

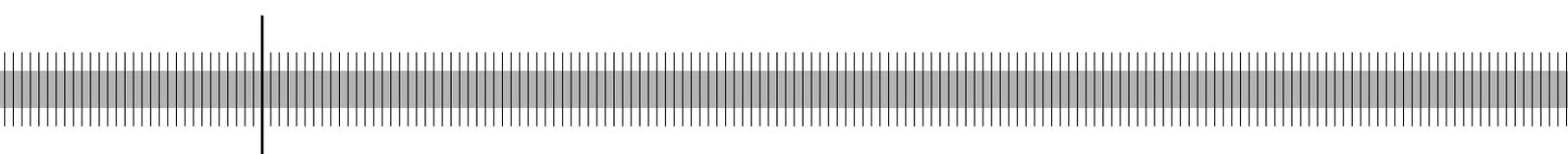
**A reappraisal of the evidence on PPP:  
a systematic investigation into MA roots in  
panel unit root tests and their implications**

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**Abstract:**

Panel unit root tests of real exchange rates – as opposed to univariate tests – usually reject non-stationarity. These tests, however, could be biased if the real exchange rate contained MA roots. Indeed, two independent arguments claim that the real exchange rate, being a sum of a stationary and a non-stationary component, is possibly an ARIMA (1, 1, 1) process. Monte Carlo simulations show, how systematic changes in the parameters of the components, of the test equation and of the correlation matrix affect the size of first and second generation panel unit root tests. Two components of the real exchange rate, the real exchange rate of a single good and a weighted sum of relative prices, are constructed from the data for a panel of countries. Computation of the relevant parameters reveals that panel unit root tests of the real exchange rate are severely oversized, usually much more so than simple ADF tests. Thus, the evidence for PPP from panel unit root tests may be merely due to extreme size biases.

**Keywords:** panel unit root test, purchasing power parity, real exchange rate, Monte Carlo simulation

**JEL-Classification:** F31, C33

## **Non Technical Summary**

In the last fifteen years, the empirical validity of purchasing power parity (PPP) over the post-Bretton Woods period has been studied extensively. As regards the methodology, unit root tests have been used to investigate whether the real exchange rate is stationary or not. While panel unit root tests usually confirm purchasing power parity, univariate unit root tests reject the PPP hypothesis in most cases. Since they combine information from several time series, panel unit root tests display greater power than univariate unit root tests. For this reason, the issue of whether purchasing power parity holds is widely held to be settled in favour of PPP.

The evidence presented in this study challenges conventional wisdom. It is shown that unit root tests are biased in favour of a rejection of the non-stationarity null if they are applied to a variable that is the sum of two components, one of which is stationary (modelled as a first order autoregressive process) and the other non-stationary (a random walk), as such a variable contains a moving average term, which is ultimately the reason for the bias. Monte Carlo simulations are used to determine, quite generally, the effect of variations in a number of parameters on the magnitude of the bias in different first and second-generation panel unit root tests. These parameters comprise the number of observations per series, the lag length in the test equation, the autoregressive parameter, the ratio between the innovation variances of the stationary and the non-stationary component, their correlation and a number of different types of cross-correlation across the series of the panel.

Two arguments from the literature are presented, both of which suggest independently that the real exchange rate consists of two such components. Based on one of the theoretical arguments, the data from the OECD's structural analysis database are used to construct the two components for eleven industrial countries. Consistent with the theory, panel unit root tests show that one of the series can be considered stationary, whereas the other most probably is non-stationary. The parameters of the two component model are estimated. For both this particular parameter constellation as well as one taken from the literature, the bias which occurs when panel unit root tests are applied to real exchange rates is determined by using Monte Carlo simulations.

It turns out that panel unit root tests of real exchange rates are biased quite substantially in favour of a rejection of the non-stationarity null if a two-component structure is assumed. The large body of empirical evidence of purchasing power parity obtained with panel unit root tests may thus simply be due to severe biases. Furthermore, the bias is found to be much larger in panel unit root tests than in simple univariate ADF tests. The commonly found result that univariate unit root tests usually reject purchasing power parity while panel unit root tests cannot reject PPP may therefore not be due to the lower power of the univariate tests (which is the commonly held view) but may be due instead to their smaller bias.

## **Nicht technische Zusammenfassung**

In den letzten anderthalb Jahrzehnten ist eine umfangreiche Literatur entstanden, die die empirische Überprüfung der Kaufkraftparitätentheorie für die Zeit seit dem Zusammenbruch des Bretton Woods Systems zum Thema hat. Methodisch wird dabei in aller Regel mit univariaten oder Panel-Einheitswurzeltests überprüft, ob der reale Wechselkurs stationär ist und damit die Kaufkraftparitätentheorie als bestätigt gelten kann oder nicht. Die Anwendung von Panel-Einheitswurzeltests führt dabei meist zur Bestätigung der Kaufkraftparitätentheorie, wohingegen sie mit univariaten Einheitswurzeltests in der Regel abgelehnt wird. Weil Panel-Einheitswurzeltests durch die Bündelung von Informationen eine höhere Macht besitzen als univariate Einheitswurzeltests, gilt die Fragestellung im Sinne der Gültigkeit der Kaufkraftparitätentheorie als beantwortet.

Die vorliegende Studie zieht diese in der Literatur vorherrschende Auffassung in Zweifel. Es wird gezeigt, dass Einheitswurzeltests in Richtung der Ablehnung der Nullhypothese Nichtstationarität verzerrt sind, wenn sie auf Variablen angewandt werden, die aus einer Summe zweier Komponenten bestehen, von denen die eine stationär (im konkreten Fall ein autoregressiver Prozess erster Ordnung) und die andere nichtstationär (hier ein random walk) ist. Denn eine solche Variable enthält einen Moving Average-Term, der letztlich die Ursache der Verzerrung ist. Mit Hilfe von Monte Carlo Simulationen wird ermittelt, welchen Einfluss eine Vielzahl von Parametern ganz generell auf die Verzerrung verschiedener Panel-Einheitswurzeltests sowohl der ersten als auch der zweiten Generation hat. Bei diesen Parametern handelt es sich um die Anzahl der Beobachtungswerte pro Reihe im Panel, die Anzahl der berücksichtigten Verzögerungen in der Testgleichung, die Höhe des autoregressiven Parameters, das Varianzverhältnis zwischen der stationären und der nichtstationären Komponente, deren Korrelation sowie verschiedene Typen von Kreuzkorrelationen zwischen den Reihen im Panel.

Es werden zwei theoretische Argumente aus der Literatur vorgestellt, die beide unabhängig voneinander nahe legen, dass reale Wechselkurse die beschriebene Zwei-Komponenten-Struktur besitzen. Aufbauend auf einem dieser Argumente werden die

zwei Komponenten für elf Industrieländer mit Hilfe von Daten der Structural Analysis Database der OECD berechnet. Panel-Einheitswurzeltests kommen tatsächlich zu dem Ergebnis, dass die eine der beiden Komponenten gut als stationäre Reihe beschrieben werden kann, während die andere höchstwahrscheinlich nichtstationär ist. Die Parameter des Zwei-Komponenten-Modells werden anschließend geschätzt. Für diese konkrete sowie für eine aus der Literatur bekannte Parameterkonstellation wird mit Simulationen die Verzerrung ermittelt, der Panel-Einheitswurzeltests realer Wechselkurse unterworfen sind.

Es zeigt sich, dass Panel-Einheitswurzeltests des realen Wechselkurses bei Vorliegen einer Zwei-Komponenten-Struktur ganz erheblich in Richtung auf Ablehnung der Nullhypothese Nichtstationarität verzerrt sind. Dass die Ergebnisse von Panel-Einheitswurzeltests der umfangreichen Literatur zufolge als Beleg für die Geltung der Kaufkraftparitätentheorie herangezogen werden, ist also möglicherweise überhaupt nicht gerechtfertigt, weil diese Ergebnisse eventuell nur auf Verzerrungen beruhen. Darüber hinaus stellt sich heraus, dass Panel-Einheitswurzeltests viel stärker verzerrt sind als ein simpler univariater ADF-Test. Dass univariate im Gegensatz zu Panel-Einheitswurzeltests die Kaufkraftparitätentheorie in aller Regel ablehnen, ist also möglicherweise nicht (wie bisher vermutet) ein ihrer geringen Macht geschuldeter Mangel der univariaten Einheitswurzeltests, sondern ein Vorzug geringerer Verzerrung. Damit muss die Gültigkeit der Kaufkraftparitätentheorie als eine weiterhin offene Frage gelten.

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# **A reappraisal of the evidence on PPP: a systematic investigation into MA roots in panel unit root tests and their implications\***

## **1. Introduction**

Since the late 1980s, the ongoing development of new unit root tests has continually spawned analyses of the validity of purchasing power parity (PPP). Meanwhile, the literature has grown so large that numerous survey studies have been published, for example Froot/Rogoff (1995), Rogoff (1996), Sarno/Taylor (2002), chapter 3, and Taylor/Taylor (2004). Notable exceptions notwithstanding, the findings of these analyses differ, interestingly, according to the type of unit root test employed, at least as long as the post-Bretton Woods era is considered. While, in most cases, the application of univariate unit root tests does not yield evidence in favour of PPP, panel unit root tests confirm the validity of PPP quite regularly (see eg Sarno/Taylor, 2002). The contradictory results are conventionally explained by the low power of univariate unit root tests in cases of a root close to unity. Panel unit root tests, it is argued, make use of more information through the pooling of several time series and thus yield more reliable results. Consequently, the debate appeared to be resolved in favour of PPP.

From a theoretical point of view, however, it can be convincingly claimed that PPP may be invalid. Permanent deviations from PPP could occur, for instance, in economies of the Balassa (1964) and Samuelson (1964) type because there will be no arbitrage-induced nominal exchange rate movements to offset internationally diverging prices of non-tradables. Based on such a model, Engel (2000) suggests that real exchange rates may consist of a sum of two components, one of which is stationary and the other non-stationary. This would, of course, imply that, empirically, one should find real exchange rates to be non-stationary, at least in the long run.

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Interestingly, Fischer (2004) shows that an alternative, less well-known argument against the validity of PPP, which has been put forward *inter alia* by Hsieh (1982), Devereux (1997) and Driver/Westaway (2005), implies a two-component structure of real exchange rates similar to that proposed by Engel (2000). In fact, the real exchange rate, by definition, consists of the real exchange rate of a single good and a weighted sum of relative prices between different goods. While it may reasonably be argued that the first of these components may be stationary because of the law of one price, there is no rationale for the second component to be mean-reverting.

In order to assess the effect of the two-component structure of the real exchange rate on univariate unit root tests, Engel (2000) assumes, for simplicity, that the stationary component is an AR (1) process and the non-stationary one a random walk. In this case, the real exchange rate is an ARIMA (1, 1, 1) process. Using Monte Carlo simulations, he demonstrates that univariate unit root tests of real exchange rates may be severely biased in favour of rejecting non-stationarity and, thus, may have difficulties in detecting the non-stationarity of the series.

The present study investigates whether recent panel unit root studies of the real exchange rate possibly have suffered from a size bias similar to the one found by Engel (2000) for univariate tests. In doing so, the implications of a two-component structure on the true size of panel unit root tests are systematically analysed. Such an analysis derives its importance from the fact that most of the evidence in favour of PPP is obtained by the application of panel unit root tests.

As a first step, the two components are constructed from the data for a panel of OECD real exchange rates. The parameters of the two component-model are estimated for each of the real exchange rates in the panel. Monte Carlo simulations show that, for these sets of parameters, both first and second-generation panel unit root tests are severely biased in favour of rejecting the non-stationarity null. Interestingly, it is exactly in the few cases (regularly characterised by a relatively high number of lags included in the test equation) in which panel unit root tests do not reject non-stationarity of real exchange rates that the size bias – while still considerable – is found to be lower. If the parameter constellation estimated by Engel (2000) is assumed to prevail for the whole panel, the size bias is even found to be so large that there is hardly any chance of not

succumbing to a type 1 error. The results imply that the evidence in favour of PPP, which has been obtained in recent panel unit root tests, rests on shaky ground. These results may simply be due to heavy size biases of the tests.

If both univariate and panel unit root tests are biased in favour of rejecting the non-stationarity null, why do the latter usually reject non-stationarity of real exchange rates and the former not? Further Monte Carlo simulations suggest that the average size bias of simple univariate ADF tests is much lower than that of panel unit root tests. The simulation results offer an alternative explanation to the opposing results of univariate and panel unit root tests of real exchange rates. It may not be the lower power of univariate unit root tests which is responsible for their rare rejection of the non-stationarity null but instead their relatively small size bias.

Finally, we investigate how the size bias in panel unit root tests is determined. This is of general importance for economists who wish to apply the tests and for econometricians who wish to improve their properties. It can, however, also be seen as a check on the robustness of the results previously obtained. After all, non-negligible cross-country differences in the estimated parameters of the processes have been found. For all the exercises, it is generally assumed that the variable to be tested is a sum of an AR (1) process and a random walk and, thus, an ARIMA (1, 1, 1) process. A Monte Carlo analysis of four first and second-generation panel unit root tests is performed in order to determine how their true size is affected by systematic changes in the ratio of the innovation variance of the two components, in the autoregressive parameter of the AR (1) process, in the correlation between the two components of the variable tested, in the cross-correlation between AR (1) components of series in different countries, in the cross-correlation between random walk components of series in different countries, in the joint cross-correlation of both components, in the number of lags in the test equation, and in the observation period.

Section 2 presents the two independent economic arguments which claim that the real exchange rate exhibits a two-component structure. Subsequently, it is shown how the two-component structure of the real exchange rate generates an MA root and, thus possibly, a bias in unit root tests. In section 3, the two components are identified and estimated. The panel unit root tests are presented and are used to test panels of real

exchange rates as well as panels of each of the two components for non-stationary. The results of Monte Carlo simulations on the extent of the bias that arises from a two-component structure are presented in section 4. Section 5 concludes.

## 2. The two-component structure of the real exchange rate

An investigation into the validity of long-run PPP amounts to testing real exchange rates for (non-)stationarity. If  $s_{it}$  denotes the log of the price of a numéraire country  $j$ 's currency expressed in units of country  $i$ 's currency at time  $t$ , and  $p_{it}$  ( $p_{jt}$ ) denotes the log of a price index of a basket of goods consumed or produced in country  $i$  ( $j$ ), then the (log of the) bilateral real exchange rate between country  $i$  and the numéraire country  $j$  is defined as

$$q_{it} \equiv s_{it} - p_{it} + p_{jt}. \quad (1)$$

A finding according to which the real exchange rate is stationary is commonly interpreted as evidence in favour of long-run PPP. Recently, however, two independent economic arguments have been proposed claiming that the real exchange rate, instead of being stationary, is the sum of two components, one of which may be non-stationary. These two arguments will be presented in the next part of the section. This is followed by an explanation as to why unit root tests of variables that consist of a stationary and a non-stationary component are often biased.

Engel (2000) puts forward an idea of a two-component real exchange rate which rests on the non-tradability of some of the goods in the baskets. In line with a conventional Balassa-Samuelson-type model, he assumes that the price index of country  $i$  (and that of  $j$ ) is a weighted average of traded and non-traded goods,

$$p_{it} = (1 - \alpha_{iN})p_{iTt} + \alpha_{iN}p_{iNt}, \quad (2)$$

where  $p_{iTt}$  ( $p_{iNt}$ ) is the log of the price index of traded (non-traded) goods and  $\alpha_{iN}$  is the fraction of non-tradable goods in the basket of country  $i$ . In this case, the real exchange rate can be expressed as a sum of two components

$$q_{it} = \bar{x}_{it} + \bar{y}_{it}, \quad (3)$$

where

$$\bar{x}_{it} = s_{it} - p_{iTt} + p_{jTt} \quad (4)$$

and

$$\bar{y}_{it} = \alpha_{jN}(p_{jNt} - p_{jTt}) - \alpha_{iN}(p_{iNt} - p_{iTt}). \quad (5)$$

While there are good reasons to believe that PPP holds for tradable goods at least, there is no economic rationale for assuming that the relative price of non-tradables in  $\bar{y}_{it}$  is stationary. Engel (2000) therefore suggests that  $\bar{x}_{it}$  may be stationary while  $\bar{y}_{it}$  (and thus  $q_{it}$ ) may rather be non-stationary.

In a recent paper, Fischer (2004) has demonstrated that the real exchange rate can be subdivided into two such components in an alternative way even without the existence of non-tradables. He formalised an old argument against PPP which eg Devereux (1997) concisely summed up in the phrase: “But the composition of price indices differs across countries, so that trend movements in relative goods prices will lead to persistent deviations from PPP.” Assume that each price index  $p_{it}$  is calculated as a geometric index, ie as the weighted sum of the log of the prices  $p_{ikt}$  of all individual goods  $k = 0, \dots, m$  in country  $i$  at time  $t$ ,

$$p_{it} \equiv \sum_{k=0}^m \alpha_{ik} p_{ikt} = p_{i0t} + \sum_{k=1}^m \alpha_{ik} (p_{ikt} - p_{i0t}), \quad (6)$$

where  $\alpha_{ik}$  denotes the weight of good  $k$  in country  $i$ 's price index,  $\sum_{k=0}^m \alpha_{ik} = 1$  for all  $i$ , and an arbitrary  $k=0$  is the numéraire good. Then, the real exchange rate by definition exhibits the two-component structure

$$q_{it} = x_{it} + y_{it}, \quad (7)$$

where

$$x_{it} = s_{it} - p_{i0t} + p_{j0t} \quad (8)$$

and

$$y_{it} = \sum_{k=1}^m [\alpha_{jk} (p_{jkt} - p_{j0t}) - \alpha_{ik} (p_{ikt} - p_{i0t})]. \quad (9)$$

Expression  $x_{it}$  can be seen as the real exchange rate for an arbitrary single good  $k = 0$ . If the law of one price (LOP) holds for good 0,  $x_{it}$  is stationary. There is, however, no economic theory that would suggest that the second component  $y_{it}$  is stationary.<sup>1</sup> This component of the real exchange rate is composed of a weighted sum of relative prices between different goods. Fischer (2004) shows for 11 OECD countries that a set of panel unit root tests cannot reject non-stationarity of relative prices or of  $y_{it}$  as a whole. This implies that Devereux's (1997) trend movements in relative goods prices do exist and, apparently, they do not cancel out. Given this evidence of non-stationarity of  $y_{it}$ , the real exchange rate would still be stationary if component  $x_{it}$  were non-stationary as well and, at the same time,  $x_{it}$  and  $y_{it}$  were cointegrated with vector  $[1, 1]'$ . Looking at it from a more economic perspective, such a "solution" may be the least plausible one, first, because it implies that PPP holds although the LOP does not and in spite of relative price trends. Second, there is no economic rationale for the two components to be cointegrated, let alone with a particular cointegrating vector (see Fischer, 2004).

Obviously, if the real exchange rate is the sum of a stationary ( $x_{it}$  or  $\bar{x}_{it}$ ) and a non-stationary component ( $y_{it}$  or  $\bar{y}_{it}$ , respectively), it is non-stationary as well. How does such a two-component structure of a variable affect the outcome of unit root tests? Intuitively, the non-stationarity of  $q_{it}$  is "blurred" by its stationary component. In order to become more specific, consider the simple assumption consistent with both Engel's (2000) and Fischer's (2004) approaches that the stationary component is an AR(1) process while the non-stationary component follows a random walk. Expressed in terms of equation (7),

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<sup>1</sup> Two mechanisms that should maintain PPP have been put forward in the literature. One approach considers PPP as an aggregation of the LOP across goods. Then, PPP is maintained by arbitrage. This mechanism can guarantee stationarity of the first component  $x_{it}$  but it does not influence the second,  $y_{it}$ . An alternative, rather macro-economic mechanism relies on the assumption that all disturbances satisfy the conditions of the homogeneity postulate of monetary theory in the sense that they leave unchanged all equilibrium relative prices, and thus lead only to an equiproportionate change in money and all prices, including the price of foreign exchange (cf. Dornbusch, 1987). This mechanism, however, by definition excludes relative price changes which drive movements in the second component  $y_{it}$ . An economic interpretation of examples of  $y_{it}$  series is given in section 3.1.

$$x_{it} = \phi_i \cdot x_{i,t-1} + \eta_{it} \quad (10)$$

and

$$y_{it} = y_{i,t-1} + \omega_{it}, \quad (11)$$

where  $\phi_i \in ]-1; 1[$  and  $\eta_{it}$  and  $\omega_{it}$  are i. i. d. but possibly contemporaneously correlated variables. From equations (7), (10), (11), one can derive that  $q_{it}$  is an ARIMA(1, 1, 1) process,<sup>2</sup>

$$\begin{aligned} \Delta q_{it} &= \phi_i \Delta q_{i,t-1} + \eta_{it} - \eta_{i,t-1} + \omega_{it} - \phi_i \omega_{i,t-1} \\ &= \phi_i \cdot \Delta q_{i,t-1} + \zeta_{it} + \mu_i \cdot \zeta_{i,t-1}, \end{aligned} \quad (12)$$

where  $\Delta q_{it} \equiv q_{it} - q_{i,t-1}$ ,  $\zeta_{it}$  is white noise,

$$\mu_i = - \frac{1 + 0.5(1 + \phi_i^2)S_i^2 + (1 + \phi_i)S_i R_i - 0.5\sqrt{(1 - \phi_i^2)^2 S_i^4 + 4(1 - \phi_i)^2 S_i^2 + 4(1 - \phi_i)(1 - \phi_i)S_i^3 R_i}}{1 + \phi_i S_i^2 + (1 + \phi_i)S_i R_i} \quad (13)$$

$S_i^2$  is the ratio between the innovation variance of the non-stationary component and that of the stationary component,  $S_i^2 = \sigma_{\omega_i}^2 / \sigma_{\eta_i}^2$ , and  $R_i$  is the correlation between the two errors,  $R_i = \sigma_{\eta_i \omega_i} / (\sigma_{\eta_i} \sigma_{\omega_i})$ . It will be demonstrated later on that, for relevant values of  $\phi_i$ ,  $S_i^2$ , and  $R_i$ , the MA parameter  $\mu_i$  is negative and larger than -1. It is well known, however, that this is exactly the case in which unit root tests of  $q_{it}$  are usually biased in favour of rejecting the non-stationarity null.<sup>3</sup>

<sup>2</sup> See e. g. Clark (1988) or Ng/Perron (2001). For the derivation of equations (12) and (13), see e. g. Hamilton's (1994) chapter on adding two moving-average processes, pp 106-7.

<sup>3</sup> The literature on this type of bias goes back to Schwert (1989), Cochrane (1991), and Blough (1992) among others. Engel (2000) demonstrates that univariate unit root tests are often severely biased if the real exchange rate behaves according to the two-component structure proposed in his study. Hlouskova/Wagner (2005) find that positive MA roots can generate considerable size biases in first-generation panel unit root tests.

### 3. Estimating a two-component model of the real exchange rate and preliminary results on (non-)stationarity

#### 3.1 Construction of the two components from the data and an economic interpretation

In order to obtain, first, an indication of whether one of the proposed components may be non-stationary and, second, an idea of the magnitude of a potential bias in unit root tests of the real exchange rate, it is necessary to quantify the respective two components. In our study, this will be done for the subdivision of the real exchange rate according to equations (7) – (9). For an estimation of the two components in the framework of the other argument, see Engel (2000).

Analogously to Fischer (2004), time series for price indices of individual goods have been replaced by price indices of sectors which can be obtained from the OECD's STAN database. For the purpose of constructing  $y_{it}$  in equation (9), the total economy is subdivided into 18 sectors. STAN provides annual data from 1977 to 1999 for all the 18 sectors of 11 OECD countries which comprise Austria, Belgium, Canada, Denmark, Finland, Germany, Italy, Japan, (South) Korea, the United Kingdom and the USA.

Since test results on PPP often depend on the choice of the numéraire country (see Coakley/Fuertes, 2000, or Papell/Theodoridis, 2001), we follow the common practice of the literature by choosing two alternative numéraire countries, the USA and Germany. Moreover, it is necessary to choose a numéraire sector for the computation of  $y_{it}$  in equation (9). We chose the “Finance, insurance, real estate and business services” sector since it is the private sector with the largest weight in the economies on average. Note that the hypothesis that  $y_{it}$  in equation (9) is non-stationary does not depend on the tradability of the numéraire sector at all.

Weights,  $\alpha_{ik}$ , can likewise be obtained from the STAN database. They have been calculated as  $\alpha_{ikt}$ , ie as possibly varying over time. Nominal exchange rates are taken from the IMF's International Financial Statistics.<sup>4</sup> The dataset is explained in more detail in Fischer (2004). The data have been used to compute  $x_{it}$ ,  $y_{it}$ , and  $q_{it}$  according to

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<sup>4</sup> Both end-of-year values and annual averages have been used. Our study presents results obtained with end-of-year data. The choice makes little difference.

equations (1), (8), and (9).<sup>5</sup> For each of the variables, a balanced panel of  $N = 10$  times series with  $T = 23$  annual observations is obtained. Thus, the number of observations per time series is low in comparison with most studies on PPP in which quarterly or even monthly data are used. It is well known, however, that higher frequency leads to no more than a small increase in the power of unit root tests, which is one of the reasons why eg Campbell/Perron (1991) even recommend using annual data.

Figure 1 depicts the series of one set of panels. Real exchange rates of the US dollar are shown in the upper two graphs. As can be seen from these and the two graphs in the middle of the figure, their movements are quite similar to those of components  $x_{it}$  because both types of variables are dominated by the rather large variations in nominal exchange rates. Components  $y_{it}$ , shown in the lower two graphs, display considerably less variation and, as expected, appear to be non-stationary with a much a higher probability than the series  $x_{it}$  and  $q_{it}$ .

How are the movements in component  $y_{it}$  to be interpreted? As shown in equation (9), the main ingredients of series  $y_{it}$  are relative prices across goods (or sectors). One important determinant of long-term trends in relative prices across goods is technological progress. The expression in square brackets in equation (9) would be positive if the increase in prices in sector  $k$  compared with prices in the numéraire sector 0 is greater in the numéraire country  $j$  than in country  $i$ , which could arise if technological progress in sector  $k$  in relation to that of sector 0 is smaller in the numéraire country than in country  $i$ . Alternatively, a positive expression in square brackets could result if relative prices across sectors rise equally in both countries, ie if the technological shock is the same in both countries, but sector  $k$  – which suffers from slower technological progress – has a larger weight  $\alpha_{jk}$  in the numéraire economy than in economy  $i$ .

Component  $y_{it}$  is a weighted average of such movements across all sectors of the economy. The lower two graphs in Figure 1 show that the average relative price in sectors other than the numéraire related to the price in the numéraire sector has

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<sup>5</sup> Since, in the real world, real exchange rates are calculated as weighted arithmetic, and not as weighted geometric averages, there is a slight difference between real exchange rates computed according to equation (1) and those computed using equation (7) (or (3)). The results are robust with respect to the alternative computation methods.

decreased in the USA compared with most other countries, possibly due to favourable technological shocks in these sectors in the USA or due to a large weight of technologically progressing sectors in the US economy. South Korea is the only country in the panel, whose relative prices fell more (whose relative technological progress may have been faster) than in the USA in the period from 1977 until 1999. It should be kept in mind, however, that the series do not show absolute price movements but, instead, a ratio of a weighted average of price movements, on the one hand, and of price movements of the numéraire sector, on the other. If the numéraire sector performs much better in terms of technological progress and price developments in one of the countries than in the others, this would show up, *ceteris paribus*, as a declining curve for this countries'  $y_{it}$  series as well. There is some indication that such a case, probably being irrelevant for most countries, may be responsible for the relatively steep decline of the German  $y_{it}$  curve in Figure 1, where the numéraire sector used is “Finance, insurance, real estate and business services”. Deutsche Bundesbank (1998) reports that unit labour costs of services compared to unit labour costs of manufacturing fell in Germany in these two decades while it rose in most other industrial countries.

### 3.2 Panel unit root tests

The issue of stationarity has been tested formally using four different panel unit root tests, which will also be used for simulations on the magnitude of the size bias in section 4. All of them are based in some way on the  $N$ -equation model

$$\Delta z_{it} = \beta_{i,0} + \rho_i z_{i,t-1} + \beta_{i,1} \Delta z_{i,t-1} + \beta_{i,2} \Delta z_{i,t-2} + \dots + \beta_{i,\ell} \Delta z_{i,t-\ell} + \varepsilon_{it} \quad (14)$$

for  $i = 1, 2, \dots, N$  countries, where  $z_{it}$  is to be tested for (non-)stationarity,  $t = 1, 2, \dots, T$  is time,  $\ell$  denotes the maximum lag, which for simplicity is assumed to be common across countries, and the error term  $\varepsilon_{it}$  is white noise but correlations across countries are allowed. As is common practice in the PPP literature, deterministic time trends are not considered because, on the one hand, they are not compatible with PPP (see, for example, Higgins/Zakrajšek, 1999, or Papell/Theodorides, 2001) and, on the other hand, real exchange rates display no indication of deterministic trends.

Two of the panel unit root tests, the Levin and Lin test (referred to below as LL, see Levin et al, 2002) and the t-bar test (IPS) developed by Im et al (2003) have become

standard panel unit root tests which can be used as a benchmark. They belong to the group of first-generation panel unit root tests which have been developed on the assumption of cross-section independence (see Breitung/Pesaran, 2005). O'Connell (1998), however, has shown that this assumption is severely violated in the case of real exchange rates and that this will bias the test results in favour of a rejection of non-stationarity. In order to allow for at least a limited degree of cross-sectional correlation, time-specific intercepts have been introduced to equation (14) in both tests by subtracting cross-sectional averages from all observations.

The other two tests, the multivariate homogeneous Dickey-Fuller (MHDF) test developed by Harvey/Bates (2002) and the robust or panel corrected standard errors (PCSE) test proposed by Jönsson (2005) and Breitung/Das (2005), are recently developed second-generation panel unit root tests which take account of cross-sectional correlations in a more general way. Since these tests are less standard than the LL and the IPS test, their application is briefly described in the following. As a first step for the computation of both the MHDF and the PCSE test, the constant term and the lagged differences of the variables  $z_{it}$  in (14) have been eliminated using a procedure suggested by Breitung/Das (2005). The initial value being the best estimator of the constant term is subtracted from each observation to yield  $\tilde{z}_{it} = z_{it} - z_{i1}$ . This approach avoids the Nickell bias. Then, an OLS estimation of equation (14) has been performed for each variable  $\tilde{z}_{it}$  separately, where  $\beta_{i,0}$  has been set to 0, of course. Using the estimated parameters  $\hat{\beta}_{i,1}, \hat{\beta}_{i,2}, \dots, \hat{\beta}_{i,\ell}$ , pre-whitened values of  $z_{it}$  are calculated as

$$\tilde{z}_{it} = \tilde{z}_{it} - \hat{\beta}_{i,1}\tilde{z}_{i,t-1} - \hat{\beta}_{i,2}\tilde{z}_{i,t-2} - \dots - \hat{\beta}_{i,\ell}\tilde{z}_{i,t-\ell}. \quad (15)$$

Breitung/Das (2005) demonstrate that the MHDF and the PCSE test statistics of the pre-whitened series  $\tilde{z}_{it}$  are asymptotically standard normally distributed. Like the LL test, the MHDF and the PCSE test assume homogeneity,  $\rho_i = \rho$ , in equation (14) for all countries  $i$ . The OLS estimator of the homogeneous parameter  $\rho$ ,

$$\hat{\rho} = \frac{\sum_{t=2+\ell}^T \tilde{z}'_{t-1} \Delta \tilde{z}_t}{\sum_{t=2+\ell}^T \tilde{z}'_{t-1} \tilde{z}_{t-1}}, \quad (16)$$

where  $\tilde{z}_t = [\tilde{z}_{1t}, \tilde{z}_{2t}, \dots, \tilde{z}_{Nt}]'$  and  $\Delta \tilde{z}_t = [\Delta \tilde{z}_{1t}, \Delta \tilde{z}_{2t}, \dots, \Delta \tilde{z}_{Nt}]'$ , can be used to compute the vector of errors  $\hat{\varepsilon}_t = [\hat{\varepsilon}_{1t}, \hat{\varepsilon}_{2t}, \dots, \hat{\varepsilon}_{Nt}]'$  where  $\hat{\varepsilon}_{it} = \Delta \tilde{z}_{it} - \hat{\rho} \tilde{z}_{i,t-1}$ . From this, an estimator of the SURE covariance matrix  $\Omega_\varepsilon$  of system (14) is obtained as  $\hat{\Omega}_\varepsilon = (T - \ell - 1)^{-1} \sum_{t=2+\ell}^T \hat{\varepsilon}_t \hat{\varepsilon}_t'$ . As in O'Connell (1998), the MHDF t statistic of Harvey/Bates (2002) is calculated in a GLS approach,

$$t_{gls} = \frac{\sum_{t=2+\ell}^T \tilde{z}'_{t-1} \hat{\Omega}_\varepsilon^{-1} \Delta \tilde{z}_t}{\sqrt{\sum_{t=2+\ell}^T \tilde{z}'_{t-1} \hat{\Omega}_\varepsilon^{-1} \tilde{z}_{t-1}}}. \quad (17)$$

In contrast, the robust PCSE t statistic of Jönsson (2005) and Breitung/Das (2005) is obtained as

$$t_{rob} = \frac{\sum_{t=2+\ell}^T \tilde{z}'_{t-1} \Delta \tilde{z}_t}{\sqrt{\sum_{t=2+\ell}^T \tilde{z}'_{t-1} \hat{\Omega}_\varepsilon \tilde{z}_{t-1}}}. \quad (18)$$

Critical values for  $N = 10$  have been taken from Harvey/Bates' (2002) Tables 2 and 1b and from Breitung/Das' (2005) Table 1. Both sets of critical values are hardly affected by variations in  $T$ .

### 3.3 Evidence on unit roots in real exchange rates and their components

Using alternative numbers of lags, Table 1 presents results of unit root tests on those real exchange rates and their components which are depicted in Figure 1. Each of the four panel unit root tests provides ample evidence for real exchange rate series ( $q_{it}$ ) to be stationary. For the two first-generation panel unit root tests, IPS and LL, however, the significance of a rejection falls below the commonly used significance levels if the number of lags is increased to three and five, respectively. When applying the standard

univariate unit root test, the ADF test, to the individual real exchange rate series of the panel, non-stationarity cannot be rejected for a single series regardless of the number of lags included. This replicates the results of the PPP literature, according to which panel unit root tests find much more evidence in favour of PPP than univariate unit root tests.

Turning to the components of the real exchange rate, the  $x_{it}$  series appear to behave fairly similarly to the respective real exchange rate series, as could be expected by visual inspection of Figure 1. The evidence for the real exchange rates  $q_{it}$  and their components  $x_{it}$  to be stationary is nearly the same. Non-stationarity of component  $y_{it}$ , in contrast, cannot be rejected in nearly all cases. The only cases which provide some evidence in favour of stationarity, the univariate ADF test and the LL test, each with five years of lags, may rather be statistical artefacts because, there, explanatory power is at its minimum.

The tests yield similar results when Germany is used as numéraire country. Panel unit root tests and the ADF test provide hardly any evidence for component  $y_{it}$  to be stationary. While panel unit root tests fiercely reject non-stationarity of the real exchange rate  $q_{it}$  and  $x_{it}$  if one lag is included in equation (14), evidence for stationarity of these variables, however, is considerably lower with three or five years of lags.

### 3.4 Estimating a two-component model of the real exchange rate

The panel unit root test results presented in the previous section suggest that real exchange rates may, indeed, consist of two components, one of which is stationary and the other non-stationary. In section 2, it has been shown that such a two-component structure causes the difference of the real exchange rate to be an ARMA (1, 1) process in the simple case where the stationary component is modelled as an AR (1) process and the non-stationary component as a random walk. The MA root, however, causes unit root tests of the real exchange rate to be biased in favour of rejecting the non-stationarity null as is well-known from the literature. In order to gauge the magnitude of this size bias, one needs to specify parameters  $\phi_i$ ,  $S_i^2$  and  $R_i$  (or, equivalently, parameters  $\phi_i$  and  $\mu_i$ ) of equations (12) and (13).

The specification of these parameters requires an estimation of the covariance matrix of the errors  $\eta_{it}$  and  $\omega_{it}$  in equations (10) and (11), which for the sake of

simplicity are assumed to describe the two components  $x_{it}$  and  $y_{it}$  adequately. Following Engel (2000), who suggests a similar but slightly more complicated approach in the framework of his two-component model of the real exchange rate, an intuitive and simple way to obtain such a covariance matrix may be the estimation of equations (10) and (11) in the form of

$$y_{it} = y_{i,t-1} + a_i u_{it} \quad (11')$$

and

$$x_{it} = \phi_i x_{i,t-1} + b_i u_{it} + c_i v_{it}, \quad (10')$$

where  $u_{it}$  and  $v_{it}$  are assumed to be i. i. d. and  $N(0; 1)$  distributed. Since  $y_{it}$  consists of relative prices between different goods (sectors), it may be sensible to assume that it is driven predominantly by real shocks to the economy, as for instance technological progress. These shocks exert a permanent influence on  $y_{it}$ , for example, if the production or consumption pattern differs between country  $i$  and the numéraire country,  $\alpha_{ik} \neq \alpha_{jk}$  in equation (9). They are represented by  $u_{it}$  in equation (11'). As can be seen from equation (8), variable  $x_{it}$ , in contrast, is the relative price of a single good (sector) in two different countries, ie the real exchange rate of a single good (sector). As modelled in equation (10'),  $x_{it}$  is affected by monetary and speculation shocks, expressed as  $v_{it}$ , as well as by real shocks,  $u_{it}$ . Because of the LOP, their effect on  $x_{it}$ , however, is only temporary.<sup>6</sup>

Parameter  $\hat{a}_i$ , the standard deviation of the series  $\Delta y_{it}$ , and, thus,  $\hat{u}_{it}$  can be computed from equation (11'). Equation (10') can be estimated consistently if  $u_{it}$  is replaced by  $\hat{u}_{it}$ .<sup>7</sup> This yields estimates of parameters  $\phi_i$ ,  $b_i$  and  $c_i$ . These can be used to compute

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<sup>6</sup> One might question the validity of the LOP, of course. Then, however, one is in need of a theory which explains why panel unit root tests reject non-stationarity of real exchange rates so often.

<sup>7</sup> In each estimation of equation (10'), a constant term has been included. There was no indication that a significant constant term could be present in equation (11'), which, accordingly, has been estimated without a constant.

$$\hat{S}_i^2 \equiv \frac{\hat{\sigma}_{\omega_i}^2}{\hat{\sigma}_{\eta_i}^2} = \frac{\hat{a}_i^2}{\hat{c}_i^2 + \hat{b}_i^2}, \quad (19)$$

$$\hat{R}_i \equiv \frac{\hat{\sigma}_{\eta_i \omega_i}}{\hat{\sigma}_{\eta_i} \hat{\sigma}_{\omega_i}} = \frac{\hat{b}_i}{\sqrt{\hat{c}_i^2 + \hat{b}_i^2}} \quad (20)$$

and, finally,  $\hat{\mu}_i$  from equation (13). Country-specific parameters have been obtained by a separate estimation of equations (10') and (11') for each country  $i$ . Alternatively, common panel estimates of parameters  $a$ ,  $b$ ,  $c$ ,  $\phi$ ,  $S^2$ ,  $R$  and  $\mu$  have been computed. For this purpose, equation (10') has been estimated using a simple fixed-effects approach. Depending on the choice of the numéraire country, the panel approach yields values of 0.70 and 0.71, respectively, for  $\hat{\phi}$ , 0.09 and 0.05 for  $\hat{S}^2$ , -0.02 and -0.19 for  $\hat{R}$ , and -0.91 and -0.93 for  $\hat{\mu}$ . When country-specific parameters are computed,  $\hat{\phi}_i$  ranges between 0.49 and 0.96,  $\hat{S}_i^2$  between 0.01 and 0.82,  $\hat{R}_i$  between -0.78 and 0.38, and  $\hat{\mu}_i$  between -0.84 and -0.99. Expressed in words, there is always a significant amount of positive autocorrelation present in real exchange rates of the numéraire sector, the innovation variance of component  $y_{it}$ , assumed to be non-stationary, is always smaller than that of component  $x_{it}$ . The only case in which the two variances are of nearly equal magnitude ( $\hat{S}_i^2 = 0.82$ ) pertains to the components of the real exchange rate between Austria and Germany, where nominal exchange rate volatility has always been especially low such that  $\hat{\sigma}_{\eta_i}^2$  is low as well. In most cases, but not always, the two components are negatively correlated. As a consequence of these parameter combinations, the differenced real exchange rate is driven by a process with a robustly negative MA parameter in equation (12).

#### **4. How large is the bias? A systematic Monte Carlo investigation into MA roots in panel unit root tests**

This section presents results of Monte Carlo simulations on the size of different first and second-generation panel unit root tests. It is generally assumed that the variable to be tested is generated as is  $q_{it}$  in equations (10) – (13). It is the sum of an AR(1) and a

random walk component and is, thus, an ARIMA (1, 1, 1) process. First, simulation results will be shown for the specific parameter combinations which have been estimated for the real exchange rate based on its subdivision into two components according to equations (7) – (9), as well as for the parameter combinations established in Engel (2000). In a second step, however, several parameters of the underlying processes and their correlation matrix will be gradually incremented in order to assess their impact on the size bias of the test. On the one hand, this can be seen as a sensitivity analysis, which is necessary because, first, it has been shown in the previous section that, in the case of the real exchange rate, the parameter combinations display considerable variations across countries, and, second, each of the various panel unit root tests in the literature, obviously, uses a different panel of real exchange rates. On the other hand, these simulation exercises produce a general set of stylised facts on the size bias of panel unit root tests of ARIMA (1, 1, 1) variables, which may be valuable in the potentially large class of cases in which it cannot be excluded that the variable under consideration follows such a process. After all, most macroeconomic variables are sums of several components, and the ARIMA (1, 1, 1) generally results if one subgroup of the components can be represented as an AR (1) process and the other as a random walk. For example, Clark (1988) suggests that real output may follow such a two-component model.

#### 4.1 Design of the Monte Carlo experiments

The Monte Carlo simulations have been performed by taking explicit account of the two-component structure of the variables to be tested in the form of equations (10) and (11). This implies that for each panel, which contains  $N$  time series of  $T$  observations of the variable  $q_{it}$ ,  $N$  series of errors of the AR (1) component (10),  $\eta_{it}$ , and  $N$  series of errors of the random walk component (11),  $\omega_{it}$ , are drawn. Each of the drawn series comprises  $T + 50$  observations, the first 50 of which are discarded. Equations (10), (11) and (7) are used to construct the two components  $x_{it}$  and  $y_{it}$  as well as their sum  $q_{it}$  from the drawn errors. The errors are drawn conditional on a  $(2N \times 1)$  vector of standard deviations

$$\sigma = \left[ \sigma_{\eta_1} \quad \sigma_{\eta_2} \quad \cdots \quad \sigma_{\eta_N} \quad \sigma_{\omega_1} \quad \sigma_{\omega_2} \quad \cdots \quad \sigma_{\omega_N} \right] \quad (21)$$

and conditional on a  $(2N \times 2N)$  correlation matrix

$$\Omega = \left[ \begin{array}{cccc|cccc} 1 & r_{\eta_1\eta_2} & \cdots & r_{\eta_1\eta_N} & R_1 = r_{\eta_1\omega_1} & r_{\eta_1\omega_2} & \cdots & r_{\eta_1\omega_N} \\ r_{\eta_1\eta_2} & 1 & & & & R_2 = r_{\eta_2\omega_2} & & \\ \vdots & & \ddots & & \vdots & & \ddots & \vdots \\ r_{\eta_1\eta_N} & & & 1 & r_{\eta_N\omega_1} & & \cdots & R_N = r_{\eta_N\omega_N} \\ \hline R_1 = r_{\eta_1\omega_1} & & \cdots & r_{\eta_N\omega_1} & 1 & r_{\omega_1\omega_2} & \cdots & r_{\omega_1\omega_N} \\ r_{\eta_1\omega_2} & R_2 = r_{\eta_2\omega_2} & & \vdots & r_{\omega_1\omega_2} & 1 & & \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \\ r_{\eta_1\omega_N} & & \cdots & R_N = r_{\eta_N\omega_N} & r_{\omega_1\omega_N} & & & 1 \end{array} \right] \quad (22)$$

where  $r_{\eta_i\omega_j} \equiv \sigma_{\eta_i\omega_j} / (\sigma_{\eta_i}\sigma_{\omega_j})$ . Vector  $\sigma$  and matrix  $\Omega$  have either been obtained from the residuals  $\eta_{it} = b_i u_{it} + c_i v_{it}$  and  $\omega_{it} = a_i u_{it}$ , which have been estimated using equations (10') and (11') (see section 3.4), or they have been constructed for given (intervals of) values of  $S_i^2 = \sigma_{\omega_i}^2 / \sigma_{\eta_i}^2$ ,  $R_i = r_{\eta_i\omega_i}$ ,  $r_{\eta_i\eta_j}$  ( $i \neq j$ ),  $r_{\omega_i\omega_j}$  ( $i \neq j$ ), where  $r_{\eta_i\omega_j}$  ( $i \neq j$ ) has generally been set to 0 and  $\sigma_{\eta_i}$  to 1 for all  $i$ . For each matrix  $\Omega$  that has been constructed, it has been verified that the smallest eigenvalue of  $\Omega$  is positive in order to ensure that  $\Omega$  is positive definite, ie a correlation matrix. The four panel unit root tests presented in section 3.2 have been applied to the panels of constructed  $q_{it}$  series and the ADF test to the individual series. The procedure has been replicated 10,000 times for each combination of parameters and the fraction of rejections given a nominal 1%, 5%, and 10% significance level has been determined.

#### 4.2 Results for Fischer's (2004) decomposition of the real exchange rate

If the real exchange rate were governed by a two-component model as estimated in section 3.4, the unit root test results of  $q_{it}$  presented in Table 1 are biased. Simulation results on the bias for this panel of real exchange rates are shown in Table 2. As may have been expected, panel unit root tests are heavily biased in favour of rejecting the non-stationarity null, in particular, if the number of lagged differences included in equation (14) is low. For a lag of one year, the probability of a type-one error is at least half. Interestingly, a comparison of Tables 1 and 2 shows that non-stationarity of the

real exchange rate has been rejected exactly in those cases in which the probability of erroneous rejection is high (at least 25%), and it has not been rejected when the probability of erroneous rejection is rather low, albeit often still not acceptable. When using, as a control, Germany instead of the USA as the numéraire country for the real exchange rates, the picture is even bleaker. With one exception, panel unit root tests reject non-stationarity for these real exchange rate panels at a nominal size of 5% only if no more than one lag is included. In these cases, the simulated true size is at least 64%.

These results may suggest that the result pervasively found in the PPP literature according to which panel unit root tests usually reject non-stationarity may simply be due to severe size biases of these tests. This applies to first-generation as well as to second-generation panel unit root tests which allow for cross-correlations. In many cases, of course, PPP studies do not use annual but rather quarterly data, and they differ in the composition of the real exchange rate panel. In the systematic analysis of the bias presented below, it will be shown, however, that the presence of severe biases in panel unit root tests is a quite robust result and that the bias actually increases further if the observation period is extended (ie in the case of a higher-than-annual frequency or of a rising time span).

The preliminary conclusion drawn in the previous paragraph is corroborated by a further piece of evidence. As a comparison, Monte Carlo simulations on the bias of simple univariate ADF tests have been calculated for each real exchange rate series in the panel. The cross-country average fraction of rejections given a nominal size of 5% is presented in the last column of Table 2. It can be seen that the average size bias is much lower in univariate ADF tests than in panel unit root tests. The low values of average size is accompanied by a low variation in size across series of the panel. In the case of one lag, for instance, the simulated true sizes range from 11.24 to 14.42.

In the last column of Table 1, it had been shown that univariate ADF tests cannot reject non-stationarity for a single real exchange rate series.<sup>8</sup> This demonstrates again that the evidence for PPP vanishes if the size bias of the test is low. The results offer a

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<sup>8</sup> Admittedly, the results are less clear-cut for the alternative panel of real exchange rates with Germany as the numéraire country, in which the ADF test produces a few rejections depending on the lag length. This result, however, is not fully in line with the bulk of the literature, in which univariate unit root tests usually cannot reject non-stationarity.

new explanation for the findings in the PPP literature that univariate unit root tests, mostly, cannot reject non-stationarity of real exchange rates whereas panel unit root tests usually do so. It is not only the power which is lower in univariate unit root tests, it is also the true size. Although the usually cited power problems of univariate unit root tests are clearly a possible explanation, one could just as well argue that real exchange rates are non-stationary because ADF tests (which claim to have found a corresponding result) are much less biased in favour of rejection than panel unit root tests which claim the opposite.

### **4.3 Results for Engel's (2000) decomposition of the real exchange rate**

Engel (2000) uses 25 years of quarterly data to decompose the bilateral real exchange rate between the USA and the UK into two components according to equations (3) – (5). He estimates the parameters determining the magnitude of the size bias in a univariate model as  $\hat{\phi} = 0.92$ ,  $\hat{S}^2 = 0.01$ , and  $\hat{R} = -0.10$ , which, according to equation (13) results in an MA parameter of  $\hat{\mu} = -0.99$  (see also Ng/Perron, 2001). Using these values, he simulates the size bias of a set of univariate unit root tests, and finds that univariate unit root tests are severely biased in favour of rejecting non-stationarity if the estimated parameters had prevailed for 100 years.

In order to assess the comparable true size of panel unit root tests, assume that the panel comprises 10 series, all of which are characterised by Engel's (2000) parameter constellation. The off-diagonal elements of the four ( $N \times N$ ) submatrices of the correlation matrix  $\Omega$  in equation (22) are set to 0. Since panel unit root tests of real exchange rates have usually been applied to the post-Bretton Woods era, we did not consider an artificial period of 100 years of data but confined the analysis, instead, to a period of  $T = 100$  (e. g. fictively quarterly) observations, for which Engel (2000) actually estimated the parameters. Moreover, instead of following Engel (2000) in determining the lag length endogenously, we continue to set the lag length exogenously as is common use when panel unit root tests are applied. Since he reports that, often, a lag length of zero has been endogenously determined, a corresponding simulation exercise has been performed additionally.

The simulated true sizes of panel unit root tests of variables that have been constructed accordingly are presented in Table 3. The size biases are extremely large. With a panel size as it is typically used in PPP studies, there is hardly any chance of not succumbing to a type 1 error if real exchange rates are adequately described by Engel's (2000) ARIMA (1, 1, 1) specification. This bias is especially large if a second-generation panel unit root test is employed. With the exception of the LL test, the inclusion of additional lags reduces the bias only very gradually. Once again, the univariate ADF test, although still severely biased in favour of rejecting the non-stationarity null, performs much better than the panel unit root tests.

#### 4.4 A systematic Monte Carlo investigation into MA roots in panel unit root tests

This section presents the results of a systematic investigation into the effects of variations in the parameters  $\phi_i$ ,  $S_i^2 = \sigma_{\omega_i}^2 / \sigma_{\eta_i}^2$ ,  $R_i = r_{\eta_i \omega_i}$ ,  $r_{\eta_i \eta_j}$  ( $i \neq j$ ),  $r_{\omega_i \omega_j}$  ( $i \neq j$ ),  $T$ , and  $\ell$  on the true size of panel unit root tests. This is recommendable for two reasons. First, it has been found in section 3 that parameter combinations display considerable variations for different real exchange rates, so that the composition of the panel matters for the bias. Second, the simulations produce a set of stylised facts on the magnitude of the bias which may be of a more general interest for econometricians or for applied economists, who cannot exclude the possibility that the variable under consideration has a two-component structure. There may be some potential for such cases since most macroeconomic variables are sums of several components and the proposed process generally results if one subgroup of these can be adequately described by an AR (1) process and the subgroup comprising the remaining ones as a random walk.

In the simulations, the panel is generally assumed to comprise  $N = 10$  time series. Corresponding to the experiments for the real exchange rates described above, each time series consists of  $T = 23$  or, alternatively, of  $T = 100$  observations. The maximum lag in equation (14) is alternatively set to  $\ell = \{1, 3, 5\}$ . An exogenous determination of the lag length is common use in the application of panel unit root tests, and it allows clear-cut results. The parameters governing the process are either set as specific values or are drawn randomly from a uniform distribution within a given interval of values. Here, only results of those experiments are presented in which the parameters have been specifically set, because drawing parameters from intervals implies that the MA

parameter  $\mu$ , which is computed endogenously from these parameters, varies across replications in a non-standard way. However, results of the two types of experiments differ only moderately from each other if the mean of the interval, from which parameters are drawn in one type of experiment, corresponds to the given parameters in the other type.

As a benchmark, a parameter constellation has been chosen which more or less corresponds to the average estimation results presented in section 3.4. For each series in the panel, the autoregressive parameter  $\phi$  is set to 0.75, the innovation variance of the random walk component is a tenth of that of the AR (1) component,  $S^2 = 0.1$ , and all the off-diagonal elements of the correlation matrix  $\Omega$  (equation (22)), including the correlation between the innovations of the two components of a given series,  $R$ , is 0.

Figure 2 shows how the true size of unit root tests of a panel of processes with such a parameter combination varies in response to changes in the number of lags included in the test equation (14). The left-hand graph considers an observation period of  $T = 23$ , the right-hand graph one of  $T = 100$ . The nominal size of 5% is indicated by a dotted line. As expected, an increasing number of lags reduces the size bias, especially in the case of the LL test. After all, the MA (1) term in the variables is equivalent to an infinite number of AR terms. The average size bias of the ADF test is usually considerably lower than that of the panel unit root tests. Their size bias is very high, especially if  $T = 100$ , where the probability of succumbing to a type 1 error is approximately 100% if the IPS, the MHDF or the PCSE panel unit root tests are applied and  $\ell \leq 3$ . The LL test is less oversized, especially if the number of lags is 5. For  $T = 23$  and  $\ell = 5$ , this test is even heavily undersized. The comparatively good size properties of the LL test have also been highlighted by Hlouskova/Wagner (2005) in their simulation study on first-generation panel unit root test. In the rest of the Figures,  $\ell = 3$  is assumed.

Figure 3 reveals that an increase in the innovation variance of the non-stationary component relative to the innovation variance of the stationary component,  $S^2$ , reduces the size bias of panel unit root tests. A corresponding result for univariate unit root tests has been found by Blough (1992) and Engel (2000). It is intuitive because, expressed metaphorically, the non-stationary component thus becomes more easily detectable.

Expressed more formally, the MA parameter  $\mu$  (whose respective value is shown in brackets in the Figure for each  $S^2$ ) rises if  $S^2$  increases. In the range between  $S^2 = 0.1$  and  $S^2 = 0.5$ , the size falls significantly for all  $\ell = \{1, 3, 5\}$ , although the true size for  $S^2 = 0.5$  and  $\ell = 3$ , for instance, is still far from acceptable.

An increase in the autoregressive parameter  $\phi$ , *ceteris paribus*, raises the MA parameter  $\mu$ . This, however, does not necessarily result in a corresponding reduction of the true size of unit root tests. As is demonstrated in Figure 4, a rising  $\phi$  causes the true size of panel unit root tests as well as that of the ADF test, first, to increase, and, after  $\phi$  has passed a level of around 0.5 – 0.8, to decrease again if  $\ell = 3$ . The turning point appears to depend on the number of lags included in the test equation. The more lags are included, the higher is the level of  $\phi$ , which is required for the true size to start decreasing. The possible non-monotonicity of the size bias in a negative MA parameter  $\mu$  is rarely recognised in simulation studies of unit root tests because, in most of these studies, either the consequences of an AR parameter or that of a MA parameter are investigated but not the presence of both at the same time (see eg Im et al., 2003, or Hlouskova/Wagner, 2005).

An observation expressed in Ng/Perron (2001) points to an explanation of the puzzling non-monotonicity of the bias. If the AR root of the ARIMA (1, 1, 1) variable  $q_{it}$  in equation (12) equals the MA root,  $\phi_i = -\mu_i$ , these two roots cancel,  $q_{it}$  is a random walk,  $\Delta q_{it} = \zeta_{it}$ , and there is no bias left. With an increase in  $\phi_i$ , the difference between the absolute values of the AR and the MA root continuously falls and the state of cancellation of the two roots approaches. Such a development might counteract the effect of the fall in  $\mu_i$  on the size bias.

Moreover, it is found that additional lags reduce the size bias much faster in the case of  $\phi = 0$  than in the benchmark case of  $\phi = 0.75$ . This may have important implications for an application of panel unit root tests because, so far, one might have hoped to avoid a bias from MA roots by including a relatively small number of lags. The usual practice of exploring the effects of MA roots only for the  $\phi = 0$  case might suggest that such a strategy could be successful. For ARIMA (1, 1, 1) variables, however, this strategy will often not be sufficient.

Figure 5 reveals that a rise in the correlation between the two components of each of the variables in the panel reduces the MA parameter and thus raises the true size of the (panel) unit root tests. The consequences of deviations from the assumption of no cross-correlations in the benchmark case are depicted in Figures 6 – 8. Size biases of different panel unit root tests are affected quite differently by cross-correlations. Often, no significant effect is discernible, however. An increase in the correlations between the innovations of the AR (1) components of two different variables, that is a rise in  $r_{\eta_i\eta_j}$  for all  $i \neq j$  – the off-diagonal elements of the upper left  $(N \times N)$  submatrix of  $\Omega$  in equation (22) –, decreases the size bias of most panel unit root tests. Rising correlations between the innovations of the random walk components, ie a rise in all the off-diagonal elements of the lower right  $(N \times N)$  submatrix of  $\Omega$ ,  $r_{\omega_i\omega_j}$  ( $i \neq j$ ), often raises the true size of the tests slightly. A notable exception to this rule is the PCSE test for  $T = 100$ , whose true size falls in this case. If both types of cross-correlations,  $r_{\eta_i\eta_j} = r_{\omega_i\omega_j}$  ( $i \neq j$ ), are raised by the same amount, the size bias of the PCSE test most often decreases, sometimes quite substantially, while the other tests are mostly unaffected.

In all the experiments, the size bias of the PCSE test is lower than that of the MHDF test if the observation period is short ( $T = 23$ ). For  $T = 100$ , these two panel unit root tests often exhibit approximately the same true size. The result according to which univariate ADF tests display, on average, a far lower size bias than any of the panel unit root tests is confirmed in almost all experiments.

## 5. Conclusions

Using a Monte Carlo investigation, the study systematically analyses the true size of several first and second-generation panel unit root tests of variables which, being a sum of two components – an AR (1) process and a random walk – are ARIMA (1, 1, 1) processes. For relevant cases, a huge size bias in favour of rejecting the non-stationarity null is found. The bias decreases with the number of lags included in the test equation, it falls if the innovation variance of the random walk component rises relative to that of the AR (1) component and – at least for the baseline parameterisation – in one of the second-generation panel unit root tests (the PCSE test), if cross-correlations increase. The bias increases with the length of the observation period, and if the correlation

between the innovation variances of the two components rises. Cross-correlations between AR (1) components and cross-correlations between random walk components can affect the bias in an opposite way. For low positive values of the AR (1) parameter, their rise increases the true size; for high positive values, they decrease the bias.

Two independent approaches from the literature convincingly claim that the real exchange rate may well be a sum of an AR (1) and a random walk component. One of them goes back to Engel (2000) and divides the real exchange rate into a real exchange rate for tradable goods (conventionally assumed to be stationary, eg an AR (1) process) and a weighted relative price between tradables and non-tradables (possibly a random walk). He estimates the parameters of the two components for one particular bilateral real exchange rate. In the present paper, it is shown that, for a panel of ten real exchange rates with the estimated properties, many panel unit root tests, as they are applied in the literature, have a less than one per cent chance of avoiding a type 1 error.

The alternative approach of Fischer (2004) maintains that real exchange rates, by definition, consist of two components, the first of which is the real exchange rate for a single good that should be stationary because of the law of one price. The second component is a weighted sum of relative prices between different goods which has no reason to be stationary. In our paper, series from the OECD's STAN database are used to construct the two components for a panel of ten real exchange rates. Panel unit root tests regularly classify the second component as non-stationary and the first as stationary in most cases. A method similar to that employed by Engel (2000) is used to estimate the parameters of the model. This allows a Monte Carlo simulation of the bias made by panel unit root tests of real exchange rates.

First, the result commonly found in the literature, namely that panel unit root tests, in most cases, reject non-stationarity of real exchange rates, is replicated. The simulations show, however, that the rejections occur exactly in those cases in which the probability of an erroneous rejection is (often very) high. In the few cases in which the true size is closer to the nominal 5% level (but still far above it), panel unit root tests cannot reject the non-stationarity null. It may therefore be necessary to reappraise the accumulated evidence on purchasing power parity, which is mainly based on a multitude of studies according to which panel unit root tests reject non-stationarity of

real exchange rates. The evidence in favour of PPP may simply be due to severe size biases of the tests.

As a further result in the empirical literature on PPP, univariate unit root tests of real exchange rates usually cannot reject non-stationarity. Until now, the contrasting results of panel unit root tests, on the one hand, and univariate unit root tests, on the other, have been explained by the much higher power of panel unit root tests. Conventionally, the conflict is, thus, resolved in favour of PPP. The systematic simulations in our study demonstrate, however, that simple univariate ADF tests, while still being oversized, suffer from a far smaller size bias than panel unit root tests in nearly all cases. One may conclude that the contrasting results of panel and univariate unit root tests concerning the stationarity of real exchange rates can just as well be resolved by dismissing panel unit root test results as suffering from overly large biases. While being superior to univariate unit root tests with respect to power, panel unit root tests are clearly inferior to the ADF test with respect to size if the real exchange rate consists of two components in the manner proposed. The validity of PPP may, thus, appear to be a still unresolved issue.

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**Table 1: Unit root tests on real exchange rates and their components;  
numéraire country: USA**

Variable	lags ( $\ell$ )	LL	IPS	MHDF	PCSE	ADF <sup>1</sup>
<i>q</i>	1	-3.96***	-2.52***	-4.76***	-2.83***	0
	3	-0.67	-2.01**	-4.15***	-2.46**	0
	5	1.90	-1.57	-3.91***	-2.26**	0
<i>x</i>	1	-4.52***	-2.52***	-4.06***	-2.46**	1
	3	-0.17	-1.65	-2.46**	-1.90**	0
	5	2.58	-1.37	-1.91**	-1.47*	0
<i>y</i>	1	-0.24	-1.14	-0.65	0.00	0
	3	-0.07	-0.92	0.78	0.58	1
	5	-2.64***	-1.51	0.80	0.86	3

Significant at the \*\*\* 1% level, \*\* 5% level, \* 10% level;

<sup>1</sup> For the univariate ADF test, the number of rejections at the 5% level is given. Critical values are taken from Fuller (1976).

**Table 2: True size of unit root tests on real exchange rates for a nominal size of 5%; numéraire country: USA; parameters of the model (10'), (11') estimated in a panel approach;  $N = 10$ ;  $T = 23$**

lags ( $\ell$ )	LL	IPS	MHDF	PCSE	ADF <sup>1</sup>
1	65.12%	60.16%	84.59%	49.60%	13.57%
3	21.98%	29.86%	67.14%	37.31%	8.29%
5	7.27%	12.77%	52.15%	25.67%	6.33%

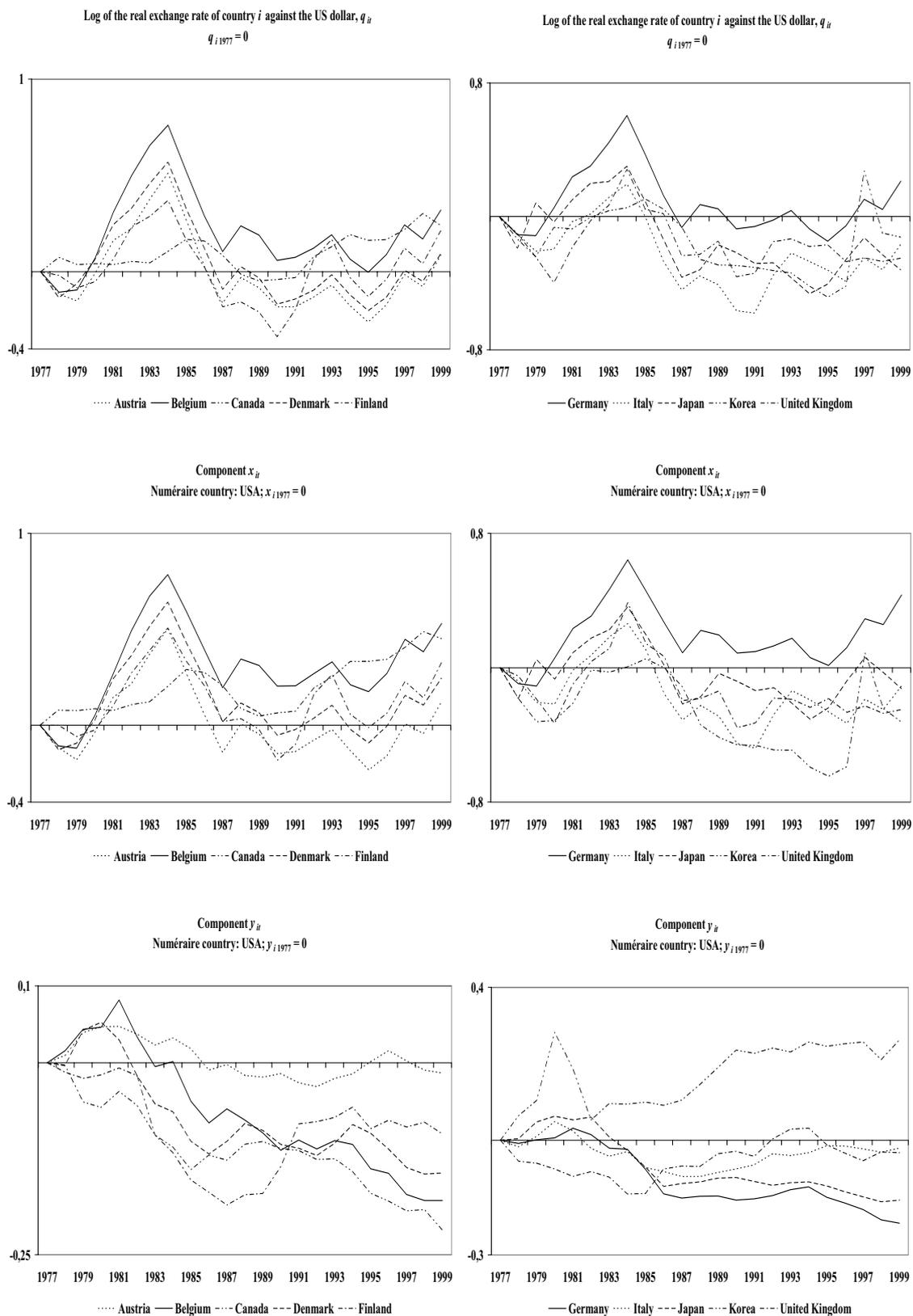
<sup>1</sup> For the univariate ADF test, an average of the true sizes that have been determined for each individual series, separately, is shown.

**Table 3: True size of unit root tests on real exchange rates for a nominal size of 5%; underlying parameters as estimated by Engel (2000);  $N = 10$ ;  $T = 100$**

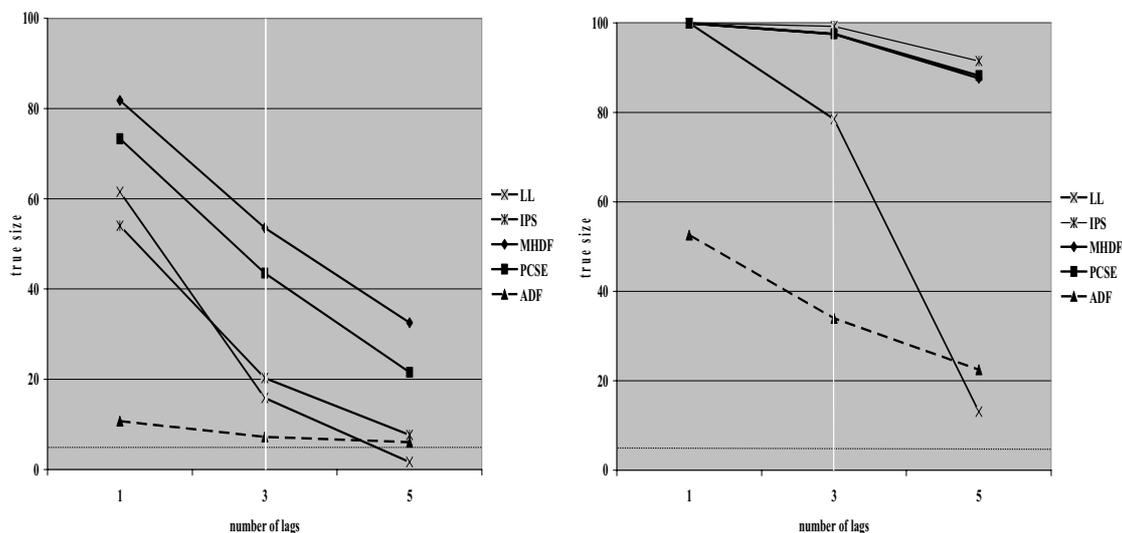
lags ( $\ell$ )	LL	IPS	MHDF	PCSE	ADF <sup>1</sup>
0	77.04%	99.05%	99.96%	99.97%	22.52%
1	63.16%	97.89%	99.88%	99.90%	20.97%
3	28.16%	93.28%	99.25%	99.42%	17.62%
5	6.34%	84.79%	97.03%	97.30%	14.88%

<sup>1</sup> For the univariate ADF test, an average of the true sizes that have been determined for each individual series, separately, is shown.

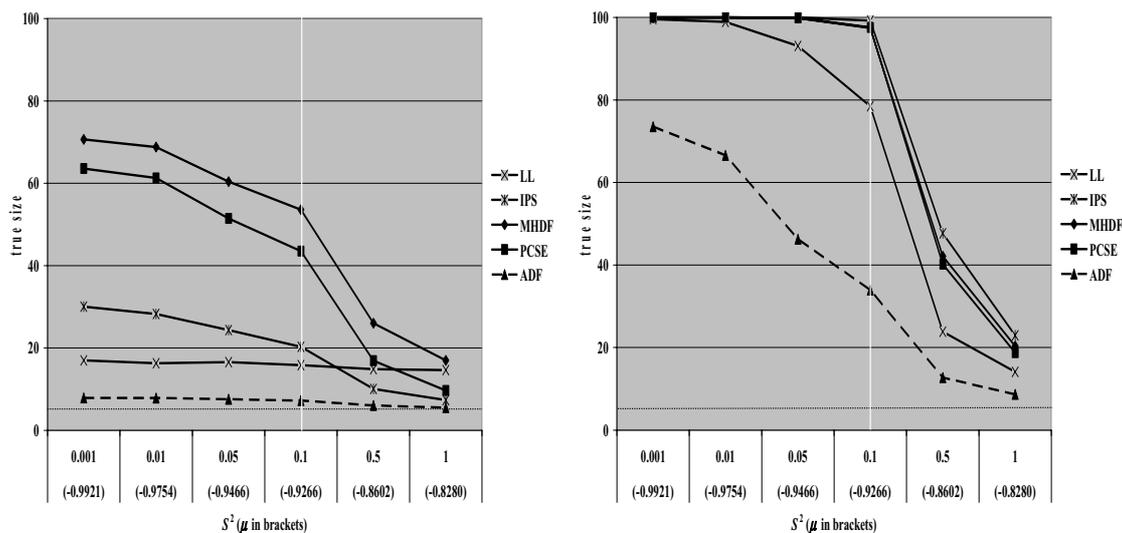
**Figure 1: Real exchange rates against the US dollar and their components**



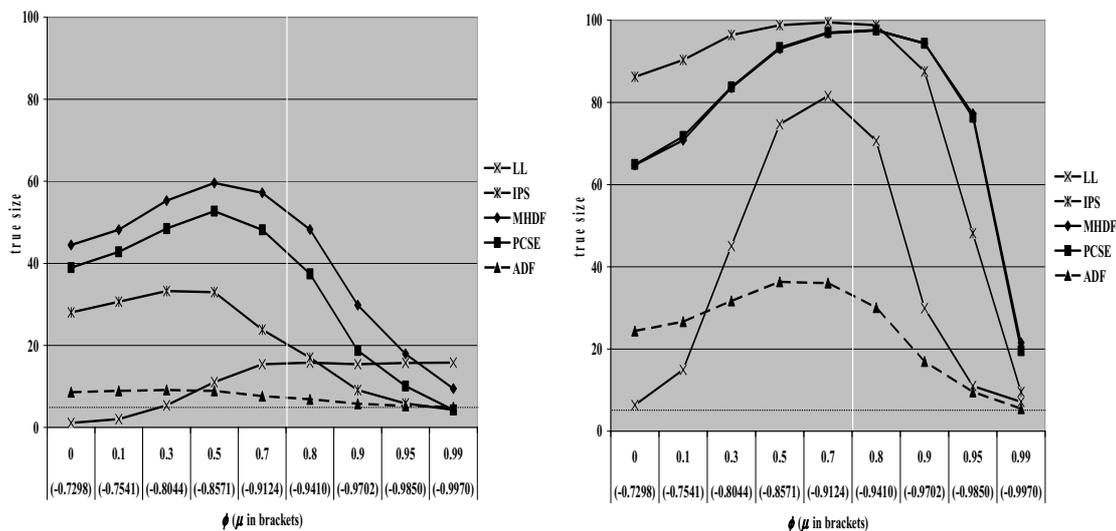
**Figure 2: Simulated true sizes of unit root tests of two-component ARIMA (1, 1, 1) variables: dependence on number of lags ( $l$ ); nominal size: 5% (dotted line);  $N = 10$ ;  $S^2 = 0.1$ ;  $\phi = 0.75$ ;  $R = 0$ ; no cross-correlations; benchmark: white vertical line; left-hand graph:  $T = 23$ , right-hand graph:  $T = 100$**



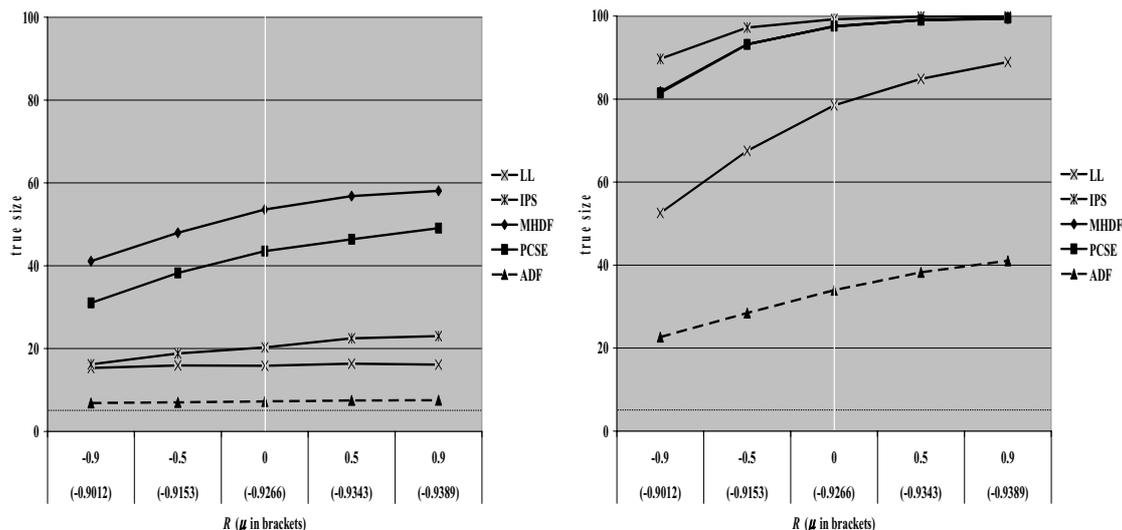
**Figure 3: Simulated true sizes of unit root tests of two-component ARIMA (1, 1, 1) variables: dependence on ratio of the two components' innovation variances ( $S^2$ ); nominal size: 5% (dotted line);  $N = 10$ ; 3 lags;  $\phi = 0.75$ ;  $R = 0$ ; no cross-correlations; benchmark: white vertical line; left-hand graph:  $T = 23$ , right-hand graph:  $T = 100$**



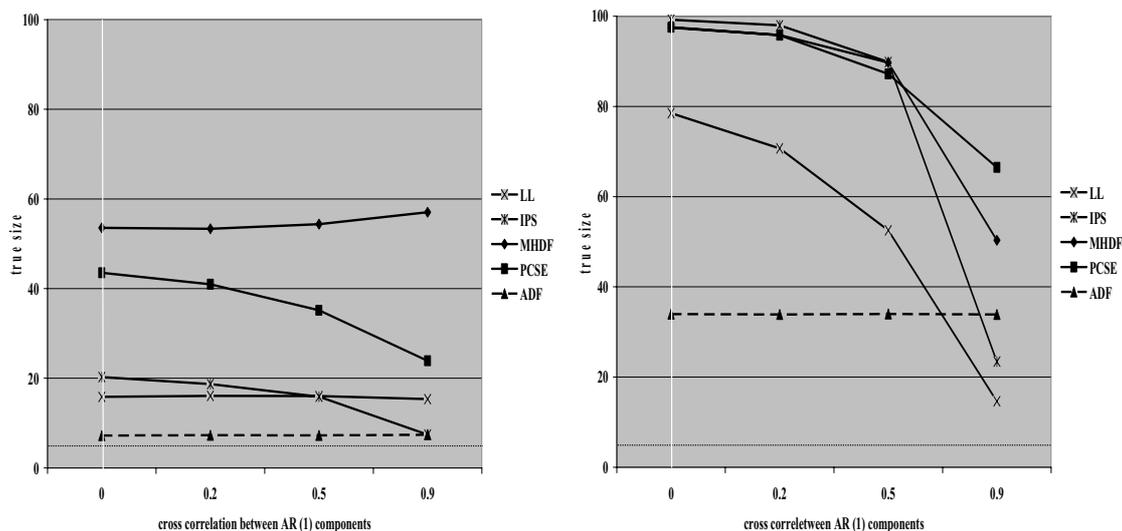
**Figure 4: Simulated true sizes of unit root tests of two-component ARIMA (1, 1, 1) variables: dependence on autoregressive parameter of the AR (1) component ( $\phi$ ); nominal size: 5% (dotted line);  $N = 10$ ; 3 lags;  $S^2 = 0.1$ ;  $R = 0$ ; no cross-correlations; benchmark: white vertical line; left-hand graph:  $T = 23$ , right-hand graph:  $T = 100$**



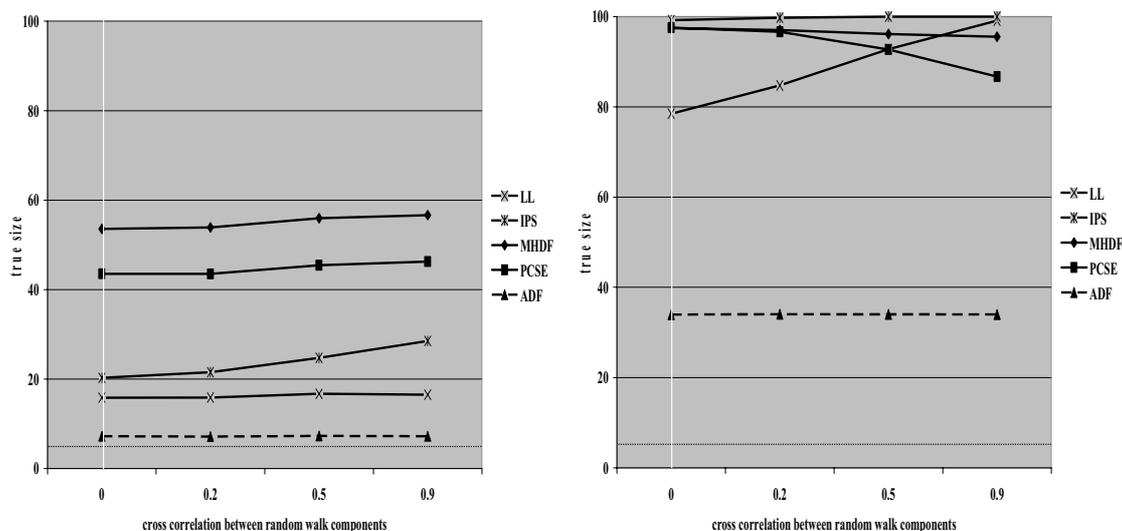
**Figure 5: Simulated true sizes of unit root tests of two-component ARIMA (1, 1, 1) variables: dependence on correlations between the two components ( $R$ ); nominal size: 5% (dotted line);  $N = 10$ ; 3 lags;  $S^2 = 0.1$ ;  $\phi = 0.75$ ; no cross-correlations; benchmark: white vertical line; left-hand graph:  $T = 23$ , right-hand graph:  $T = 100$**



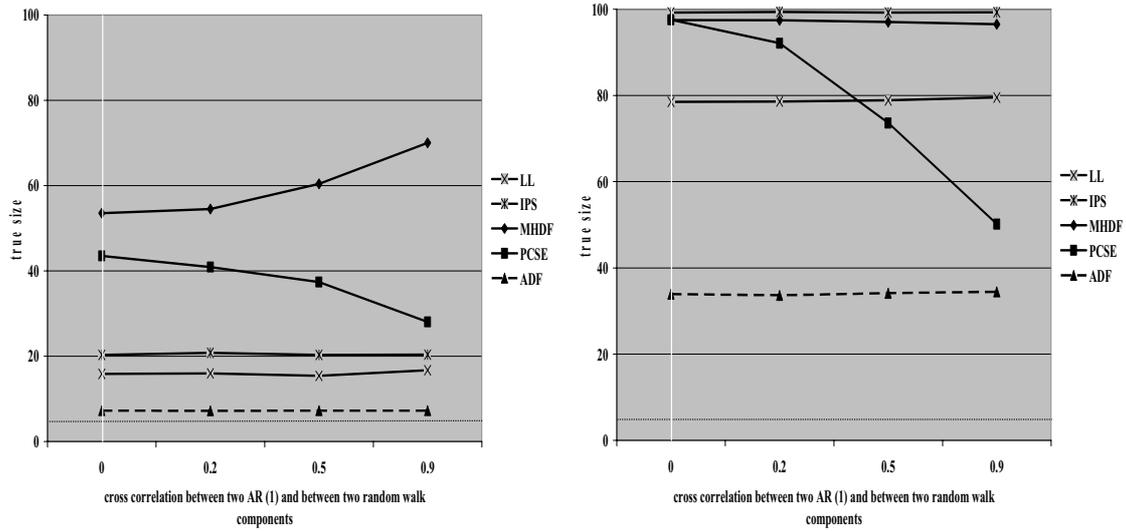
**Figure 6: Simulated true sizes of unit root tests of two-component ARIMA (1, 1, 1) variables: dependence on cross-correlations between innovations of AR (1) components ( $r_{\eta_i\eta_j}, i \neq j$ ); nominal size: 5% (dotted line);  $N = 10$ ; 3 lags;  $S^2 = 0.1$ ;  $\phi = 0.75$ ;  $R = 0$ ; no cross-correlations between random walk components; benchmark: white vertical line; left-hand graph:  $T = 23$ , right-hand graph:  $T = 100$**



**Figure 7: Simulated true sizes of unit root tests of two-component ARIMA (1, 1, 1) variables: dependence on cross-correlations between innovations of random walk components ( $r_{\omega_i\omega_j}, i \neq j$ ); nominal size: 5% (dotted line);  $N = 10$ ; 3 lags;  $S^2 = 0.1$ ;  $\phi = 0.75$ ;  $R = 0$ ; no cross-correlations between AR (1) components; benchmark: white vertical line; left-hand graph:  $T = 23$ , right-hand graph:  $T = 100$**



**Figure 8: Simulated true sizes of unit root tests of two-component ARIMA (1, 1, 1) variables: dependence on cross-correlations between innovations of two random walk and two AR (1) components ( $r_{\omega_i\omega_j} = r_{\eta_i\eta_j}, i \neq j$ ); nominal size: 5% (dotted line);  $N = 10$ ; 3 lags;  $S^2 = 0.1$ ;  $\phi = 0.75$ ;  $R = 0$ ; benchmark: white vertical line; left-hand graph:  $T = 23$ , right-hand graph:  $T = 100$**



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