, which can be evaluated as a high quality forecast. The idea of using such a fuzzy logic model predicting macroeconomic risk of bankruptcies in a country is new in the literature. Due to the fact that the fuzzy logic model is an “open” application, a person interested in its use can not only use

¹ Due to the limited size of the article, the author did not submit the figures with membership functions for all variables (the author can send them by e-mail to all interested readers).

Figure 3: The membership functions of variable X5



it in their current forms, but can also easily modify it for his own needs. The number of adaptations of the model is virtually unlimited by transforming the set of decision rules for individual needs (for example changing Polish macroeconomic variables into Spanish macroeconomic variables or any other European country) without changing the core of the model.

CONCLUSIONS

The author hopes that this study will be of interest to economic analysts, entrepreneurs, students and researchers in economic sciences. Financial crisis in a company does not appear suddenly – from day to day, but is the result of the accumulation of many factors and

symptoms ignored by managers, which deteriorate the economic situation of a company and its environment over an extended period of time. This is not a sudden phenomenon, impossible to predict. Therefore, the earlier warning signals are detected, the more time managers will have to prepare and react in subsequent phases of the crisis. For this reason, the article's author hopes that the presented research will contribute to future increases in the effectiveness of analysis of this important phenomenon in macro and micro-economics in two perspectives: 1) identifying the symptoms and causes of bankruptcy in individual companies (micro-perspective), 2) forecasting the risk of conducting business in a country with the use of two econometric equations (macro-perspective) and an innovative method – fuzzy logic.

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